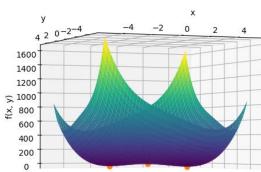
NOTES:

- With sumpy the result inludes the imginary and real roots, however in assignment I only found the **real roots and plotted them**.
- To plot the derivatives use the **plot_all** function that take **get_derivatives** returned value (**deri**) as a parameter.
- Sympy cannot show the graph for the **function = constant**, for 3D that is a plane.

Q1.

a) Plot of function with critical points:

$$f(x,y) = x^4 + y^4 + 16xy$$

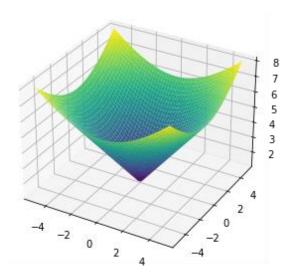


 $critical\ points = (-2, 2), (0, 0), (2, -2)$

Derivatives					
$\frac{df}{dx} = 4x^3 + 16y$	$\frac{df}{dy} = 16x + 4y^3$	$\frac{df}{dx^2}12x^2$	$\frac{df}{df^2}12y^2$	$\frac{df}{dx} = 16$	
10000 5000 0 -5000 10000 10000 15 -15	10000 5000 0 -5000 10000 15 15 10 15 10 15 10 15	2500 2000 1500 1000 500 15 15 10 15 10 15 15	2500 2000 1500 1000 500 15 15 10 15 15	The graph is plane in 3 dimension where the value of function $df dxy = 16$	

b) Plot of function with critical points:

$$f(x,y) = \sqrt{x^2 + y^2} + 1$$

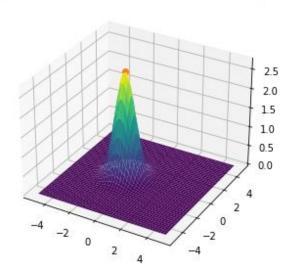


There is no critical point, because at no point derivative is zero and smooth.

Derivatives				
df_{-} x	df_{-} y	df x^2	df y^2	df xy
$\frac{1}{dx} - \frac{1}{\sqrt{x^2 + y^2}}$	$\frac{dy}{dy} - \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2}}$	$\frac{1}{dx^2} = -\frac{1}{(x^2 + y^2)^{\frac{3}{2}}}$	$\frac{dy^2}{dy^2} = -\frac{1}{(x^2 + y^2)^{\frac{3}{2}}}$	$\frac{1}{dxy} = -\frac{1}{(x^2 + y^2)^{\frac{3}{2}}}$
, ,	, ,	$(x + y)^2$	$(x + y)^2$	(2 1 3)-
		$+\frac{1}{\sqrt{x^2+y^2}}$	$+\frac{1}{\sqrt{x^2+y^2}}$	
0.75 0.25 0.00 0.25 0.00 0.75 0.5 0.00 0.75 0.5 0.00 0.75 0.5 0.00 0.75 0.5 0.00 0.75 0.00 0.05 0.05	0.75 0.50 0.25 0.00 0.25 0.00 0.75 0.50 0.00 0.75 0.50 0.75 0.50 0.75 0.50 0.75 0.50 0.75 0.50 0.75 0.50 0.75 0.50 0.75 0.50 0.75 0.75	10 08 06 04 02 15 10 15 10 15 10 15	10 08 06 0.4 0.2 15 10 15 10 15 10 15	10 05 00 -0.5 -1.0 15 10 15 15

c) Plot of function with critical points: $f(x,y) = e^{x^2 + y^2 + 2x}$

$$f(x,y) = e^{x^2 + y^2 + 2x}$$

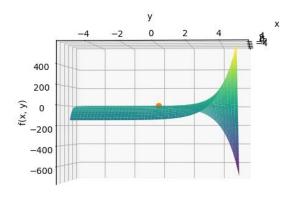


There is one critical point (-1,0) that is very much obvious from the graph.

Derivatives				
$\frac{df}{dx} = (-2x - 2)e^{-x^2 - 2x - y^2}$	$\frac{df}{dy} = -2ye^{-x^2 - 2x - y^2}$	$ \frac{df}{dx^2} = (-2x - 2)^2 e^{-x^2 - 2x - y^2} - 2e^{-x^2 - 2x - y^2} $	$\frac{df}{dy^{2}}$ $= 4y^{2}e^{(-x^{2}-2x-y^{2})}$ $-2e^{-x^{2}-2x-y^{2}}$	$\frac{df}{dxy}$ $= -2y(-2x$ $-2)e^{-x^2-2x-y^2}$
1 0 -1 -1 -2 15 10 15 10 15 15 15 15 15 15 15 15 15 15 15 15 15	2 1 0 -1 -1 -2 15 5 10 15 15 10 15 15	2 1 0 -1 -2 -3 -4 15 10 15 -15	1 0 -1 -2 -3 15 10 15 -15 10 15 -15	15 10 05 00 00 00 00 00 00 -1.5 -1.0 -1.5 10 15 -15

a) Plot of function with critical points: $f(x,y) = xe^{y} - e^{x}$

$$f(x,y) = xe^y - e^x$$



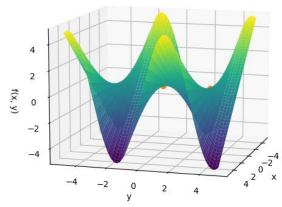
Derivatives				
$\frac{df}{dx} = -e^x + e^y$	$\frac{df}{dy} = xe^y$	$\frac{df}{dx^2} = -e^x$	$\frac{df}{dy^2}xe^y$	$\frac{df}{dxy} = e^y$
-15 -10 -5 0 5 10 15 -15	-15 ₋₁₀ ₋₅ ₀ ₅ _{10₁₅-15}	-15 -10 -5 10 15 -15	4 2 2 0 -2 -4 15 10 15 -15 10 15 -15	30 25 20 15 10 0.5 0.0 0.5 0.0 15 10 15 10 15

There is one critical point at (0,0).

Discriminant = -1, so it is a saddle point.

b) Plot of function with critical points:

$$f(x,y) = x sin(y)$$

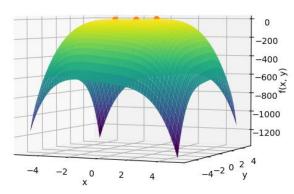


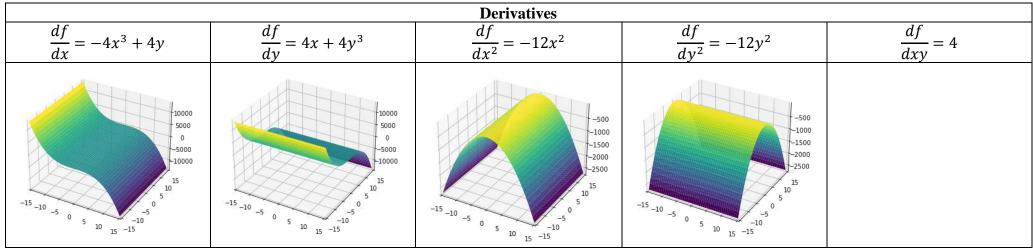
Derivatives				
$\frac{df}{dx} = \sin(y)$	$\frac{df}{dy} = x\cos(y)$	$\frac{df}{dx^2} = 0$	$\frac{df}{dy^2} = -x * \sin(y)$	$\frac{df}{dy^2} = -x\sin(y)$
0.75 0.50 0.25 0.00 0.00 0.025 0.05 0.05 0.	-15 ₋₁₀ -5 -5 -10 15 10		10 5 0 -5 -10 15 10 15 10 15	0.75 0.50 0.25 0.00 0.00 0.05 0.05 0.05 0.0

There are multiple critical points (0,0), $(0,\pi)$. There could be other multiple points following sequence of $k\pi$ because it is function of sin. Discriment =-1 for both point, so both points are saddle points.

c) Plot of function with critical points:

$$f(x,y) = 4xy - x^4 - y^4$$





There are three critical points: (-1,-1), (0,0), (1,1) for point (0,0), Discriminant < 0, so it is a saddle point. for point (1,1), Discriminat > 0, and $f_{xx} < 0$, so it is a local maximum for point (-1,-1), Discriminant > 0 and $f_{xx} < 0$, so it is a local maximum