# CSE 221: Algorithms

## Sorting lower bounds and Linear time sorting

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**References**

1. T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms, Second Edition*. The MIT Press, September 2001.
2. Erik Demaine and Charles Leiserson, *6.046J Introduction to Algorithms*. MIT OpenCourseWare, Fall 2005. Available from: `ocw.mit.edu/OcwWeb/Electrical-Engineering-and-Computer-Science/6-046JFall-2005/CourseHome/index.htm`

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# Contents

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# What's the best we can do?

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- Heapsort, mergesort . . . $O(n \lg n)$

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Can a sorting algorithm do better than $O(n \lg n)$ in the worst-case?

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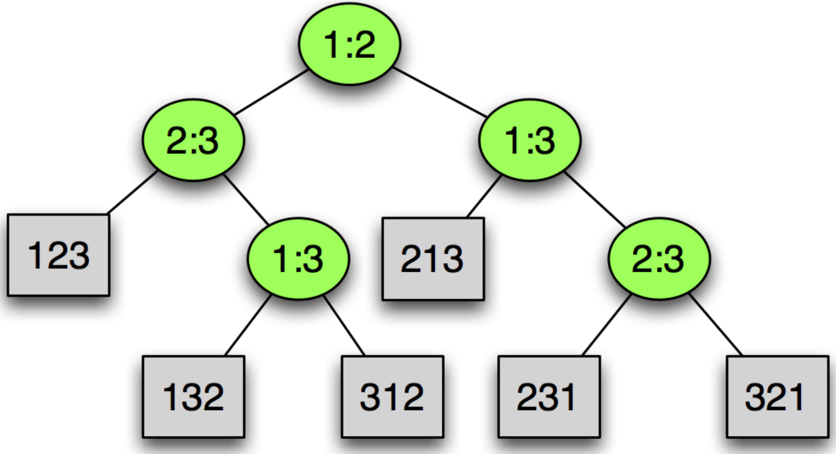
Can a sorting algorithm do better than $O(n \lg n)$ in the worst-case?

We can use a decision tree to answer this question.

# Decision tree for comparison based sorting

Sequence $A = \langle 9, 4, 6 \rangle$



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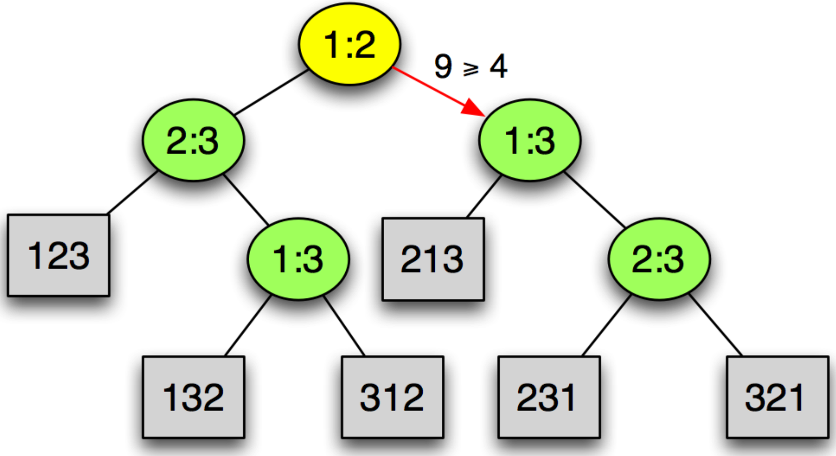

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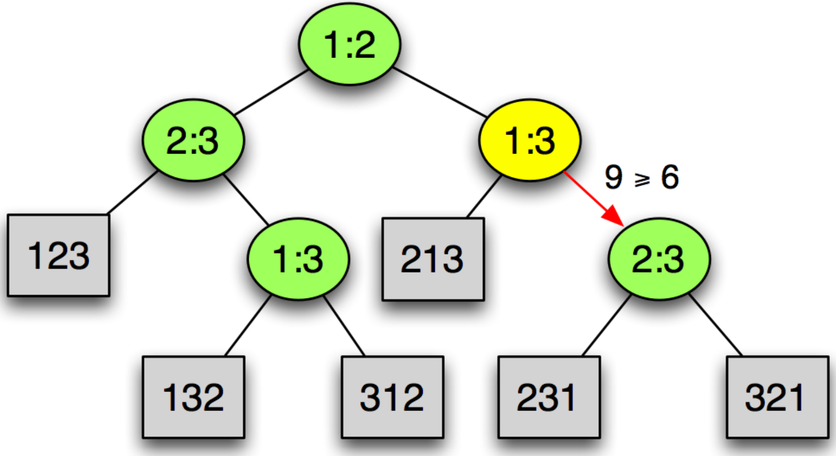

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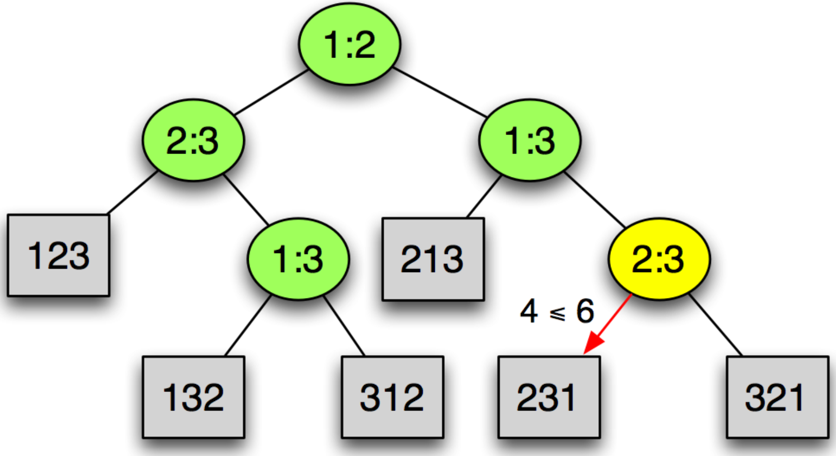
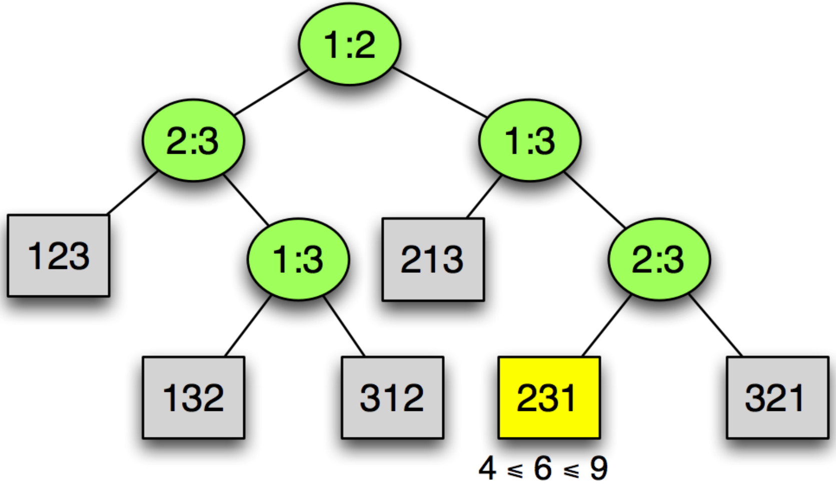


# Decision tree for comparison based sorting

Sequence $A = \langle 9, 4, 6 \rangle$ $\Longrightarrow$ Sequence $B = \langle 4, 6, 9 \rangle$



# Lower bound on comparison based sorting

**Question**

What is the best that we can do with comparison-based sorting?

- $n!$ possible permutations, one of which is the sorted sequence.
- Minimum number of comparisons is the path from root of the decision tree to one of the $n!$ leaves.
- Height of a binary tree with $n!$ leaves is $\lceil \lg n! \rceil$. (Note: by Stirling's approximation $n! \geq (n/e)^n$, where $e$ is Euler's constant.)

$$\begin{aligned} h = \lceil \lg n! \rceil &\geq \lceil \lg((n/e)^n) \rceil = \lceil n \lg n - n \lg e \rceil \\ &= \Omega(n \lg n) \end{aligned}$$

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$$h = \lceil \lg n! \rceil \geq \lceil \lg((n/e)^n) \rceil = \lceil n \lg n - n \lg e \rceil$$
$$= \Omega(n \lg n)$$

**Theorem**

*The worst-case asymptotic time complexity for any comparison-based sorting algorithm is* $\boxed{\Omega(n \lg n)}$.

# Contents

# Sorting in linear time: Counting sort

**Counting sort**: No comparisons between elements.

- **Input**: $A[1 \mathinner{.\,.} n]$, where $A[j] \in \{1, 2, \ldots, k\}$.
- **Output**: $B[1 \mathinner{.\,.} n]$, sorted.
- **Auxiliary storage**: $C[1 \mathinner{.\,.} k]$.

# Sorting in linear time: Counting sort

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- **Auxiliary storage**: $C[1 \,..\, k]$.

## Counting sort algorithm

$\textbf{for } i \leftarrow 1 \textbf{ to } k$

$\quad \textbf{do } C[i] \leftarrow 0$

$\textbf{for } j \leftarrow 1 \textbf{ to } n$

$\quad \textbf{do } C[A[j]] \leftarrow C[A[j]] + 1 \qquad \triangleright\ C[i] = |\{key = i\}|$

$\textbf{for } i \leftarrow 2 \textbf{ to } k$

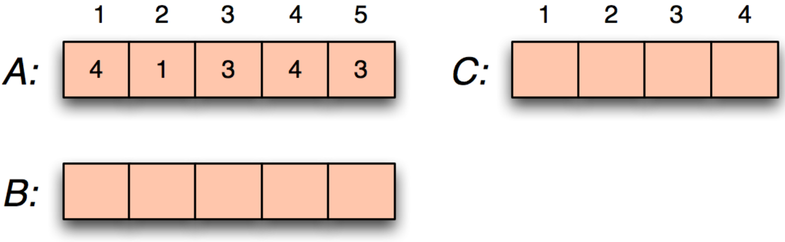
$\quad \textbf{do } C[i] \leftarrow C[i] + C[i-1] \qquad \triangleright\ C[i] = |\{key \le i\}|$

$\textbf{for } j \leftarrow n \textbf{ downto } 1$

$\quad \textbf{do } B[C[A[j]]] \leftarrow A[j]$

$\quad \phantom{\textbf{do }} C[A[j]] \leftarrow C[A[j]] - 1$

# Counting sort example



# Counting sort example: loop 1

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| *A*: | 4 | 1 | 3 | 4 | 3 |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| *C*: | 0 | 0 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| *B*: | | | | | |

**for** $i \leftarrow 1$ **to** $k$
 **do** $C[i] \leftarrow 0$

# Counting sort example: loop 2

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| A: | 4 | 1 | 3 | 4 | 3 |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| C: | 0 | 0 | 0 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| B: | | | | | |

**for** $j \leftarrow 1$ **to** $n$
    **do** $C[A[j]] \leftarrow C[A[j]] + 1$    $\triangleright\ C[i] = |\{key = i\}|$

# Counting sort example: loop 2

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| *A:* | 4 | 1 | 3 | 4 | 3 |

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| | | | | | |
|---|---|---|---|---|---|
| *B:* | | | | | |

$$\textbf{for } j \leftarrow 1 \textbf{ to } n$$
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| | | | | | |
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**for** $j \leftarrow 1$ **to** $n$

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|---|---|---|---|---|---|
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| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| *C:* | 1 | 0 | 1 | 2 |

| | | | | | |
|---|---|---|---|---|---|
| *B:* | | | | | |

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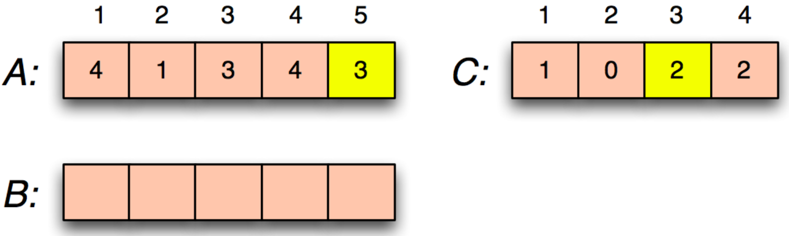


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# Counting sort example: loop 3

A:

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 4 | 1 | 3 | 4 | 3 |

B:

| | | | | |
|---|---|---|---|---|
| | | | | |

C:

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| 1 | 0 | 2 | 2 |

C′:

| | | | |
|---|---|---|---|
| 1 | 1 | 2 | 2 |

**for** $i \leftarrow 2$ **to** $k$
  **do** $C[i] \leftarrow C[i] + C[i-1]$  $\triangleright\ C[i] = |\{key \leq i\}|$

# Counting sort example: loop 3

A:

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 4 | 1 | 3 | 4 | 3 |

C:

| 1 | 2 | 3 | 4 |
|---|---|---|---|
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B:

| | | | | |
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| *A*: | 4 | 1 | 3 | 4 | 3 |
| *B*: | | | | | |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| *C*: | 1 | 0 | 2 | 2 |
| *C'*: | 1 | 1 | 3 | 5 |

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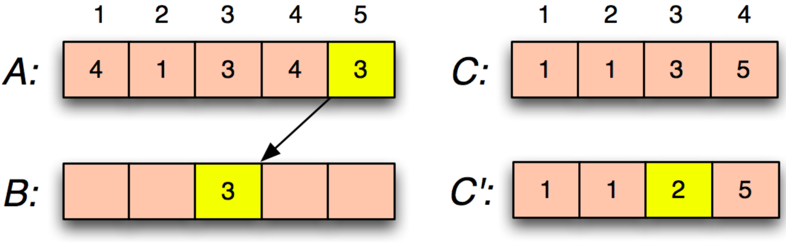


**for** $j \leftarrow n$ **downto** $1$
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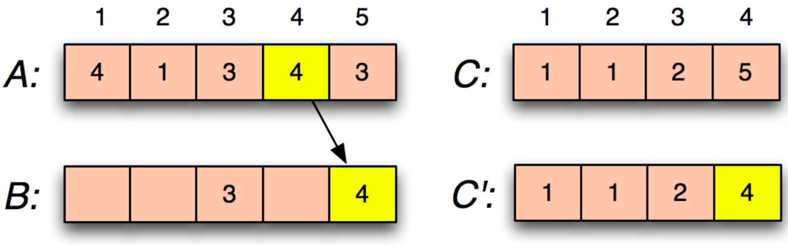


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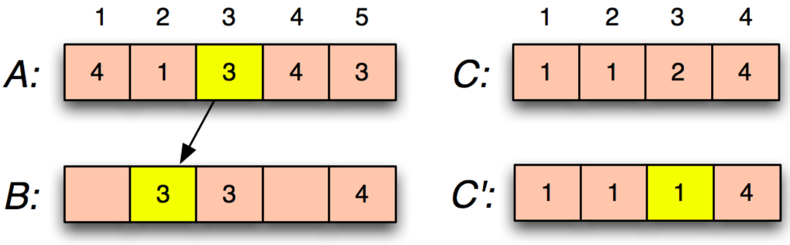


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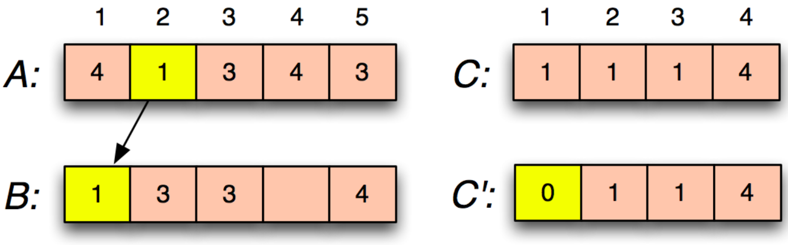


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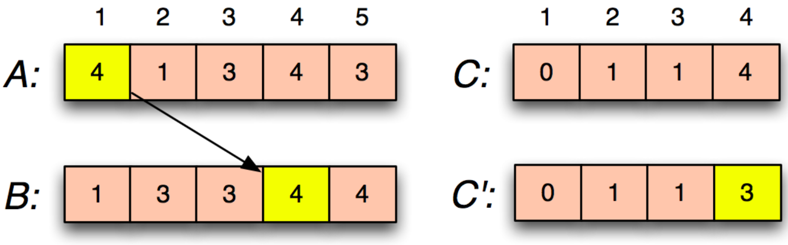


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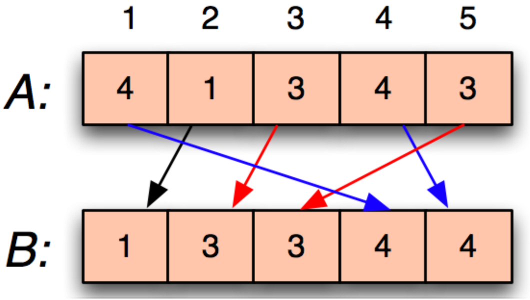

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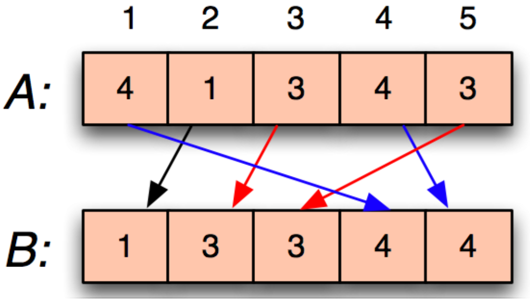


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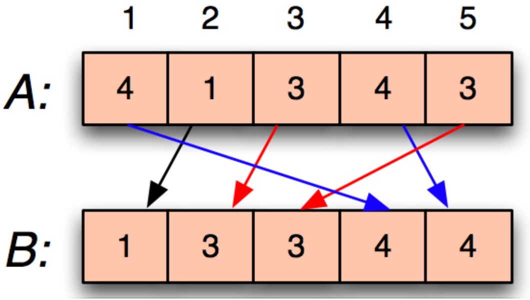


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- Would it still be stable if we had used **for** $j \leftarrow 1$ **to** $n$ instead of **for** $j \leftarrow n$ **downto** $1$?
- What other sort algorithms that you've seen so far are stable?

# Counting sort complexity

$$
\begin{aligned}
&\textbf{for } i \leftarrow 1 \textbf{ to } k \\
&\qquad \textbf{do } C[i] \leftarrow 0 \\
&\textbf{for } j \leftarrow 1 \textbf{ to } n \\
&\qquad \textbf{do } C[A[j]] \leftarrow C[A[j]] + 1 \\
&\textbf{for } i \leftarrow 2 \textbf{ to } k \\
&\qquad \textbf{do } C[i] \leftarrow C[i] + C[i-1] \\
&\textbf{for } j \leftarrow n \textbf{ downto } 1 \\
&\qquad \textbf{do } B[C[A[j]]] \leftarrow A[j] \\
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\end{aligned}
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\Theta(k) \quad \left\{ \begin{array}{l} \textbf{for } i \leftarrow 1 \textbf{ to } k \\ \qquad \textbf{do } C[i] \leftarrow 0 \end{array} \right.
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\hline
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\end{array}
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- So what's wrong with this picture?

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- But didn't we just prove that sorting takes $\Omega(n \lg n)$ time?
- So what's wrong with this picture?

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The worst-case running time of Counting sort is $O(n + k)$.

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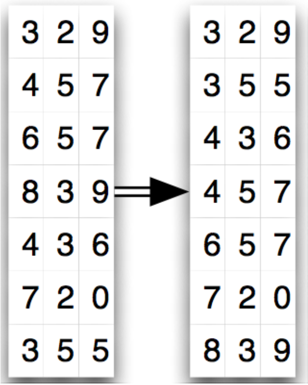
- In fact, counting sort does not use a single comparison.

# Radix sort




## Radix sort basics

- Digit by digit sort.
- Can be either *most-significant* digit first, or *least-significant* digit first.
- A good way is to stably sort *least-significant* digit first.

# Radix sort in action

| | | |
|---|---|---|
| 3 | 2 | 9 |
| 4 | 5 | 7 |
| 6 | 5 | 7 |
| 8 | 3 | 9 |
| 4 | 3 | 6 |
| 7 | 2 | 0 |
| 3 | 5 | 5 |

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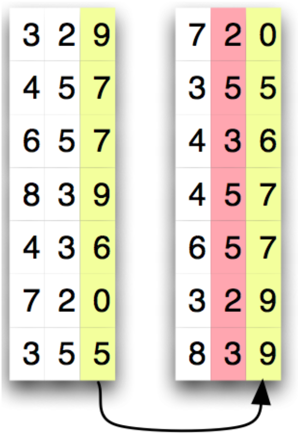

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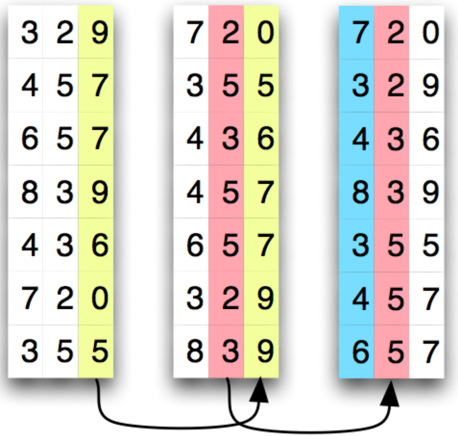


  

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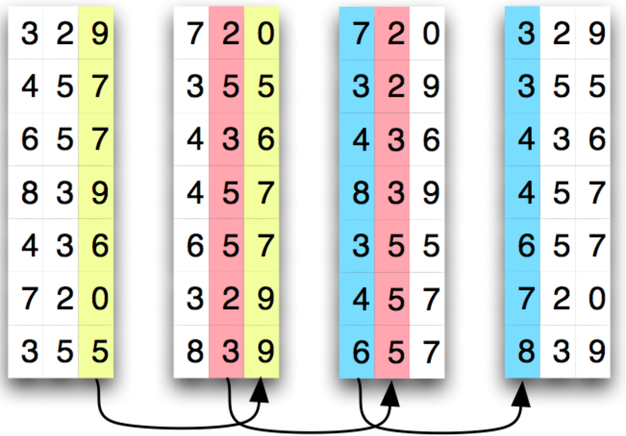

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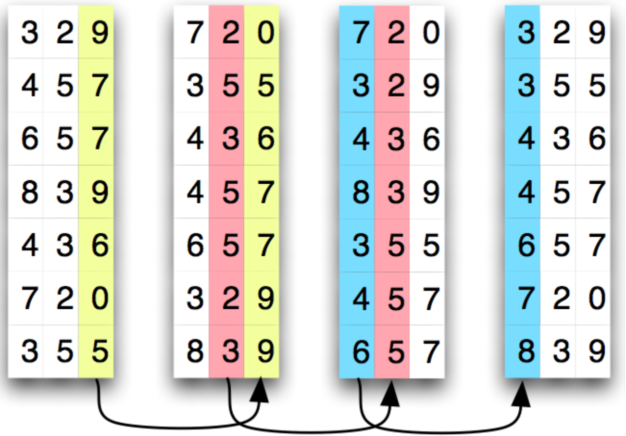


Analysis: For numbers in the range $[0 \,..\, n^d - 1]$, radix sort runs in $\Theta(dn)$ time.

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- Why do we recommend sorting *least-significant* digits first in radix sort?