Turbulent channel flow tests

May 18, 2018

Channel Flow tests

The idea for this investigation is to check the cause for non-existant eddy viscosity in the channel for the $Re_\tau = 395$. We would perform two set of simulations on the mesh resolution 192x64x96 (Mesh1). We would write out all the quantities i.e. gradients $\frac{\partial v_i}{\partial x_i}$, strain rates contraction and the traceless tensor formulated with the square of the velocity gradient tensor. In short, it would be same as we did previously in the poiseuille flow simulations and the Taylor green vortex simulations.

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i. Mesh 1 AA2016Wale. We will perform the simulation for total 300,000 time steps and write out the data only for the $300,000^{th}$ time-step. In this tests we will not perform any kind of averaging, but write out all the details as discussed above. The idea here is to observe the order for the different quantities written out and to estimate the orgin of the error.

ii. Mesh 1 AA2016Wale - smaller perturbation number (ϵ). Here we will perform the same simulation as above, but with the smaller ϵ added to make the system well-poised. The definition of which is as follows:

$$\nu_t = (C_w\Delta)^2 \frac{\left(S^d_{ij}S^d_{ij}\right)^{3/2}}{\left(\bar{S_{ij}}\bar{S_{ij}}\right)^{5/2} + \left(S^d_{ij}S^d_{ij}\right)^{5/4} + \epsilon} \tag{1}$$

where, ϵ in our case is of the order $\sim 10^{-8}$.

In this simulation we will change the value of ϵ to $\sim 10^{-30}$ or lower as the order of the terms in the numerator and the denominator are in between $\sim 10^{-18}$ to $\sim 10^{-25}$. Adding the epsilon of $\sim 10^{-8}$ in the denominator, which is relatively a larger number, is the reason we do not see the eddy viscosity in the channel. The purpose of this test is to see if we get the same orders of magnitude of the eddy viscosity as obtained in the tests performed using Matlab.

1.1 Discussion 9^{th} May

I. Test 1. Here we have to improve the conditioning for the eddy viscosity in a more general way, rather than adding a constant approximately. The idea as discussed by professor is as follows:

$$\begin{aligned} A^{3/2} &= \left(S^d_{ij}S^d_{ij}\right)^{3/2}, \\ A^{5/4} &= \left(S^d_{ij}S^d_{ij}\right)^{5/4}, \\ B^{5/2} &= \left(\bar{S_{ij}}\bar{S_{ij}}\right)^{5/2} \end{aligned} \tag{2}$$

When terms of the equation 6 are replaced in the equation for 1 it looks as follows (not considering ϵ now):

$$\nu_t = (C_w\Delta)^2 \frac{A^{3/2}}{B^{5/2} + A^{5/4}}, \qquad OP = \frac{A^{3/2}}{B^{5/2} + A^{5/4}} \tag{3}$$

Now in the equation 7 let's divide by $A^{5/4}$ and OP would like:

$$OP = \frac{A^{3/2}A^{-5/4}}{\left(B^{5/2}/A^{5/4}\right) + 1} \tag{4}$$

Some sort of this modification can give a general conditioning of the eddy viscosity.
II. Channel flow. Pressure and the density are negative i.e. mass deficit in the channel
III. Taylor green vortex. Finer resolution with changed value of ϵ.

$$OP = \frac{A^{1/4}}{\left(B^{1/2}/\left(A^{1/4} + 10^{-10}\right)\right)^5 + 1} \tag{5}$$

1.2 Next steps $15^{th}May$

Since the reason for the missing eddy viscosity is found, $\epsilon = 10^{-30}$, and an appropriate wale model co-efficient, $C_W = 0.55$, we should start the wale model simulations on the channel flow.

i. AA2016Wale and OneWale
ii. AA2016wale with double precision. This will be a comparison to see if any significant changes are present in the flow.

2 Results and discussion

3 Introduction Turbulence modelling

i DNS.
Direct numerical simulation (DNS) method uses a modelling-free approach for approximation of the exact solution, as the navier-stokes equation comprises of all turbulence mechanisms. Since the turbulence is three dimensional and inherently instationary it has to take in to account all the components present in the navier stokes equation.Turbulence involves the interaction in between the scales of various different sizes and thus to represnt the exact solution or the characteristics of the turbulent flow considered correctly, the DNS method emphasises on the fine spatial and temporal resolution of the meshes. Apart from that this method requires the discretization methods to have low level of numerical dissipation and dispersion. In order to correctly determine the effects of the turbulence intensity, the fluctuating flow variables which satisfy the navier stokes equation are provided as inputs.
ii DES.
Detached eddy simulation, DES, combines the approach of RANS and the LES method; it therefore belongs to the class of the hybrid RANS-LES simulations. The basic idea of this method is that the near wall boundary layer would be resolved using the RANS approach, while the usuage of LES to (correctly) resolve such regions requires a very fine spatial and temporal resolution, as the turbulent scales to be resolved in the boundary layers are very small. On the other hand, the detached regions, wakes and the free shear layers are the regions, where the RANS model frequently fails, are handled by the LES.

DES was first introduced by *Spalart et al.*. The goal was to compute the aerodynamic flows involving massive seperation with a moderate increase in the grid resolution compared to the RANS method. The idea behind the DES is to segregate the flow, using suitable detectors, in to the domains where the entire energy spectrum will be modelled using the RANS (near wall boundary layer) or the large energy containing structures are resolved using the LES method (detached flow region). Therefore in this approach, a suitable method is chosen, for each domain, in terms of efficiency and accuracy.

In comparison with LES, for the computation of the flows involving the boundary layer, DES reduces the computational effort significantly. Since the flows involved are unsteady and three dimensional turbulent flows, the computational effort will be relatively higher when compared with RANS.

Avery good compilation of the hybrid RANS-LES approach is found in *Frhlich and von Tersi (2008)*

$$\begin{aligned} A^{3/2} &= \left(S^d_{ij} S^d_{ij}\right)^{3/2}, \\ A^{5/4} &= \left(S^d_{ij} S^d_{ij}\right)^{5/4}, \\ B^{5/2} &= \left(\bar{S_{ij}} \bar{S_{ij}}\right)^{5/2} \end{aligned} \tag{6}$$

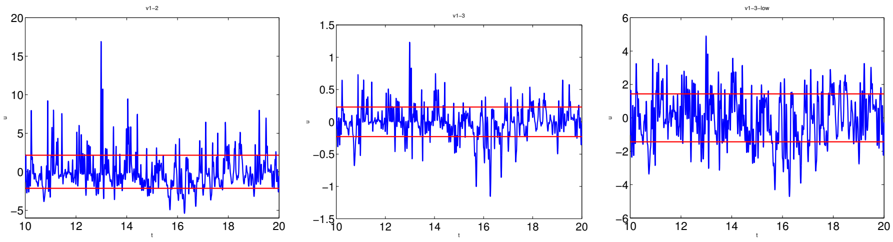
When terms of the equation 6 are replaced in the equation for 1 it looks as follows (not considering ϵ now):

$$\nu_t = (C_w \Delta)^2 \frac{A^{3/2}}{B^{5/2} + A^{5/4}}, \qquad (7)$$
$$OP = \frac{A^{3/2}}{B^{5/2} + A^{5/4}}$$

Now in the equation 7 let's divide by $A^{5/4}$ and OP would like:

$$OP = \frac{A^{3/2} A^{-5/4}}{\left(B^{5/2}/A^{5/4}\right) + 1} \qquad (8)$$

Some sort of this modification can give a general conditioning of the eddy viscosity.
II. Channel flow. Pressure and the density are negative i.e. mass deficit in the channel
III. Taylor green vortex. Finer resolution with changed value of ϵ.



(a) $S = 1.51$ and $F = 7.68$ (b) $S = -0.27$ and $F = 5.81$ (c) $S = -0.09$ and $F = 2.77$

Figure 1: Time history of v_1'. Horisontal red lines show $\pm v_{1,rms}$.

4 Turbulent mean flow

There is no definition on turbulent flow, but it has a number of characteristic features (see Pope [1] and Tennekes & Lumley [2]) such as:

I. Irregularity. Turbulent flow is irregular, random and chaotic. The flow consists of a spectrum of different scales (eddy sizes). We do not have any exact definition of an *turbulent eddy*, but we suppose that it exists in a certain region in space for a certain time and that it is subsequently destroyed (by the cascade process or by dissipation, see below). It has a characteristic velocity and length (called a velocity and length scale). The region covered by a large eddy may well enclose also smaller eddies. The largest eddies are of the order of the flow geometry (i.e. boundary layer thickness, jet width, etc). At the other end of the spectra we have the smallest eddies which are dissipated by viscous forces (stresses) into thermal energy resulting in a temperature increase. Even though turbulence is chaotic it is deterministic and is described by the Navier-Stokes equations.

turbulent eddy

$S = 1.51$, -0.27 and -0.09. The flatness are $F = 7.68$, 5.81 and 2.77.

Consider the probability density functions of the fluctuations. The second moment corresponds to the variance of the fluctuations (or the square of the RMS, i.e.

$$\overline{v'^2} = \int_{-\infty}^{\infty} v'^2 f(v')dv'$$

$\overline{v'^2}$ is usually computed by integrating in time.

5 Turbulent mean flow

5.1 Time averaged Navier-Stokes

When the flow is turbulent it is preferable to decompose the instantaneous variables (for example the velocity components and the pressure) into a mean value and a fluctuating value, i.e.

$$\begin{aligned} v_i &= \bar{v}_i + v_i' \\ p &= \bar{p} + p' \end{aligned} \tag{9}$$

where the bar, $\bar{\cdot}$, denotes the time averaged value. One reason why we decompose the variables is that when we measure flow quantities we are usually interested in their mean values rather than their time histories. Another reason is that when we want to solve the Navier-Stokes equation numerically it would require a very fine grid to resolve all turbulent scales and it would also require a fine resolution in time (turbulent flow is always unsteady).

The continuity equation and the Navier-Stokes equation for incompressible flow with constant viscosity read

$$\begin{aligned} &\frac{\partial v_i}{\partial x_i} = 0 \\ &\rho\frac{\partial v_i}{\partial t} + \rho\frac{\partial v_i v_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 v_i}{\partial x_j \partial x_j} \end{aligned} \tag{10}$$

The gravitation term, $-\rho g_i$, has been omitted which means that the p is the *hydrostatic* pressure. Inserting Eq. 9 at p. viii into the continuity equation (10) and the Navier-Stokes equation we obtain the *time averaged* continuity equation and Navier-Stokes equation

$$\frac{\partial \bar{v}_i}{\partial x_i} = 0 \tag{11}$$

$$\rho\frac{\partial \bar{v}_i}{\partial t} + \rho\frac{\partial \bar{v}_i\bar{v}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu\frac{\partial \bar{v}_i}{\partial x_j} - \rho\overline{v'_i v'_j}\right) \tag{12}$$

This equation is the time-averaged Navier-Stokes equation and it is often called the *Reynolds equation*. A new term $\rho\overline{v'_i v'_j}$ appears on the right side of Eq. 12 which is called the *Reynolds stress tensor*. The tensor is symmetric (for example $\overline{v'_1 v'_2} = \overline{v'_2 v'_1}$). It represents correlations between fluctuating velocities. It is an additional stress term due to turbulence (fluctuating velocities) and it is unknown. We need a model for $\overline{v'_i v'_j}$ to close the equation system in Eq. 12. This is called the *closure problem*: the number of unknowns (ten: three velocity components, pressure, six stresses) is larger than the number of equations (four: the continuity equation and three components of the Navier-Stokes equations).

Reynolds equations

closure problem

5.1.1 Boundary-layer approximation

For steady ($\partial/\partial t = 0$), two-dimensional ($\bar{v}_3 = \partial/\partial x_3 = 0$) boundary-layer type of flow (i.e. boundary layers along a flat plate, channel flow, pipe flow, jet and wake flow, etc.) where

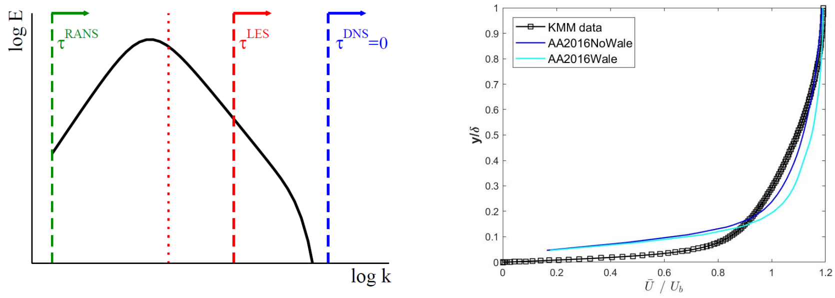
$$\bar{v}_2 \ll \bar{v}_1, \; \frac{\partial}{\partial x_1} \ll \frac{\partial}{\partial x_2}, \tag{13}$$

Eq. 12 reads

$$\rho\frac{\partial \bar{v}_1\bar{v}_1}{\partial x_1} + \rho\frac{\partial \bar{v}_2\bar{v}_1}{\partial x_2} = -\frac{\partial \bar{p}}{\partial x_1} + \frac{\partial}{\partial x_2}\underbrace{\left[\mu\frac{\partial \bar{v}_1}{\partial x_2} - \rho\overline{v'_1 v'_2}\right]}_{\tau_{tot}} \tag{14}$$

x_1 and x_2 denote the streamwise and wall-normal coordinate, respectively.

If you want to learn more how to derive transport equations of turbulent quantities, see [5] which can be downloaded here
`http://www.tfd.chalmers.se/~lada/allpapers.html`



(a) Zoom near the wall (b) Outer region

Figure 2: Channel flow at $Re_\tau = 2000$. Terms in the $\overline{v'_1 v'_2}$ equation scaled by u_τ^4/ν. DNS data [3, 4]. —— : P_{12}; - - - : $-\varepsilon_{12}$; ∇ : $-\partial \overline{v'p'}/\partial x_2$; - · - : Π_{12}; +: $-\partial(\overline{v'_1 v'^2_2})/\partial x_2$; ○: $\nu \partial^2 \overline{v'_1 v'_2}/\partial x_2^2$.

i	n	j	k	$\varepsilon_{inm}\varepsilon_{mjk}$	$\delta_{ij}\delta_{nk} - \delta_{ik}\delta_{nj}$
1	2	1	2	$\varepsilon_{12m}\varepsilon_{m12} = \varepsilon_{123}\varepsilon_{312} = 1 \cdot 1 = 1$	$1 - 0 = 1$
2	1	1	2	$\varepsilon_{21m}\varepsilon_{m12} = \varepsilon_{213}\varepsilon_{312} = -1 \cdot 1 = -1$	$0 - 1 = -1$
1	2	2	1	$\varepsilon_{12m}\varepsilon_{m21} = \varepsilon_{123}\varepsilon_{321} = 1 \cdot -1 = -1$	$0 - 1 = -1$
1	3	1	3	$\varepsilon_{13m}\varepsilon_{m13} = \varepsilon_{132}\varepsilon_{213} = -1 \cdot -1 = 1$	$1 - 0 = 1$
3	1	1	3	$\varepsilon_{31m}\varepsilon_{m13} = \varepsilon_{312}\varepsilon_{213} = 1 \cdot -1 = -1$	$0 - 1 = -1$
1	3	3	1	$\varepsilon_{13m}\varepsilon_{m31} = \varepsilon_{132}\varepsilon_{231} = -1 \cdot 1 = -1$	$0 - 1 = -1$
2	3	2	3	$\varepsilon_{23m}\varepsilon_{m23} = \varepsilon_{231}\varepsilon_{123} = 1 \cdot 1 = 1$	$1 - 0 = 1$
3	2	2	3	$\varepsilon_{32m}\varepsilon_{m23} = \varepsilon_{321}\varepsilon_{123} = -1 \cdot 1 = -1$	$0 - 1 = -1$
2	3	3	2	$\varepsilon_{23m}\varepsilon_{m32} = \varepsilon_{231}\varepsilon_{132} = 1 \cdot -1 = -1$	$0 - 1 = -1$

Table 1: The components of the $\varepsilon - \delta$ identity which are non-zero.

A $\varepsilon - \delta$ identity

The $\varepsilon - \delta$ identity reads

$$\varepsilon_{inm}\varepsilon_{mjk} = \varepsilon_{min}\varepsilon_{mjk} = \varepsilon_{nmi}\varepsilon_{mjk} = \delta_{ij}\delta_{nk} - \delta_{ik}\delta_{nj}$$

In Table 1 the components of the $\varepsilon - \delta$ identity are given.

References

[1] S.B. Pope. *Turbulent Flow.* Cambridge University Press, Cambridge, UK, 2001.

[2] H. Tennekes and J.L. Lumley. *A First Course in Turbulence.* The MIT Press, Cambridge, Massachusetts, 1972.

[3] S. Hoyas and J. Jiménez. Scaling of the velocity fluctuations in turbulent channels up to $Re_\tau = 2003$. *Physics of Fluids A*, 18(011702), 2006.

[4] S. Hoyas and J. Jiménez. http://torroja.dmt.upm.es/ftp/channels/data/statistics/. 2006.

[5] L. Davidson. Transport equations in incompressible URANS and LES. Report 2006/01, Div. of Fluid Dynamics, Dept. of Applied Mechanics, Chalmers University of Technology, Göteborg, Sweden, 2006.