# Turbulence

Turbulence occurs when the inertia forces in the fluid become significant compared to the viscous forces, and is characterized by the high Reynolds number. Turbulence consists of fluctuations in the flow field in time and space. It is a complex process, mainly because it is three dimensional and unsteady. Rotational flow structures, so called turbulent eddies, with a wide range of length scales are found in turbulent flows. Large turbulent eddies transfer the energy to the small eddies and the small eddies in turn dissipate the energy into thermal energy because of the action of viscous stresses.

In principle Navier-Stokes equations describe both laminar and turbulent flows without the need for additional information. But performing the calculations directly by solving these equations makes this approach, computationally, a lot expensive. An alternative approach to the approach mentioned before would be to have, some mathematical models, turbulence models which can account for all the effects of turbulence without the requirements of fine mesh and avoid solving the navier-stokes equation directly.

The following are the categories of turbulence models:

1. RANS (Reynolds averaged navier stokes equation) :

Here the attention is focused on the mean flow and the effects of turbulence on mean flow properties. Prior to the application of numerical methods the Navier–Stokes equations are time averaged (or ensemble averaged in flows with time-dependent boundary conditions). Extra terms appear in the time-averaged (or Reynolds averaged) flow equations due to the interactions between various turbulent fluctuations. These extra terms are modeled with classical turbulence models: among the best known ones are the *k*–ε model and the Reynolds stress model. The computing resources required for reasonably accurate flow computations are modest, so this approach has been the mainstay of engineering flow calculations over the last three decades.

1. LES (Large Eddy Simulation):

This is an intermediate form of turbulence calculations which tracks the behavior of the larger eddies. The method involves space filtering of the unsteady Navier–Stokes equations prior to the computations, which passes the larger eddies and rejects the smaller eddies. The effects on the resolved flow (mean flow plus large eddies) due to the smallest, unresolved eddies are included by means of a so-called sub-grid scale model. Unsteady flow equations must be solved, so the demands on computing resources in terms of storage and volume of calculations are large, but (at the time of writing) this technique is starting to address CFD problems with complex geometry.

1. DNS (Direct Numerical Simulation):

In this method the simulations compute the mean flow and all turbulent velocity fluctuations. The unsteady Navier–Stokes equations are solved on spatial grids that are sufficiently fine that they can resolve the Kolmogorov length scales at which energy dissipation takes place and with time steps sufficiently small to resolve the period of the fastest fluctuations. These calculations are highly costly in terms of computing resources, so the method is not used for industrial flow computations [10].

## RANS Models:

The continuity and the momentum equations are described as below:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |
|  |  |  |

where, is the molecular stress tensor, is the momentum source term

When the time scales, which are much larger that the time scales of turbulent fluctuations, are considered the turbulent flow is said to exhibit the average characteristics, but with an additional time-varying and fluctuating component i.e. velocity component may be divided in to an average and a time varying component

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

where the averaged component is given by:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

Substituting the averaged quantities, eq (1.3) , in the original transport equations, eq (1.1) - (1.2), results in the Reynolds averaged equations given below. In the following equations, the bar is dropped for averaged quantities, except for products of fluctuating quantities:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |

The continuity equation has not been altered but the momentum equation has additional turbulent flux terms in addition to the molecular diffusive fluxes. These terms - are the Reynolds stresses. They have to be modeled by additional equations of known quantities in order to achieve “closure”. Closure means that there should be sufficient number of equations for all the unknowns, including the unknowns which appear from the averaging equations.

The most common RANS turbulent models are classified depending on the number of additional equations that are required to be solved along with the averaged transport equations [10]:

|  |  |
| --- | --- |
| No. of additional transport equations | Name |
| Zero | Mixing length model |
| One | Spalart-Allmaras model |
| Two | k-ε  k-ω  Algebraic stress model |
| Seven | Reynolds stress model |

Table

### Two equation models

A proposal from boussinesq in 1877 suggested that the reynolds stresses can be related to the mean velocity gradients and turbulent viscosity by the gradient diffusion hypothesis, in a manner analogus to the relationship between the stress and strain tensors in laminar newtonian flow:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

where is the turbulent viscosity which must be modeled and k is the turbulent kinetic energy defined as:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

In the equation stating the boussinesq hypothesis above, indicates that only the normal components of reynolds stresses will be considered. This implies an isotropic assumption for the normal reynolds stresses. Analogus to the turbulent viscosity hypothesis is the turbulent diffusivity hypothesis, which states that the Reynolds fluxes of a scalar are linearly related to the mean scalar gradient.

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

where is defined as the turbulent diffusivity and it is written as,

|  |  |  |
| --- | --- | --- |
|  | = | (.) |

Or

|  |  |  |
| --- | --- | --- |
|  | = | (.) |

where and are the turbulence Prantl and Schmidt number respectively

Applying the hypothesis, eq (1.7), to the averaged momentum equation, eq (1.6), becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |
|  |  | (.) |

Similary the scalar transport equations are written as :

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

#### k-ε model:

This model uses two additional equations, in the form of turbulence kinetic energy (k) m2/s2 and turbulence eddy dissipation (ε) m2/s3, defined as the rate at which the velocity fluctuations dissipate. This model uses the assumption, that the turbulence viscosity is linked to the turbulence kinetic energy and dissipation via the relation:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

where is the dimensionless constant.

The transport equations for the k and ε, which are obtained by performing certain algebraic manipulations, are shown below:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |

where,, are constants. is the turbulence production term due to the viscous forces. and represent the influence of the buoyancy forces.

This model does not perform well in the weak shear layers and the spreading rate of the axisymmetric jets in stagnant surroundings is severely over-predicted. Such difficulties can be overcome by modifying the model constant C in eq (1.18). Since this model is based on the boussinesq`s assumption, of isotropic eddy viscosity, it delivers poor performance in flows with swirl, with curved boundary layers.

#### Realizable k-ε model:

Realizable k-ε model aims to improve the limitations of the standard k-ε model, in certain flow types viz. flows with high strain rates. The formulation of this model differs from the standard k-ε model in the following aspects:

New eddy-viscosity formula, involving a variable , which was originally proposed by reynolds [13]. New model equation for the dissipation (ε) based on the dynamic equation of the mean-square vorticity fluctuation

The realizability here implies positivity of the normal stresses and fullfiling the Schwarz inequality for the shear stresses [13]. It is achieved by the first aspect of the realizable k-ε model. This realiability is only achieved by the realizable k-ε model and not by the other, previously discussed, types of k-ε model.

The two equations for k and εfor this model are as shown below:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |

where C1 = max, , S = , *Gk* represents the production of the turbulent kinetic energy, *Gb* is the generation of turbulence kinetic energy due to buoyancy. One limitation of this model is that it produces non-physical turbulent viscosities in situations where the rotating and stationary fluid zones exist.

#### Standard k-ω model:

This model does not require complex non-linear damping function, as required in the k-ε model, which makes it more robust and accurate. This model assumes that the turbulent viscosity is associated with the turbulence kinetic energy and turbulence frequency via the relation :

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

In this model the two equations used are turbulence kinetic energy (k) and turbulence eddy frequency (ω) respectively:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |
|  |  | (.) |

, , , and are constants

The problem with this model is its strong sensitivity towards the free-stream condition. Also depending on the value of ω at inlet, a significant variation in the results was obtained and which was not desirable in case of external aerodynamics and aerospace [12].

#### Shear stress transport model (SST):

Considering the better performance of k-ε model in the free stream and better performance of the standard k-ω model in the near wall region it was suggested in the literature [22] to have an hybrid model:

1. Transformation of the k-ε model to k-ω model in the near wall region
2. Standard k-ε model in the fully turbulent region far from the wall

In this model a series of modifications have been suggested by Menter [22] in order to optimize the performance of the SST model. It included modification in the model constants, Blending functions and proper limiters. Blending function was implemented in such a fashion that in the near wall region it turned to 1, means k-ω is activated there, and 0 in the free stream region, means k-ε model is activated there.

### Reynolds Stress Models (RSM):

These models do not use the boussinesq`s hypothesis, but solves the transport equations for each and every reyolds stress component. They are based on solving of all the stress component equations along with the dissipation equation. Exact production term and modelling of stress anisotropies makes these models suited to more complex flows. But they are not always good compared to the two equation model [10].

Reynolds Stress Models are considered to be the ‘simplest’ type of model with the potential to describe all the mean flow properties and Reynolds stresses without case-by-case adjustment. Apart from that, model is known for its large computing costs i.e. solving seven extra PDE`s (3D) and five (2D). Models face the problem in converging for e.g. in flows with axisymmetric jets and unconfined recirculating flows

#### Base line reynolds stress ω based (BSLRSM):

In this model Omega and BSL reynolds stress model are related wih each other. The reynolds stress – ω model is the reynolds stress model based on the ω-equation [12]. The advantage of the ω based formulation here is a better near wall treatment.

The modeled equation for the reynolds stresses is as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

Here indicates the reynolds stresses, and indicate the shear and bouyancy turbulence production terms of the reynolds stresses respectively, is the pressure strain tensor.

The Omega reynolds stress model

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

where = 2, = 2, = 0.075, are all defined as constants

The BSL Reynolds stress model

The coefficients and of the -equation, as well as both the turbulent Prandtl numbers and , are blended between values from the two sets of constants, corresponding to the -based model constants and the -based model constants transformed to an -formulation:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

There are two set of constants for the -zone and -zone:

Set 1 (SMC -zone):

= 2, = 0.075,

The values of correspond to the value from model.

Set 2 (SMC -zone):

= 0.856, = 0.0828,

The blending of the coefficients is done by the following equation:

|  |  |  |
| --- | --- | --- |
|  |  | (.) |

where

|  |  |  |
| --- | --- | --- |
| and |  | (.) |

#### Explicit Algebraic Reynolds stress (EARSM) model:

Explicit Algebraic Reynolds stress models (EARSM) represent an extension of the standard two-equation models. They are derived from the Reynolds stress transport equations and give a nonlinear relation between the Reynolds stresses and the mean strain-rate and vorticity tensors. Due to the higher order terms, many flow phenomena are included in the model without the need to solve transport equations. The EARSM enables an extension of the current ( and BSL) turbulence models to capture the following flow effects [12]:

* Secondary flows
* Flows with streamline curvature and system rotation

In calculations the EARSM formulation was used with the BSL model (BSL EARSM).

# Additional Theories:

## Turbulent Schmidt number (Sct):

For a multicomponent fluid, scalar transport equations, ref Eq (1.15) , are solved for velocity, pressure, temperature and other quantities of the fluid. However, additional equations must be solved to determine how the components of the fluid viz. propane mass fraction and other species are transported within the fluid. The additional turbulent transport equations which are solved for the components of fluid are of the form:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1) |

where,

*U* is the fluid velocity

is the mixture density, mass per unit volume

is the conserved quantity per unit volume, or concentration

, is the conserved quantity per unit mass

is a volumetric source term, with units of conserved quantity per unit volume per unit time

is the Turbulent Schmidt number

is the kinematic diffusivity for the scalar

Since the turbulent transport of momentum, heat or mass is due to the same mechanism – eddy mixing - we expect that the value of the turbulent diffusivity is fairly close to that of the turbulent viscosity . This assumption is known as Reynolds analogy [10].Hence turbulent diffusion is expressed by . The molecular diffusion term is expressed by.

is introduced as a proportionality factor for the turbulent diffusivity, as the time scale at which the molecular diffusion occurs is different from that in the diffusion of momentum (viscosity). Generally the value of is found between 0.7 - 0.9 in most CFD software`s [23].

## Round jet anomaly:

The standard k-ε model with the standard coefficients delivers the velocity field quite accurately in two-dimensional plane jet, but large errors occur for the axisymmetric jets. Specifically, the spreading rate of round jet is overestimated by 40% [5]. The reason for this “round jet/ plane jet anomaly” is the considered mainly due to the modeled dissipation (ε) equation, refer eq (1.18). Several modifications of constants, in the dissipation equation, have been suggested in the literature by Pope [5]. This shows the non-universality of turbulence [4].