hw1

September 13, 2016

1 HW1 (S.Mottahedi)

2 Chapter 1

3 Problem 1

3.1 (a)

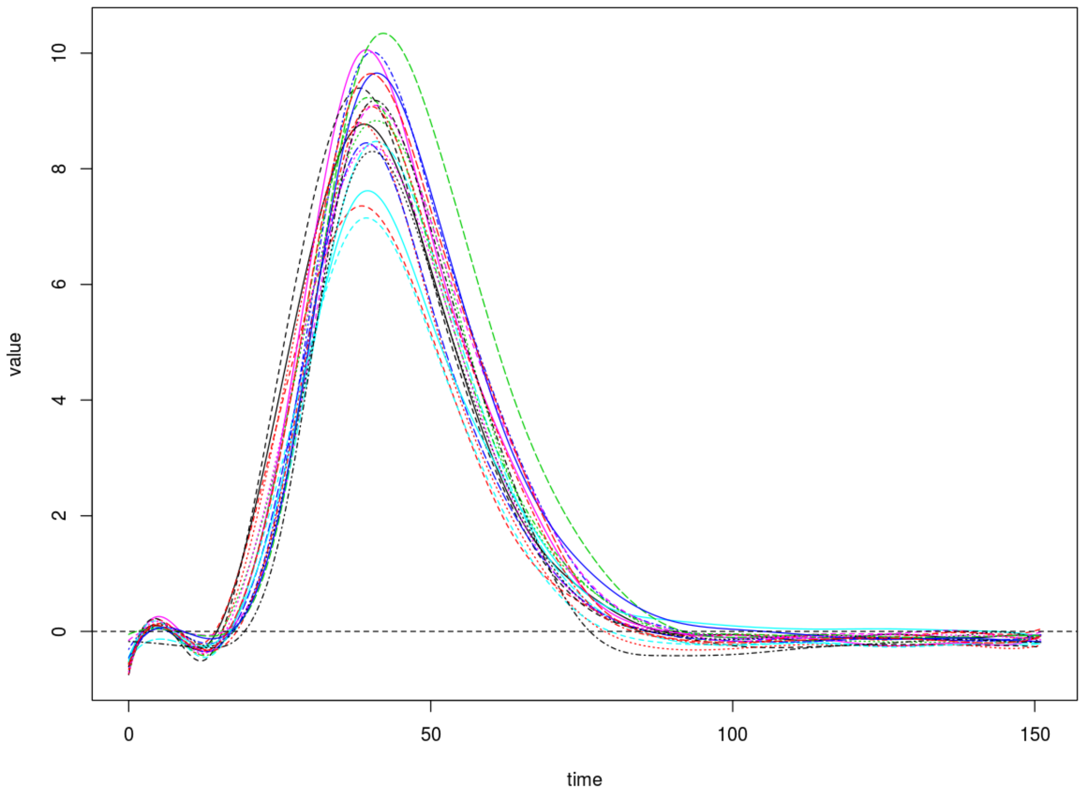
```
In [2]: library(fda)
        library(ggplot2)
        library(tidyr)
        library(dplyr)
        library(fds)

        options(repr.plot.width=10, repr.plot.height=8)
```

```
In [23]: df <- data.frame(pinch)
         names(df) <- 1:20
```

```
In [42]: bs.basis <- create.bspline.basis(rangeval=c(0, 151),
                                          nbasis=15, norder=4)
         pinch.fd = smooth.basis(y=pinch, fdParobj=bs.basis)
         # pinch.fd <- Data2fd(argvals=1:151, y=pinch, basisobj=bs.basis)
         plot(pinch.fd)
```

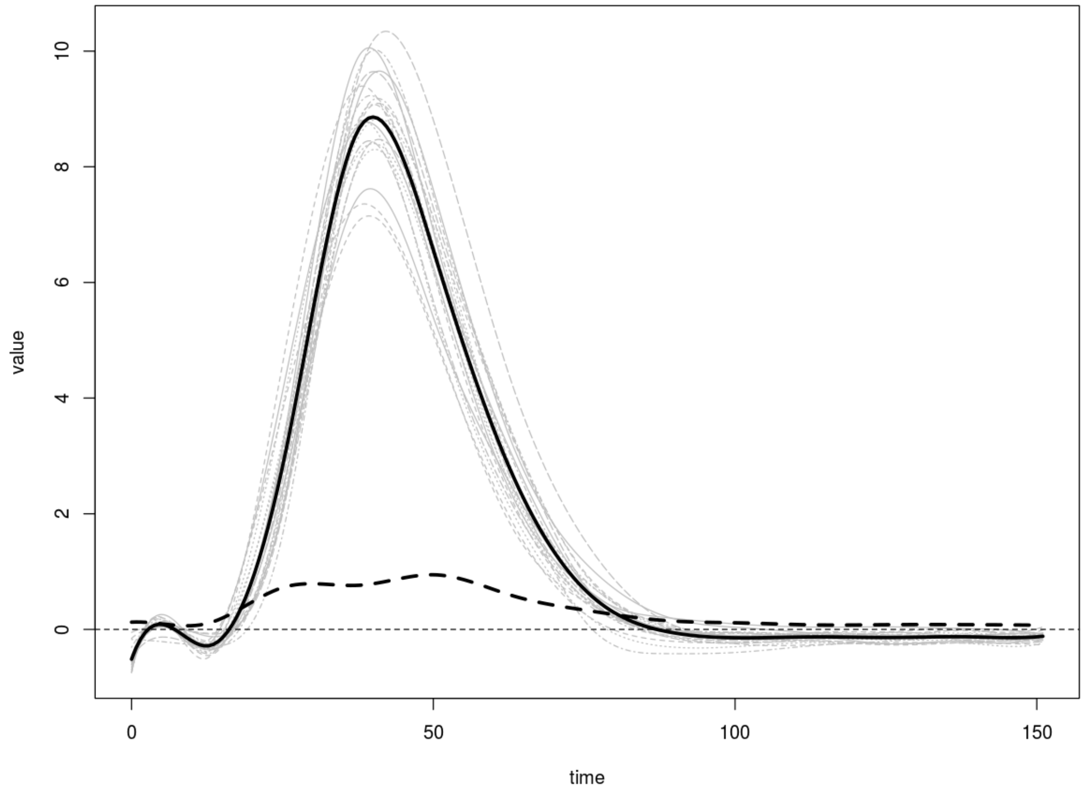
'done'



3.2 (b)

```
In [44]: pinch.mean = mean(pinch.fd$fd)
         pinch.sd = std.fd(pinch.fd$fd)
         plot(pinch.fd, col='gray')
         lines(pinch.mean, lwd=3)
         lines(pinch.sd, lty=2, lwd=3)
```

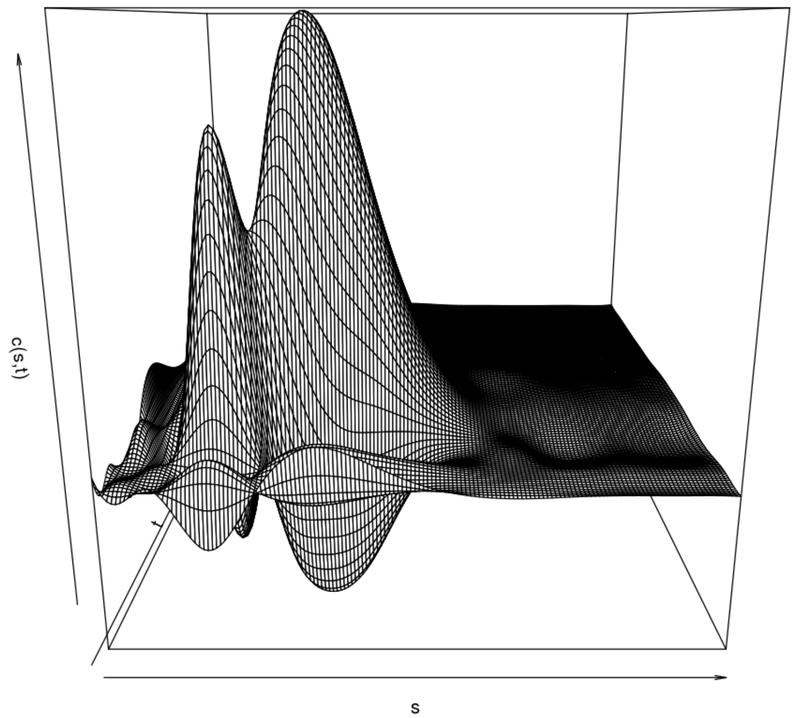
'done'



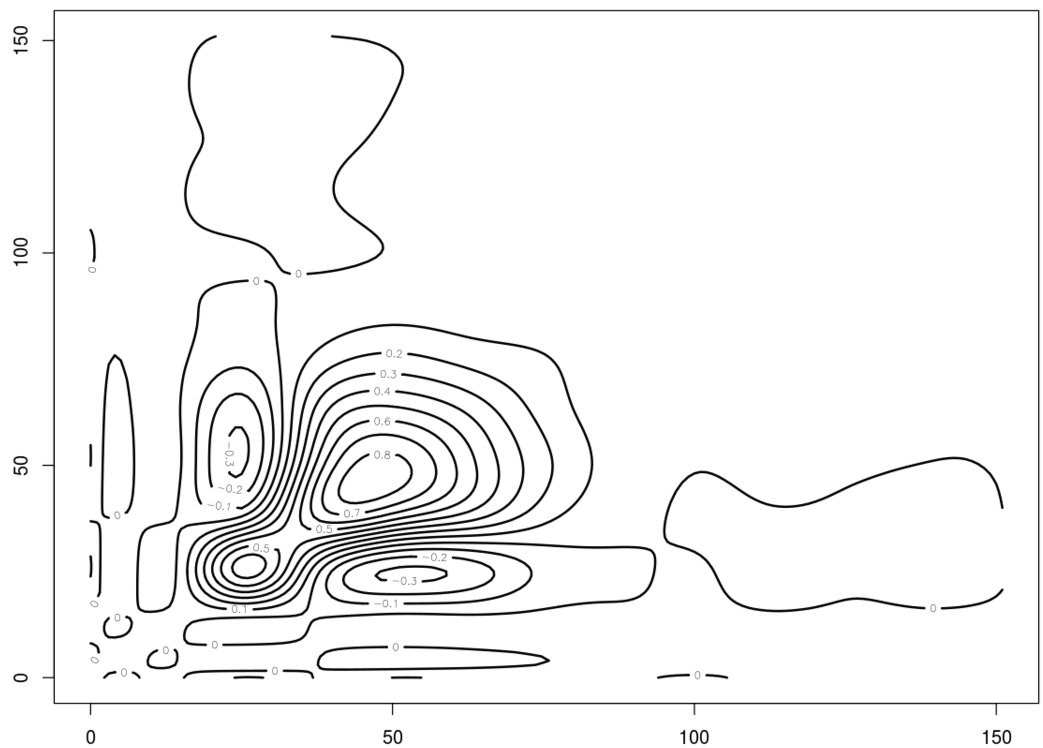
3.3 (c)

```
In [49]: pinch.cov <- var.fd(pinch.fd$fd)
         grid <- 0:151
         cov.mat <- eval.bifd(grid, grid, pinch.cov)

In [52]: persp(grid, grid, cov.mat,
               xlab='s', ylab='t', zlab='c(s,t)')
```



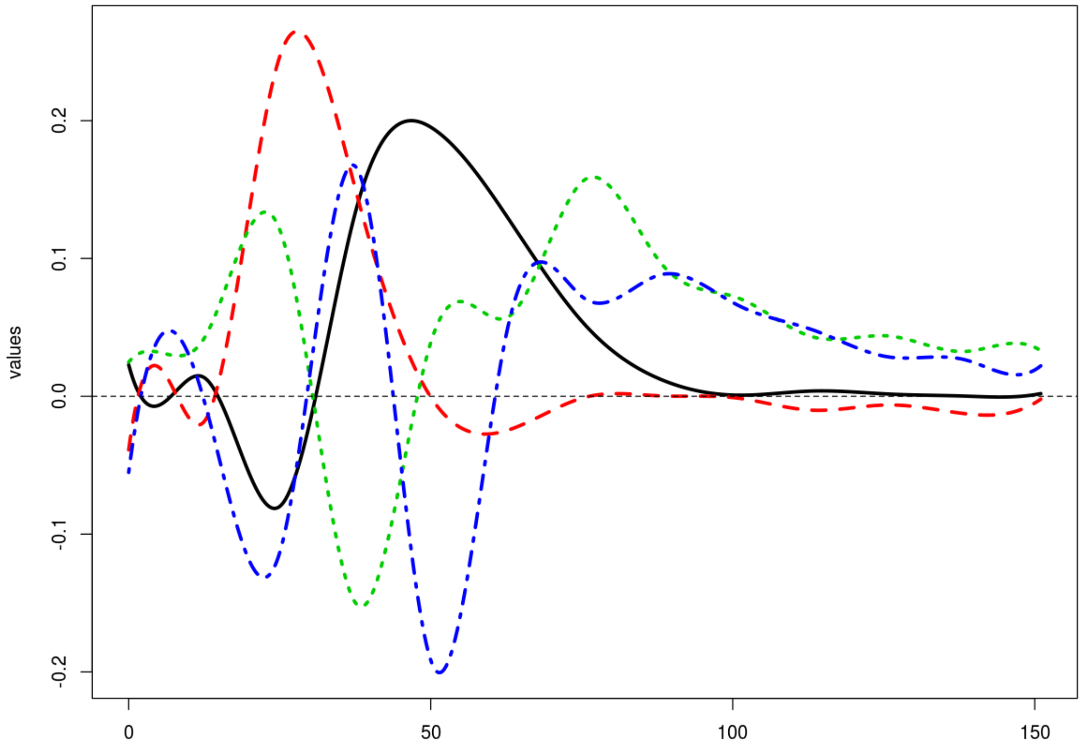

```
In [54]: contour(grid, grid, cov.mat, lwd=2)
```



3.4 (d)

```
In [57]: pinch.pca = pca.fd(pinch.fd$fd, nharm=4)
         plot(pinch.pca$harmonics, lwd=3)
```

'done'



the first two EFPC can explain 92% of variability.

```
In [61]: sum(pinch.pca$varprop[1:2])
```

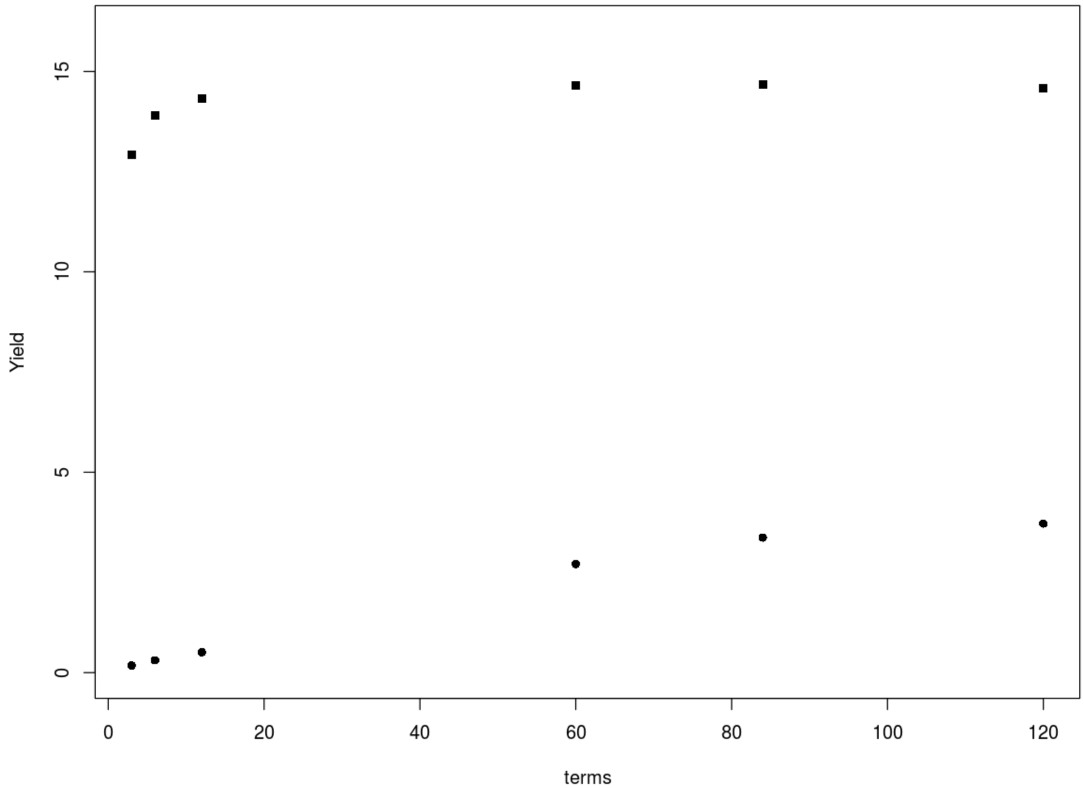
0.919562437559035

4 Problem 2

4.1 (a)

In general Tereseries yield is higher in 1982 compared to 2009. The 2009 curve is a $NormalYieldCurve$ which has higher return for long term investments and lower return for short term investments. This kind of yield curve is a sign of expansionary economic policies. The 1982 yield curve is $HumpedYieldCurve$, the highest rate of return is for 60 month investment rather than longer term maturities which is a sign of slowing economic growth.

```
In [10]: yield = FedYieldcurve; terms = yield$x
         plot(terms, yield$y[,1], pch=15, ylab="Yield", ylim=c(0,16))
         points(terms, yield$y[,330], pch=16)
```



4.2 (b)

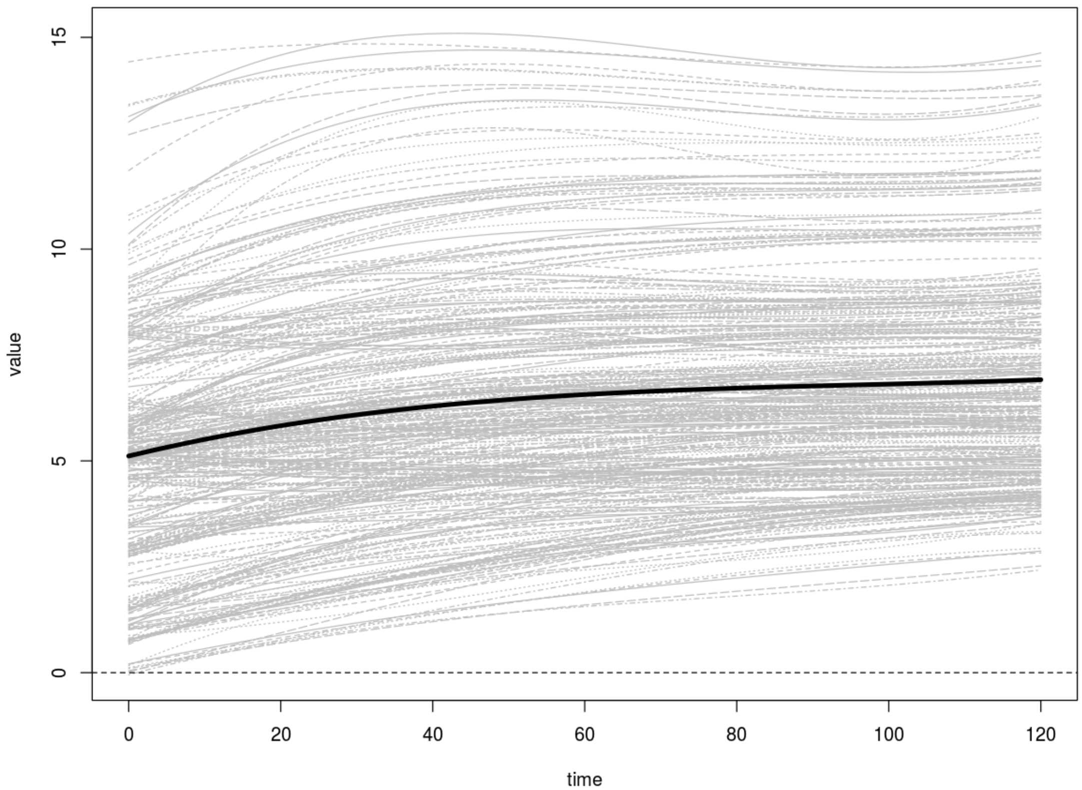
the average yield has a positive slope with lowest return for short and highest return for long term investments.

```
In [25]: bs.basis <- create.bspline.basis(rangeval=c(0, 120), nbasis=4)
         yield.fd = smooth.basis(y=yield$y, argvals=yield$x, fdParobj=bs.basis)

         yield.mean = mean(yield.fd$fd)

         plot(yield.fd, col='gray')
         lines(yield.mean, lwd = 4)
```

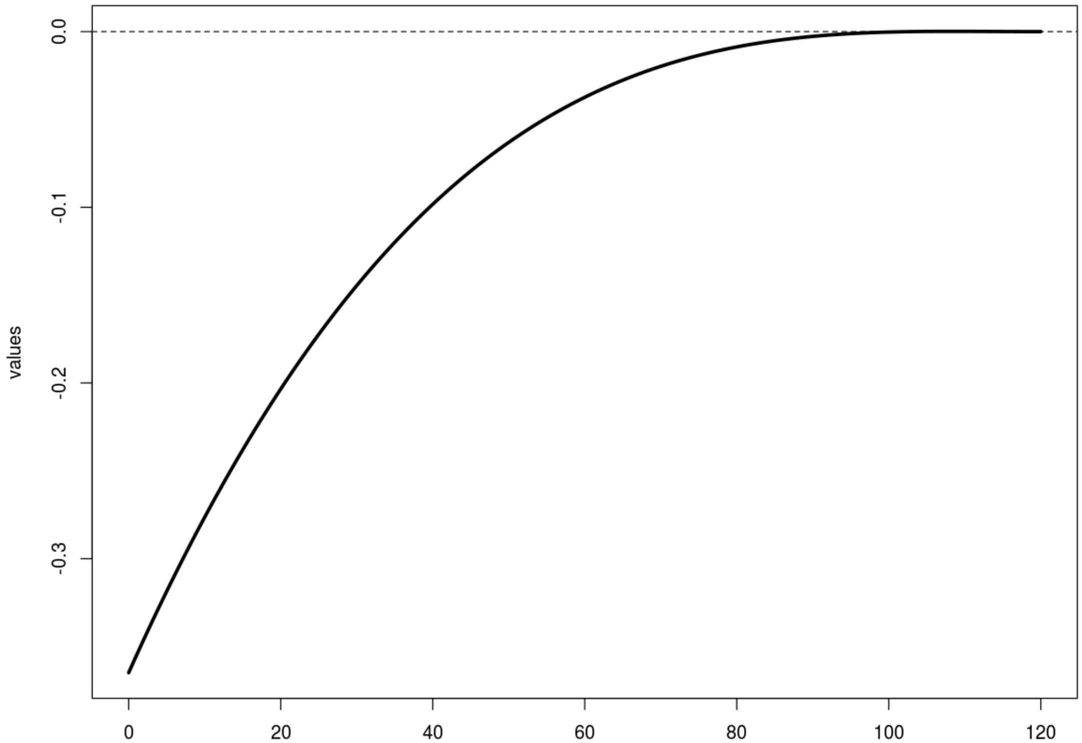
'done'



4.3 (c)

```
In [30]: yield.pca = pca.fd(yield.fd$fd, nharm=1, centerfns=T)
         plot(yield.pca$harmonics, lwd=3)
```

'done'



the first principle component explains 99.99% variability in the data. The first pricinple components shows that the yield increases with investment target maturity.

```
In [31]: yield.pca$varprop
```

0.999978266493213

5 Problem 6

$X_n(t) = \sum_m c_{nm} B_m(t)$

5.1 (a) $\bar{x}_{N(t)}(t) = \sum_{m=1}^{M} a_m B_m(t)$

$\bar{x}(t) := \frac{1}{N} \sum_{n=1}^{N} X_n(t)$
$\bar{x}(t) = \frac{1}{N} \sum_n \sum_m c_{mn} B_m(t) = \sum_m \bar{a}_m B_m(t)$
$\downarrow$
$\bar{a}_m = \frac{1}{N} \sum c_{nm}$

5.2 (b) $\hat{c}(t,s) = \sum_{m=1}^{M} \sum_{k=1}^{M} b_{mk} B_m(t) B_k(s)$

$\hat{C}(t,s) = \frac{1}{N-1} \sum_{n=1}^{N} (X_n(t) - \hat{\mu(t)})(X_n(s) - \hat{\mu}(s))$
$\tilde{c}_{nm} = c_{nm} - \bar{c}_m$
$\hat{C}(t,s) = \frac{1}{N-1} \sum_n \sum_{m1} \sum_{m2} \tilde{c}_{nm_1} \tilde{c}_{nm_2} B_{m_1}(t) B_{m_2}(s)$
$\hat{C}(t,s) = \frac{1}{N-1} \sum_{m1} \sum_{m2} (\tilde{c}^T \tilde{c})_{m_1,m_2} B_{m_1}(t) B_{m_2}(s)$
$= \sum_{m1} \sum_{m2} (\Sigma_c)_{m_1 m_2} B_{m_1}(t) B_{m_2}(s)$
$\downarrow$
$b_{mk} = (\tilde{c}^T \tilde{c})_{m_1,m_2} = (\Sigma_c)_{m_1 m_2}$

6 Chapter 2

7 Problem 1

Verify equality

$$\int_0^T [L(x)(t)]^2 dt = \pi \omega^5 \sum_{j=2}^{J} j^2 (j^2-1)^2 (a_j^2 + b_j^2) \quad (1)$$

$x_j(t) = c_0 + \sum_{j=1}^{J} [a_j + sin(\omega jt) + b_j cos(\omega jt)]$
$x^{(1)} = \sum_{j=1}^{J} [a_j \omega j + sin(\omega jt) + b_j \omega j cos(\omega jt)]$
$x^{(2)} = \sum_{j=1}^{J} [-a_j \omega^2 j^2 + sin(\omega jt) - b_j \omega^2 j^2 cos(\omega jt)]$
$x^{(3)} = \sum_{j=1}^{J} [-a_j \omega^3 j^3 + sin(\omega jt) + b_j \omega^3 j^3 cos(\omega jt)]$
$L(x)(t) = \sum \omega^3 (j^2-1)[bsin(\omega jt) - acos(\omega jt)]$
$[L(x)(t)]^2 = \sum \omega^5 (j^2-1)^2 [b^2 sin^2(\omega jt) + a^2 cos^2(\omega jt) - 2abcos(\omega jt) sin(\omega jt)]$
$\int_0^T [L(x)(t)]^2 dt = \pi \omega^2 \sum j^2 (j^2-1)^2 (a_j^2 + b_j^2)$

8 Problem 2

9 Problem 5