

CSC 411. Spring 2023

Homework #3

Due: Feb 21, 2022 (Tuesday) 11:59 pm

1. Textbook #3.4-10 (a) (b) (30 Points)

- 10. *Magic squares*** A magic square of order n is an arrangement of the integers from 1 to n^2 in an $n \times n$ matrix, with each number occurring exactly once, so that each row, each column, and each main diagonal has the same sum.
- a.** Prove that if a magic square of order n exists, the sum in question must be equal to $n(n^2 + 1)/2$.
 - b.** Design an exhaustive-search algorithm for generating all magic squares of order n .

2. Textbook #3.4-11 (a) (b) (30 Points)

- 11. *Famous alphametic*** A puzzle in which the digits in a correct mathematical expression, such as a sum, are replaced by letters is called *cryptarithm*; if, in addition, the puzzle's words make sense, it is said to be an *alphametic*. The most well-known alphametic was published by the renowned British puzzlist Henry E. Dudeney (1857–1930):

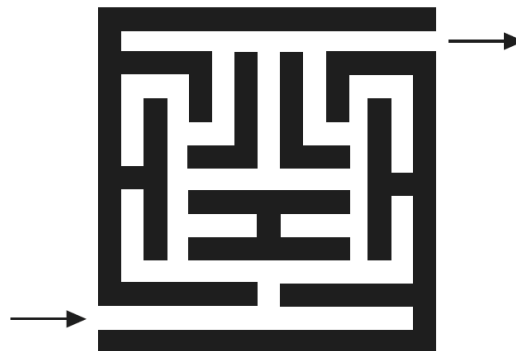
$$\begin{array}{r} \text{S E N D} \\ + \text{M O R E} \\ \hline \text{M O N E Y} \end{array}$$

Two conditions are assumed: first, the correspondence between letters and decimal digits is one-to-one, i.e., each letter represents one digit only and different letters represent different digits. Second, the digit zero does not appear as the left-most digit in any of the numbers. To solve an alphametic means to find which digit each letter represents. Note that a solution's uniqueness cannot be assumed and has to be verified by the solver.

- a. Write a program for solving cryptarithms by exhaustive search. Assume that a given cryptarithm is a sum of two words.
- b. Solve Dudeney's puzzle the way it was expected to be solved when it was first published in 1924.

3. Textbook #3.5-10 (a) (b) (20 Points)

10. One can model a maze by having a vertex for a starting point, a finishing point, dead ends, and all the points in the maze where more than one path can be taken, and then connecting the vertices according to the paths in the maze.
 - a. Construct such a graph for the following maze.



- b. Which traversal—DFS or BFS—would you use if you found yourself in a maze and why?

4. Textbook #3.5-11 (20 Points)

11. *Three Jugs* Siméon Denis Poisson (1781–1840), a famous French mathematician and physicist, is said to have become interested in mathematics after encountering some version of the following old puzzle. Given an 8-pint jug full of water and two empty jugs of 5- and 3-pint capacity, get exactly 4 pints of water in one of the jugs by completely filling up and/or emptying jugs into others. Solve this puzzle by using breadth-first search.