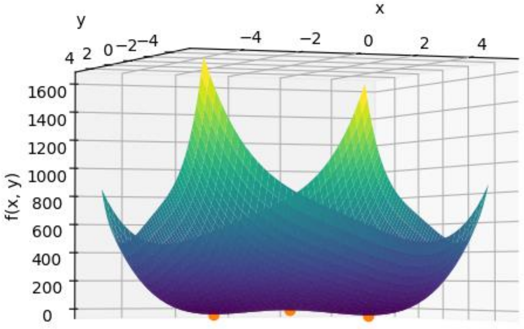
Muhammad Muneeb Ur Rahman **BSCS19057**

NOTES:

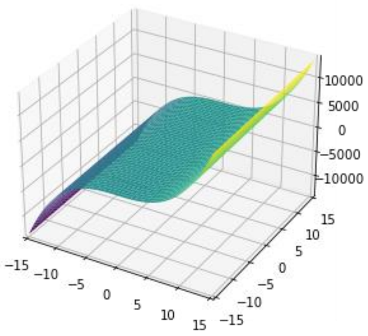
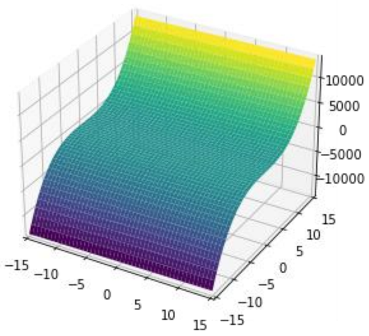
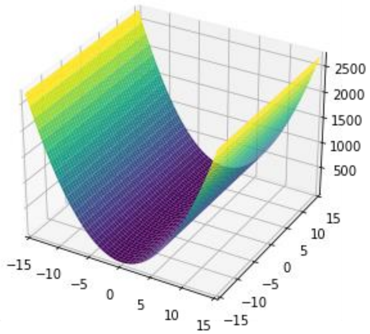
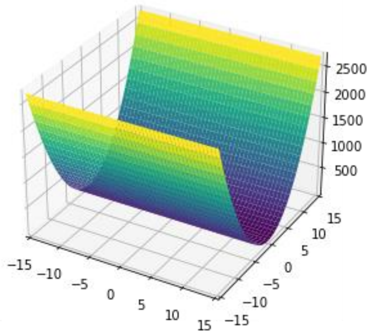
- With sumpy the result inludes the imginary and real roots, however in assignment I only found the **real roots and plotted them**.
- To plot the derivatives use the **plot_all** function that take **get_derivatives** returned value (**deri**) as a parameter.
- Sympy cannot show the graph for the **function = constant**, for 3D that is a plane.

Q1.

a) Plot of function with critical points:
$f(x, y) = x^4 + y^4 + 16xy$

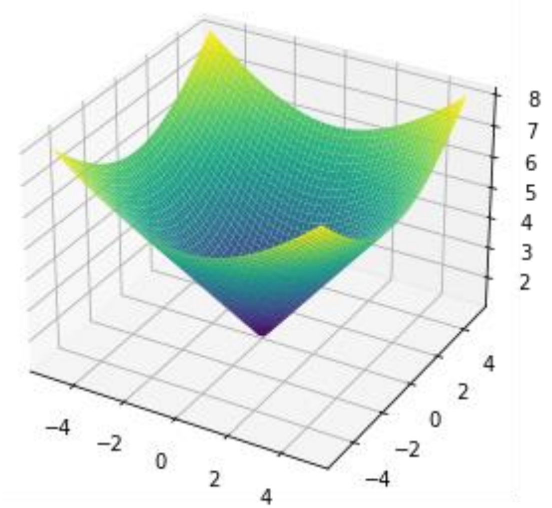


$critical\ points = (-2, 2), (0, 0), (2, -2)$

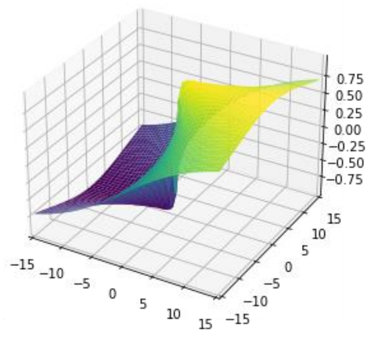
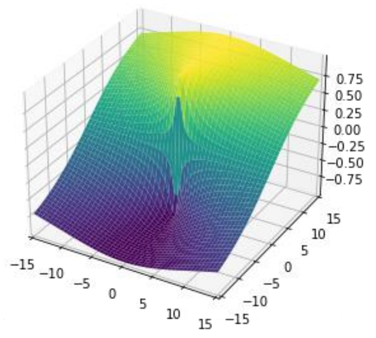
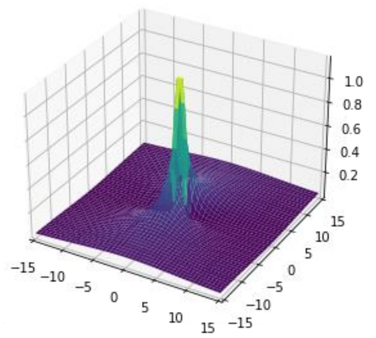
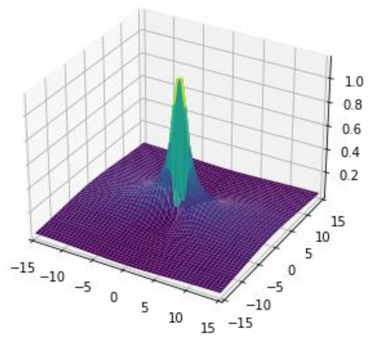
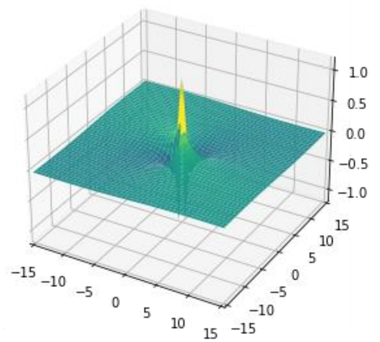
Derivatives				
$\frac{df}{dx} = 4x^3 + 16y$	$\frac{df}{dy} = 16x + 4y^3$	$\frac{df}{dx^2}12x^2$	$\frac{df}{df^2}12y^2$	$\frac{df}{dx} = 16$
				The graph is plane in 3 dimension where the value of function $dfdxy = 16$

b) Plot of function with critical points:

$f(x,y) = \sqrt{x^2 + y^2} + 1$

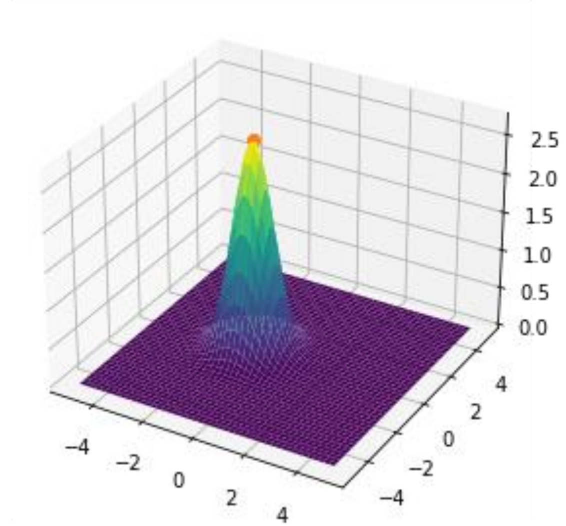


$There\ is\ no\ critical\ point, because\ at\ no\ point\ derivative\ is\ zero\ and\ smooth.$

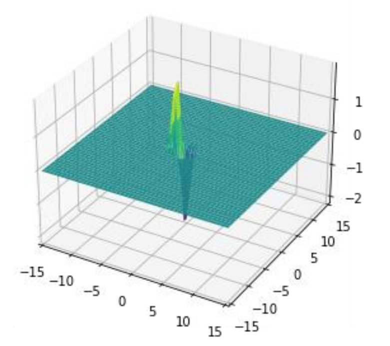
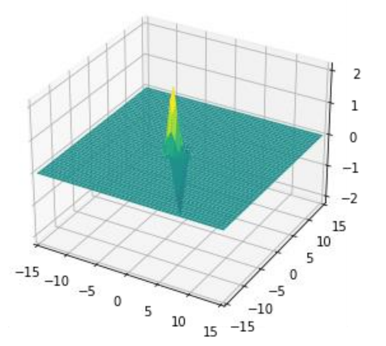
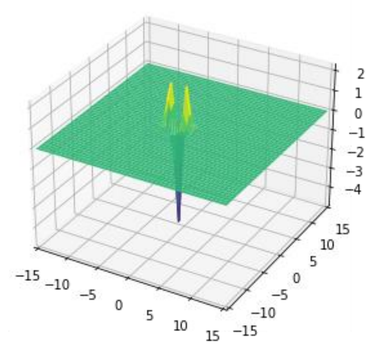
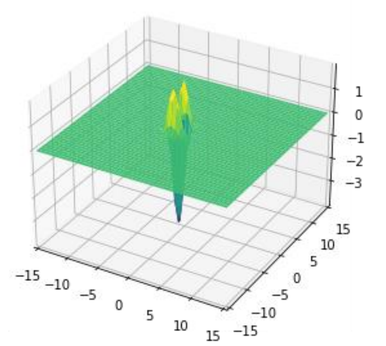
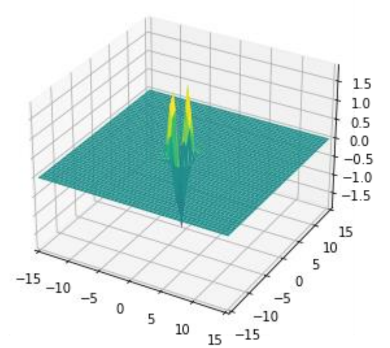
Derivatives				
$\frac{df}{dx} = \frac{x}{\sqrt{x^2 + y^2}}$	$\frac{df}{dy} = \frac{y}{\sqrt{x^2 + y^2}}$	$\frac{df}{dx^2} = -\frac{x^2}{(x^2 + y^2)^{\frac{3}{2}}} + \frac{1}{\sqrt{x^2 + y^2}}$	$\frac{df}{dy^2} = -\frac{y^2}{(x^2 + y^2)^{\frac{3}{2}}} + \frac{1}{\sqrt{x^2 + y^2}}$	$\frac{df}{dxy} = -\frac{xy}{(x^2 + y^2)^{\frac{3}{2}}}$
				

c) Plot of function with critical points:

$f(x, y) = e^{x^2+y^2+2x}$



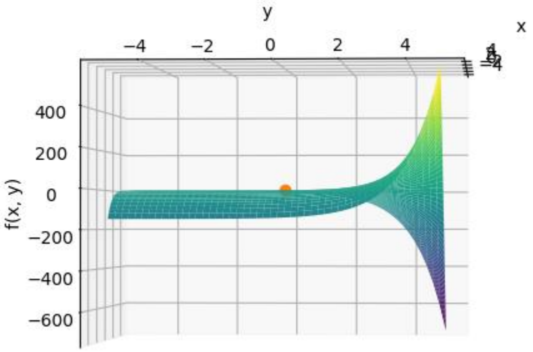
$There\ is\ one\ critical\ point\ (-1, 0)\ that\ is\ very\ much\ obvious\ from\ the\ graph.$

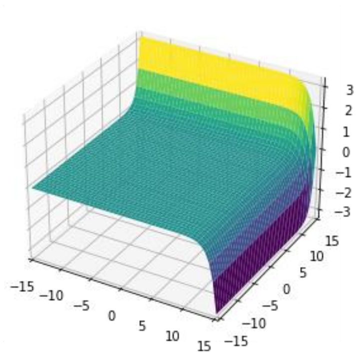
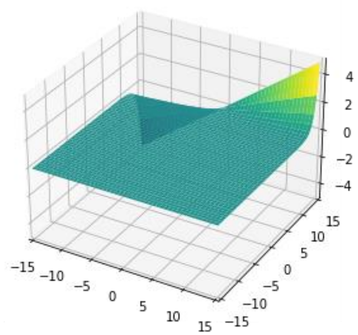
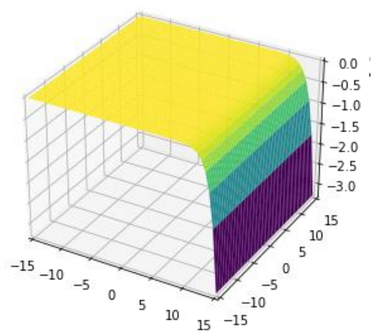
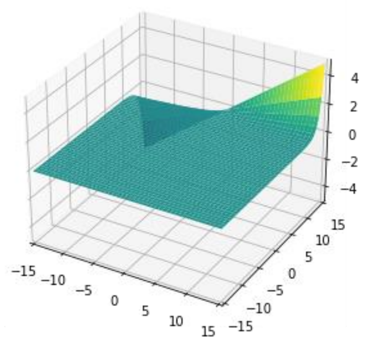
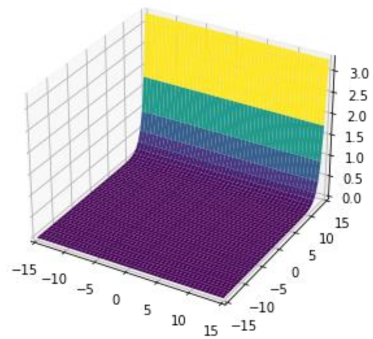
Derivatives				
$\frac{df}{dx} = (-2x-2)e^{-x^2-2x-y^2}$	$\frac{df}{dy} = -2ye^{-x^2-2x-y^2}$	$\frac{df}{dx^2} = (-2x-2)^2e^{-x^2-2x-y^2} - 2e^{-x^2-2x-y^2}$	$\frac{df}{dy^2} = 4y^2e^{(-x^2-2x-y^2)} - 2e^{-x^2-2x-y^2}$	$\frac{df}{dxy} = -2y(-2x-2)e^{-x^2-2x-y^2}$
				

Q2.

a) Plot of function with critical points:

$f(x, y) = xe^{y} - e^{x}$

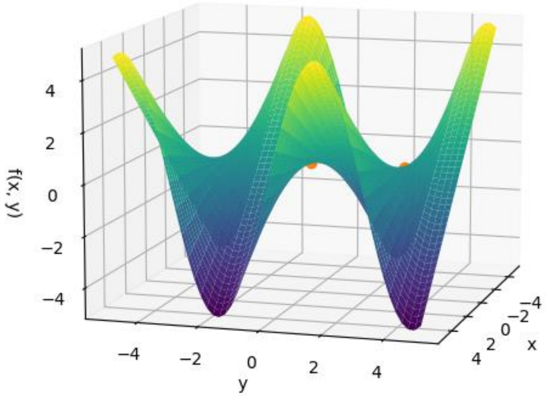


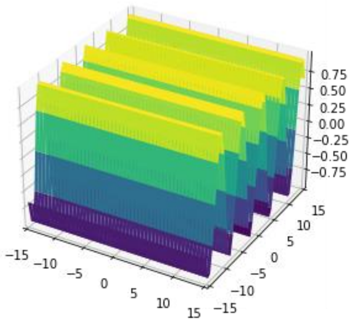
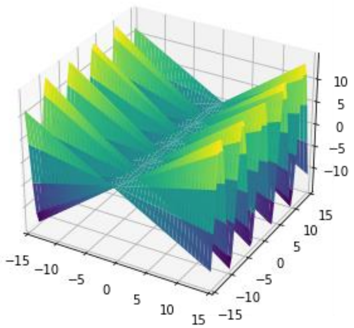
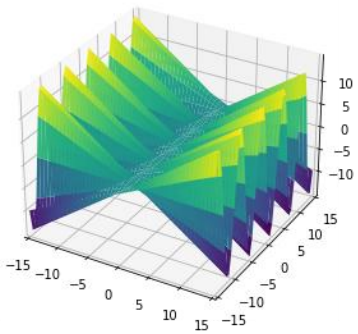
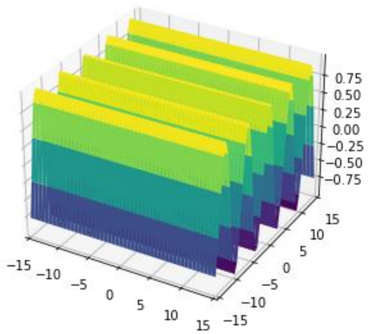
Derivatives				
$\frac{df}{dx} = -e^{x} + e^{y}$	$\frac{df}{dy} = xe^{y}$	$\frac{df}{dx^2} = \ -e^{x}$	$\frac{df}{dy^2} xe^{y}$	$\frac{df}{dxy} = e^{y}$
				

$There\ is\ one\ critical\ point\ at\ (0, 0).$

$Discriminant = \ -1, so\ it\ is\ a\ saddle\ point.$

b) Plot of function with critical points:
$f(x, y) = x sin(y)$

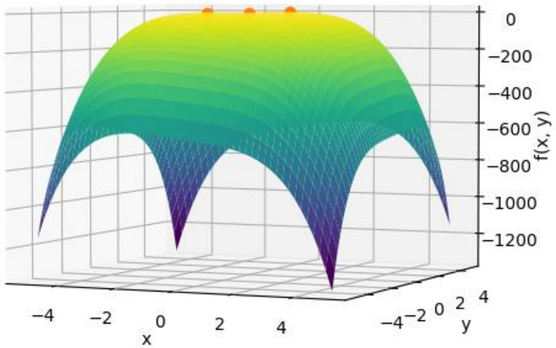


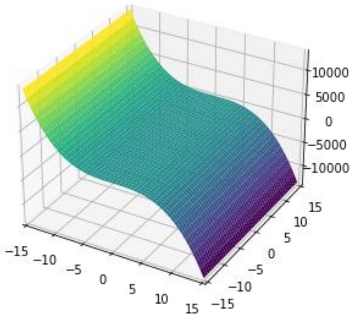
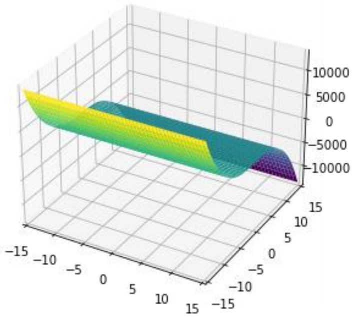
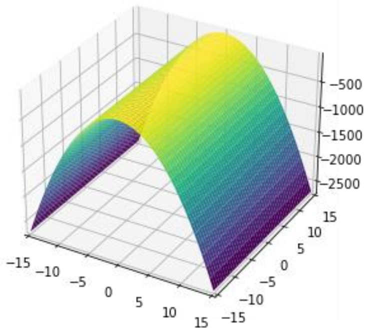
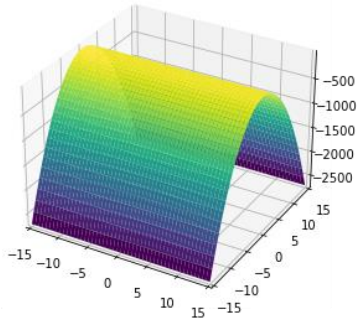
Derivatives				
$\frac{df}{dx} = \sin(y)$	$\frac{df}{dy} = x cos(y)$	$\frac{df}{dx^2} = 0$	$\frac{df}{dy^2} = -x * \sin(y)$	$\frac{df}{dy^2} = -x sin(y)$
				

$There\ are\ multiple\ critical\ points\ (0,0), (0,\pi). There\ could\ be\ other\ multiple\ points\ following\ sequence\ of\ k\pi\ becuase\ it\ is\ function\ of\ sin.$
$Discriment = -1\ for\ both\ point, so\ both\ points\ are\ saddle\ points.$

c) Plot of function with critical points:

$f(x, y) = 4xy - x^4 - y^4$



Derivatives				
$\frac{df}{dx} = -4x^3 + 4y$	$\frac{df}{dy} = 4x + 4y^3$	$\frac{df}{dx^2} = -12x^2$	$\frac{df}{dy^2} = -12y^2$	$\frac{df}{dxy} = 4$
				

There are three critical points: $(-1, -1), (0, 0), (1, 1)$

$for\ point(0, 0), Discriminant < 0, so\ it\ is\ a\ saddle\ point.$

$for\ point(1, 1), Discriminatn > 0, and\ f_{xx} < 0, so\ it\ is\ a\ local\ maximum$

$for\ point(-1, -1), Discriminant > 0\ and\ f_{xx} < 0, so\ it\ is\ a\ local\ maximum$