Lattice-Boltzmann Methods for turbulent flow simulations

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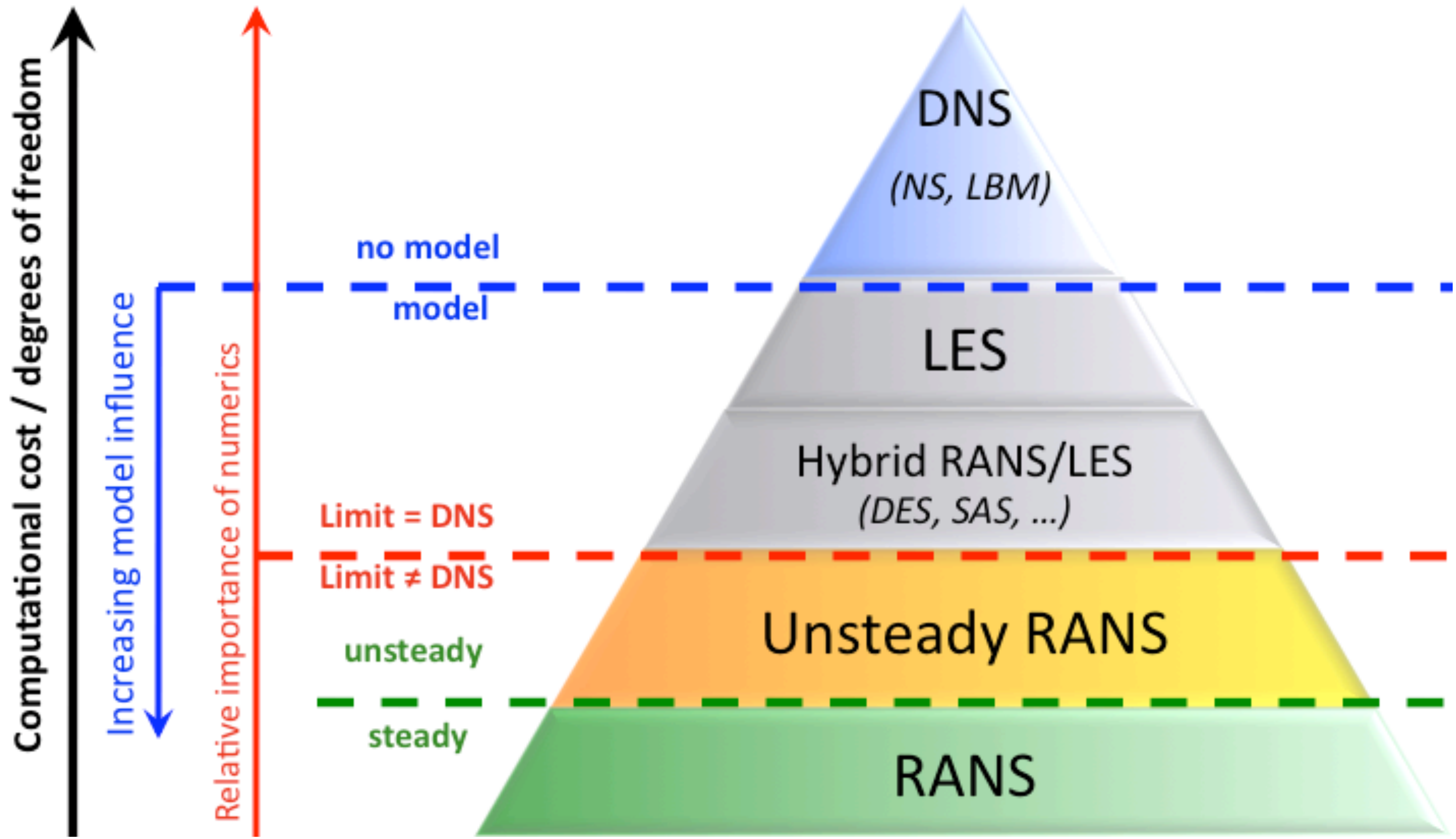
***Thanks to* : O. Malaspinas (*UPMC*), E. Vergnault (*UPMC*), Hui Xu (*UPMC*)**

EUROMECH Spring Festival
Toulon
9-10 avril 2013

Hierarchy of CFD methods

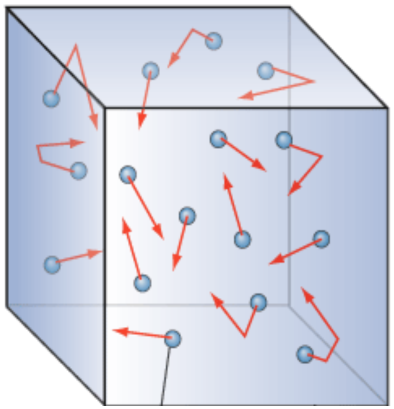
« ***Multiscale & Multiresolution approaches for turbulence, 2nd edn*** »

Sagaut, Deck & Terracol, Imperial College Press, 2013



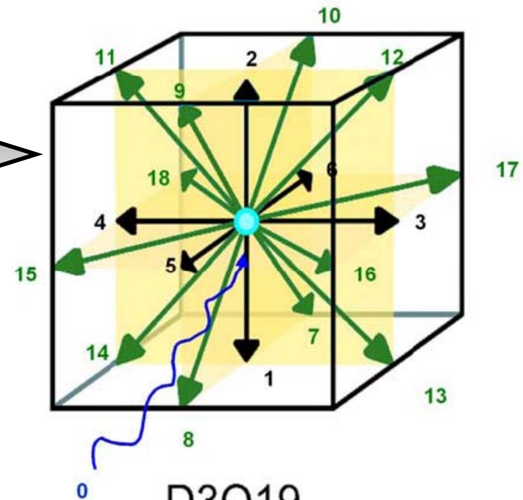
Lattice-Boltzmann Methods

Statistical Mechanics	*Continuum Mechanics*
• **Boltzmann Eq.**	• **Navier-Stokes Eqs.**
• **Molecules**	• **Continuous medium**
– **Kinetic energy**	– **Temperature**
– **Momentum**	– **Pressure**
– **Collisions**	– **Density**
– **Mean free path**	– **Viscosity**



Lattice-Boltzmann

- **Volumic mesh**
- **Discrete velocities**
- **Probabilistic description**



D3Q19

Derivation of LBM

Starting point: continuous Boltzmann-BGK Eq.

f(ξ,x,t)
Single-particule distribution function

$$\frac{\partial f}{\partial t} + \vec{\xi} \cdot \nabla f + \overbrace{\vec{g} \cdot \nabla_\xi f}^{\textit{External force}} = \overbrace{-\frac{1}{\tau}\left(f - f^{(0)}\right)}^{\textit{Collision term (Bhatnagar-Gross-Krook, 1954)}}$$

velocity

Post-collision Relaxation time

Equilibrium distribution (Maxwell-Boltzmann)

Expansion of the equilibrium distribution

$$f^{(0)} = \frac{\rho}{(2\pi c_T^2)^{D/2}} \exp\left(\frac{-(\vec{\xi} - \vec{u})^2}{2c_T^2}\right)$$

$$\Leftrightarrow \quad f^{(0)} = \frac{\rho}{(2\pi c_T^2)^{D/2}} \exp\left(\frac{-\xi^2}{2c_T^2}\right) \exp\left(\frac{2\vec{\xi}\cdot\vec{u} - u^2}{2c_T^2}\right)$$

Taylor series expansion

$$f^{(0)} \simeq \frac{\rho}{(2\pi c_T^2)^{D/2}} \exp\left(\frac{-\xi^2}{2c_T^2}\right)\left(1 + \frac{(\vec{x}\cdot\vec{u})}{c_T^2} + \frac{(\vec{x}\cdot\vec{u})^2}{2c_T^4} - \frac{u^2}{2c_T^2} + \frac{(\vec{x}\cdot\vec{u})^3}{2c_T^6} - \frac{(\vec{x}\cdot\vec{u})u^2}{2c_T^4} + O(u^4)\right)$$

Rem: *Mach number expansion*

continued

- Hermite polynomial expansion

$$f(x,\vec{\xi},t) = \underbrace{\frac{1}{(2\pi)^{D/2}} e^{-\xi^2/2}}_{\omega(\vec{\xi})} \sum_{n=0}^{\infty} \frac{1}{n!} a^{(\vec{n})} H_n(\vec{\xi})$$

Advantage: it allows for **exact reconstruction** of macroscopic variables

$$\begin{aligned}
a^{(0)} &= \int f d\vec{\xi} = \rho \\
a^{(1)} &= \int f \vec{\xi} d\vec{\xi} = \rho \vec{u} \\
a^{(2)} &= \int f (\vec{\xi}\vec{\xi} - \delta) d\vec{\xi} = P + (\vec{u}\vec{u} - \delta) \\
a^{(3)} &= \int f (\vec{\xi}\vec{\xi}\vec{\xi} - \delta\vec{\xi}) d\vec{\xi} = Q + \vec{u} a^{(2)} - (D-1)\rho\vec{u}\vec{u}\vec{u}
\end{aligned}$$

Classical BGK models

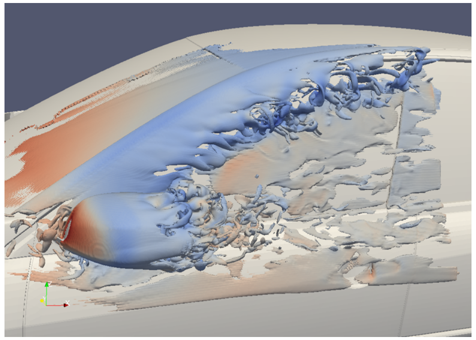
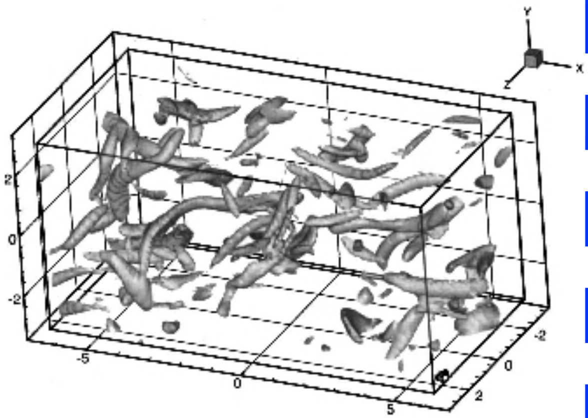
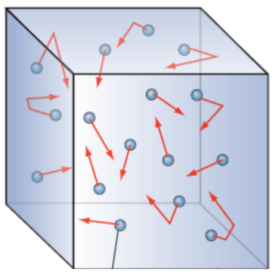
Nth-order LBM with *d* discrete velocities in dimension D

$$\frac{\partial f_l}{\partial t} + \vec{\xi}_l \cdot \nabla f_l = -\frac{1}{\tau}\left(f_l - f_l^{(0)}\right) + F_l \quad l = 1, d$$

$$\begin{aligned} f_l^{(0)} &= w_l \rho \Big\{ 1 + (\vec{\xi}_l \cdot \vec{u}) + \frac{1}{2}\Big((\vec{\xi}_l \cdot \vec{u})^2 - u^2 + (\theta - 1)(\xi_l^2 - D)\Big) \\ &\quad + \frac{1}{6}(\vec{\xi}_l \cdot \vec{u})\Big((\vec{\xi}_l \cdot \vec{u})^2 - 3u^2 + 3(\theta - 1)(\xi_l^2 - D - 2)\Big) + \ldots \Big\} \end{aligned}$$

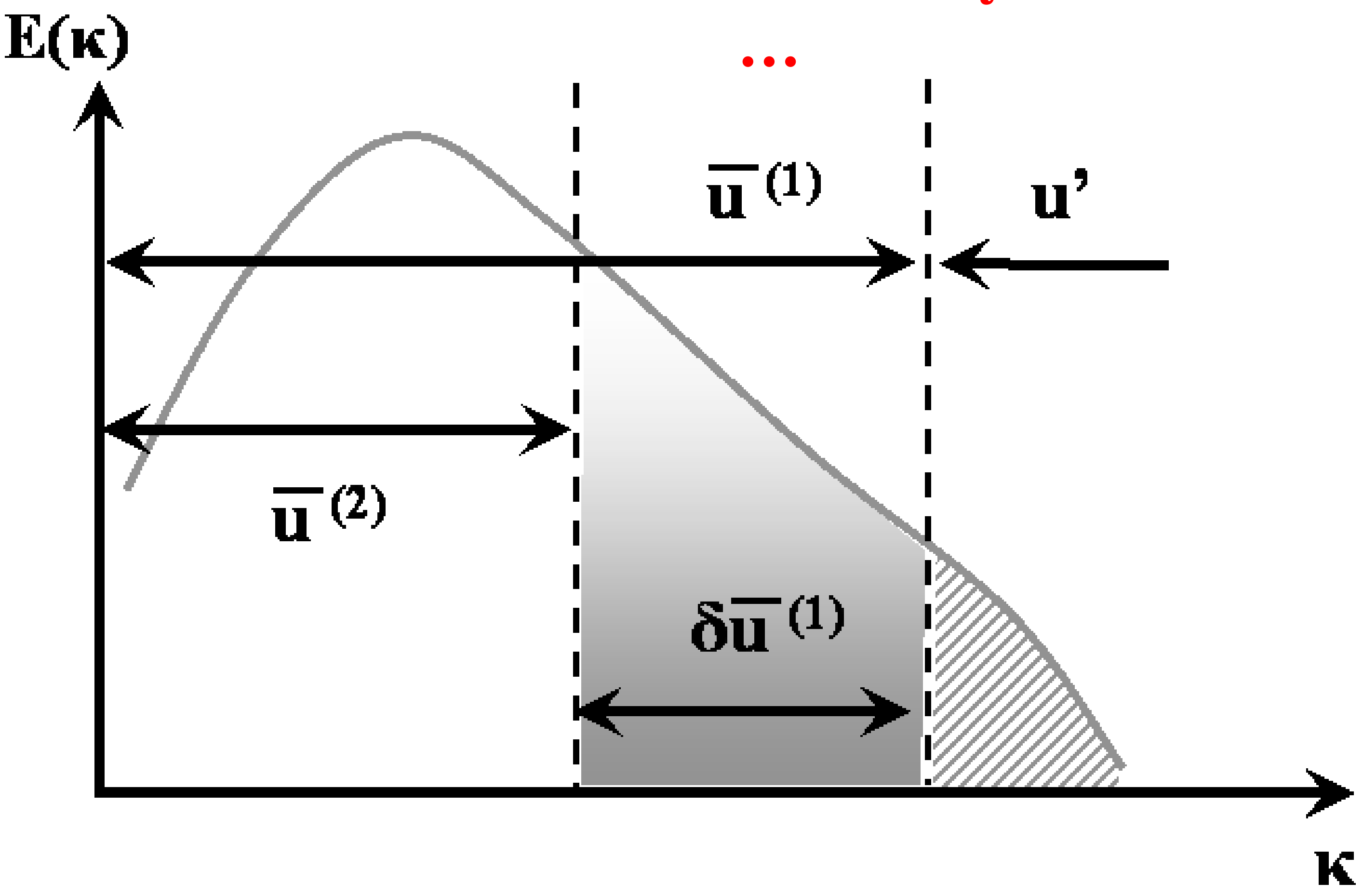
$$\begin{aligned} F_l &= w_l \rho \Big\{ (\vec{\xi}_l \cdot \vec{g}) + (\vec{\xi}_l \cdot \vec{g})(\vec{\xi}_l \cdot \vec{u}) - (\vec{g} \cdot \vec{u}) \\ &\quad + \frac{1}{2\rho} a^{(2)} \Big((\vec{\xi}_l \cdot \vec{g}) H_2(\vec{\xi}_l) - 2\vec{g}\vec{\xi}_l\Big) + \ldots \Big\} \end{aligned}$$

LBM-LES schematic view

	NS-LES	LBM-LES	Non-linear terms
	Filtered macroscopic $(\bar{u}, \bar{p})$	Filtered microscopic $(\bar{f}, \bar{\rho}, \tilde{u})$	inherited
	macroscopic (u, p)		advection (quadratic)
	microscopic f	microscopic f	collision (exponential form/ Nth-order Hermite)

3-range decomposition

VMS
ADM
Germano identity
…



A fully general formulation

[Malaspinas & Sagaut, J. Fluid Mech 700, 2012]

Generic LB equation

$$\underbrace{\frac{\partial}{\partial t} f + v \cdot \nabla f}_{\frac{Df}{Dt}} = C(f)$$

General formulation for filtered LBM

$$\frac{\partial}{\partial t}\bar{f} + v \cdot \nabla \bar{f} = \frac{D\bar{f}}{Dt} = \overline{C(f)} = G \star C\,(f)$$

Residual formulation for filtered LBM

$$\frac{D\bar{f}}{Dt} - C(\bar{f}) = \underbrace{G \star C\,(f) - C(\bar{f})}_{R}$$

Approximate Deconvolution Method

[Stolz, Adams, Phys. Fluids 11, 1999]
[Sagaut, Comput. Maths. App. 59, 2010]
[Malaspinas & Sagaut, Phys. Fluids 23, 2011]

Defining approximate inverse filter Q: $(Q \star G) = I + O(h^l)$

$$\Rightarrow \quad f^* \equiv Q \star \bar{f} = f + error \simeq f$$

Subgrid residual can be split as

$$\begin{aligned} R &\equiv G \star C(f) - C(\bar{f}) \\ &= \underbrace{\left[G \star C(f^*) - C(\bar{f})\right]}_{R_1} + \underbrace{\left[G \star C(f) - G \star C(f^*)\right]}_{R_2} \end{aligned}$$

Directly computable (R1) — Closure needed (R2)

High-order closure

Closing R_2 via Taylor series expansion:

$$
\begin{aligned}
R_2 &= G\star[C(f)-C(f^*)] = G\star\left[\frac{\partial C}{\partial f}|_f(f-f^*)+O(f-f^*)^2\right] \\
&\simeq G\star\left[\frac{\partial C}{\partial f}|_{f^*}(I-Q\star G)\star f^*\right]
\end{aligned}
$$

Closed LBM-LES equation:

$$
\frac{D\bar{f}}{Dt} - G\star C(f^*) = G\star\left[\frac{\partial C}{\partial f}|_{f^*}(I-Q\star G)\star f^*\right]
$$

linearity

$$
G\star\left(\frac{Df^*}{Dt} - C(f^*)\right) = G\star\left[\frac{\partial C}{\partial f}|_{f^*}(I-Q\star G)\star f^*\right]
$$

Practical 2-step implementation

1. Resolution step $$\frac{Df^*}{Dt} - C(f^*) = \frac{\partial C}{\partial f}\Big|_{f^*} (I - Q \star G) \star f^*$$

2. Filtering step $$f^{*(n+1)} = (Q \star G) \star f^{*(n+1)}$$

Remarks:

- resolution step based on modified LBE equation
- Jacobian term in LHS not trivial
- single filter: $Q \star G$

Navier-Stokes implementation:

[Matthew, Lechner, Foysi, Sesterhenn, Friedrich, Phys. Fluids, 15, 2003]

Simplified closure

Motivations:

- works based on Navier-Stokes show that R_2 has small influence
- R_2 induces a significant increase in model complexity (vs. usual LBM)

$$\Rightarrow \quad G \star \left(\frac{Df^*}{Dt} - C(f^*) \right) = 0$$

1. Resolution step $\quad \frac{Df^*}{Dt} - C(f^*) = 0 \quad$ ***(classical LBM method !)***

2. Filtering step $\quad f^{*(n+1)} = (Q \star G) \star f^{*(n+1)}$

Macroscopic variable reconstruction

Direct simulation

$$
\begin{aligned}
\rho(x,t) &\equiv mn(x,t) = m\int f dv \\
\rho u(x,t) &= m\int f v dv \\
\rho e(x,t) &= \frac{1}{2}m\int f|\underbrace{v-u}_{c}|dv
\end{aligned}
$$

Filtered variables

$$
\begin{aligned}
\bar{\rho}(x,t) &= m\int \bar{f} dv \\
\overline{\rho u}(x,t) &= m\int \bar{f} v dv \\
\overline{\rho e}(x,t) &= \frac{1}{2}m\int \overline{f c^2} dv
\end{aligned}
$$

Computable variables in LES

$$\widetilde{u}(x,t) = \frac{1}{\bar{\rho}}m\int \bar{f} v dv$$

$$\widetilde{c}(x,t) = v - \widetilde{u}(x,t)$$

$$\widetilde{\rho e}(x,t) = \frac{1}{2}m\int \bar{f}\widetilde{c}^2 dv$$

Comment on LES-LBM EOS

Classical LBM Equation Of State (perfect gas): $$p = c_s^2 \rho$$

EOS for LES-LBM (computable variables):

$$(\widetilde{p} + p') = c_s^2(\widetilde{\rho} + \rho') \quad \Longrightarrow \quad \widetilde{p} = c_s^2 \widetilde{\rho} + \underbrace{\left[c_s^2 \rho' - p'\right]}_{\text{Subgrid residual}}$$

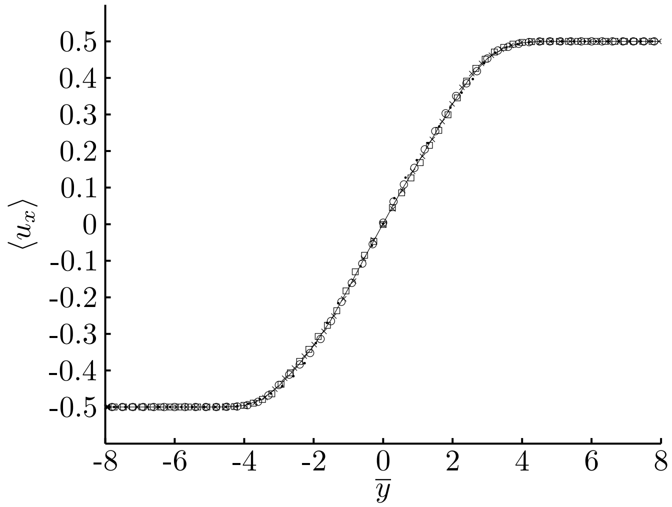
Equivalent modified speed of sound formulation

$$\widetilde{p} = (c_s^2 + {c'_s}^2)\widetilde{\rho}, \qquad {c'_s}^2 \equiv \frac{1}{\widetilde{\rho}}\left[c_s^2 \rho' - p'\right]$$

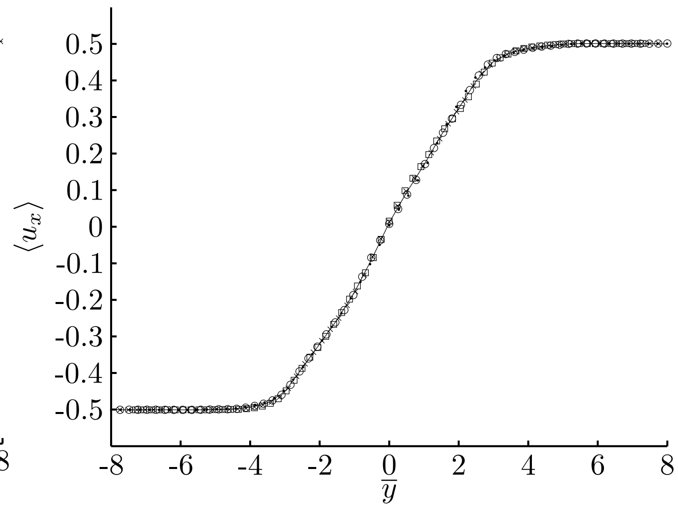
Equivalent Regularized ADM

Time developing plane mixig layer: Self-similar solution

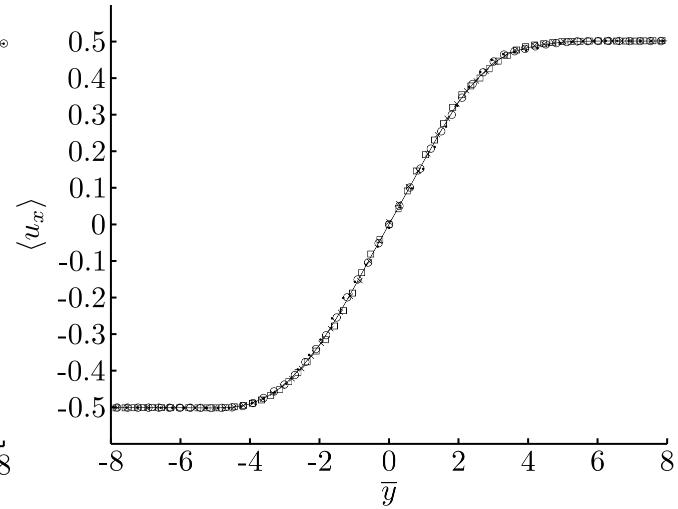
(a)



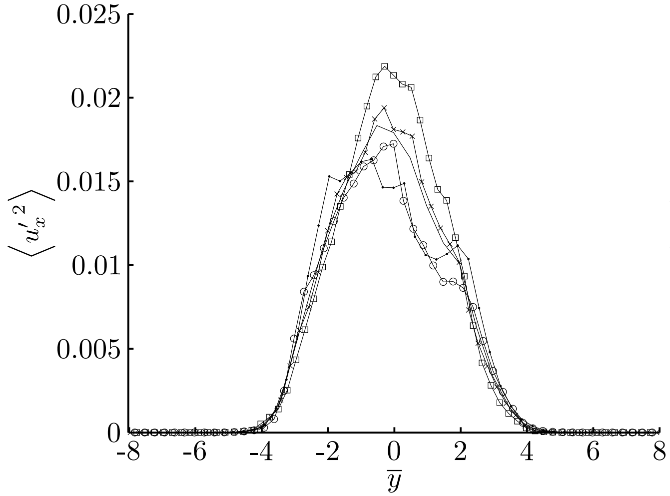
(b)



(c)



(a)



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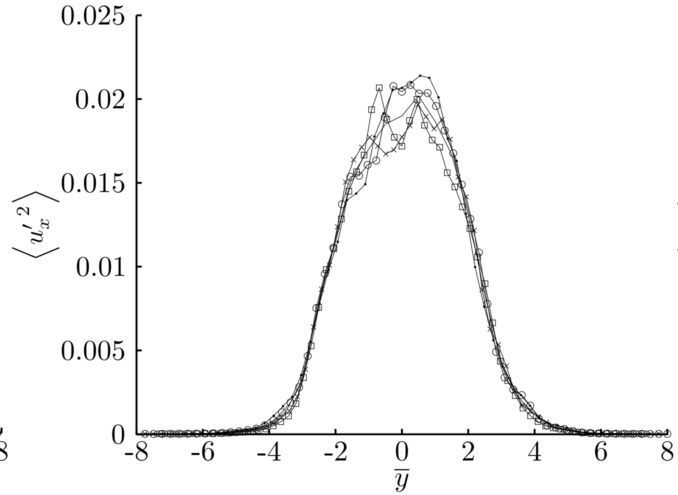

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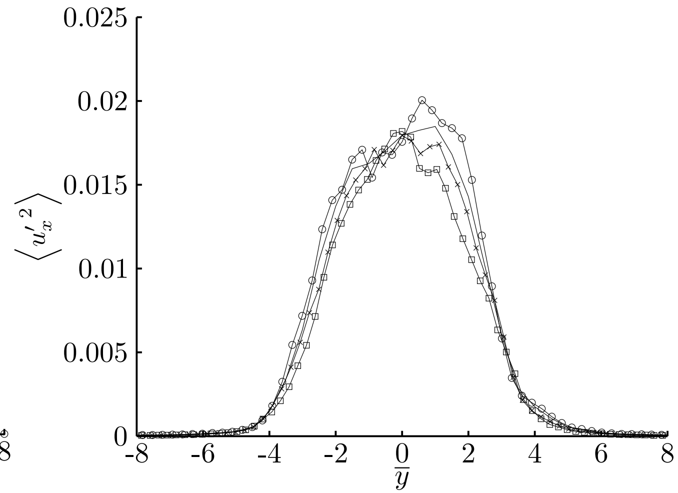
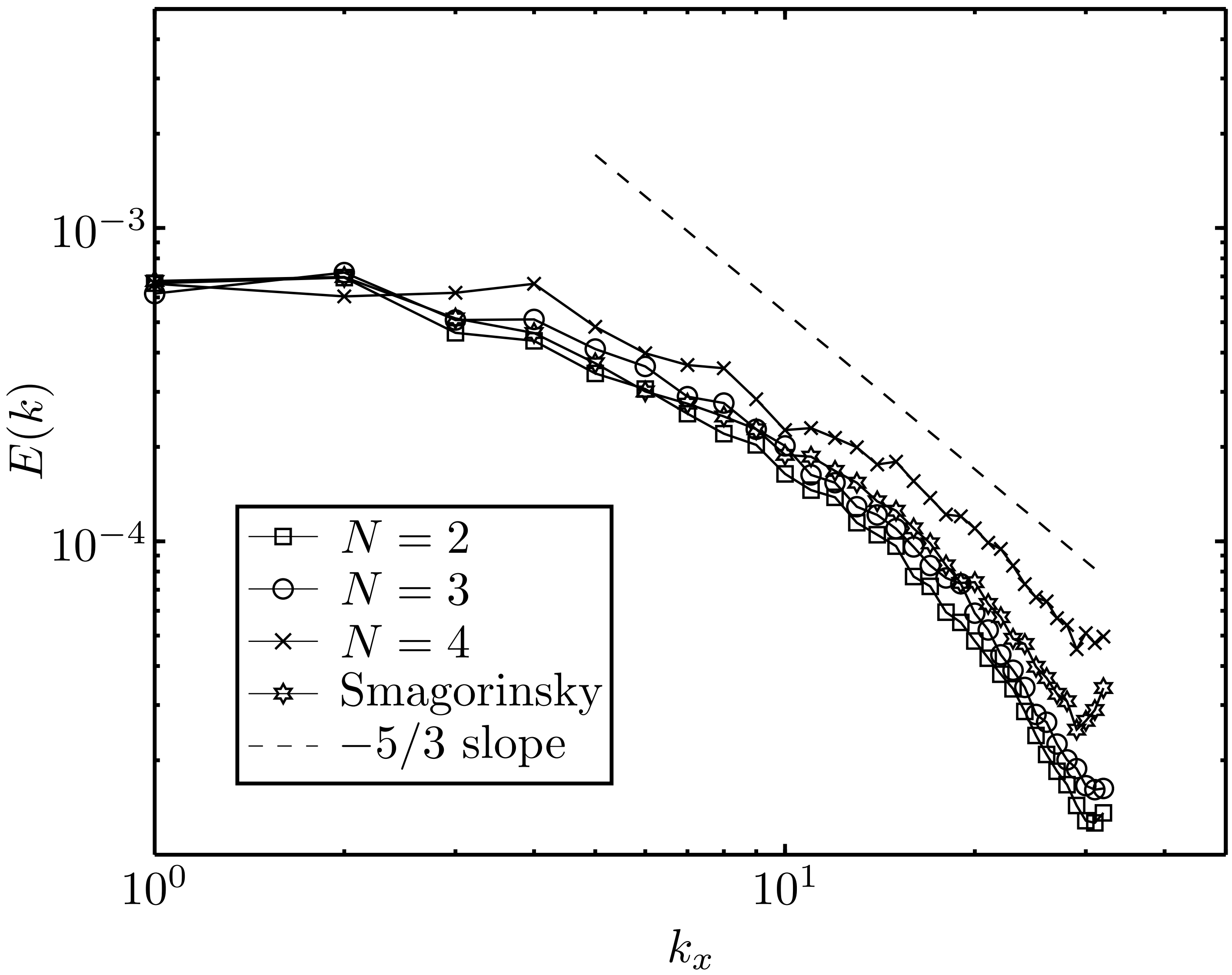


Equivalent Regularized LBM-ADM

Spectral content



Wall Model for LBM-LES

(O. Malaspinas & P. Sagaut, UPMC)

- Issue: to use large first off-wall cells

- Proposed strategy :
  - RANS-type solution within 1st cell
    - Empirical mean velocity/temperature profile
    - Solving simplified RANS Turbulent Boundary Layer Eqs.
  - Reconstruction of LBM variables in the 1st cell

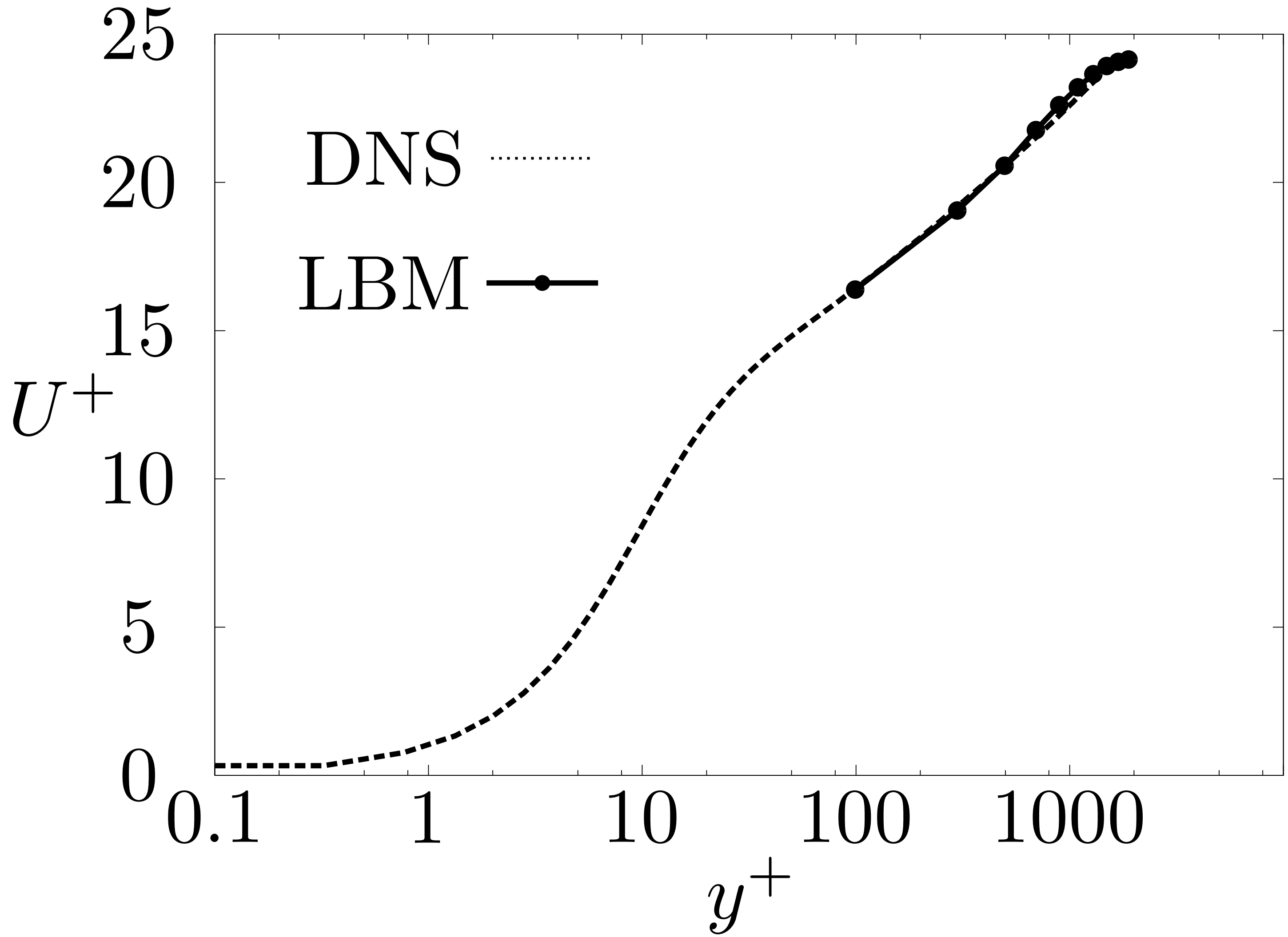
$$f_i = w_i \rho \left(1 + \frac{\xi_i \cdot u}{c_s^2} + \frac{1}{2c_s^4} Q_i : uu \right) - \frac{w_i \rho}{\omega c_s^2} Q_i : S - \frac{1}{4c_s^4} Q_i : (ug + gu)$$

$$Q_i \equiv \xi_i \xi_i - c_s^2 I$$

Wall Model for LBM-LES

Channel flow at $Re_\tau = 2003$

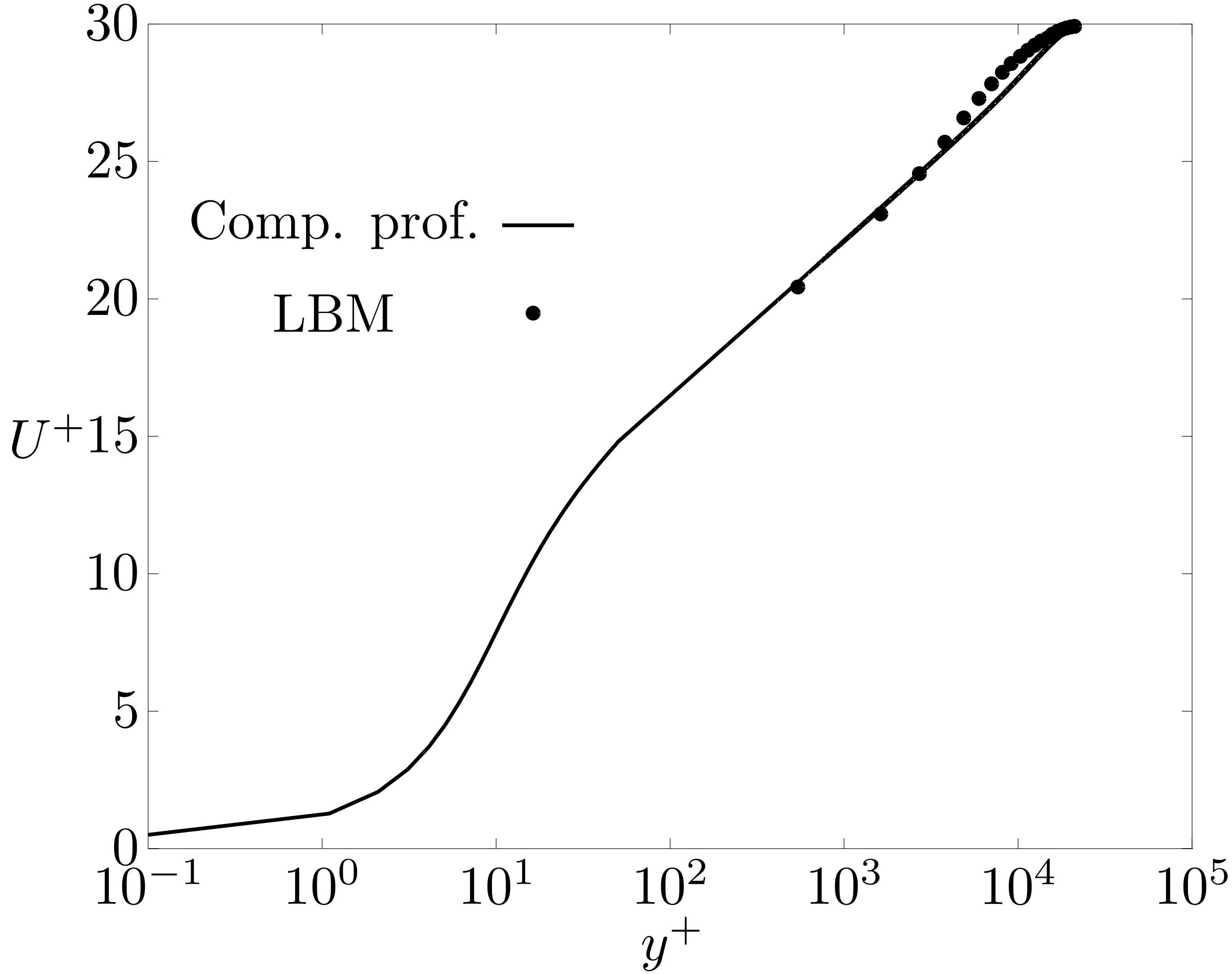
$$\Delta x^+ = \Delta y^+ = \Delta z^+ = 200$$



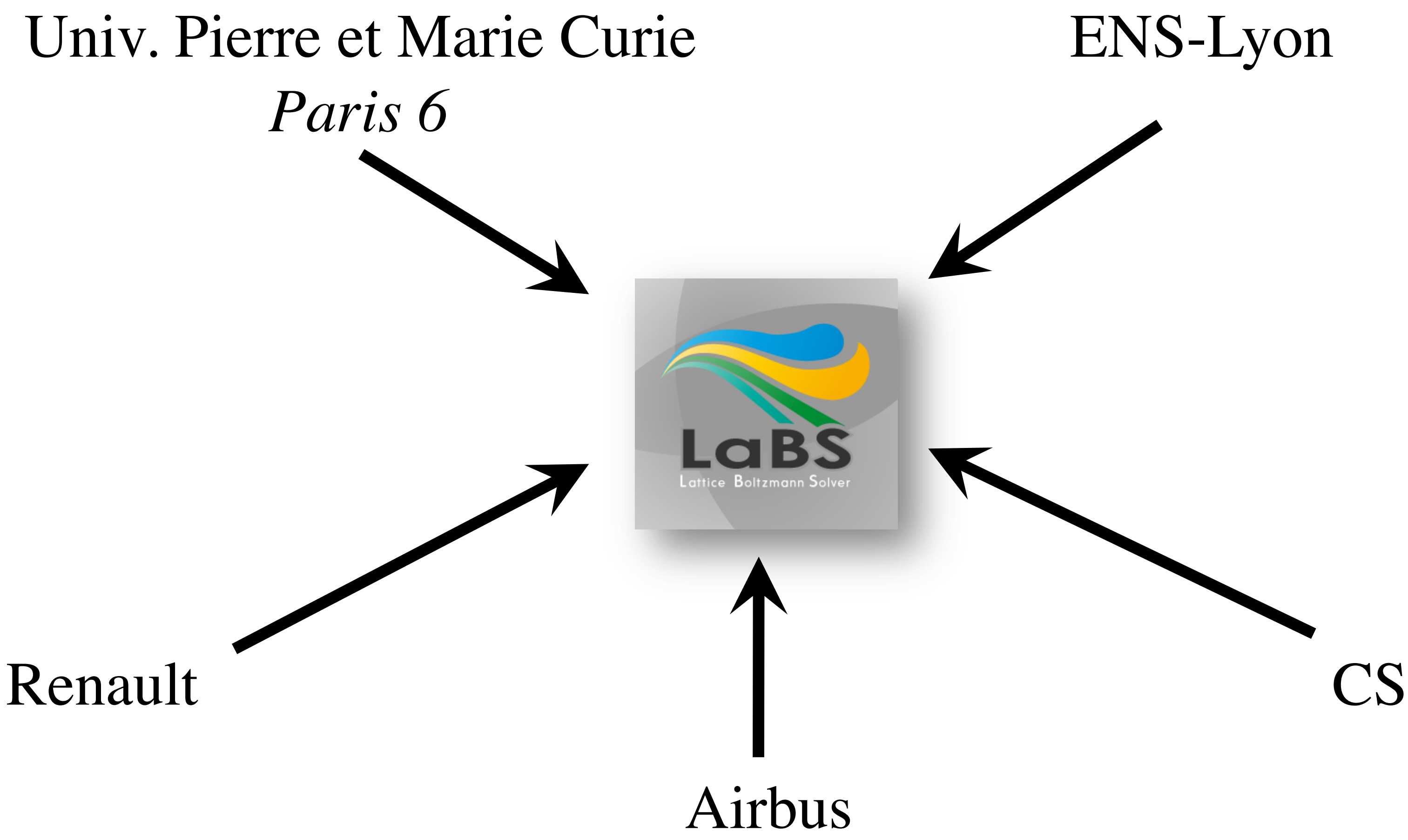
Wall Model for LBM-LES

Channel flow at $Re_\tau = 20,000$

$$\delta x^+ = \Delta y^+ = \Delta z^+ = 1000$$

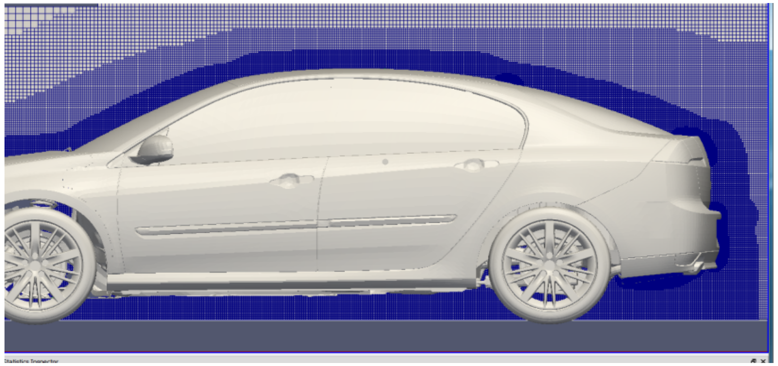


LAttice-Boltzmann Solver: a new industrial CFD tool



http://www.labs-project.org/

Validation on full-scale vehicles



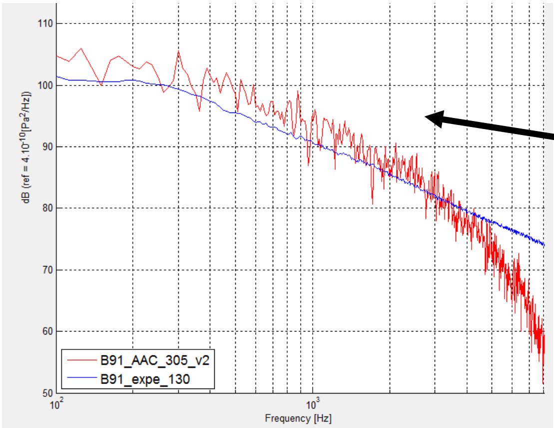
- Full scale vehicle simulation
- 186 surfaces (2,3 millions surface triangles)
- 10 levels of grid refinement, 88.6 millions cells
- dx_min=1.25mm
- 300 000 time steps → 0.96 sec
- U0 = 44.4 m/s
- Wall Model in first cell LES
- LBM-ADM model

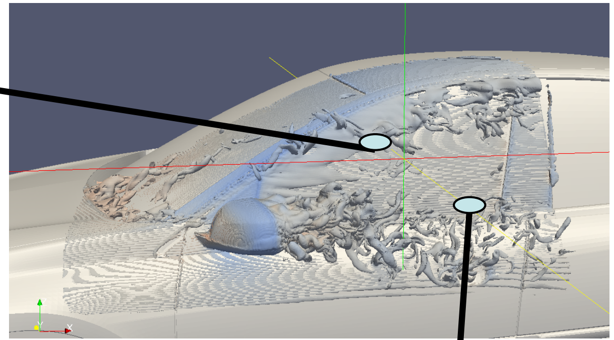
Validation on full-scale vehicles

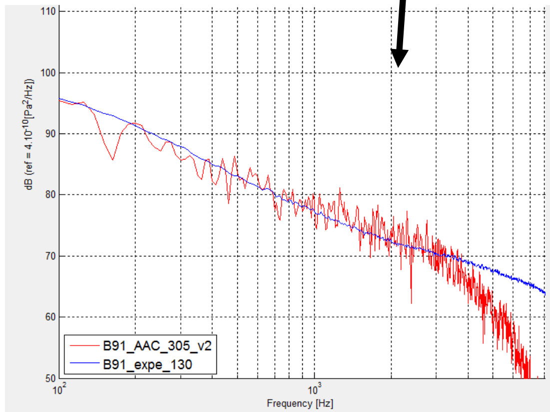
LaBS

Measurements

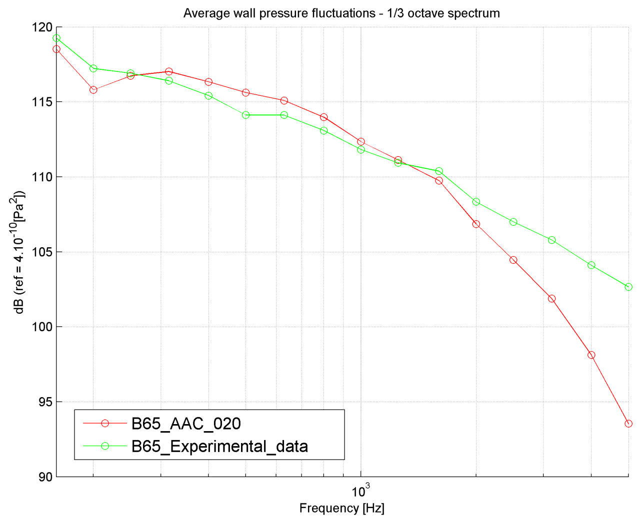
Laguna case : fine band spectra

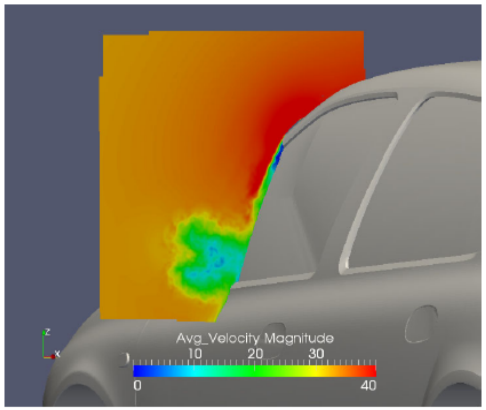






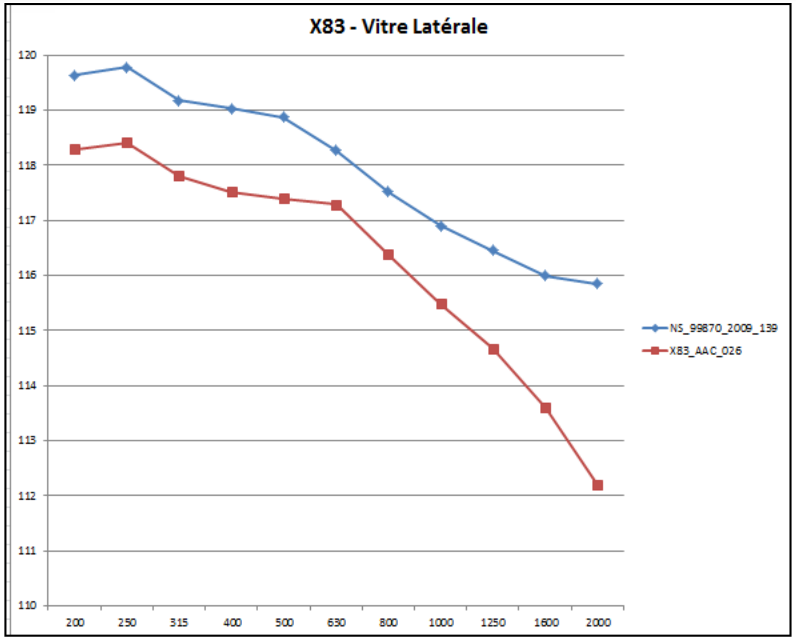
Clio case : third-octave band spectra, averaged on the whole surface of the side window

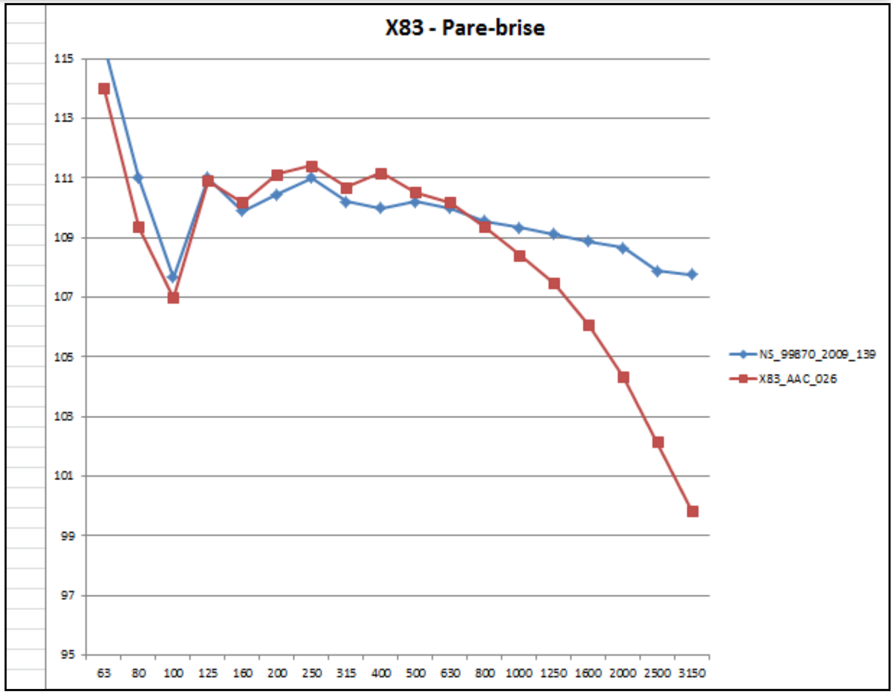


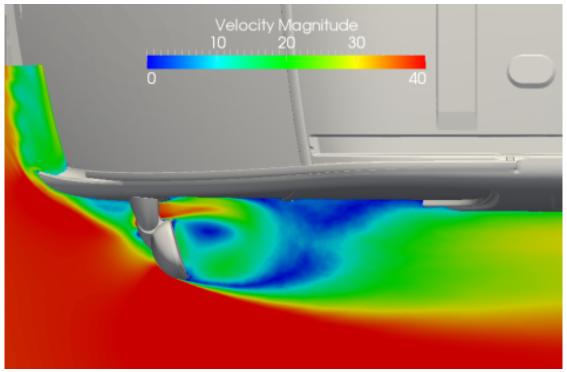


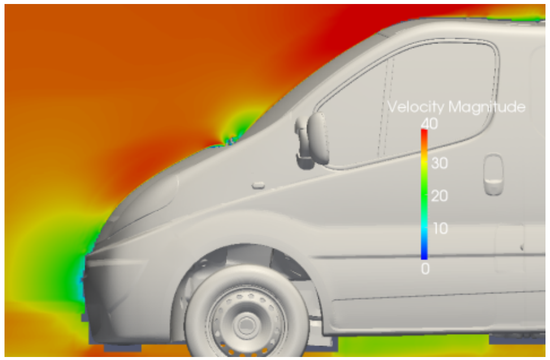
Validation on full-scale vehicles

1 dB









X83 – Trafic – pressure spectrum on side window & windshield

Conclusions

- A fully general formulation for LBM-LES has been proposed
    - macroscopic variables are not filtered as f
    - macroscopic filter is variable-dependent

- A general ADM-LBM closure strategy has been derived
    - no explicit eddy-viscosity assumption
    - fast implementation possible

- Implementation in LaBS for real industrial applications

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Thank you!