Module 2 – The LP Model

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Purpose

Formulate linear programming (LP) models for two scenarios by defining decision variables, the objective function, constraints, and a full mathematical formulation.

Problem 1 — Back Savers Backpacks

Introduction

Back Savers plans to offer two models, Collegiate and Mini produced with the same nylon fabric and labor pool. The task is to determine weekly production quantities that maximize profit while honoring material supply, labor availability, and sales forecasts.

a) Decision Variables

Let x1 be the number of Collegiate backpacks produced per week, and x2 be the number of Mini backpacks produced per week. These are the controllable choices management must set to achieve the best outcome.

b) Objective Function

Each Collegiate yields $32 profit; each Mini yield $24. The objective is to maximize total weekly profit from both models:
Maximize Z = 32*x1 + 24*x2

c) Constraints

Fabric (sq ft): Collegiate uses 3; Mini uses 2; total nylon available is 5,000 sq ft per week.

3*x1 + 2*x2 <= 5,000

We cannot consume more fabric than the weekly shipment.

Labor (minutes): Collegiate requires 45; Mini requires 40. There are 35 laborers × 40 hours/week = 1,400 hours = 84,000 minutes.

45*x1 + 40*x2 <= 84,000

Production time is limited by available labor-hours.

Demand caps (units): At most 1,000 Collegiates and 1,200 Minis can be sold per week.

x1 <= 1,000; x2 <= 1,200

Making beyond forecasted sales does not increase profit.

Non-negativity:

x1 >= 0; x2 >= 0

d) Full Mathematical Formulation

Maximize Z = 32*x1 + 24*x2

subject to 3*x1 +
2*x2 <= 5,000
45*x1 + 40*x2 <=
84,000 x1 <=
1,000 x2 <=
1,200 x1, x2 >= 0

Produce 1,000 Collegiates (the sales max) and 975 Minis with the remaining labor. This plan exactly uses the available labor, stays within the fabric and demand limits, and yields about $55,400 in weekly profit. In short: max out Collegiates, then fill the rest with Minis for the best profit.

Problem 2 — Weigelt Corporation

Introduction

Weigelt operates three plants with excess capacity and can produce a new product in Large (L), Medium (M), and Small (S) sizes. Profits and storage needs differ by size. All plants can produce any size. To avoid layoffs, each plant must use the same percentage of its excess capacity. The goal is to maximize daily profit subject to capacity, storage, and sales constraints.

a) Decision Variables

Let x1L, x1M, x1S be the units of L, M, S produced at Plant 1 per day; x2L, x2M, x2S at Plant 2; x3L, x3M, x3S at Plant 3. Introduce u ∈ [0,1] as the common utilization fraction so that each plant uses the same percentage of its excess capacity.

b) Linear Programming Model

Net unit profits are $420 (L), $360 (M), and $300 (S). The objective is to maximize total daily profit across plants:
Maximize Z = 420*(x1L+x2L+x3L) + 360*(x1M+x2M+x3M) + 300*(x1S+x2S+x3S)

Equal utilization of excess capacity (linearized):
x1L + x1M + x1S = 750*u x2L +
x2M + x2S = 900*u x3L + x3M +
x3S = 450*u
Enforces the same percentage of capacity usage at every plant.

In-process storage limits (sq ft):

20*x1L + 15*x1M + 12*x1S <= 13,000

20*x2L + 15*x2M + 12*x2S <= 12,000

20*x3L + 15*x3M + 12*x3S <= 5,000

Daily storage space caps limit how many units can be staged.

Sales forecasts (units/day):

x1L + x2L + x3L <= 900 (Large) x1M +
x2M + x3M <= 1,200 (Medium) x1S +
x2S + x3S <= 750 (Small)

Do not plan to sell more than the market will accept.

Bounds and non-negativity:

0 <= u <= 1; xij >= 0 for all i ∈ {1,2,3}, j ∈ {L,M,S}

Maximize Z = 420(x1L + x2L + x3L) + 360(x1M + x2M + x3M) + 300(x1S + x2S + x3S)

subject to x1L + x1M +
x1S = 750*u x2L + x2M
+ x2S = 900*u x3L +
x3M + x3S = 450*u
20*x1L + 15*x1M + 12*x1S <= 13,000
20*x2L + 15*x2M + 12*x2S <= 12,000
20*x3L + 15*x3M + 12*x3S <=
5,000 x1L + x2L + x3L <= 900
x1M + x2M + x3M <= 1,200 x1S
+ x2S + x3S <= 750
0 <= u <= 1; xij >= 0

Run all three plants at the same utilization rate and, within each plant’s total, choose the size mix that best fits storage and sales caps while maximizing profit. If storage is the tightest limit, lean more on Small/Medium (better profit per square foot); if storage is ample and sales allow, lean more on Large (highest unit profit).