LECTURE NOTES

Professor Anita Wasilewska

NEURAL NETWORKS

Neural Networks Classification Introduction

- **INPUT:** classification data, i.e. it contains an classification (class) attribute.
- **WE** also say that the class label is known for all data.
- **DATA** is divided, as in any classification problem, into **TRAINING and TEST** data sets.

Neural Networks Classifier

– **ALL DATA must be normalized,** i.e. all values of attributes in the dataset has to be changed to contain values in the interval [0,1], or [-1,1].

TWO BASIC normalization techniques:

– Max- Min normalization and

– Decimal Scaling normalization.

Data Normalization

- Max-Min Normalization

Performs a linear transformation on the original data.

- Given an attribute A, we denote by

*min*A, *max*A the minimum and maximum values of the values of the attribute A.

- Max-Min Normalization maps a value v

of A to v' in the range

- [*new_min*A, *new_max*A]

as follows.

Data Normalization

Max- Min normalization formula is as follows:

$$\boxed{v' = \frac{v - \min A}{\max A - \min A}(new\_\max A - new\_\min A) + new\_\min A}$$

Example: we want to normalize data to range of the interval [-1,1].
We put: new_max A= 1, new_minA = -1.
In general, to normalize within interval [a,b], we put:
new_max A= b, new_minA = a.

Example of Max-Min Normalization

Max- Min normalization formula

$$v' = \frac{v - \min A}{\max A - \min A}(new\_\max A - new\_\min A) + new\_\min A$$

Example: We want to normalize data to range of the interval [0,1].
We put: new_max A= 1, new_minA =0.

Say, max A was 100 and min A was 20 (That means maximum and minimum values for the attribute A).

Now, if v = 40 (If for this particular pattern , attribute value is 40), v' will be calculated as , v' = (40-20) x (1-0) / (100-20) + 0

=> v' = 20 x 1/80

=> v' = 0.4

Decimal Scaling Normalization

Normalization by decimal scaling normalizes by moving the decimal point of values of attribute A.

A value v of A is normalized to v' by computing

$$\boxed{v' = \frac{v}{10^j}}$$

where j is the smallest integer such that max|v'|<1.

Example :

A – values range from -986 to 917. Max |v| = 986.

v = -986 normalize to v' = -986/1000 = -0.986

Neural Network

- Neural Network is a set of connected INPUT/OUTPUT UNITS, where each connection has a WEIGHT associated with it.
- Neural Network learning is also called CONNECTIONIST learning due to the connections between units.
- Neural Network is always fully connected.
- It is a case of SUPERVISED, INDUCTIVE or CLASSIFICATION learning.

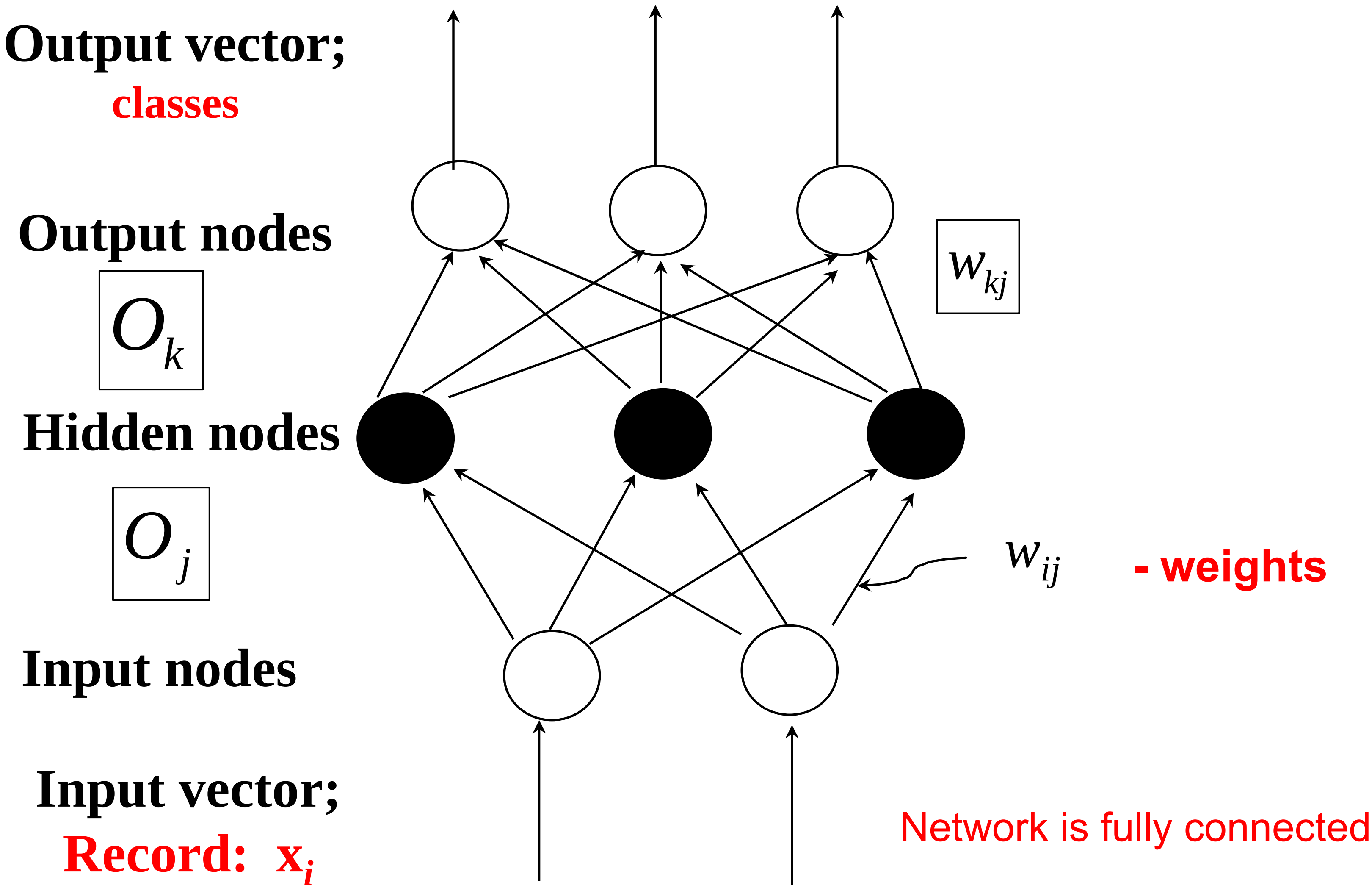
Neural Network Learning

- Neural Network learns by adjusting the weights so as to be able to correctly classify the training data and hence, after testing phase, to classify unknown data.
- Neural Network needs long time for training.
- Neural Network has a high tolerance to noisy and incomplete data.

Neural Network Learning

- Learning is being performed by a back propagation algorithm.
- The inputs are fed simultaneously into the input layer.
- The weighted outputs of these units are, in turn, are fed simultaneously into a “neuron like” units, known as a hidden layer.
- The hidden layer’s weighted outputs can be input to another hidden layer, and so on.
- The number of hidden layers is arbitrary, but in practice, usually one or two are used.
- The weighted outputs of the last hidden layer are inputs to units making up the output layer.

A Multilayer Feed-Forward (MLFF) Neural Network



MLFF Neural Network

- The units in the hidden layers and output layer are sometimes referred to as neurones, due to their symbolic biological basis, or as output units.
- A multilayer neural network shown on the previous slide has two layers of output units.
- Therefore, we say that it is a two-layer neural network.

MLFF Neural Network

- A network containing two hidden layers is called a three-layer neural network, and so on.
- The network is feed-forward - it means that none of the weights cycles back to an input unit or to an output unit of a previous layer.

MLFF Neural Network

Output vector;
3 classes here

Output nodes

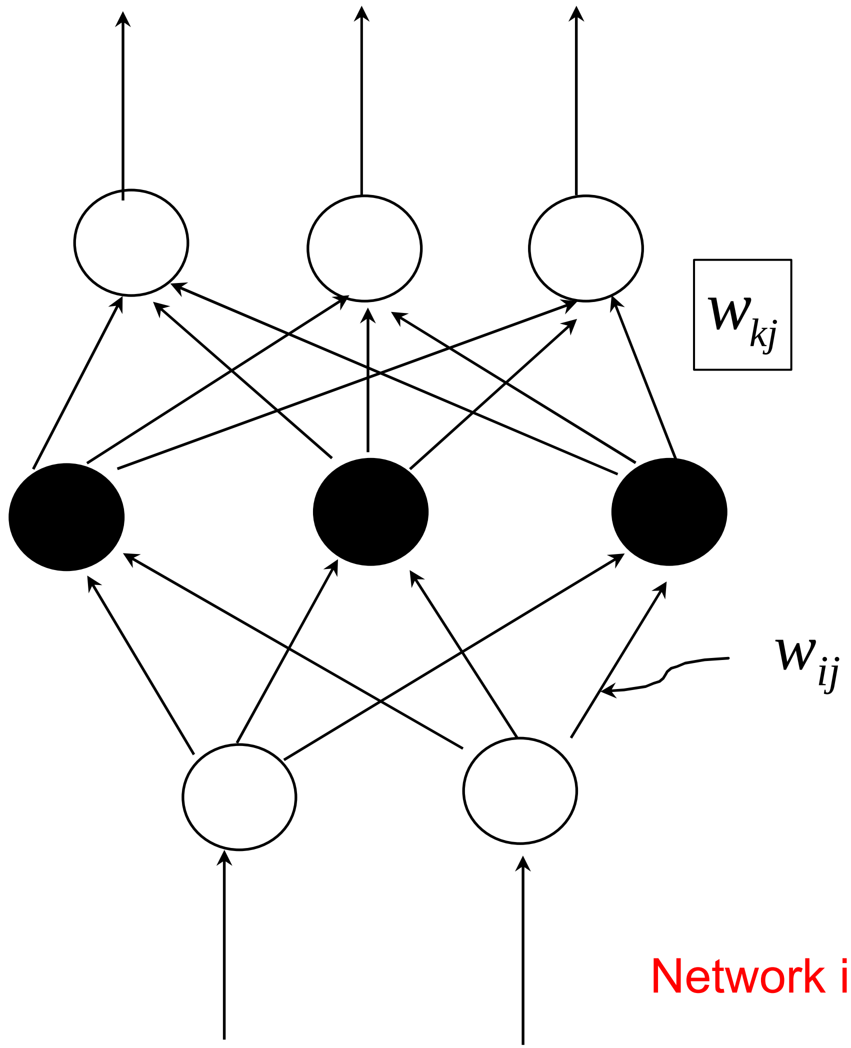
$\boxed{O_k}$

Hidden nodes

$\boxed{O_j}$

Input nodes

Input vector;
Record: $\mathbf{x}_i$
2 attributes here



- weights

Network is fully connected

MLFF Network Input

- INPUT: records without class attribute with normalized attributes values. We call it an input vector.
- INPUT VECTOR:

X = { x1, x2, …. xn}

where n is the number of (non class) attributes.

Observe that {,} do not denote a SET symbol here!

NN network people like use that symbol for a vector;

Normal vector symbol is [x1, … xn]

MLFF Network Topology

- INPUT LAYER – there are as many nodes as non-class attributes i.e. as the length of the input vector.

- HIDDEN LAYER – the number of nodes in the hidden layer and the number of hidden layers depends on implementation.

$\boxed{O_j}$ j=1, 2 ..#hidden nodes

MLFF Network Topology

- OUTPUT LAYER – corresponds to the **class attribute.**
- There are as many nodes as classes (values of the class attribute).

$$\boxed{O_k}$$

k= 1, 2,.. #classes

- Network is fully connected, i.e. each unit provides input to each unit in the next forward layer.

Classification by Backpropagation

- Backpropagation is a neural network learning algorithm.

- It learns by iteratively processing a set of training data (samples), comparing the network's classification of each record (sample) with the actual known class label (classification).

Classification by Backpropagation

- For each training sample, the weights are
- first set random, and then
- modified as to minimize the mean squared error between the network's classification (prediction) and actual classification (value of the class attribute).
- These weights modifications propagated in "backwards" direction, that is, from the output layer, through each hidden layer down to the first hidden layer.
- Hence the name backpropagation.

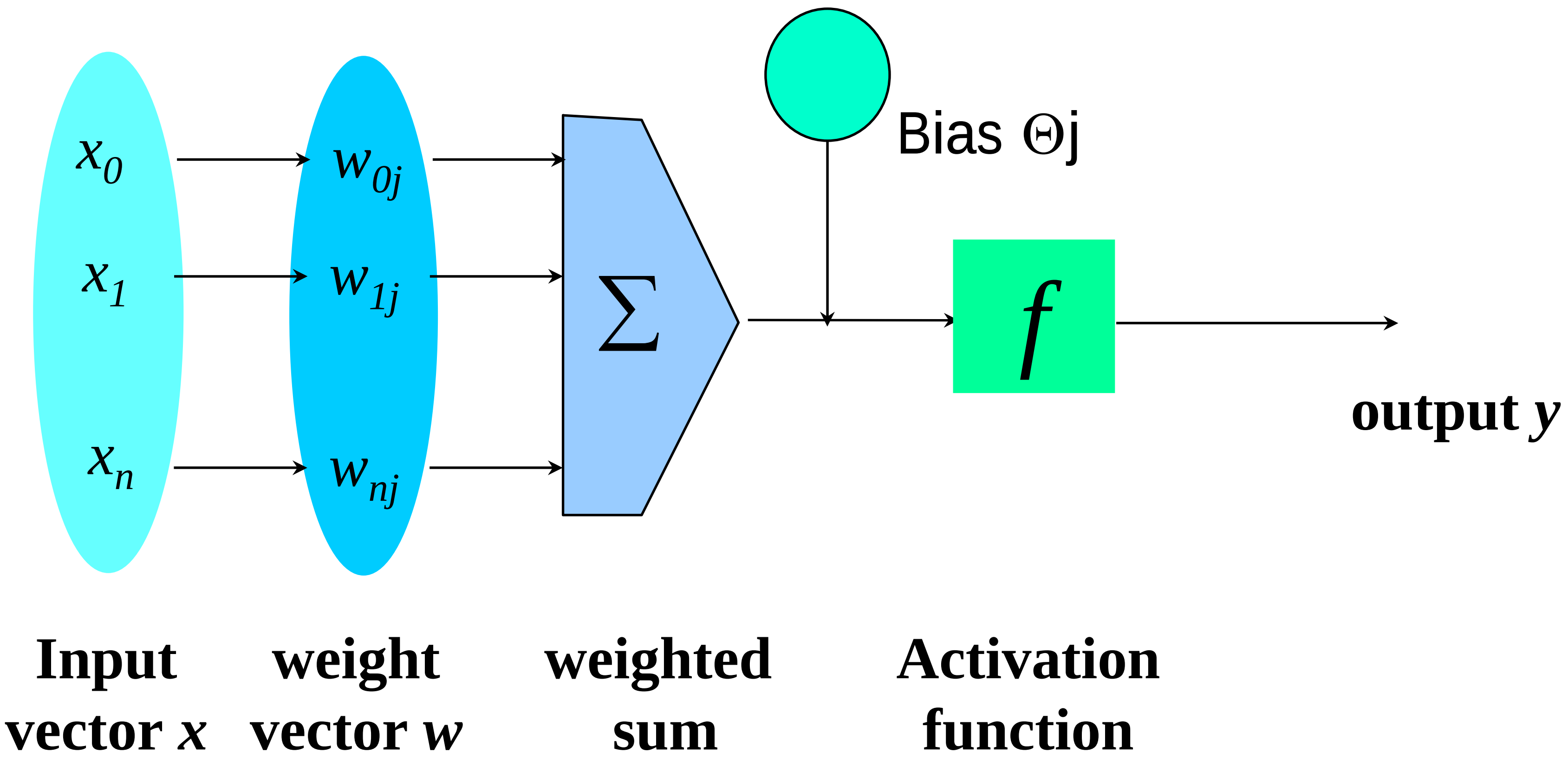
Steps in Backpropagation Algorithm

- **STEP ONE:** initialize the **weights and biases.**
- **The weights** in the network are initialized to small random numbers ranging for example from -1.0 to 1.0, or -0.5 to 0.5.
- Each unit has a **BIAS** associated with it (see next slide).
- **The biases** are similarly initialized to small random numbers.
- **STEP TWO:** feed the training sample (record)

Steps in Backpropagation Algorithm

- **STEP THREE:** propagate the inputs forward; we compute the net input and output of each unit in the hidden and output layers.
- **STEP FOUR:** back propagate the error.
- **STEP FIVE:** update weights and biases to reflect the propagated errors.
- **STEP SIX:** repeat and apply terminating conditions.

A Neuron; a Hidden, or Output Unit j



- The inputs to unit j are outputs from the previous layer. These are multiplied by their corresponding weights in order to form a weighted sum, which is added to the bias associated with unit j.
- A nonlinear activation function f is applied to the net input.

Step Three: propagate the inputs forward

- For unit j in the **input layer**, its **output** is equal to its **input**, that is,

$$\boxed{O_j = I_j}$$

for **input unit** j.

• The net input to each unit in the hidden and output layers is computed as follows.

•Given a unit j in a **hidden** or **output** layer, the **net input** is

$$\boxed{I_j = \sum_i w_{ij} O_i + \theta_j}$$

where wij is the weight of the connection from unit i in the previous layer to unit j; Oi is the output of unit i from the previous layer;

$\boxed{\theta_j}$ is **the bias** of the unit

Step Three: propagate the inputs forward

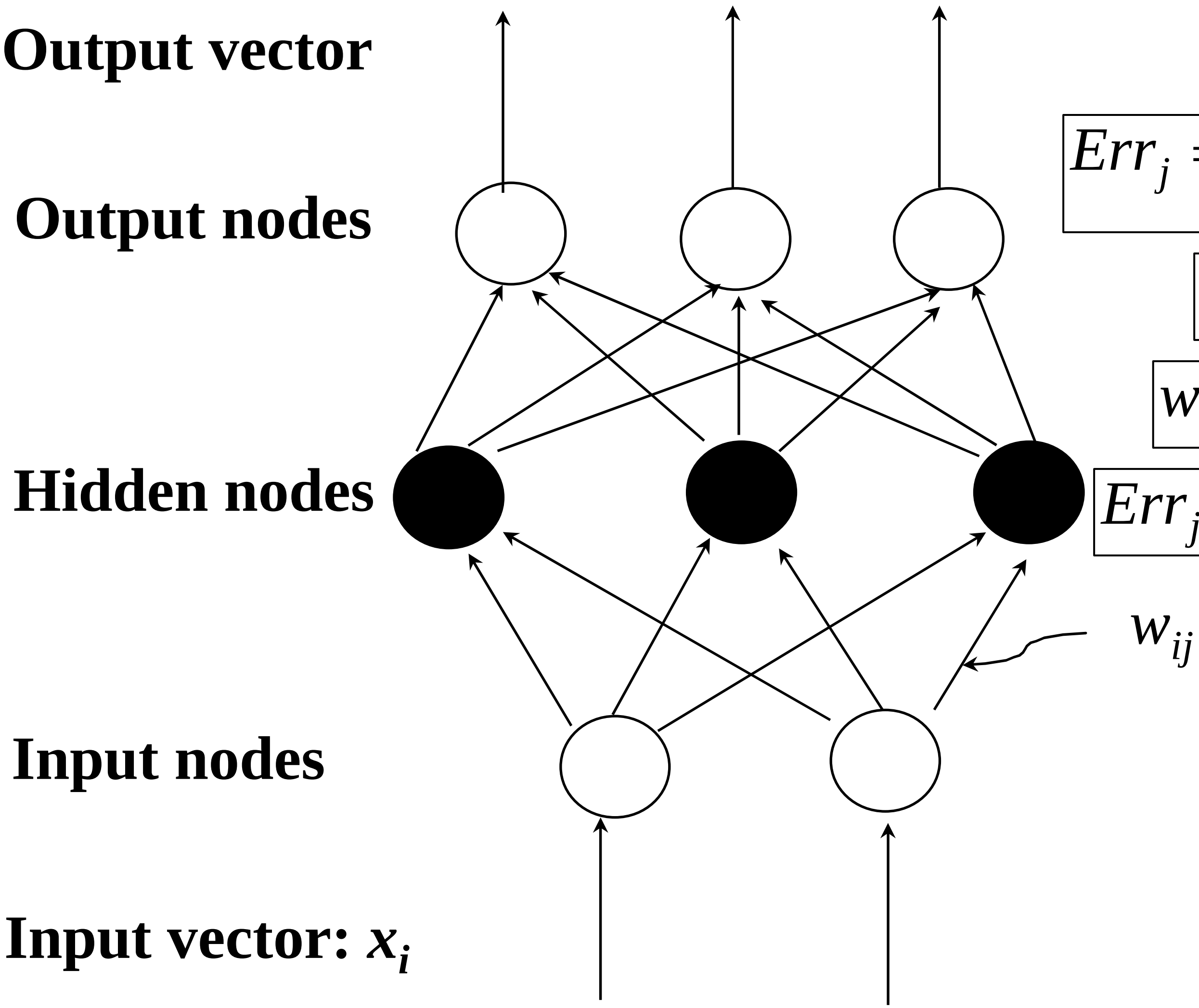
- Each **unit in the hidden and output layers** takes its net input and then applies **an activation function**.
- The function symbolizes the activation of the neuron represented by the unit. It is also called a logistic, sigmoid, or squashing function.
- Given a net input I_j to unit j, then

$$O_j = f(I_j),$$

the output of unit j, is computed as

$$\boxed{O_j = \frac{1}{1+e^{-I_j}}}$$

Back propagation Formulas



$$Err_j = O_j(1-O_j)\sum_k Err_k w_{jk}$$

$$\theta_j = \theta_j + (l)Err_j$$

$$w_{ij} = w_{ij} + (l)Err_j O_i$$

$$Err_j = O_j(1-O_j)(T_j - O_j)$$

$$O_j = \frac{1}{1+e^{-I_j}}$$

$$I_j = \sum_i w_{ij} O_i + \theta_j$$

Step 4: Back propagate the error

- When reaching the **Output layer**, the error is computed and propagated backwards.
- For a unit k in the output layer the error is computed by a formula:

$$\boxed{Err_k = O_k(1-O_k)(T_k - O_k)}$$

Where Ok is the actual output of unit k (computed by activation function.

$$\boxed{O_k = \frac{1}{1+e^{-I_k}}}$$

Tk is the TRUE output based of known class label of training sample

Observe: Ok(1-Ok) is a Derivative (rate of change) of activation function.

Step 4: Back propagate the error

- **The error** is propagated backwards by updating weights and biases to reflect the error of the network classification.
- For a unit j in the hidden layer the error is computed by a formula:
- $$\boxed{Err_j = O_j(1-O_j)\sum_k Err_k w_{jk}}$$

where wjk is the weight of the connection from unit j to unit k in the next higher layer, and Errk is the error of unit k.

Step 5: Update weights and biases

Weights update

- **Weights** are updated by the following equations, where *l* is a constant between 0.0 and 1.0 reflecting the learning rate, this learning rate is fixed for implementation.

$$\boxed{\Delta w_{ij} = (l) Err_j O_i}$$

$$\boxed{w_{ij} = w_{ij} + \Delta w_{ij}}$$

The rule of thumb is to set the learning rate to
l = 1/k where k is the number of iterations through the training set so far.

Step 5: Update weights and biase

Learning Rate

- **Backpropagation learns using a method of gradient descent to search for a set of weights that fit the training data so as to minimize the mean squared distance between the network's class prediction and known target value of the records.**

- **The learning rate helps avoid getting stuck at local mimimum (i.e. where the weights appear to converge, but are not optimum solution).**
- **The learning rate encourages finding the global minimum.**
- **If the learning rate is too small, then learning will occur at a very slow pace.**
- **If the learning rate is too large, then oscillation between inadequate solutions may occur.**

Step 5: Update weights and biases

Bias update

Biases are updated by the following equations

$$\boxed{\Delta\theta_j = (l)Err_j}$$

$$\boxed{\theta_j = \theta_j + \Delta\theta_j}$$

Where $\boxed{\Delta\theta_j}$ is the change in the bias

Weights and Biases Updates

- Case updating: we are updating weights and biases after the presentation of each sample (record).
- Epoch: One iteration through the training set is called an epoch.

- Epoch updating:
- **The weight and bias increments are accumulated in variables and the weights and biases are updated after all of the samples of the training set have been presented.**

- **Case updating is more accurate**

Terminating Conditions

- Training stops when
  - All $\boxed{\Delta w_{ij}}$ in the previous epoch are below some threshold, or

•The percentage of samples misclassified in the previous epoch is below some threshold, or

• a pre- specified number of epochs has expired.

• In practice, several hundreds of thousands of epochs may be required before the weights will converge.

Back Propagation Formulas

Output vector

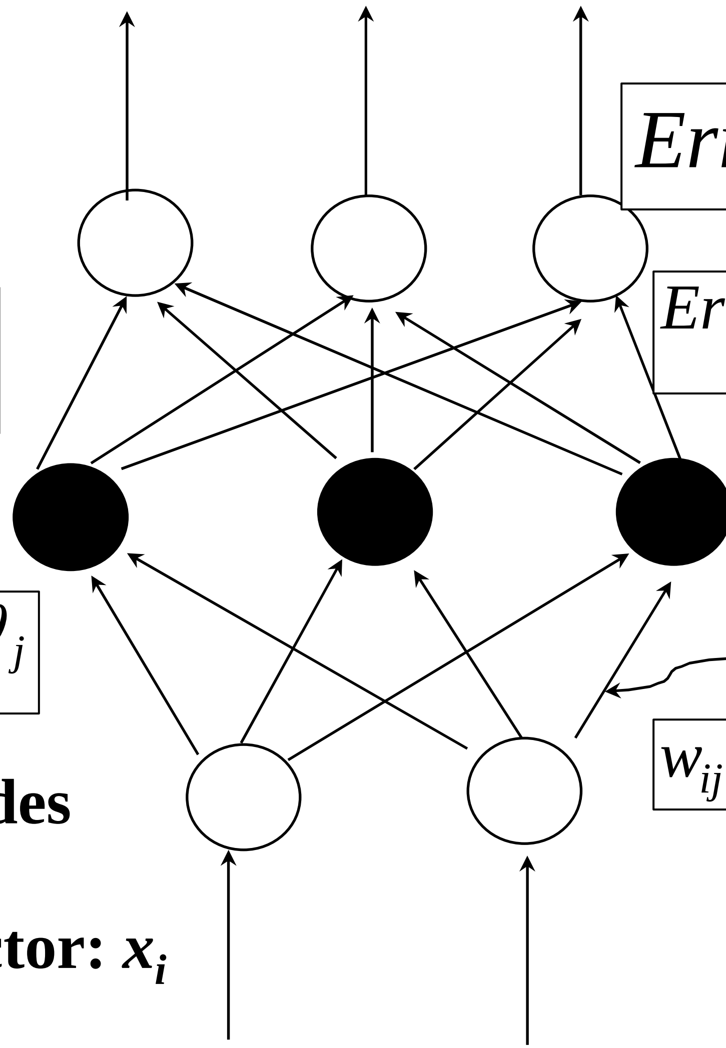
$$Err_k = O_k(1-O_k)(T_k - O_k)$$

Output nodes

$$Err_j = O_j(1-O_j)\sum_k Err_k w_{jk}$$

$$O_j = \frac{1}{1+e^{-I_j}}$$

Hidden nodes



$$I_j = \sum_i w_{ij} O_i + \theta_j$$

w_{ij}

$$\theta_j = \theta_j + (l) Err_j$$

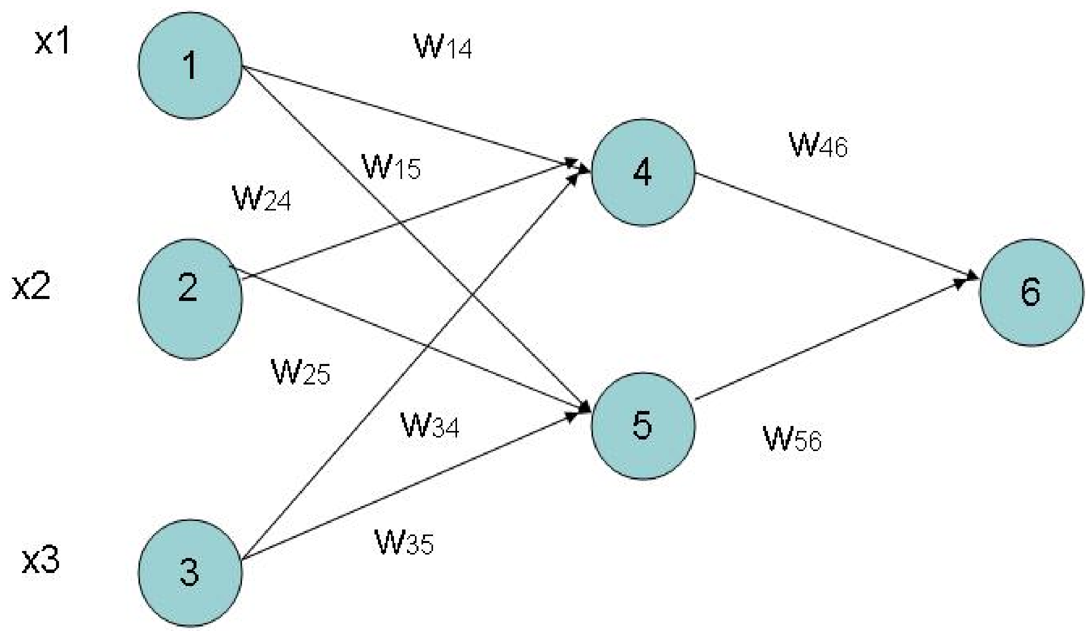
$$w_{ij} = w_{ij} + (l) Err_j O_i$$

Input nodes

Input vector: x_i

Example of Back Propagation

Input = 3, Hidden Neuron = 2 Output =1



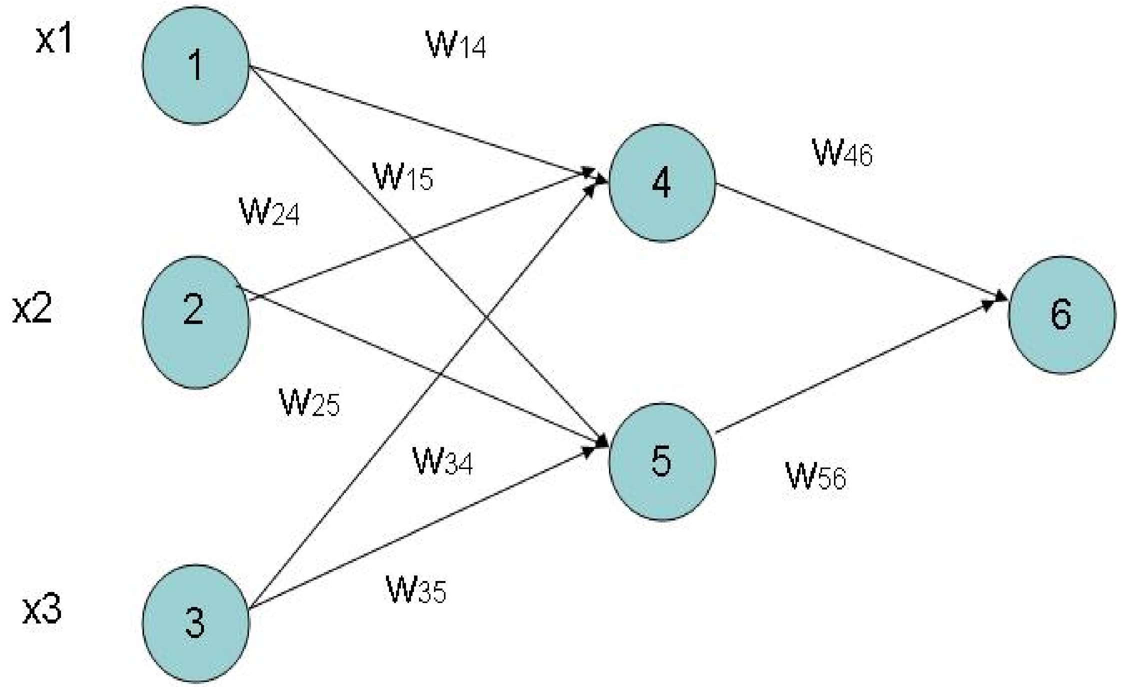
Initialize weights :

Random Numbers from -1.0 to 1.0

Initial Input and weight

x1	x2	x3	w_{14}	w_{15}	w_{24}	w_{25}	w_{34}	w_{35}	w_{46}	w_{56}
1	0	1	0.2	- 0.3	0.4	0.1	- 0.5	0.2	-0.3	-0.2

Example of Back Propagation



- **Bias added to Hidden and output nodes**
- **Initialize Bias**
- **Bias: Random Values from**
- **-1.0 to 1.0**
- **Bias (Random)**

θ_4	θ_5	θ_6
-0.4	0.2	0.1

Net Input and Output Calculation

Unit j	Net Input I_j	Output O_j
4	**0.2 + 0 + 0.5 -0.4 = -0.7**	$O_j = \frac{1}{1+e^{0.7}}$ = 0.332
5	**-0.3 + 0 + 0.2 + 0.2 =0.1**	$O_j = \frac{1}{1+e^{-0.1}}$ = 0.525
6	**(-0.3)0.332-(0.2)(0.525)+0.1= -0.105**	$O_j = \frac{1}{1+e^{0.105}}$ = 0.475

Calculation of Error at Each Node

Unit j	Error j
6	**0.475(1-0.475)(1-0.475) =0.1311** **We assume T_6 = 1**
5	**0.525 x (1- 0.525)x 0.1311x** **(-0.2) = 0.0065**
4	**0.332 x (1-0.332) x 0.1311 x** **(-0.3) = -0.0087**

Calculation of weights and Bias Updating

Learning Rate l =0.9

Weight	New Values
w_{46}	**-0.3 + 0.9(0.1311)(0.332) = - 0.261**
w_{56}	**-0.2 + (0.9)(0.1311)(0.525) = - 0.138**
w_{14}	**0.2 + 0.9(-0.0087)(1) = 0.192**
w_{15}	**-0.3 + (0.9)(-0.0065)(1) = - 0.306**
……..similarly	**………similarly**
θ_6	**0.1 +(0.9)(0.1311)=0.218**
……..similarly	**………similarly**

Some Facts to be Remembered

- NNs perform well, generally better with larger number of hidden units
- More hidden units generally produce lower error
- Determining network topology is difficult
- Choosing single learning rate impossible
- Difficult to reduce training time by altering the network topology or learning parameters
- NN with Subsets (see next slides) learning often produce better results

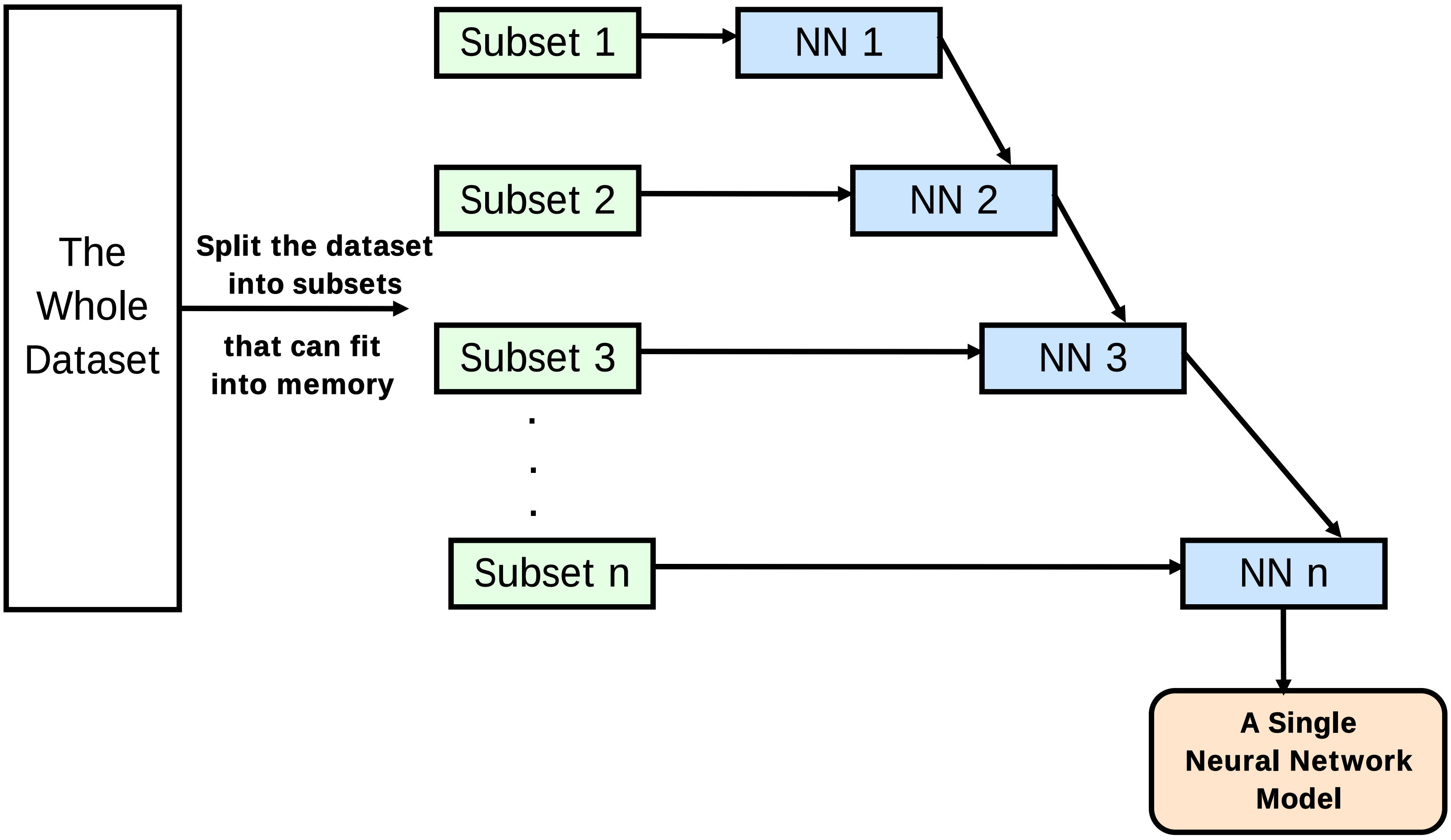
Advanced Features of Neural Network (to be covered by students presentations)

- Training with Subsets
- Modular Neural Network
- Evolution of Neural Network

Training with Subsets

- Select subsets of data
- Build new classifier on subset
- Aggregate with previous classifiers
- Compare error after adding a classifier
- Repeat as long as error decreases

Training with subsets



Modular Neural Network

- Modular Neural Network
  - Made up of a combination of several neural networks.

  The idea is to reduce the load for each neural network as opposed to trying to solve the problem on a single neural network.

Evolving Network Architectures

- Small networks without a hidden layer can’t solve problems such as XOR, that are not linearly separable.
  - Large networks can easily overfit a problem to match the training data, limiting their ability to generalize a problem set.

Constructive vs Destructive Algorithm

- Constructive algorithms take a minimal network and build up new layers nodes and connections during training.

- Destructive algorithms take a maximal network and prunes unnecessary layers nodes and connections during training.

Faster Convergence

- Back propagation requires many epochs to converge
- (An epoch is one presentation of all the training examples in the dataset)
- Some ideas to overcome this are:
  - *Stochastic learning:* updates weights after each example, instead of updating them after one epoc
  - *Momentum:* This optimization is due to the fact that it speeds up the learning when the weight are moving in a single direction continuously by increasing the size of steps
  - The closer this value is to one, the more each weight change will not only include the current error, but also the weight change from previous examples

(which often leads to faster convergence)