Linear Independence & Linear Dependence

A collection of vector is either linearly independent or linearly dependent.

The vector $\{\bar{v}_1, \bar{v}_2, \cdots, \bar{v}_n\}$ are linearly independent if the equation involving linear combinations

$$a_1\bar{v}_1 + a_2\bar{v}_2 + \cdots + a_n\bar{v}_n = 0$$

is true only when the scalars a_i are all equal to zero. The vectors are linearly dependent if the equation has a solution when at least one of the scalars is not zero.

To test for linear independence, we can write the corresponding matrix is echelon form.

So - are the following vector linearly independent or dependent?

ⓐ $\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} \right\}$ ⇒ • Nontrivial
• Linearly dependent.

ⓑ $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 7 \end{bmatrix} \right\}$ ⇒ • Trivial solution
• Linearly Independent.

Solution:

$$a_1 \begin{bmatrix} 1 \\ 2 \\ 1 \\ 4 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} + a_3 \begin{bmatrix} 2 \\ 0 \\ 1 \\ 7 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 4 & 1 & 7 & 0 \end{array}\right]$$

- - - - - - - - - -

Lec: 06

The Basis of a vector space:

A set of vector $\{\bar{v}_1, \bar{v}_2, ----, \bar{v}_n\}$ is said to form a basis for a vector space if

① The vectors $\bar{v}_1, \bar{v}_2, ----, \bar{v}_n$ span the vector space

② The vectors $\bar{v}_1, \bar{v}_2, ---, \bar{v}_n$ are linearly independent.

For example: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ are a basis for $\mathbb{R}^3$

Do the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ form a basis for $\mathbb{R}^3$?

1st step:

Independent:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 0 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow$$ Trivial Solution.

Linearly independent.

2nd step:

$$a_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + a_3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$