Lagoa Fall 2016

EE 587/ME 559 - Homework #1
Due Monday, September 12

1. Consider a two-dimensional system

$$\dot{x} = Ax$$

where both eigenvalues are located at zero.

   a) Sketch the phase portrait for all possibilities; i.e., for the cases where there does exist two linearly independent eigenvectors and for the case where that exists only one.

   b) Verify the results using Matlab.

2. Consider the Van der Pol oscillator whose state equations are given by

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + (1 - x_1^2)x_2\end{aligned}$$

   a) Show that the origin is an unstable focus for this system.

   b) Verify this results by simulating this system in Matlab.

   c) Verify, using again Matlab simulations, that there exists a periodic orbit and that this periodic orbit is stable.

3. Khalil problem 2.5.

4. Download and install Yalmip and Sedumi. The first one is a nice interface to define optimization problems. The second one is a nice numerical solver of so-called Linear matrix inequalities

   `http://users.isy.liu.se/johanl/yalmip/pmwiki.php`
   `http://sedumi.ie.lehigh.edu/`

5. Consider the system

$$\dot{x} = Ax$$

where

$$A = \begin{bmatrix} -7 & 3 & -7 & 0 \\ -2 & -5 & 2 & -9 \\ 6 & 0 & -1 & -11 \\ 0 & 3 & -5 & -6 \end{bmatrix}$$

We are going to check stability of this system by using the concept of positive definite matrix. A matrix P is said to be positive definite, denoted by $P \succ 0$, if

$$P = P^T \quad \text{and} \quad x^T P x > 0 \text{ for all } x \neq 0.$$

Show that the system above is stable by using Yalmip and Sedumi to find a matrix P satisfying

$$P \succ 0 \text{ and } -A^T P - PA \succ 0 \text{ and } \mathrm{trace}(P) = 1.$$

where $\mathrm{trace}(P)$ is the sum of elements in the diagonal of P.

Argue that the system is asymptotically stable by showing that for the P computed above and for any initial condition the function

$$V[x(t)] = x(t)^T P x(t)$$

is a decreasing function of t and, therefore, $x(t) \to 0$.