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Large Eddy Simulation using Lattice Boltzmann Method based on Sigma Model

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Abstract

The Lattice Boltzmann Method (LBM) is a novel method for simulating convection-diffusion type of problems. It has direct advantages over other available methods. One advantage of LBM is that it can be easily parallelized and hence can be used to simulate fluid flow in multi-core computers using parallel computing. In this paper we have used the Lattice Boltzmann Method to simulate turbulent flow in a channel using the Sigma Model for Large Eddy Simulation (LES). LES is widely used in simulating turbulent flows in industries because of its less computational needs compared to Direct Numerical Simulation (DNS) where the Kolmogorov Eddies have to be resolved. Unlike the static Smagorinsky Model this model has the potential to simulate turbulent flow in complex geometries. The eddy viscosity in the sigma model becomes zero at the boundary $O(y^3)$ and thus eliminates the use of artificial damping function that is used with the static Smagorinsky model. The sigma model also ensures a positive eddy viscosity which ensures that there are no numerical instabilities. The eddy viscosity also vanishes when the flow is two dimensional or axisymmetric.

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Keywords: Large Eddy Simulation; Lattice Boltzmann Method; Sigma Model; Turbulent Flow.

1. Introduction

The Lattice Boltzmann Method (LBM) [1, 2, 3] has gained popularity in the recent years because of its simplicity. The method has the ability to operate on multiple cores/threads or in a graphics processing unit (GPU). In LBM boundary conditions (BCs) are also easily implemented. Schemes like bounce back are used in which the density

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functions are simply bounced back to implement no-slip wall BCs, pressure outlet BCs, and velocity inlet BCs. The concept of LBM came from the Lattice Gas Automata (LGA) [4] which was dropped because of its intensive computational need and noise in velocity and pressure field. However, the fluid particles used in LGA was latter replaced with probability density function. This resulted in less noise and faster computation.

Recently, a lot of research has been done on this method. LBM finds its application in Multiphase Flows [5], Micro-Scale Flows, and Flows in Porous Medium. One such application of LBM is to simulate Turbulent Flows. Large Eddy Simulation is widely used as a tool to simulate Turbulent Flows. The LES offers lot more advantages over Reynolds Averaged Navier-Stokes (RANS) modelling of turbulent flows. In LES spatial variation of the turbulent structures can be resolved unlike the RANS models where we solve for the averaged quantities over a time scale.

In LES filtering is applied to the Navier–Stokes equations to eliminate small scales of the solution which are computationally expensive to resolve. The Navier-Stokes equation and Continuity equation are transformed to a set of filtered velocity fields. However, the main problem arrives due to the implicitness of the convective term which is resolved into two parts. For models like the Smagorinsky Model, the model coefficient varies from problem to problem. Dynamic procedures are used to get the model coefficients. However, they are computationally expensive and fail to give correct results in complex geometries. The new Sigma Model [6, 7] has the ability to self-adjust to the flow geometry and give accurate result, and provide better numerical stability by remaining positive at a less computation cost. The turbulent viscosity is also of the order of third power of distance from the wall, which is not present in the dynamic or static Smagorinsky model. The eddy viscosity also vanishes when the flow is 2D or axisymmetric. The sigma model also self-adjusts to the geometry of the flow and reduces the variation of the model coefficient. These all properties of the sigma model make it an ideal model for LES of turbulent flow.

Nomenclature

ρ	Density
$\vec{u}$	Velocity vector
P	Pressure
ν	Kinematic viscosity
τ_{ij}^{SGS}	Subgrid-scale (SGS) stress tensor
Δ	Characteristic length scale
g_{ij}	Velocity gradient tensor
f_i	Particle distribution function in i^{th} direction
$f_i^{(0)}$	Equilibrium particle distribution function in i^{th} direction
F	Source term
τ	Relaxation parameter related to viscosity
c	Speed of sound
c_s	Speed of sound in medium
δ	Channel half width
Re_τ	Reynolds number based on friction velocity
u_τ	Friction velocity

2. Numerical Method

The governing equations for fluid flow are the Navier-Stokes Equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla .\left(\rho \vec{u}\vec{u}\right) = -\nabla . P + \nabla .\left(\rho \nu \nabla \vec{u}\right)$$

Applying a filtering operation and considering flow to be incompressible we get,

$$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} + \frac{\partial \tilde{w}}{\partial z} = 0$$

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \tilde{u}\tilde{u}_i}{\partial x} + \frac{\partial \tilde{v}\tilde{u}_i}{\partial y} + \frac{\partial \tilde{w}\tilde{u}_i}{\partial z} = -\frac{\partial \tilde{P}}{\rho \partial x_i} + \frac{\partial}{\partial x}\left(\nu \frac{\partial \tilde{u}_i}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu \frac{\partial \tilde{u}_i}{\partial y}\right) + \frac{\partial}{\partial z}\left(\nu \frac{\partial \tilde{u}_i}{\partial z}\right) - \frac{\partial \tau_{ij}^{SGS}}{\partial x_j}$$

$\tau_{ij}^{SGS} = \overline{u_i u_j} - \overline{u_i}\,\overline{u_j}$, is the effect of small scales on large scales.

This term is modelled using the Boussinesq approximation as, $\tau_{ij}^{SGS} = -2\nu_T \overline{S_{ij}}$ where $\overline{S_{ij}}$ is the filtered strain rate tensor.
Referring to the sigma model [5],

$$\nu_T = \left(\Delta C_\sigma\right)^2 D_\sigma$$

Δ is the characteristic length scale which is taken to be the cube root of the volume of the cell and C_σ is the model constant and D_σ is defined as follows,

$$D_\sigma = \frac{\sigma_3\left(\sigma_1 - \sigma_2\right)\left(\sigma_2 - \sigma_3\right)}{{\sigma_1}^2}$$

σ_1, σ_2, σ_3 are the square roots of Eigen-values of the matrix $G = g^T g$ where elements of the g matrix are given by $g_{ij} = \frac{\partial u_i}{\partial x_j}$.
The new formulation for sigma model using LBM can be written as,

$$f_i\left(\vec{x} + \vec{c}_i \delta t\ , t + \delta t\right) = f_i\left(\vec{x}\ , t\right) + \Phi_i$$

$$\Phi_i = \left(\frac{f_i^{(0)}\left(\vec{x}\ , t\right) - f_i\left(\vec{x}\ , t\right)}{\tau}\right) + \left(1 - \frac{1}{2\tau}\right)\frac{w_i \delta t}{{c_s}^2}\left[\left(\vec{c}_i - \vec{u}\right) + \frac{\left(\vec{c}_i . \vec{u}\right)\vec{c}_i}{{c_s}^2}\right].\vec{F}$$

$\vec{F}$ is the source term and τ is the relaxation parameter and the total viscosity is related to the relaxation parameter by this relation,

$$\nu_T + \nu = {c_s}^2\left(\tau - 0.5\right)\delta t$$

In our simulation we have used the D3Q19 lattice model for simulation. The velocity directions are,

$$\vec{c}_i = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

And the corresponding weights are,

$$w_i = \begin{bmatrix} \frac{1}{3} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} & \frac{1}{36} \end{bmatrix}$$

The corresponding macro-parameters are calculated by

$$\rho = \sum_i f_i^{(0)}$$

$$\vec{u} = \frac{\left(\sum_i f_i^{(0)} \vec{c}_i + \frac{1}{2} \vec{F} \right)}{\rho}$$

To find the velocity derivatives we use the following scheme,

$$\vec{\nabla} u_i = \frac{3}{2c\delta t} \begin{Bmatrix} \hat{i} \sum_1^{18} w_i (\vec{c}_i.\hat{i}) \left[u_i \left(\vec{r} + \vec{c}\delta t \right) - u_i \left(\vec{r} - \vec{c}\delta t \right) \right] \\ \hat{j} \sum_1^{18} w_i (\vec{c}_i.\hat{j}) \left[u_i \left(\vec{r} + \vec{c}\delta t \right) - u_i \left(\vec{r} - \vec{c}\delta t \right) \right] \\ \hat{k} \sum_1^{18} w_i (\vec{c}_i.\hat{k}) \left[u_i \left(\vec{r} + \vec{c}\delta t \right) - u_i \left(\vec{r} - \vec{c}\delta t \right) \right] \end{Bmatrix}$$

Where c is equal to 1, and δt is the time step.

$f_i^{(0)}$ is the equilibrium distribution function given by

$$f_i^{(0)} = \begin{cases} w_i \rho \left[1 - \dfrac{3}{2} \dfrac{\vec{u}.\vec{u}}{c^2} \right] \ldots \quad when\ i = 0 \\ w_i \rho \left[1 + 3 \dfrac{\vec{c}_i.\vec{u}}{c^2} + \dfrac{9}{2} \dfrac{(\vec{c}_i.\vec{u})^2}{c^4} - \dfrac{3}{2} \dfrac{\vec{u}.\vec{u}}{c^2} \right] \ldots \quad when\ i \neq 0 \end{cases}$$

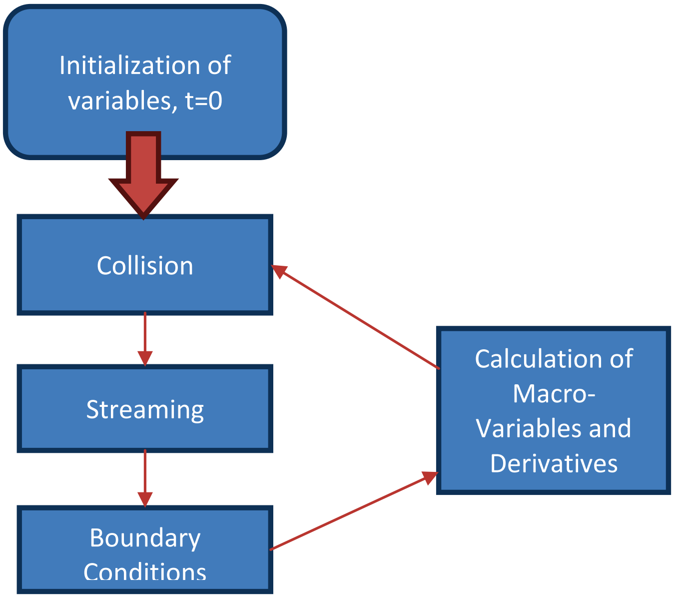


Fig.1. Flow-Chart of the Algorithm

3. Results

To test this new Lattice Boltzmann Method formulation we try to set up the channel flow with $Re_\tau = 395$. The Re_τ is defined as,

$$Re_\tau = \frac{u_\tau \delta}{\nu} \quad u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

δ , is the channel half width and we define appropriate scaling factor for y and u based on friction velocity and viscous length scale,

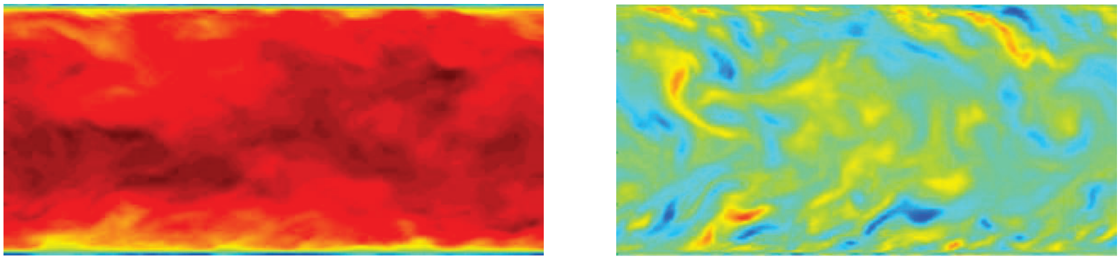
$$y^+ = \frac{u_\tau y}{\nu} \quad U^+ = \frac{U}{u_\tau}$$

For our simulation we consider a grid size of 100 x 200 x 100 with 200 grid points along stream-wise direction. Periodic condition has been applied on two faces normal to the stream-wise direction. The flow is forced by adding a pressure gradient term as a body force along the stream-wise direction which decides the final Reynolds number. To adjust the pressure gradient term automatically we define the pressure gradient term in the following manner for a new time step.

$$\vec{F} = \left(\frac{dp}{dx}\right)_{new} = \left(\frac{dp}{dx}\right)_{old} + a\left(1 - \frac{Re_{present}}{Re_{final}}\right)$$

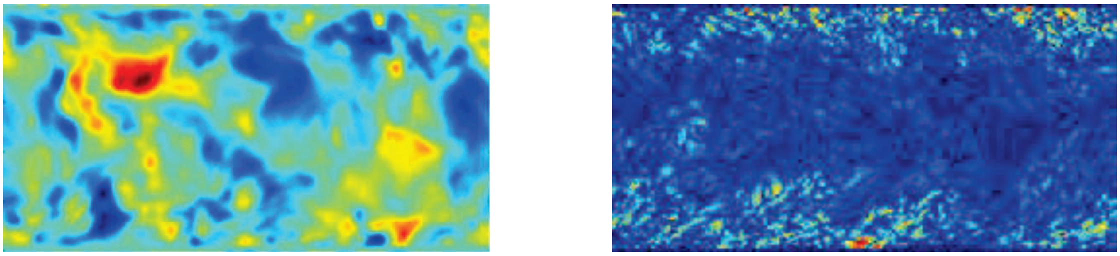
Here 'a' is the constant which can be appropriately adjusted.

In the next section contours of the velocity field and turbulent field has been plotted for the simulation showing turbulent velocity fluctuations in a plane. A quantitative validation of the new Lattice Boltzmann formulation of Sigma Model has been done in the next sections to see whether the LBM discretisation of the filtered Navier-Stokes Equation preserves all the features of the Sigma Model.



(a) (b)

Fig. 2. (a) Contour of velocity in x direction; (b) Contour of velocity in y direction



(a) (b)

Fig.3. (a) Contour of velocity in z direction;(b) Contour of turbulent viscosity

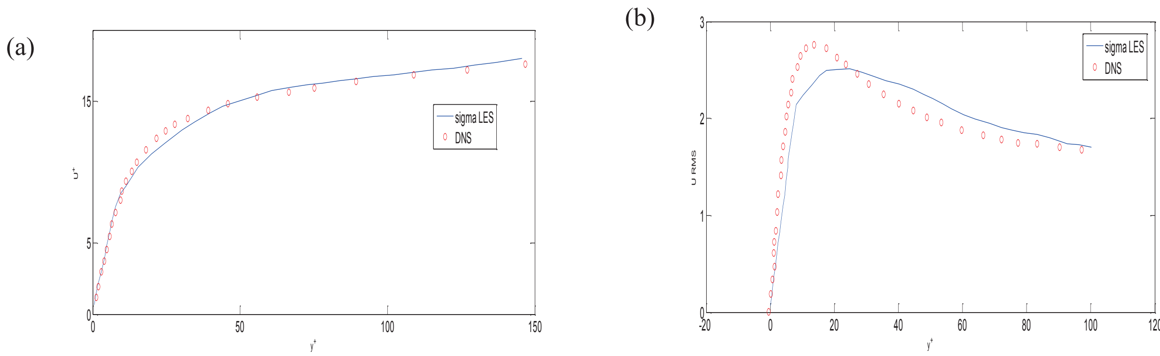


Fig.4. (a)Comparison of mean velocity profile from the wall towards the channel half width with DNS [8].(b)Comparison of RMS velocity profile from the wall towards the channel half width with DNS [8].

The viscosity of the fluid is set to be 0.0004 in lattice units, channel half width is 50 lattice length units and the model coefficient C_σ is set to be 1.5. Doing appropriate calculations we get $u_\tau = 0.00316,\ y^+ = 7.9y$, $U^+ = U / 0.00316$. Next, we plot the $U_{Mean}\ vs\ y^+$ and compare it with DNS simulation by [8].

The profile of mean velocity found by ensemble average of the velocity data points using the new sigma LBM model was found to be matching with the DNS data. In this simulation we have considered a uniform grid for the sake of implementation and validation of the model. However, we expect better results if number of lattices are increased near the walls by considering lattices of different sizes. This is the main reason the RMS profile from the simulation didn't match to our expectations.

The simulation was done on Intel Xeon Server with clock speed of 2.4 GHz using 12 threads for parallel processing. Open MP directives were used for parallelizing the C++ code. The total computation time on the server was 5 hours 30 minute and the memory requirement was 3 GB.

4. Conclusion

The new method for LES was found to be promising method for simulation of flow in complex geometry using LBM. The new method was able to reproduce correct mean velocity and RMS velocity profile for a Channel flow with frictional Reynolds number = 395. Thus, the capability of model to simulate turbulent flows in complex geometry along with the easiness to implement LBM in complicated geometry makes this an ideal method for simulation of turbulent flows. In our future work we wish to solve the same problem on a finer grid with more number of points near the wall to get better statistics and check grid dependence. Graphics Processing Unit (GPU) based parallel computing can be also be done to get faster results using CUDA.

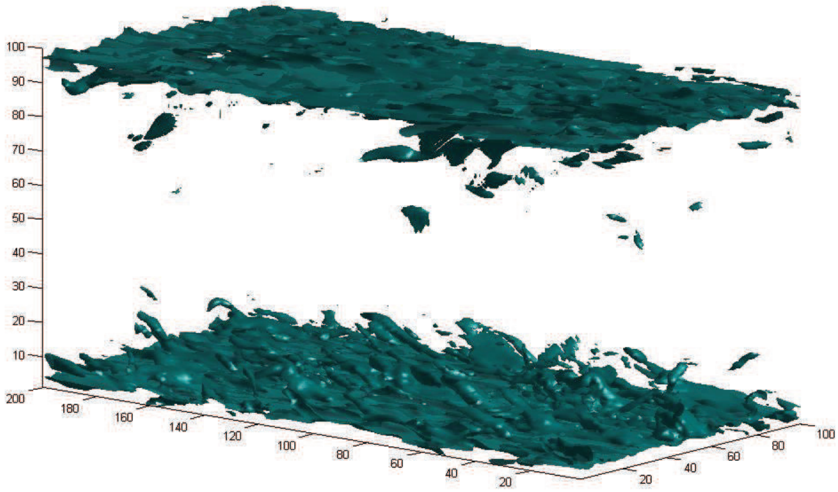


Fig.5. Contour of Vorticity Magnitude = 0.00005 showing evolving turbulent structures.

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