Numerical study of the models which describe the heart rhythm

Marharyta TOMINA

Supervisor: Lyudmyla YUSHCHENKO

Purposes

Understand the biological and mathematical aspects of the components of the heart and the heartbeat;

Overview existing types of models to describe the heart rhythms;

Consider the numerical description of the Aliev-Panfilov model.

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The heart and its components;

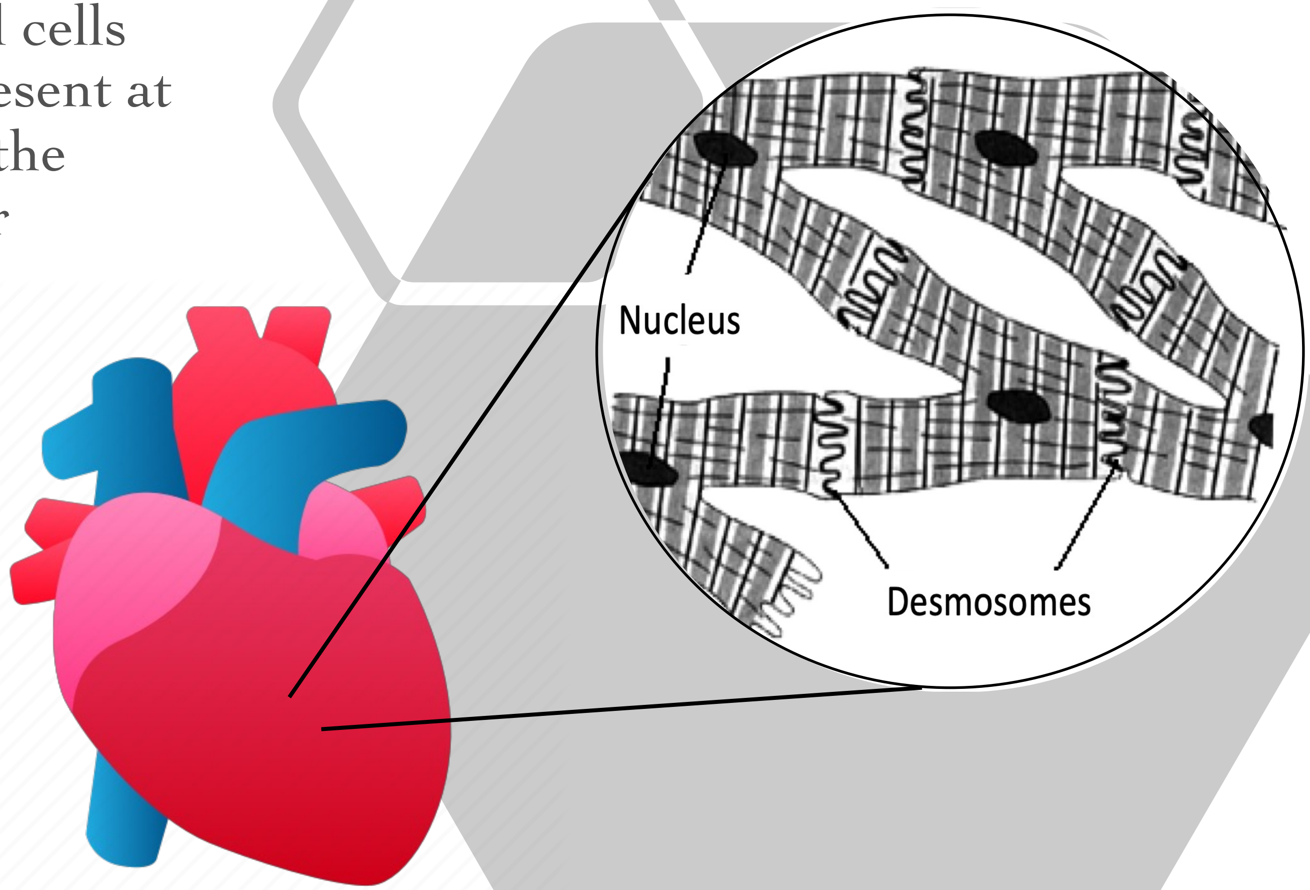
Types of models to describe the heart rhythms;

- First models
  - ✓ Hodgkin and Huxley model
- Ionic models;
  - ✓ Noble model
- Simplified heart tissue models;
  - ✓ FitzHugh-Nagumo model
  - ✓ Aliev-Panfilov model
- Mechanical models;
  - ✓ Excitation-contraction coupling model

Aliev-Panfilov 1D and 2D model;

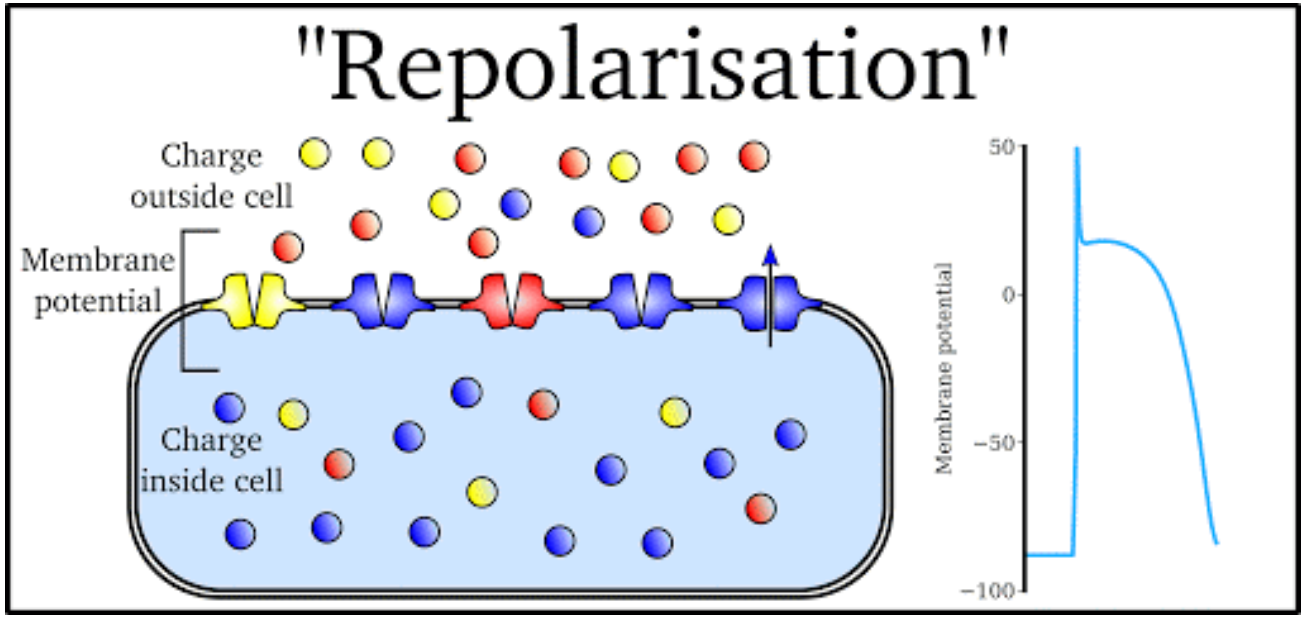
Reaction-diffusion equations.

The heart muscle is made up of a complex network of interconnected **cardiomyocytes** (cardiac muscle cells). These cylindrical cells interconnect at the bifurcations they present at their ends. Several studies have shown the importance of connections in the proper functioning of the heart.



The cardiomyocyte is able to contract constantly without getting tired during the approximately three billion heartbeats in an individual's lifetime.

The electrical signals that arise inside biological organisms are due to the movement of ions. The most important cations for the action potential are sodium Na^+ and potassium K^+ cations;



The membrane potential is the difference in electrical potential arising between the charges of the inner and outer sides of the semipermeable membrane. It has 5 phases.

Simplified heart tissue models

These models belong to the group of simplified or so-called analytical models, which are based on the chemical reaction in the heart tissue. These are notably the first models to have been created and which do not incorporate a physiological basis.

$$\begin{cases} \epsilon\partial_t u = \epsilon^2 d\Delta u + f(u,v), \quad d > 0 \\ \partial_t v = g(u,v) \end{cases}$$

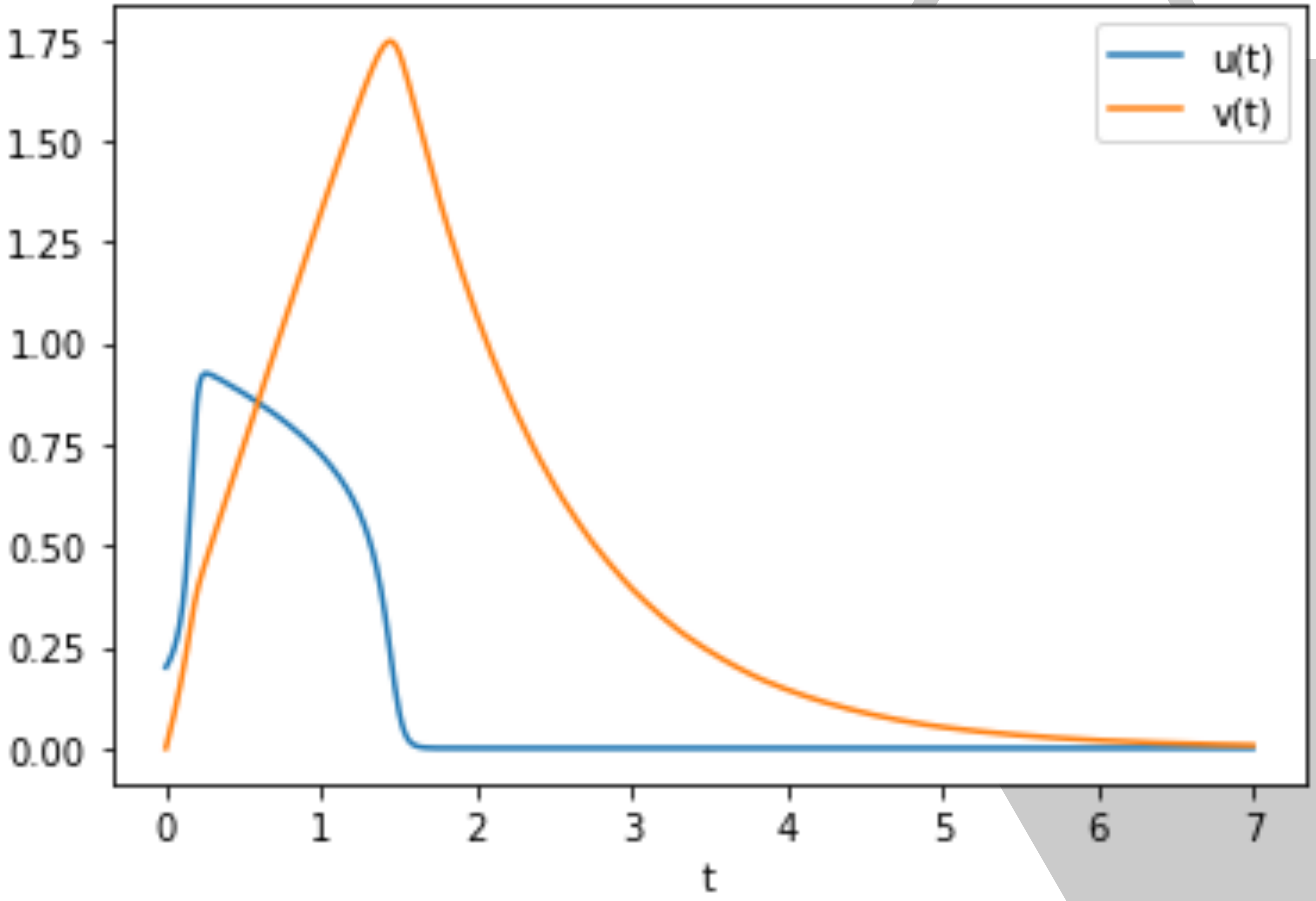
Aliev-Panfilov model

$$f(u,v) = -ku(u-1)(u-a) - uv,$$
$$g(u,v) = ku(1+a-u) - v$$

FHN model

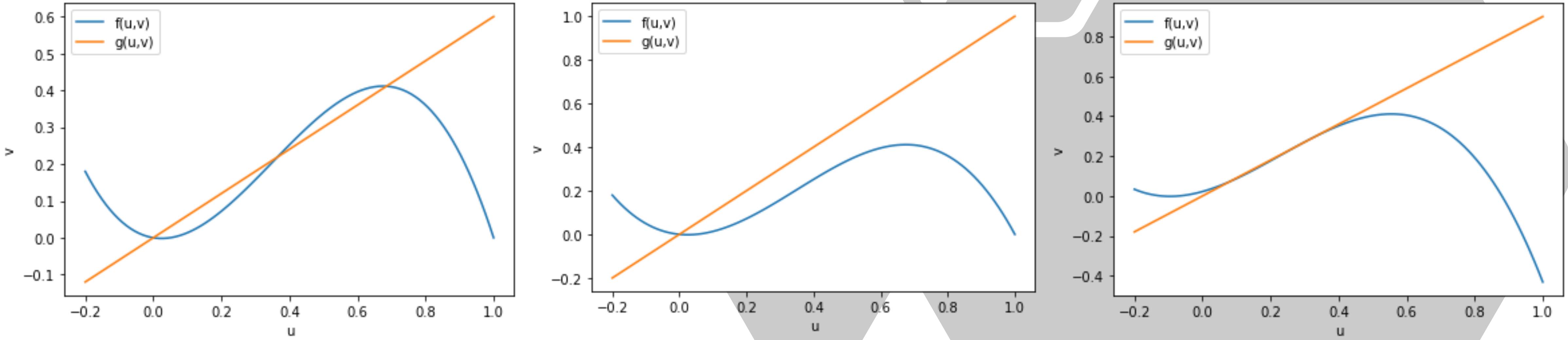
$$f(u,v) = -u(u-1)(u-a) - v,$$
$$g(u,v) = ku - v$$

Types of models do describe the heart rhythm



u(t)
v(t)
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0
1
2
3
4
5
6
7
t

Dynamics and stability of FHN



Ionic models

Ionic models accurately reproduce most of the basic properties of cardiac tissue. These include the depolarization and repolarization phases of the action potential, restitution properties, dynamical changes in ionic concentration, etc.

The Noble model

$$C_m \frac{dV}{dt} = (I_K + I_{Na} + I_{An})$$

Sodium current

$$I_{Na} = (400000 m^3 h + 140)(V - E_{Na})$$

A current of anions chloride

$$I_{An} = 75(V - E_{An})$$

Types of models do describe the heart rhythm

Ionic models

Potassium current

$$I_{K1} = (1200\exp\left(\frac{-V-90}{50}\right) + 15\exp\left(\frac{V+90}{60}\right))(V-E_K)$$

$$I_{K2} = 1200n^4(V-E_K)$$

Mechanical models

Compared to existing models, despite the complexity, mechanical models have proven their worth in simulating the heart.

Excitation-contraction coupling model

Types of models do describe the heart rhythm

Numerical description of 1D Aliev-Panfilov model

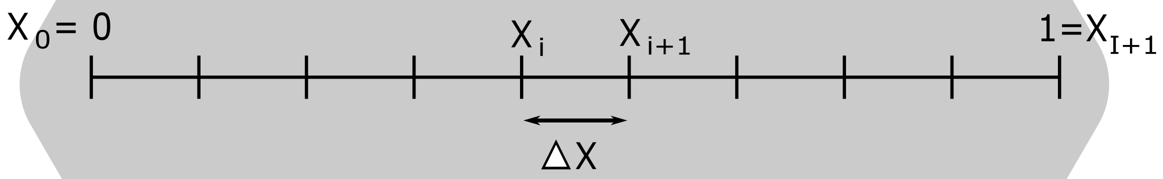
Boundary conditions

$$\partial_x u(0) = 0, \quad \partial_x u(1) = 0$$

Initial conditions

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x)$$

$$\begin{cases} \epsilon \dfrac{u_j^{n+1} - u_j^n}{\triangle t} = \dfrac{\epsilon^2 d}{\triangle x^2} A u^n + f(u_j^n, v_j^n) \\ \dfrac{v_j^{n+1} - v_j^n}{\triangle t} = g(u_j^n, v_j^n) \end{cases}$$



Discretization of 1D space

Numerical description of 2D Aliev-Panfilov model

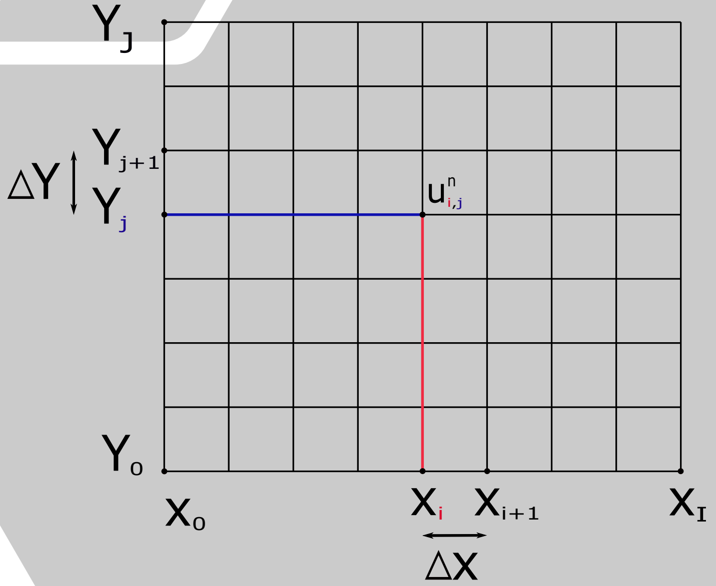
For $1 < i < I + 1$ and $1 < j < J + 1$

$$\begin{cases} \epsilon \dfrac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\triangle t} = \epsilon^2 d \dfrac{u_{i-1,j}^{n} - 2u_{i,j}^{n} + u_{i+1,j}^{n}}{\triangle x} + \epsilon^2 d \dfrac{u_{i,j-1}^{n} - 2u_{i,j}^{n} + u_{i,j+1}^{n}}{\triangle y} + f(u_{i,j}^{n}, v_{i,j}^{n}) \\ \dfrac{v_{i,j}^{n+1} - v_{i,j}^{n}}{\triangle t} = g(u_{i,j}^{n}, v_{i,j}^{n}) \end{cases}$$

For $i = 1$ and $j = 1$

or $i = I + 1$ and $j = J + 1$

$$\begin{cases} \epsilon \dfrac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\triangle t} = \epsilon^2 d \dfrac{u_{i+1,j}^{n} - u_{i,j}^{n}}{\triangle x} + \epsilon^2 d \dfrac{u_{i,j+1}^{n} - u_{i,j}^{n}}{\triangle y} + f(u_{i,j}^{n}, v_{i,j}^{n}) \\ \dfrac{v_{i,j}^{n+1} - v_{i,j}^{n}}{\triangle t} = g(u_{i,j}^{n}, v_{i,j}^{n}) \end{cases}$$



Discretization of 2D space

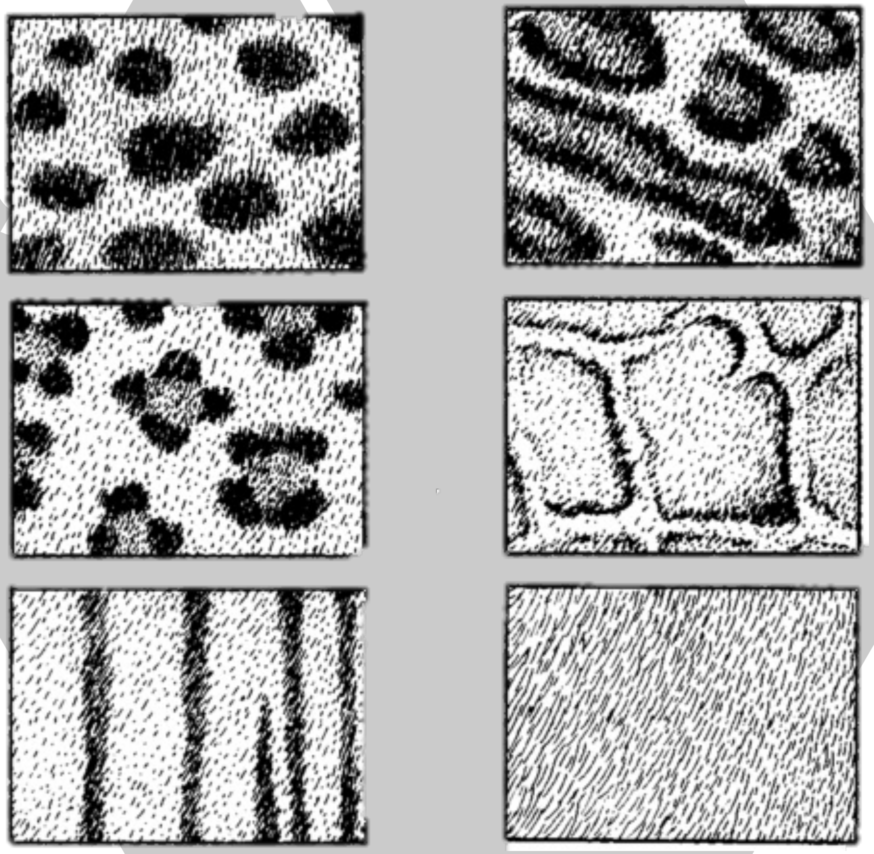
Reaction-Diffusion Equations

$$\partial_t u = D \triangle u + f(u)$$

$$\begin{bmatrix} \text{Time rate of} \\ \text{change} \\ \text{of concentration} \\ \text{of chemical component} \end{bmatrix} = \begin{bmatrix} \text{Change in} \\ \text{component} \\ \text{due to} \\ \text{diffusion} \end{bmatrix} + \begin{bmatrix} \text{Rate of formation} & & \text{Rate of} \\ \text{of component} & - & \text{cosumption} \\ & & \text{of component} \end{bmatrix}$$

Reaction-diffusion equations and the spots

$$\begin{cases} \partial_t u = \Delta u + f(u,v) \\ \partial_t v = d\Delta v + g(u,v), \end{cases}$$



$$f(u,v) = \gamma\left(a - u - \frac{\rho u v}{1 + u + K u^2}\right)$$

$$g(u,v) = \gamma\left(\alpha(b - v) - \frac{\rho u v}{1 + u + K u^2}\right)$$

Conclusions

- Got an understanding of interdisciplinary cooperation works, understood how near mathematics and the real world problems are;
- Was able to got an overview of an existing models and make up a clear vision of modeling of the heartbeat;
- Had an opportunity to improve my programming skills, using the knowledge I got this year from Approximation of PDEs course.