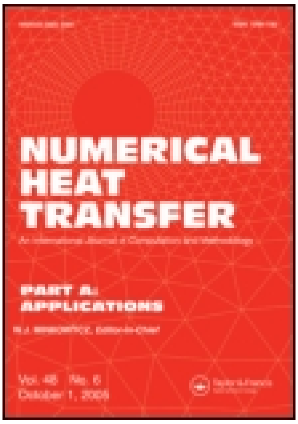

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A Review on the Application of the Lattice Boltzmann Method for Turbulent Flow Simulation

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A REVIEW ON THE APPLICATION OF THE LATTICE BOLTZMANN METHOD FOR TURBULENT FLOW SIMULATION

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The Lattice Boltzmann Method (LBM) is a potent numerical technique based on kinetic theory, which has been effectively employed in various complicated physical, chemical, and fluid mechanics problems. In recent years, turbulent flow simulation by using this new class of computational fluid dynamics technique has attracted more attention. In this article, a review of previous studies on turbulence in the frame of LBM is presented. Recent extensions of this method are categorized based on three main groups of turbulence simulation: DNS, LES and RANS methods.

1. INTRODUCTION

The turbulence problem is difficult to figure out in terms of physical understanding and mathematical solutions, or in light of the engineering accuracy needed for different applications that strongly relied on viscid fluid dynamics. Industrial field of global concerns such as the automobile industries, airliner, and turbo machineries are of these groups. Researchers make serious attempts to determine a method which may transfer from the research field to real engineering application. Recently, microscopic dynamics approaches have attracted significant attention. The computational viewpoint of digital fluid dynamic methods such as LBM is vastly different from traditional CFD methods. The idea of digital fluid dynamics relies on the fact that fluid hydrodynamics is not sensitive to the underlying details in microscopic physics. Fluid hydrodynamics are the result of the collective behavior of numerous molecules in the system [1, 2]. In recent years, considerable progress has been made to derive turbulence models from discrete kinetic theory [3–5].

The lattice Boltzmann method was applied to improve lattice gas cellular automata (LGCA) modeling [6–8]. In this approach, fluid flow is simulated by tracing the movement of particles on a discrete lattice. However, the same streaming and collision processes are used as the modern LBM, but the collision operator, instead of a

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NOMENCLATURE

c	speed of sound	w_i	weighting factor in the lattice fluid density
e_i	discrete particle velocity	δ_{ij}	Kronecker delta function
f	single particle distribution function	τ	nondimensional single relaxation time parameter
$\widetilde{f}_i$	post-collision single particle mass density distribution	τ_e	effective turbulence time relaxation parameter
f_i^{eq}	discrete single particle equilibrium mass density distribution	τ_{ij}	discrete Reynolds stress tensor
S_{ij}	discrete large scale strain rate tensor	ω	vorticity stream function
$\overline{u_i u_j}$	i,jth component of the Reynolds stress tensor		

relaxation process, is based on a discrete set of collision rules [9]. This method is capable of recovering the macroscopic motion of the fluid, but is overwhelmed by noise. Pursuing this idea, McNamara and Zanetti [10] proposed substituting the Boolean occupation numbers with a single-particle distribution function to solve the problems with noise, but kept the same collision rules as the LGCA. However, it was unsuccessful to change the viscosity, since this method uses the collision rules from LGCA models [11]. Numerical instability is a critical unconvinced difficulty in the adoption of LGCA methods for realistic flows. Turbulent flows are described by a small viscosity. This is because a small viscosity implies that there are distinguishable structures in a very large range of scales; too many to be directly captured by a simulation. LGCA shows extremely less convenient numerical stability whenever the viscosity is made too small [12]. To solve this problem, Higuera and Jimenez [13] suggested a linear collision operator that is capable of changing in viscosity. The proposed collision operator relaxes the pre-collision particle distributions at a particular site towards their local equilibrium distribution. The relaxation times are related to the transport coefficients, such as viscosity. Eventually, the LBM was established as a numerical method completely independent of the LGCA. The noticeable difference between the LBM and the LGCA method is that the LBM can be used for smaller viscosities without losing stability. Hence, the LBM could be utilized for direct numerical simulation of fluid flows at high Reynolds number [14]. During the last two decades, a noticeable amount of research reported different approaches for the linearized collision operator and equation of the equilibrium distribution function [15, 16]. These advances allow the extension of the LBM to three dimensions, which was previously difficult for the LGCA models.

This improvement process during the last decade has continued and established the LBM as an alternative and successful method for simulation of turbulence effect in complicated physical, chemical, and fluid mechanics problems. The parallel computing nature of the LBM and the simplicity of the boundary condition implementation have then been utilized to propose new methods for complex physical problems, especially turbulence modeling. Simulations of the sedimentation of solid particles in turbulent flows [17, 18], flows around Bluff bodies [19], or particle laden flow calculating [20–23] which presents the influence of the particle phase on the turbulence spectrum and on particle collisions are some examples of such application. The lattice Boltzmann method has also been applied as a powerful method for heat transfer simulation in turbulent flow [24–26]. Biferale et al. [27] presented the results of a high resolution numerical study of 2-D Rayleigh–Taylor turbulence. They utilized a 3-D thermal LBM to investigate velocity and temperature fluctuations. Sajjadi et al. [28] investigated the turbulent natural convection flow through large eddy simulation in the frame of LBM. In this research, streamlines, local and average Nusselt numbers, and isotherm counters have been studied in different Rayleigh numbers. Liu et al. [29] reported the 2-D cavity natural convention turbulent flow simulation with an LBGK method based on LES. Chen and Krafczyk [30] investigated the entropy generation in turbulent natural convection due to internal heat generation in a square cavity varying from the steady laminar symmetric state to the fully turbulent state. Van Treek [31] employed the hybrid lattice Boltzmann method to simulate a 3-D turbulent heat transfer by using a large eddy subgrid scale. The mentioned studies employed a different turbulence modeling suggested in the LBM structure. In the following sections, we will present the recent progress on turbulence modeling in the frame of LBM. In this article, variant extensions of this method will be summarized and discussed based on three main groups of turbulence simulation: DNS, LES, and RANS methods.

2 LATTICE BOLTZMANN METHOD

The lattice Boltzmann method is a developing alternative to Navier–Stokes (NS)-based methods for flow computation [14, 32–34]. The practical approach interpreted in the LBM, consists of solving the lattice Boltzmann equation for the evolution of a single distribution function $f\,(x,\ v,\ t)$ of particles as they move and collide on a lattice. The solution of the equation includes two main steps; the stream step propagates information through the lattice cells, while the collision step normalizes the distribution functions to the equilibrium distribution function. The number of discrete velocity directions standing for the lattice is chosen to respect certain symmetry requirements to recover the isotropy of the viscous stress tensor of the fluid flow [35].

Stream step

$$f_i(x + e_i\Delta t, t + \Delta t) = \widetilde{f}_i(x, t + \Delta t) \tag{1}$$

Collision step

$$\widetilde{f}_i(x, t + \Delta t) = f_i(x, t) - \frac{1}{\tau}\left(f_i(x, t) - f_i^{eq}(x, t)\right) \tag{2}$$

Where τ is the relaxation time, Δt is the time step, $e_i(=\Delta x/\Delta t)$ is the particle velocity in the i-direction, and f_i^{eq} is the equilibrium single-particle distribution.

$$f_i^{eq} = w_i\rho\left[1 + 3\frac{e_i u}{c^2} + \frac{9}{2}\frac{(e_i u)^2}{c^4} - \frac{3}{2}\frac{uu}{c^2}\right] \tag{3}$$

Where $\rho(=\Sigma f_i)$ is the fluid density, $u(\rho u = \Sigma f_i e_i)$ is the fluid velocity, and w is the weighting factor in the lattice fluid density. The relaxation time is related to the kinematic viscosity of the fluid via the following relation:

$$\tau = 3\nu + \frac{1}{2} \tag{4}$$

3. DIRECT NUMERICAL SIMULATION

In its simplest order, fluid turbulence could be described by a single factor, the Reynolds number ($\mathrm{Re} = \frac{UL}{\nu}$), where U is the macroscopic velocity of flow and L is a typical macroscopic scale. Turbulence occurs when $\mathrm{Re} >> 1$, and there is an energy cascade from large scales (L) energy containing flow to small scales (of order $\mathrm{L}/\mathrm{Re}^{\frac{3}{4}}$) where eddy motion is stable, and molecular viscosity is effective in dissipating the kinetic energy [33]. Direct numerical simulation (DNS) resolves all these relevant excited degrees of freedom for turbulent flow. This is a practically impossible task in high Reynolds numbers. For example, computation of flow past an automobile at $\mathrm{Re} \approx 10^6$ requires at least 10^{14} degrees of freedom, which is completely related to our computational ability.

The DNS approach was not reasonable until the 1970s, after computers of sufficient power were created. Direct numerical simulations are often carried out in fully-spectral or pseudo-spectral approaches. These methods are well accepted because of their exceptional accuracy. Navier-Stokes equations are transformed by spectral methods into spectral space for characterization of the flow field as a finite set of basic functions. The governing equations are then solved in spectral space. In this procedure, since the derivatives in the Navier-Stokes equations do not need to be approximated, they are essentially free of truncation errors, and, thus, are very accurate. Another benefit of spectral methods is that the solution of the Poisson equation for pressure, which is an expensive operation computed, is reduced to a simple division in spectral space. Homogeneous isotropic turbulence in three dimensions (3-D) is an approved case in turbulence theory. One of the earliest simulations of turbulence in the frame of LBM was carried out by Benzi and Succi [36]. They have simulated two-dimensional forced isotropic turbulence. In the referred study, by comparing the time evolution of total energy, enstrophy and the energy spectra, the potential of the LBM has been examined against the spectral methods. The LBM recovered the inertial range in the energy spectrum quite similar to the spectral method. They found that the computational efficiency of LBM and spectral code are approximately equal. Martinez et al. [37] made a similar validation for LBM. The initial Reynolds number was 10,000. Martinez et al. had successfully compared the time evolution of the stream functions, spatial distribution, and the vorticity contour plots at a specific time. Small-scale quantities, energy spectra, and fourth-order enstrophy as the functions of time were also studied. Energy spectra comparing have shown that the findings were similar to those found by Benzi and Succi [34] and Suo et al. [36, 38]. Furthermore, they found that the LBM matched very well to the spectral results early in the simulation, but drifted somewhat over time. However, the same vortical structures were observed in both simulations, even if the exact positions and times were slightly different. Waleed et al. [39] carried out a study based on the LBM with a forcing scheme to simulate homogeneous isotropic turbulence. Two models, D3Q15 and D3Q19, were employed with various resolutions. The results of high resolution, regardless of which model was used, show the same turbulence characteristics of the flow compared with the spectral method. In order to create forced turbulent flows, different kinds of forcing methods have been proposed. Cate et al. [40] employed the spectral forcing scheme of Alvelius [41], which controls the power input by getting rid of the force velocity correlation in the Fourier domain and produces anisotropic forcing.

Chen et al. [32] have proposed a new 14-direction lattice Boltzmann model which requires less storage and computational time for solving real three-dimensional problems, based on the linearized Boltzmann approximation with a single time relaxation. This study had utilized a new equilibrium distribution function, leading to a correct equation of state, which has a Galilean-invariant convection term. The numerically measured results agree well with the spectral method for several tests, including the time evolution of energy, enstrophy decay, and vortex evolution in space, except for a small inconsistency at high wave numbers. This evaluation was echoed by Treviño and Higuera [42], who studied the nonlinear stability of Kolmogorov flows applying the pseudo-spectral method and the two types of lattice Boltzmann simulations. Results of streamline and vorticity patterns were presented for different Reynolds and wave numbers.

To date, pseudo-spectral (PS) methods remain as the most accurate numerical tool for direct numerical simulations (DNS) of homogeneous isotropic turbulence (HIT) [43, 44]. Direct numerical simulations of isotropic flows using pseudo-spectral methods began with the work of Orszag and Patterson [43]. The limitation of this method due to the simplicity of the boundary conditions causes it to be preferred in simple geometries, particularly those that can be implemented with periodic boundaries such as channel flow. With regards to this fact, there are many attempts to validate the lattice Boltzmann method for DNS of decaying turbulence to apply for flows in complex geometries of engineering interests.

Yan Peng et al. [45] reported a detailed comparison of the lattice Boltzmann and the pseudo-spectral methods for direct numerical simulations of decaying turbulence in three dimensions. In this research study, instantaneous flow fields such as velocity and vorticity field and low-order statistical quantities like spectrum, the dissipation rate, the total kinetic energy are quantified in both methods. The results show that the LBE method achieves good agreements in comparison with the PS method regarding to efficiency and accuracy. The major difference between statistical quantities computed by these two methods is the pressure spectrum, due to significantly different treatments of the pressure used in LBE and PS methods. The pressure field obtained by using the LBE method is much less satisfactory. The results specify that the resolution required for the LBE method is approximately twice of the requirement for PS methods. Overall, the LBE method is shown to be a reliable and accurate method for the DNS of decaying turbulence.

Luo et al. [38] studied three-dimensional decaying isotropic turbulence in the frame of LBM; the energy spectrum, mean kinetic energy, and the dissipation rate have been compared with regards to the results from a pseudo-spectral method. They found a disagreement between the two methods for high wave numbers, which was related to the fact that the LBM is only second-order accurate in time and space. Therefore, it is slightly more dissipative than the spectral code. Xu et al. [46] had carried out a 3-D decaying isotropic turbulence study based on an analogous Galerkin filter and focused on the fundamental statistical isotropic property. This regularized method is produced based on orthogonal Hermite polynomial space, and the study has achieved a range of well accepted results.

Djenidi [47] carried out an LBM direct numerical simulation of grid-generated turbulence. He reported that the limitations of the simulation, particularly the boundary conditions and mesh resolution, can lead us to incorrect or only imprecise results. These inaccuracies were obvious in the turbulence kinetic energy and the longitudinal velocity spectra studied. While the performed simulation was not large enough to capture the largest scale structures, and the resolution was not sufficient to resolve the smallest scales, Djenidi reported a good agreement with their experimental results.

The first DNS of turbulent channel flow was performed by Kim et al. [48]. They studied a turbulent channel flow at Reynolds number of 3,300 using a fully spectral method. Their main concern is the behavior of turbulence correlations near the wall. The Fourier series was used for the main functions in the homogeneous direction, and Chebychev polynomials were employed for wall-normal direction functions. Moreover, a number of statistical correlations were reported as complementary to the existing experimental data for the first time. Subsequently, Moser et al. [49] recreated the results of numerical simulations of fully developed turbulent channel flow at three Reynolds numbers up to $Re = 590$. These simulations turned into a standard database used by the DNS community to analyze the performance of other DNS methods for wall-bounded turbulent flows. By then, the turbulent channel flow simulation has been repeated in many research studies. Bespalko et al. [50] compared mean velocity and pressure profiles, Reynolds stress profiles, skewness and flatness factors, the turbulence kinetic-energy budget, and one-dimensional energy spectra in fully-developed turbulent channel flow using the results of the spectral data from Moser et al.'s work [49]. In general, they had observed good agreements of these two studies but some differences were also reported in Reynolds stresses and kinetic-energy budget.

Eggles [51] had carried out a fully developed channel flow simulation with a thermal LBM, and included a temperature gradient across the channel in the wall-normal direction. Amati et al. [52] presented a high resolution lattice-Boltzmann simulation of turbulent channel flow. The vorticity structures which appeared near the wall layer and their influence on the scaling laws were studied. From this, a similar study of channel flow with a Reynolds number of 3,300 had been reported later [53], which showed good agreement with the recorded result of Moser et al. [49]. Recently, Lammers et al. [54] had used a standard Chebyshev pseudo-spectral method for fully developed, incompressible, pressure-driven turbulence channel flow simulation. The results illustrated that even the simplest lattice Boltzmann method would show statistics of the same quality as pseudo-spectral methods at a similar resolution and, in general, better than those of the Chebyshev pseudo-spectral results, but this could be reached at a competitive computational cost. The most considerable discrepancy between the two methods was that the LBM over predicted the moments of fluctuating pressure approximately by 25%.

4. LARGE EDDY SIMULATION

Large eddy simulation (LES) is a well-accepted method for turbulent flow simulation [55]. The connotation of the Kolmogorov (1941) theory of self-similarity is that the large eddies of the flow are dependent on the geometry, while the smaller scales are more universal. This special aspect allows one to explicitly account for the large eddies in calculation and implicitly solve for the small eddies by using a subgrid-scale model (SGS). Mathematically, the velocity field could be separated into the resolved and sub-grid part. The smallest scales in the flow are replaced with a model, while the large scales are accurately resolved. The dissipation of the smallest scales is accounted for by introducing an eddy viscosity.

To merge LES with LBM, there are two methods of filtering. The first method needs filtering of the LBE, after which the nonlinear term in the collision brought the nonclosure term that needs to be modeled at the microscopic level. The second method is an easy to handle approach. It filters the Navier–Stokes equations (NSE), and then computes the filtered NSE and nonclosure term using LBM. The nonclosure term is modeled by the sub-grid stress model and introduced into the relaxation times. Most of the present LBM–LES works follow this approach [56]. The basic idea of all subgrid models is to make an assumption to include the physical effects of unresolved motion on the resolved fluid motion. These models often take a simple form of eddy-viscosity models for the Reynolds stress that provide to damp short-wavelength oscillations [57]. At high Reynolds number, for simulating the large scale resolved fluid problems, a space filtering operation is usually initiated. The NS continuity and momentum equations in the incompressible limit will be changed through the filtering operation of density, pressure, and velocity as follows.

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{5}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{n}\frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\nu\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\right) \tag{6}$$

Where τ_{ij} is the Reynolds stress tensor which depicted the effects of the unresolved scales on the resolved scales.

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j \tag{7}$$

The standard Smagorinsky model could be mentioned as the simplest and most popular subgrid models [58], in which the anisotropic part of the Reynolds stress term is calculated as follows.

$$\tau_{ij} - \frac{\delta_{ij}}{3}\tau_{kk} = -2\nu_t \overline{S}_{ij} = -2C\Delta^2 |\overline{S}| \overline{S}_{ij} \tag{8}$$

Where C is the Smagorinsky constant, δ_{ij} is the Kronecker delta function, and $|\overline{S}| = \sqrt{2\overline{S}_{ij}\overline{S}_{ij}}$ is the magnitude of the large scale strain rate tensor.

$$\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) \tag{9}$$

A noticeable number of LES studies have been reported in LBM frame work during the last decade. The calculation of the strain rate tensor is the key step in the performance of LES in LBM.

Jun Li et al. [59] have proposed an alternative explicit scheme using the average of the two distinct values as the distribution function after the relaxation and after the motion subroutines. This derivation, leads to an explicit scheme for computing

S_{ij} based on physical analysis of the momentum transport process. In this procedure, stress tensor T_{ij} is calculated first, and then S_{ij} is resolved from T_{ij} using the constitutive relationship for Newtonian fluid. The verification of the LBM-LES algorithm based on this explicit scheme had shown good agreement with the N-S equation-based results of velocity profiles in lid-driven flow in a cubic cavity. This method gains the same computational time and cost as the traditional method.

Sheng Chen [60, 61] reported a simple large eddy simulation based on LBM for two-dimensional turbulence modeling. In contrast to the existing primitive-variables large-eddy-based Lattice Boltzmann models, the target macroscopic equations of this model are vorticity-stream function equations. Owing to inherent features of vorticity-stream function equations, the computation of $|\overline{S}|$ is very simple ($|\overline{S}| = \omega$). Furthermore, the calculation of relaxation time (τ_e) of this method is much easier and more computationally efficient.

Huidan Yu [1], assessed the consistency of the lattice Boltzmann as a computational tool in performing direct numerical simulations (DNS) and large-eddy simulations (LES) of turbulent flows. LBE-LES and NS-LES (of a similar order of numerical accuracy) results of HIT were compared and it was established that the LBE-LES simulations seem to preserve instantaneous flow fields to a certain extent more precisely. The comparison between LBE-LES and LBE-DNS results demonstrated that LBE-LES correctly captures the large-scale flow behavior. In this research, the Smagorinsky constant C_s in LBE-LES has chosen smaller than the typical value which is used in Navier–Stokes (NS) LES approaches. The results from this study obviously show that the LBE method can accurately capture important features of decaying homogeneous isotropic turbulence and is potentially a consistent computational tool for turbulence simulations.

Dong and Sagaut [62] analyzed the act of various Smagorinsky models in terms of their time correlation properties at decaying isotropic turbulence with two different initial energy spectra. They investigated the Eulerian time correlations with the classical Lilly–Smagorinsky model and the inertial-range consistent Smagorinsky model, and a detailed comparison was made for the correlations evaluated by LBM–LES and NS–DNS.

The majority of the existing subgrid closures employed in LBM-LES are straightforward extensions of the Smagorinsky eddy viscosity model [63–72]. Pioneering work was carried out by Sagaut [73], in which he addressed a subgrid model for large-eddy simulations (LES) of turbulent flows. He extended the approximate deconvolution method (ADM), formerly proposed by Stolz and Adams [74–76] in NS framework, within the LBM. Since the subgrid closure problem is the consequence of the absence of commutativity of the LES filter and nonlinear terms, and the nonlinearities in LBM equations are totally different from those of the Navier–Stokes equations, the use of an eddy-viscosity model can be taken as an appropriate but not as an optimal way to close the LBM-LES equations. The mentioned LBM-LES approach does not rely upon the eddy-viscosity concept. This new method by using the approximate defiltering approach becomes more general.

The Smagorinsky model, due to its excessive eddy viscosity and isotropy assumption, was unsuccessful to correctly predict the flow near the solid walls and the transitional region. For flows with complex geometries, the accuracy of the wall boundary conditions is of main importance and must be validated. In enclosure flows, turbulence changes into anisotropic near the walls, and its length scales turn into the smaller scale gradually and disappearing over the wall. Due to this fact, the SGS coefficient depends upon the distances from the wall. It should be decreased and approaching zero at the wall to account for near-wall turbulence anisotropy, so the model coefficient could not be constant. There have been some significant progresses made recently to improve the LES-LBM near wall turbulence modeling [77, 78]. In particular, in some earlier studies, Premnath et al. [79] discussed the incorporation of a dynamic procedure for large-eddy simulation of complex turbulent flows in LBM frame work with the common Smagorinsky eddy-viscosity model. In this approach, the local value of the model coefficient of the eddy viscosity, obtained from sampling of the smallest super-grid or resolved scales, was used as the test-filtered scales, and scale-invariance was assumed at these two levels. In other attempts, Premnath et al. [80] reported a study based on the generalized lattice-Boltzmann equation via multiple relaxation times with forcing term. In this research, the SGS turbulence effects had been accounted by standard Smagorinsky eddy–viscosity model modified by using the van Driest wall-damping function to simulate a wall bounded turbulent flow. Two canonical bounded flows, 3-D driven cavity flow and a fully-developed turbulent channel flow had been studied. Detailed computed near-wall turbulent flow structure, such as mean velocity and components of root-mean-square velocity and vorticity fluctuations and Reynolds stress, had also been compared. The results showed good agreement with existing data from experimental studies and direct numerical simulations (DNS), and the stability of the proposed method was more reliable than the SRT-LES methods.

Jafari and Rahnama [81] proposed a shear-improved Smagorinsky model (SISM) based on Lèvêque's et al. [82] research work, which is capable of projecting turbulent near wall region precisely without any wall function or any kind of dynamic adjustment. In this model, Smagorinsky eddy viscosity was computed by subtracting the magnitude of the mean shear from the magnitude of the instantaneous resolved rate of strain tensor. Their results indicated excellent agreement with DNS and dynamic Smagorinsky model. Recently, an LES wall adapting local eddy-viscosity (WALE) model [83] was reported within the lattice Boltzmann framework [84]. This model is based upon the square of the velocity gradient tensor, which takes into account both strain-rate and vorticity. The model resulted in a fast and efficient method due to its algebraic character. Additionally, its prediction of the transition from laminar to turbulent regimes has revealed valuable results without applying a damping function or dynamic procedures. A while later, Ming et al. [85] had compared the wall-adapting local eddy-viscosity and Vreman [86] subgrid scale models by simulating a fully developed turbulent channel flow in a generalized LBM–LES frame work. The Vreman model is indicated in first-order derivatives. It is rotationally invariant for isotropic filter widths and does not desire averaging, explicit filtering, or clipping procedures. The obtained results are very similar, showing a maximum inconsistency of 1% and both matched the DNS calculations quite well. With regards to the computational cost, the Vreman model saves about 40% in comparison with the WALE model. Of wall bounded studies, the Kang and Hassan [70] work could be mentioned. They had performed a comprehensive study of boundary treatment techniques and collision models, and also grid sizes in the simulation of the turbulent pipe flows. The Smagorinsky LES model was utilized for turbulence simulation in two geometries, a circular pipe and a square duct with and without 45° rotation to study the effect of lattice models in the LBM. They compared the two different models of D3Q19 and D3Q27 lattices. It was remarked that the D3Q27 lattice model could achieve the rotational invariance in terms of turbulence statistics and produced the comparable results to the DNS data, while the D3Q19 lattice model broke the rotational invariance and generated unreasonable data.

Recently, Pirker et al. [87] proposed a hybrid turbulence model to study cyclone short-cut flow. They combined the fine-grid lattice Boltzmann-based large eddy simulation with a coarse-grid finite volume-based RANS turbulence model.

5. REYNOLDS AVERAGE NAVIER-STOKES

It will be costly for primary industrial design or for interdisciplinary studies, such as turbulent multi-phase flow, to solve the flow directly without any mathematical modeling. RANS models offer the most economical approach for computing complex turbulent industrial flows. The Spalart-Allmaras (SA), k-ϵ (k-epsilon), and k-ω (k–omega) models which employs the Boussinesq hypothesis are some examples of RANS models. The SA model uses only one additional equation to model turbulence viscosity transport, while the k models use two. SA was basically developed to model aerodynamic flows. In recent years, some research studies have reported on RANS turbulence modeling in the frame of LBM. Chen [88] had applied MRT-LBM coupled with Spalart-Allmaras to model a two dimensional flow around a NACA0012 airfoil. The results of this study illustrated that with similar grid resolution, traditional CFD methods fail to capture the flow separations with SA turbulence model while the proposed method was able to catch the location of separation correctly. According to the result, it is evident that LBM gives very good predictions, and flexible domain size can be accepted if the far-field boundary condition is well modeled. This 2-D LBM–SA model compared to 3-D LES-LBM simulations is more efficient computationally without losing accuracy. Kai et al. [89] reported a similar 2-D LBM–SA model intended to do numerical simulation of highly turbulent flow around obstacles of curved geometry employing non-body-fitted Cartesian meshes.

The two equation k-ϵ model remains the most popular choice for the simulation of turbulent flows. In the frame of LB, it could be solved by two different approaches. A research study [90] suggested the solution of k-ϵ equations within an LB structure by creating two additional populations, with components in the same directions as the particle distribution for each of the turbulent properties while some use LB in conjunction with finite difference schemes for the solution of k-ε [91, 92]. Pertinently, an investigation of two–dimensional turbulent flows in an axial compressor cascade was carried out by Fillipova et al. [93]. The two-equation turbulent kinetic energy k-ϵ model was extended to the multi-scale based on the LBGK framework to get better resolution in the regions of high gradients. In this study the combined finite difference–LB model was employed. The LBGK equations and equations for k and ϵ are considered to be decoupled during a single time step. Simultaneously, similar type of combined LB–finite difference schemes have been effectively developed for the problems of low Mach number combustion [94, 95].

Shu et al. [96] merged the Taylor series expansion and least-squares based LBM with S-A and k–ω turbulence models to simulate a turbulent channel flow and the turbulent flow over a backward facing step at $\mathrm{Re} = 44{,}000$. The achieved results indicated a good similarity in comparison with the experimental data and analytical solution by Kim et al. [97]. In the referred study, k–ω showed more agreement with the experimental results than the S-A model.

6. CONCLUDING REMARKS

This review is planned to be an overview on the applications of lattice Boltzmann method in turbulent flow modeling. Related studies are so diverse and interdisciplinary that it is not achievable to cover all proposed methods and specified topics. In the first part of the article, the evolution of LCGA as a discrete velocity model to LBM has been explained briefly to illustrate how this method has matured as a proper option for turbulent flow simulation. Regarding the reported research, compared results of NS spectral method with LBM are quite similar. This similarity has been shown in the instantaneous flow fields such as velocity and vorticity field and low-order statistical quantities like the spectrum, dissipation rate, and total kinetic energy. However, the computational efficiency of LBM and spectral code are approximately equal. Meanwhile, the simplicity of boundary implementation in LBM and the capability of parallel computing make the LBM more convenient. The pressure spectrum was addressed as the less satisfied field of LBM, due to significantly different treatments of the pressure. LBM is computed from the ideal gas law, and so it does not require the solution of a Poisson equation to determine the pressure. On the other hand, this feature of LBM made it more appealing on the competition of computational cost. Implementing the subgrid scale simulation to the LBM structure and performing a large eddy simulation of turbulent flows are areas of intense research, which are briefly explained here. Due to the literature, most of the LES-LBM research studies have utilized the standard Smagorinsky eddy-viscosity model. The results of these researches indicated that besides the near wall turbulence effects, this method has produced acceptable results in comparison with NS spectral approaches and even more accurate than NS-LES. The RANS methods received less attention than LES or DNS turbulence simulating schemes.

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