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CSE330: Numerical Methods **Quiz 4 SET B**
Full Marks: 25 Time: 35 Mins

12.5

1) Let $f(x) = 2x^2 + \cos(x) + \ln x$. Find Forward difference approximation of second derivative of $f(2)$ with $\Delta x = 0.4$. Find the exact value of $f''(x)$ at $x=2$. Also find the absolute relative true error.
Formula: $f''(x) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{(\Delta x)^2}$ [6.5+3+3]

1. $f(x) = 2x^2 + \cos x + \ln x$

$x_i = 2$
$x_{i+1} = 2 + 0.4 = 2.4$
$x_{i+2} = 2.4 + 2.8 = 2.8$

$f(x_i) = 8.277000344$ (i) ✓
$f(x_{i+1}) = 11.65807502$ (ii) ✓
$f(x_{i+2}) = 15.76739708$ (iii) ✓

x^{-1}
$\frac{1}{x} = -1 x^{-2}$

$$f''(x) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{(\Delta x)^2}$$

$$f''(2) = \frac{f(2.8) - 2f(2.4) + f(2)}{(0.4)^2}$$ ✓

$f''(2) = 4.551546106$ (Ans) ✓

2nd part

$f(x) = 2x^2 + \cos x + \ln x$

$f'(x) = 4x - \sin x + \frac{1}{x}$

$f''(x) = 4 - \cos x - \frac{1}{x^2}$

$\therefore f''(2) = 4 - \cos 2 - \frac{1}{4}$

$f''(2) = 4.166146837$ (Ans) ✓

2nd part

R. True error $= \left| \frac{\text{Exact value} - \text{Approx.}}{\text{Exact value}} \right| \times 100\%$

$= \left| \frac{4.166146837 - 4.551546106}{4.166146837} \right| \times 100\%$

$= 9.250736573\ \%$

R. True Error = 9.2507 % (Ans) ✓

2) Let $f(x) = \int_1^{25} (2x + \cos(2\sqrt{x}))dx$ Find numerically approximate the integral using Simpsons' 1/3 rule with $n=6$. Find exact solution. Also find the absolute relative true error.

$\int_a^b f(x)dx = \frac{h}{3}[f(x_0) + 4\sum_{i=1,i=odd}^{n-1} f(x_i) + 2\sum_{i=2,i=even}^{n-2} f(x_i) + f(x_n)]$ [6.5+3+3]

1st part

$$h = \frac{b-a}{n} = \frac{25-1}{6} = 4$$

$a = x_0 = 1$

$x_1 = 1+4 = 5$

$x_2 = 5+4 = 9$

$x_3 = 9+4 = 13$

$x_4 = 13+4 = 17$

$x_5 = 17+4 = 21$

$b = x_6 = 21+4 = 25$

$f(x_0) = 1.583853163$ (A)

$f(x_1) = 9.762051608$ (A)

$f(x_2) = 18.96017029$ (A)

$f(x_3) = 26.59956226$ (B)

$f(x_4) = 33.61775034$ (C)

$f(x_5) = 41.03351409$ (D)

$f(x_6) = 49.16092847$ (X)

$$\int_a^b f(x)dx = \frac{h}{3}\left[f(x_0) + 4\{f(x_1)+f(x_3)+f(x_5)\} + 2\{f(x_2)+f(x_4)\} + f(x_6)\right]$$

$$= \frac{4}{3}\left[f(1) + 4\{f(5)+f(13)+f(21)\} + 2\{f(9)+f(17)\} + f(25)\right]$$

$$= \frac{4}{3}\left[1.583853163 + 4\{77.39506796\} + 2\{52.57792063\} + 49.16092847\right]$$

$$= \frac{4}{3}[465.4808947]$$

$$\int_a^b f(x)dx = 620.641193 \text{ (Ans)}$$

2nd part

$$\int_1^{25} 2x + \cos(2x^{1/2})\,dx$$

$\int x x^{1/2} = \frac{2x^{3/2}}{3/2} = \frac{4}{3}x^{3/2}$

$$= \left[x^2 + \frac{\sin(2x^{1/2})}{2 \cdot \frac{1}{2} x^{-1/2}}\right]_1^{25}$$

$$= \left[x^2 + \frac{\sin(2x^{1/2})}{x^{-1/2}}\right]_1^{25}$$

$= 623.3147628$ (Ans)

Exact value $= 620.1591347$ (Ans)

-1.004

$$\text{True error} = \left| \frac{\text{True} - \text{Appr.}}{\text{True}} \right| \times 100\%$$

$$= 0.07773199193\%$$

$$\text{R.T.E} = 0.07773\%$$

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Quiz 1 SET A

1. Solve the following system using LU decomposition method: (12.5)

$$3a + 8b + 14c = 13$$
$$2a + 6b + 13c = 4$$
$$a + 2b + 4c = 3$$

$$\begin{pmatrix} 3 & 8 & 14 \\ 2 & 6 & 13 \\ 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 13 \\ 4 \\ 3 \end{pmatrix}$$

$[A] = [L] \times [U]$

$$\begin{bmatrix} 3 & 8 & 14 \\ 2 & 6 & 13 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \times \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$l_{21} = 0.6667$

. Determine the matrix form of all the equations using quadratic spline (show all the stapes)

Sl. No.	X	Y
1	3	4.0
2	3.7	5.2
3	4.3	6.4
4	4.9	7.4