Definition of Emp

G be a set of elements with operation.

and satisfy:

closure: 0.6 EG 4 a,6 EG

identity: e ∈ G such mont e. a = a. e = a, ba ∈ G

inverse : bis inverse of a if a-b=b-a=e, & a \in G

associettive: (a.b).c=a.(b.c) +a,b,c = G,

(G,.) is a goup.

Outline	Closure	
1) Definition of group		
2) Finite us infinite		7
3) Abelian us non-al	belian	
t) Operation table		
5) Direct product		
6) Ismorphism		

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Examples
(1) (226, t) is a group of size 26
(Zn,+) is a group of size n.
    Z_n = set of remainders modulo n = 30, 1, 2, \dots, n-3
(3) (Z, +) is a group of infrite size:
      Closure: Ya, b EZ, a+b EZ
      identity: OEZ and Ota = 9+0 = a Yaez
      inverse: YaETZ, -a EZZ and a+(-a) =0
      associative: Ya, b, C EZ : (afb) + C is equal to 9 + Cb+c)
  (I, *) is not a grup.

L'integre multiplication
       identry = 1, 1 * a = a * 1 = a \ \ a \ \ Z
       2 e Z 2*b=1 if b= 1 € Z
       There is no invece for 2.
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Finite group US Infinite group Definition: If (G, .) is a group of size o, then the order of G is n. Abotion Vs non-abelian In (Z>6,+): GHO = B+0 + 0,6 E Z>6 commentative A group that satisfy commention is a abolish group. Homework  $G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \text{ and } ad - bc \neq 0 \right\}$ With operation \* = matix multiplication is a non-abelian group (matix multipliated) is not commutated: A \* B ≠ B \* A for some mornices A.B

Z6 = 10,12,3,4,53	
+ 10 11 2 3 14 6	
0 0 1 2 3 4 5 0 is identity	
1 1 2 3 4 5 0 each row has ident	tren 0
2 2 3 4 5 0 1 Dosenation:	
B3 4 5 0 1 2 Quinque identità	
4 4 5 0 1 2 3 6 Urique inverse	2
5 5 0 1 2 3 4	

Lemand

Given a group (G, .), show that

(a) the identity of (a, .) is unique

If there are two identities e, f

e. 0 = a. e = a

f. a = a f = a

Show that e is equal to f

(b) Y a e a, the inverse of a is unique.

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Direct product
(\mathbb{Z}_2,+)\times(\mathbb{Z}_3,+)= \{(a,b)|a\in\mathbb{Z}_2,b\in\mathbb{Z}_3\}
             = \frac{1}{1}(0,0), (0,1), (0,2), (1,6),
                  (1,1), (1,2)} of order 6
(0,6), (7,0) (7,1) (7,1)
(0,b) \times (c,d) = (a+c,b+d)
 TWO groups (G, ), (H, *), We can create a
 5001P (H, *)
   = 1 (g,h) | g ∈ q, h ∈ H}
 with opports of x such that
       (9,h) × (8,h) = (9.8, h *h)
  Order of (E, ) × (H, *) = Grder of (E, ) ×
                               oder of (H,*)
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Proof that (E, , ) × (H, \*) is a grop crosure: (g, h), (g', h') & (f, \*) (g,h) x (g',h') = (9.9', n\*h') & GxH 9.9' € 9 MWN EH identity: (eg. eH) x (g, b) = (eg. g, eH\*b) 4geG, *h* ∈ *H* = (9,h)(g,h) x (eg,ey) = (g,eg, h, +eh) (N,B) = VGEG, HEH, [ NUP ( Se : I inverse for g denoted as g inverse for h denoted as h- $(g,h) \times (g',h') = (g \cdot g',h \times h')$ = ( eg, eh)  $(G', h') \times (g,h) = (e_{H}, e_{H})$ 

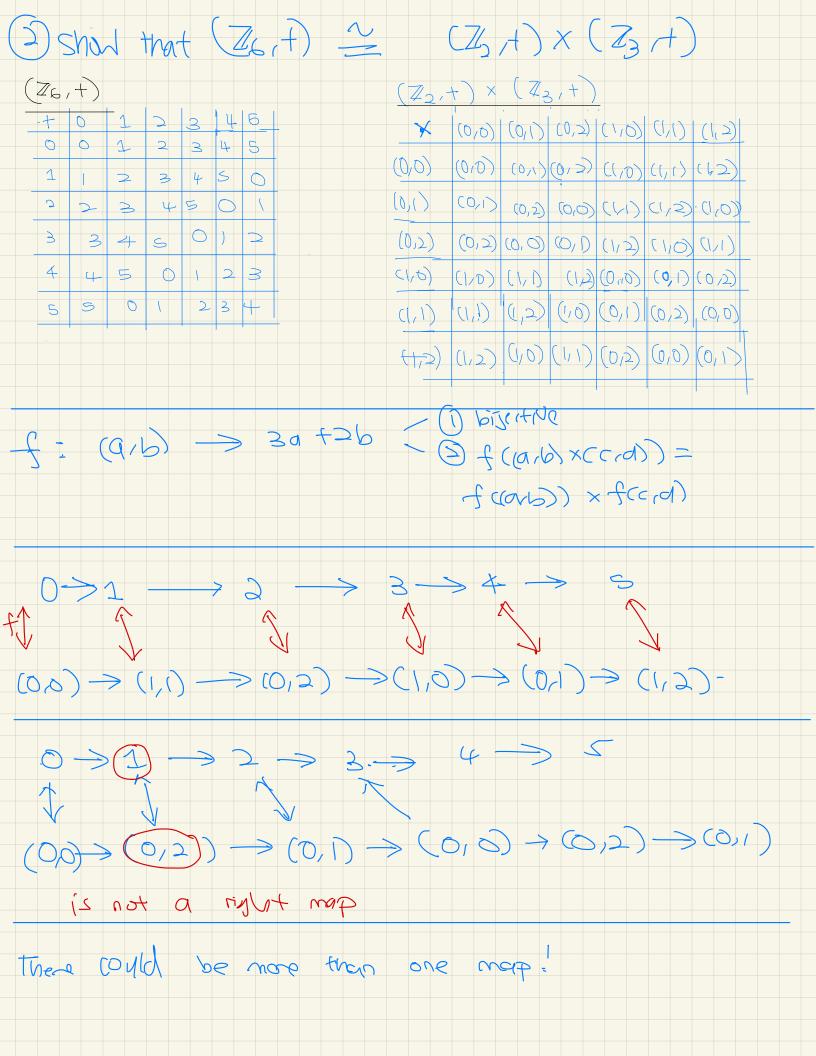
associuting: operation is element - wise

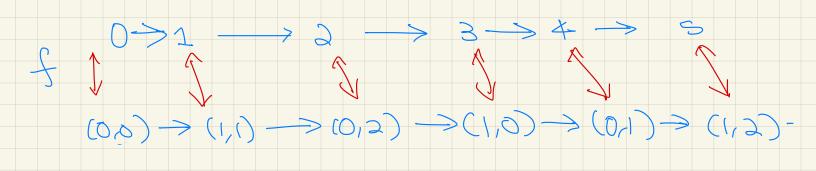
mindromaz Two graps G, H are isomorphic it there exits a bigerue map from elements in & to clarents in H that present the operations of the group elements. \$ : (6, ·) -> (H, \*) 74 9.5' = 9 ₩ 9, 9' € € fa + f(g') = f(a) f(g.g') = f(g)\* f(g') Example S 1) (Z2,+) x (Z3,+) is isomorphic to (Z3,+) x (Z2,+) 2) (72,7) X(23,1+) is isomorphic to (76,1) 3) (Z, t) x (Z, t) is not isomorphic to (Z4, t)

 $(\mathbb{Z}_2,+)\times(\mathbb{Z}_3,+)$  is isomorphic with  $(\mathbb{Z}_3,+)\times(\mathbb{Z}_2,+)$ + > (0,6)Let f ((a,b)) = (b,a) (0,0) show that f preserves the C(,0) (01) opertos. (0/2) $\rightarrow (2.0)$ w.t.s f((c,d)) x f((c,d)) ((0))  $\rightarrow$  (0,1)((1)) > (1/1) = f((a,b) x (c,d)) (1,2,) > (2,1)  $= (p,a) \times (q,c)$ f(a,b)) x f(c(d))

 $f(a,b) \times f(c(a)) = (b,a) \times (a,c)$  = (b+d, a+c) = f(a+c, b+d)  $= f(a,b) \times (c,d)$ 

Suppose G ~ H (G is isomorphic to H). and f is an isomorphism of Ge and H Show that f(PG) = PH 2) Let g' be the inverse of g E G f(a') is the inverse of fcasEH W.t.s:5(eg)-h = h-f(eg) = h, WhEH f(eg). h = f(eg). f(o) where fcq)=h = f(eq.9) = 5(5) h.f(PG) = f(g).f(eG) = f(9.86) fcg) = h Q', q = 9, 8 = e by desintion w.t.s fcg).fcg)=fcg)=e f(g')-f(g)=f(e)=en f(q).fd) = f(e)= e+





(72,-	+) × (Z <sub>3</sub> , +)
×	(0,1) (0,1) (2,0) (1,1) (1,2)
(0,0)	(0,0) (0,1) (0,0) (1,1) (42)
[0,()	(0,1) (0,2) (0,0) (41) (1,2) (1,0) {
(0,2)	(0,2) (0,0) (0,1) (1,2) (1,0) (1,1)
(1,6)	(1,0) (1,1) (1,2) (0,1) (0,1)
((,1)	(1,1) $(1,2)$ $(1,0)$ $(0,1)$ $(0,2)$ $(0,0)$
(1/2)	(1,2) $(1,0)$ $(1,1)$ $(0,2)$ $(0,0)$ $(0,1)$

		4-	2	3	1	5
0	0	4	2	3	)	5
4	4	2	0	1	6	3
2	2	0	4	5	3	1
3	3	1	5	0	4	2
1		67	3	4	2	0
5	5 1	3	1	2	0	4-

This operator table represents (Z6,+)!

