BRAC UNIVERSITY

Inspiring Excellence

Department of Mathematics and Natural Sciences

Mid-term Examination

Semester: Fall 2015

Course Title: Linear Algebra and Fourier Analysis

Course No.: MAT216

Section: 04

Time: 1 hour
Total Marks: 40

Date: Oct 26, 2015

Answer any <u>FOUR</u>:

1. Define a system of linear equations, and consistent and inconsistent systems. Determine the values of the parameter, λ, such that the following system has: (i) no solution, (ii) unique solution, (iii) infinite solutions. Also solve the system. [10]

$$\begin{aligned} x + y + \lambda z &= 2 \\ 3x + 4y + 2z &= \lambda \\ 2x + 3y - z &= 1. \end{aligned}$$

2. Define elementary row operations. Transform the following system into the matrix equation of the form $AX = B$ and solve by using $X = A^{-1}B$. [10]

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 6 \\ 2x_1 + 4x_2 + 3x_3 &= 3 \\ x_1 + 3x_2 + 3x_3 &= 4 \end{aligned}$$

3. Define vector space and subspace with examples. Show that the set of all 3×2 matrices, $M_{3\times 2}$, is a vector space under the matrix addition and scalar multiplication. [10]

4. Define basis and dimension of a vector space. Determine whether the following set of vectors is a basis of $\mathbb{R}^3$. [10]

$$S = \{(1,1,2), (1,0,1), (1,-1,2)\}$$

Determine whether the vector $(2,-1,3)$ is a linear combination of vectors in S.

5. Find the bases for the row space, column space, and nullspace of [10]

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 3 & 6 & -5 & 4 \\ 1 & 2 & 0 & 3 \end{pmatrix}.$$

Also find the rank and nullity of A.