Adequacy of Linear Regression Models

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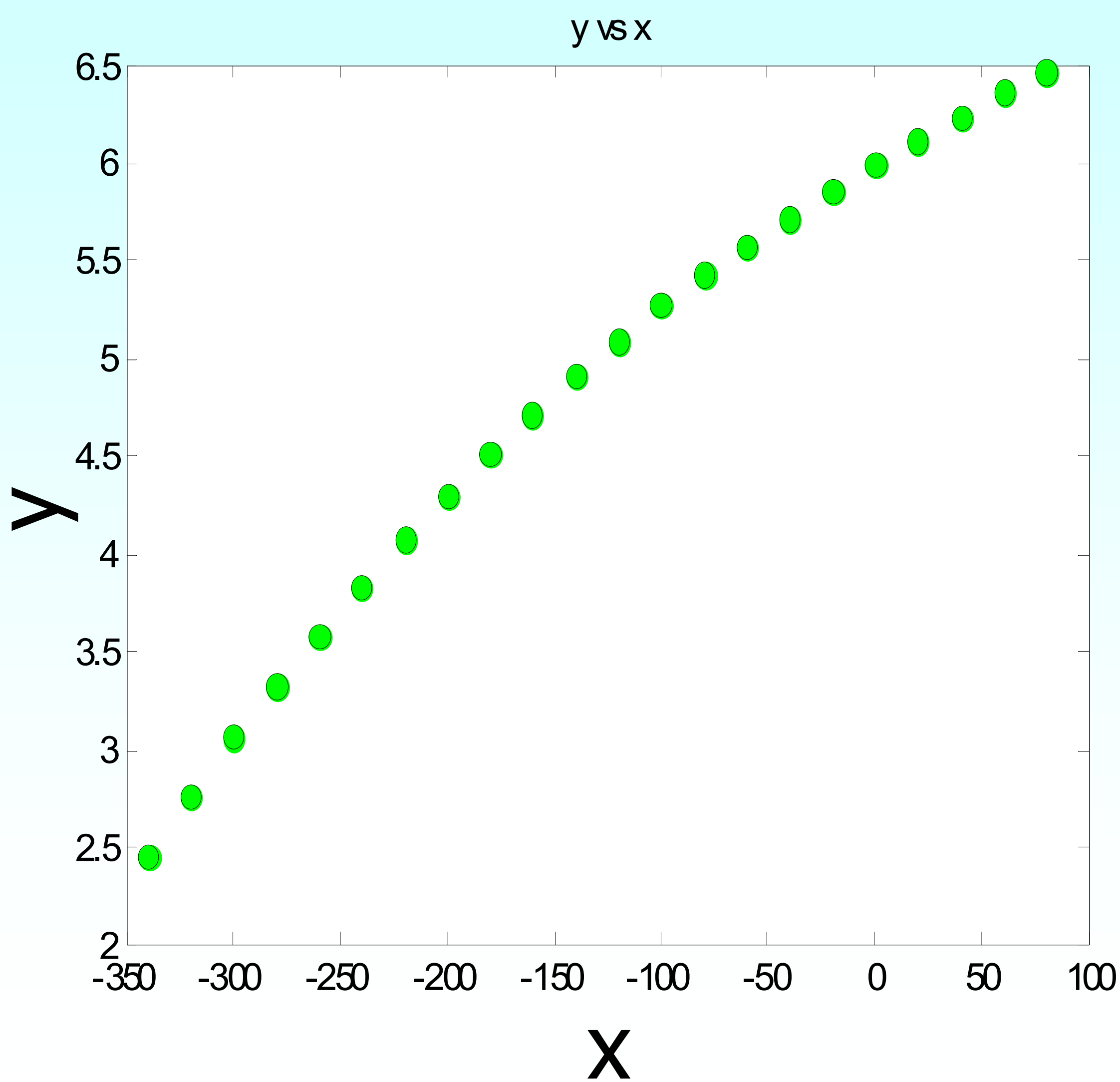
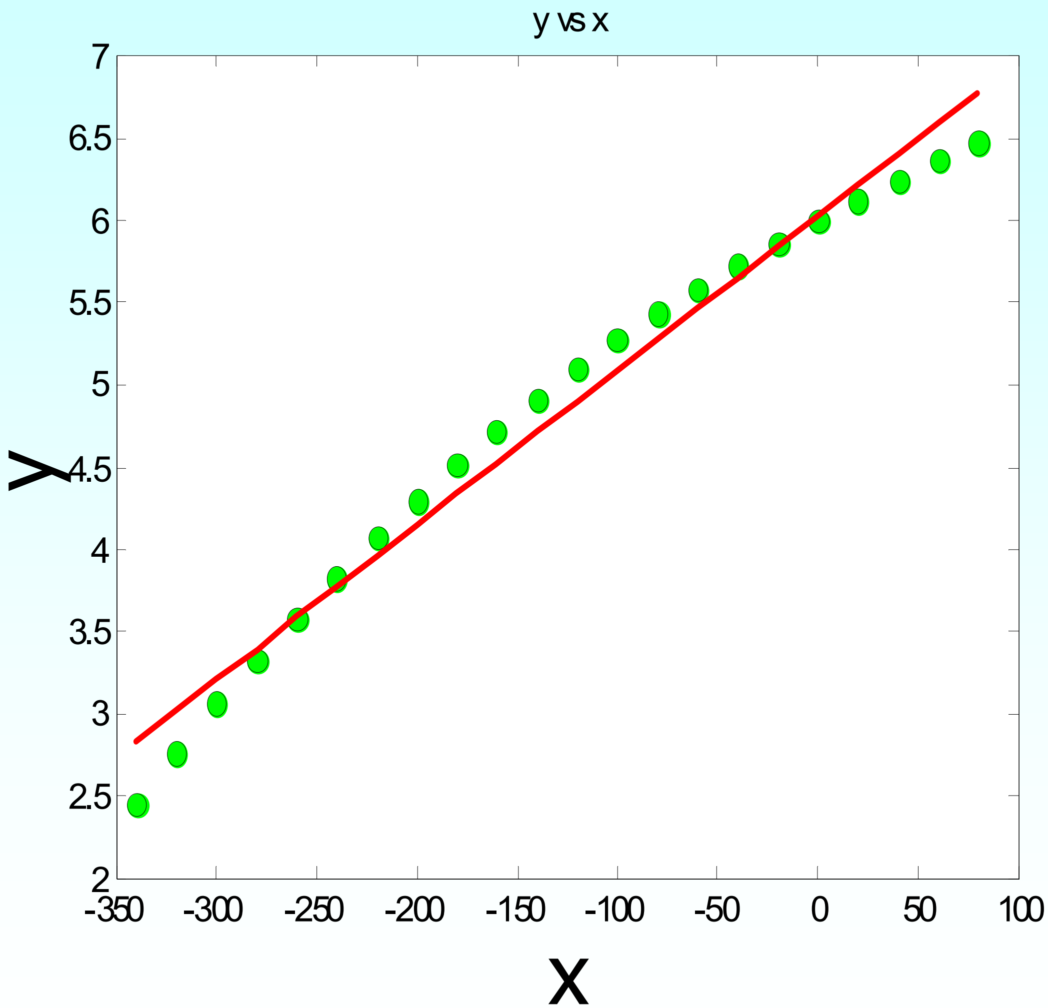
Transforming Numerical Methods Education for STEM Undergraduates

Data



Is this adequate?



Straight Line Model

Quality of Fitted Data

- Does the model describe the data adequately?

- How well does the model predict the response variable predictably?

Linear Regression Models

- Limit our discussion to adequacy of straight-line regression models

Four checks

1. Plot the data and the model.
2. Find standard error of estimate.
3. Calculate the coefficient of determination.
4. Check if the model meets the assumption of random errors.

Example: Check the adequacy of the straight line model for given data

$$\alpha = a_0 + a_1 T$$

T (F)	α (μin/in/F)
-340	2.45
-260	3.58
-180	4.52
-100	5.28
-20	5.86
60	6.36

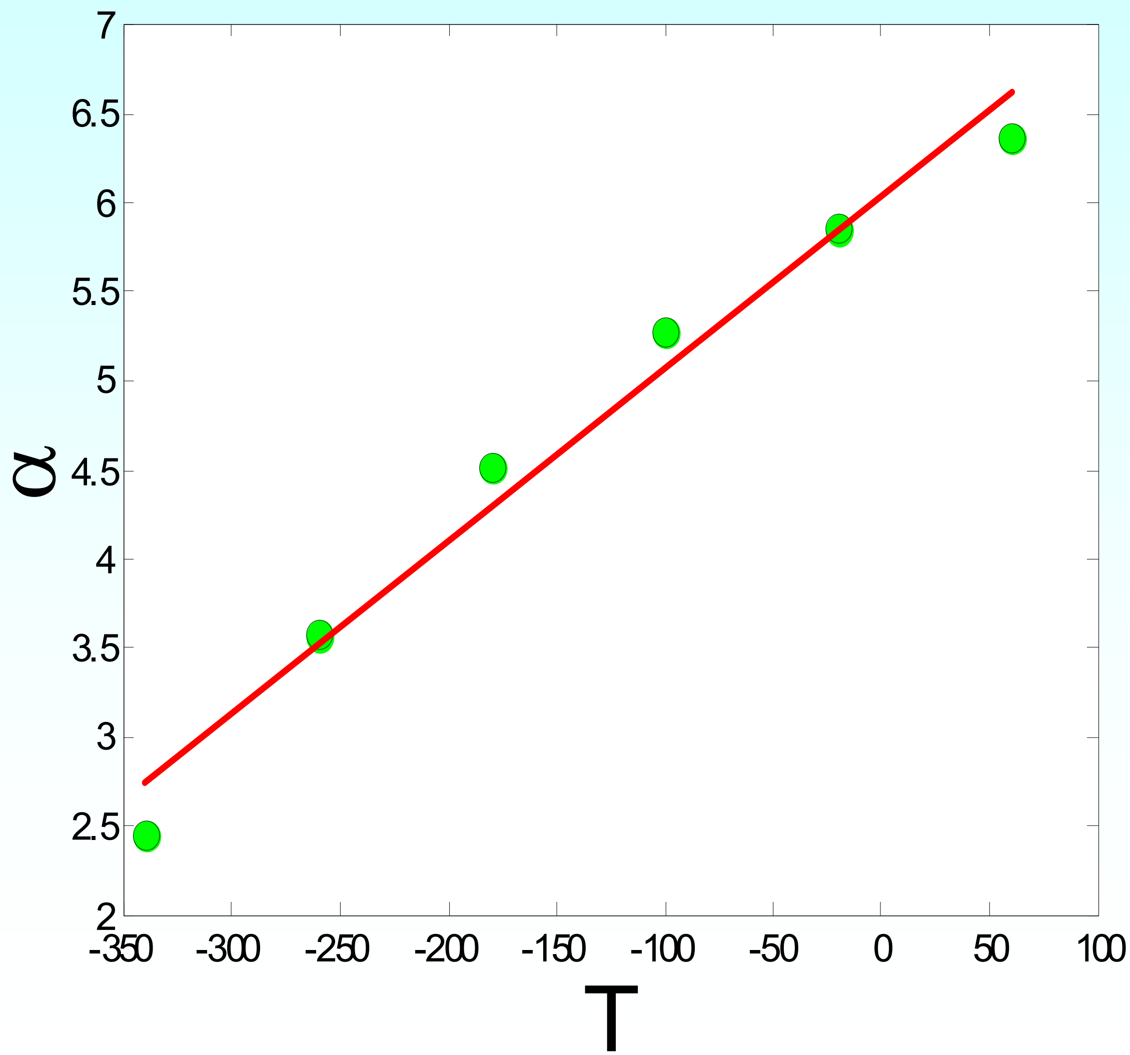
END

1. Plot the data and the model

Data and model

$$\alpha(T) = 6.0325 + 0.0096964T$$

T (F)	α (μin/in/F)
-340	2.45
-260	3.58
-180	4.52
-100	5.28
-20	5.86
60	6.36



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2. Find the standard error of estimate

Standard error of estimate

$$s_{\alpha/T} = \sqrt{\frac{S_r}{n-2}}$$

$$S_r = \sum_{i=1}^{n} (\alpha_i - a_0 - a_1 T_i)^2$$

Standard Error of Estimate

$$\alpha(T) = 6.0325 + 0.0096964T$$

T_i	α_i	$a_0 + a_1 T_i$	$\alpha_i - a_0 - a_1 T_i$
-340	**2.45**	**2.7357**	**-0.28571**
-260	**3.58**	**3.5114**	**0.068571**
-180	**4.52**	**4.2871**	**0.23286**
-100	**5.28**	**5.0629**	**0.21714**
-20	**5.86**	**5.8386**	**0.021429**
60	**6.36**	**6.6143**	**-0.25429**

Standard Error of Estimate

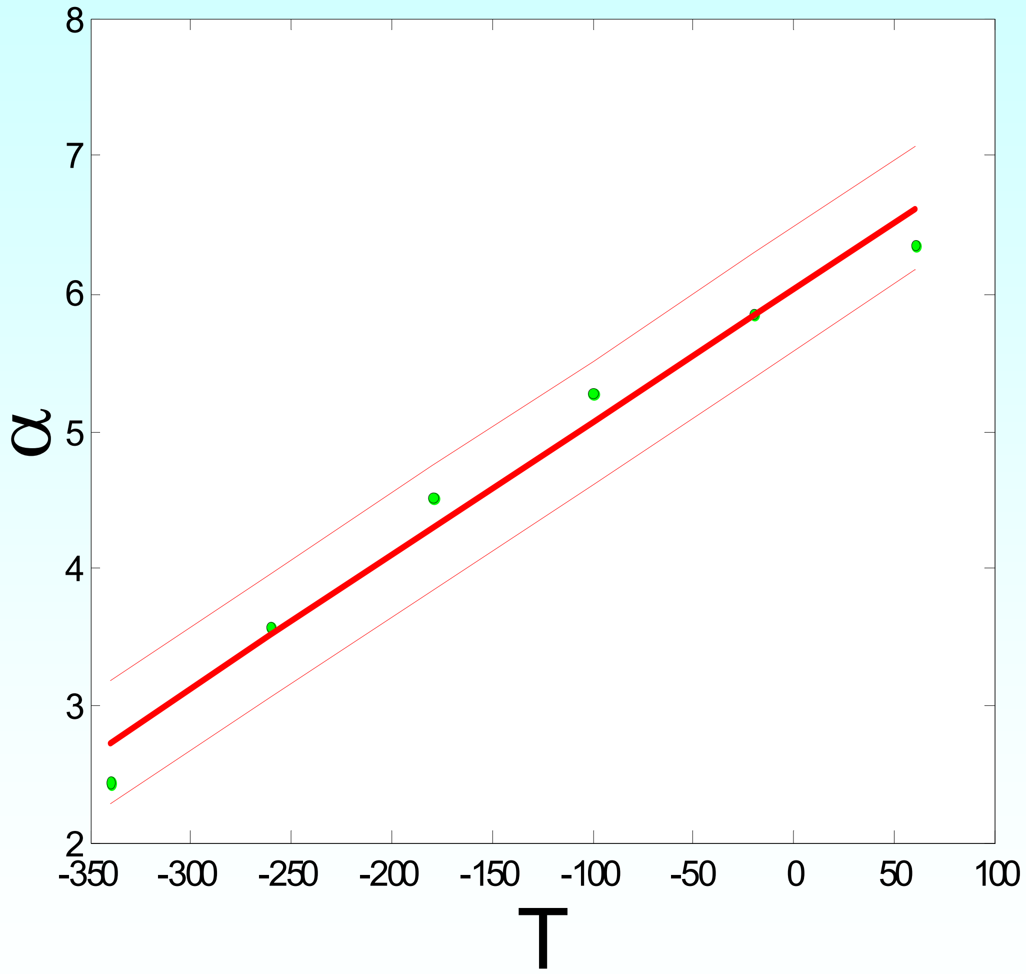
$$S_r = 0.25283$$

$$s_{\alpha/T} = \sqrt{\frac{S_r}{n-2}}$$

$$= \sqrt{\frac{0.25283}{6-2}}$$

$$= 0.25141$$

Standard Error of Estimate



$$\text{Scaled Residual} = \frac{\alpha_i - a_0 - a_1 T_i}{s_{\alpha/T}}$$

Scaled Residuals

$$\text{Scaled Residual} = \frac{\text{Residual}}{\text{Standard Error of Estimate}}$$

$$\text{Scaled Residual} = \frac{\alpha_i - a_0 - a_1 T_i}{s_{\alpha/T}}$$

95% of the scaled residuals need to be in [-2,2]

Scaled Residuals

$$s_{\alpha/T} = 0.25141$$

T_i	α_i	Residual	Scaled Residual
-340	2.45	-0.28571	-1.1364
-260	3.58	0.068571	0.27275
-180	4.52	0.23286	0.92622
-100	5.28	0.21714	0.86369
-20	5.86	0.021429	0.085235
60	6.36	-0.25429	-1.0115

END

3. Find the coefficient of determination

Coefficient of determination

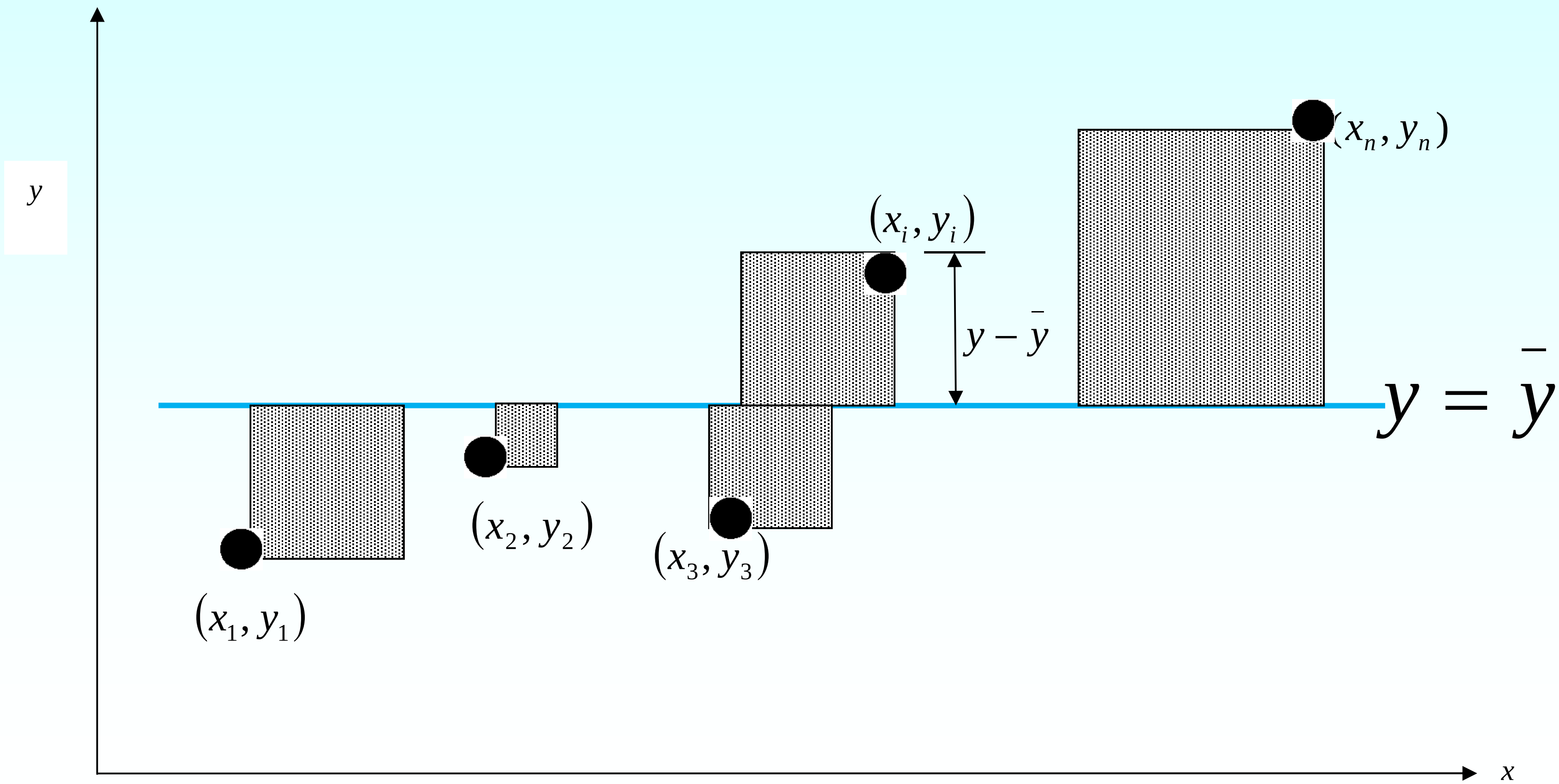
$$S_t = \sum_{i=1}^{n} \left(\alpha_i - \overline{\alpha} \right)^2$$

$$S_r = \sum_{i=1}^{n} \left(\alpha_i - a_0 - a_1 T_i \right)^2$$

$$r^2 = \frac{S_t - S_r}{S_t}$$

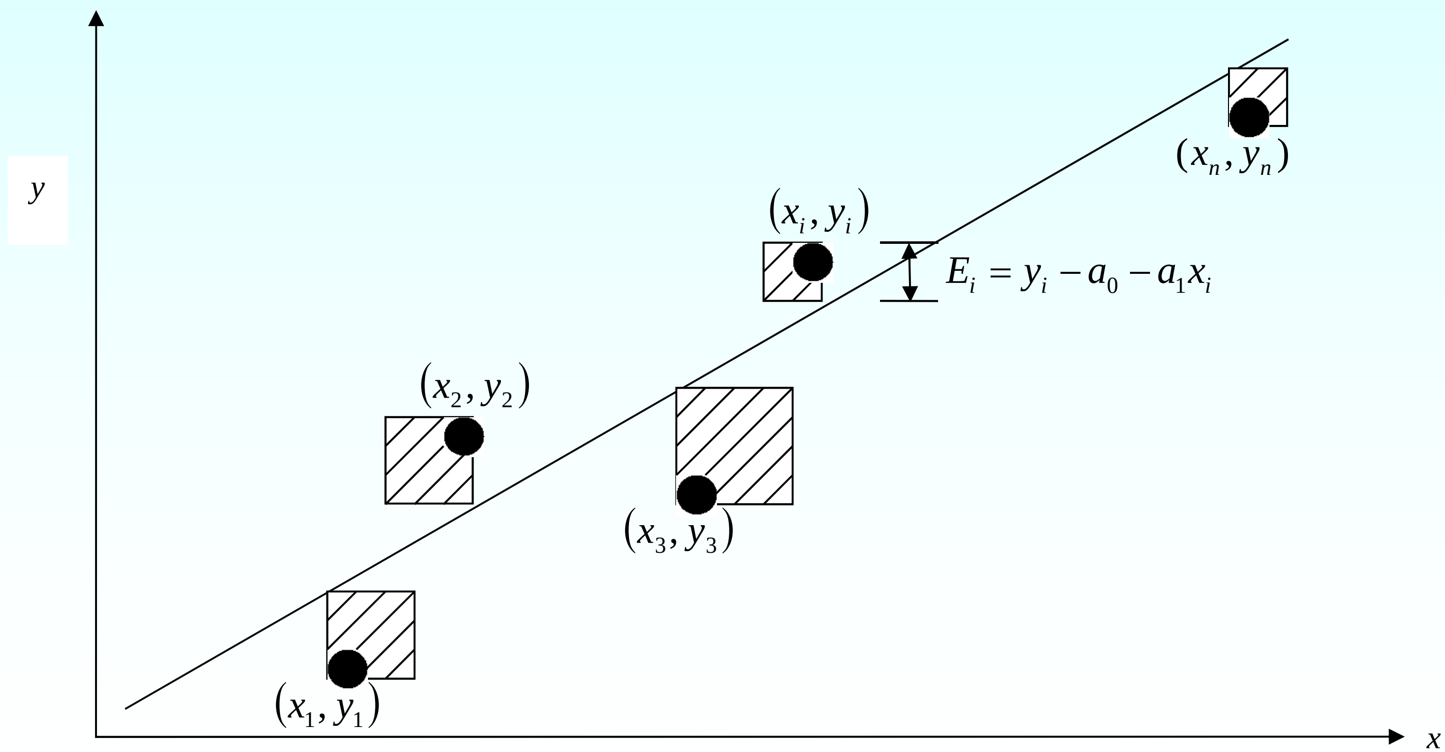
Sum of square of residuals between data and mean

$$S_t = \sum_{i=1}^{n} \left(\alpha_i - \overline{\alpha}\right)^2$$



Sum of square of residuals between observed and predicted

$$S_r = \sum_{i=1}^{n} \left(\alpha_i - a_0 - a_1 T_i\right)^2$$



Limits of Coefficient of Determination

$$r^2 = \frac{S_t - S_r}{S_t}$$

$$0 \le r^2 \le 1$$

Calculation of S_t

T_i	α_i	$\alpha_i - \overline{\alpha}$
-340	**2.45**	**-2.2250**
-260	**3.58**	**-1.0950**
-180	**4.52**	**0.15500**
-100	**5.28**	**0.60500**
-20	**5.86**	**1.1850**
60	**6.36**	**1.6850**

$$\overline{\alpha} = 4.6750$$

$$S_t = 10.783$$

Calculation of S_r

T_i	α_i	$a_0 + a_1 T_i$	$\alpha_i - a_0 - a_1 T_i$
-340	**2.45**	**2.7357**	**-0.28571**
-260	**3.58**	**3.5114**	**0.068571**
-180	**4.52**	**4.2871**	**0.23286**
-100	**5.28**	**5.0629**	**0.21714**
-20	**5.86**	**5.8386**	**0.021429**
60	**6.36**	**6.6143**	**-0.25429**

$$S_r = 0.25283$$

Coefficient of determination

$$r^2 = \frac{S_t - S_r}{S_t}$$

$$= \frac{10.783 - 0.25283}{10.783}$$

$$= 0.97655$$

Correlation coefficient

$$r = \sqrt{\frac{S_t - S_r}{S_t}}$$

$$= 0.98820$$

How do you know if r is positive or negative ?

What does a particular value of r mean?

0.8 to 1.0 - Very strong relationship

0.6 to 0.8 - Strong relationship

0.4 to 0.6 - Moderate relationship

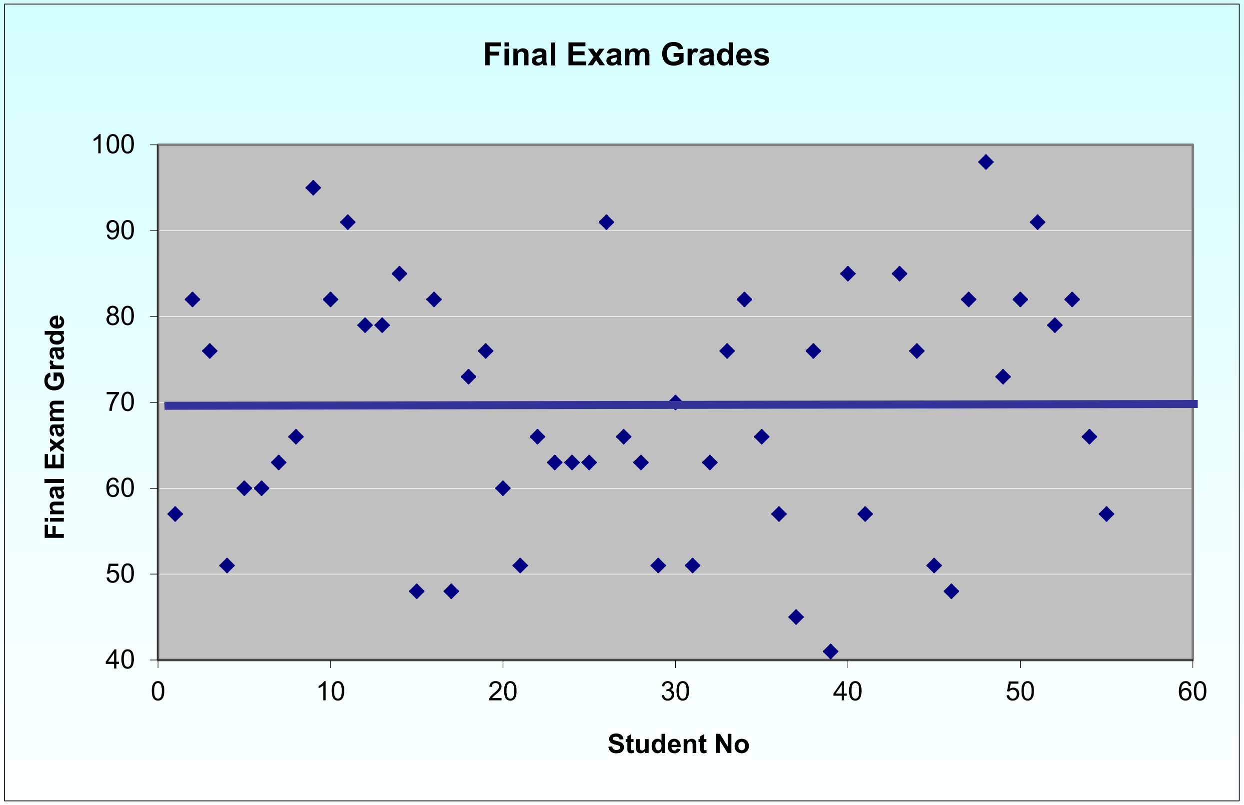
0.2 to 0.4 - Weak relationship

0.0 to 0.2 - Weak or no relationship

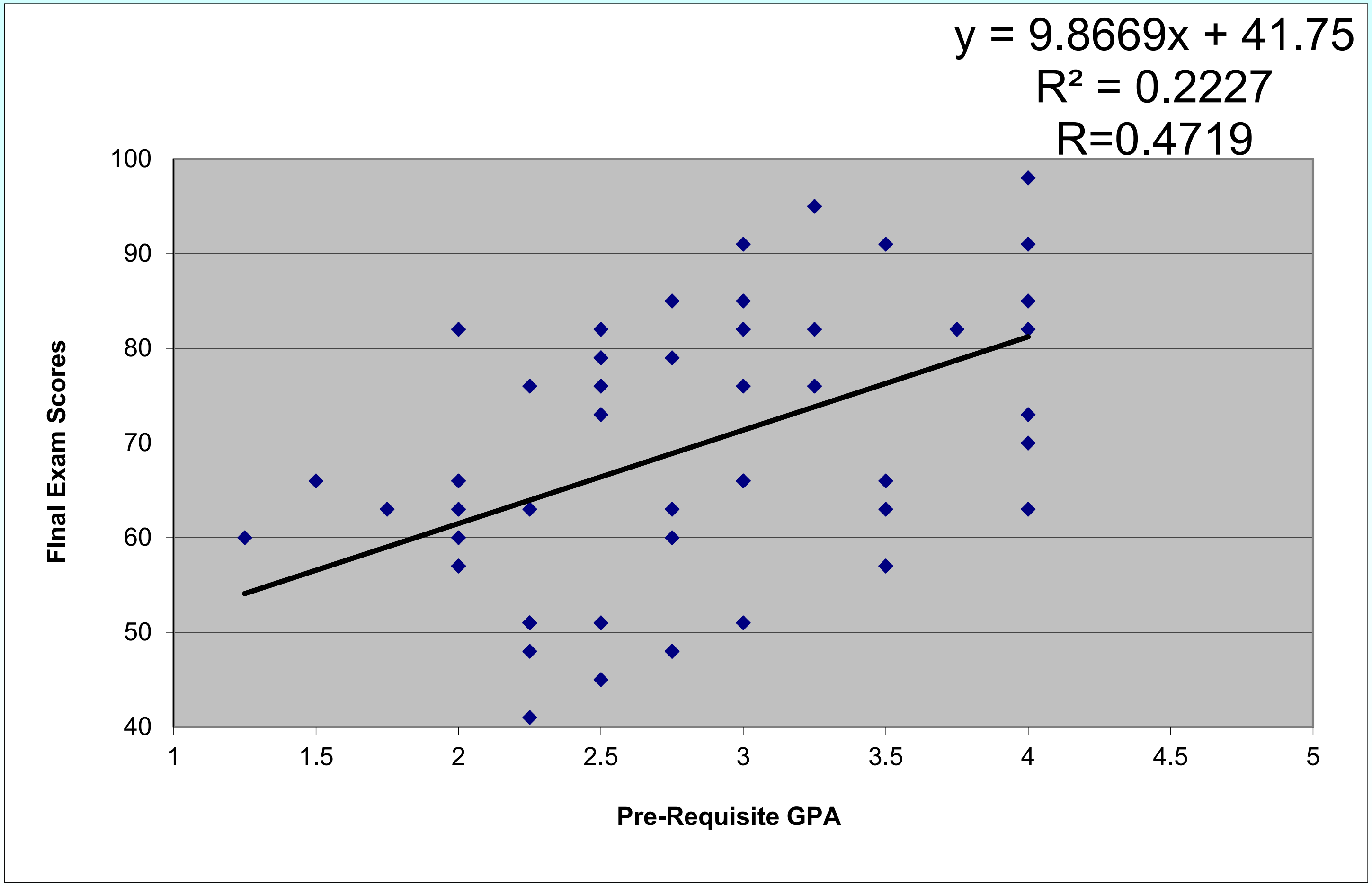
Caution in use of r^2

- Increase in spread of regressor variable (x) in y vs. x increases r^2
- Large regression slope artificially yields high r^2
- Large r^2 does not measure appropriateness of the linear model
- Large r^2 does not imply regression model will predict accurately

Final Exam Grade



Final Exam Grade vs Pre-Req GPA



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4. Model meets assumption of random errors

Model meets assumption of random errors

- Residuals are negative as well as positive
- Variation of residuals as a function of the independent variable is random
- Residuals follow a normal distribution
- There is no autocorrelation between the data points.

Therm exp coeff vs temperature

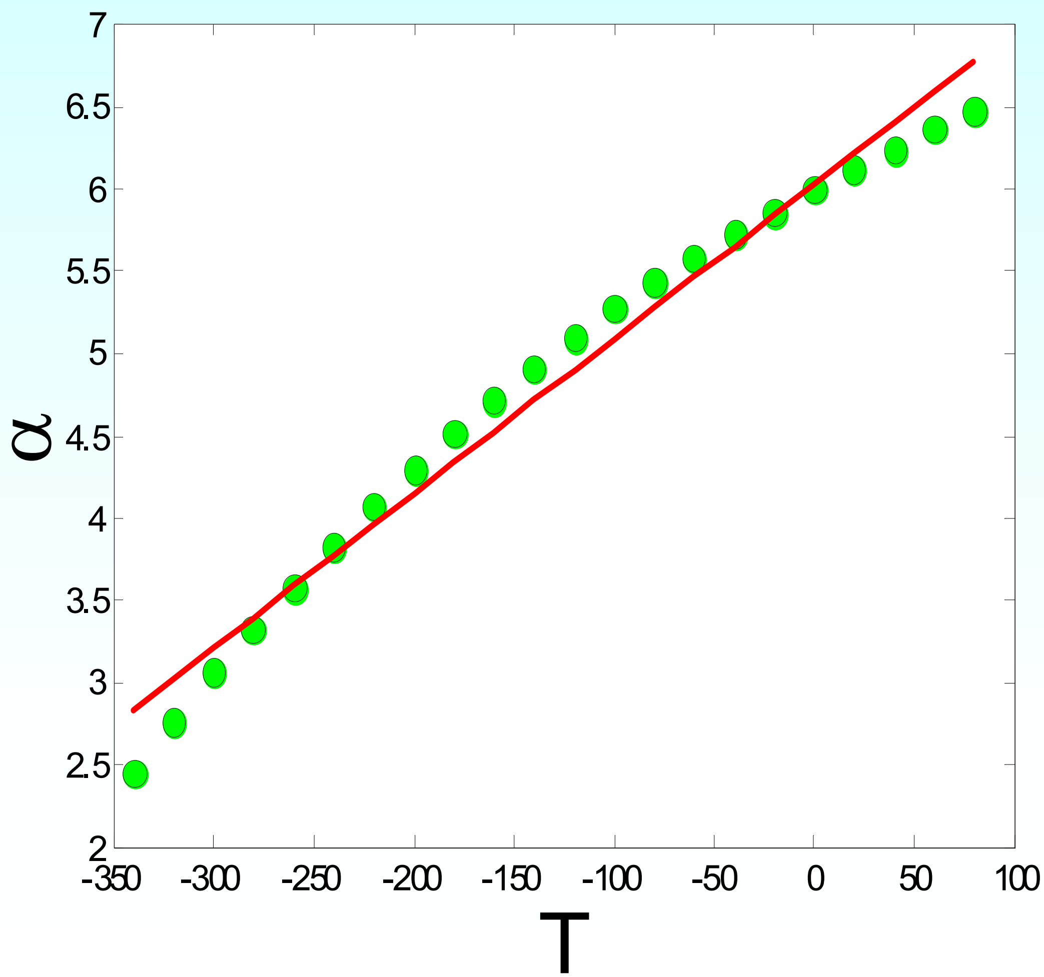
T	α
60	6.36
40	6.24
20	6.12
0	6.00
-20	5.86
-40	5.72
-60	5.58
-80	5.43

T	α
-100	5.28
-120	5.09
-140	4.91
-160	4.72
-180	4.52
-200	4.30
-220	4.08
-240	3.83

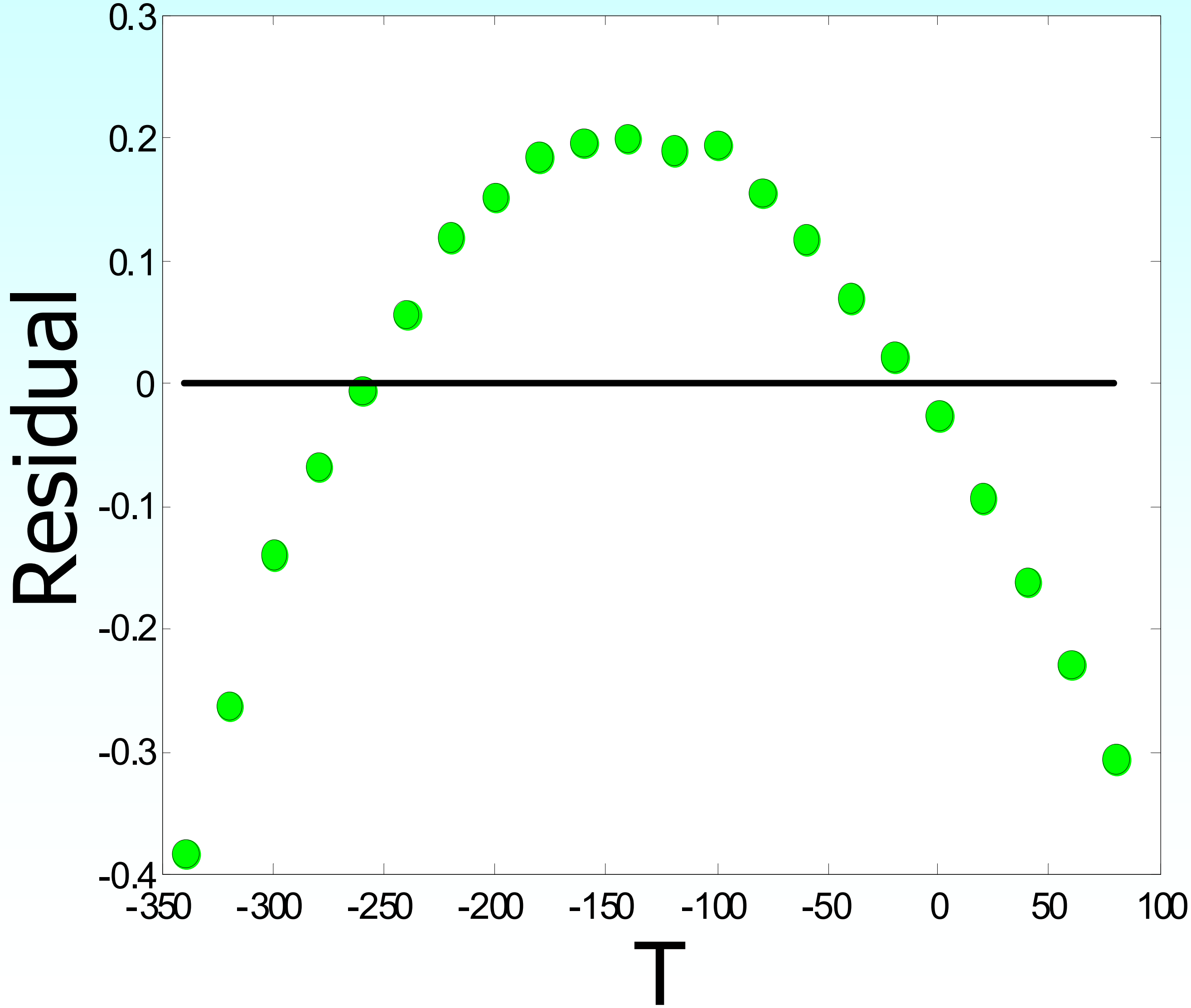
T	α
-280	3.33
-300	3.07
-320	2.76
-340	2.45

Data and model

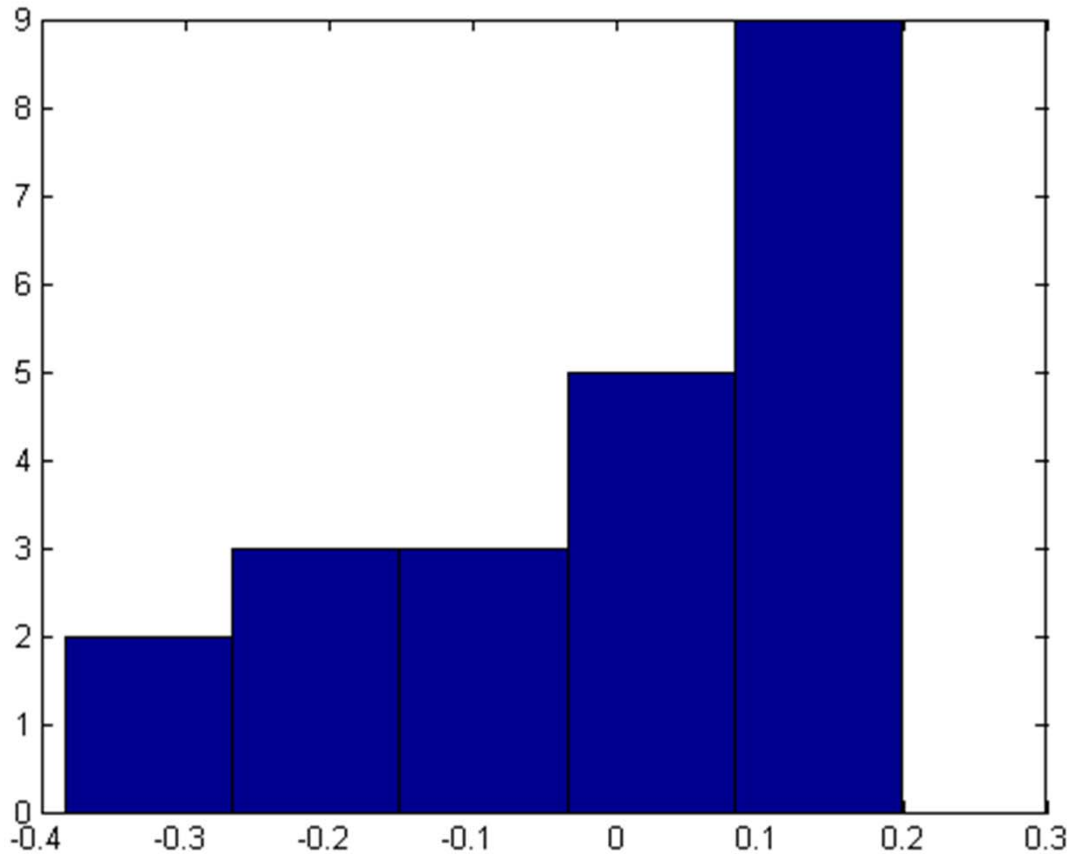
$$\alpha = 6.0248 + 0.0093868T$$



Plot of Residuals



Histograms of Residuals



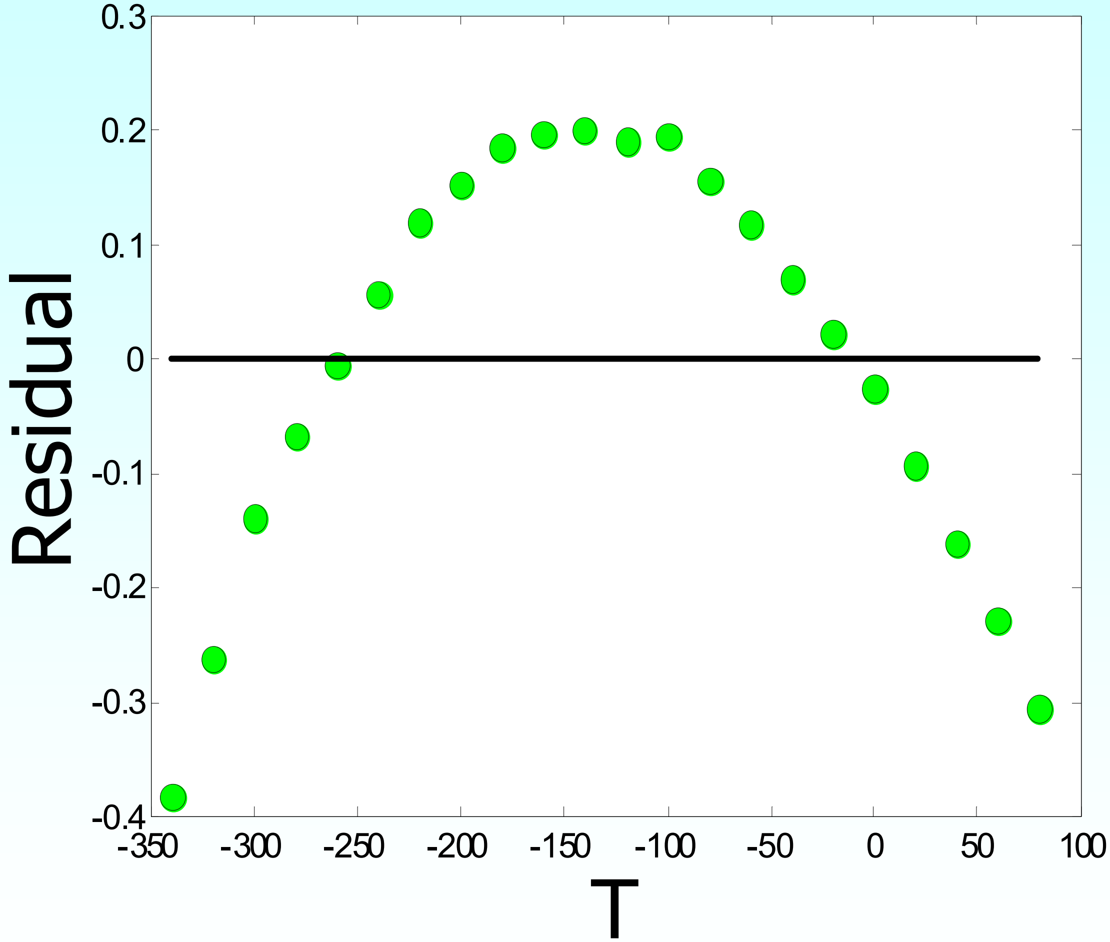
Check for Autocorrelation

- Find the number of times, q the sign of the residual changes for the *n* data points.
- If $(n-1)/2-\sqrt{(n-1)} \leq q \leq (n-1)/2+\sqrt{(n-1)}$, you most likely do not have an autocorrelation.

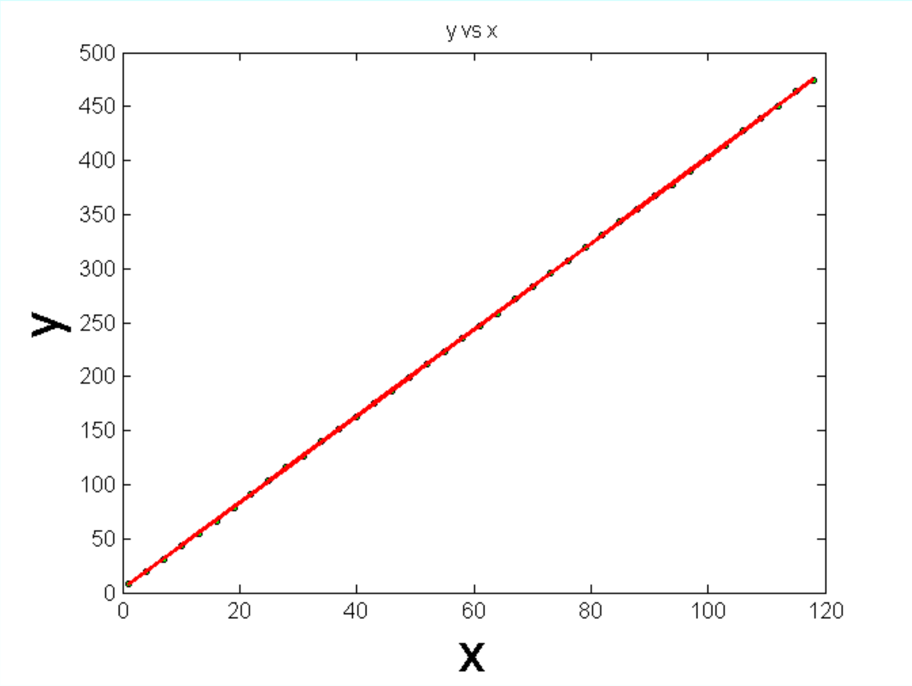
$$\frac{(22-1)}{2}-\sqrt{22-1} \leq q \leq \frac{22-1}{2}+\sqrt{22-1}$$

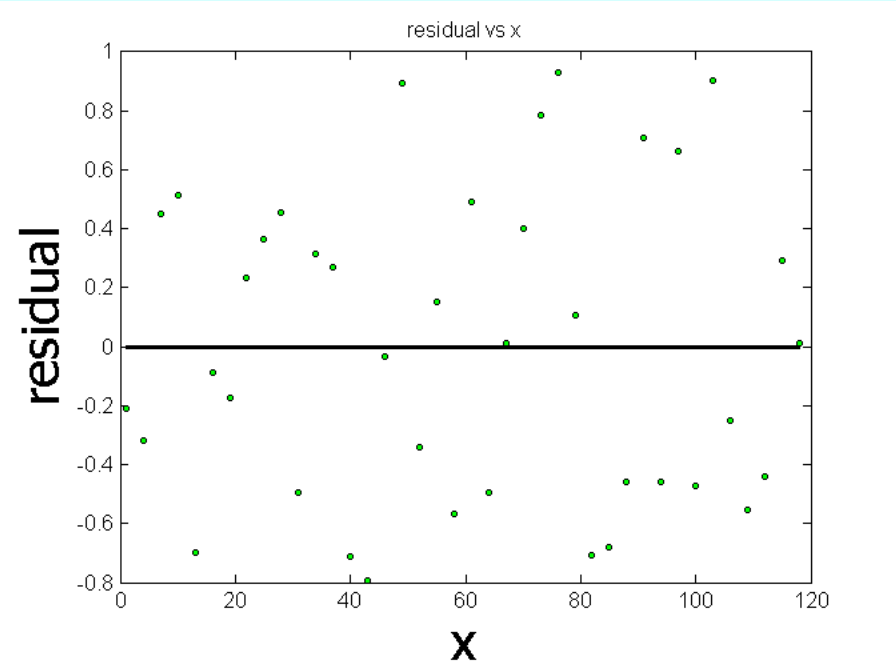
$$5.9174 \leq q \leq 15.083$$

Is there autocorrelation?



y vs x fit and residuals



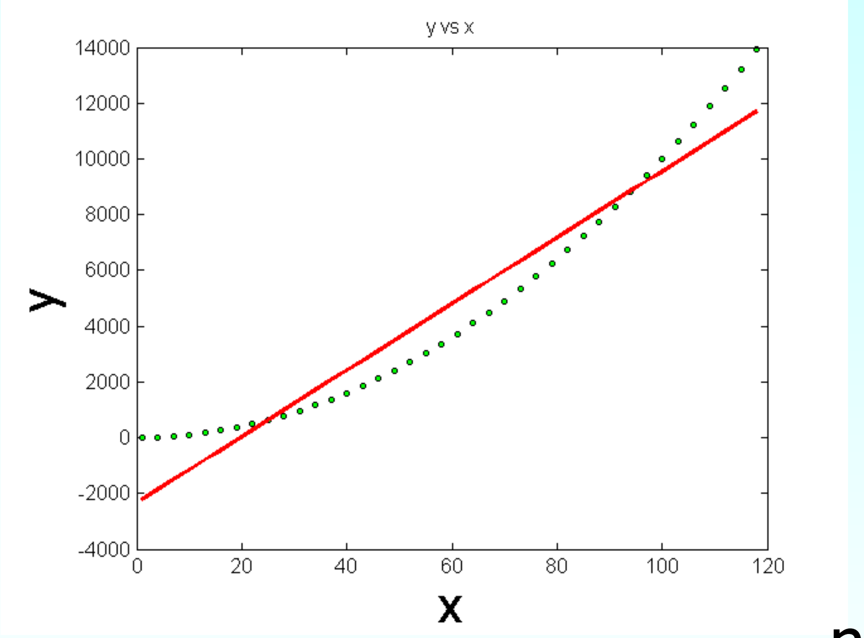


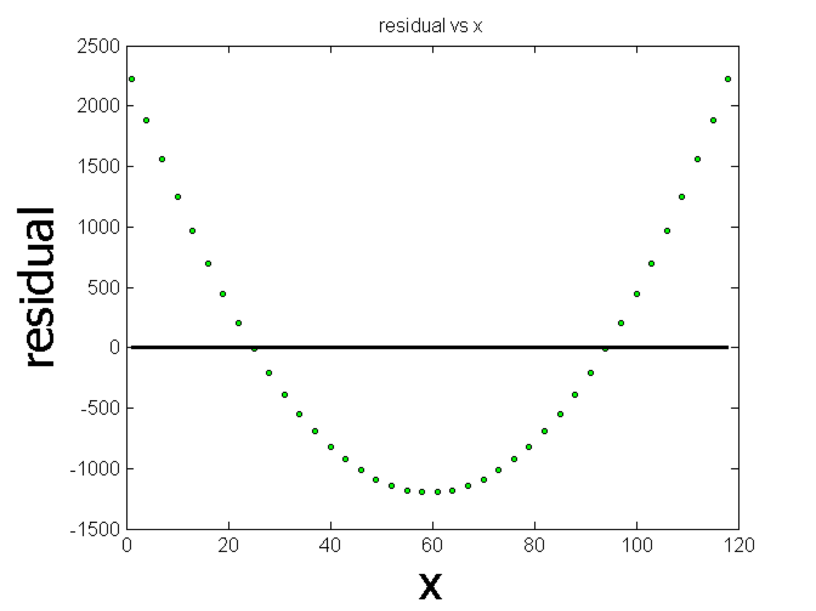
n=40

(n-1)/2-√(n-1) ≤p≤ (n-1)/2+√(n-1)

Is 13.3≤21≤ 25.7? Yes!

y vs x fit and residuals





n=40

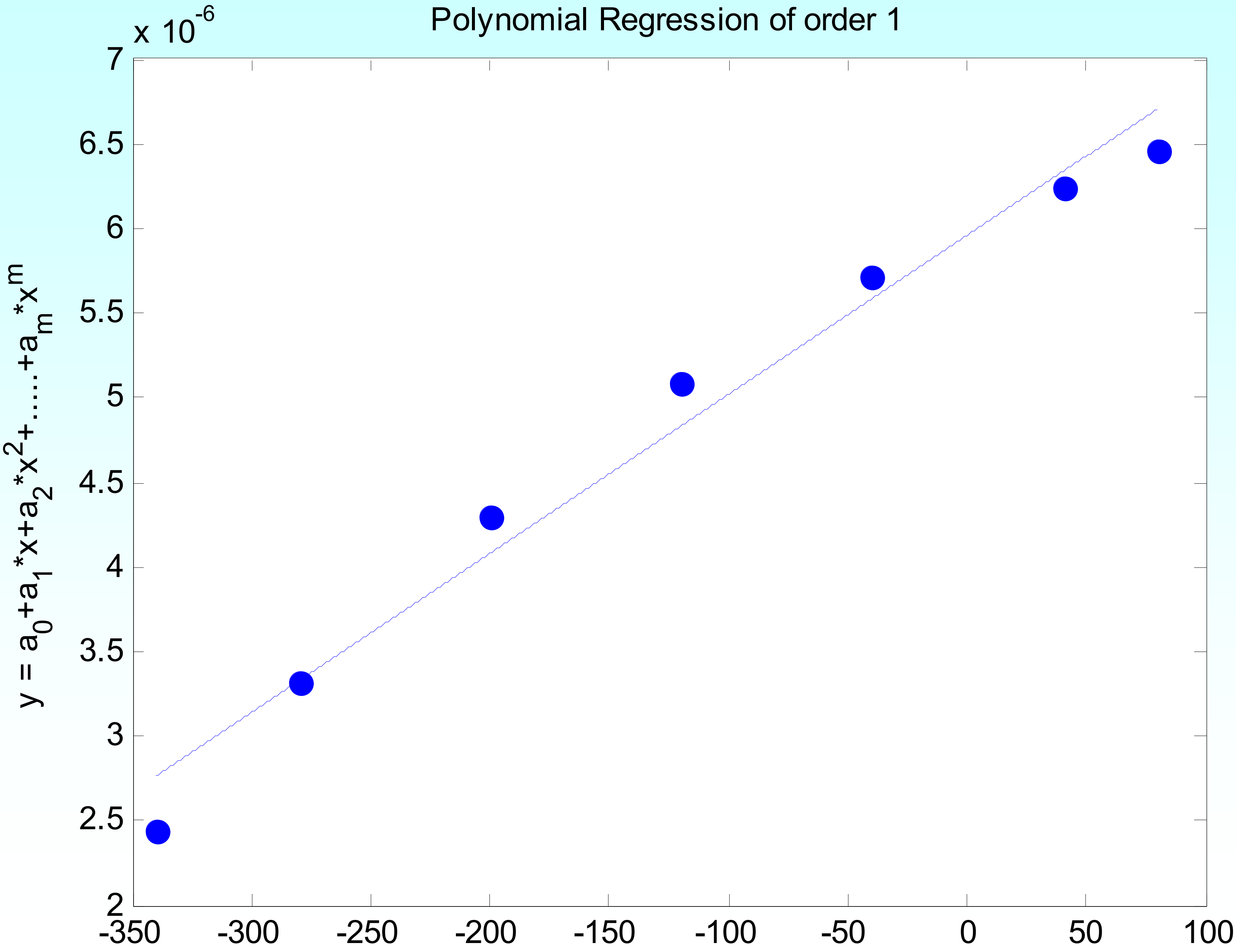
(n-1)/2-√(n-1) ≤p≤ (n-1)/2+√(n-1)

Is 13.3≤2≤ 25.7? No!

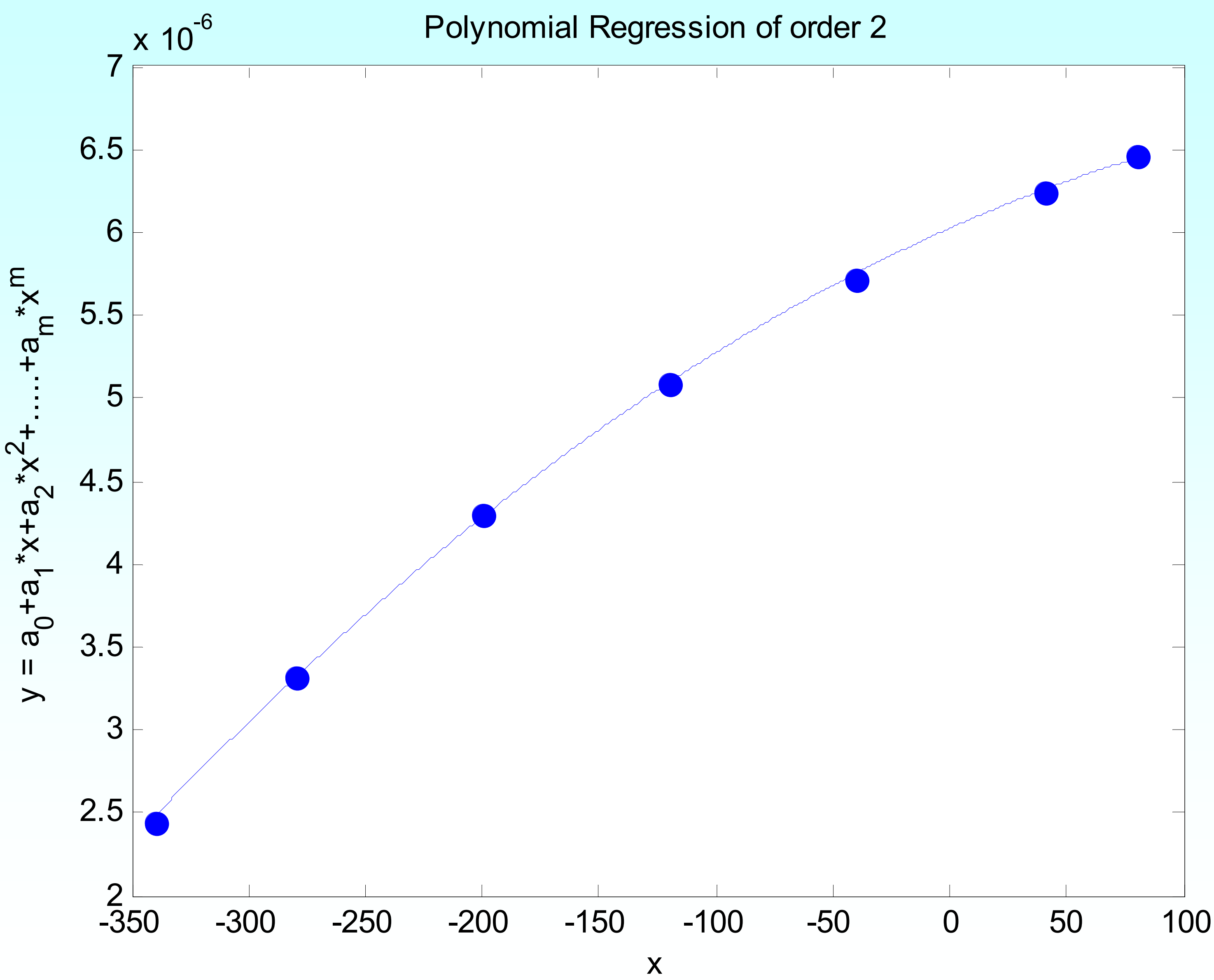
END

What polynomial model to choose if one needs to be chosen?

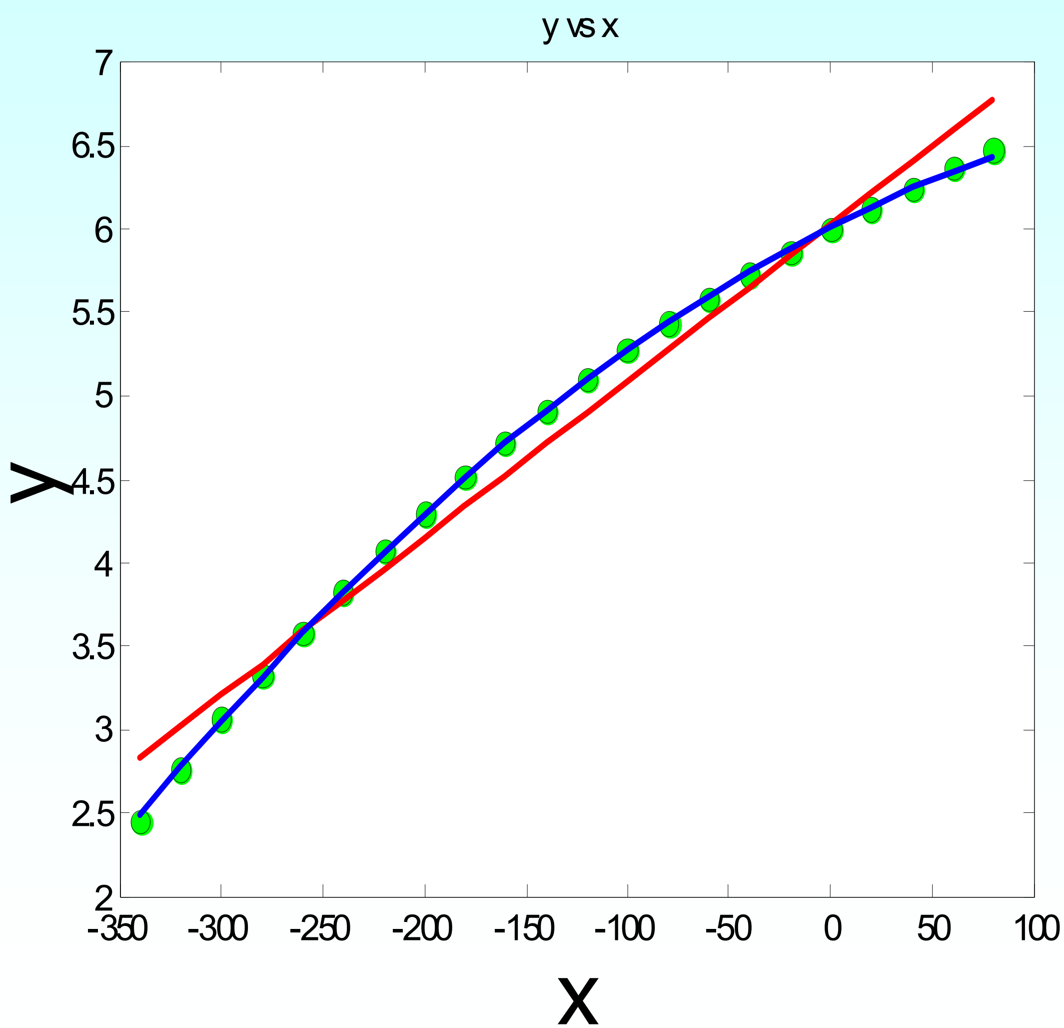
First Order of Polynomial



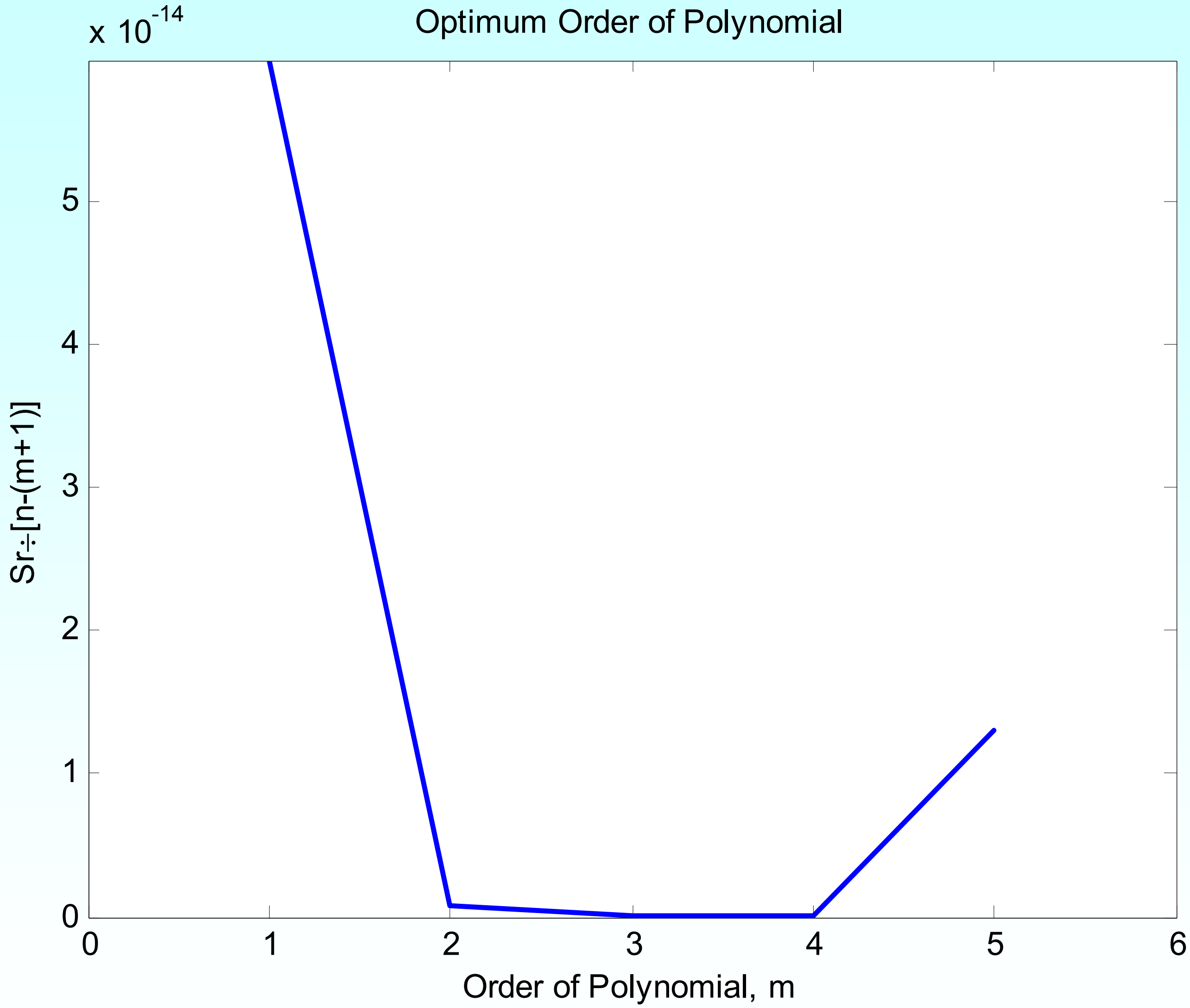
Second Order Polynomial



Which model to choose?



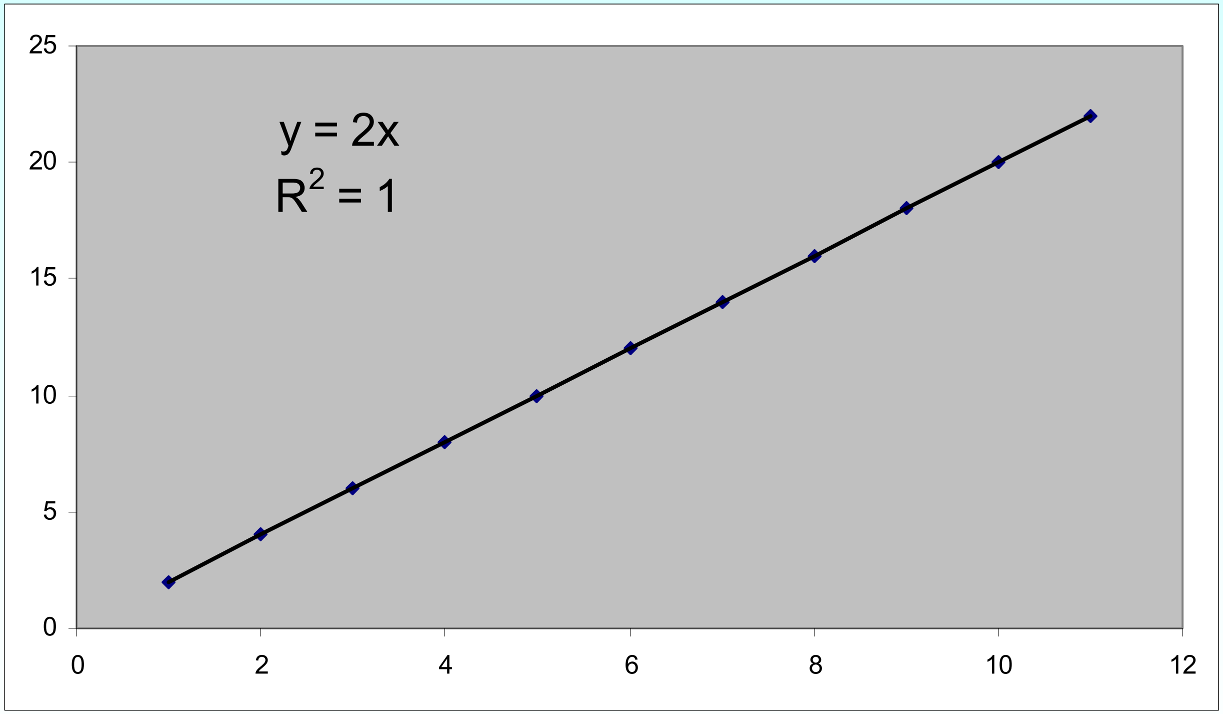
Optimum Polynomial



THE END

Effect of an Outlier

Effect of Outlier



Effect of Outlier