

# Homework 1

1. Prove that

$$G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \text{ and } ad - bc \neq 0 \right\}$$

With operation  $*$  = matrix multiplication  
is a non-abelian group.

2a. Make multiplication table for  $\mathbb{Z}_5$ .

b. Show that  $(\mathbb{Z}_5, \cdot)$  is not a group.

↑ Integer multiplication

c. Show that  $(\mathbb{Z}_5 \setminus \{0\}, \cdot)$  is a group.

↑ omit the 0 element

3a. Make multiplication table for  $\mathbb{Z}_6$

b. Show that  $(\mathbb{Z}_6 \setminus \{0\}, \cdot)$  is not a group.

4a. Make operation tables for  $(\mathbb{Z}_{10}, +)$  and  $(\mathbb{Z}_2, +) \times (\mathbb{Z}_5, +)$ .

b. Show that they are isomorphic.

5a. Make operation tables for  $(\mathbb{Z}_4, +)$  and  $(\mathbb{Z}_2, +) \times (\mathbb{Z}_2, +)$ .

b. Show that they are not isomorphic.