Department of Mathematics and Natural Sciences
Mid-term Examination
Semester: Summer 2015
Course Title: Linear Algebra and Fourier Analysis
Course No.: MAT216
Section: 06

Time: 1 hour
Total Marks: 40

Date: June 24, 2015

Answer any <u>FOUR</u>:

1. Define system of linear equations, consistent, and inconsistent systems. Determine the values of parameter, λ, such that the following system has: (i) no solution, (ii) unique solution, (iii) infinite solutions. Also solve the system. [10]

$$\begin{aligned} x + y + \lambda z &= 2 \\ 3x + 4y + 2z &= \lambda \\ 2x + 3y - z &= 1 \end{aligned}$$

2. Define elementary row operations. Transform the following system into the matrix form $AX = B$ and solve using $X = A^{-1}B$. [10]

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 6 \\ x_1 + 3x_2 + 3x_3 &= 4 \\ 2x_1 + 4x_2 + 3x_3 &= 3 \end{aligned}$$

3. (a) Define vector space and subspace with examples. Show that the set of all 2×3 matrices, $M_{2\times 3}$, is a vector space under the matrix addition and scalar multiplication. [5]

   (b) Determine whether the following set of vectors is a basis of $\mathbb{R}^3$. [5]

$$S = \{(1, 1, 2), (1, -1, 2), (1, 0, 1)\}$$

4. Define basis and dimension of a vector space. Find the bases for the row space and column space of [10]

$$A = \begin{pmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ -3 & 0 & 6 & -1 \\ 3 & 4 & -2 & 1 \\ 2 & 0 & -4 & 2 \end{pmatrix}.$$

   Also find the rank(A).

5. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation defined by [10]

$$T(x_1, x_2, x_3, x_4) = (x_1 + 4x_2 + 5x_3 + 2x_4, 2x_1 + x_2 + 3x_3, -x_1 + 3x_2 + 2x_3 + 2x_4)\,.$$

Find the standard matrix for this transformation. Also find the basis and dimension for $\ker(T)$.