

Zero Knowledge Proofs: Homework 1

Meek Msaki

February 9, 2023

Question 1

$$S = \mathbb{Z}_7$$

- a) $4 + 4 \equiv 1 \pmod{7}$. Answer is 1.
- b) $3 \cdot 5 \equiv 1 \pmod{7}$. Answer is 1.
- c) $3^{-1} \equiv 3^{7-2} \pmod{7} \equiv 243 \pmod{7} \equiv 5 \pmod{7}$. Verify $3 \cdot 5 \pmod{7} \equiv 1 \pmod{7}$. Using Fermat's little theorem, answer is 5.

Question 2

Answer is Yes. $(\mathbb{Z}_7, +)$ is a group, it has a set of elements $0, 1, 2, 3, 4, 5, 6$ plus an operator $+$.

- 1. It is closed. For all $a, b \in \mathbb{Z}_7$, the results of the operation is also in \mathbb{Z}_7 .
- 2. Associativity, for all $a, b, c \in \mathbb{Z}_7$ $(a + b) + c = a + (b + c)$
- 3. There is an identity element e , which for each element $a \in \mathbb{Z}_7$ $a + e = e + a = a$, where the identity element $e = 0$.
- 4. There exists an inverse element e , for $a \in \mathbb{Z}_7$, such that $a + (-a) = (-a) + a = e$, where the inverse identity element $e = 0$.

Question 3

$-13 \pmod{5} \equiv 2 \pmod{5}$. Answer is 2.

Question 4

We can simplify our function $f(x) = x^3 - x^2 + 4x - 12$ to factors $f(x) = (x - 2) \cdot (x^2 + x + 6)$

Using Polynomial Remainder Theorem $\frac{P(x)}{x-a} \rightarrow r = P(a)$ where $P(x) = x^3 - x^2 + 4x - 12$. If we divided $P(x)$ with a first degree polynomial $(x - a)$, and we don't get a remainder $r = 0$, this verifies $(x - a)$ is a valid factor of our polynomial $P(x)$. We can solve this with our factor $(x - 2)$ for $P(a)$. When $x = 2$, $P(2) = 2^3 - 2^2 + 4 \cdot 2 - 12 = 0$. Solving for x using $(x - 2) = 0$ or $(x^2 + x + 6) = 0$. Answer is 2. The degree of this polynomial is 3.

See: Remainder theorem: checking factors