(brief) Review of gravity

Calculate F_g between an electron ($m_e = 9.11\times10^{-31}$ kg) and a positron ($m_p = m_e$) 1 m apart.

(recall: $G = 6.67\times10^{-11}\ N\cdot m^2/kg^2$, $F_g = G\frac{m_1 m_2}{r^2}$)

$$F_g = \frac{(6.67\times10^{-11})(9.11\times10^{-31})(9.11\times10^{-31})}{1^2}$$

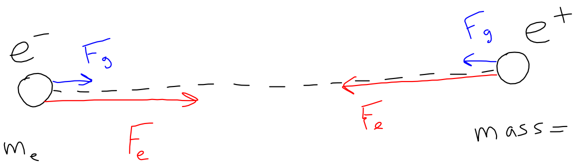
$$= \boxed{5.54\times10^{-71}\ N}$$

Actual experiment:

$$F = 2.3\times10^{-28}\ N$$

4-million-trillion-trillion-trillion times larger!

Explanation: In addition to mass, some objects carry another fundamental property that we call electric charge. Just like masses give rise to the gravitational force, electric charges give rise to the electrostatic (Coloumb) force.



mass = m_e mass = m_e

charge = $q = -e$ charge = $q = +e$

where $m_e = 9.11\times10^{-31}$ kg

$e = 1.6\times10^{-19}$ Coulombs (C)

new fundamental unit

$$F_g = G\frac{m_1 m_2}{r^2} \qquad F_e = k\frac{q_1 q_2}{r^2}$$

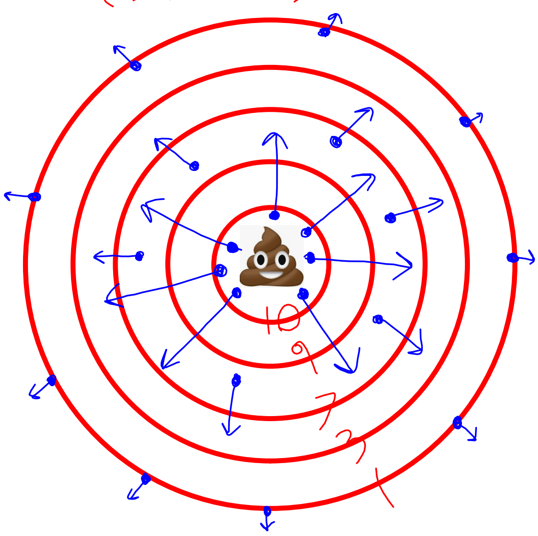
Discovered by Newton discovered by Coulomb

$G = 6.67\times10^{-11}\frac{N\cdot m^2}{kg^2}$ $k = 8.99\times10^{9}\ N\cdot m^2/C^2$

Additional findings:

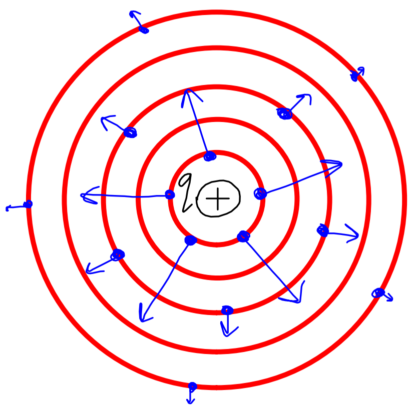
- charges can be (+) or (−)
- opposite charges attract
- like charges repel

red = how bad is the smell? scale 0-10
(scalar)



blue = which direction to run in (vector)

Charges



Electric potential:

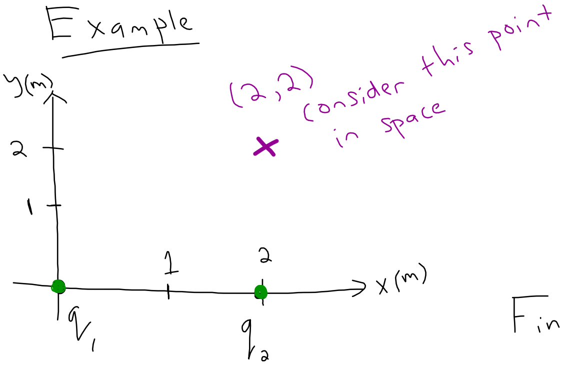
$$V = \frac{kq}{r} \quad \text{(scalar)}$$

Electric Field: (vector)

$$\vec{E} = -\vec{\nabla} V$$

$$|\vec{E}| = \frac{kq}{r^2}$$

Example

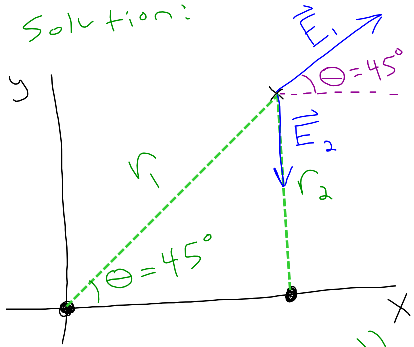


$q_1 = 2 \times 10^{-4}$ C

$q_2 = -1 \times 10^{-4}$ C

Find V and $\vec{E}$ at the point

Solution:



$r_1 = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ m

$r_2 = 2$ m

$$V_1^{(\text{calculated at } (2,2))} = \frac{k q_1}{r_1}$$

$$V_2^{(\text{calculated at } (2,2))} = \frac{k q_2}{r_2}$$

$$V_{total}^{(2,2)} = V_1 + V_2 = \boxed{1.86 \times 10^5 \underbrace{\frac{N \cdot m}{C}}_{\text{Volts (V)}}}$$

$$|\vec{E}_1| = \left|\frac{k q_1}{r_1^2}\right| = 2.24 \times 10^5 \text{ N/C}$$

$$|\vec{E}_2| = \left|\frac{k q_2}{r_2^2}\right| = 2.25 \times 10^5 \text{ N/C}$$

$$\vec{E}_1 = (2.24 \times 10^5 \text{ N/C}) \cos(45°)\,\hat{i} + (2.24 \times 10^5 \text{ N/C}) \sin(45°)\,\hat{j}$$

$$\vec{E}_2 = 0\,\hat{i} - 2.25 \times 10^5 \text{ N/C}\,\hat{j}$$

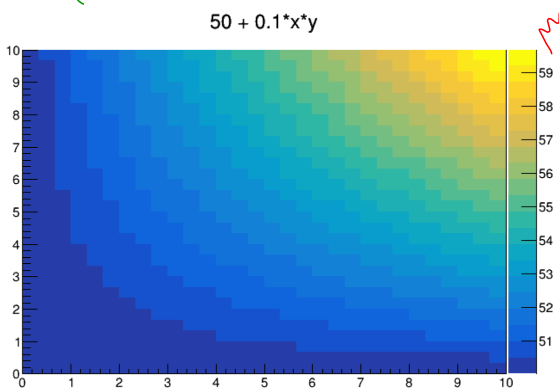
$$\boxed{\vec{E}_{total} = (1.59 \times 10^5\,\hat{i} - 0.66 \times 10^5\,\hat{j}) \text{ N/C}}$$

Aside: Field → scalar fields
→ Vector fields

Scalar fields – Every point in space has a number (scalar)

e.g. temperature distribution in a room

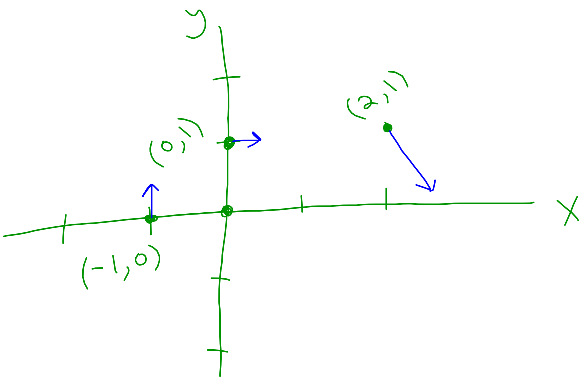
$$T(x,y,z) = 50 + (0.1)xy + z$$

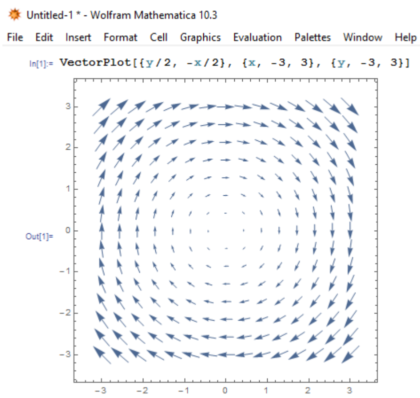


fireplace

Vector fields – Every point in space has a vector

2-D example: $\vec{f}(x,y) = \Big(\underbrace{\frac{y}{2}}_{x\text{-component}}, \underbrace{\frac{-x}{2}}_{y\text{-component}}\Big)$





$$\vec{f}(0,0) = (0,0)$$
$$\vec{f}(0,1) = (\tfrac{1}{2}, 0)$$
$$\vec{f}(2,1) = (\tfrac{1}{2}, -1)$$
$$\vec{f}(-1,0) = (0, \tfrac{1}{2})$$