HW 11

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Contents

Problem 1

a

```
temp <- read.ftable(textConnection("Z z1 z2
Y y1 y2 y3
X
x1 125 7 11 5 106 18
x2 124 6 22 3 109 9
x3 146 6 0 2 111 0"))

df <- data.frame(temp)


# saturated model
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z + Y*Z + X*Y*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + Y * Z + X *
##     Y * Z, family = poisson(link = "log"), data = df)
##
## Deviance Residuals:
##  [1]  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.828e+00  8.944e-02  53.982  < 2e-16 ***
```

```
## Xx2         -8.032e-03  1.267e-01  -0.063   0.9495
## Xx3          1.553e-01  1.219e-01   1.274   0.2025
## Yy2         -2.882e+00  3.884e-01  -7.421 1.16e-13 ***
## Yy3         -2.430e+00  3.145e-01  -7.728 1.09e-14 ***
## Zz2         -3.219e+00  4.561e-01  -7.058 1.69e-12 ***
## Xx2:Yy2     -1.461e-01  5.706e-01  -0.256   0.7979
## Xx3:Yy2     -3.094e-01  5.695e-01  -0.543   0.5869
## Xx2:Yy3      7.012e-01  3.904e-01   1.796   0.0725 .
## Xx3:Yy3     -2.586e+01  6.965e+04   0.000   0.9997
## Xx2:Zz2     -5.028e-01  7.412e-01  -0.678   0.4976
## Xx3:Zz2     -1.072e+00  8.455e-01  -1.267   0.2050
## Yy2:Zz2      5.936e+00  6.002e-01   9.890  < 2e-16 ***
## Yy3:Zz2      3.711e+00  5.954e-01   6.234 4.56e-10 ***
## Xx2:Yy2:Zz2  6.849e-01  9.368e-01   0.731   0.4647
## Xx3:Yy2:Zz2  1.272e+00  1.021e+00   1.245   0.2130
## Xx2:Yy3:Zz2 -8.835e-01  9.233e-01  -0.957   0.3386
## Xx3:Yy3:Zz2  5.791e-01  9.851e+04   0.000   1.0000
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1.1759e+03  on 17  degrees of freedom
## Residual deviance: 3.0325e-10  on  0  degrees of freedom
## AIC: 113.92
##
## Number of Fisher Scoring iterations: 21
```

```
# homogenous model
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z + Y*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + Y * Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##        1         2         3         4         5         6         7
## -0.03739  -0.05946   0.08975   0.19267   0.41545  -0.65888   1.30773
##        8         9        10        11        12        13        14
## -0.68341  -0.41758  -0.28057   0.17658   0.10301  -0.70228   0.55381
##       15        16        17        18
## -0.00010   0.61805  -0.76566  -0.00008
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept)    4.83166    0.08869  54.481  < 2e-16 ***
## Xx2           -0.00604    0.12523  -0.048   0.9615
## Xx3            0.14451    0.12087   1.196   0.2319
## Yy2           -3.42432    0.36088  -9.489  < 2e-16 ***
## Yy3           -2.22923    0.26518  -8.407  < 2e-16 ***
## Zz2           -3.30964    0.37304  -8.872  < 2e-16 ***
## Xx2:Yy2        0.65707    0.37818   1.737   0.0823 .
```

```
## Xx3:Yy2        0.40568    0.44897   0.904   0.3662
## Xx2:Yy3        0.37422    0.32803   1.141   0.2540
## Xx3:Yy3      -21.85471 6620.02069  -0.003   0.9974
## Xx2:Zz2       -0.66721    0.35779  -1.865   0.0622 .
## Xx3:Zz2       -0.54102    0.44503  -1.216   0.2241
## Yy2:Zz2        6.59287    0.40584  16.245  < 2e-16 ***
## Yy3:Zz2        3.44829    0.43416   7.942 1.98e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.873  on 17  degrees of freedom
## Residual deviance:    4.897  on  4  degrees of freedom
## AIC: 110.82
##
## Number of Fisher Scoring iterations: 17
```

```
# independece model
model <- glm(Freq ~ X + Y + Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.0596   -9.1703   -0.0282    5.2241    7.4939
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.31818    0.07690  56.153  < 2e-16 ***
## Xx2          0.00367    0.08567   0.043  0.96583
## Xx3         -0.02607    0.08631  -0.302  0.76260
## Yy2         -0.16034    0.07326  -2.189  0.02863 *
## Yy3         -1.90954    0.13833 -13.804  < 2e-16 ***
## Zz2         -0.20816    0.07065  -2.946  0.00322 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.87  on 17  degrees of freedom
## Residual deviance:  849.93  on 12  degrees of freedom
## AIC: 939.85
##
## Number of Fisher Scoring iterations: 6
```

```
# joint indep model (XY,Z)
model <- glm(Freq ~ X + Y + Z + X*Y, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.7073   -9.0275   -0.0011    5.7728    7.0259
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   4.27306    0.09325  45.825  < 2e-16 ***
## Xx2          -0.02335    0.12477  -0.187  0.85156
## Xx3           0.12968    0.12020   1.079  0.28067
## Yy2          -0.14015    0.12862  -1.090  0.27586
## Yy3          -1.50024    0.20537  -7.305 2.77e-13 ***
## Zz2          -0.20816    0.07065  -2.946  0.00322 **
## Xx2:Yy2       0.04089    0.18197   0.225  0.82220
## Xx3:Yy2      -0.09489    0.17845  -0.532  0.59490
## Xx2:Yy3       0.09004    0.28689   0.314  0.75364
## Xx3:Yy3     -17.07691  539.13663  -0.032  0.97473
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.9  on 17  degrees of freedom
## Residual deviance:  799.4  on  8  degrees of freedom
## AIC: 897.32
##
## Number of Fisher Scoring iterations: 12
```

```
# joint independece model (XZ,Y)
model <- glm(Freq ~ X + Y + Z + X*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -9.8476  -9.2742   0.0684   5.5459   7.7311
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.26970    0.09071  47.072   <2e-16 ***
## Xx2          0.06104    0.11650   0.524   0.6003
## Xx3          0.06104    0.11650   0.524   0.6003
## Yy2         -0.16034    0.07326  -2.189   0.0286 *
## Yy3         -1.90954    0.13833 -13.804   <2e-16 ***
## Zz2         -0.10303    0.12143  -0.848   0.3962
## Xx2:Zz2     -0.12506    0.17201  -0.727   0.4672
```

```
## Xx3:Zz2     -0.19346    0.17371  -1.114   0.2654
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.87  on 17  degrees of freedom
## Residual deviance:  848.65  on 10  degrees of freedom
## AIC: 942.57
##
## Number of Fisher Scoring iterations: 6
```

```
# joint independece model (YZ,X)
model <- glm(Freq ~ X + Y + Z + Y*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + Y * Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -4.6468  -0.5859  -0.0853   0.3739   2.8735
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.88765    0.07052  69.305   <2e-16 ***
## Xx2          0.00367    0.08567   0.043    0.966
## Xx3         -0.02607    0.08631  -0.302    0.763
## Yy2         -3.03445    0.23487 -12.920   <2e-16 ***
## Yy3         -2.48238    0.18120 -13.699   <2e-16 ***
## Zz2         -3.67630    0.32021 -11.481   <2e-16 ***
## Yy2:Zz2      6.51876    0.39778  16.388   <2e-16 ***
## Yy3:Zz2      3.47563    0.41215   8.433   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.873  on 17  degrees of freedom
## Residual deviance:   59.215  on 10  degrees of freedom
## AIC: 153.13
##
## Number of Fisher Scoring iterations: 5
```

```
# cond independece model (XY,XZ)
model <- glm(Freq ~ X + Y + Z + X*Z + X*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Z + X * Z, family = poisson(link = "log"),
```

```
##     data = df)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -9.8476  -9.2742   0.0684   5.5459   7.7311
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.26970    0.09071  47.072   <2e-16 ***
## Xx2          0.06104    0.11650   0.524   0.6003
## Xx3          0.06104    0.11650   0.524   0.6003
## Yy2         -0.16034    0.07326  -2.189   0.0286 *
## Yy3         -1.90954    0.13833 -13.804   <2e-16 ***
## Zz2         -0.10303    0.12143  -0.848   0.3962
## Xx2:Zz2     -0.12506    0.17201  -0.727   0.4672
## Xx3:Zz2     -0.19346    0.17371  -1.114   0.2654
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.87  on 17  degrees of freedom
## Residual deviance:  848.65  on 10  degrees of freedom
## AIC: 942.57
##
## Number of Fisher Scoring iterations: 6
```

```
# cond independece model (XY,XZ)
model <- glm(Freq ~ X + Y + Z + X*Z + X*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Z + X * Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -9.8476  -9.2742   0.0684   5.5459   7.7311
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.26970    0.09071  47.072   <2e-16 ***
## Xx2          0.06104    0.11650   0.524   0.6003
## Xx3          0.06104    0.11650   0.524   0.6003
## Yy2         -0.16034    0.07326  -2.189   0.0286 *
## Yy3         -1.90954    0.13833 -13.804   <2e-16 ***
## Zz2         -0.10303    0.12143  -0.848   0.3962
## Xx2:Zz2     -0.12506    0.17201  -0.727   0.4672
## Xx3:Zz2     -0.19346    0.17371  -1.114   0.2654
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
```

```
##
##     Null deviance: 1175.87  on 17  degrees of freedom
## Residual deviance:  848.65  on 10  degrees of freedom
## AIC: 942.57
##
## Number of Fisher Scoring iterations: 6
```

```r
# cond independece model (XY,YZ)
model <- glm(Freq ~ X + Y + Z + X*Y + Y*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + Y * Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -1.41824  -0.15293  -0.00015   0.11361   1.29499
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept)    4.84253    0.08806  54.990   <2e-16 ***
## Xx2           -0.02335    0.12477  -0.187    0.852
## Xx3            0.12968    0.12020   1.079    0.281
## Yy2           -3.01425    0.25756 -11.703   <2e-16 ***
## Yy3           -2.07307    0.23638  -8.770   <2e-16 ***
## Zz2           -3.67630    0.32021 -11.481   <2e-16 ***
## Xx2:Yy2        0.04089    0.18197   0.225    0.822
## Xx3:Yy2       -0.09489    0.17845  -0.532    0.595
## Xx2:Yy3        0.09004    0.28689   0.314    0.754
## Xx3:Yy3      -21.10242 4034.85186  -0.005    0.996
## Yy2:Zz2        6.51876    0.39778  16.388   <2e-16 ***
## Yy3:Zz2        3.47563    0.41215   8.433   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.8734  on 17  degrees of freedom
## Residual deviance:    8.6798  on  6  degrees of freedom
## AIC: 110.6
##
## Number of Fisher Scoring iterations: 16
```

```r
# cond independece model (XZ,YZ)
model <- glm(Freq ~ X + Y + Z + X*Z + Y*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Z + Y * Z, family = poisson(link = "log"),
##     data = df)
```

```
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -4.7374  -0.5530  -0.0608   0.6330   2.8398
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.83917    0.08537  56.687   <2e-16 ***
## Xx2          0.06104    0.11650   0.524    0.600
## Xx3          0.06104    0.11650   0.524    0.600
## Yy2         -3.03445    0.23487 -12.920   <2e-16 ***
## Yy3         -2.48238    0.18120 -13.699   <2e-16 ***
## Zz2         -3.57118    0.33509 -10.657   <2e-16 ***
## Xx2:Zz2     -0.12506    0.17201  -0.727    0.467
## Xx3:Zz2     -0.19346    0.17371  -1.114    0.265
## Yy2:Zz2      6.51876    0.39778  16.388   <2e-16 ***
## Yy3:Zz2      3.47563    0.41215   8.433   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.873  on 17  degrees of freedom
## Residual deviance:   57.933  on  8  degrees of freedom
## AIC: 155.85
##
## Number of Fisher Scoring iterations: 5
```

The best model is conditional independece model (XY,YZ).

The estimated errors are high.

dropping the cells with zero counts.

```
df = df[df$Freq != 0,]

# saturated model
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z + Y*Z + X*Y*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + Y * Z + X *
##     Y * Z, family = poisson(link = "log"), data = df)
##
## Deviance Residuals:
##  [1]  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
##
## Coefficients: (2 not defined because of singularities)
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.828314   0.089443  53.982  < 2e-16 ***
## Xx2         -0.008032   0.126746  -0.063   0.9495
## Xx3          0.155293   0.121858   1.274   0.2025
## Yy2         -2.882404   0.388403  -7.421 1.16e-13 ***
```

```
## Yy3         -2.430418   0.314498  -7.728 1.09e-14 ***
## Zz2         -3.218876   0.456070  -7.058 1.69e-12 ***
## Xx2:Yy2     -0.146119   0.570603  -0.256   0.7979
## Xx3:Yy2     -0.309444   0.569538  -0.543   0.5869
## Xx2:Yy3      0.701179   0.390420   1.796   0.0725 .
## Xx3:Yy3            NA         NA      NA       NA
## Xx2:Zz2     -0.502793   0.741214  -0.678   0.4976
## Xx3:Zz2     -1.071584   0.845487  -1.267   0.2050
## Yy2:Zz2      5.936405   0.600243   9.890  < 2e-16 ***
## Yy3:Zz2      3.711352   0.595369   6.234 4.56e-10 ***
## Xx2:Yy2:Zz2  0.684853   0.936766   0.731   0.4647
## Xx3:Yy2:Zz2  1.271825   1.021184   1.245   0.2130
## Xx2:Yy3:Zz2 -0.883501   0.923270  -0.957   0.3386
## Xx3:Yy3:Zz2        NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 9.8506e+02  on 15  degrees of freedom
## Residual deviance: 1.3545e-14  on  0  degrees of freedom
## AIC: 109.92
##
## Number of Fisher Scoring iterations: 3
```

```r
# homogenous model
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z + Y*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + Y * Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##        1         2         3         4         5         6         7
## -0.03739  -0.05946   0.08975   0.19267   0.41545  -0.65888   1.30773
##        8         9        10        11        12        13        14
## -0.68341  -0.41758  -0.28057   0.17658   0.10301  -0.70228   0.55381
##       16        17
##  0.61805  -0.76566
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.83166    0.08869  54.481  < 2e-16 ***
## Xx2         -0.00604    0.12523  -0.048   0.9615
## Xx3          0.14451    0.12087   1.196   0.2319
## Yy2         -3.42432    0.36087  -9.489  < 2e-16 ***
## Yy3         -2.22923    0.26518  -8.407  < 2e-16 ***
## Zz2         -3.30964    0.37304  -8.872  < 2e-16 ***
## Xx2:Yy2      0.65707    0.37818   1.737   0.0823 .
## Xx3:Yy2      0.40568    0.44897   0.904   0.3662
## Xx2:Yy3      0.37422    0.32803   1.141   0.2540
## Xx3:Yy3           NA         NA      NA       NA
```

```
## Xx2:Zz2     -0.66721    0.35779  -1.865   0.0622 .
## Xx3:Zz2     -0.54102    0.44503  -1.216   0.2241
## Yy2:Zz2      6.59287    0.40584  16.245  < 2e-16 ***
## Yy3:Zz2      3.44829    0.43416   7.942 1.98e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 985.065  on 15  degrees of freedom
## Residual deviance:   4.897  on  3  degrees of freedom
## AIC: 108.82
##
## Number of Fisher Scoring iterations: 4
```

```
# independece model
model <- glm(Freq ~ X + Y + Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.5066   -9.0970   -0.0544    5.8943    7.1901
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.27850    0.07773  55.044  < 2e-16 ***
## Xx2          0.00367    0.08567   0.043  0.96583
## Xx3          0.09057    0.08762   1.034  0.30131
## Yy2         -0.16034    0.07326  -2.189  0.02863 *
## Yy3         -1.47362    0.14094 -10.456  < 2e-16 ***
## Zz2         -0.20816    0.07065  -2.946  0.00322 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 985.06  on 15  degrees of freedom
## Residual deviance: 800.08  on 10  degrees of freedom
## AIC: 890
##
## Number of Fisher Scoring iterations: 6
```

```
# joint indep model (XY,Z)
model <- glm(Freq ~ X + Y + Z + X*Y, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
```

```
## glm(formula = Freq ~ X + Y + Z + X * Y, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -10.707   -9.117   -0.097    5.952    7.026
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.27306    0.09325  45.825  < 2e-16 ***
## Xx2         -0.02335    0.12477  -0.187  0.85156
## Xx3          0.12968    0.12020   1.079  0.28067
## Yy2         -0.14015    0.12862  -1.090  0.27586
## Yy3         -1.50024    0.20537  -7.305 2.77e-13 ***
## Zz2         -0.20816    0.07065  -2.946  0.00322 **
## Xx2:Yy2      0.04089    0.18197   0.225  0.82220
## Xx3:Yy2     -0.09489    0.17845  -0.532  0.59490
## Xx2:Yy3      0.09004    0.28689   0.314  0.75364
## Xx3:Yy3           NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 985.06  on 15  degrees of freedom
## Residual deviance: 799.40  on  7  degrees of freedom
## AIC: 895.32
##
## Number of Fisher Scoring iterations: 6
```

```
# joint independece model (XZ,Y)
model <- glm(Freq ~ X + Y + Z + X*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.2161   -9.2452   -0.1456    6.1503    7.2805
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.23002    0.09141  46.275   <2e-16 ***
## Xx2          0.06104    0.11650   0.524   0.6003
## Xx3          0.17767    0.11747   1.513   0.1304
## Yy2         -0.16034    0.07326  -2.189   0.0286 *
## Yy3         -1.47362    0.14094 -10.456   <2e-16 ***
## Zz2         -0.10303    0.12143  -0.848   0.3962
## Xx2:Zz2     -0.12506    0.17201  -0.727   0.4672
## Xx3:Zz2     -0.19346    0.17371  -1.114   0.2654
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 985.06  on 15  degrees of freedom
## Residual deviance: 798.80  on  8  degrees of freedom
## AIC: 892.72
##
## Number of Fisher Scoring iterations: 6
```

```
# joint independece model (YZ,X)
model <- glm(Freq ~ X + Y + Z + Y*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + Y * Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -1.43514  -0.36146  -0.09861   0.39213   1.27981
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.84798    0.07143  67.873   <2e-16 ***
## Xx2          0.00367    0.08567   0.043    0.966
## Xx3          0.09057    0.08762   1.034    0.301
## Yy2         -3.03445    0.23487 -12.920   <2e-16 ***
## Yy3         -2.04645    0.18320 -11.170   <2e-16 ***
## Zz2         -3.67630    0.32021 -11.481   <2e-16 ***
## Yy2:Zz2      6.51876    0.39778  16.388   <2e-16 ***
## Yy3:Zz2      3.47563    0.41215   8.433   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 985.0649  on 15  degrees of freedom
## Residual deviance:   9.3619  on  8  degrees of freedom
## AIC: 103.28
##
## Number of Fisher Scoring iterations: 4
```

```
# cond independece model (XY,XZ)
model <- glm(Freq ~ X + Y + Z + X*Z + X*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Z + X * Z, family = poisson(link = "log"),
##     data = df)
##
```

```
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.2161   -9.2452   -0.1456    6.1503    7.2805
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.23002    0.09141  46.275   <2e-16 ***
## Xx2          0.06104    0.11650   0.524   0.6003
## Xx3          0.17767    0.11747   1.513   0.1304
## Yy2         -0.16034    0.07326  -2.189   0.0286 *
## Yy3         -1.47362    0.14094 -10.456   <2e-16 ***
## Zz2         -0.10303    0.12143  -0.848   0.3962
## Xx2:Zz2     -0.12506    0.17201  -0.727   0.4672
## Xx3:Zz2     -0.19346    0.17371  -1.114   0.2654
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 985.06  on 15  degrees of freedom
## Residual deviance: 798.80  on  8  degrees of freedom
## AIC: 892.72
##
## Number of Fisher Scoring iterations: 6
```

```r
# cond independece model (XY,XZ)
model <- glm(Freq ~ X + Y + Z + X*Z + X*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Z + X * Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.2161   -9.2452   -0.1456    6.1503    7.2805
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.23002    0.09141  46.275   <2e-16 ***
## Xx2          0.06104    0.11650   0.524   0.6003
## Xx3          0.17767    0.11747   1.513   0.1304
## Yy2         -0.16034    0.07326  -2.189   0.0286 *
## Yy3         -1.47362    0.14094 -10.456   <2e-16 ***
## Zz2         -0.10303    0.12143  -0.848   0.3962
## Xx2:Zz2     -0.12506    0.17201  -0.727   0.4672
## Xx3:Zz2     -0.19346    0.17371  -1.114   0.2654
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 985.06  on 15  degrees of freedom
```

```
## Residual deviance: 798.80  on  8  degrees of freedom
## AIC: 892.72
##
## Number of Fisher Scoring iterations: 6
```

```
# cond independece model (XY,YZ)
model <- glm(Freq ~ X + Y + Z + X*Y + Y*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + Y * Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##        1         2         3         4         5         6         7
## -0.15936   0.01220   0.13743   0.92293  -0.07725  -0.94741   0.30523
##        8         9        10        11        12        13        14
## -0.13364  -0.17677  -0.07527   0.03196   0.04215  -1.31362   1.14685
##       16        17
##  1.29499  -1.41824
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.84253    0.08806  54.990   <2e-16 ***
## Xx2         -0.02335    0.12477  -0.187    0.852
## Xx3          0.12968    0.12020   1.079    0.281
## Yy2         -3.01425    0.25756 -11.703   <2e-16 ***
## Yy3         -2.07307    0.23638  -8.770   <2e-16 ***
## Zz2         -3.67630    0.32020 -11.481   <2e-16 ***
## Xx2:Yy2      0.04089    0.18197   0.225    0.822
## Xx3:Yy2     -0.09489    0.17845  -0.532    0.595
## Xx2:Yy3      0.09004    0.28689   0.314    0.754
## Xx3:Yy3           NA         NA      NA       NA
## Yy2:Zz2      6.51876    0.39778  16.388   <2e-16 ***
## Yy3:Zz2      3.47563    0.41215   8.433   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 985.0649  on 15  degrees of freedom
## Residual deviance:   8.6798  on  5  degrees of freedom
## AIC: 108.6
##
## Number of Fisher Scoring iterations: 4
```

```
# cond independece model (XZ,YZ)
model <- glm(Freq ~ X + Y + Z + X*Z + Y*Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
```

```
## glm(formula = Freq ~ X + Y + Z + X * Z + Y * Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -1.32462  -0.45396   0.00099   0.42914   1.15866
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.79723    0.08681  55.261   <2e-16 ***
## Xx2          0.06104    0.11650   0.524    0.600
## Xx3          0.17967    0.11832   1.519    0.129
## Yy2         -3.03445    0.23487 -12.920   <2e-16 ***
## Yy3         -2.02486    0.18503 -10.943   <2e-16 ***
## Zz2         -3.56624    0.33585 -10.619   <2e-16 ***
## Xx2:Zz2     -0.12506    0.17201  -0.727    0.467
## Xx3:Zz2     -0.19780    0.17631  -1.122    0.262
## Yy2:Zz2      6.51876    0.39778  16.388   <2e-16 ***
## Yy3:Zz2      3.42805    0.41567   8.247   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 985.0649  on 15  degrees of freedom
## Residual deviance:   8.0597  on  6  degrees of freedom
## AIC: 105.98
##
## Number of Fisher Scoring iterations: 4
```

b

```
delta1 <- c(0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,  0, 1, 0, 0, 0)
delta2 <- c(0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,  0, 0, 0, 0, 1)

df <- data.frame(temp)


# saturated model
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z + Y*Z + X*Y*Z + delta1 + delta2, data = df, family = poisson(
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + Y * Z + X *
##     Y * Z + delta1 + delta2, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##  [1]  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
##
## Coefficients: (2 not defined because of singularities)
```

```
##                Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.828e+00  8.944e-02  53.982  < 2e-16 ***
## Xx2         -8.032e-03  1.267e-01  -0.063   0.9495
## Xx3          1.553e-01  1.219e-01   1.274   0.2025
## Yy2         -2.882e+00  3.884e-01  -7.421 1.16e-13 ***
## Yy3         -2.430e+00  3.145e-01  -7.728 1.09e-14 ***
## Zz2         -3.219e+00  4.561e-01  -7.058 1.69e-12 ***
## delta1      -2.586e+01  6.965e+04   0.000   0.9997
## delta2      -2.528e+01  6.965e+04   0.000   0.9997
## Xx2:Yy2     -1.461e-01  5.706e-01  -0.256   0.7979
## Xx3:Yy2     -3.094e-01  5.695e-01  -0.543   0.5869
## Xx2:Yy3      7.012e-01  3.904e-01   1.796   0.0725 .
## Xx3:Yy3             NA         NA      NA       NA
## Xx2:Zz2     -5.028e-01  7.412e-01  -0.678   0.4976
## Xx3:Zz2     -1.072e+00  8.455e-01  -1.267   0.2050
## Yy2:Zz2      5.936e+00  6.002e-01   9.890  < 2e-16 ***
## Yy3:Zz2      3.711e+00  5.954e-01   6.234 4.56e-10 ***
## Xx2:Yy2:Zz2  6.849e-01  9.368e-01   0.731   0.4647
## Xx3:Yy2:Zz2  1.272e+00  1.021e+00   1.245   0.2130
## Xx2:Yy3:Zz2 -8.835e-01  9.233e-01  -0.957   0.3386
## Xx3:Yy3:Zz2         NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1.1759e+03  on 17  degrees of freedom
## Residual deviance: 3.0331e-10  on  0  degrees of freedom
## AIC: 113.92
##
## Number of Fisher Scoring iterations: 21
```

```
# homogenous model
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z + Y*Z + delta1 + delta2, data = df, family = poisson(link = "
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + Y * Z + delta1 +
##     delta2, family = poisson(link = "log"), data = df)
##
## Deviance Residuals:
##        1         2         3         4         5         6         7
## -0.03739  -0.05946   0.08975   0.19267   0.41545  -0.65888   1.30773
##        8         9        10        11        12        13        14
## -0.68341  -0.41758  -0.28057   0.17658   0.10301  -0.70228   0.55381
##       15        16        17        18
## -0.00009   0.61805  -0.76566  -0.00009
##
## Coefficients: (1 not defined because of singularities)
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept)    4.83166    0.08869  54.481  < 2e-16 ***
## Xx2           -0.00604    0.12523  -0.048   0.9615
## Xx3            0.14451    0.12087   1.196   0.2319
```

```
## Yy2           -3.42432    0.36088  -9.489  < 2e-16 ***
## Yy3           -2.22923    0.26518  -8.407  < 2e-16 ***
## Zz2           -3.30964    0.37304  -8.872  < 2e-16 ***
## delta1       -22.04952 9426.61685  -0.002   0.9981
## delta2       -21.64715 9426.61686  -0.002   0.9982
## Xx2:Yy2        0.65707    0.37818   1.737   0.0823 .
## Xx3:Yy2        0.40568    0.44897   0.904   0.3662
## Xx2:Yy3        0.37422    0.32803   1.141   0.2540
## Xx3:Yy3             NA         NA      NA       NA
## Xx2:Zz2       -0.66721    0.35779  -1.865   0.0622 .
## Xx3:Zz2       -0.54102    0.44503  -1.216   0.2241
## Yy2:Zz2        6.59287    0.40584  16.245  < 2e-16 ***
## Yy3:Zz2        3.44829    0.43416   7.942 1.98e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.873  on 17  degrees of freedom
## Residual deviance:    4.897  on  3  degrees of freedom
## AIC: 112.82
##
## Number of Fisher Scoring iterations: 17
```

```r
# independece model
model <- glm(Freq ~ X + Y + Z, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.0596   -9.1703   -0.0282    5.2241    7.4939
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.31818    0.07690  56.153  < 2e-16 ***
## Xx2          0.00367    0.08567   0.043  0.96583
## Xx3         -0.02607    0.08631  -0.302  0.76260
## Yy2         -0.16034    0.07326  -2.189  0.02863 *
## Yy3         -1.90954    0.13833 -13.804  < 2e-16 ***
## Zz2         -0.20816    0.07065  -2.946  0.00322 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.87  on 17  degrees of freedom
## Residual deviance:  849.93  on 12  degrees of freedom
## AIC: 939.85
##
```

```
## Number of Fisher Scoring iterations: 6
```

```
# joint indep model (XY,Z)
model <- glm(Freq ~ X + Y + Z + X*Y + delta1 + delta2, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + delta1 + delta2, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.7073   -9.0275   -0.0011    5.7728    7.0259
##
## Coefficients: (1 not defined because of singularities)
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   4.27306    0.09325  45.825  < 2e-16 ***
## Xx2          -0.02335    0.12477  -0.187  0.85156
## Xx3           0.12968    0.12020   1.079  0.28067
## Yy2          -0.14015    0.12862  -1.090  0.27586
## Yy3          -1.50024    0.20537  -7.305 2.77e-13 ***
## Zz2          -0.20816    0.07065  -2.946  0.00322 **
## delta1      -17.20508  773.78386  -0.022  0.98226
## delta2      -16.99693  773.78386  -0.022  0.98248
## Xx2:Yy2       0.04089    0.18197   0.225  0.82220
## Xx3:Yy2      -0.09489    0.17845  -0.532  0.59490
## Xx2:Yy3       0.09004    0.28689   0.314  0.75364
## Xx3:Yy3            NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.9  on 17  degrees of freedom
## Residual deviance:  799.4  on  7  degrees of freedom
## AIC: 899.32
##
## Number of Fisher Scoring iterations: 12
```

```
# joint independece model (XZ,Y)
model <- glm(Freq ~ X + Y + Z + X*Z+ delta1 + delta2, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Z + delta1 + delta2, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -10.2161   -9.0303   -0.0011    5.9168    7.2805
##
```

```
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   4.23002    0.09141  46.275   <2e-16 ***
## Xx2           0.06104    0.11650   0.524   0.6003
## Xx3           0.17767    0.11747   1.513   0.1304
## Yy2          -0.16034    0.07326  -2.189   0.0286 *
## Yy3          -1.47362    0.14094 -10.456   <2e-16 ***
## Zz2          -0.10303    0.12143  -0.848   0.3962
## delta1      -17.23666  773.78385  -0.022   0.9822
## delta2      -16.94017  773.78385  -0.022   0.9825
## Xx2:Zz2      -0.12506    0.17201  -0.727   0.4672
## Xx3:Zz2      -0.19346    0.17371  -1.114   0.2654
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.9  on 17  degrees of freedom
## Residual deviance:  798.8  on  8  degrees of freedom
## AIC: 896.72
##
## Number of Fisher Scoring iterations: 12
```

```r
# joint independece model (YZ,X)
model <- glm(Freq ~ X + Y + Z + Y*Z+ delta1 + delta2, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + Y * Z + delta1 + delta2, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -1.43514  -0.33344  -0.03137   0.33929   1.27981
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   4.84798    0.07143  67.873   <2e-16 ***
## Xx2           0.00367    0.08567   0.043    0.966
## Xx3           0.09057    0.08762   1.034    0.301
## Yy2          -3.03445    0.23487 -12.920   <2e-16 ***
## Yy3          -2.04645    0.18320 -11.170   <2e-16 ***
## Zz2          -3.67630    0.32021 -11.481   <2e-16 ***
## delta1      -21.19467 5717.53214  -0.004    0.997
## delta2      -20.99400 5717.53214  -0.004    0.997
## Yy2:Zz2       6.51876    0.39778  16.388   <2e-16 ***
## Yy3:Zz2       3.47563    0.41215   8.433   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.8734  on 17  degrees of freedom
```

```
## Residual deviance:    9.3619  on  8  degrees of freedom
## AIC: 107.28
##
## Number of Fisher Scoring iterations: 16
```

```
# cond independece model (XY,XZ)
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z+ delta1 + delta2, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + delta1 + delta2,
##     family = poisson(link = "log"), data = df)
##
## Deviance Residuals:
##        1         2         3         4         5         6         7
##   6.1340    5.7201    6.0097   -9.3908   -9.4333  -10.4121   -8.6531
##        8         9        10        11        12        13        14
##  -9.3620   -9.6563    6.3069    7.0457    7.4373   -1.1449    1.0939
##       15        16        17        18
##  -0.0011    1.0926   -1.3654   -0.0011
##
## Coefficients: (1 not defined because of singularities)
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   4.22458    0.10492  40.264  < 2e-16 ***
## Xx2           0.03402    0.14764   0.230    0.818
## Xx3           0.21679    0.14343   1.511    0.131
## Yy2          -0.14015    0.12862  -1.090    0.276
## Yy3          -1.50024    0.20537  -7.305 2.77e-13 ***
## Zz2          -0.10303    0.12143  -0.848    0.396
## delta1      -17.24371  773.78386  -0.022    0.982
## delta2      -16.94722  773.78387  -0.022    0.983
## Xx2:Yy2       0.04089    0.18197   0.225    0.822
## Xx3:Yy2      -0.09489    0.17845  -0.532    0.595
## Xx2:Yy3       0.09004    0.28689   0.314    0.754
## Xx3:Yy3            NA         NA      NA       NA
## Xx2:Zz2      -0.12506    0.17201  -0.727    0.467
## Xx3:Zz2      -0.19346    0.17371  -1.114    0.265
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.87  on 17  degrees of freedom
## Residual deviance:  798.12  on  5  degrees of freedom
## AIC: 902.03
##
## Number of Fisher Scoring iterations: 12
```

```
# cond independece model (XY,YZ)
model <- glm(Freq ~ X + Y + Z + X*Y + Y*Z+ delta1 + delta2, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + Y * Z + delta1 + delta2,
##     family = poisson(link = "log"), data = df)
##
## Deviance Residuals:
##        1         2         3         4         5         6         7
## -0.15936   0.01220   0.13743   0.92293  -0.07725  -0.94741   0.30523
##        8         9        10        11        12        13        14
## -0.13364  -0.17677  -0.07527   0.03196   0.04215  -1.31362   1.14685
##       15        16        17        18
## -0.00015   1.29499  -1.41824  -0.00015
##
## Coefficients: (1 not defined because of singularities)
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept)    4.84253    0.08806  54.990   <2e-16 ***
## Xx2           -0.02335    0.12477  -0.187    0.852
## Xx3            0.12968    0.12020   1.079    0.281
## Yy2           -3.01425    0.25756 -11.703   <2e-16 ***
## Yy3           -2.07307    0.23638  -8.770   <2e-16 ***
## Zz2           -3.67630    0.32021 -11.481   <2e-16 ***
## delta1       -21.20172 5717.53214  -0.004    0.997
## delta2       -21.00105 5717.53214  -0.004    0.997
## Xx2:Yy2        0.04089    0.18197   0.225    0.822
## Xx3:Yy2       -0.09489    0.17845  -0.532    0.595
## Xx2:Yy3        0.09004    0.28689   0.314    0.754
## Xx3:Yy3             NA         NA      NA       NA
## Yy2:Zz2        6.51876    0.39778  16.388   <2e-16 ***
## Yy3:Zz2        3.47563    0.41215   8.433   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.8734  on 17  degrees of freedom
## Residual deviance:    8.6798  on  5  degrees of freedom
## AIC: 112.6
##
## Number of Fisher Scoring iterations: 16
```

```
# cond independece model (XZ,YZ)
model <- glm(Freq ~ X + Y + Z + X*Z + Y*Z+ delta1 + delta2, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Z + Y * Z + delta1 + delta2,
##     family = poisson(link = "log"), data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -1.32462  -0.41394  -0.00015   0.39802   1.15866
##
## Coefficients:
```

```
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept)    4.79723    0.08681  55.261   <2e-16 ***
## Xx2            0.06104    0.11650   0.524    0.600
## Xx3            0.17967    0.11832   1.519    0.129
## Yy2           -3.03445    0.23487 -12.920   <2e-16 ***
## Yy3           -2.02486    0.18503 -10.943   <2e-16 ***
## Zz2           -3.56624    0.33585 -10.618   <2e-16 ***
## delta1       -21.25463 5717.53214  -0.004    0.997
## delta2       -20.91864 5717.53214  -0.004    0.997
## Xx2:Zz2       -0.12506    0.17201  -0.727    0.467
## Xx3:Zz2       -0.19780    0.17631  -1.122    0.262
## Yy2:Zz2        6.51876    0.39778  16.388   <2e-16 ***
## Yy3:Zz2        3.42805    0.41567   8.247   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1175.8734  on 17  degrees of freedom
## Residual deviance:    8.0597  on  6  degrees of freedom
## AIC: 109.98
##
## Number of Fisher Scoring iterations: 16
```

Problem 2

```
temp <- read.ftable(textConnection("TonsilSize Normal Slightl-enlarged Very-enlarged
CarrierCond
Carrier 19 29 24
Non-Carrier 497 560 269"))

df <- data.frame(temp)

options(contrast=c("contr.treatment","contr.poly"))

#independece model
model <- glm(Freq ~ CarrierCond + TonsilSize, data = df, family = poisson(link = "log"))

summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ CarrierCond + TonsilSize, family = poisson(link = "log"),
##     data = df)
##
## Deviance Residuals:
##        1         2         3         4         5         6
## -1.54914   0.34153  -0.24416   0.05645   2.11019  -0.53736
##
## Coefficients:
##                            Estimate Std. Error z value Pr(>|z|)
```

```
## (Intercept)                 3.27997    0.12293  26.682  < 2e-16 ***
## CarrierCondNon-Carrier      2.91326    0.12101  24.075  < 2e-16 ***
## TonsilSizeSlightl-enlarged  0.13232    0.06030   2.194   0.0282 *
## TonsilSizeVery-enlarged    -0.56593    0.07315  -7.737 1.02e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 1487.2173  on 5  degrees of freedom
## Residual deviance:    7.3209  on 2  degrees of freedom
## AIC: 53.992
##
## Number of Fisher Scoring iterations: 4
```

```
xCarrier <- rep(c(1,2), c(3,3))
ySize <- rep(c(1,2,3), 2)

#linear by linear
model <- glm(Freq ~ CarrierCond + TonsilSize + xCarrier * ySize,
             data = df, family = poisson(link = "log"))

summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ CarrierCond + TonsilSize + xCarrier * ySize,
##     family = poisson(link = "log"), data = df)
##
## Deviance Residuals:
## [1]  0  0  0  0  0  0
##
## Coefficients: (1 not defined because of singularities)
##                             Estimate Std. Error z value Pr(>|z|)
## (Intercept)                   5.7904     0.9750   5.939 2.87e-09 ***
## CarrierCondNon-Carrier        4.1116     0.5138   8.003 1.22e-15 ***
## TonsilSizeSlightl-enlarged    2.1178     0.8353   2.535  0.01123 *
## TonsilSizeVery-enlarged       4.7746     2.0690   2.308  0.02101 *
## xCarrier                     -1.9985     0.8424  -2.372  0.01767 *
## ySize                            NA         NA      NA       NA
## xCarrier:ySize               -0.8475     0.3163  -2.680  0.00737 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance:  1.4872e+03  on 5  degrees of freedom
## Residual deviance: -2.9754e-14  on 0  degrees of freedom
## AIC: 50.671
##
## Number of Fisher Scoring iterations: 3
```

the linear by linear model fits better compared to independet model.

both, hw4 and linear by linear model show that being a carrier and tonsil is related.

Problem 3

a

```
count <- c(425, 17, 80, 36, 10, 555, 74, 47, 7, 34, 771, 33, 5, 14, 29, 452)

at04 <- rep(c("N", "MW", "S", "W"), c(4,4,4,4))
at16 <- rep(c("N", "MW", "S", "W"), 4)

iN <- (at04 == "N") * (at16 == "N")
iMW <- (at04 == "MW") * (at16 == "MW")
iS <- (at04 == "S") * (at16 == "S")
iW <- (at04 == "W") * (at16 == "W")


at04_N <- (at04 == "N")
at04_MW <- (at04 == "MW")
at04_S <- (at04 == "S")
at04_W <- (at04 == "W")


at16_N <-  (at16 == "N")
at16_MW <- (at16 == "MW")
at16_S <-  (at16 == "S")
at16_W <-  (at16 == "W")

symm1 <- 1 * (at04 == "N") * (at16 == "N")
symm2 <- 2 * (at04 == "MW") * (at16 == "MW")
symm3 <- 3 * (at04 == "S") * (at16 == "S")
symm4 <- 4 * (at04 == "S") * (at16 == "S")

symm5 <- 5 * (at04 == "N") *  (at16 == "MW") + 5 * (at04 == "MW") * (at16 == "N")
symm6 <- 6 * (at04 == "S")  * (at16 == "N") + 6*   (at04 == "N") *(at16 == "S")
symm7 <- 7 * (at04 == "W") * (at16 == "N") + 7 *(at04 == "N") * (at16 == "W")
symm8 <- 8 * (at04 == "S")*  (at16 == "MW")+ 8 * (at04 == "MW")* (at16 == "S")
symm9  <- 9 *(at04 == "MW") * (at16 == "W") + 9  * (at04 == "W")* (at16 == "MW")
symm10  <- 10 *(at04 == "S") * (at16 == "W")+ 10 * (at04 == "W")* (at16 == "S")

symm=symm3+symm1+symm4+symm6+symm2+symm5+symm6 + symm7 + symm8 + symm9 + symm10

df <- data.frame(at04, at16, count, iN, iMW, iW , iS , symm)

df
```

```
##    at04 at16 count iN iMW iW iS symm
## 1     N    N   425  1   0  0  0    1
## 2     N   MW    17  0   0  0  0    5
## 3     N    S    80  0   0  0  0   12
## 4     N    W    36  0   0  0  0    7
## 5    MW    N    10  0   0  0  0    5
## 6    MW   MW   555  0   1  0  0    2
## 7    MW    S    74  0   0  0  0    8
## 8    MW    W    47  0   0  0  0    9
```

```
## 9     S    N     7  0   0  0  0   12
## 10    S   MW    34  0   0  0  0    8
## 11    S    S   771  0   0  0  1    7
## 12    S    W    33  0   0  0  0   10
## 13    W    N     5  0   0  0  0    7
## 14    W   MW    14  0   0  0  0    9
## 15    W    S    29  0   0  0  0   10
## 16    W    W   452  0   0  1  0    0
```

symmetry model

```
model=glm(count~symm1+symm4+symm6+symm2+symm5+symm3+symm7 + symm8+ symm9+ symm10,data=df,family=poisson
summary(model)
```

```
##
## Call:
## glm(formula = count ~ symm1 + symm4 + symm6 + symm2 + symm5 +
##     symm3 + symm7 + symm8 + symm9 + symm10, family = poisson(link = log),
##     data = df)
##
## Deviance Residuals:
##    Min      1Q  Median      3Q     Max
## -6.886  -1.480   0.000   1.330   4.948
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  6.11368    0.04704 129.979  < 2e-16 ***
## symm1       -0.06159    0.06757  -0.912  0.36199
## symm4        0.13350    0.01481   9.014  < 2e-16 ***
## symm6       -0.39015    0.01951 -19.995  < 2e-16 ***
## symm2        0.10264    0.03168   3.240  0.00119 **
## symm5       -0.70220    0.03962 -17.722  < 2e-16 ***
## symm3             NA         NA      NA       NA
## symm7       -0.44189    0.02330 -18.965  < 2e-16 ***
## symm8       -0.26559    0.01339 -19.837  < 2e-16 ***
## symm9       -0.29955    0.01516 -19.765  < 2e-16 ***
## symm10      -0.26797    0.01354 -19.786  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 4481.79  on 15  degrees of freedom
## Residual deviance:  134.45  on  6  degrees of freedom
## AIC: 246.96
##
## Number of Fisher Scoring iterations: 5
```

$df = 6$, $G^2 = 134.45$, p-value $= 0$

symmetry model doesnot seem to be a good fit.

Quasi-symmetry Model

```
model=glm(count~at04_N + at04_W+ at04_MW+ at04_S + at16_N+ at16_W+ at16_MW+ at16_S +symm1+symm4+symm6+s
summary(model)
```

```
##
## Call:
## glm(formula = count ~ at04_N + at04_W + at04_MW + at04_S + at16_N +
##     at16_W + at16_MW + at16_S + symm1 + symm4 + symm6 + symm2 +
##     symm5 + symm3 + symm7 + symm8 + symm9 + symm10, family = poisson(link = log),
##     data = df)
##
## Deviance Residuals:
##        1         2         3         4         5         6         7
##  0.00000  -0.66161   0.40293  -0.11169   1.01493   0.00000  -0.45446
##        8         9        10        11        12        13        14
##  0.16214  -1.16490   0.71091   0.00000  -0.07427   0.31630  -0.28734
##       15        16
##  0.07997   0.00000
##
## Coefficients: (6 not defined because of singularities)
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.74814    0.25878   2.891  0.00384 **
## at04_NTRUE   0.09267    0.21685   0.427  0.66913
## at04_WTRUE   2.60427    0.16321  15.957  < 2e-16 ***
## at04_MWTRUE  0.31699    0.19401   1.634  0.10227
## at04_STRUE        NA         NA      NA       NA
## at16_NTRUE  -1.88784    0.29337  -6.435 1.23e-10 ***
## at16_WTRUE   2.76127    0.17233  16.023  < 2e-16 ***
## at16_MWTRUE -0.63753    0.21904  -2.911  0.00361 **
## at16_STRUE        NA         NA      NA       NA
## symm1        7.09913    0.38605  18.389  < 2e-16 ***
## symm4        1.47489    0.06532  22.580  < 2e-16 ***
## symm6        0.58264    0.04131  14.105  < 2e-16 ***
## symm2        2.94568    0.14219  20.717  < 2e-16 ***
## symm5        0.55724    0.06211   8.971  < 2e-16 ***
## symm3             NA         NA      NA       NA
## symm7             NA         NA      NA       NA
## symm8        0.41141    0.02713  15.164  < 2e-16 ***
## symm9             NA         NA      NA       NA
## symm10            NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 4481.7873  on 15  degrees of freedom
## Residual deviance:    3.9324  on  3  degrees of freedom
## AIC: 122.44
##
## Number of Fisher Scoring iterations: 4
```

$df = 3$, $G^2 = 3.9324$, p-value $= 0.2688568$

the quasi-symmetry model fits moderatlely well.

b

$G^2(marginal homogenety) = G^2(symmtery) - G^2(quasi - symmtery) = 130.518$

$df = 3$

p-value= 0$

the marginal homgeneity model does not fit the data.