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FACULTY OF ENGINEERING AND THE BUILT ENVIRONMENT

DEPARTMENT OF ELECTRICAL ENGINEERING

SIGNAL PROCESSING 4

SEMESTER GROUP PROJECT

FILTERATION OF SIGNALS

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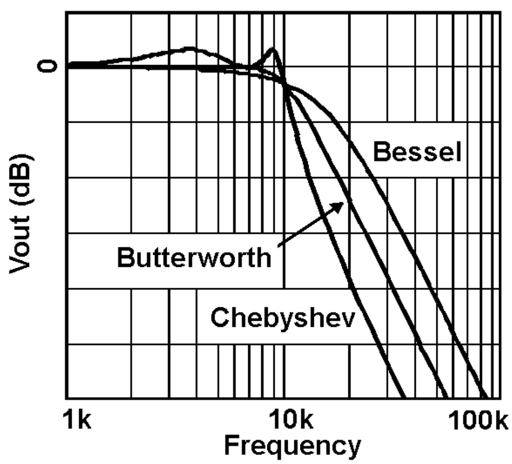
Project A

Butterworth Filter

What is a Butterworth Filter?

The term Butterworth refers to a type of filter response, not a type of filter. It is sometimes called the Maximally Flat approximation, because for a response of order n, the first (2n-1) derivatives of the gain with respect to frequency are zero at frequency = 0. There is no ripple in the passband, and DC gain is maximally flat.

The following figures are representative of a low pass filter. The response characteristics are mirror imaged for high pass filters.



Amplitude

The designer can see that there is no ripple in the passband of a Butterworth filter. The Butterworth filter, however, has a flatter response in the passband. There is a continuum of filter characteristics, of which Bessel is one, and Butterworth another; but anything in between is possible - it just wouldn't have a name. The Chebyshev response continues the continuum beyond Butterworth, which is the last characteristic for which there is a flat passband.

Low Pass Butterworth Filter Design

The frequency response of the **Butterworth Filter** approximation function is also often referred to as "maximally flat" (no ripples) response because the pass band is designed to have a frequency response which is as flat as mathematically possible from 0Hz (DC) until the cut-off frequency at -3dB with no ripples. Higher frequencies beyond the cut-off point rolls-off down to zero in the stop band at 20dB/decade or 6dB/octave. This is because it has a "quality factor", "Q" of just 2^0.2.

However, one main disadvantage of the Butterworth filter is that it achieves this pass band flatness at the expense of a wide transition band as the filter changes from the pass band to the stop band. It also has poor phase characteristics as well. The ideal frequency response, referred to as a "brick wall" filter, and the standard Butterworth approximations

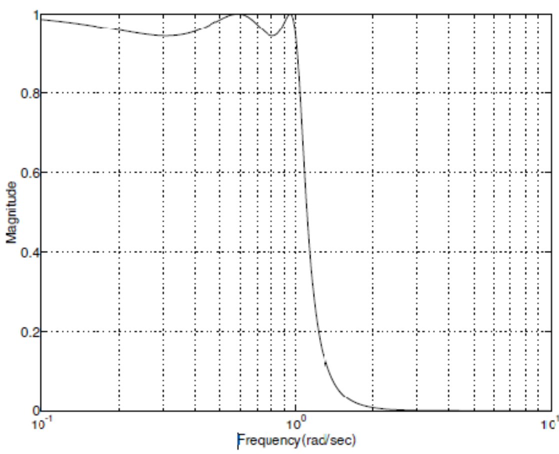
Butterworth maximally flat magnitude

Advantage is that it has the most flat passband meaning that it is very good at simulating the passband of an ideal filter.

The disadvantage is that it has a horrible stopband because it gradually goes to zero so some parts of the stopband are still passed. However, for an nth-order Butterworth Filter, as n increases, the closer it is to an ideal filter. However, it is highly impractical to build a ridiculously high order Butterworth filter.

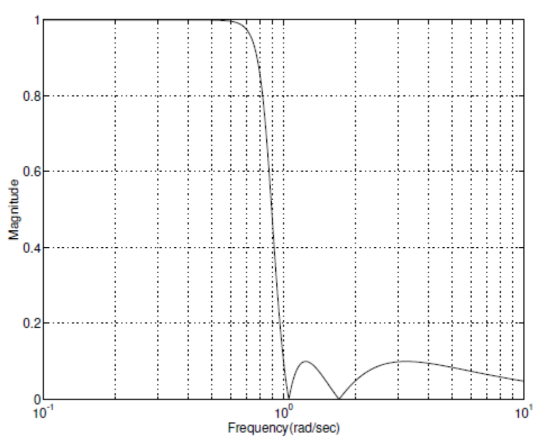
Chebyshev Type I Filter

The Chebyshev Type I filter minimizes the absolute difference between the Ideal and actual frequency response over the entire passband by incorporating An equal ripple of Rp dB in the passband. Stopband response is maximally flat. The transition from passband to stopband is more rapid than for the Butterworth filter. Chebyshev type I has ripple in the passband, shaper transition band compared to Butterworth and poorer group delay. Allowing more ripples in the passband causes Narrow transition band, shaper cut-off and poor phase response



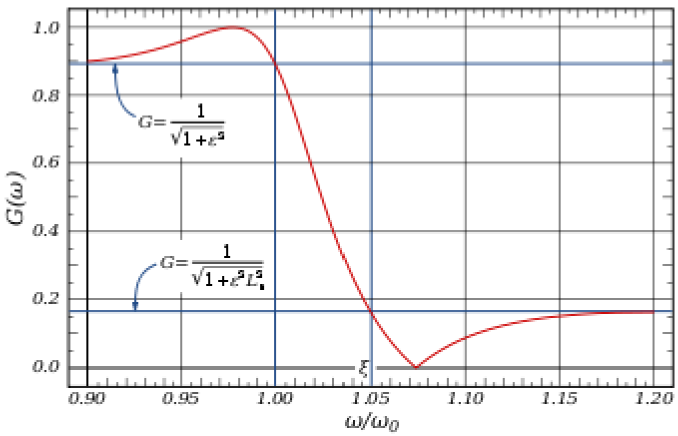
Chebyshev Type II Filter

The Chebyshev Type II filter minimizes the absolute difference between the ideal and actual frequency response over the entire stopband by incorporating an equal ripple of Rs dB in the stopband. Passband response is maximally flat.
The stopband does not approach zero as quickly as the type I filter (and does Not approach zero at all for even-valued filter order n). The absence of ripple in The passband, however, is often an important advantage.
Chebyshev II filter has ripple in the stopband, shaper transition band compared to Butterworth and passband phase is more linear compared to Chebyshev I



Characteristics of Elliptical Filter

The elliptical filter contains a ripple, defined by the variance in output voltage from either the maximum or minimum output. However, unique to the elliptical filter is an equal dB ripple in the pass-band, the frequency region that is passed with minimal attenuation, and the stop-band, the frequency region that is severely attenuated. In a filter with a ripple on the stop-band, the voltage rise after cut-off may detrimental effects on connected circuitry if incorrectly designed. Because of this, the elliptic filter is also classified by it's A min value. This value, in decibels, indicates the guaranteed minimum drop from DC response such that the output may never rise past this threshold after cut-off. Finally, the filter is characterized by the ws/wc value. This value defines the roll-off, or how fast the filter drops from the cut-off frequency to the stop-band.

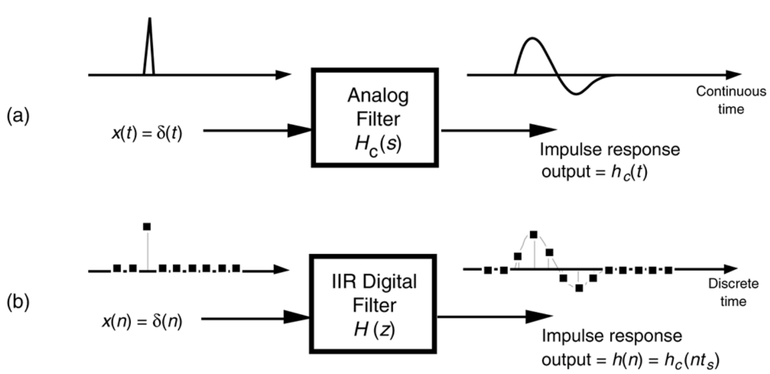


Elliptic Filter Advantages

The elliptical filter has an extremely sharp cut-off frequency, which makes it ideally suited for filter design cases where there must be severe attenuation in frequencies just entering the stop-band of the filter. Further, because the rippling effect is distributed across both the pass- and stop-bands in the elliptic filter, it makes it an excellent candidate for a low pass filter where the amount of error needs to be minimized on both sides of the cut-off frequency. The Chebyshev filter, with a slower roll-off and an imbalanced ripple, does not offer either of these advantages. Therefore, in cases where there are signals that are very close and must be cut off at exactly or very close to one particular frequency, the elliptic filter is recommended.

Impulse Invariance Iir Filter Design Method

The impulse invariance method of IIR filter design is based upon the notion that we can design a discrete filter whose time-domain impulse response is a sampled version of the impulse response of a continuous analogue filter. If that analogue filter has some desired frequency response, then our IIR filter will yield a discrete approximation of that desired response. The impulse response equivalence of this design method is depicted (see below)



where we use the conventional notation of δ to represent an impulse function and $h_c(t)$ is the analogue filter's impulse response. We use the subscript “*c*” in **Figure A (**analogue filter continuous IR**)** to emphasize the continuous nature of the analogue filter. **Figure B(**digital filter discrete IR**)** illustrates the definition of the discrete filter's impulse response: the filter's time-domain output sequence when the input is a single unity-valued sample (impulse) preceded and followed by all zero-valued samples. Our goal is to design a digital filter whose impulse response is a sampled version of the analogue filter's continuous impulse response. Implied in the correspondence of the continuous and discrete impulse responses is the property that we can map each pole on the *s*-plane for the analogue filter's $H_c(s)$ transfer function to a pole on the *z*-plane for the discrete IIR filter's$H(z)$ transfer function. What designers have found is that the impulse invariance method does yield useful IIR filters, as long as the sampling rate is high relative to the bandwidth of the signal to be filtered. In other words, IIR filters designed using the impulse invariance methods are susceptible to aliasing problems because practical analogue filters cannot be perfectly band-limited. Aliasing will occur in an IIR filter's frequency response.

Bilinear Transform Iir Filter Design Method

There's a popular analytical IIR filter design technique known as the *bilinear transform* method. Like the impulse invariance method, this design technique approximates a prototype analogue filter defined by the continuous Laplace transfer function $H_c(s)$ with a discrete filter whose transfer function is $H(z)$. However, the bilinear transform method has great utility because

- it allows us simply to substitute a function of *z* for *s* in $H_c(s)$ to get $H(z)$, thankfully, eliminating the need for Laplace and *z*-transformations as well as any necessity for partial fraction expansion algebra;
- it maps the entire *s*-plane to the *z*-plane, enabling us to completely avoid the frequency-domain aliasing problems we had with the impulse invariance design method; and
- it induces a nonlinear distortion of $H(z)$'s frequency axis, relative to the original prototype analogue filter's frequency axis, that sharpens the final roll-off of digital low-pass filters.

Section B: Filter Design in MATLAB

The following parameters were specified for the implementation of the different types of low pass discrete filters:

Stop-band edge frequency : 4000 Hz

Pass-band edge frequency : 2500 Hz

Nyguist frequency = Fs/2 Hz

Sampling frequency Fs = 44100 Hz

Minimum gain in the pass-band GpbMin: 37 dB

Minimum gain in the stop-band GsbMax : -55 dB

Maximum gain in the pass-band GpbMax : 40 dB

Express the magnitude of the pass-band ripple δ_{IIR}, the stop-band attenuation ξ_{IIR} , and the scaling factor K_{IIR} in terms of the GpbMax, GpbMin and GsbMax.

The three quantities are determined as follows:

$$\text{GpbMin} = 10^{\frac{-\delta_{IIR}}{20}} \tag{1}$$

Multiply both sides by log_{10}, then equation (1) is

$$log_{10}\,\text{GpbMin}= log_{10}10^{\frac{-\delta_{IIR}}{20}} \tag{2}$$

$$20log_{10}\,\text{GpbMin}= -\delta_{IIR} \tag{3}$$

Hence in dB, the equation is expressed as

$$\delta_{IIR} = -\ \text{GpbMin} \tag{4}$$

From
$$\text{GpbMin} = 10^{\frac{-\xi_{IIR}}{20}} \tag{5}$$

Multiply both sides of the equation (5) by log_{10},

$$\log_{10}\text{GpbMin} = \log_{10} 10^{\frac{-\xi_{IIR}}{20}} \tag{6}$$

Since $\log_{10} 10 = 1$, therefore $-\xi_{IIR} = 20\log_{10}\text{GpbMin}$ (7)

In linear scale, the maximum gain in the pass-band is equal to 1. This means GpbMax =1, therefore the scaling factor can be expressed as

$$K_{IIR} = {\text{GpbMin}}/{\text{GpbMax}} = 10^{\frac{-\delta_{IIR}}{20}}/1 = 10^{\frac{-\delta_{IIR}}{20}} \tag{8}$$

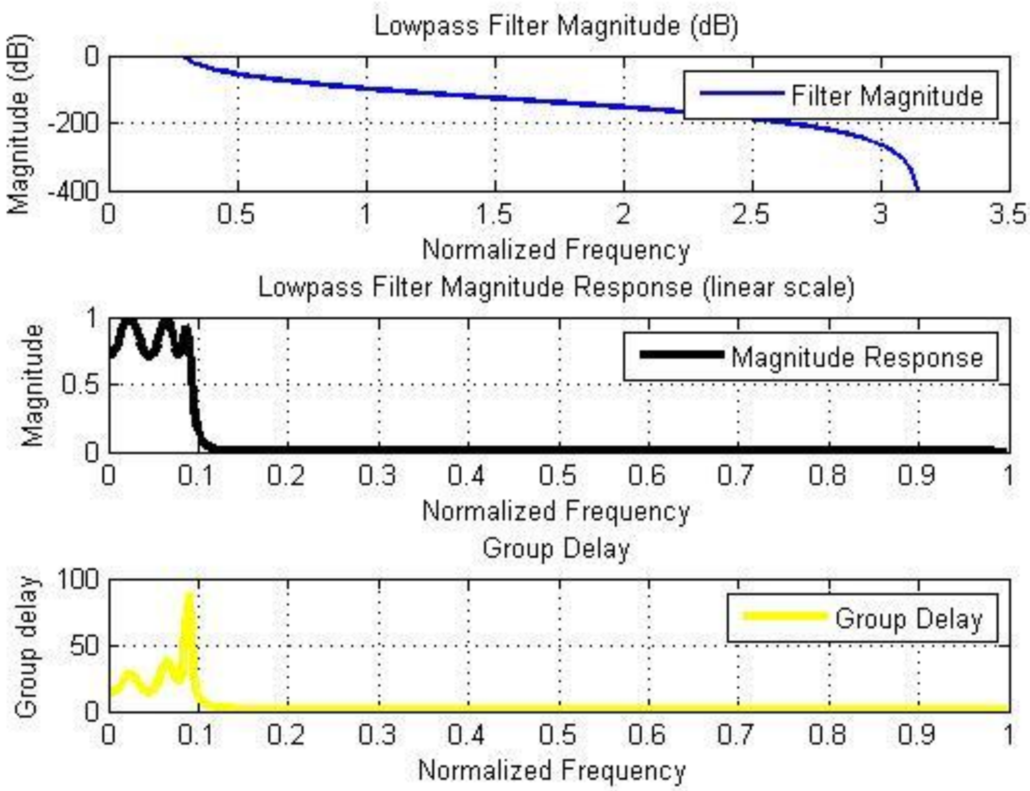
By taking the log of both sides of the equation (8), then

$$20\log_{10}K_{IIR} = -\delta_{IIR} \tag{9}$$

Thus in dB, $K_{IIR} = -\delta_{IIR}$ dB

A. <u>Chebyshev Type I Filter Design</u>

The Operation of each filter was perfomed using MATLAB.The following graphs are for the Chebyshev 1- Magnitude and Normalized Frequency, As we know the the ripples are on the Passband instead of the StopBand



<u>Figure 1: Magnitude responses and Group delay of the Chebyshev Type 1 filter</u>

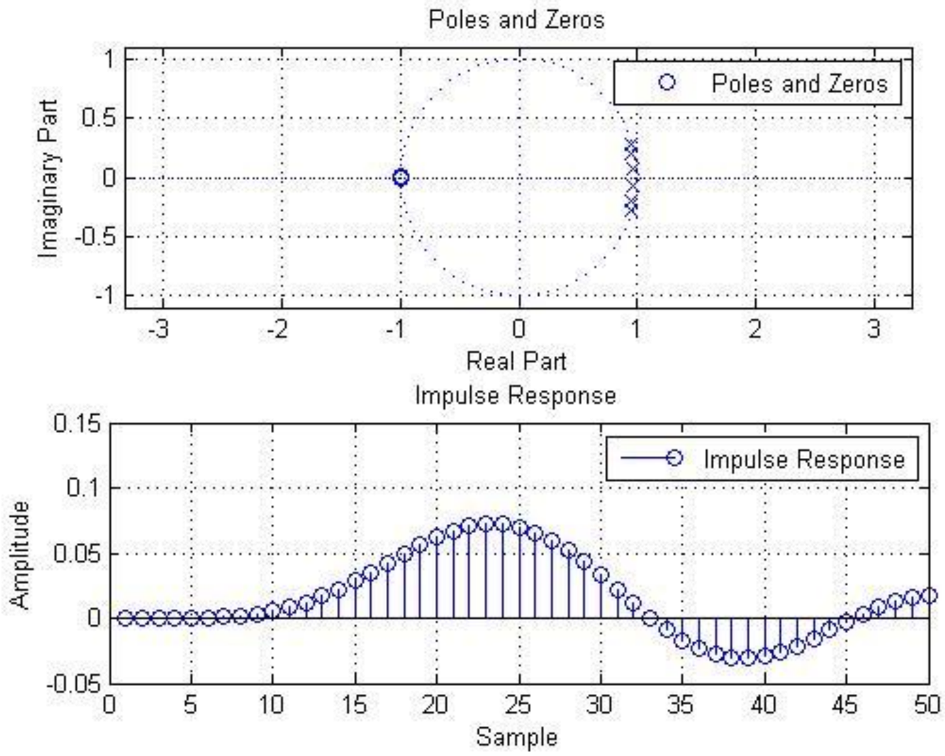


Figure 2: Filter Poles and Zeros, and Impulse response of the Chebyshev Type I

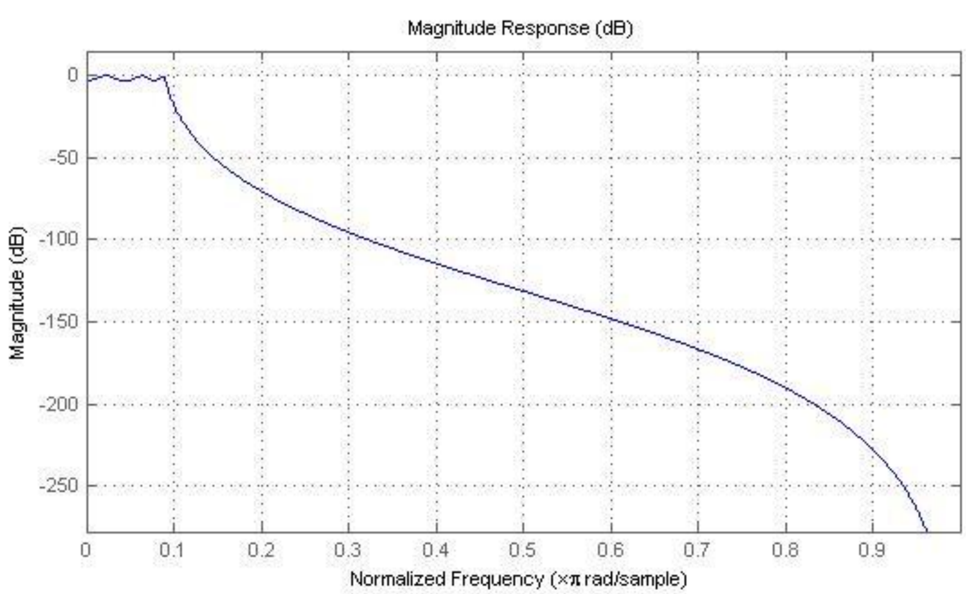


Figure 3: Filter Visualization Tool

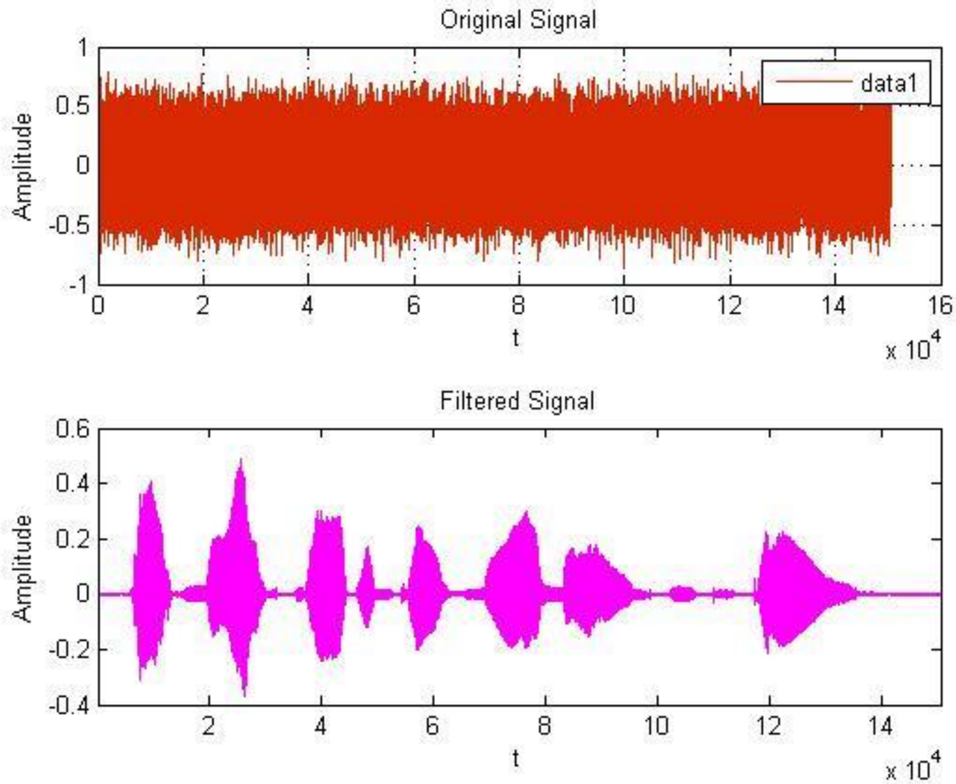


Figure 4: Original Noisy Signal and Filtered Signal using Chebyshev Type I filter

B. Chebyshev Type II Filter Design

The magnitude responses and group delay for the Chebyshev Type II filter are shown in figures below.

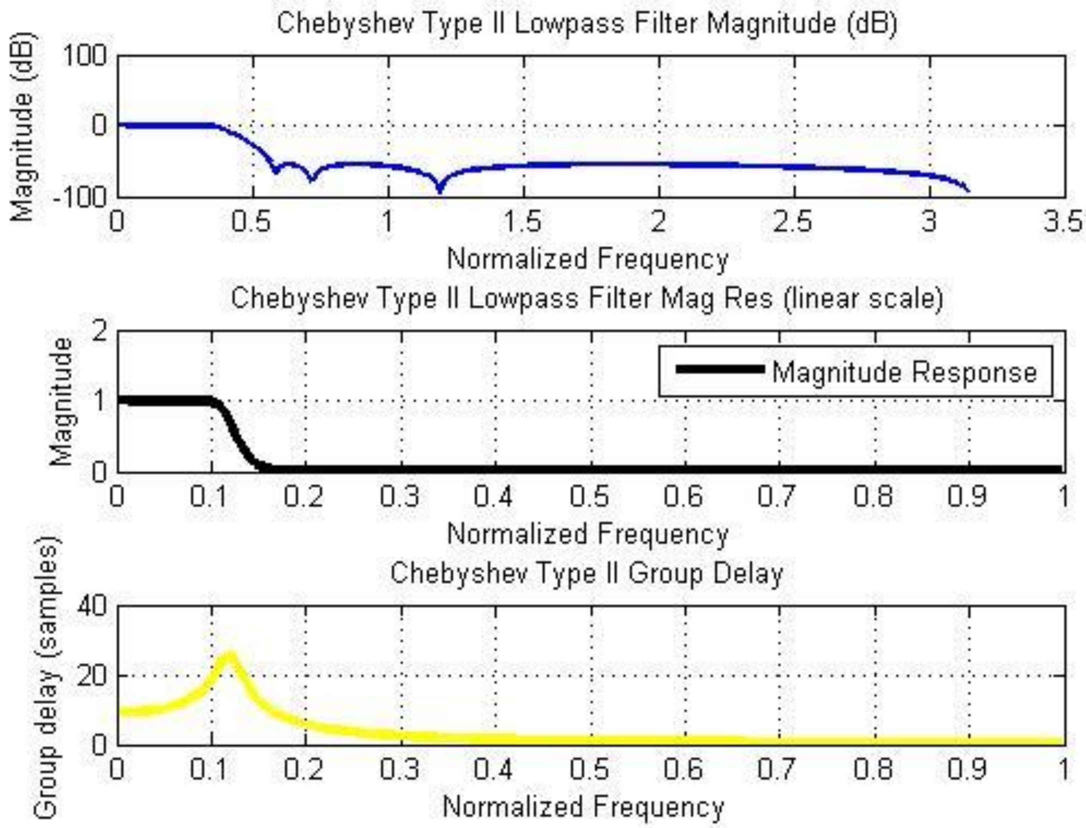


Figure 5: Magnitude responses and group delay of the Chebyshev Type II filter

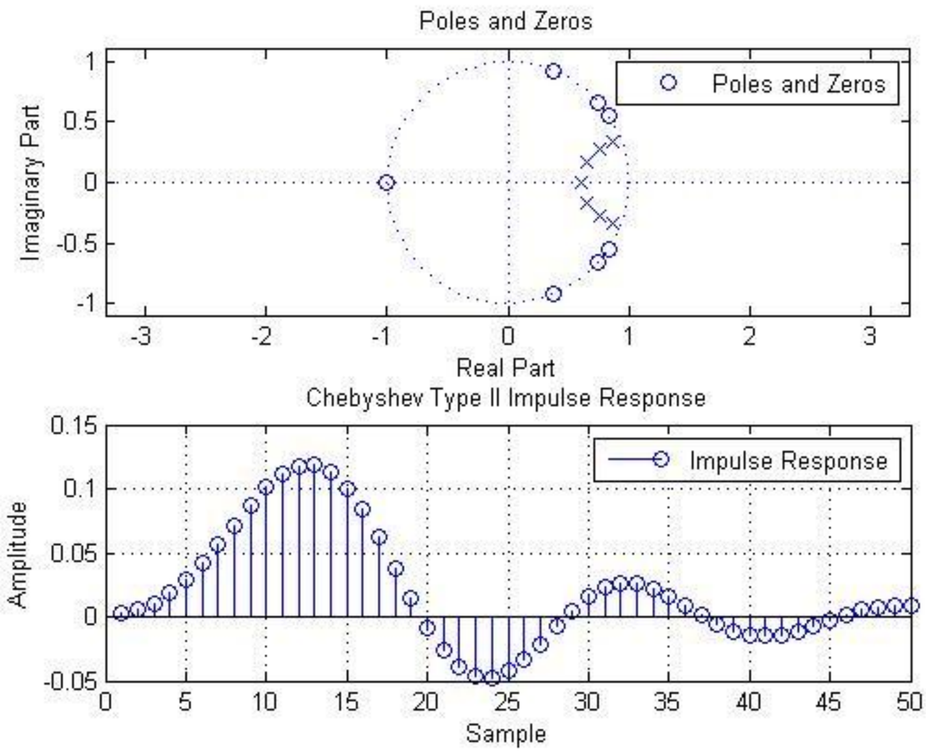


Figure 6: Poles, Zeros, and the Impulse response of the Chebyshev Type II filter

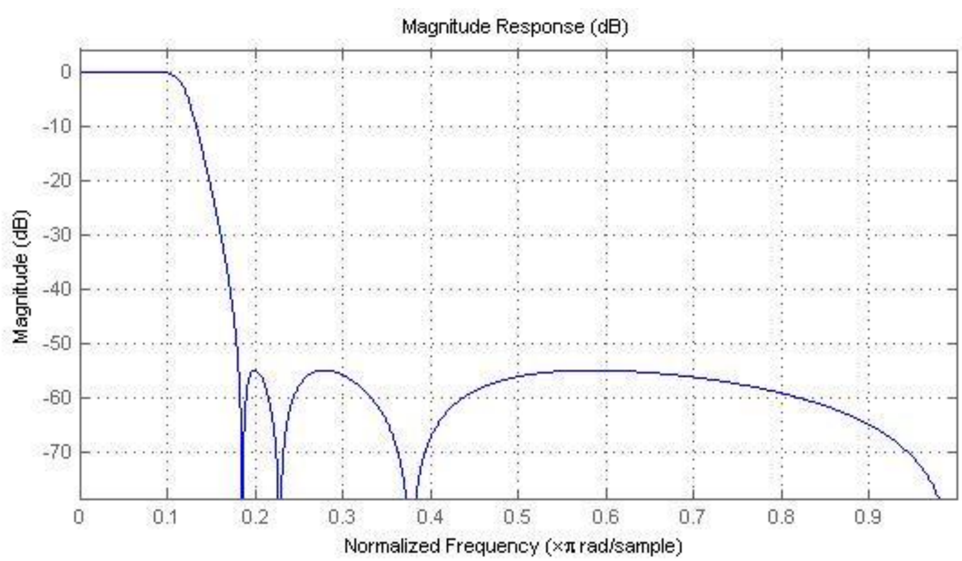


Figure 7: Filter Visualization Tool

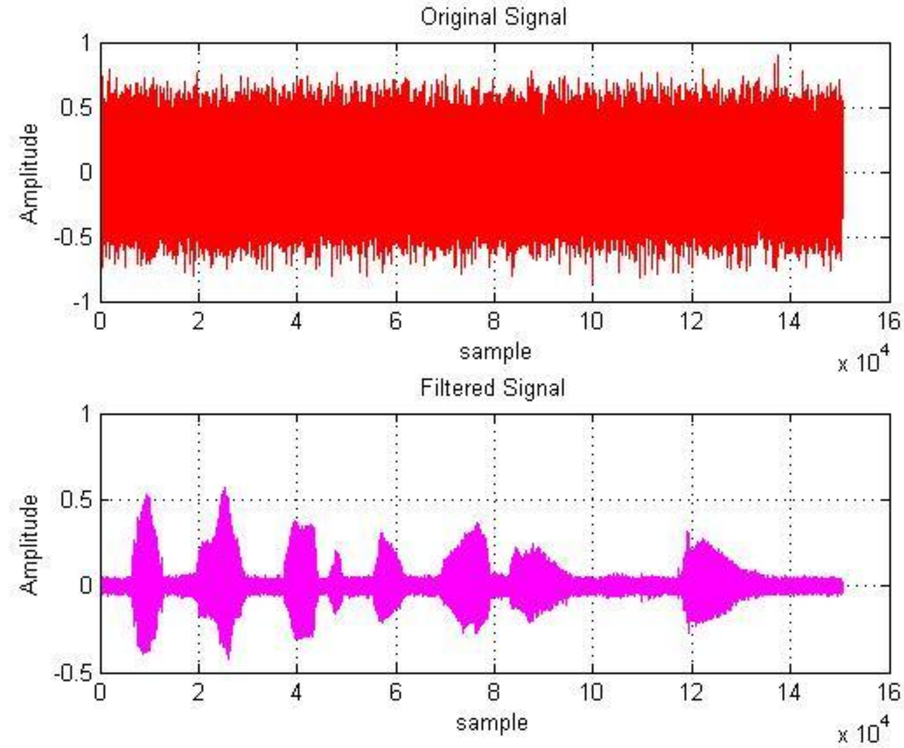


Figure 8: Original and Filtured Signal using the Chebyshev Type II filter

C. <u>Elliptic Filter Design</u>

The Elliptic filter produced the smallest filter order as shown in the table. This means that it is the most suitable design for the required specifications.

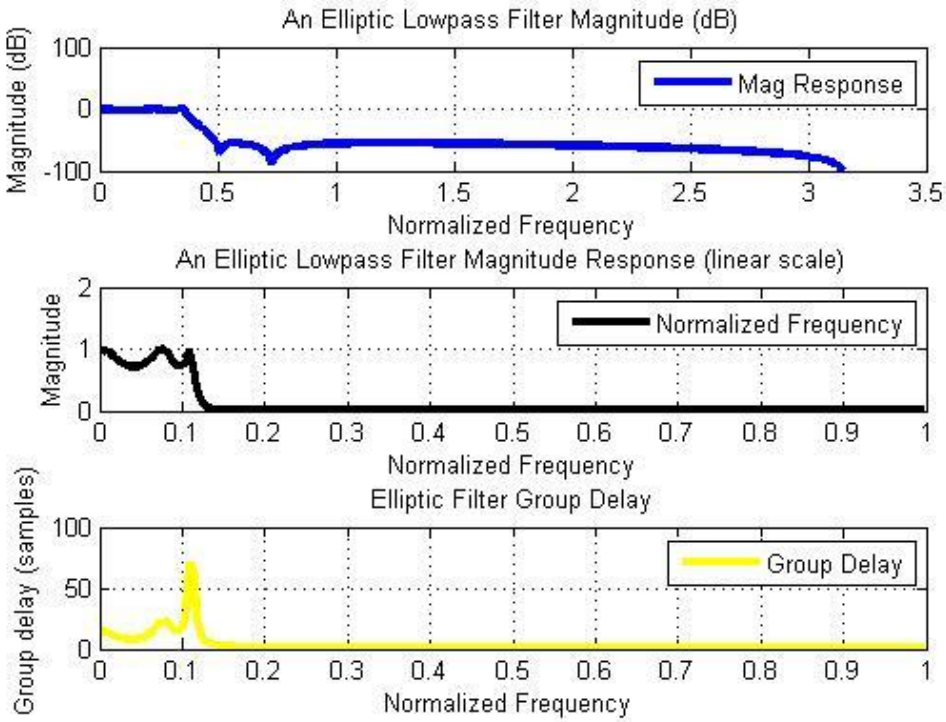


Figure 9: Magnitude responses and group delay of the Elliptic filter

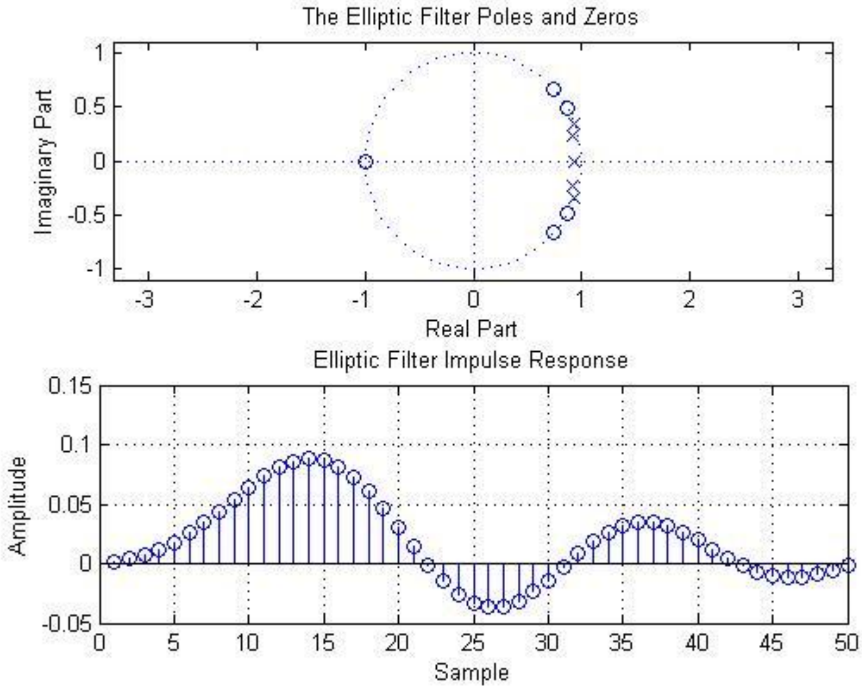


Figure 10: Poles ,Zeros, and the Impulse response of the Elliptic filter

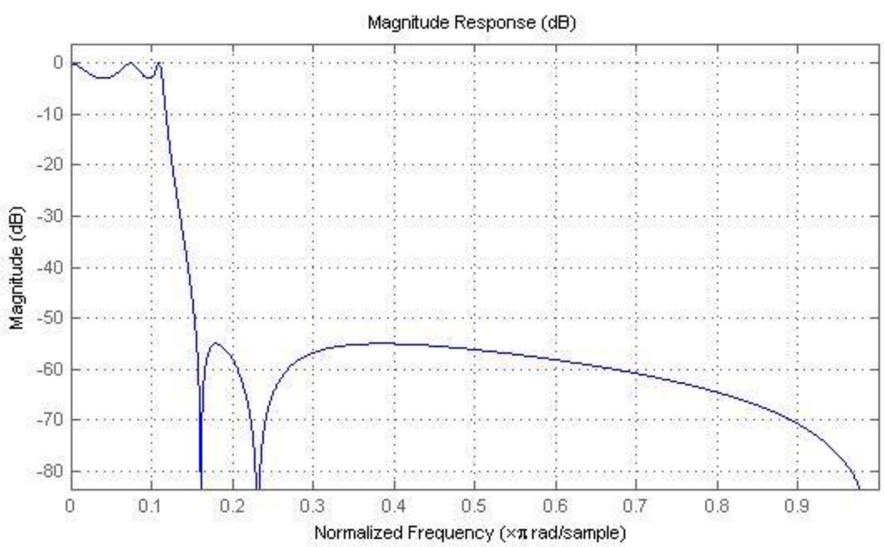


Figure 11: Filter Virtualizatial Tool

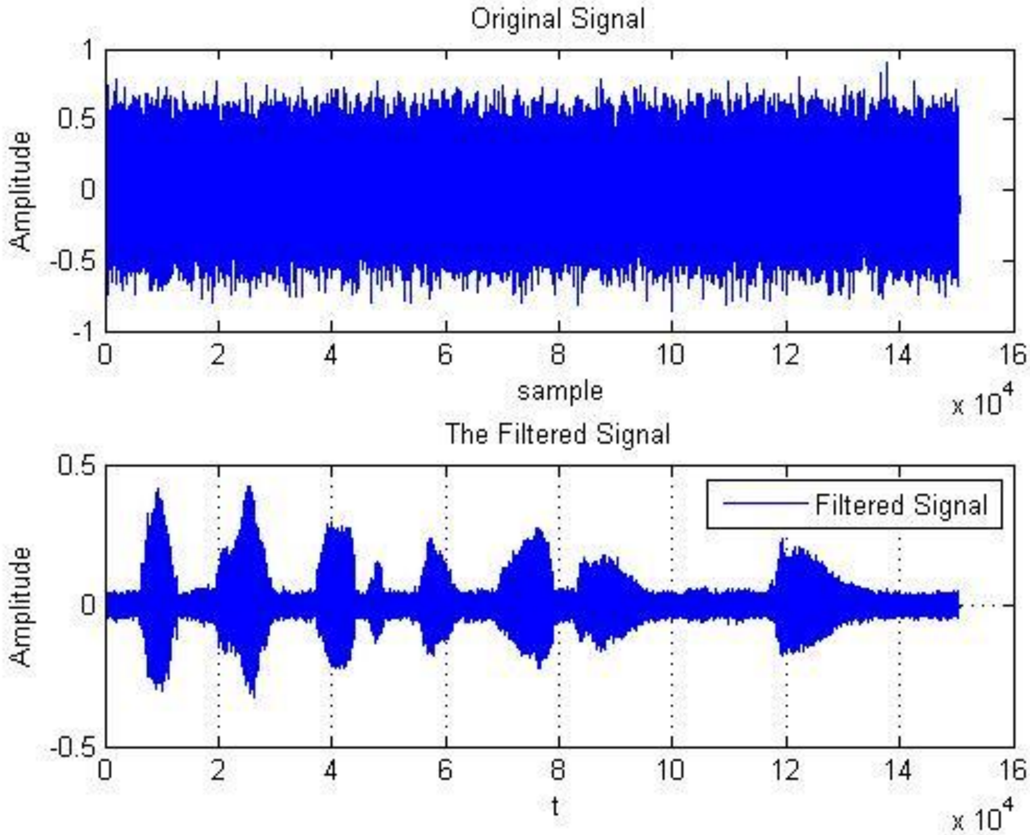


Figure 12: Filtering performance of the Elliptic filter

D. **Butterworth Filter Design**

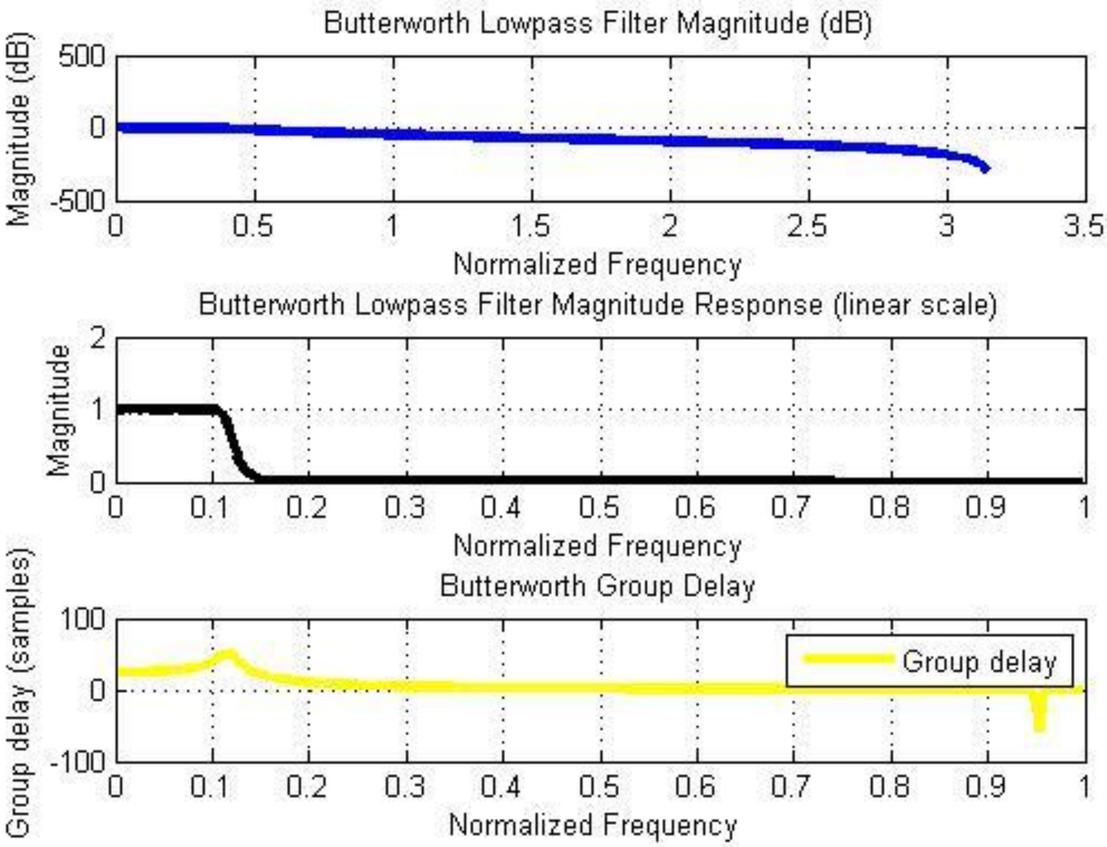


Figure 13: Magnitude responses and group delay of the Butterworth filter

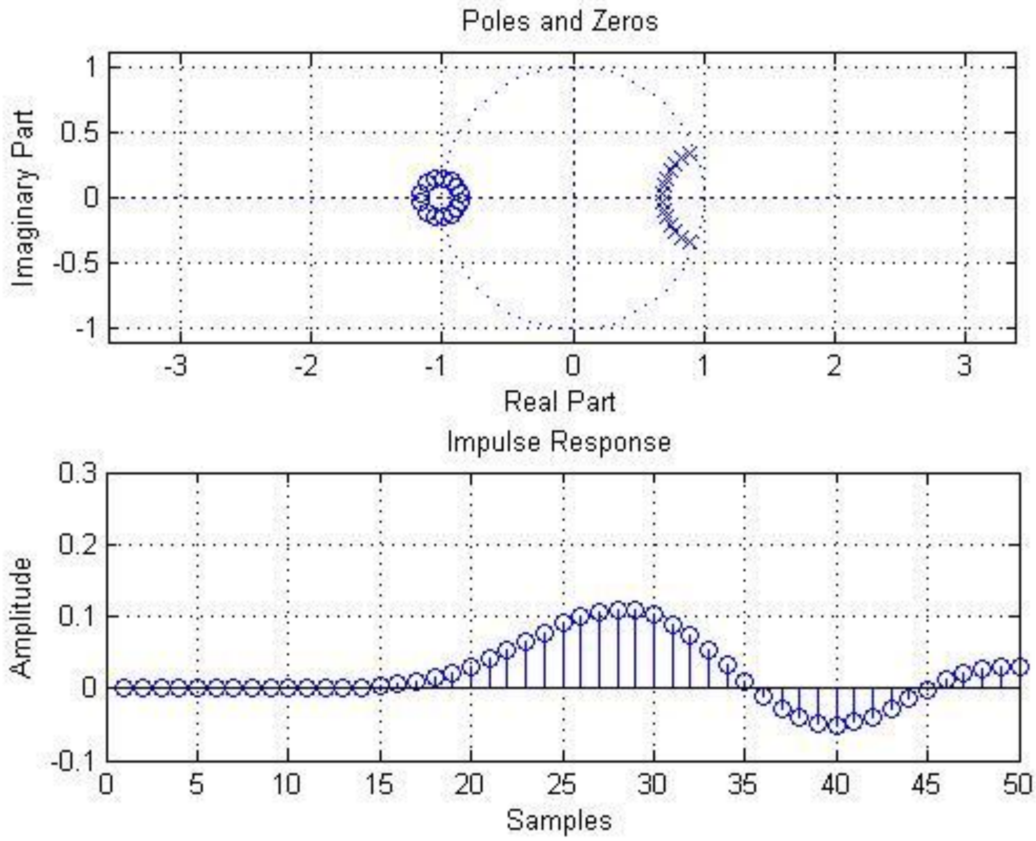


Figure 14: Butterworth filter Poles and Zeros, and the Impulse response

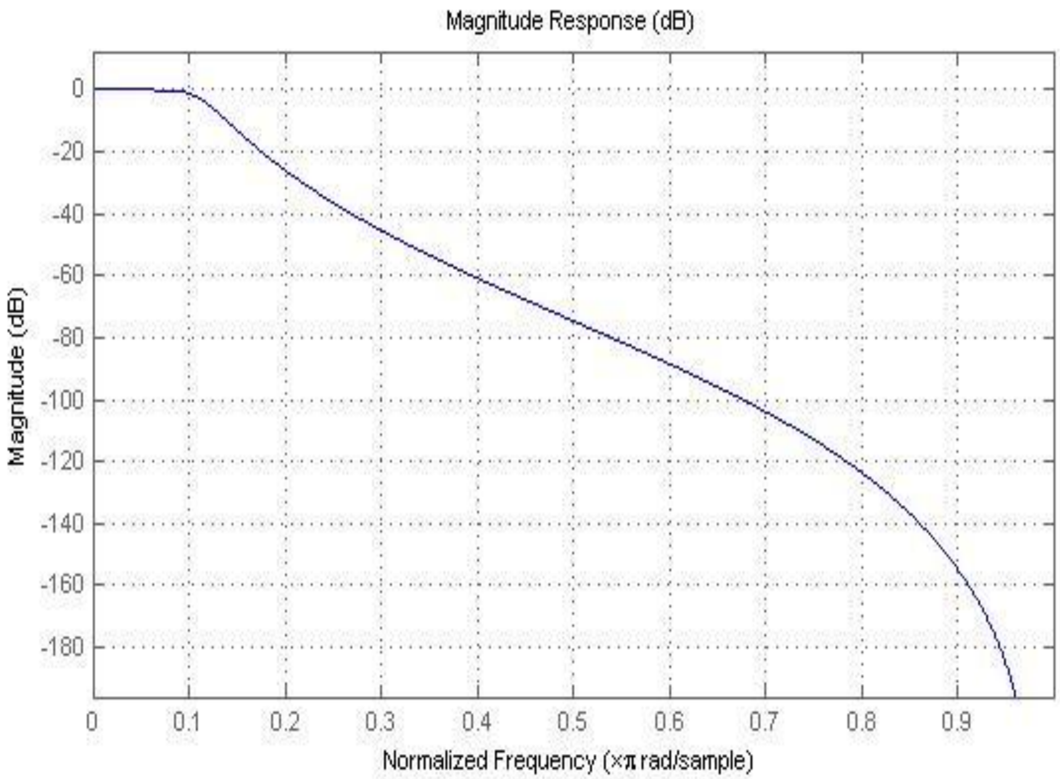


Figure 15: Virtual Tool for the Butterworth filter

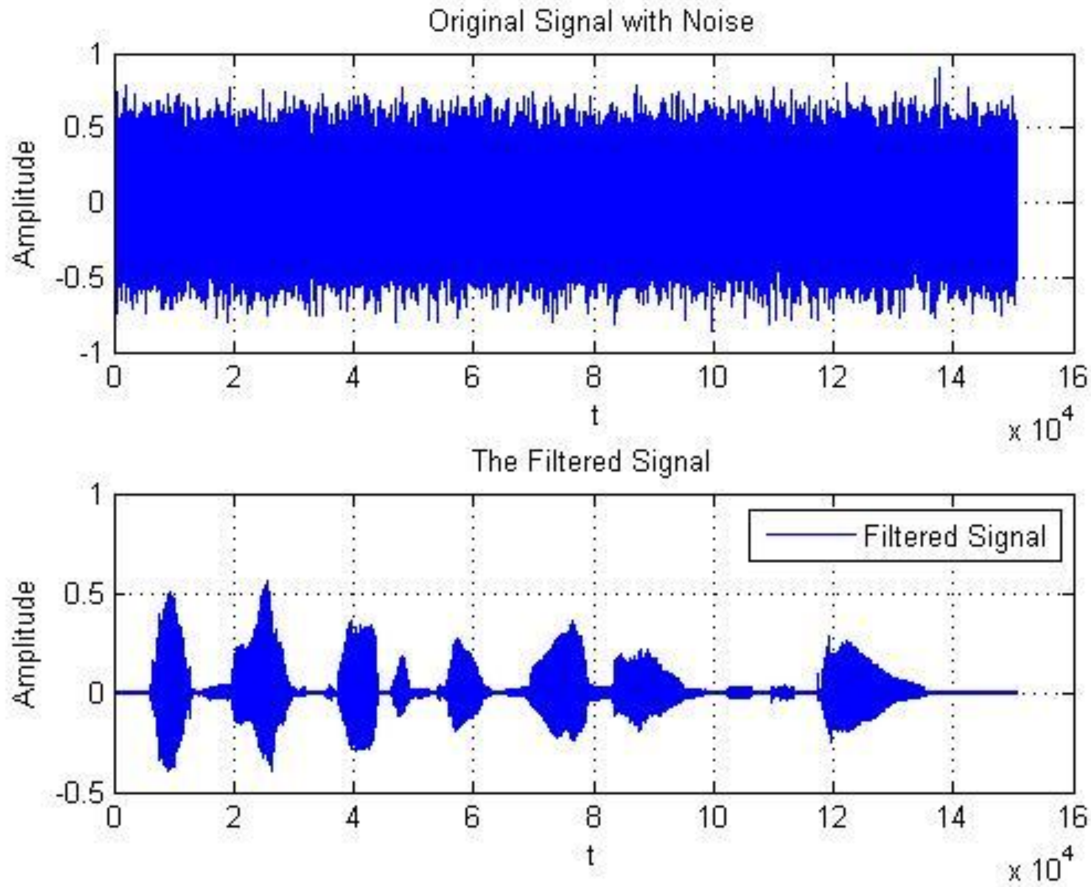


Figure 16: Original and Filtured Signal of the Butterworth filter

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