Quantitative Climate Science: Data Centric Methods, Wavelet Methods

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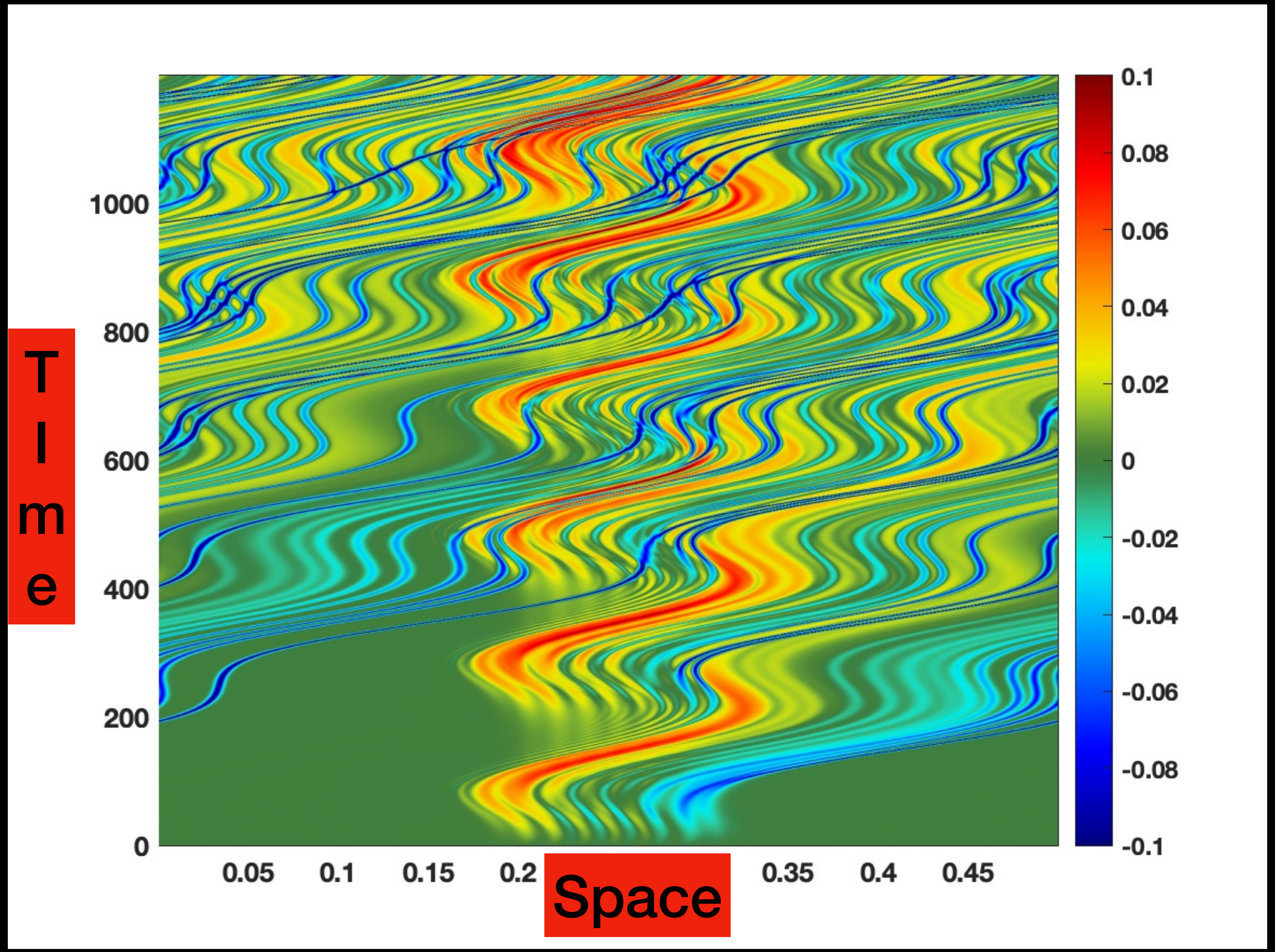
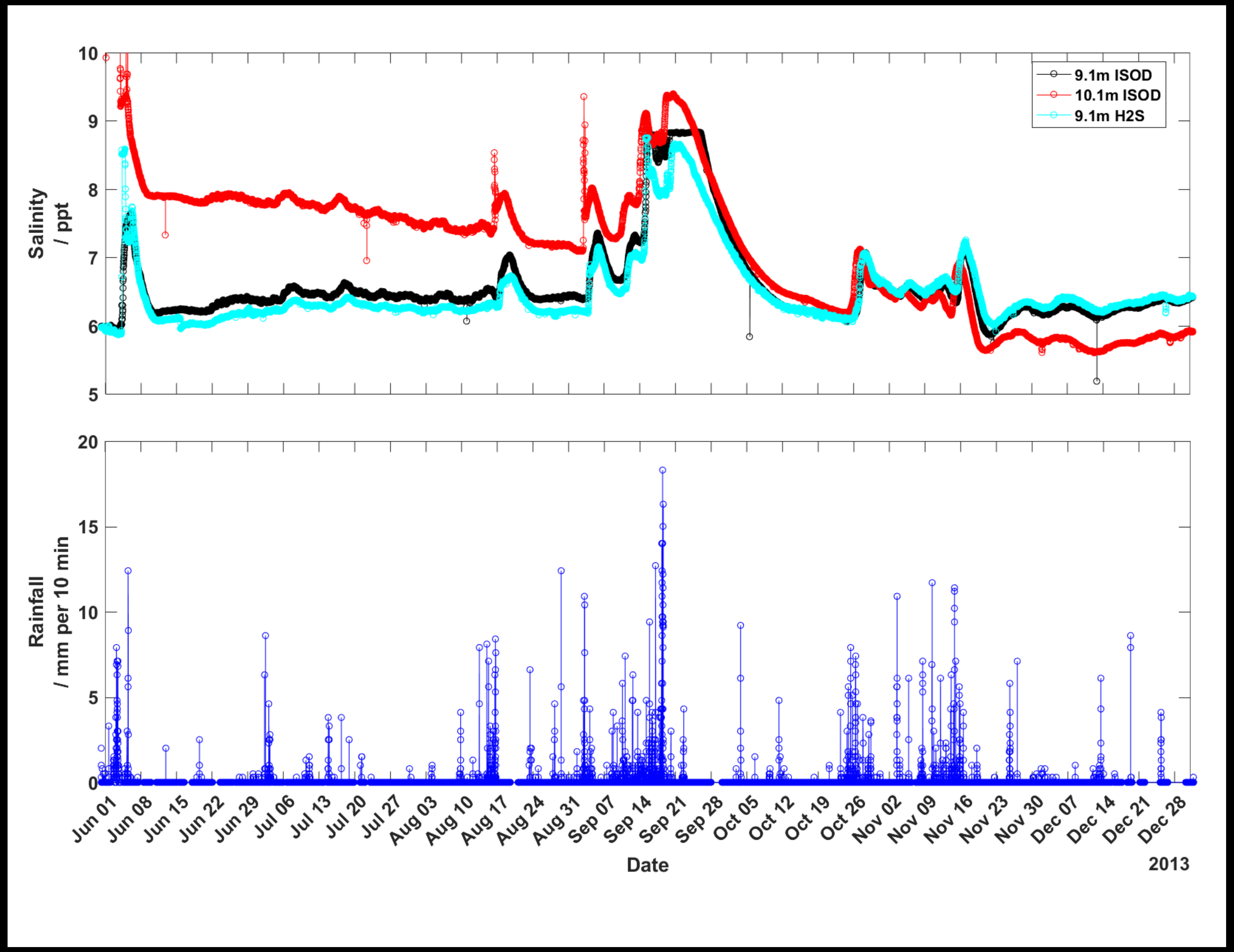
The data (review)

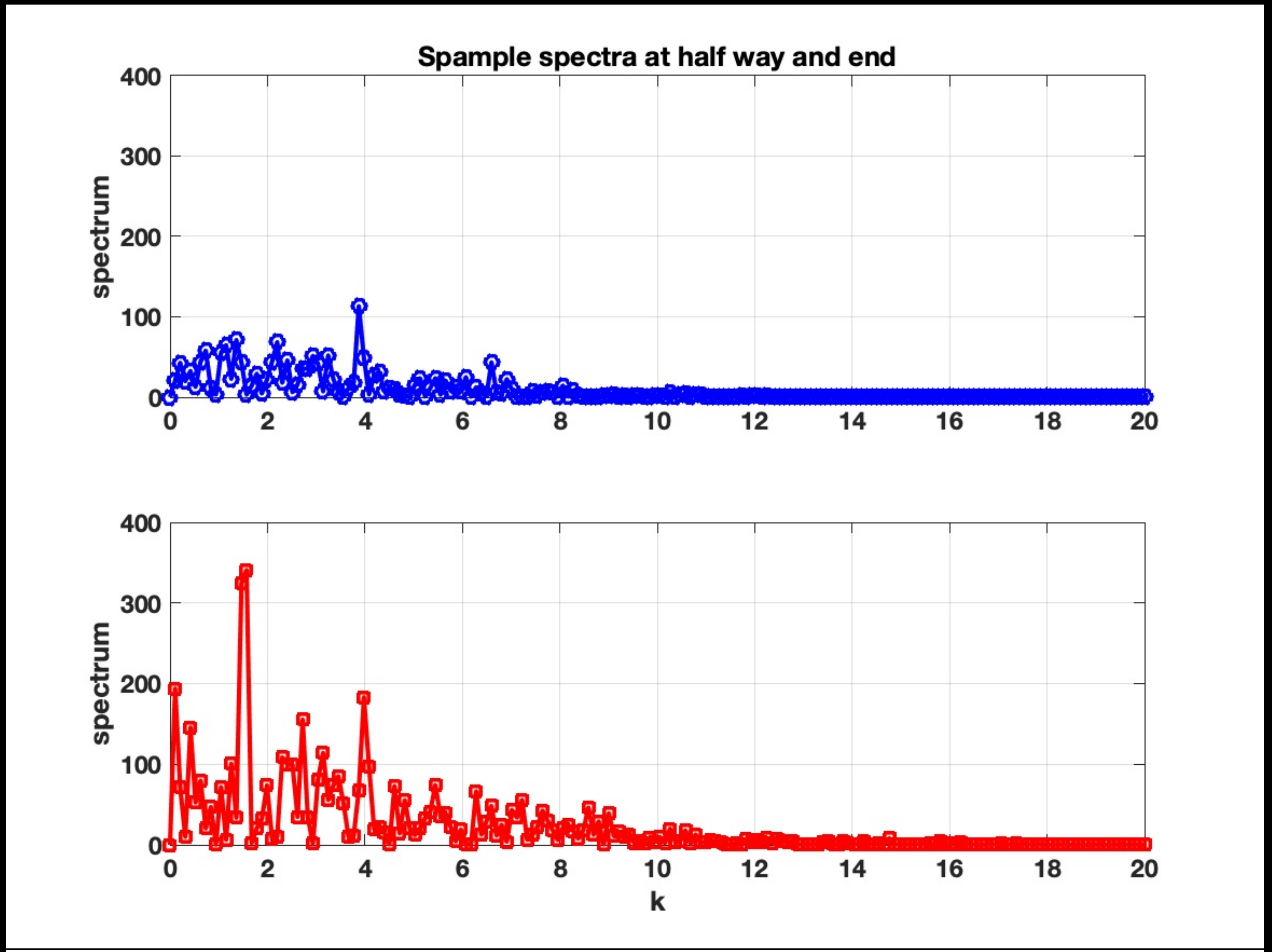
- We started this module with data from a simulation.
- As is typical of such situations the number of spatial points was larger than the number of time measurements.
- For field data we often have sparse coverage in space and a large number of points in time, as in the Salinity measurements from a cave system shown on the lower right.
- Field data is often noise with possible sensor failures (the weird outliers on the lower right)

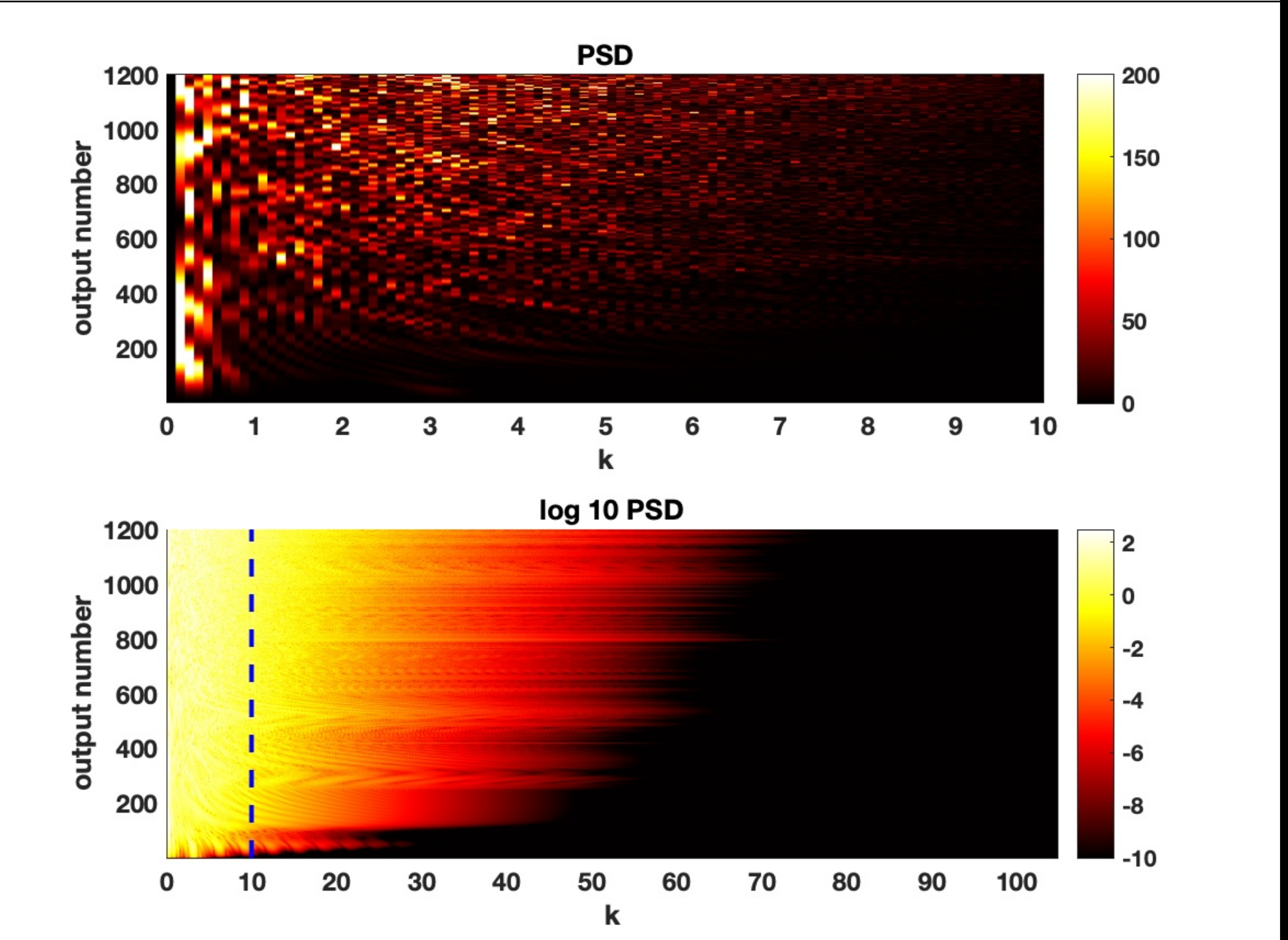




Sample spectra I

- We were able to get some sense of the peaks in wavenumber space using snapshots of the simulaton data set at different times.
- We got somewhat different pictures by stacking the spectra at different times and colouring in the regions where there was a lot of spectral power.
- The truth was that these pictures were not super easy to interpret.



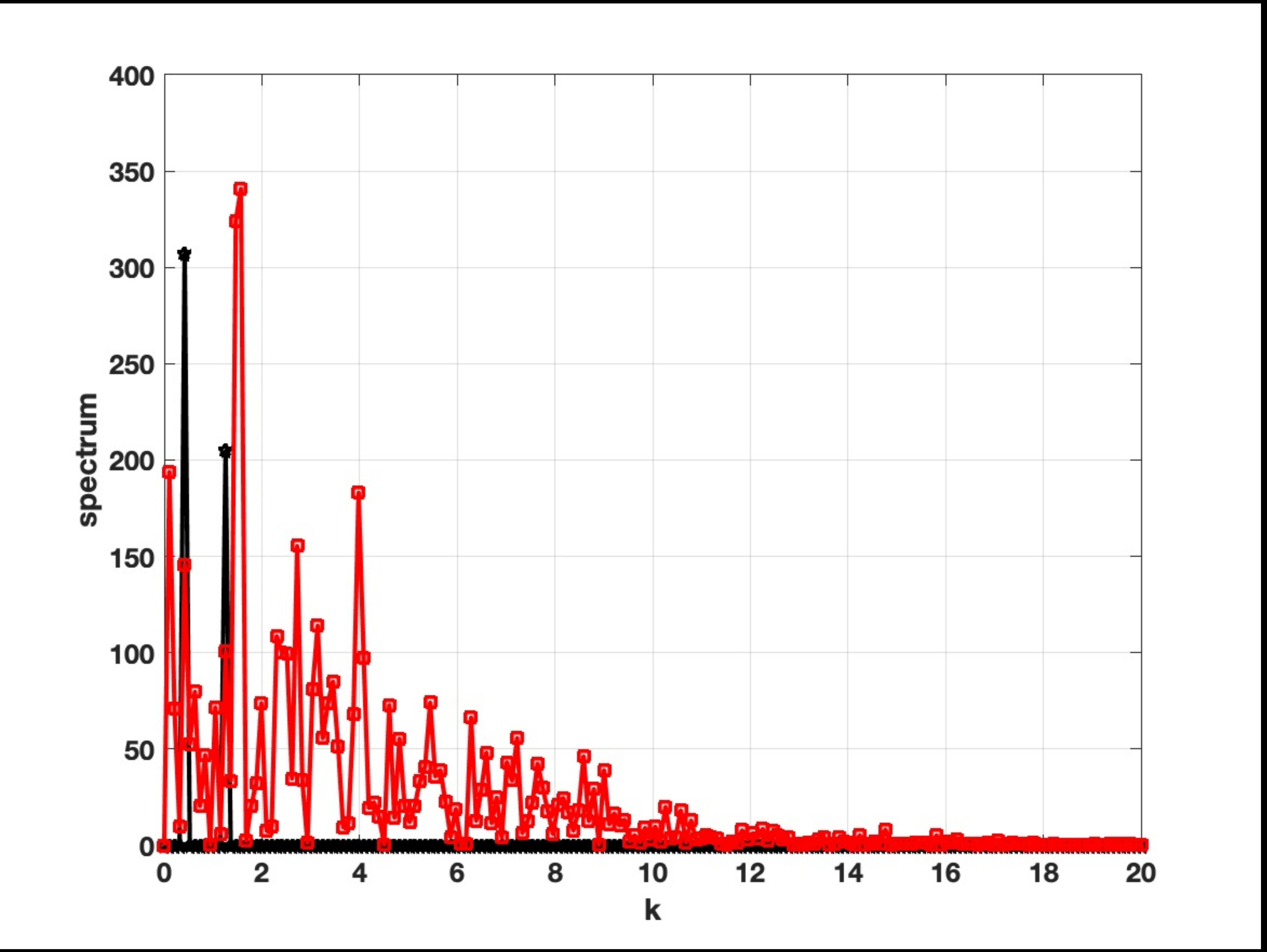


Spectra intuition

- By ploting the spectrum from data in red, and the spectrum of g(x) in black we see that a specturm at least potentially tells us the relevant scales.

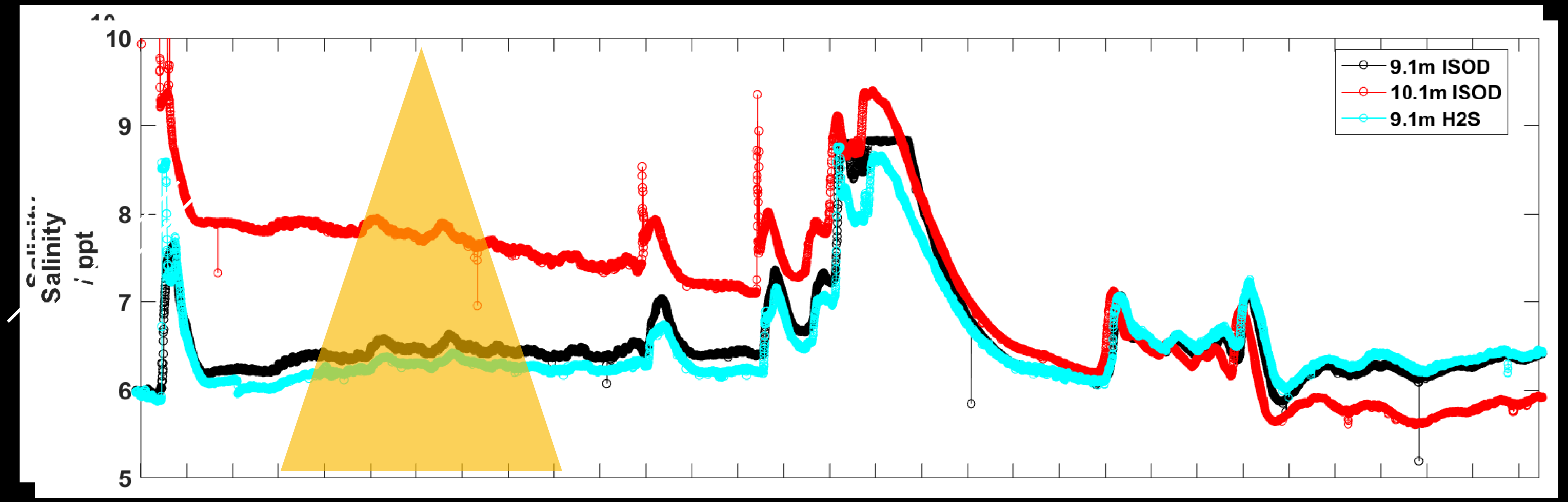
$$g(x) = 0.3\sin\left(\frac{2\pi x}{15}\right) + 0.2\cos\left(\frac{2\pi x}{5}\right)$$

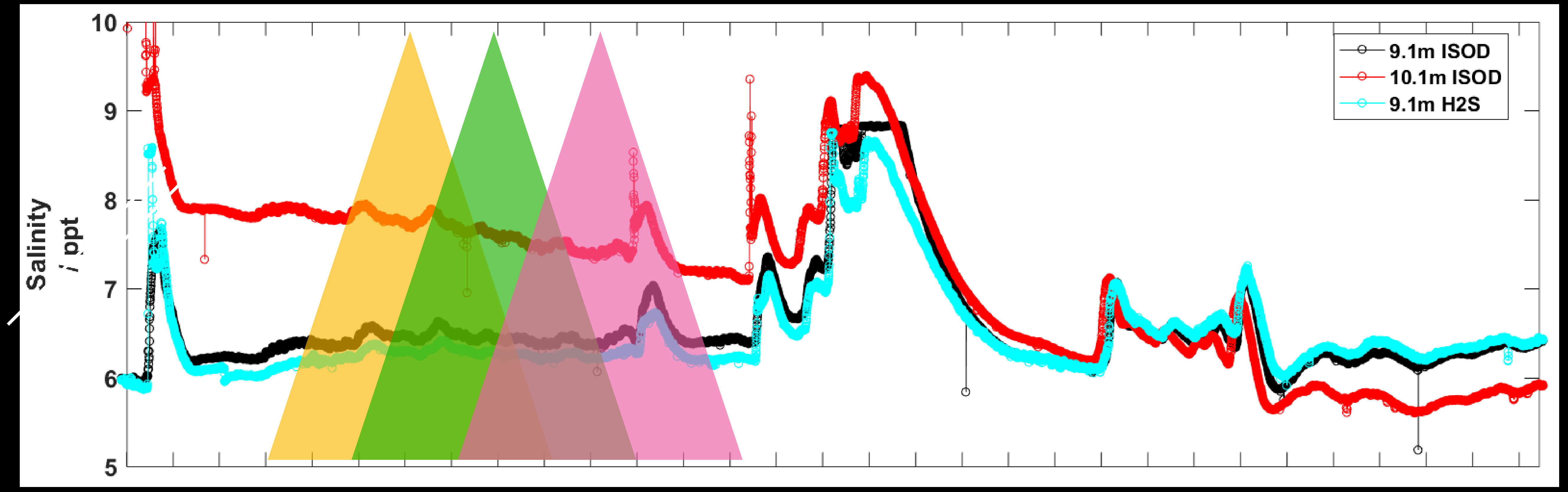
- For a long time series we could ask whether we could get information that tells us how the spectrum changes in time.



"Local" spectra

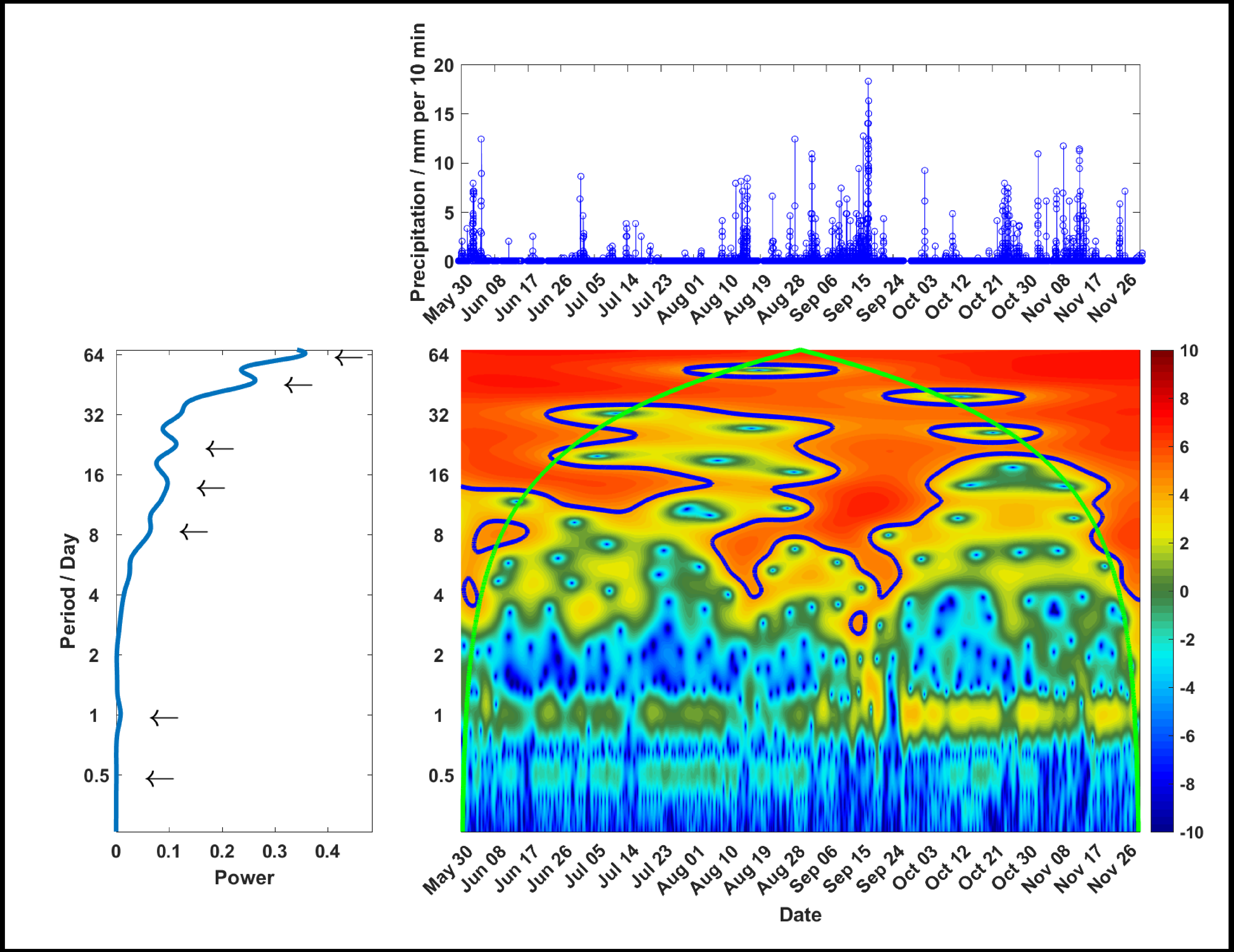
- To get local spectra at some point in time we can return to how we dealt with time series that were not periodic.
- We define a window (the yellow triangle in the upper right) that is centered about the time of interest.
- We multiply the time series by the window and compute the spectrum, which will have fewer frequencies than the one for the whole time series but with frequent measurements that is OK.
- Finally we move the window (lower panel) and repeat the process.





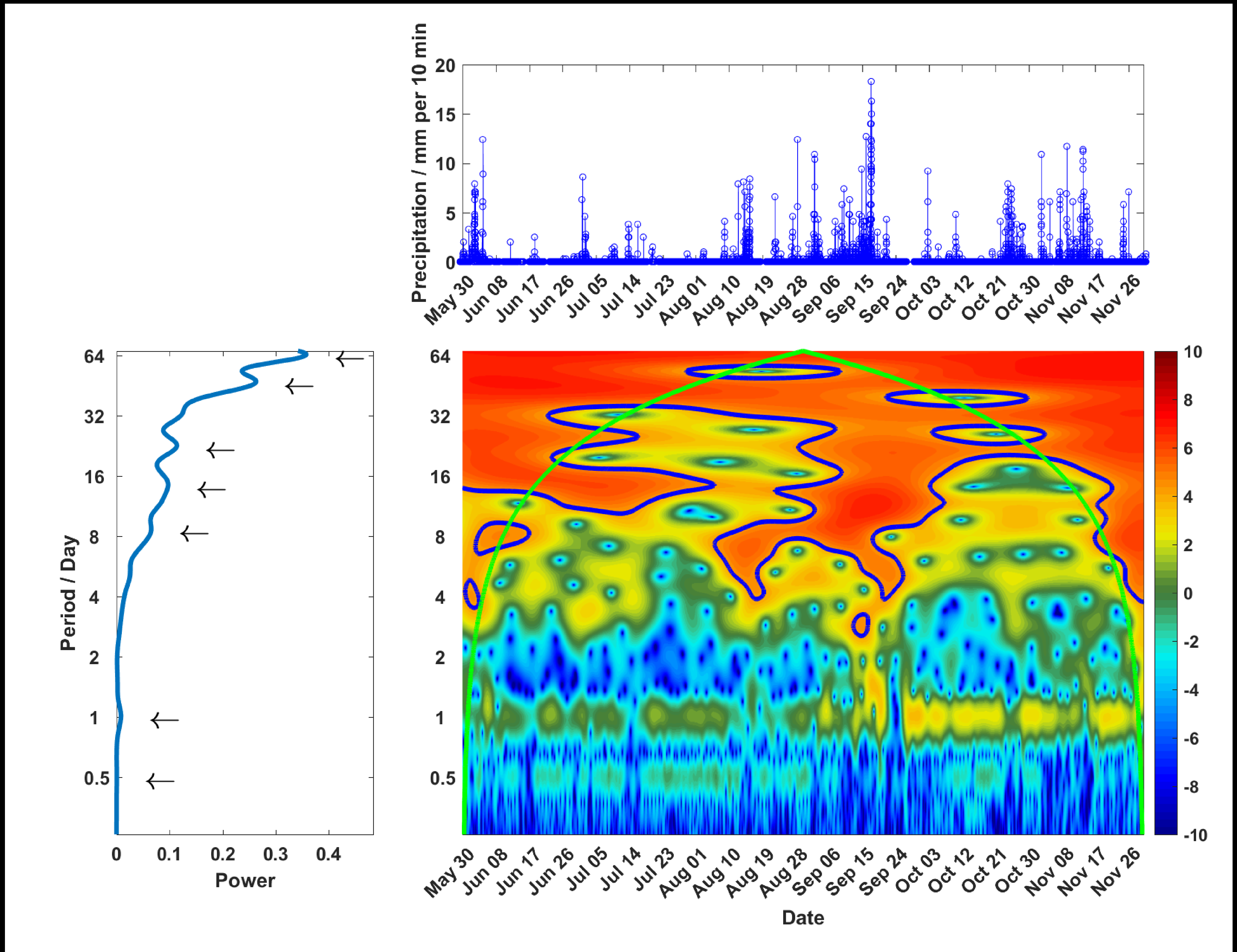
Wavelets: a more systematic alternative

- There is nothing wrong with the sliding spectrum method and indeed corners of the oceanographic community use it to this day.
- However there are more systematic alternatives, such as the wave let transform on the right.
- It is shown as a coloured in plot with the rain gauge data above it, and an average on the left.
- The time runs along the horizontal axis for the two plots on the right so you can see how the wavelet power lines up (or doesn't) with the rain data.



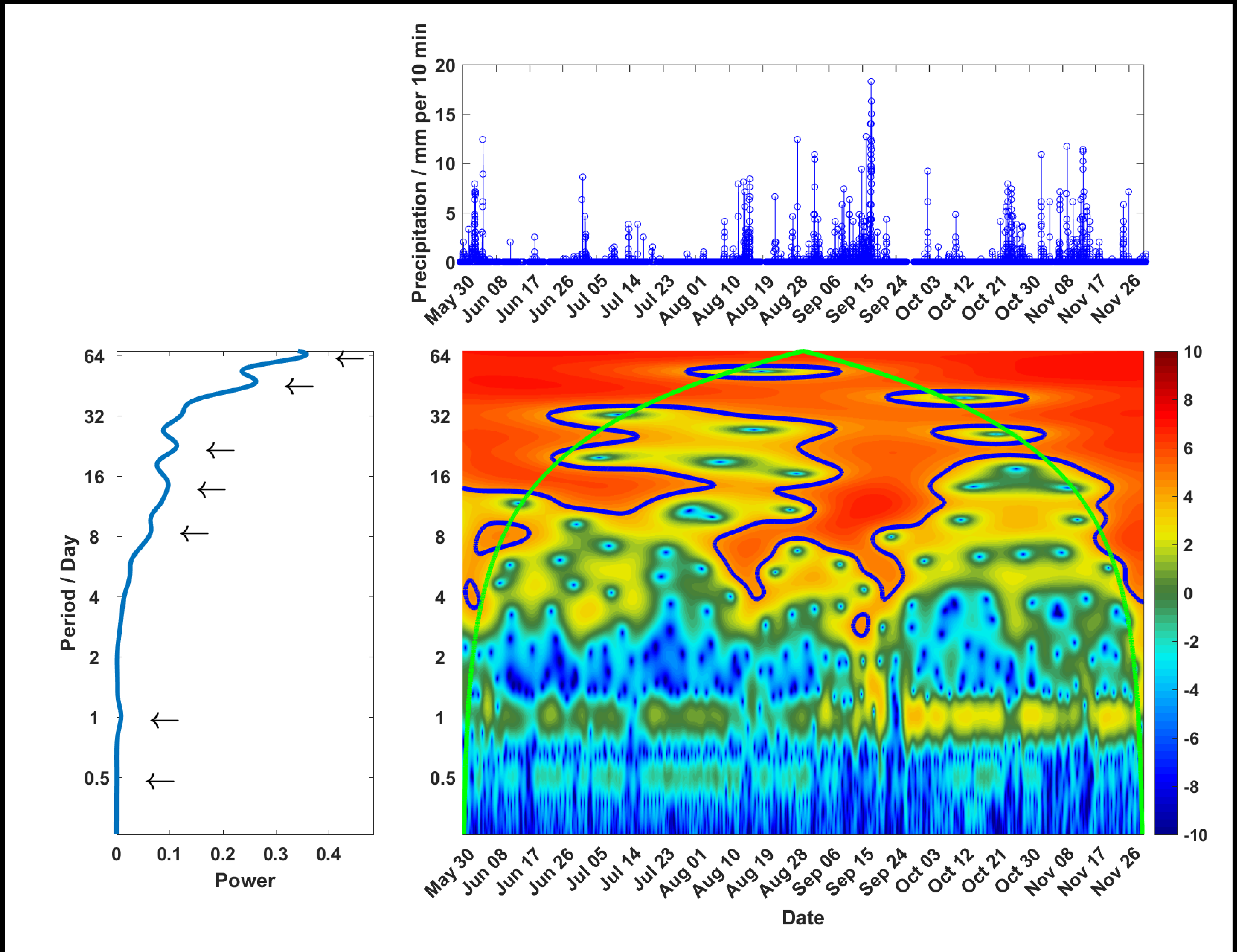
Wavelet plots: making sense of the colours

- Let's look at the wvelet plot (lower right) in more detail.
- The colours tell you the wavelet power, which is an analogy for spectral power.
- As is the case with spectra one generally finds that longer periods (lower frequencies) have more power.
- Physicists call this the "red shift" of natural data.
- Since the y-axis has been massaged to give the period in days you can also see that there are occasional peaks at one day, as well as half day.
- The big rain event around Sept 15 also seems to create a new regime where there is more power at 1 day. This regime etends well into November.



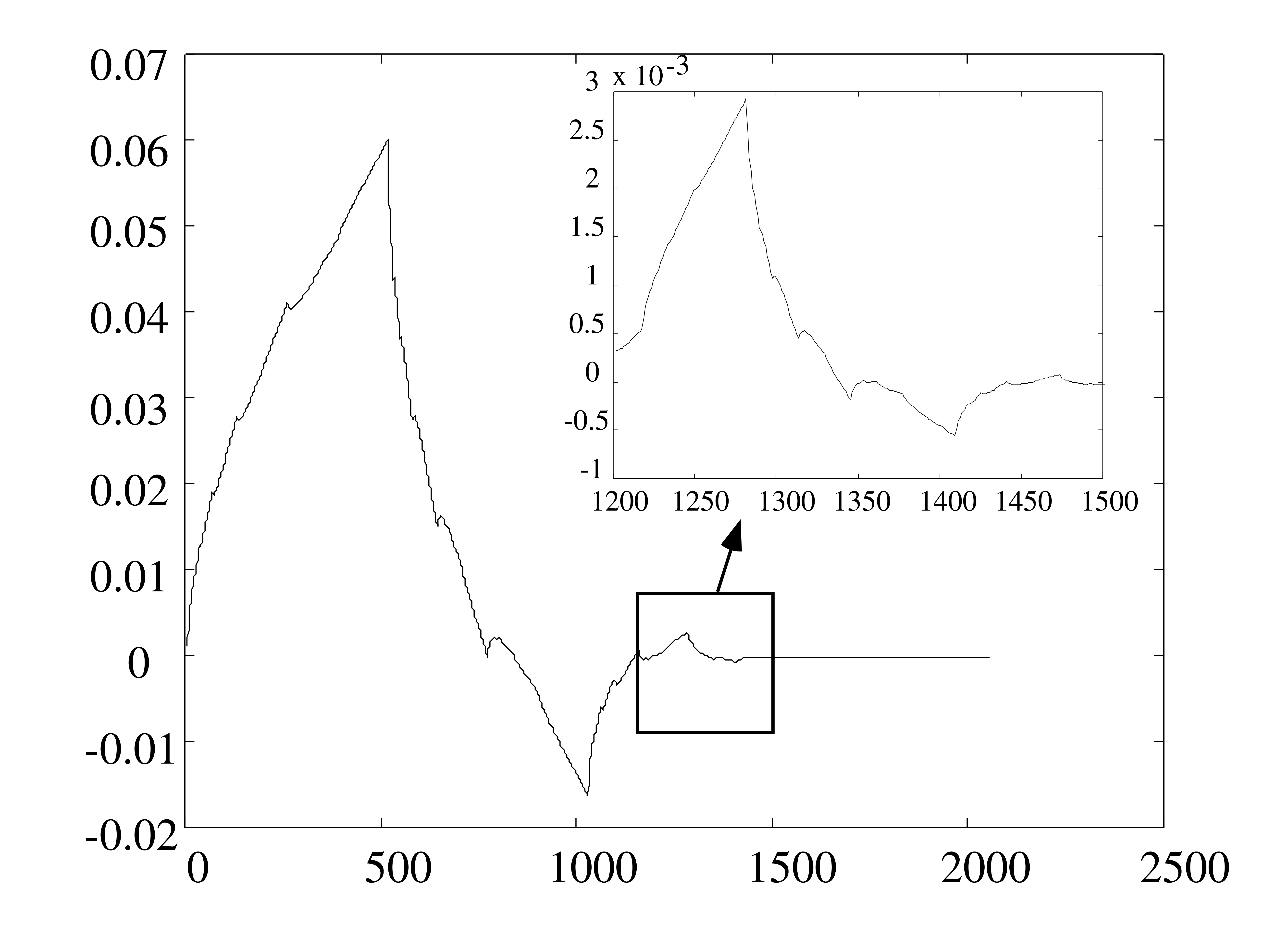
Wavelet plots: averages

- There are many ways to average the wavelet plot and the figure on the left shows one.
- The Power is along the x axis and the Period along the y axis.
- You can see this looks like a much smoother version of the Fourier spectra from the simulation.
- The red arrows have been added to show some of the local maxima.
- The 1 day signal is tiny compared to the red shift, but is important in practice.



What are Wavelets?

- Wavelets are the to the wavelet transform what sine and cosine are to the Fourier transform.
- There is a key difference in that wavelets are meant to represent a systematic change of scale (what is called self-similarity).
- In the picture on the right this is evident when you magnify the square box and plot it in the inset.
- The assigned reading shows the self-similarity of the Daubechies wavelet.



Wavelets in Practice

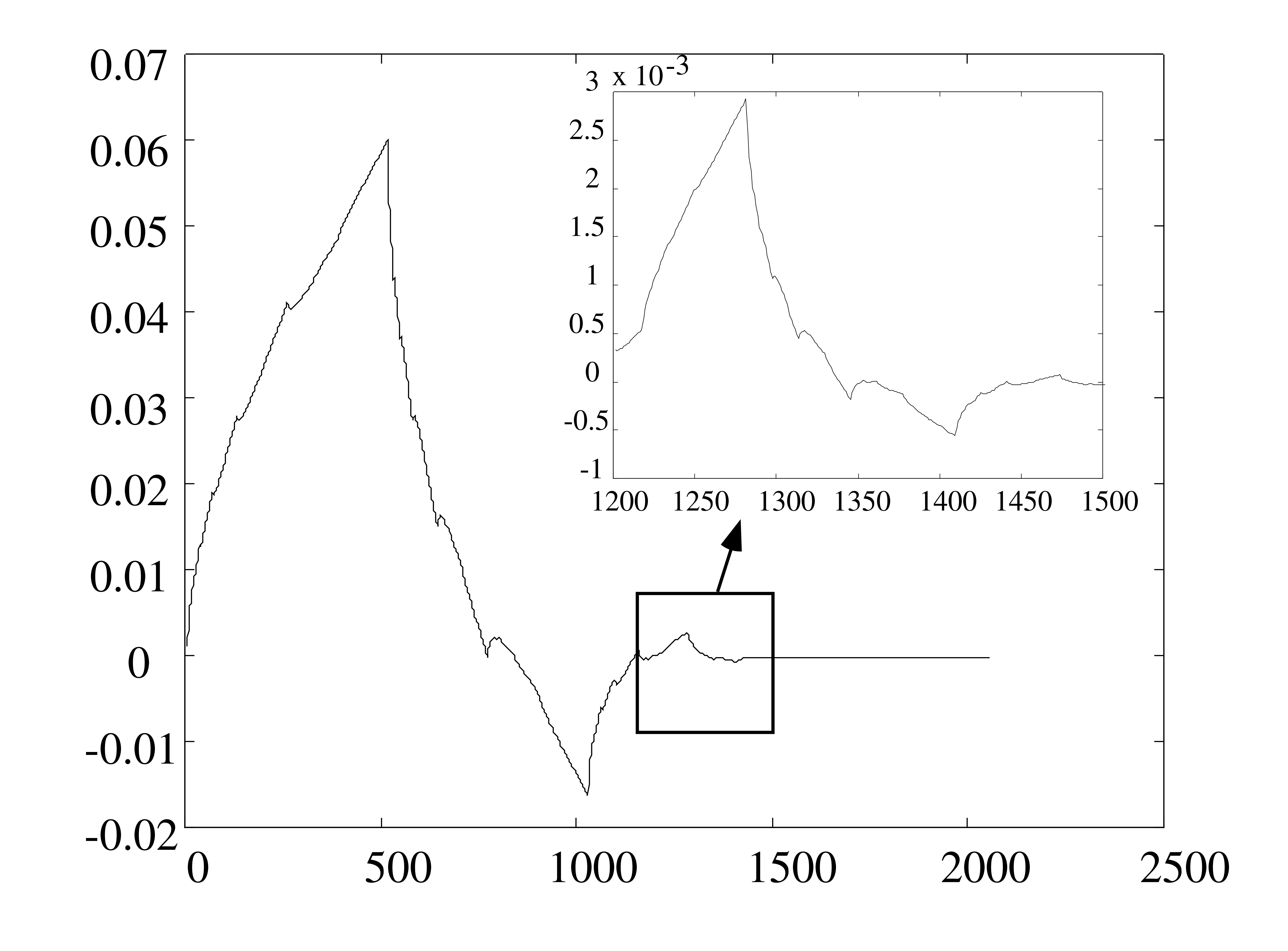
- You don't need to be a wavelet theorist to use wavelets any more than you need to have studied Fourier Analysis.

- There are a few things to keep in mind:

1. Wavelet transforms give MORE data than Fourier transforms so that making useful plots takes practice.

2. Wavelet spectra often smooth and are not as choppy as Fourier spectra, but they may have issues resolving sharp peaks.

3. Because there are many types of wavelets it may be worth redoing the same plot with different wavelets



- Matlab's cwt code does a lot of the heavy lifing and has significant documentation. Below I give an example of my own use.

- Generally it is getting the axis labels to look right that is the challenge because the built in plot type has attractive features (like shading the near start/end regions in which information is dubiout) that one wants to keep.

```
figure(12)
clf
colormap hot
cwt(myf1,'amor',days(time(2)-time(1)),'VoicesPerOctave',24)
% Now get it to return values
[wt, period, coi]=cwt(myf1,'amor',days(time(2)-
time(1)),'VoicesPerOctave',24);
% sum over fast timescales
wtsub=wt(1:100,:);
myhigh1=sum(abs(wtsub).^2,1);
% sum over medium timescales
wtsub=wt(101:150,:);
mymid1=sum(abs(wtsub).^2,1);
```

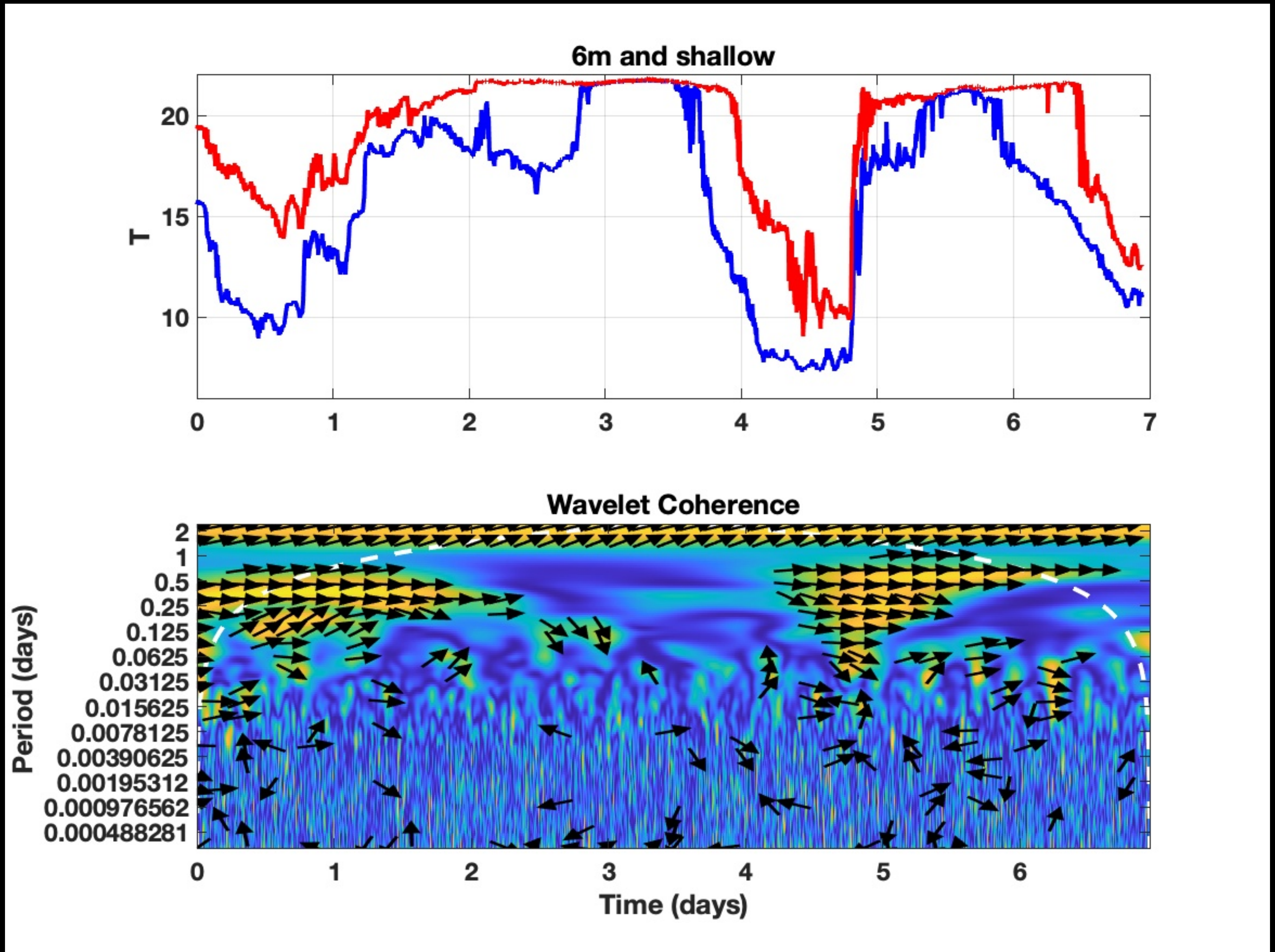
This is the wavelet transform with plotting

This is the wavelet transform without plotting

This is how you sum up over a range of periods/frequencies in your data

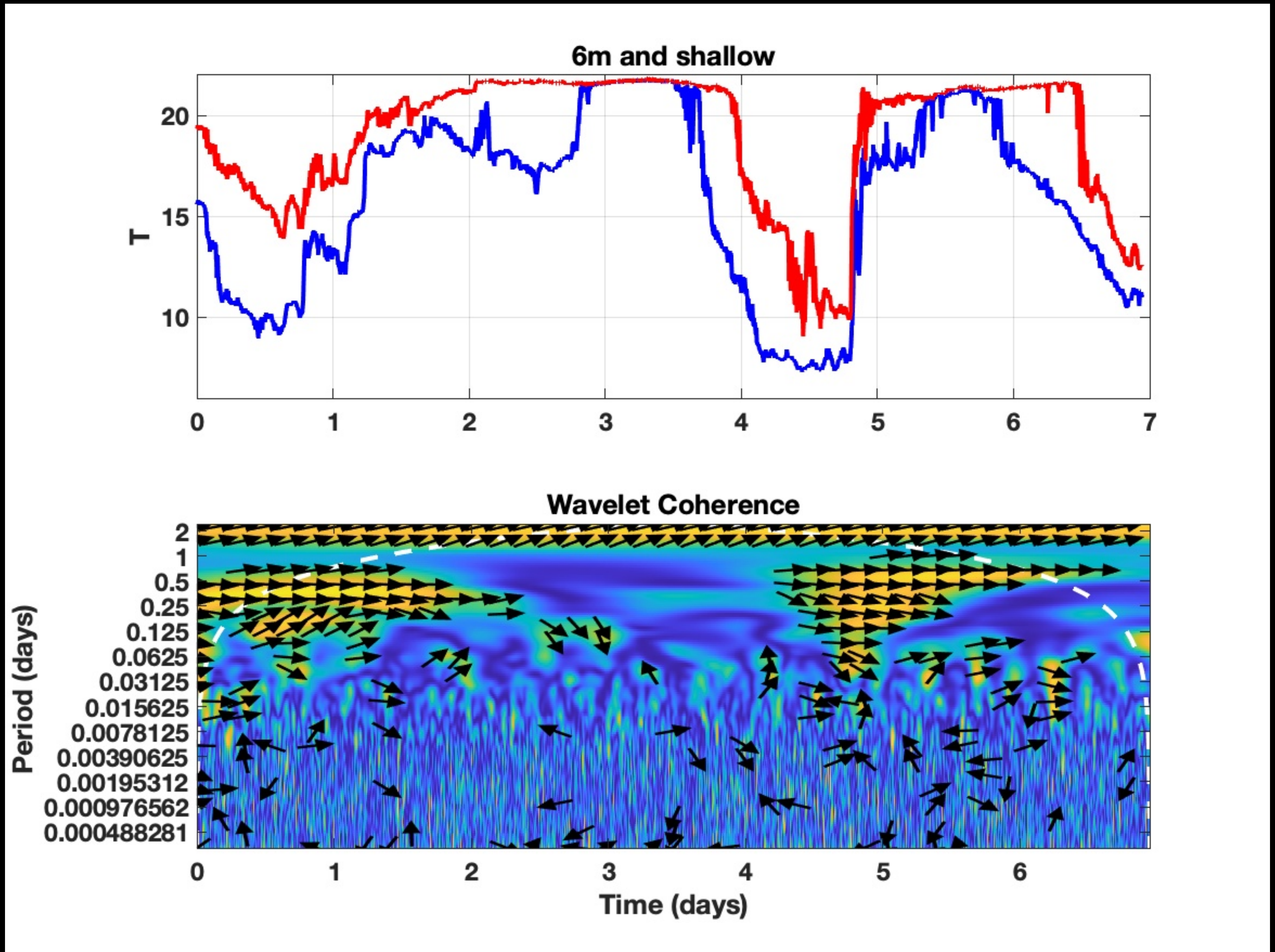
Wavelet coherence 1

- If we have two signals, $\vec{f}_1$ and $\vec{f}_2$ we have found that computing their correlation gave us useful quantitative information.
- Because wavelets give us information in spectral space and time we could ask whether wavelets can generalize the notion of correlation.
- On the right I show two time series of temperature in a lake and in the bottom panel the wavelet coherence.
- The wavelet plot is quite busy!



Wavelet coherence 2

- The bright spots tell us when in time the two signals are coherent, with the scale on the left telling us over what time scales.
- You can see that the analysis tells us that signals are coherent for two epochs in the 1/4 to 1/2 day band and are always coherent over the 2 day band.
- The arrows tell us which signal leads or lags the other.
- To help interpret it help start with some basic functions.



https://www.mathworks.com/help/wavelet/ref/wcoherence.html

Wavelet coherence 3

- You can see that the direction of the arrows corresponds to the phase shift.
- Scaling the sinusoid has no effect at all; a nice sanity check!
- Youn can also see that adding white noise leads to a loss of coherence but ONLY for the short periods. The longer periods don't "see" the noise which is a very nice property of wavelets.

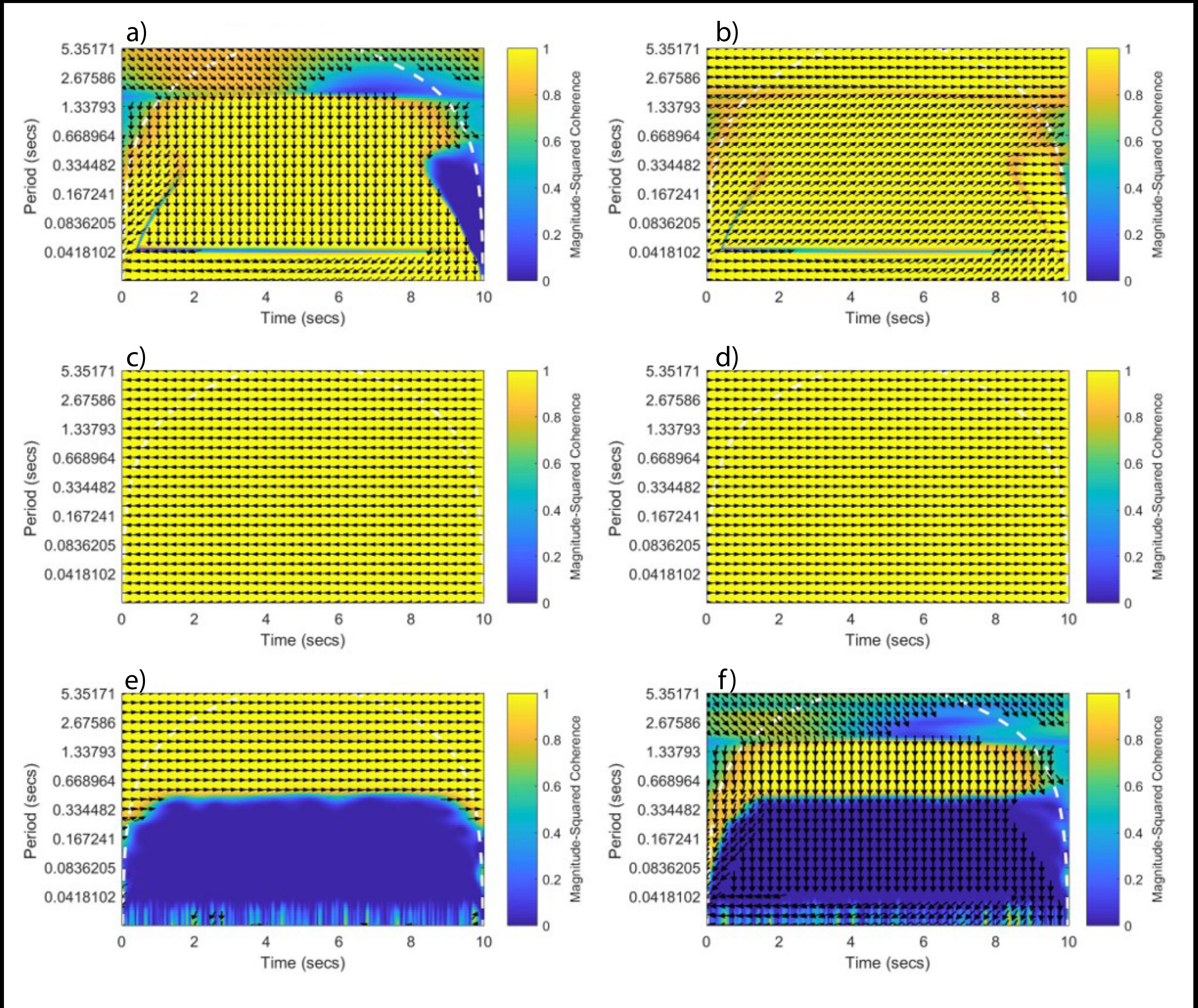


Figure 1.9: Panel (a) shows the wavelet coherence of two sine waves that are off-set by $\pi/2$. All phase angles are shifted by $\pi/2$. Panel (b) is the wavelet coherence of two sine waves off-set by $\pi/4$. Panel (c) is the wavelet coherence of two sine waves off-set by π. Panel (d) is the wavelet coherence of two sine waves with differing amplitudes. Panel (e) is the wavelet coherence of two sine waves where one has had white noise added to it. The final panel (f) is sine waves where one has white noise added and a $\pi/2$ phase shift.

Wavelet coherence 4

- When signals have peaks, only peaks in both signals lead to coherence.
- Here only the peak around 6 occurs in both time series.
- It takes a bit of time to understand the broad yellow blob, but basically as the periods over which the wavelets give coherence increases so too does the region in time over which it is predicted.
- The noise shows up as some random splotches of coherence with short time period.

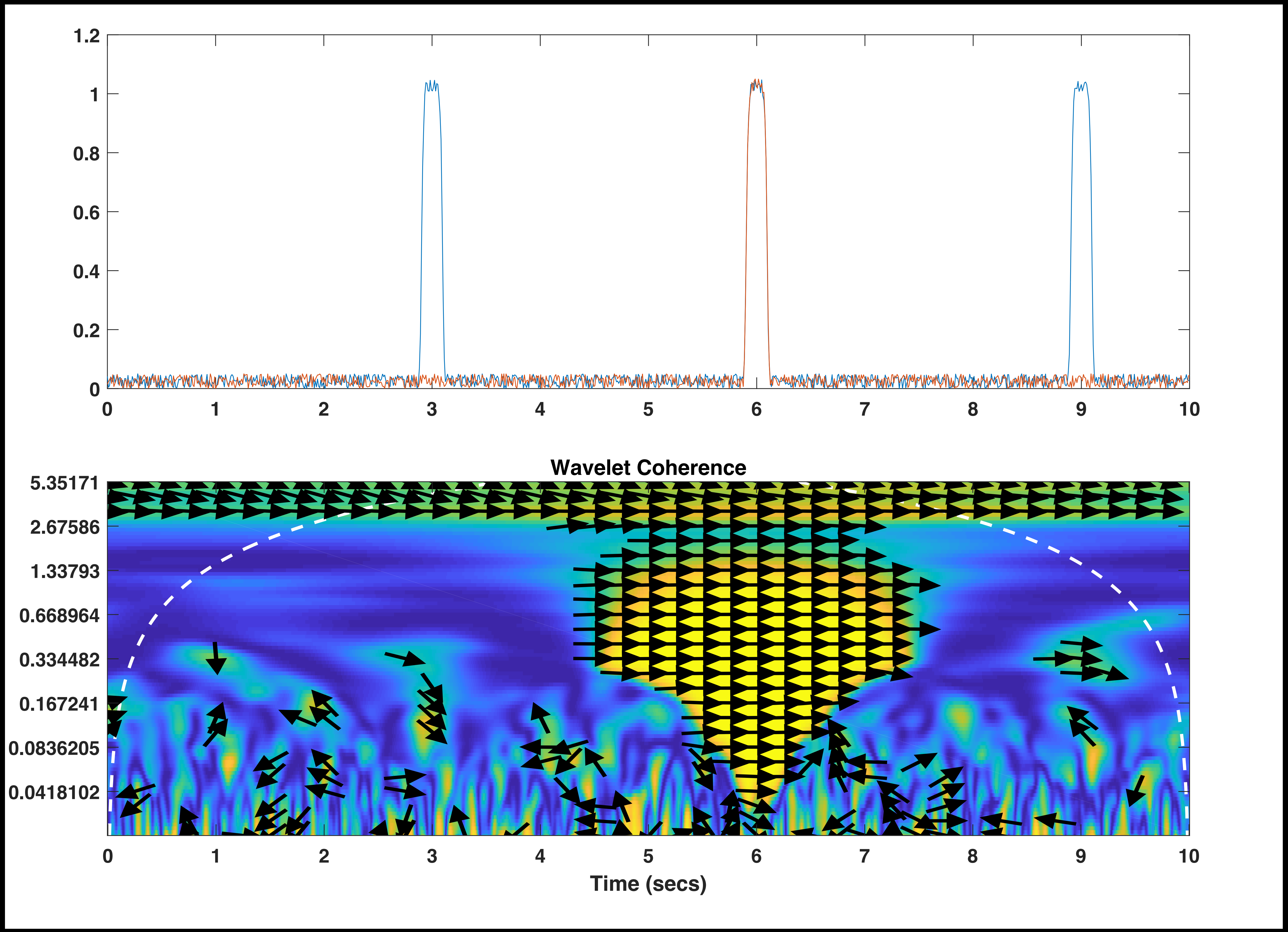


Figure 1.10: In the case of localized wave packets or events the wavelet coherence plot (bottom) only picks up the event where both signals occurred simultaneously (time 6; top).