HW 11

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Contents

Problem 1																	1														
a																		 													-
b																		 	•												15
$\mathbf{Problem}$	2																													2	22
Problem	3																													•	24
a																		 													24
b																		 													27

Problem 1

 \mathbf{a}

##

Deviance Residuals:

Coefficients:

```
temp <- read.ftable(textConnection("Z z1 z2
Y y1 y2 y3
X
x1 125 7 11 5 106 18
x2 124 6 22 3 109 9
x3 146 6 0 2 111 0"))
df <- data.frame(temp)

# saturated model
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z + Y*Z + X*Y*Z, data = df, family = poisson(link = "log"))
summary(model)

##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + Y * Z + X *
## Y * Z, family = poisson(link = "log"), data = df)</pre>
```

[1] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

(Intercept) 4.828e+00 8.944e-02 53.982 < 2e-16 ***

Estimate Std. Error z value Pr(>|z|)

```
## Xx2
              -8.032e-03 1.267e-01 -0.063
                                              0.9495
## Xx3
                                              0.2025
               1.553e-01 1.219e-01
                                     1.274
## Yy2
              -2.882e+00 3.884e-01 -7.421 1.16e-13 ***
              -2.430e+00 3.145e-01 -7.728 1.09e-14 ***
## Yy3
## Zz2
              -3.219e+00 4.561e-01 -7.058 1.69e-12 ***
## Xx2:Yy2
              -1.461e-01 5.706e-01 -0.256
                                             0.7979
## Xx3:Yy2
              -3.094e-01 5.695e-01 -0.543
                                             0.5869
## Xx2:Yy3
               7.012e-01 3.904e-01
                                      1.796
                                             0.0725 .
## Xx3:Yy3
              -2.586e+01 6.965e+04
                                     0.000
                                              0.9997
## Xx2:Zz2
              -5.028e-01 7.412e-01 -0.678
                                             0.4976
## Xx3:Zz2
              -1.072e+00 8.455e-01 -1.267
                                              0.2050
## Yy2:Zz2
                                    9.890 < 2e-16 ***
               5.936e+00 6.002e-01
## Yy3:Zz2
               3.711e+00 5.954e-01
                                    6.234 4.56e-10 ***
                                            0.4647
## Xx2:Yy2:Zz2 6.849e-01 9.368e-01
                                    0.731
                                     1.245
                                              0.2130
## Xx3:Yy2:Zz2 1.272e+00 1.021e+00
## Xx2:Yy3:Zz2 -8.835e-01 9.233e-01
                                     -0.957
                                              0.3386
## Xx3:Yy3:Zz2 5.791e-01 9.851e+04
                                      0.000
                                              1.0000
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 1.1759e+03 on 17 degrees of freedom
## Residual deviance: 3.0325e-10 on 0 degrees of freedom
## AIC: 113.92
## Number of Fisher Scoring iterations: 21
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z + Y*Z, data = df, family = poisson(link = "log")
summary(model)
##
## glm(formula = Freq \sim X + Y + Z + X * Y + X * Z + Y * Z, family = poisson(link = "log"),
##
       data = df
##
## Deviance Residuals:
                                                                    7
         1
                   2
                             3
                                       4
                                                 5
                                                           6
## -0.03739
            -0.05946
                       0.08975
                                 0.19267
                                           0.41545 -0.65888
                                                               1.30773
         8
                   9
                            10
                                      11
                                                12
## -0.68341
            -0.41758
                      -0.28057
                                 0.17658
                                           0.10301 -0.70228
                                                               0.55381
##
         15
                   16
                            17
## -0.00010
             0.61805 -0.76566
                                -0.00008
##
## Coefficients:
                Estimate Std. Error z value Pr(>|z|)
                            0.08869 54.481 < 2e-16 ***
## (Intercept)
                 4.83166
## Xx2
                -0.00604
                            0.12523 -0.048
                                              0.9615
## Xx3
                 0.14451
                            0.12087
                                     1.196
                                              0.2319
## Yy2
                -3.42432
                            0.36088 -9.489 < 2e-16 ***
## Yy3
                -2.22923
                            0.26518 -8.407 < 2e-16 ***
## Zz2
                -3.30964
                            0.37304 -8.872 < 2e-16 ***
## Xx2:Yy2
                 0.65707
                            0.37818
                                     1.737 0.0823 .
```

```
## Xx3:Yv2
                 0.40568
                            0.44897
                                    0.904
                                              0.3662
## Xx2:Yy3
                 0.37422
                            0.32803
                                            0.2540
                                     1.141
## Xx3:Yy3
               -21.85471 6620.02069 -0.003
                                             0.9974
## Xx2:Zz2
                -0.66721
                            0.35779 -1.865
                                             0.0622 .
## Xx3:Zz2
                -0.54102
                            0.44503 -1.216
                                             0.2241
                 6.59287
                            0.40584 16.245 < 2e-16 ***
## Yy2:Zz2
                 3.44829
                            0.43416
                                     7.942 1.98e-15 ***
## Yy3:Zz2
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 1175.873 on 17 degrees of freedom
## Residual deviance: 4.897 on 4 degrees of freedom
## AIC: 110.82
##
## Number of Fisher Scoring iterations: 17
# independece model
model <- glm(Freq ~ X + Y + Z, data = df, family = poisson(link = "log"))</pre>
summary(model)
##
## Call:
## glm(formula = Freq ~ X + Y + Z, family = poisson(link = "log"),
       data = df
##
## Deviance Residuals:
                        Median
##
       Min
              1Q
                                      3Q
                                               Max
## -10.0596 -9.1703 -0.0282
                                  5.2241
                                            7.4939
##
## Coefficients:
##
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.31818 0.07690 56.153 < 2e-16 ***
               0.00367
                          0.08567
                                   0.043 0.96583
## Xx2
## Xx3
              -0.02607
                          0.08631 -0.302 0.76260
## Yy2
              -0.16034
                          0.07326 -2.189 0.02863 *
                          0.13833 -13.804 < 2e-16 ***
## Yy3
              -1.90954
              -0.20816
## Zz2
                          0.07065 -2.946 0.00322 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 1175.87 on 17 degrees of freedom
## Residual deviance: 849.93 on 12 degrees of freedom
## AIC: 939.85
## Number of Fisher Scoring iterations: 6
model <- glm(Freq ~ X + Y + Z + X*Y, data = df, family = poisson(link = "log"))
summary(model)
```

```
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y, family = poisson(link = "log"),
      data = df
## Deviance Residuals:
                       Median
       Min 10
                                     30
                                             Max
## -10.7073 -9.0275
                      -0.0011
                                 5.7728
                                          7.0259
##
## Coefficients:
               Estimate Std. Error z value Pr(>|z|)
              4.27306 0.09325 45.825 < 2e-16 ***
## (Intercept)
                        0.12477 -0.187 0.85156
## Xx2
               -0.02335
               ## Xx3
## Yy2
                          0.12862 -1.090 0.27586
              -0.14015
## Yy3
               -1.50024
                          0.20537 -7.305 2.77e-13 ***
## Zz2
              -0.20816
                          0.07065 -2.946 0.00322 **
## Xx2:Yv2
               0.04089
                          0.18197
                                  0.225 0.82220
## Xx3:Yy2
              -0.09489
                          0.17845 -0.532 0.59490
## Xx2:Yy3
               0.09004
                          0.28689
                                   0.314 0.75364
## Xx3:Yy3
             -17.07691 539.13663 -0.032 0.97473
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
      Null deviance: 1175.9 on 17 degrees of freedom
## Residual deviance: 799.4 on 8 degrees of freedom
## AIC: 897.32
## Number of Fisher Scoring iterations: 12
# joint independece model (XZ,Y)
model \leftarrow glm(Freq \sim X + Y + Z + X*Z, data = df, family = poisson(link = "log"))
summary(model)
##
## glm(formula = Freq ~ X + Y + Z + X * Z, family = poisson(link = "log"),
##
      data = df
##
## Deviance Residuals:
      Min
               1Q
                    Median
                                 3Q
                                        Max
## -9.8476 -9.2742
                    0.0684
                             5.5459
                                     7.7311
##
## Coefficients:
              Estimate Std. Error z value Pr(>|z|)
                         0.09071 47.072 <2e-16 ***
## (Intercept) 4.26970
## Xx2
              0.06104
                         0.11650 0.524
                                          0.6003
## Xx3
                                 0.524
              0.06104
                         0.11650
                                          0.6003
## Yy2
              -0.16034
                         0.07326 -2.189
                                          0.0286 *
## Yy3
             -1.90954
                         0.13833 -13.804
                                         <2e-16 ***
## Zz2
              -0.10303
                         0.12143 -0.848
                                         0.3962
                         0.17201 -0.727
## Xx2:Zz2
             -0.12506
                                         0.4672
```

```
## Xx3:Zz2
             -0.19346
                          0.17371 -1.114 0.2654
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 1175.87 on 17 degrees of freedom
## Residual deviance: 848.65 on 10 degrees of freedom
## AIC: 942.57
##
## Number of Fisher Scoring iterations: 6
model <- glm(Freq ~ X + Y + Z + Y*Z, data = df, family = poisson(link = "log"))
summary(model)
##
## Call:
## glm(formula = Freq ~ X + Y + Z + Y * Z, family = poisson(link = "log"),
       data = df
##
## Deviance Residuals:
      Min 1Q Median
                                  3Q
                                          Max
## -4.6468 -0.5859 -0.0853 0.3739
                                       2.8735
##
## Coefficients:
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.88765
                         0.07052 69.305 <2e-16 ***
## Xx2
               0.00367
                          0.08567
                                   0.043
                                             0.966
## Xx3
              -0.02607
                          0.08631 -0.302
                                            0.763
## Yy2
              -3.03445
                          0.23487 -12.920
                                          <2e-16 ***
## Yy3
              -2.48238
                          0.18120 -13.699
                                           <2e-16 ***
## Zz2
              -3.67630
                          0.32021 -11.481
                                            <2e-16 ***
## Yy2:Zz2
              6.51876
                          0.39778 16.388
                                          <2e-16 ***
## Yy3:Zz2
              3.47563
                          0.41215 8.433 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 1175.873 on 17 degrees of freedom
## Residual deviance: 59.215 on 10 degrees of freedom
## AIC: 153.13
##
## Number of Fisher Scoring iterations: 5
model <- glm(Freq ~ X + Y + Z + X*Z + X*Z, data = df, family = poisson(link = "log"))
summary(model)
##
## Call:
## glm(formula = Freq \sim X + Y + Z + X * Z + X * Z, family = poisson(link = "log"),
```

```
##
       data = df
##
## Deviance Residuals:
           1Q Median
                                3Q
##
      Min
                                         Max
## -9.8476 -9.2742 0.0684 5.5459
                                      7.7311
##
## Coefficients:
              Estimate Std. Error z value Pr(>|z|)
##
## (Intercept) 4.26970
                         0.09071 47.072 <2e-16 ***
                          0.11650 0.524
## Xx2
              0.06104
                                           0.6003
              0.06104
                                  0.524 0.6003
## Xx3
                          0.11650
## Yy2
              -0.16034
                          0.07326 -2.189 0.0286 *
              -1.90954
## Yy3
                          0.13833 -13.804 <2e-16 ***
                          0.12143 -0.848 0.3962
## Zz2
              -0.10303
## Xx2:Zz2
             -0.12506
                          0.17201 -0.727
                                           0.4672
## Xx3:Zz2
              -0.19346
                          0.17371 -1.114
                                          0.2654
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 1175.87 on 17 degrees of freedom
## Residual deviance: 848.65 on 10 degrees of freedom
## AIC: 942.57
##
## Number of Fisher Scoring iterations: 6
model \leftarrow glm(Freq \sim X + Y + Z + X*Z + X*Z, data = df, family = poisson(link = "log"))
summary(model)
##
## Call:
## glm(formula = Freq \sim X + Y + Z + X * Z + X * Z, family = poisson(link = "log"),
##
       data = df)
## Deviance Residuals:
      Min 10 Median
                                 30
                                         Max
## -9.8476 -9.2742 0.0684 5.5459
                                      7.7311
##
## Coefficients:
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.26970
                         0.09071 47.072
                                           <2e-16 ***
## Xx2
              0.06104
                          0.11650 0.524
                                           0.6003
## Xx3
              0.06104
                          0.11650
                                  0.524
                                          0.6003
## Yy2
              -0.16034
                          0.07326 -2.189 0.0286 *
## Yy3
              -1.90954
                          0.13833 -13.804
                                           <2e-16 ***
## Zz2
              -0.10303
                          0.12143 -0.848
                                          0.3962
## Xx2:Zz2
              -0.12506
                          0.17201 - 0.727
                                           0.4672
## Xx3:Zz2
              -0.19346
                          0.17371 -1.114
                                          0.2654
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
```

```
##
       Null deviance: 1175.87 on 17 degrees of freedom
## Residual deviance: 848.65 on 10 degrees of freedom
## AIC: 942.57
## Number of Fisher Scoring iterations: 6
model <- glm(Freq ~ X + Y + Z + X*Y + Y*Z, data = df, family = poisson(link = "log"))</pre>
summary(model)
##
## glm(formula = Freq \sim X + Y + Z + X * Y + Y * Z, family = poisson(link = "log"),
##
       data = df)
##
## Deviance Residuals:
       Min
                  1Q
                        Median
                                      3Q
## -1.41824 -0.15293 -0.00015 0.11361
                                           1.29499
## Coefficients:
                Estimate Std. Error z value Pr(>|z|)
## (Intercept)
               4.84253 0.08806 54.990 <2e-16 ***
## Xx2
                -0.02335 0.12477 -0.187
                                            0.852
                          0.12020
## Xx3
                 0.12968
                                     1.079
                                              0.281
## Yy2
                -3.01425
                          0.25756 -11.703
                                             <2e-16 ***
## Yy3
                -2.07307
                          0.23638 -8.770
                                            <2e-16 ***
## Zz2
                -3.67630
                            0.32021 -11.481
                                            <2e-16 ***
## Xx2:Yy2
                 0.04089
                            0.18197
                                     0.225
                                              0.822
## Xx3:Yy2
                            0.17845 -0.532
                                             0.595
                -0.09489
## Xx2:Yy3
                 0.09004
                            0.28689 0.314
                                            0.754
               -21.10242 4034.85186 -0.005
                                             0.996
## Xx3:Yy3
## Yv2:Zz2
                 6.51876
                          0.39778 16.388
                                              <2e-16 ***
                 3.47563
                            0.41215 8.433
## Yy3:Zz2
                                            <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 1175.8734 on 17 degrees of freedom
## Residual deviance: 8.6798 on 6 degrees of freedom
## AIC: 110.6
## Number of Fisher Scoring iterations: 16
model <- glm(Freq ~ X + Y + Z + X*Z + Y*Z, data = df, family = poisson(link = "log"))</pre>
summary(model)
##
## Call:
## glm(formula = Freq \sim X + Y + Z + X * Z + Y * Z, family = poisson(link = "log"),
      data = df)
##
```

```
##
## Deviance Residuals:
      Min
                1Q
                     Median
                                   30
## -4.7374 -0.5530 -0.0608
                                        2.8398
                              0.6330
##
## Coefficients:
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.83917
                           0.08537 56.687
                                             <2e-16 ***
## Xx2
               0.06104
                           0.11650
                                     0.524
                                              0.600
## Xx3
               0.06104
                           0.11650
                                     0.524
                                              0.600
## Yy2
              -3.03445
                           0.23487 -12.920
                                             <2e-16 ***
## Yy3
               -2.48238
                           0.18120 -13.699
                                             <2e-16 ***
## Zz2
              -3.57118
                           0.33509 -10.657
                                             <2e-16 ***
                                              0.467
## Xx2:Zz2
              -0.12506
                           0.17201 - 0.727
## Xx3:Zz2
                           0.17371 -1.114
                                              0.265
              -0.19346
## Yy2:Zz2
               6.51876
                           0.39778 16.388
                                             <2e-16 ***
## Yy3:Zz2
               3.47563
                           0.41215
                                    8.433
                                             <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
##
      Null deviance: 1175.873 on 17 degrees of freedom
## Residual deviance:
                       57.933 on 8 degrees of freedom
## AIC: 155.85
## Number of Fisher Scoring iterations: 5
```

The best model is conditional independence model (XY,YZ).

The estimated errors are high.

dropping the cells with zero counts.

```
df = df[df$Freq != 0,]
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z + Y*Z + X*Y*Z, data = df, family = poisson(link = "log"))
summary(model)
##
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + Y * Z + X *
       Y * Z, family = poisson(link = "log"), data = df)
##
##
## Deviance Residuals:
   [1] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##
## Coefficients: (2 not defined because of singularities)
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.828314
                          0.089443 53.982 < 2e-16 ***
## Xx2
              -0.008032
                          0.126746 -0.063
                                             0.9495
## Xx3
               0.155293
                          0.121858
                                     1.274
                                             0.2025
                          0.388403 -7.421 1.16e-13 ***
## Yy2
              -2.882404
```

```
## Yv3
               -2.430418
                           0.314498 -7.728 1.09e-14 ***
## Zz2
                           0.456070 -7.058 1.69e-12 ***
               -3.218876
               -0.146119
## Xx2:Yy2
                           0.570603 -0.256
                                              0.7979
## Xx3:Yy2
               -0.309444
                           0.569538 -0.543
                                              0.5869
## Xx2:Yy3
                0.701179
                           0.390420
                                     1.796
                                              0.0725
## Xx3:Yy3
                      NA
                                 NA
                                         NA
                                                  NA
                           0.741214 -0.678
## Xx2:Zz2
               -0.502793
                                              0.4976
                           0.845487 -1.267
## Xx3:Zz2
               -1.071584
                                              0.2050
## Yy2:Zz2
                5.936405
                           0.600243
                                     9.890 < 2e-16 ***
## Yy3:Zz2
                3.711352
                           0.595369
                                    6.234 4.56e-10 ***
## Xx2:Yy2:Zz2 0.684853
                           0.936766
                                    0.731
                                              0.4647
                                     1.245
## Xx3:Yy2:Zz2 1.271825
                           1.021184
                                              0.2130
## Xx2:Yy3:Zz2 -0.883501
                           0.923270 -0.957
                                              0.3386
## Xx3:Yy3:Zz2
                                 NA
                                         NA
                                                  NA
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 9.8506e+02 on 15 degrees of freedom
## Residual deviance: 1.3545e-14 on 0 degrees of freedom
## AIC: 109.92
##
## Number of Fisher Scoring iterations: 3
model \leftarrow glm(Freq \sim X + Y + Z + X*Y + X*Z + Y*Z, data = df, family = poisson(link = "log")
summary(model)
##
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + Y * Z, family = poisson(link = "log"),
       data = df)
##
##
## Deviance Residuals:
          1
                    2
                              3
                                        4
                                                  5
                                                            6
                                                                      7
## -0.03739 -0.05946
                        0.08975
                                  0.19267
                                            0.41545 -0.65888
                                                                1.30773
          8
                    9
                             10
                                       11
                                                 12
                                                           13
                                                                     14
                      -0.28057
                                  0.17658
                                            0.10301 -0.70228
## -0.68341
            -0.41758
                                                                0.55381
##
         16
                   17
##
   0.61805 -0.76566
##
## Coefficients: (1 not defined because of singularities)
##
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.83166
                           0.08869 54.481 < 2e-16 ***
               -0.00604
                           0.12523
                                    -0.048
## Xx2
                                             0.9615
## Xx3
                0.14451
                           0.12087
                                     1.196
                                            0.2319
## Yy2
                           0.36087
                                    -9.489 < 2e-16 ***
               -3.42432
## Yv3
               -2.22923
                           0.26518
                                    -8.407 < 2e-16 ***
## Zz2
               -3.30964
                           0.37304 -8.872 < 2e-16 ***
## Xx2:Yy2
                0.65707
                           0.37818
                                     1.737
                                             0.0823 .
## Xx3:Yy2
                0.40568
                           0.44897
                                     0.904
                                            0.3662
                0.37422
                           0.32803
                                    1.141
                                             0.2540
## Xx2:Yy3
## Xx3:Yy3
                                NA
                                        NA
                                                 NΑ
                     NA
```

```
## Xx2:Zz2
             -0.66721
                          0.35779 -1.865 0.0622 .
## Xx3:Zz2
             -0.54102
                          0.44503 -1.216 0.2241
              6.59287
## Yy2:Zz2
                          0.40584 16.245 < 2e-16 ***
               3.44829
                                  7.942 1.98e-15 ***
## Yy3:Zz2
                          0.43416
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 985.065 on 15 degrees of freedom
## Residual deviance: 4.897 on 3 degrees of freedom
## AIC: 108.82
## Number of Fisher Scoring iterations: 4
model <- glm(Freq ~ X + Y + Z, data = df, family = poisson(link = "log"))</pre>
summary(model)
##
## Call:
## glm(formula = Freq ~ X + Y + Z, family = poisson(link = "log"),
       data = df
##
## Deviance Residuals:
       Min 1Q
                       Median
                                      3Q
                                              Max
## -10.5066 -9.0970 -0.0544
                                 5.8943
                                           7.1901
## Coefficients:
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.27850 0.07773 55.044 < 2e-16 ***
## Xx2
              0.00367
                          0.08567
                                   0.043 0.96583
## Xx3
                                   1.034 0.30131
               0.09057
                          0.08762
## Yv2
              -0.16034
                          0.07326 -2.189 0.02863 *
                          0.14094 -10.456 < 2e-16 ***
## Yy3
              -1.47362
## Zz2
              -0.20816
                          0.07065 -2.946 0.00322 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 985.06 on 15 degrees of freedom
##
## Residual deviance: 800.08 on 10 degrees of freedom
## AIC: 890
## Number of Fisher Scoring iterations: 6
model <- glm(Freq ~ X + Y + Z + X*Y, data = df, family = poisson(link = "log"))</pre>
summary(model)
##
```

Call:

```
## glm(formula = Freq ~ X + Y + Z + X * Y, family = poisson(link = "log"),
##
       data = df
##
## Deviance Residuals:
                 1Q
                     Median
                                   3Q
                                           Max
## -10.707
           -9.117
                     -0.097
                                5.952
                                         7.026
## Coefficients: (1 not defined because of singularities)
               Estimate Std. Error z value Pr(>|z|)
                          0.09325 45.825 < 2e-16 ***
## (Intercept) 4.27306
## Xx2
               -0.02335
                           0.12477 -0.187 0.85156
                                    1.079 0.28067
## Xx3
               0.12968
                           0.12020
## Yv2
               -0.14015
                           0.12862 -1.090 0.27586
                           0.20537 -7.305 2.77e-13 ***
## Yy3
               -1.50024
## Zz2
               -0.20816
                           0.07065 -2.946 0.00322 **
## Xx2:Yy2
               0.04089
                           0.18197
                                    0.225 0.82220
               -0.09489
                           0.17845
                                   -0.532 0.59490
## Xx3:Yy2
## Xx2:Yv3
               0.09004
                           0.28689
                                     0.314 0.75364
## Xx3:Yy3
                                       NA
                                NΑ
                                                 NΑ
                    NΑ
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
       Null deviance: 985.06 on 15 degrees of freedom
##
## Residual deviance: 799.40 on 7 degrees of freedom
## AIC: 895.32
## Number of Fisher Scoring iterations: 6
# joint independece model (XZ,Y)
model <- glm(Freq ~ X + Y + Z + X*Z, data = df, family = poisson(link = "log"))</pre>
summary(model)
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Z, family = poisson(link = "log"),
       data = df
##
## Deviance Residuals:
        \mathtt{Min}
                   1Q
                         Median
                                       3Q
                                                Max
## -10.2161
            -9.2452
                       -0.1456
                                   6.1503
                                             7.2805
##
## Coefficients:
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.23002
                           0.09141 46.275
                                            <2e-16 ***
## Xx2
                0.06104
                           0.11650
                                    0.524
                                             0.6003
## Xx3
               0.17767
                           0.11747
                                    1.513
                                            0.1304
## Yv2
               -0.16034
                           0.07326 - 2.189
                                            0.0286 *
                           0.14094 -10.456
## Yy3
               -1.47362
                                             <2e-16 ***
## Zz2
               -0.10303
                           0.12143 -0.848
                                             0.3962
## Xx2:Zz2
              -0.12506
                           0.17201 -0.727
                                             0.4672
## Xx3:Zz2
               -0.19346
                           0.17371 -1.114
                                            0.2654
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 985.06 on 15 degrees of freedom
## Residual deviance: 798.80 on 8 degrees of freedom
## AIC: 892.72
## Number of Fisher Scoring iterations: 6
# joint independece model (YZ,X)
model <- glm(Freq ~ X + Y + Z + Y*Z, data = df, family = poisson(link = "log"))</pre>
summary(model)
##
## Call:
## glm(formula = Freq ~ X + Y + Z + Y * Z, family = poisson(link = "log"),
       data = df
##
## Deviance Residuals:
        Min
              1Q
                                      3Q
                         Median
                                               Max
## -1.43514 -0.36146 -0.09861
                                0.39213
##
## Coefficients:
##
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.84798
                          0.07143 67.873 <2e-16 ***
## Xx2
               0.00367
                          0.08567
                                   0.043
                                             0.966
## Xx3
               0.09057
                          0.08762
                                   1.034
                                             0.301
## Yy2
               -3.03445
                          0.23487 -12.920
                                            <2e-16 ***
## Yy3
               -2.04645
                          0.18320 -11.170
                                           <2e-16 ***
## Zz2
               -3.67630
                          0.32021 -11.481
                                            <2e-16 ***
                          0.39778 16.388 <2e-16 ***
## Yy2:Zz2
               6.51876
## Yy3:Zz2
               3.47563
                          0.41215
                                   8.433
                                           <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 985.0649 on 15 degrees of freedom
## Residual deviance: 9.3619 on 8 degrees of freedom
## AIC: 103.28
##
## Number of Fisher Scoring iterations: 4
model \leftarrow glm(Freq \sim X + Y + Z + X*Z + X*Z, data = df, family = poisson(link = "log"))
summary(model)
##
## Call:
## glm(formula = Freq \sim X + Y + Z + X * Z + X * Z, family = poisson(link = "log"),
       data = df
##
```

```
## Deviance Residuals:
##
       Min 1Q Median
                                     30
                                              Max
                                           7.2805
## -10.2161 -9.2452 -0.1456
                                 6.1503
##
## Coefficients:
              Estimate Std. Error z value Pr(>|z|)
##
## (Intercept) 4.23002
                        0.09141 46.275 <2e-16 ***
                                  0.524
## Xx2
              0.06104
                          0.11650
                                           0.6003
                                  1.513 0.1304
## Xx3
              0.17767
                          0.11747
## Yy2
              -0.16034
                          0.07326 -2.189 0.0286 *
## Yy3
              -1.47362
                          0.14094 -10.456 <2e-16 ***
                          0.12143 -0.848
              -0.10303
                                          0.3962
## Zz2
## Xx2:Zz2
              -0.12506
                          0.17201 -0.727
                                          0.4672
## Xx3:Zz2
                          0.17371 -1.114 0.2654
             -0.19346
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 985.06 on 15 degrees of freedom
## Residual deviance: 798.80 on 8 degrees of freedom
## AIC: 892.72
##
## Number of Fisher Scoring iterations: 6
# cond independece model (XY,XZ)
model \leftarrow glm(Freq \sim X + Y + Z + X*Z + X*Z, data = df, family = poisson(link = "log"))
summary(model)
##
## glm(formula = Freq \sim X + Y + Z + X * Z + X * Z, family = poisson(link = "log"),
       data = df)
##
##
## Deviance Residuals:
       \mathtt{Min}
                  1Q
                        Median
                                     3Q
                                              Max
## -10.2161 -9.2452 -0.1456
                                           7.2805
                                  6.1503
## Coefficients:
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.23002 0.09141 46.275 <2e-16 ***
## Xx2
              0.06104
                          0.11650
                                  0.524 0.6003
## Xx3
              0.17767
                          0.11747
                                  1.513
                                           0.1304
## Yy2
              -0.16034
                          0.07326 -2.189 0.0286 *
## Yy3
              -1.47362
                          0.14094 -10.456
                                          <2e-16 ***
## Zz2
              -0.10303
                          0.12143 -0.848
                                           0.3962
## Xx2:Zz2
              -0.12506
                          0.17201 - 0.727
                                           0.4672
## Xx3:Zz2
              -0.19346
                          0.17371 -1.114
                                          0.2654
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 985.06 on 15 degrees of freedom
##
```

```
## Residual deviance: 798.80 on 8 degrees of freedom
## AIC: 892.72
##
## Number of Fisher Scoring iterations: 6
# cond independece model (XY,YZ)
model \leftarrow glm(Freq \sim X + Y + Z + X*Y + Y*Z, data = df, family = poisson(link = "log"))
summary(model)
##
## Call:
## glm(formula = Freq \sim X + Y + Z + X * Y + Y * Z, family = poisson(link = "log"),
       data = df
##
##
## Deviance Residuals:
                                                  5
         1
## -0.15936
                        0.13743
                                  0.92293 -0.07725 -0.94741
              0.01220
                                                                0.30523
##
                    9
                             10
                                                 12
          8
                                       11
## -0.13364
            -0.17677 -0.07527
                                  0.03196
                                            0.04215 -1.31362
                                                                1.14685
         16
                   17
## 1.29499 -1.41824
## Coefficients: (1 not defined because of singularities)
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.84253
                          0.08806 54.990
                                            <2e-16 ***
## Xx2
              -0.02335
                           0.12477 -0.187
                                             0.852
## Xx3
                                   1.079
                                             0.281
               0.12968
                           0.12020
## Yy2
              -3.01425
                           0.25756 -11.703
                                           <2e-16 ***
## Yy3
               -2.07307
                           0.23638 -8.770
                                            <2e-16 ***
                           0.32020 -11.481
## Zz2
              -3.67630
                                           <2e-16 ***
## Xx2:Yy2
              0.04089
                           0.18197
                                    0.225
                                            0.822
## Xx3:Yy2
               -0.09489
                           0.17845 -0.532
                                            0.595
## Xx2:Yv3
               0.09004
                           0.28689
                                     0.314
                                              0.754
## Xx3:Yy3
                                       NA
                     NA
                               NA
                                                NA
## Yy2:Zz2
                6.51876
                           0.39778 16.388
                                             <2e-16 ***
                                   8.433
## Yy3:Zz2
               3.47563
                           0.41215
                                            <2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 985.0649 on 15 degrees of freedom
## Residual deviance: 8.6798 on 5 degrees of freedom
## AIC: 108.6
## Number of Fisher Scoring iterations: 4
model <- glm(Freq ~ X + Y + Z + X*Z + Y*Z, data = df, family = poisson(link = "log"))
summary(model)
##
## Call:
```

```
## Deviance Residuals:
       \mathtt{Min}
                  1Q
                       Median
                                     3Q
                                              Max
## -1.32462 -0.45396
                      0.00099
                               0.42914
                                          1.15866
## Coefficients:
##
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.79723
                         0.08681 55.261
                                           <2e-16 ***
## Xx2
              0.06104
                         0.11650
                                  0.524
                                            0.600
## Xx3
               0.17967
                         0.11832
                                  1.519
                                            0.129
## Yy2
              -3.03445
                         0.23487 -12.920
                                          <2e-16 ***
              -2.02486
                         0.18503 -10.943
## Yy3
                                          <2e-16 ***
## Zz2
                         0.33585 -10.619
              -3.56624
                                           <2e-16 ***
## Xx2:Zz2
              -0.12506
                         0.17201 -0.727
                                            0.467
## Xx3:Zz2
              -0.19780
                         0.17631 -1.122
                                           0.262
## Yv2:Zz2
              6.51876
                         0.39778 16.388
                                          <2e-16 ***
              3.42805
                         0.41567
## Yy3:Zz2
                                  8.247
                                           <2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 985.0649 on 15 degrees of freedom
## Residual deviance:
                      8.0597 on 6 degrees of freedom
## AIC: 105.98
## Number of Fisher Scoring iterations: 4
b
delta1 \leftarrow c(0,0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0)
df <- data.frame(temp)</pre>
# saturated model
model \leftarrow glm(Freq \sim X + Y + Z + X*Y + X*Z + Y*Z + X*Y*Z + delta1 + delta2, data = df, family = poisson(
summary(model)
##
```

$glm(formula = Freq \sim X + Y + Z + X * Z + Y * Z, family = poisson(link = "log"),$

##

##

Call:

##

##

data = df)

Deviance Residuals:

data = df

glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + Y * Z + X *

[1] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Coefficients: (2 not defined because of singularities)

Y * Z + delta1 + delta2, family = poisson(link = "log"),

```
Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.828e+00 8.944e-02 53.982 < 2e-16 ***
## Xx2
               -8.032e-03 1.267e-01 -0.063
                                              0.9495
## Xx3
                1.553e-01 1.219e-01
                                      1.274
                                              0.2025
## Yy2
               -2.882e+00 3.884e-01 -7.421 1.16e-13 ***
               -2.430e+00 3.145e-01 -7.728 1.09e-14 ***
## Yy3
               -3.219e+00 4.561e-01 -7.058 1.69e-12 ***
## Zz2
## delta1
               -2.586e+01 6.965e+04
                                     0.000
                                              0.9997
## delta2
               -2.528e+01 6.965e+04
                                     0.000
                                              0.9997
## Xx2:Yy2
               -1.461e-01 5.706e-01 -0.256
                                             0.7979
## Xx3:Yy2
               -3.094e-01 5.695e-01 -0.543
                                             0.5869
                                             0.0725
## Xx2:Yy3
               7.012e-01 3.904e-01
                                      1.796
## Xx3:Yy3
                      NΑ
                                 NΑ
                                         NΑ
                                                  NA
## Xx2:Zz2
               -5.028e-01 7.412e-01
                                     -0.678
                                             0.4976
## Xx3:Zz2
                                     -1.267
                                              0.2050
               -1.072e+00 8.455e-01
## Yy2:Zz2
                5.936e+00 6.002e-01
                                      9.890 < 2e-16 ***
## Yy3:Zz2
                3.711e+00 5.954e-01
                                      6.234 4.56e-10 ***
## Xx2:Yy2:Zz2 6.849e-01
                          9.368e-01
                                      0.731
                                              0.4647
## Xx3:Yy2:Zz2 1.272e+00 1.021e+00
                                      1.245
                                              0.2130
## Xx2:Yy3:Zz2 -8.835e-01 9.233e-01
                                     -0.957
                                              0.3386
## Xx3:Yy3:Zz2
                      NA
                                 NA
                                         NA
                                                  NΑ
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 1.1759e+03 on 17 degrees of freedom
## Residual deviance: 3.0331e-10 on 0 degrees of freedom
## AIC: 113.92
##
## Number of Fisher Scoring iterations: 21
# homogenous model
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z + Y*Z + delta1 + delta2, data = df, family = poisson(link = "
summary(model)
##
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + Y * Z + delta1 +
##
       delta2, family = poisson(link = "log"), data = df)
##
## Deviance Residuals:
##
          1
                    2
                              3
                                                 5
                                                           6
                                                                     7
            -0.05946
                                                               1.30773
## -0.03739
                        0.08975
                                 0.19267
                                            0.41545
                                                   -0.65888
          8
                    9
                             10
                                                 12
                                       11
                       -0.28057
                                  0.17658
## -0.68341
            -0.41758
                                            0.10301 -0.70228
                                                               0.55381
                   16
                             17
         15
## -0.00009
             0.61805
                      -0.76566
                                -0.00009
## Coefficients: (1 not defined because of singularities)
                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                  4.83166
                             0.08869 54.481 < 2e-16 ***
                 -0.00604
                             0.12523 -0.048
## Xx2
                                             0.9615
```

1.196

0.2319

0.14451

Xx3

0.12087

```
## Yv2
                -3.42432
                            0.36088 -9.489 < 2e-16 ***
## Yy3
                -2.22923
                            0.26518 -8.407 < 2e-16 ***
## Zz2
                -3.30964
                            0.37304 -8.872 < 2e-16 ***
               -22.04952 9426.61685
                                    -0.002
## delta1
                                             0.9981
## delta2
               -21.64715 9426.61686 -0.002
                                             0.9982
## Xx2:Yy2
                 0.65707
                           0.37818
                                             0.0823 .
                                     1.737
## Xx3:Yy2
                 0.40568
                            0.44897 0.904
                                             0.3662
## Xx2:Yy3
                 0.37422
                            0.32803
                                              0.2540
                                     1.141
## Xx3:Yy3
                      NA
                                 NA
                                         NA
                                                  NA
## Xx2:Zz2
                -0.66721
                            0.35779 -1.865
                                              0.0622 .
## Xx3:Zz2
                -0.54102
                            0.44503 -1.216
                                              0.2241
                            0.40584 16.245 < 2e-16 ***
## Yy2:Zz2
                 6.59287
## Yy3:Zz2
                 3.44829
                            0.43416
                                     7.942 1.98e-15 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 1175.873 on 17 degrees of freedom
## Residual deviance:
                        4.897 on 3 degrees of freedom
## AIC: 112.82
## Number of Fisher Scoring iterations: 17
model <- glm(Freq ~ X + Y + Z, data = df, family = poisson(link = "log"))</pre>
summary(model)
##
## Call:
## glm(formula = Freq ~ X + Y + Z, family = poisson(link = "log"),
       data = df
##
## Deviance Residuals:
                  1Q
                        Median
        Min
                                      ЗQ
                                               Max
                                            7.4939
## -10.0596 -9.1703
                       -0.0282
                                  5.2241
##
## Coefficients:
              Estimate Std. Error z value Pr(>|z|)
                          0.07690 56.153 < 2e-16 ***
## (Intercept) 4.31818
## Xx2
               0.00367
                          0.08567
                                   0.043 0.96583
## Xx3
              -0.02607
                          0.08631 -0.302 0.76260
## Yy2
                          0.07326 -2.189 0.02863 *
               -0.16034
              -1.90954
## Yy3
                          0.13833 -13.804 < 2e-16 ***
## Zz2
              -0.20816
                          0.07065 -2.946 0.00322 **
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 1175.87 on 17 degrees of freedom
## Residual deviance: 849.93 on 12 degrees of freedom
## AIC: 939.85
##
```

```
## Number of Fisher Scoring iterations: 6
```

```
# joint indep model (XY,Z)
model <- glm(Freq ~ X + Y + Z + X*Y + delta1 + delta2, data = df, family = poisson(link = "log"))
summary(model)
##
## Call:
\#\# glm(formula = Freq \sim X + Y + Z + X * Y + delta1 + delta2, family = poisson(link = "log"),
       data = df
##
##
## Deviance Residuals:
        Min
                   1Q
                         Median
                                       3Q
                                                Max
## -10.7073
              -9.0275
                        -0.0011
                                   5.7728
                                             7.0259
##
## Coefficients: (1 not defined because of singularities)
                Estimate Std. Error z value Pr(>|z|)
##
## (Intercept)
                4.27306
                            0.09325 45.825 < 2e-16 ***
## Xx2
                -0.02335
                            0.12477 -0.187 0.85156
## Xx3
                0.12968
                            0.12020
                                     1.079 0.28067
## Yy2
                            0.12862 -1.090 0.27586
                -0.14015
                -1.50024
                                    -7.305 2.77e-13 ***
## Yy3
                            0.20537
                -0.20816
## Zz2
                            0.07065 -2.946 0.00322 **
## delta1
               -17.20508 773.78386 -0.022 0.98226
## delta2
               -16.99693 773.78386 -0.022 0.98248
## Xx2:Yy2
                 0.04089
                            0.18197
                                      0.225 0.82220
## Xx3:Yy2
                -0.09489
                            0.17845 -0.532 0.59490
## Xx2:Yy3
                 0.09004
                            0.28689
                                     0.314 0.75364
## Xx3:Yy3
                      NA
                                 NA
                                         NA
                                                  NA
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 1175.9 on 17 degrees of freedom
## Residual deviance: 799.4 on 7 degrees of freedom
## AIC: 899.32
##
## Number of Fisher Scoring iterations: 12
model <- glm(Freq ~ X + Y + Z + X*Z+ delta1 + delta2, data = df, family = poisson(link = "log"))
summary(model)
##
## glm(formula = Freq ~ X + Y + Z + X * Z + delta1 + delta2, family = poisson(link = "log"),
##
       data = df
##
## Deviance Residuals:
##
        Min
                   1Q
                         Median
                                       3Q
                                                Max
## -10.2161 -9.0303
                        -0.0011
                                   5.9168
                                             7.2805
##
```

```
## Coefficients:
##
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.23002
                           0.09141 46.275
                                             <2e-16 ***
## Xx2
                0.06104
                           0.11650
                                    0.524
                                             0.6003
## Xx3
                0.17767
                           0.11747
                                     1.513
                                             0.1304
## Yy2
               -0.16034
                           0.07326 - 2.189
                                            0.0286 *
                           0.14094 -10.456
## Yy3
               -1.47362
                                             <2e-16 ***
                           0.12143 -0.848
## Zz2
               -0.10303
                                             0.3962
## delta1
              -17.23666 773.78385 -0.022
                                             0.9822
## delta2
              -16.94017 773.78385 -0.022
                                             0.9825
## Xx2:Zz2
               -0.12506
                           0.17201 -0.727
                                             0.4672
               -0.19346
                           0.17371 -1.114 0.2654
## Xx3:Zz2
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 1175.9 on 17 degrees of freedom
## Residual deviance: 798.8 on 8 degrees of freedom
## AIC: 896.72
##
## Number of Fisher Scoring iterations: 12
model <- glm(Freq ~ X + Y + Z + Y*Z+ delta1 + delta2, data = df, family = poisson(link = "log"))</pre>
summary(model)
##
## Call:
  glm(formula = Freq ~ X + Y + Z + Y * Z + delta1 + delta2, family = poisson(link = "log"),
##
       data = df)
##
## Deviance Residuals:
                  1Q
                        Median
                                               Max
## -1.43514 -0.33344 -0.03137
                                0.33929
                                           1.27981
##
## Coefficients:
                Estimate Std. Error z value Pr(>|z|)
                          0.07143 67.873
                                             <2e-16 ***
## (Intercept)
                 4.84798
## Xx2
                 0.00367
                            0.08567
                                      0.043
                                               0.966
## Xx3
                 0.09057
                            0.08762
                                      1.034
                                               0.301
## Yy2
                -3.03445
                            0.23487 -12.920
                                             <2e-16 ***
                                              <2e-16 ***
## Yy3
                -2.04645
                            0.18320 - 11.170
## Zz2
                -3.67630
                            0.32021 -11.481
                                              <2e-16 ***
## delta1
               -21.19467 5717.53214 -0.004
                                              0.997
                -20.99400 5717.53214 -0.004
                                               0.997
## delta2
## Yy2:Zz2
                 6.51876
                            0.39778 16.388
                                              <2e-16 ***
## Yy3:Zz2
                 3.47563
                            0.41215
                                      8.433
                                              <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 1175.8734 on 17 degrees of freedom
##
```

```
## AIC: 107.28
##
## Number of Fisher Scoring iterations: 16
model <- glm(Freq ~ X + Y + Z + X*Y + X*Z+ delta1 + delta2, data = df, family = poisson(link = "log"))
summary(model)
##
## Call:
## glm(formula = Freq ~ X + Y + Z + X * Y + X * Z + delta1 + delta2,
       family = poisson(link = "log"), data = df)
##
##
## Deviance Residuals:
##
          1
                    2
                              3
                                         4
                                                   5
                                                             6
##
     6.1340
               5.7201
                         6.0097
                                  -9.3908
                                             -9.4333
                                                     -10.4121
                                                                 -8.6531
##
          8
                    9
                             10
                                       11
                                                  12
                                                            13
                                                                      14
                                                       -1.1449
##
   -9.3620
              -9.6563
                         6.3069
                                   7.0457
                                             7.4373
                                                                  1.0939
##
         15
                   16
                             17
                                       18
##
   -0.0011
               1.0926
                        -1.3654
                                  -0.0011
##
## Coefficients: (1 not defined because of singularities)
##
                Estimate Std. Error z value Pr(>|z|)
                            0.10492 40.264 < 2e-16 ***
## (Intercept)
                 4.22458
## Xx2
                 0.03402
                            0.14764
                                     0.230
                                               0.818
## Xx3
                 0.21679
                            0.14343
                                     1.511
                                               0.131
                            0.12862 -1.090
## Yy2
                -0.14015
                                               0.276
## Yy3
                -1.50024
                            0.20537 -7.305 2.77e-13 ***
                            0.12143 -0.848
## Zz2
                -0.10303
                                               0.396
## delta1
               -17.24371 773.78386 -0.022
                                               0.982
## delta2
               -16.94722 773.78387 -0.022
                                               0.983
## Xx2:Yy2
                 0.04089
                            0.18197
                                     0.225
                                               0.822
                -0.09489
                            0.17845 -0.532
## Xx3:Yy2
                                               0.595
## Xx2:Yy3
                 0.09004
                                     0.314
                                               0.754
                            0.28689
## Xx3:Yy3
                      NA
                                 NA
                                         NA
                                                   NA
## Xx2:Zz2
                -0.12506
                            0.17201 - 0.727
                                               0.467
## Xx3:Zz2
                -0.19346
                            0.17371 -1.114
                                               0.265
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 1175.87 on 17 degrees of freedom
## Residual deviance: 798.12 on 5 degrees of freedom
## AIC: 902.03
##
## Number of Fisher Scoring iterations: 12
# cond independece model (XY,YZ)
model <- glm(Freq ~ X + Y + Z + X*Y + Y*Z+ delta1 + delta2, data = df, family = poisson(link = "log"))
summary(model)
```

9.3619 on 8 degrees of freedom

Residual deviance:

```
##
## Call:
## glm(formula = Freq \sim X + Y + Z + X * Y + Y * Z + delta1 + delta2,
       family = poisson(link = "log"), data = df)
## Deviance Residuals:
          1
                               3
                                                   5
## -0.15936
              0.01220
                        0.13743
                                   0.92293
                                            -0.07725 -0.94741
                                                                 0.30523
##
          8
                    9
                             10
                                        11
                                                  12
                                                            13
                                                                      14
             -0.17677
                       -0.07527
                                   0.03196
## -0.13364
                                             0.04215 -1.31362
                                                                 1.14685
         15
                   16
                             17
                                        18
## -0.00015
              1.29499
                       -1.41824
                                  -0.00015
## Coefficients: (1 not defined because of singularities)
                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                  4.84253
                             0.08806 54.990
                                                <2e-16 ***
## Xx2
                 -0.02335
                             0.12477 -0.187
                                                 0.852
## Xx3
                  0.12968
                             0.12020
                                        1.079
                                                 0.281
## Yy2
                 -3.01425
                             0.25756 -11.703
                                                <2e-16 ***
## Yy3
                 -2.07307
                             0.23638 -8.770
                                                <2e-16 ***
## Zz2
                 -3.67630
                             0.32021 -11.481
                                                <2e-16 ***
## delta1
                -21.20172 5717.53214 -0.004
                                                0.997
## delta2
                -21.00105 5717.53214 -0.004
                                                 0.997
## Xx2:Yy2
                  0.04089
                             0.18197
                                       0.225
                                                 0.822
## Xx3:Yy2
                 -0.09489
                             0.17845 - 0.532
                                                 0.595
## Xx2:Yy3
                  0.09004
                             0.28689
                                       0.314
                                                0.754
## Xx3:Yy3
                                                    NA
                       NA
                                   NA
                                           NA
                  6.51876
                              0.39778
                                      16.388
                                                <2e-16 ***
## Yy2:Zz2
## Yy3:Zz2
                  3.47563
                             0.41215
                                       8.433
                                                <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 1175.8734 on 17 degrees of freedom
## Residual deviance:
                         8.6798 on 5 degrees of freedom
## AIC: 112.6
##
## Number of Fisher Scoring iterations: 16
# cond independece model (XZ,YZ)
model <- glm(Freq ~ X + Y + Z + X*Z + Y*Z+ delta1 + delta2, data = df, family = poisson(link = "log"))</pre>
summary(model)
##
## Call:
## glm(formula = Freq \sim X + Y + Z + X * Z + Y * Z + delta1 + delta2,
       family = poisson(link = "log"), data = df)
##
##
## Deviance Residuals:
        Min
                   1Q
                         Median
                                                 Max
                                        30
## -1.32462 -0.41394 -0.00015
                                  0.39802
                                             1.15866
## Coefficients:
```

```
Estimate Std. Error z value Pr(>|z|)
                4.79723 0.08681 55.261
## (Intercept)
                                           <2e-16 ***
                           0.11650 0.524
                                             0.600
## Xx2
                 0.06104
                                             0.129
## Xx3
                 0.17967
                           0.11832
                                    1.519
## Yy2
               -3.03445
                           0.23487 -12.920
                                            <2e-16 ***
               -2.02486
                         0.18503 -10.943
## Yy3
                                           <2e-16 ***
## Zz2
               -3.56624
                         0.33585 -10.618
                                           <2e-16 ***
             -21.25463 5717.53214 -0.004
## delta1
                                             0.997
              -20.91864 5717.53214 -0.004
## delta2
                                             0.997
## Xx2:Zz2
               -0.12506
                          0.17201 -0.727
                                             0.467
## Xx3:Zz2
               -0.19780
                           0.17631 -1.122
                                             0.262
                           0.39778 16.388
                                            <2e-16 ***
## Yy2:Zz2
                 6.51876
## Yy3:Zz2
                 3.42805
                           0.41567 8.247
                                           <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for poisson family taken to be 1)
##
      Null deviance: 1175.8734 on 17 degrees of freedom
## Residual deviance:
                       8.0597 on 6 degrees of freedom
## AIC: 109.98
## Number of Fisher Scoring iterations: 16
```

Problem 2

```
temp <- read.ftable(textConnection("TonsilSize Normal Slightl-enlarged Very-enlarged
CarrierCond
Carrier 19 29 24
Non-Carrier 497 560 269"))

df <- data.frame(temp)

options(contrast=c("contr.treatment","contr.poly"))

#independece model
model <- glm(Freq ~ CarrierCond + TonsilSize, data = df, family = poisson(link = "log"))
summary(model)</pre>
```

```
##
## Call:
## glm(formula = Freq ~ CarrierCond + TonsilSize, family = poisson(link = "log"),
## data = df)
##
## Deviance Residuals:
## 1 2 3 4 5 6
## -1.54914 0.34153 -0.24416 0.05645 2.11019 -0.53736
##
## Coefficients:
## Estimate Std. Error z value Pr(>|z|)
```

```
## (Intercept)
                               3.27997
                                           0.12293 26.682 < 2e-16 ***
## CarrierCondNon-Carrier
                               2.91326
                                           0.12101
                                                    24.075 < 2e-16 ***
## TonsilSizeSlightl-enlarged 0.13232
                                           0.06030
                                                     2.194
                                                             0.0282 *
                                                   -7.737 1.02e-14 ***
## TonsilSizeVery-enlarged
                              -0.56593
                                           0.07315
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
   (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 1487.2173
                                 on 5
                                        degrees of freedom
## Residual deviance:
                         7.3209
                                 on 2
                                       degrees of freedom
## AIC: 53.992
##
## Number of Fisher Scoring iterations: 4
xCarrier \leftarrow rep(c(1,2), c(3,3))
ySize \leftarrow rep(c(1,2,3), 2)
model <- glm(Freq ~ CarrierCond + TonsilSize + xCarrier * ySize,</pre>
             data = df, family = poisson(link = "log"))
summary(model)
##
## Call:
## glm(formula = Freq ~ CarrierCond + TonsilSize + xCarrier * ySize,
##
       family = poisson(link = "log"), data = df)
##
## Deviance Residuals:
## [1] 0 0 0 0 0 0
##
## Coefficients: (1 not defined because of singularities)
##
                              Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                                5.7904
                                            0.9750
                                                     5.939 2.87e-09 ***
                                4.1116
                                                     8.003 1.22e-15 ***
## CarrierCondNon-Carrier
                                            0.5138
## TonsilSizeSlightl-enlarged
                                2.1178
                                            0.8353
                                                     2.535 0.01123 *
## TonsilSizeVery-enlarged
                                4.7746
                                            2.0690
                                                     2.308
                                                            0.02101 *
## xCarrier
                                -1.9985
                                            0.8424
                                                   -2.372 0.01767 *
## ySize
                                    NA
                                                NA
                                                        NA
                                                   -2.680 0.00737 **
## xCarrier:ySize
                               -0.8475
                                            0.3163
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
  (Dispersion parameter for poisson family taken to be 1)
##
##
       Null deviance: 1.4872e+03 on 5 degrees of freedom
## Residual deviance: -2.9754e-14 on 0 degrees of freedom
## AIC: 50.671
## Number of Fisher Scoring iterations: 3
the linear by linear model fits better compared to independet model.
```

both, hw4 and linear by linear model show that being a carrier and tonsil is related.

Problem 3

 \mathbf{a}

```
count <- c(425, 17, 80, 36, 10, 555, 74, 47, 7, 34, 771, 33, 5, 14, 29, 452)
at04 <- rep(c("N", "MW", "S", "W"), c(4,4,4,4))
at16 <- rep(c("N", "MW", "S", "W"), 4)
iN \leftarrow (at04 == "N") * (at16 == "N")
iMW <- (at04 == "MW") * (at16 == "MW")</pre>
iS <- (at04 == "S") * (at16 == "S")
iW \leftarrow (at04 == "W") * (at16 == "W")
at04_N <- (at04 == "N")
at04_MW <- (at04 == "MW")
at04_S <- (at04 == "S")
at04_W <- (at04 == "W")
at16_N <- (at16 == "N")
at16_MW <- (at16 == "MW")
at16_S <- (at16 == "S")
at16 W <- (at16 == "W")
symm1 <- 1 * (at04 == "N") * (at16 == "N")
symm2 <- 2 * (at04 == "MW") * (at16 == "MW")
symm3 < -3 * (at04 == "S") * (at16 == "S")
symm4 <- 4 * (at04 == "S") * (at16 == "S")
symm5 <- 5 * (at04 == "N") * (at16 == "MW") + 5 * (at04 == "MW") * (at16 == "N")
symm6 <- 6 * (at04 == "S") * (at16 == "N") + 6* (at04 == "N") *(at16 == "S")
symm7 < -7 * (at04 == "W") * (at16 == "N") + 7 * (at04 == "N") * (at16 == "W")
symm8 <- 8 * (at04 == "S")* (at16 == "MW")+ 8 * (at04 == "MW")* (at16 == "S")
symm9 < -9 * (at04 == "MW") * (at16 == "W") + 9 * (at04 == "W") * (at16 == "MW")
symm10 <- 10 *(at04 == "S") * (at16 == "W")+ 10 * (at04 == "W")* (at16 == "S")
symm=symm3+symm1+symm4+symm6+symm2+symm5+symm6 + symm7 + symm8 + symm9 + symm10
df <- data.frame(at04, at16, count, iN, iMW, iW , iS , symm)</pre>
df
```

```
##
     at04 at16 count iN iMW iW iS symm \,
## 1
      N
           N
               425
                   1
                       0 0 0
## 2
       N
          MW
                   0
                       0 0 0
                17
                                 5
## 3
      N
          S
                80
                   0
                       0 0 0
                                12
## 4
           W
                36 0
                       0 0 0
                                 7
      N
## 5
      MW
           N
                10 0
                       0 0 0
                                 5
## 6
      MW
               555 0
                       1 0 0
                                 2
         MW
## 7
      MW
          S
                74 0
                       0 0 0
                       0 0 0
## 8
      MW
                47 0
                                 9
           W
```

```
## 9
         S
                    7 0
                           0 0 0
                                     12
## 10
         S
             MW
                   34
                       0
                           0 0 0
                                      8
## 11
         S
              S
                  771
                                      7
                   33
## 12
         S
              W
                      0
                           0 0 0
                                     10
## 13
         W
             N
                   5
                      0
                           0 0 0
                                     7
## 14
         W
             MW
                   14
                      0
                           0 0 0
                                      9
## 15
                   29
                      0
                           0 0 0
         W
              S
                                     10
## 16
                  452 0
                           0 1 0
                                      0
```

symmetry model doesnot seem to be a good fit.

symmetry model

model=glm(count~symm1+symm4+symm6+symm2+symm5+symm3+symm7 + symm8+ symm9+ symm10,data=df,family=poisson summary(model)

```
##
## Call:
## glm(formula = count ~ symm1 + symm4 + symm6 + symm2 + symm5 +
       symm3 + symm7 + symm8 + symm9 + symm10, family = poisson(link = log),
##
       data = df)
##
## Deviance Residuals:
     Min
              1Q Median
                               3Q
                                      Max
## -6.886 -1.480 0.000
                                    4.948
                            1.330
##
## Coefficients: (1 not defined because of singularities)
##
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 6.11368
                          0.04704 129.979 < 2e-16 ***
## symm1
              -0.06159
                           0.06757 -0.912 0.36199
## symm4
               0.13350
                          0.01481
                                     9.014 < 2e-16 ***
## symm6
              -0.39015
                          0.01951 -19.995 < 2e-16 ***
## symm2
               0.10264
                           0.03168
                                     3.240 0.00119 **
              -0.70220
                           0.03962 -17.722 < 2e-16 ***
## symm5
## symm3
                     NA
                                NA
                                       NA
                                                 NA
              -0.44189
## symm7
                          0.02330 -18.965
                                           < 2e-16 ***
## symm8
              -0.26559
                           0.01339 -19.837
                                           < 2e-16 ***
              -0.29955
                           0.01516 -19.765 < 2e-16 ***
## symm9
              -0.26797
                           0.01354 -19.786 < 2e-16 ***
## symm10
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 4481.79 on 15 degrees of freedom
## Residual deviance: 134.45 on 6 degrees of freedom
## AIC: 246.96
## Number of Fisher Scoring iterations: 5
df = 6, G^2 = 134.45, p-value = 0
```

Quasi-symmetry Model

model=glm(count~at04_N + at04_W+ at04_MW+ at04_S + at16_N+ at16_W+ at16_MW+ at16_S +symm1+symm4+symm6+ssummary(model)

```
##
## glm(formula = count ~ at04_N + at04_W + at04_MW + at04_S + at16_N +
       at16_W + at16_MW + at16_S + symm1 + symm4 + symm6 + symm2 +
##
       symm5 + symm3 + symm7 + symm8 + symm9 + symm10, family = poisson(link = log),
##
       data = df
##
## Deviance Residuals:
##
          1
                              3
                                                   5
                                                             6
##
   0.00000
             -0.66161
                        0.40293
                                 -0.11169
                                            1.01493
                                                       0.00000
                                                                -0.45446
##
          8
                    9
                             10
                                       11
                                                  12
                                                            13
   0.16214
             -1.16490
                        0.71091
                                  0.00000
                                           -0.07427
                                                       0.31630
##
                                                                -0.28734
##
         15
              0.00000
##
  0.07997
##
## Coefficients: (6 not defined because of singularities)
              Estimate Std. Error z value Pr(>|z|)
## (Intercept) 0.74814
                           0.25878
                                     2.891 0.00384 **
## at04 NTRUE
               0.09267
                           0.21685
                                     0.427 0.66913
                                    15.957 < 2e-16 ***
## at04 WTRUE
                2.60427
                           0.16321
## at04 MWTRUE 0.31699
                           0.19401
                                     1.634 0.10227
## at04_STRUE
                     NA
                                NA
                                        NA
                                                 NA
                                    -6.435 1.23e-10 ***
## at16_NTRUE -1.88784
                           0.29337
## at16_WTRUE
                2.76127
                           0.17233
                                    16.023 < 2e-16 ***
## at16_MWTRUE -0.63753
                           0.21904
                                    -2.911 0.00361 **
## at16_STRUE
                     NA
                                NA
                                        NA
                                                 NA
## symm1
                7.09913
                           0.38605
                                    18.389 < 2e-16 ***
                                    22.580 < 2e-16 ***
## symm4
                1.47489
                           0.06532
## symm6
                                    14.105 < 2e-16 ***
                0.58264
                           0.04131
## symm2
                2.94568
                           0.14219
                                    20.717
                                            < 2e-16 ***
## symm5
                0.55724
                           0.06211
                                     8.971
                                            < 2e-16 ***
## symm3
                     NA
                                NA
                                        NA
                                                 NA
## symm7
                     NΑ
                                NA
                                        NA
                                                 NA
## symm8
                0.41141
                           0.02713
                                    15.164
                                            < 2e-16 ***
## symm9
                                NA
                                        NA
                                                 NA
                     NA
## symm10
                     NA
                                NA
                                        NA
                                                 NA
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
       Null deviance: 4481.7873 on 15 degrees of freedom
## Residual deviance:
                         3.9324 on 3 degrees of freedom
## AIC: 122.44
##
## Number of Fisher Scoring iterations: 4
df = 3, G^2 = 3.9324, p-value = 0.2688568
```

the quasi-symmetry model fits moderatlely well.

\mathbf{b}

$$G^2(marginal homogenety) = G^2(symmtery) - G^2(quasi-symmtery) = 130.518$$

$$df = 3$$
 p-value= 0\$

the marginal homgeneity model does not fit the data.