θ 1.3

$x \to$ this is the variable
$z \to$ latent factors related to x
(conditioning variable)

The joint probility –

$$P(x, z) = p(x|z) \cdot P(z)$$
$$= P(z|x) \cdot P(x)$$
$$P(z|x) = \frac{p(x|z) \cdot p(z)}{p(x)}$$

Now, "prior" of $p(z) = \pi_z$ {marginal distribution}

Likelihood: $p(x|z) = \mathcal{N}(x|\mu_z, \Sigma_z)$

So, posterior –

$$p(z|x) = \frac{\pi_z \cdot N(x|\mu_z, \Sigma_z)}{\sum_{z'} \pi_{z'} N(x|\mu_{z'}, \Sigma_{z'})}$$

Now, $\tau_k^i = p(z^i = k | x^i, \theta^t)$

$$= \frac{p(x^i | z^i = k)\, p(z^i = k)}{}$$

$$= \frac{\pi_k \cdot \mathcal{N}(x^i | \mu_k, \Sigma_k)}{\sum_{k'=1 \ldots k} \pi_{k'} N(x^i | \mu_{k'}, \Sigma_{k'})}$$

Which is same as posterior from baye's.