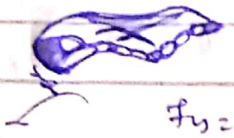
Muneeb BSCS19057

Q1) a) $f(x,y) = x^4 + y^4 + 16xy$

$f_x = 4x^3 + 16y$

$f_y = 4y^3 + 16x$



$f_x = 0$

$4x^3 + 16y = 0$

$4(x^3 + 4y) = 0$

$x^3 + 4y = 0$

Critical points.

$(0, 0)$ Ans

~~$(-1, \frac{1}{4})$~~

$f_y = 0$

$4y^3 + 16x = 0$

$y^3 + 4x = 0$

Critical point

$(0, 0)$

b) $f(x,y) = \sqrt{x^2 + y^2} + 1$

$f_x = \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} 2x$

$f_y = \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} 2y$

$f_x = 0$

$\frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} 2x = 0$

$x(x^2 + y^2)^{-\frac{1}{2}} = 0$

$(x^2 + y^2)^{-\frac{1}{2}} \neq 0$

So $x \neq 0$

$\frac{x}{\sqrt{x^2 + y^2}} \neq 0$

$f_y = 0$

$\frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}} 2y = 0$

$y(x^2 + y^2)^{-\frac{1}{2}} = 0$

$(x^2 + y^2)^{-\frac{1}{2}} \neq 0$

So ~~$y \neq 0$~~ $\frac{y}{\sqrt{x^2 + y^2}} \neq 0$

~~Critical point (0,0) Ans~~

~~No critical point~~ $\boxed{(0,0)}$

$x = y$ $y = 0$

$\frac{x}{\sqrt{x^2 + y^2}} = 0$

$\frac{y}{\sqrt{x^2 + y^2}} = 0$

$(0, 0)?$

$x = 0$

Derivative undefined at $(0,0)$ So no Critical Point.

c) $f(x,y) = e^{-(x^2+y^2+2x)}$

$f_x = e^{-(x^2+y^2+2x)} \cdot -(2x+2)$

$f_x = 0$.

$-(2x+2)\, e^{-(x^2+y^2+2x)} = 0$.

$e^{-(x^2+y^2+2x)} \neq 0$.

So,

$-(2x+2) = 0$

$2x = -2$

$x = -1$

$f_y = e^{-(x^2+y^2+2x)} \cdot -(2y)$

$f_y = 0$. for critical point.

$e^{-(x^2+y^2+2x)} \cdot -(2y) = 0$.

$e^{-(x^2+y^2+2x)} \neq 0$.

So,

$-2y = 0$

$y = 0$.

Critical point $(-1, 0)$

Scanned with CamScanner

Q2

a) $f(x,y) = xe^y - e^x$

$f_x = e^y - e^x$

$f_x = 0$

$e^y = e^x$

$y = x$

$y = 0$

$f_y = xe^y$

$f_y = 0$

$e^y \neq 0$

So,

$x = 0$

So, critical point $(0,0)$

$f_{xx} = -e^x$

$f_{yy} = xe^y$

$f_{xy} = e^y$

$D = f_{xx} f_{yy} - (f_{xy})^2 = -e^x \times e^y \; \cancel{=} - (e^0)^2 = -1$

$D<0$ Saddle point at $(0,0)$.

b) $f(x,y) = x\sin(y)$

$f_x = \sin(y)$

$f_x = 0$

$\sin(y) = 0$

$y = \sin^{-1}(0)$

$y = 0$

$f_y = x$

$x = 0$.

Critical point $(0,0)$ $(0,\pi)$

$f_{xx} = 0$

$f_{yy} = 0$

$f_{xy} = \cos(y)$

$D = f_{xx} \cdot f_{yy} - f_{xy}^2$

$= 6 \cdot 0 - (\cos(0))^2 = (1)^2$

$= -1.$

~~Both~~ $D<0$, so Saddle point (Both Points)
are Saddle because $D<0$ for both.

Q4c)

~~Q3~~

~~$f(x,y) = 4xy - x^4 - y^4$~~

~~$f_x = 4x - 4x^3$~~ ~~$f_y = 4y - 4y^3$~~

~~$f_x = 0$~~

~~$4x - 4x$~~

$f(x,y) = 4xy - x^4 - y^4$

$f_x = 4y - 4x^3$ $f_y = 4x - 4y^3$

$f_x = 0$

$4y - 4x^3 = 0$

$y - x^3 = 0$

$(0, 0)$

$(1, 1)$

$(-1, -1)$

$f_y = 0$

$x - y^3 = 0$

$(0,0)$

$(1,1)$

$(-1,-1)$

$f_{xx} = -12x^2$ $\quad$ $f_{yy} = -12y^2$

$f_{xy} = 4$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$D = -12x^2 - 12y^2 - 16$

$D = 144x^2y^2 - 16$

for $P(0,0)$

$D = -16$ Saddle point

for $P(1,1)$

$D = 144 - 16$

$D = 128$ $\quad$ $f_{xx} = -12$

$D > 0$ and $f_{xx} < 0$ so local Maximum

for $P(-1,-1)$

$D = 144 - 16$ $\quad$ $f_{xx} = -12$

$= 128$

$D > 0$ and $f_{xx} < 0$ so local Maximum

Extreme value for $(1,1)$

$f(1,1) = 4(1)(1) - 1 - 1 = 2$

$f(-1,-1) = 4 - 1 - 1 = \underline{\underline{2}}$