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An Essay On The Concept Of
Collective Rationality

by

Jerry W. Gustafson

A dissertation submitted to
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## Abstract

This dissertation is a critical review of litera-
ture which has sought the meaning of rationality in the
context of collective choice. Although the concept of
collective rationality is central to choice theory and
logically prerequisite to policy formation, adequate
definition is lacking. The purpose of this study is
to clarify some leading problems encountered in the
attempt to definitively interpret this concept. The
approach is to propose two conditions for rationality
of social choice and to assess the possibility of
satisfying them in light of results achieved in the
literature.

The conditions are that individual preferences must
acceptably aggregate to a social ordering of social
states and that a decision procedure must exist which
reliably selects the optimum state. The consistency
of a choice mechanism does not imply reliability if
individual choice is made under uncertainty or if
social choice proceeds by a sequence of decisions on
separate issues, particularly when the issues are
interdependent. Therefore, consistency of social choices
bears no normative significance, and must be distin-
guished from transitivity of the social preference struc-
ture, which does.

ii

Market behavior does not permit inference of a social preference ordering. Market rules become especially unreliable in the presence of second-best problems, externality, and public goods. Non-market choice procedures must supplement market rules in order both to identify and achieve an optimum.

Arrow has questioned the consistency of any choice system which satisfies several value-conditions obeyed by market rules. His theorem is applicable to rules for aggregation of choice orderings, preference orderings of either social states or decision rules, and to the consistency of decision procedures. Evaluation of the theorem requires recognition that the ethical validity of the value-conditions varies with the particular application involved.

The independence of irrelevant alternatives may be sacrificed by allowing vote-trading to account for preference intensity. But vote-trading is possible only when choice behavior may not reflect true preferences; the consistency of decision so achieved is normatively ambiguous.

The condition of unrestricted domain may be weakened by introducing conditions sufficient for a social ordering which require unanimous agreement upon both the criteria for evaluating outcomes and the natural ordering of alternatives in terms of the criteria, and severe

iii

restrictions on tastes. Such conditions have little empirical basis. Neither does the mere existence of a social ordering guarantee reliability of the choice process.

Satisfaction of the rationality conditions appears unlikely. Collective rationality might alternatively be interpreted as the use of decision rules which reliably account for such consensus as does exist, even in the absence of a social ordering of states. Vote-trading might be allowed to generate consistency on issues lacking consensus so long as decisions so achieved are consistent with outcomes reached by consensus. Thus different value-conditions may be appropriately relaxed at different stages of the decision process; one condition need not be sacrificed for decisions of all types. If such a procedure, evaluated in terms of its properties, commands sufficient consensus, Arrow's theorem is solved at the level of constitutional choice and conflicts as to outcomes achieved by the process may be taken as optimally resolved.

## Acknowledgments

I wish to thank Professors Edwin S. Mills, Allan Kirman and William H. Oakland for their patient supervision of this study. I benefitted greatly from numerous conversations with them and their criticisms of earlier drafts were not only helpful but essential. I also greatly appreciated the substantial encouragement offered by Professor Charles Blackorby during the formative stages of the paper.

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v

## Table of Contents

vi

## Table of Figures

# CHAPTER 1

## Introduction

An extensive literature has developed recently which has attempted to elucidate both the mechanisms by which societies may determine standards for evaluation of social states and those which may be utilized to effect institutional changes necessary to alter social states in desirable ways. The social state which actually obtains at any given time may be considered to be the result of the interaction of decisions made by citizens in various institutional contexts such as markets, voting systems, and complex organizations. That is, the actual social state of affairs can be conceived as emerging from an extremely complex process of individual choice behavior. The literature which has investigated the manifestations of the relation between structures of individual decision and emergent properties of social states, therefore, has been categorized with the rubric "the theory of social choice."

The social choice literature at present, however, is not in an entirely coherent state. It consists of a number of works by authors having various disciplinary backgrounds and differing interpretations of and approaches to problems involving social choice-making. Their efforts are loosely related insofar as they have

the common theme of investigation of ways in which
social outcomes are attached to particular patterns of
choice behavior.  But the scope of such an inquiry must
necessarily be broad and the contributions made have
dealt with such widely different issues as discovery of
sufficient conditions for aggregation of individual
preferences, properties of voting systems, the relation
of market choice to democratic theory, the relation of
the theory of groups to collective decision, and perhaps
most familiar, the relation of the theory of externality
and public goods to proper formulation of economic pol-
icy.  The body of theory dealing with the crucial in-
vestigation of processes of social decision and their
associated normative properties, thus, exists in a
deeply fragmented state.

The purpose of this paper is to review, interpret,
and criticize some leading contributions to the social
choice literature and, in so doing, to synthesize and
in some cases, reformulate, the results of others.  My
intention is that this overview might add some greater
measure of coherence and logical integrity to the
literature than currently exists.  As a discipline
reaches a certain stage of maturity, it is appropriate
that attempts be made to re-evaluate earlier literature
and review the fundamental issues to which the disci-
pline is addressed.  In a field such as social choice,

where mutual understanding is made difficult by the wide variety of possible approaches, it seems especially appropriate that such re-evaluation occur concurrently with further creative analysis. Some writers in their haste to achieve new analytical results, have neglected either to discuss carefully the exact nature of the problems which they have addressed or to interpret the significance of their findings in the context of previous contributions. I attempt, at least partially, to fill that gap, as its existence has led to unnecessary disputation.

As my interest is in the attempt to synthesize existing literature, I have not tried to achieve original analytical results. In the final chapter I suggest a somewhat novel formulation of the problem of social choice, but even there my remarks are intended as conjectural and serve mainly to outline a potentially profitable direction for future work. The pursuit of matters along the lines suggested is beyond the scope of the present inquiry which is intended to ferret out old confusions rather than to raise new ones.

This review is not a comprehensive bibliographical essay, however. It is focused upon a central issue which recurs throughout the literature. The issue is that of what meaning can be ascribed to the concept of rationality in collective choice. I have discussed

works which seemed particularly to bear upon this topic.
I have not intended to include all relevant literature;
however, I have intended to include a widely represen-
tative sampling of opinion concerning the nature of
collective rationality. Of course, this selection has
been made by one with training in economics and is
inevitably biased. Social choice theory is by nature
interdisciplinary but I have tried not to stray unduly
from the boundaries of welfare economics. For example,
no account is taken of theories of administrative
behavior in complex organizations or of statistical
decision theory.

The paper is focused upon the concept of rationality
of social decision for two reasons. First, the notion
of rationality seems to me to be the keystone of the
theory of social choice. It therefore provides a
unifying theme for drawing together a body of literature
some of whose separate elements seem only vaguely rela-
ted. More important, if social science is to be used
to inform social policy, one of its central tasks must
be to determine how social arrangements can be evalua-
ted and how desirable arrangements may be implemented.
If rationality of collectivities is to be loosely
interpreted as the maximization of specified desiderata
under certain constraints, then the formation of guides
to social action require careful investigation of the

desiderata and constraints; that is, precise under-
standing of the meaning of collective rationality is
prerequisite to adequate policy formation. The social
choice literature has not arrived at such an understand-
ing. It is, as I will show below, rife with confusion
about the very structure of the problem of rational
social choice. The basic outlines of that structure,
however, have been generated in economic welfare theory.
A brief glance at the origins of the problem in welfare
theory will serve as further motivation for the main
body of the study.

Perhaps the central theme in economics since its
inception has been the attempt to discover a trustworthy
guide to rational social action. Normative economic
theory has not succeeded, however, in finding a set of
institutional arrangements which would, if established,
permit the attainment of optimal welfare. The actual
prescription for policy has necessitated the inclusion
of best judgment along with economic analysis.

Beginning with Robbins[1] and culminating with
Bergson's compact 1938 statement,[2] normative economists

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[1] Lionel Robbins, An Essay on the Nature and Signi-
ficance of Economic Science, 2nd Edition (London,
McMillan, 1935).

[2] Abram Bergson [Burk], "A Reformulation of Certain
Aspects of Welfare Economics," Quarterly Journal of
Economics, LII (Feb., 1938).

have tried to reduce to a minimum the role of sheer
value judgment in the recommendation of policy. The
thirties were marked by a flurry of interest in extend-
ing the value judgment of Pareto to encompass situations
where some persons might be losers from a change in
which others gained. The use of compensation criteria
as devised by Kaldor,[3] Hicks,[4] and Scitovsky[5] would
have allowed policy to be recommended without recourse
to value judgments other than the Pareto principle,
but as Samuelson[6] and Graaff[7] soon demonstrated, this
attempt failed. These last interpretations of Bergson's
earlier work showed that the compensation argument had
not properly accounted for distributional considera-
tions; when considering a change in the economic envi-
ronment, the initial environment together with all
possible distributions under it must be compared with
the changed environment together with all conceivable

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[3]N. Kaldor, "Welfare Propositions of Economics and
Interpersonal Comparisons of Utility," Economic Journal,
XLIX (Sept., 1938).

[4]John Hicks, "The Foundations of Welfare Economics,"
Economic Journal, XLIX (Dec., 1939).

[5]Tibor Scitovsky, "A Note on Welfare Propositions
in Economics," Review of Economic Studies, IX (1941-
1942).

[6]Paul Samuelson, Foundations of Economic Analysis
(Cambridge, Harvard University Press, 1947), Chapter 8.

[7]J. de V. Graaff, Theoretical Welfare Economics
(Cambridge, Eng., Cambridge University Press, 1967).

distributions. The compensation controversy culminated
in Samuelson's concept of "utility possibility frontier"
which, in the absence of a social welfare function,
based on an additional explicit value judgment as to
distribution, allowed the economist only to state the
"potential benefit" of changes which shift this frontier
outward along all its axes.

This rule left economics with no workable standard
for recommending change and led to Graaff's nihilistic
injunction that since no policy prescriptions may be
made in the absence of a social welfare function, econ-
omists should leave the area of normative theory and
concentrate on positive analysis.[8]

Economists of a practical persuasion were once more
left to their own judgmental devices in assessing the
desirability of proposed changes. Since only the Pareto
criterion was available, it was expedient to separate
the allocational and distributional problems entirely.
It seemed advisable to endorse changes which would lead
society to its transformation locus and to consign
distributional matters to whatever forces are responsible
for them. Additionally, economics could emphasize means
of enlarging the national product, thus making possible
increases in "potential welfare." In any case, the

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[8]_Ibid._, p. 179.

distributional judgment would be provisionally ignored;[9] welfare theory would render unto political science what belonged to political science.

This pragmatic approach also incorporated the Pigovian approach to problems of market failure. The primary reliance on the market mechanism could be supplemented by tax-subsidy schemes designed to account for differences in private and social cost and benefit where common sense dictates such policies are appropriate. Additionally, cost-benefit analysis could be used to order social priorities and increase the rationality of public expenditure.

But this pragmatic position is rather disquieting. "Common sense" has been an untrustworthy ally of science in the past. If criteria for public action exist, it should be possible to determine them logically. Indeed, the pragmatic approach has been seriously questioned by recent contributions to welfare theory by Coase,[10]

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[9]This ignores the problem of evaluating the size of output independently of distribution. "A satisfactory theory of welfare . . . must dispense with the time-honored device of drawing a distinction between the size and the distribution of the national income and saying that welfare depends upon them both. It depends on size only--and we do not know what the size is until we know the distribution." Ibid., p. 92.

[10]Ronald Coase, "The Problem of Social Cost," Journal of Law and Economics, III (Oct., 1960).

Buchanan and Kafoglis,[11] Baumol,[12], and Lancaster and Lipsey,[13] among others. These contributions serve to make more ambiguous the correct rules to follow in suggesting social improvements; they indicate that "common-sense" notions may be dangerously inadequate.

These contributions are within the mainstream of normative economic analysis. They raise the fundamental question of whether there exists a procedure for decision which might allow societies to make rational adjustments in their economic arrangements. Other major contributions have been made which consist essentially in the explicit introduction of political variables into welfare models. The purpose of this approach is to investigate whether a complete description of optimal social arrangements might emerge from such a widened model of social choice. The examination of political manifestations of choice models followed mainly from the Bergson formulation which emphasized the role of the social welfare function in normative theory. When

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[11] James M. Buchanan and Milton Kafoglis, "A Note on Public Goods Supply," *American Economic Review*, LIII (June, 1963).

[12] William Baumol, "External Economies and the Second-Order Optimality Conditions," *American Economic Review*, LIV (June, 1964).

[13] Kelvin Lancaster and Richard Lipsey, "The General Theory of the Second Best," *Review of Economic Studies*, XXIV(1) (1956).

it was clearly understood that a set of welfare judgments must be made in order to define an optimum and that nothing in positive economics could determine those judgments, it was logical to explore other means of making them. Since under democratic values a definition of an optimal social state must result from a process of political choice, choice theorists turned to an investigation of that process in order to attempt to complete the system which pure welfare economics had carried so far.

One of the early results achieved by this approach was Arrow's finding that no method of aggregation of individual orderings of social states existed which was consistent with several intuitively persuasive conditions. The conclusion was widely drawn from Arrow's work that it is impossible for political choice methods to supplement market procedures in such a way as to render social choices normatively unambiguous. The implication of this conclusion is that groups are incapable of collective rationality if this concept is taken as meaning the ability of groups to make decisions which correspond to a social ordering of the alternatives for choice.

Such a conclusion is scarcely acceptable unless one is willing to believe in the impossibility of developing a normative basis for policy recommendation. But in

spite of the great effort of many to surmount the serious difficulty that Arrow's theorem raises, an adequate solution has not been found. Arrow's contribution still occupies the central place in the social choice litera- ture, therefore, and his work will be given great empha- sis in the chapters that follow.

I have organized the discussion of topics in the order with which they unfold in theoretical welfare economics. Chapter 2 attempts to specify with some rigor the problem of rational collective choice. Chapter 3 consists of a review of formal welfare econo- mics which emphasizes the role of the social welfare function and the impact of interdependence upon tradi- tional rules of policy formation. In Chapter 4, Arrow's General Possibility Theorem is raised as a logical out- growth of the formal economics literature and four distinct interpretations of its significance are suggested. Chapter 5 relates Arrow's theorem to the concept of the social welfare function and examines the possibility of achieving a social welfare function by admitting revealed preference intensity in non-market choice. Chapter 6 examines the possibility of achieving a social welfare function by relying upon the similari- ty of individual preferences. Chapter 7 summarizes conclusions from previous chapters concerning rational methods of social choice, suggests an unexplored means

of achieving reliability of social choice and closes
with a brief summary of recent literature which empha-
sizes the search for properties of decision techniques
rather than preference aggregation procedures.

CHAPTER 2

The Problem of Rational Collective Choice

A central thrust in modern economic theory has been
to set out axioms on personal preference and choice
from which empirically verifiable behavior may be
deduced.  Generally speaking, these axioms specify the
existence of some field of alternatives, the nature of
the individual's preferences, the constraints upon his
behavior, and that the individual will choose the most
preferred attainable alternative.  It is customary to
refer to behavior consistent with the axioms as
"rational."  Rational personal behavior, therefore,
implies the choice of the best alternative given all
constraints and rationality is defined strictly with
reference to the set of axioms.

The pattern of individual decisions, both through
markets and other group decision procedures, such as
voting, results in a social outcome.  It is often useful
to conceive of the social outcome as the result of a
collective choice.  The question arises whether rational
collective behavior may be defined in a manner analogous
to the individual case.  Without exception, modern
writers on social choice theory have considered that
collective behavior results from particular aggregation
procedures on individual decisions.  Therefore, rather

than specifying axioms for collective choice, these writers have tended to specify conditions which must be satisfied by the aggregation procedure. These conditions are "reasonable" in the sense that they are either met by known actual aggregation rules or receive wide support on ethical grounds, and they customarily include some rationality postulate, such as that the procedure must yield a complete, reflexive, and transitive social preference relation. Thus, the term "collective rationality" is given content in the context of the particular conditions placed upon rules of aggregation of individual choices. Rationality is not a property of collective decision which may be defined a priori; the meaning of "rational social choice" must ultimately be found in some particular theoretical setting.

It is important to recognize this point from the outset, for the literature concerned with social choice has tended to focus upon conditions specified for the aggregation procedure which are mutually inconsistent. Since these constraints are inconsistent, it is impossible for processes of actual group choice to be consistent with them. Rational social behavior cannot be defined in the context of such a set of conditions. To assert otherwise would be to hold that rational social behavior is a logical impossibility. This conclusion is certainly inappropriate if "rationality" is to imply

the selection of the best alternative taking all con-
straints into account. One task of choice theory is to
arrive at a consistent set of restrictions on the
aggregation procedure which allow a meaningful interpre-
tation of the concept of collective rationality. The
history of social choice theory may be viewed as an
attempt to find such a set of restrictions.

A full discussion of particular conditions which
have received much attention in the literature will be
made in Chapters 3-5 below. It will be apparent that no
widely acceptable set of consistent restrictions on
aggregation procedures has yet been found. In order to
begin to clarify some of the issues which arise in the
course of passing from individual preferences to social
outcomes, it is suitable to proceed heuristically by
stating two conditions which are intended to embody the
intuitive meaning of the term "rationality." The
approach will be to accept these conditions provisionally
and to investigate their compatibility with other
reasonable conditions often specified for aggregation
procedures. The property of collective rationality will
be defined by these conditions. But it is implicit that
if these should be inconsistent with other restrictions
which must ultimately be obeyed by aggregation proce-
dures, they must be modified or discarded.

The purpose of the present chapter is to set out

the two conditions and to discuss their implications for
the nature of the problem of social choice. The follow-
ing concepts and assumptions will be employed through-
out the paper:

1. There exists a finite field of alternatives, X,
   from which society must choose some alternative,
   x.

2. Society consists of n individuals, all of whom
   possess orderings over X.

3. A binary relation, R (is at least as good as).
   R is complete and reflexive, where
      Completeness: For all (x,y) in X, either
         xRy or yRx.
      Reflexiveness: For all x in X, xRx.
   If R is also transitive, R is an ordering, where
      Transitivity: For all (x,y,z) in X, xRy
         and yRz implies xRz.

4. A binary relation P (strictly preferred to).
   P is irreflexive and asymmetric, where
      Irreflexiveness: For all x in X, not (xPx)
      Asymmetry: For all (x,y) in X, if xPy, then
         not (yPx)
   If P is also complete and transitive, P is a
   strong ordering.

5. A binary relation I (indifference). I is
   reflexive, symmetric, and transitive.

6. A binary relation C (is chosen over). C is
   irreflexive and asymmetric. If C is also
   complete and transitive, it is a choice order-
   ing.

7. An aggregation procedure is a rule which selects
   some alternative from X given a configuration

of individual orderings:

$$R = F(R_1, R_2, \ldots, R_n), \text{ where } R_i \text{ indicates}$$
the ordering of the ith individual of the
$x_i$ in X.

## A. The Nature of the Problem of Collective Choice.

Suppose that a society of individuals must choose among several possible alternative social states. Each of the individuals has an ordering of the alternatives based upon whatever criteria he deems appropriate. This society faces the problem of choosing the best feasible alternative. Such a choice raises two distinct problems. First, the notion of best social state must be defined unambiguously. This issue is entirely normative. It must be possible, conceptually, for a neutral observer to identify some social state as optimal, taking only the individual preferences as data. It is assumed that there is no standard by which social states may be ranked apart from individual attitudes toward them. Second, given that an optimal state is identified, it must be that the actual choice mechanisms employed by the society give rise to that optimal state. This issue is essentially positive; it entails a knowledge of the properties of various procedures of group choice. Much confusion in the social choice literature stems from a failure to distinguish carefully between these two problems.

An examination of each of these problems leads to a plausible rationality condition for collective choice. Let us take them in turn.

The problem of specification of an optimal social state has been of extreme interest to welfare theorists; it is the central issue in the vast literature dealing with social welfare functions. To make a judgment between alternatives, it is required that there be some rule for determining a social preference from any configuration of individual preferences $(R_1,\ldots,R_n)$. There are, of course, many such rules. For example, the rule might simply be to let the social preference be identical with that of the first individual for any pair of alternatives $(x,y)$. It is widely desirable, however, to have some rule which will take into account the preferences of all individuals. Then a rule for transforming individual preferences into a social preference will be some procedure of aggregating individual preferences such as voting.[1] It is well known, however, that for some configurations of individual preferences, voting rules will lead to normatively ambiguous results. An example is given by the paradox of voting. Suppose individuals A, B, and C order alternatives x, y, and z,

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[1] Although "voting" in this context means a purely hypothetical operation and not actual choice behavior. The reason for this distinction will be made clear below.

in the following fashion:

$$xP_a yP_a z$$
$$yP_b zP_b x$$
$$zP_c xP_c y$$

Then, if the aggregation procedure is majority rule, for society, xPy and yPz but zPx. The social preference relation does not possess the transitivity property; there is no social ordering. It is not possible to say which outcome is socially desirable. In order to say that some outcome should be attained, it must be that that outcome can gain a majority over each of the others. For the configuration of individual preferences in the example, the procedure of majority rule cannot give rise to meaningful social normative comparison between alternatives.

Such comparison could be made, however, if the aggregation procedure did yield a social ordering. The ability to make such comparison, to have some standard by which it can be said that society should achieve some outcome and not some other, is clearly involved in the notion of rationality. The social choice literature has generally adopted the transitivity of social preference as a condition for rationality and has sought to find aggregation procedures which might yield social orderings. Arrow was the first to view the problem of collective rationality as the derivation of a transitive social

ordering from any admissible individual orderings in some acceptable way.[2]  His generalization of the voting paradox demonstrated that no voting rule will generate a social ordering for all conceivable patterns of individual preference while also satisfying other reasonable restrictions.  This result still serves as the focal point in the literature.

Since the interpretation of collective rationality as the internal consistency of social preference is so widely used and since it seems intuitively compelling that rationality can have meaning only if there is some means by which society may discriminate normatively between alternatives, let us adopt the following rationality condition, which, for convenience, may be called the principle of internal consistency.

Internal consistency:  An aggregation procedure for individual preferences which yields a social ordering must exist.

The literature which has followed Arrow's approach to social welfare functions has been so deeply concerned with finding conditions for aggregation procedures which imply internal consistency that it creates the impression that this is the only problem of rationality of collective choice.  This impression is misleading.  In the

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[2]Kenneth Arrow, _Social Choice and Individual Values_, 2nd Ed., (New York, Wiley, 1963), p. 98 and _passim_.

first place, the existence of a social ordering is slightly stronger than necessary to insure that an optimal social state may be identified. More important- ly, the solution to the problem of identification of an optimal social state is not directly concerned with actual mechanisms of choice and does not, of itself, guarantee that the optimal state will, in fact, be chosen.

That full transitivity of the social ordering is not necessary to find an optimal state is illustrated by the following simple example: suppose there are four alternatives and three individuals and that the indi- vidual orderings are

          A:  wRxPyPz
          B:  wRyPzPx
          C:  wRzPxPy

In order to define an optimal social state, it is necessary to find one (or more) alternative(s) which is socially at least as good as any others in X. This group does not possess a social ordering for xPy, yPz but zPx. Nevertheless, each individual considers w to be at least as good as any other alternative, and an aggregation rule such as majority decision would indi- cate w as the socially preferred alternative.[3]  Generali-

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[3]The fact that the existence of a social ordering is not necessary for the existence of a choice set has been proved and developed by P. K. Pattanaik. See

zing, we may define the <u>choice set</u>, C(S) of a set of alternatives S in X as that subset of alternatives which are at least as good as any others in S:[4]

   C(S) = all x in X such that for all y in S,

      xRy.

   For social choice to bear normative significance, a non-empty choice set must exist for any subset of alternatives. Then the following rationality condition might be substituted for the principle of internal consistency:

   An aggregation procedure for individual preferences which yields a non-empty C(S) for any S in X must exist.

   This condition also stipulates only that there must be some basis in individual preferences for society to do one thing rather than some other. It is obvious from the definition of an ordering that a social ordering will guarantee the existence of a choice set. But the set of restrictions placed on aggregation procedures necessary to yield a choice set are weaker than those

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"A Note on Democratic Decision and the Existence of Choice Sets," <u>Review of Economic Studies</u>, XXXV(1) (Jan., 1968).

[4]C(S) indicates the set of alternatives from which a social choice should be made, not the set from which a choice must be made. The term "preference set" would be better, perhaps, but since the notation C(S) is commonly used, it is advisable to maintain that usage.

necessary to yield an ordering. Most of the literature to be discussed in later chapters has been concerned with the existence of a social ordering, however, and it will simplify that discussion if the stronger rationality condition is maintained, although we bear in mind that that condition may be weakened somewhat, if necessary.

The first condition, however, does not fully embody the intuitive notion of rationality. The existence of an aggregation procedure which gives rise to a social ordering does not insure that there is a social choice mechanism which behaves consistently with that ordering. Social choice is the outcome of configurations of individual behavior which take place in various different contexts and which may vary with respect to changes in institutional circumstances. Properties of actual group decision procedures must be investigated to find when individual behavior will lead to the optimal social outcome. Let us define a social decision procedure (SDP) as a rule

$$C = F(C_1, \ldots, C_n)$$

where C indicates a social outcome derived from the pattern of individual choices $(C_1, \ldots, C_n)$. Then we may define the property of reliability:

An SDP is reliable if and only if $F(C_1, \ldots, C_n)$ implies xCy for all $y$ in X when $F(R_1, \ldots, R_n)$ implies xRy for all $y$ in X.

With this definition, the second rationality condition may be stated:

> The SDP employed by society must be reliable.

This condition stipulates only that if there is a socially preferred alternative within some set, then that alternative must be chosen. To suggest that a society is collectively rational only if it chooses its most preferred alternative would appear to raise little controversy. It is worth noting that in the economic theory of individual choice, reliability is explicitly assumed. The variables of choice (levels of consumption of various commodities) are subject to the control of the individual consumer. He allocates his budget so as to maximize his own utility function. But it is improper to make the analogous assumption that the existence of social preferences must imply the proper choices. Social choice results from a pattern of individual behaviors which are motivated by individual, rather than by social, preference orderings. The variables of choice are not subject to the control of some social entity which can choose the proper outcome. In any situation of collective choice, it is necessary to determine explicitly that the pattern of privately motivated behavior is consistent with a social optimum, if one exists.

B.  Some Examples of Unreliable Decision Procedures.

The matter of reliability has received almost no attention in the social choice literature. Most writers have apparently assumed that reliability is implied by internal consistency. There is a considerable body of writing which is concerned essentially with the properties of group decision procedures.[5] But that literature has mainly focused upon the social choice relation, C. In particular, these writers have sought an SDP which will yield a transitive C, and they have suggested that if a choice ordering for society can be found, the problem of rational collective choice is solved.[6] But it is possible that an SDP may yield a transitive C and still be unreliable. Theorists interested in non-market decision making have tended to confuse the transitivity of C, which ordinarily will not have normative significance, with the transitivity of R, which does. This issue will be clarified in Chapters 4 and 5.

Theorists concerned with social welfare functions, on the other hand, have followed Arrow in assuming that

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[5]Some leading examples are James M. Buchanan and Gordon Tullock, The Calculus of Consent (Ann Arbor, University of Michigan Press, 19620; Tullock, "The General Irrelevance of the General Impossibility Theorem," Quarterly Journal of Economics, LXXXI (May, 1967); James Coleman, "The Possibility of a Social Welfare Function," American Economic Review, LVI (Dec., 1966).

[6]See Coleman, "The Possibility of a Iocial Welfare Function," in particular.

social choice and social preference are identical.
Arrow is explicit on this issue:

> I prefer to locate (social values) in the
> actions taken by society through its rules
> for making social decisions. This position
> is a natural extension of the ordinalist
> view of values; just as it identifies values
> and choices for the individual, so I regard
> social values as meaning nothing more than
> social choices.[7]

If it is assumed to be true that the choice relation
and preference relation yield identical orderings for
individuals and if it is also assumed that the aggrega-
tion procedure for preferences is identical to the social
decision procedure, then if the individual preferences
aggregate to a social ordering, the optimal outcome will
be identical with the actual social choice. The problem
of reliability will not arise. But such assumptions do
not take realistic account of the possibilities of actual
choice mechanisms. In many situations, an individual's
choice behavior may not be predicted from knowledge of
his preference ordering over alternatives. In those
situations, social choice may not coincide with the
social preference.

For example, suppose three individuals, A, B, and
C are to vote directly on three distinct and completely

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[7]Arrow, _Social Choice_, p. 106. This statement,
however, is inconsistent with other positions he adopts
and does not bear heavily on the importance of his
general possibility theorem in any event.

defined social states, x, y, and z. The preference orderings of A, B, and C may be of such a nature that they do aggregate to a social preference. Let the orderings be

A:  x y z

B:  y z x

C:  z y x

If the aggregation procedure for preferences is majority rule, a social ordering y P z P x is generated. The social choice should be y, but it may not be. Each individual knows that the outcome from voting will depend upon others' votes as well as his own. If he is not sure how others are to vote, then his actual choice behavior may be assumed to take place under uncertainty and it might be appropriate for one or more of the individuals to employ a strategy of voting inconsistently with his own preferences in order to increase the likelihood of an outcome favorable to himself. Preference orderings and choice orderings may not be identical for individuals if choice is under uncertainty, and therefore the social choice may not be optimal.

Suppose that majority decision is the method of actually choosing one of the alternative social states; the social choice is an alternative which is chosen by a majority over each of the others. Suppose alternatives (x,y) are voted on first. C might believe (correctly)

that his most preferred alternative, z, would more likely defeat x than y. His expected utility of a vote for x might exceed that of a vote for y. He therefore violates his preferences and votes insincerely for x, which is, then, chosen by a majority over y. Then in a vote between (x,z), z will win. But between (y,z), y will win.[8] None of the alternatives can gain a majority over both of the others; the vote will cycle as long as C votes insincerely between (x,y).

Actual rules of procedure, however, often specify that only two votes will be taken between three alternatives. The winner of the first pair is compared to the third alternative, and the winner of that pair is accepted as the social choice. If this procedure is applied to the example, the final outcome will depend on the order in which the votes are taken. If (x,y) is the first vote, z will be the outcome even though y is the socially preferred alternative.

This example is most plausible in the context of decision processes of small groups. If there are many voters, then the expectation tht strategic behavior may influence the outcome of a single decision is reduced.

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[8]Note that on votes (x,z) and (y,z), A and B must vote sincerely. Between (x,z) B must be sincere or x would be the social choice; between (y,z) A must be sincere or z would be the social choice.

But actual collective decision does not usually take the form of a single choice between completely defined social states. Citizens often choose a social outcome indirectly by means of a series of choices over many separate issues. When this procedure is employed, the SDP is a rule of the form

$$C = F_1(C_1^1,\ldots,C_n^1), \; F_2(C_1^2,\ldots,C_n^2),\ldots,F_m(C_1^m,\ldots,C_n^m)$$

where the group of n individuals must choose one alternative for each of m issues. When social outcomes are achieved by such a series of choices, new possibilities for strategic behavior arise. The outcome may depend not only upon orderings of alternatives in the m issues but upon the intensity of concern over various issues. If there are at least two issues and if one individual is intensely concerned about the first and some other individual the second, then these two persons might find it mutually advantageous to "exchange" votes on the issues. The "exchange" of votes consists of a mutual agreement to vote in violation of one's preferences on some issue. Such exchanges may lead to an outcome other than the socially preferred outcome. This result can be illustrated with the following example.

Suppose that there are two choices, C and C', to be made over two issues. The rule employed for deciding each issue is that of majority decision. C and C' are

independent issues where independence is defined as
follows:

> Two issues C and C' are independent if and
> only if $F(R_1,\ldots,R_n,\ R_1',\ldots,R_n') =$
>
> $F_1(R_1,\ldots,R_n) + F_2(R_1',\ldots,R_n')$

where $(R_1^m,\ldots,R_n^m)$ are the individual orderings of alter-
natives in the mth issue. If the issues are independent
any $C_i C_i'$ may be chosen and it is possible to attain the
socially preferred outcome for both issues by making a
choice over each issue separately.

Let us denote the two issues S and S' and assume
that the orderings of three individuals A, B, and C over
the three alternatives in each issue are as follows:

|     | S |   |   | S' |    |    |
|-----|---|---|---|----|----|----|
| A:  | x | y | z | x' | y' | z' |
| B:  | y | z | x | y' | z' | x' |
| C:  | z | y | x | z' | y' | x' |

An aggregation procedure of majority rule will yield
social orderings for each issue, and since the issues
are independent, the optimum indicated by this procedure
is (y,y'). If each individual were constrained to vote
on S and S' consistently with his own (ordinal) prefer-
ences, (y,y') would be chosen. But since there are two
issues, the outcome may be affected by the intensity of
preference of the individuals over the two issues.
Suppose that A is intensely concerned about S' and that
C is intensely concerned about S. Both A and C might

consider the combination of outcomes (z,x') better than
(y,y'). If so, then A and C would agree to exchange
votes, A voting for z over x and y and C voting for x'
over z' and y'. In fact, (z,x') would not be the final
outcome, however. For B can offer a better outcome to
either A or C depending upon whether he is more con-
cerned about S or S'. If B is more concerned about S,
he can offer the combination (y,x') to A. A would accept
as he prefers (y,x') to (z,x'). If B were more concerned
about S', he can offer (z,y') to C which C would prefer
to (z,x'). The final outcome, then, will be either
(y,x') or (z,y'). The socially optimal outcome (y,y')
is preferred by a majority to either of the possible
outcomes. But (y,y') cannot be chosen in fact, because
a majority prefer (z,x') to it.

These relationships may be clarified by writing out
preferences of the individuals over all nine possible
outcomes under the assumptions that A is intensely con-
cerned about S', C about S, and B about both S and S'.
Possible orderings under these assumptions are

A: (x,x')(y,x')(z,x')(x,y')(y,y')(z,y')(x,z')(y,z')(z,z')
B: (y,y')(y,z')(z,y')(y,x')(x,y')(z,z')(z,x')(x,z')(x,x')
C: (z,z')(z,y')(z,x')(y,z')(y,y')(Y,x')(x,z')(x,y')(x,x')

One can verify by inspection that each of the coali-
tion alternatives (z,x'), (x,y'), and (y,x') are preferred
by a majority to any other alternative except (y,y').
We need only investigate orderings of these four. Among

the coalitions, the orderings are

           A:  (y,x')  (z,x')  (z,y')
           B:  (z,y') (y,x')  (z,x')
           C:  (z,y')  (z,x')  (y,x')

and the social ordering is (z,y') C (y,x') C (z,x').

Even though the social choice ordering is transitive, the optimal choice is not made. Examination of the complete preference orderings reveals that (y,y') is preferred to (z,y') by A and B, and is preferred to (y,x') by B and C. But (y,y') cannot be chosen because (z,x') is preferred to it by A and C. Thus, when social states arise from a series of independent decisions, vote trading strategies may lead individuals to violate their preferences in choosing among some alternatives thus allowing social choice to deviate from social preference.[9]

In the preceding example, it was assumed that the two issues were independent and that the group used the same decision process (majority voting) for deciding each issue. A realistic interpretation of this situation might be the case of a legislature determining the level

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[9]One may wonder in what sense (y,y') is an optimum in the above example as long as it does not dominate (z,x') as well as the other coalitions. The answer is that in aggregating preferences separately over the two independent issues, only preference order is taken into account. The orderings of coalitions take into account preference intensity as well. While (z,x') is chosen over (y,y') we would only deem it better of we were willing to make interpersonal comparisons among the voters: A's preference for x' over y' and z's preference for z over y are "more important" than B's preference for y over z and y' over z'.

of appropriation for agencies requesting public funds.

Society must sometimes decide issues which are not independent, however. As one example, decisions with respect to government fiscal policy and level of welfare transfers are interdependent; as high employment is achieved, the level of transfers might fall. If two issues are interdependent, it is not possible to find a social optimum by aggregating preferences over each issue separately. The aggregation procedure must aggregate individual orderings of pairs of alternatives, for the aggregation rule is of the form

$$R = F((R_1^1,\ldots,R_n^1),\ldots,(R_1^m,\ldots,R_n^m)) \neq$$
$$F_1(R_1^1,\ldots,R_n^1) + \ldots + F_m(R_1^m,\ldots,R_n^m)$$

Thus, the aggregation rule must treat two interdependent issues as if they were, in effect, a single issue. Furthermore, since the decision of one of the issues will affect the outcome of the other, not all pairs of alternatives are feasible. Only feasible pairs must be socially ordered.

In some situations, society may not recognize the interdependence between issues and may try to choose separately with respect to each. Under such a procedure, social choice may deviate from social preference even if individuals choose in accordance with their own preferences. The following example makes the essential

point. Consider two interdependent issues, S and S',
which consist of three alternatives each, x, y, z, and
x', y', z', respectively. Let the interdependence be of
an extreme form: x' can be chosen in S' if and only if
x is chosen in S. However, if z or y are chosen in S,
either y' or z' may be chosen in S'. Assume that the
individuals are unaware of this interdependence. The
SDP specifies that S and S' are to be determined by maj-
ority vote, in turn.

The individual orderings of A, B, and C over al-
ternatives in each of the issues are

|  | S |  |  | S' |  |  |
|---|---|---|---|---|---|---|
| A: | x | z | y | x' | y' | z' |
| B: | y | z | x | x' | y' | z' |
| C: | z | y | x | x' | y' | z' |

Since the issues are not independent, we cannot
aggregate preferences over each separately in order to
determine an optimum. We must aggregate pairs of out-
comes. The individuals might have the following order-
ings of pairs (which are consistent with the partitioned
preferences):[10]

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[10]The orderings may be assumed to be on the basis
of comprehensively defined utility functions; that is,
the arguments of individual utility include values of
justice, aesthetic appreciation, and so on, as well as
own-consumption. As constructed, the utility functions
of A and B indicate a predominate sensitivity to issue
S while that of C indicates a predominate sensitivity to
issue S'. But in no individual case are alternatives in
S or S' ordered on different bases. In particular, there

A:  (x,x')(x,y')(x,z')(z,x')(z,y')(z,z')(y,x')(y,y')(y,z')
B:  (y,x')(y,y')(y,z')(z,x')(z,y')(z,z')(x,x')(x,y')(x,z')
C:  (z,x')(y,x')(x,x')(z,y')(y,y')(x,y')(z,z')(y,z')(x,z')

By the assumption of interdependence, the alterna-
tives (y,x') and (z,x') are unfeasible. If we eliminate
these from the orderings above, we see that A and C
both consider (x,x') to be preferred to all remaining
alternatives. The optimum by majority aggregation rule
is (x,x').

The optimum will not be chosen, however, as long
as individuals do not recognize the interdependence of
the two issues and continue to divide the question. If
all individuals vote sincerely on S, the outcome of S
will be z. When making a choice from S', it is discovered
that x' is no longer feasible. The social choice will
be y' and the final outcome is (z,y'). The social choice
deviates from the social preference even though all
voting is sincere. This result is achieved because the
individual orderings which aggregate to a social pre-
ference are defined over the space of alternatives (S,
S'). But the choice from S is derived from individual
preferences over only alternatives in S. Because the
method of decision allows choice to be taken from a

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is no distinction here between orderings on the basis of
"tastes" and orderings on the basis of "values." The
nature of this distinction will be discussed in Chapter
4 below.

space of alternatives different from the space of alternatives which may aggregate to a social preference, a sub-optimal outcome is achieved.

Some added significance may be given the example by suggesting a specific interpretation of it. Let S' consist of alternative amounts of some public good which is enjoyed equally by A, B, and C. Let S consist of alternative means of financing the production of the public good; x and y are specific taxing schemes while z represents a collection of voluntary contributions. Only x, in fact, will yield sufficient revenue to produce x' amount of the good. But the individuals do not know this; they believe any desired amount may be produced whatever the method of financing. The means of financing is determined first. Then the level of production of the good will be determined subject to the (initially unknown) revenue constraint. The preferences of A, B, and C indicate (x,x') as the optimum outcome, but for the choice procedure specified, the actual outcome will be (z,y').

Of course, in the real world, the revenue to be generated by various methods is usually known. With slight modification, however, the example characterizes a wide class of actual choice situations. Social choice with respect to some issues is often implicit. That is, the outcome is not chosen directly but simply results

from the pattern of private utility maximizing behavior of individuals who behave under such constraints as exist. For example, the amount of effluent emitted into the atmosphere results from private consumption and production decisions. Problems associated with any externality or insufficiency of public good supply arise in such a context. In any case where some environmental characteristic is associated with some particular pattern of individual choices, there are two interdependent social decisions which must be made: to what degree should that characteristic be altered and what additional constraints should be placed upon individual behavior in order to achieve the desired alteration? If the social decision procedure specifies that these choices are to be made separately and if the individuals do not recognize the interdependence between them, a sub-optimal social choice may result.

Assume individual orderings are as in the previous example, but interpret S' as containing alternative amounts of some quality of environment. Let S consist of alternative sets of constraints on individual behavior. Suppose that an alternative in S will be chosen by majority rule but that the rule for choosing from S' is to accept whichever alternative results from the pattern of private behavior which occurs in response to the choice in S. Assume that private utility maximizing

will lead to x' only if x is chosen, y' only if y is
chosen, and z' only if z is chosen. The "choice" of S'
is fully determined by the choice of S, but A, B, and C
do not know this. They treat the two issues as inde-
pendent. Each individual, in effect, assumes that his
choice in S will have no impact on the outcome of S'.
Under this assumption, a majority believe they can achieve
higher utility under the set of constraints z than under
sets y or x. Then z is chosen and the social outcome
is (z,z').

The optimum could be attained if the individuals
were aware of the interdependence and accounted for it
in their choice of constraints. For example, if C
were aware that x' would result only if x were chosen,
he would realize that (z,x') and (y,x') were not possible
outcomes and accordingly, he would vote for x in S. The
knowledge of interdependence entails the rather strin-
gent requirement that C be able to forecast the behavior
of A and B under sets of constraints y and z.

The optimum outcome could not be guaranteed by mak-
ing the choice of alternatives in S' explicit. If x'
were, in fact, selected by a voting procedure, there
would still be no reason why the individuals would select
a set of constraints such that private behavior would
be consistent with it as long as they remain ignorant of
the interdependence.

It is instructive to compare this situation with the case of a social choice of an income distribution and a production set. These two issues are interdependent; the socially preferred state, in general, cannot be indicated by aggregating individual preferences over production sets and income distributions separately.[11] Suppose that individual preferences are as in the example above, but that S' represents the choice of distribution, to be selected by majority rule, while alternatives in S represent Pareto optimal production sets, which are determined by market rules. A leading feature of market choice is that it does allow decisions to be made separately with respect to these issues[12] provided that the distributional matter is settled first. If an optimal distribution, x', is chosen, prices will adjust in such a manner that society will be led to the production set x, which is consistent with the social optimum. In effect, the market "factors" the socially optimal production decision into the correct individual choices by rendering alternatives y and z unattainable with distribution x'. In this case society need not be aware of the interdependence of the two issues; even if

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[11]This point is developed in Chapter 3.

[12]Under the assumption that there are no external relations as well as other familiar restrictions. See Chapter 3.

they are decided separately the social choice will coin-
cide with the social preference. Of course, if the opti-
mum distribution is not chosen, then the resulting pro-
duction set will not be socially preferred.

In the presence of externality or public goods,
market rules will not lead to an optimum production set
even if the chosen distribution is correct. In this
case, society must make a further choice with respect
to the level of externality or production of the public
good. An explicit choice among additional constraints
on individual behavior is required so that the constrain-
ed pattern of market behavior will achieve the preferred
level. There is nothing implied by the existence of an
optimum level which will automatically lead individuals
to choose to impose a set of constraints which will
lead to the optimal outcome. The point is not that
societies cannot choose the proper constraints on pri-
vate behavior such that implicit choices are optimal; it
is only that societies may fail to do so when individuals
do not foresee the consequences of private behavior un-
der alternative sets of constraints.[13] This problem
may arise in any situation where externality or public
goods are present. In such situations, then, there is

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[13]William J. Baumol, Welfare Economics and the
Theory of the State, 2nd Ed. (Cambridge, Harvard Univer-
sity Press, 1965) has presented an extended discussion
of this matter.

a presumption that social choice may deviate from social preference.

The preceding examples are intended only to illustrate the point that the existence of a social preference among some set of alternatives does not, of itself, guarantee that social choice will be consistent with that preference. The examples are not intended to exhaust the possibilities for such deviation but only to establish that whether social choice is consistent with social preference depends upon the nature of alternatives to be chosen from and upon the particular mechanism by which choice is made. Internal consistency of social preference defined over a set of alternatives is not a sufficient condition for rational social choice. It is also necessary that the means by which the social choice is made be reliable.

## C. Summary and Implications.

The provisional conditions of collective rationality employed in this paper, then, are that society must possess an internally consistent social preference ordering of social states and it further must possess a reliable social decision procedure. With this notion of rationality in mind, we shall proceed in later chapters to review and, in some cases, to reinterpret some leading contributions to the theory of social choice.

The twofold problem of identification and choice of a social optimum may be clarified with the aid of a diagram by Newman.[14] Our goal is to generate an optimal
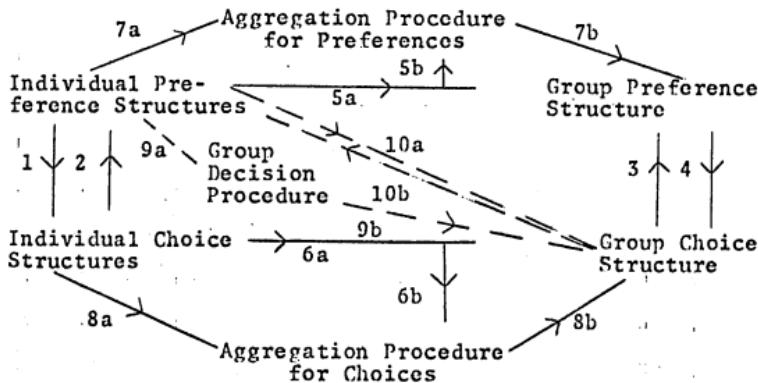


Fig. 1

A Schematic Summary of Aggregation Problems

social state given only individual preferences as data. In other words, we wish to pass from individual preference structures to group choice structures. Any set of individual preferences will imply some set of individual choices, and those choices will result in some group outcome. We want to know not only how an actual social decision is achieved, however, but also how that choice may be made in optimal fashion.

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[14]Peter Newman, The Theory of Exchange (Englewood Cliffs, N.J., Prentice-Hall, 1965), p. 174.

First, we consider the problems involved in passing from individual preferences to group preferences which will indicate the social optimum. We are given the preference structures of the individuals. In order to derive the group structure it is necessary to aggregate the individual orderings. We must move along path 7 in the diagram. But not all procedures of aggregation will yield a social ordering. For some configurations of preferences, a majority rule of aggregation, for example, will yield an intransitive group preference structure. In light of this realization, we might assume that a social ordering does exist and proceed along path 5, asking if there is any aggregation procedure which can derive that social ordering from the individual orderings in an acceptable manner. This problem, of course, is the focus of Arrow's General Possibility Theorem. The conditions of acceptability of the procedure are subject to much controversy. It is clear we must settle on some aggregation procedure, however, for otherwise no social ordering will be determined and there will be no standard by which to judge the social outcome or inform public policy.

Let us assume, therefore, that society does possess an ordering of social states as well as an aggregation method which will reveal it. Then society must choose the best social state attainable. We must pass from the

group preference structure to the group choice structure.
But we cannot do so directly by moving along path 4.
The existence of route 4 would imply that there is some
social entity (the general will, perhaps) other than
individuals which makes choices. That view is rejected
here.

Consequently, we must consider not only the aggre-
gation of individual preferences; we must also consider
the parallel problem of aggregation of individual choice
orderings to a group choice structure. Path 8 exists;
some outcome will result from a pattern of individual
choices. We wish, however, to determine the normative
significance of such an outcome. Assuming that there
is an optimal group choice, we must ask if there is an
acceptable aggregation procedure for individual choice
structures which will yield the optimal outcome. If we
seek an aggregation rule for choice structures which
yields a transitive group ordering, then Arrow's theorem
is again germane. For example, an aggregation procedure
of majority decision can yield intransitive group choices
for some sets of transitive individual orderings.
Similarly, an aggregation procedure of voluntary ex-
change guided by market rules will not yield a social
choice ordering in the absence of the prior choice of
correct income distribution.[15]

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[15]This result is implied by the intersection of

Suppose that an acceptable procedure is found for generating consistent social choices from individual choice structures. The question still remains whether the social choice thereby implied from individual choice orderings is the same as the social preference derived from individual preference structures. We would wish to know the conditions under which we might assume the existence of path 3, the inference of social preference from social choice.

Path 3 will exist, and the social choice will be optimal, if paths 1 and 2 exist. If individual orderings of alternatives are identical whether in terms of a choice or preference relation, then the reliability of social choice is logically implied. The existence of paths 1 and 2 is implicitly assumed by Arrow and by most other contributors to the theory of social choice. But as the examples given above illustrate, the identity of choice and preference may not hold if two conditions are not met.

First, the space of alternatives which are ordered by the preference relation must be defined identically to that ordered by the choice relation. If preference relations are defined on social states while choice or-

Scitovsky indifference curves as will be explained in the following chapter.

derings are defined on various component issues which
together determine a social outcome, there is no guaran-
tee that an individual's choice with respect to those
separate issues will imply his preferences over social
states.[16]  For example, if preference orderings are of
completely specified social states while choices are
ordered separately with respect to commodity distribu-
tions, on the one hand, and all other aspects of social
states on the other, the social choice in commodity space
may not turn out to be the one implied by the socially
preferred state.[17]  The correspondence between an in-
dividual's preference ordering of social states and his
choice behavior would depend upon the nature of the
issues which are separately decided, the mechanism of
choice in each case, and the degree of knowledge held
by the individual with respect to the effect of a choice

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[16] As Newman points out, "If the underlying field
of choice is not a commodity space, . . . the concepts
of preference structure and choice structure would have
to be interpreted differently . . . "Equivalence" of
preference and choice structures would have to be proved
or disproved in each particular case.  Newman, p. 174 n.

[17] Relatively little attention has been given to
the properties of group structures corresponding to
individual orderings over alternatives in different spaces.
Negishi, however, has shown that if an individualistic
welfare function is quasi-concave in utility space and
if individual preferences are convex in commodity space,
then community indifference curves will be convex in
commodity space.  See Takashi Negishi, "On Social Wel-
fare Function," Quarterly Journal of Economics, LXXVII,
(Feb., 1963), p. 156.

of some alternative in one issue upon the outcome of other issues.

Second, individual choice structures may deviate from preference structures when actual choice occurs under uncertainty. If indivuduals realize that the social outcome with respect to some features of the social state depends not only upon their own private behavior but also that of others, uncertainty is implied and individuals may not choose in accordance with their own preferences. This point was illustrated in the examples given above. Then we may not be sure that the combined paths 1 and 8 will lead to optimal social choice implied by the aggregation of individual preferences. In such a circumstance, Newman points out that social choice may not result from a procedure of aggregating choice structures at all, but from a set of rules which are intended to determine a social outcome directly from individual preferences. Newman calls such sets of rules a "group decision procedure."[18] Such procedures generally entail voting and so appear to be similar to aggregation procedures for individual orderings which also may take

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[18]Note that the definition of social decision procedure used in this paper is not the same as Newman's. As defined here, a social decision procedure includes both mechanisms of explicit social choice (Newman's group decision procedure) as well as the implicit social choice implied by an aggregation of a series of individual choices.

the form of (hypothetical) voting. But the two are different. As Newman writes[19]

> The essential aspect of a group decision procedure is that the persons involved realize that what emerges from their actions is not a series of individual choices (which would then be aggregated in some way, as in route 8) but actually a group decision. The decision procedure specifies in detail what group decision follows from what individual actions, and these rules are known in advance (or should be) to all the participants.

Several of the writers on social choice are concerned with this type of group decision procedure and are not concerned with aggregation procedures per se. Buchanan and Tullock,[20] for example, attempt to explain the logic underlying particular forms of such processes by locating the roots of constitutional decision rules in an individual utility calculus. A set of rules for decision are established by utility maximizing individuals (9a) and these rules give rise to group choice in specific situations (9b).[21]

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[19]Newman, p. 175. Such rules must imply choice under uncertainty for, since outcomes depend upon the behavior of all voters, individual voters are unsure of the effect of their votes upon the social choice, and may, therefore, attempt to maximize expected utility and occasionally vote in violation of their true preferences.

[20]Buchanan and Tullock, The Calculus of Consent.

[21]Paths 9 and 10 are shown as broken lines in Newman's diagram to suggest that these paths do not belong in exchange theory, proper. The effort of Buchanan and Tullock to derive particular group decision procedures from an individual utility calculus at the level of constitution formation suggests that analysis of group decision procedures might belong in exchange theory after all.

Whether the social outcome is derived by a group decision procedure, in Newman's sense, or by an aggregation procedure for choice structures, or by some combination of both (an SDP as defined above), we have no guarantee of reliability. The aggregation procedure will only be reliable in situations where the alternatives to be ordered are such that the choice ordering and preference ordering will be the same for individuals. If that identity does not hold, society may employ a group decision procedure in order to generate a social outcome. But there is no assurance that the particular form of the decision rules will be such that the procedure yields the social outcome which is socially preferred.

The foregoing discussion outlines in schematic form most of the problems associated with rational collective choice as defined in terms of the two rationality conditions developed above. A more satisfactory and conclusive treatment must be given in the context of specific situations. But even this abstract and informal discussion is helpful insofar as it suggests the basic questions which motivate the following chapters and allows those questions to be clearly distinguished. We wish to find the conditions under which society might be collectively rational in terms of the two conditions. The diagram suggests that collectively rational choice

entails the following dequence of problems. First we
must pass from individual preferences on social states
to group preferences on social states. If we have an
aggregation procedure which yields an internally consis-
tent social ordering, we then have an unambiguous stan-
dard by which actual social choice may be evaluated.
Second, we must pass from individual preferences to
either individual choice structures, which will be
aggregated in some fashion to an implicit social choice;
or to a group decision procedure which will yield a soc-
ial choice directly from expressions of individual pre-
ference. But it must be that the method by which the
social outcome is achieved is reliable. We must then
analyze what types of social decision procedures would
result in the optimal social choice providing that one
exists. In other words, we must inquire what restric-
tions must be placed on the form of the decision proce-
dure such that the social outcome would imply social
preference. Third, therefore, we must, in effect, pass
from the social optimum to the methods of choice which
are to be socially employed.

The remainder of this paper will consist of a
review and evaluation of social choice literature in
light of the separate problems indicated in the last
paragraph. First, developments in pure welfare economics
will be discussed. It will be shown that several problems

in welfare economics of current interest are concerned
with the matter of reliability. Then an interpretation
of Arrow's theorem will be presented. The status of the
theorem will be evaluated in each of the situations
where it is relevant. The remaining chapters will deal
with literature concerned with acceptability of various
conditions which might be placed on aggregation and
decision procedures.

## CHAPTER 3

### Market Choice and Collective Rationality

The discussion in this paper is directed toward literature on social choice which is concerned with economic welfare theory. It is therefore appropriate to begin this examination of collective rationality with a survey of aspects of welfare economics. There are two important reasons for initiating the discussion in this familiar territory. First, the body of welfare theory is well enough developed so that it clearly suggests certain tasks which must be performed by a political choice system in order for the totality of social choice to be rational in the sense developed in the previous chapter. Second, welfare economics constitutes a self-contained theory of social choice, one which does not satisfy the two conditions for collective rationality even under very strict assumptions about the real world. Its very limitations, however, serve to indicate ways in which the political system must differ from the economic in terms of individual motivation, behavior, and decisionmaking "rules of the game." In other words, the welfare model serves as a paradigm with which the political model may be compared and contrasted, while the search for ways in which to improve the subtle gestures of the "invisible hand" continues. It suggests,

but, of course, does not prove, that purely self interest-
ed behavior of individuals and groups, in general as
well as in strictly economic theory, is not a reliable
method for social choice.

The purpose of this chapter, then, is to review
some of the well known results of economic welfare
analysis, while maintaining a focus on the particular
issue of collective rationality. Even in simple models,
the decentralized choice system of traditional interest
in economics cannot give rise to socially rational deci-
sions; that is, the market, in general, does not yield
an internally consistent social choice structure. And
when at least one market cannot achieve the Pareto
conditions, problems concerned with the reliability of
market rules are introduced; apparently logical rules
aimed at correcting for market error may not have the
effect intended. Problems of reliability grow worse as
public goods and externality are introduced into the
model. On the other hand, the presence of public goods
and externality may reduce the likelihood of internal
inconsistency.[1] And it follows that the agreement genera-

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[1]Insofar as "publicness" increases the grounds for
consensus, it may reduce the probability of internal
inconsistency of a social _preference_ ordering. It ob-
viously does not reduce internal inconsistency in social
_choice_ structures if the market is looked upon as an
aggregation procedure for individual choice.

ted by such interdependence must be transmitted to the
social choice function via some political, or at least
extra-economic, means.

## A. Preliminary Remarks.

Before beginning the discussion of welfare economics,
a preliminary observation must be made. Welfare theory
has always dealt, at least implicitly, with collective
rationality in its full sense. Historically, economists
who have been interested in normative theory have grap-
pled with two fundamental and distinct questions. The
first is "what constitutes the ideal output?" The sec-
ond is "what kind of social institutions, or rules for
individual behavior, can provide the ideal output?"
Finding a satisfactory answer to both of these questions
would satisfy the collective rationality conditions
suggested by Chapter 2. Once both are answered, society
has only to implement the proper institutions.[2]

But the distinction between these two sets of
questions has not always been clear, and progress along
both fronts has not proceeded evenly. Due to their
willingness to make particular ethical judgments, the
"old welfare" economists believed they had a workable
concept of "optimum." Either Cambridge equality or the

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[2]Answering these questions does not, however, root
the social choice of the institutions in individual
choice structures, as any truly satisfactory answer prob-
ably must.

"natural" marginal productivity income distribution were most usually selected as proper, and given that distribution, the forces of competition could be relied upon to achieve the optimum, except insofar as a system of subsidies and taxes were to aid in enforcing the competitive solution in cases of externality, increasing returns, and so forth. Thus, before Bergson, the main emphasis was on policy--the devising of a set of interferences which would make the world behave more as the competitive model did. Since the overwhelming bulk of such policy was to rely upon the automatic decentralized process of market choice, the "old welfare economics" was able to avoid the entanglements of involved political analysis.

This situation changed markedly with Bergson's 1938 article.[3] The exposure of the value judgments implicit in the "old welfare economics" together with the supplanting of cardinal utility analysis by ordinal analysis, virtually obliterated all previous constructs of the "optimum." The distributional judgment was seen to be purely ethical, and both the "natural" and the egalitarian schemes were correspondingly seen to be, in Samuelson's phrase, "shibboleth distributions." Without an income distribution the ideal output could not be

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[3]Bergson, "A Reformulation of Certain Aspects of Welfare Economics."

specified; it could not even be defined unambiguously. Therefore, the "new welfare economics" brought with it a shift in emphasis from policy to a new investigation of the nature of the optimum. It is simply illogical to ask which social-decisionmaking institutions would be best to attain an optimum which is not defined. Thus, pure competition lost much of its status as an institutional norm, and no modified set of rules has replaced it. In fact, little recent analysis has dealt with the question of how the proper rules, as distinguished from conditions for an optimum, may be achieved. This problem has been so nearly forgotten that it has been suggested that lump-sum transfers of income may permit an optimum, without considering the fact that no authority or decentralized mechanism is yet conceived which can estimate or effect such transfers.[4] The reluctance to deal with institutional decision processes as well as the optimal social state which such processes may permit remains a serious inadequacy in the structure of welfare economics.

Of course, some of the most important assertions of modern welfare analysis deal with the correspondence of "efficiency" conditions with the results attained by

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[4]Paul Samuelson, "Social Indifference Curves," *Quarterly Journal of Economics*, LXX (Feb., 1956).

free market institutions. In fact, that the perfectly competitive solution, attained by allowing individuals and firms to pursue utility and profit maximization in a fashion unrestrained by governmental influence, will satisfy the Pareto conditions, is still probably the chief working hypothesis of welfare economists. This hypothesis forms the basic structure to which qualifications and complications which arise are tacked on in a rather patchwork manner. So welfare economics is yet at least marginally attached to an investigation of decision procedures. What is needed, however, is the simple recognition that the successful definition of an optimum is not sufficient to secure collective rationality. Individual choice behavior must be "processed" by a decision procedure which leads the patterns of individual behavior to be consistent with the optimum.

What is implied then, is that the extensive investigation of failure of the market must be matched by a systematic investigation of choice procedures which may account for the phenomena which cause those failures. This area of the "new welfare economics" is yet in its infancy.


B.  Welfare Analysis In The Private Goods Model.

The main welfare theorems are well known. But it '

is necessary to set them down briefly.[5] Their derivation begins with several convenient assumptions about individuals (and firms), chiefly that they maximize utility (profits), that they know what alternative combinations of goods (factors) are available and can compare them, that the rate at which they will substitute one good (factor) for another diminishes as they have more of the second and less of the first. Since the economist does not wish to impose his own value judgments on his analysis of the ideal output, he makes one explicit and widely acceptable, value judgment. This value is the Pareto Principle, which constitutes a definition of efficiency: "most efficient" implies a situation such that no redistribution of any factor of production or final product can be carried out amongst any combination of firms or consumers which can leave at least one person better off, and no person worse off, than before the redistribution. From this normative axiom, the optimality conditions readily flow:

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[5] For discussion of the main welfare theorems see Francis Bator, "Simple Analytics of Welfare Maximization," American Economic Review, XLVII (March, 1957). For the purposes at hand, nothing is lost by ignoring most of the technical difficulties which may interfere with "duality." Throughout this paper all the convenient assumptions, i.e., strict convexity and continuity of preference and transformation functions, absence of external or corner tangencies, the existence of a unique competitive outcome, will be made. Notice will be taken of one other difficulty: interdependence effects (which may disturb convexity). Otherwise, everything is assumed to be "well-behaved."

1. Consumers must not be able to profit from further exchange, therefore all PRS's (personal rates of substitution) of all pairs of goods must be equal for all consumers.

2. Producers must not be able to expand production by a mere reallocation of factors, thus RTS's (rates of technical substitution) must be equal, implying that the ratio of marginal productivity of a factor in one line to its marginal productivity in another line must equal the ratio of marginal productivity of a second factor in the first line to its marginal productivity in the second.

3. The common rate of personal substitution between any two goods must be equal to the rate at which the first good may be transformed into the second by the productive process. The PRS = MRT (marginal rate of transformation). This implies that it is impossible for a consumer to carry out "trades" with the producer; the amount of a good x which a producer may create at a cost of a unit of good y foregone must be equal to the number of units of x which the consumer would be willing to accept in exchange for one unit of y.

These conditions hold regardless of the type of social system which engineers their attainment. One of the most powerful conclusions of welfare economics, however, is that the institutions of pure competition under familiar strong assumptions will satisfy these conditions. Thus, it is theoretically unnecessary for

any central authority either to force consumers to take certain actions or to try to devise a particular and explicit set of rewards and sanctions to which consumers may respond in order to achieve a Pareto optimum. The proper (in the sense of Pareto optimal) signals to optimal choice will emerge from the competitive system itself in the form of relative prices. Since the market establishes unique prices for all goods and factors, individuals and firms which respond rationally to the prices will create a social state which is characterized by the required marginal equalities.

### 1. Consumers' Sovereignty

The marginal equalities only state necessary conditions for an optimum, however; they do not constitute sufficient conditions. To find an optimum, one must know that the correct configuration of goods and services is produced. It is not enough to know that the method of production and allocation is efficient.

The market will determine the particular array of goods and services which will be available. Each individual has an initial claim on the goods produced which is indicated by his income. The utility functions of individuals, together with their incomes, imply the amounts of various goods each will purchase at any set of price ratios. The cost functions of firms imply the amount of each good offered by producers at any set

of price-ratios. Furthermore, all markets must be cleared
in order to achieve equilibrium. For the particular
distribution of claims initially given, one set of
price-ratios will exist which will result in the
clearing of all markets. The market process will lead
to the attainment of this set of prices.[6]

In such a system, each dollar expenditure is, in
effect, a "vote" for the production of the good pur-
chased. The market responds to consumer tastes by
means of adjustment to prices which reflect the pattern
of those tastes. The ability of the market to cater to
individual tastes is denoted by the phrase "consumers'
sovereignty," and has been treated approvingly, for
the most part, in economic theory.

Even though markets respond to consumer taste,
however, one may not conclude that the array of goods
resulting from market behavior is the optimal one. For
under the assumptions we are making, any Pareto optimum
can exist as a competitive equilibrium for the appropri-
ate prices. And any set of prices may be achieved for
the appropriate initial distribution of claims. The
responsiveness of markets to the configuration of
individual tastes is of slight normative significance as

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[6]To achieve this result, it is necessary to posit
some "convergence" mechanism such as a tatonnement process,
in addition to the other assumptions mentioned in the
previous note.

long as the initial distribution of claims is arbitrary.

All efficiently produced arrays of goods, then, are ethically equivalent on the Paretian axiom. In order to determine the correct amounts of goods to be produced, some further ethical axiom is required. If markets are, in fact, responsive to consumer desires, then the additional value judgment may be with respect to the distribution of income. If the optimal distribution, in terms of the additional ethical judgment, is attained, then, in a world of private goods, the market system will calculate prices correctly and call forth the optimal constellation of goods.[7]

Alternatively, it would be possible to attain the optimum collection of goods by introducing explicit value judgments about the goods themselves. One might wish to formulate judgments at this level if he believed that there were reasons other than a mal-distribution of claims which would cause competitive prices to fail to yield maximum social welfare. Such reasons have been adduced by writers who have criticized the doctrine of consumers' sovereignty. If one supposes either that markets should not respond to consumer taste or that they

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[7]It is not required, of course, that positive amounts of every conceivable good be produced. Zero production in some lines, that is corner solutions and implied inequality constraints, may be handled without problem with modern tools of analysis, as shown by Arrow, Koopmans, and others.

should but, in fact, do not do so, the logical conse-
quence is to require value judgments concerning the
nature of the goods produced. Both arguments have been
made and since they bear heavily on the concept of ratio-
nal social choice, they require comment. A familiar
criticism of consumers' sovereignty is that the market
evaluates too heavily transitory consumer notions of
good and bad. Concepts of social welfare should not
derive completely from individual tastes for persons do
not always understand what is best for them. In other
words, some goods are inherently better than others
apart from individuals' preferences for them.[8] A con-
sequence of this view is that a rational state must
somehow supply individuals with goods which "it" knows
to be best, in spite of the fact that its citizens
might deny the validity of this belief. Under such a
notion, the concept of rational collective choice would
imply the pursuit of social goals which were inherently
worthy. But it seems there is no way to observe the
inherent goodness of such goals. All that we can observe
is the behavior of individuals. And from that behavior
we may try to infer their attitudes toward particular
goods or goals. Until we can observe the inherent

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[8]This view has been argued vigorously in John
Kenneth Galbraith, The Affluent Society (Boston, Houghton
Mifflin, 1958).

qualities of goods, it appears we must locate values in persons' beliefs about them, and interpret the concept of collective rationality accordingly. Thus, however plausible such an argument may seem, it is not relevant to an explanation of actual social behavior. For that purpose notions of welfare should be located explicitly in individual preferences, and, accordingly, it is appropriate to judge that whether consumers are sovereign or not, they <u>should</u> be.

It has also been argued that, while consumers' sovereignty is a laudable ideal, the market process, in fact, is not responsive to the full range of consumer taste. Scitovsky,[9] for example, contends that markets respond to notions of mass appeal to the exclusion of the desires of minorities:

> Today, in the age of mass production, it usually seems more profitable to design every product for the majority, however saturated the major- ity demand may be already, and to expand the market by moulding minority preferences through advertising or the offer of fringe benefits and added features. Hence the secular increase in the uniformity and decline in the range of products in most every field. This need not always lead to a loss of welfare but it does render the market less liberal. The individual is still

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[9]Tibor Scitovsky, "What Price Economic Progress," "A Critique of Present and Proposed Standards," and "On the Principle of Consumers' Sovereignty," all in <u>Papers on Welfare and Growth</u> (Stanford, Calif., Stanford University Press, 1964). The same belief is present in Galbraith, <u>The Affluent Society</u>.

> free to fill his shopping bag with whatever
> goods he wishes; but the nature of goods
> from which he chooses is imposed by the tastes
> and wants of the majority.[10]

This contention presupposes a dynamic interaction between the price system and the formation of tastes. Advertising need not play a crucial role in this interaction. Any set of prices will implicitly reward those who derive high utility from relatively inexpensive goods and penalize those whose tastes are relatively expensive. It is always possible that persons whose tastes are satisfied only at relatively great expense may be motivated to cultivate less expensive tastes in themselves. If individual tastes are, in fact, transmuted and leavened by the process of market aggregation of choice, then one may question the wisdom of accepting an aggregation of those tastes as a concept of legitimate group desire. From this viewpoint, the question is not so much how well the market responds to the aggregate profile of tastes, but what the leavening process is and what its impact upon welfare is.

The assumptions made in the last paragraph violate those usually made in explaining consumer behavior. Since these assumptions are violated (consumers may alter their tastes in response to the particular

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[10]Scitovsky, "A Critique of Present and Proposed Standards," p. 237.

configuration of market prices) we would not expect to
be able to aggregate them and get the desirable group
properties. Then value judgments with respect to the
distribution of claims alone would not be sufficient to
insure that corresponding market behavior which satis-
fies the marginal equalities of a Pareto optimum is the
true social optimum.

The usual assumptions concerning consumer preferences
will be maintained in this paper. Therefore, arguments
concerned with social welfare which turn on changing
consumer tastes in response to the market process are
not of direct concern. The point of view adopted here
is that such arguments can be handled appropriately in
a general discussion of social welfare functions which
may consider explicitly the degree to which all indi-
vidual preferences are to count in reaching a social
ordering, under the assumption of constant tastes. Only
in such a context may the response of markets to consumer
desires be given a precise enough meaning to make
discussion useful.

2. The Role of the Social Welfare Function.

Under the assumptions employed, then, we may find
sufficient conditions for a welfare optimum by introducing
some value judgment with respect to the distribution
of claims. Because of the difficulties involved in
judging the inherent value of goods apart from individual

preferences for them, it is customary to take this approach. Thus, the set of additional value judgments is shifted from the "rightness" of the goods themselves to the weighting to be given each individual in choosing the goods. This procedure takes the "worthiness" of goods to be located exclusively in the preference orderings of individuals. Specification of the distribution of claims in addition to efficient prices will determine a constellation of goods which is optimal in the sense that each individual's budget is fixed at such a level that the contribution of each to total welfare at the margin is equal. The logic of this procedure is most easily understood by referring to some standard diagrams of welfare analysis.

Suppose we have found the locus of points of production of two goods, x and y, which are efficient in the production sense. These points are indicated by TT in Fig. 2. Each point on this transformation curve may or may not be Pareto optimal depending upon how the levels of production at each point are allocated to the two consumers. Pick any point B and draw the corresponding box diagram with efficiency locus AB. If we assume a unique competitive equilibrium exists, there must be one point on AB where the PRS's for A and B are equal and equal to the MRT shown by the slope of TT at B. If α is that point, then the situation shown by B
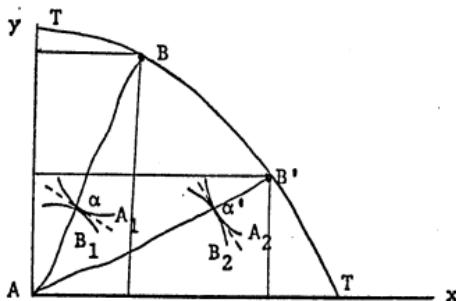
Fig. 2: Two Representative Pareto Optima

production and α allocation is a Pareto optimum.

But a point like B' may also represent a Pareto optimum situation if associated with allocation α'. How can one compare all points such as B and B' which are (potential) Pareto optima?    We cannot.   The Pareto principle is too weak an ethical judgment to select a best B.   Given a distribution, the price system can find the most efficient solution to the production allocation problem.   If the distribution is wrong, the price system cannot correct it.   Newman has put this well-known point succinctly:

> (It is false) to attribute to prices
> redistributive functions that they do not
> possess and to suppose that the free work-
> ings of the untrammeled price system would
> necessarily turn out to the advantage of
> everybody, regardless of the initial commodi-
> ty distribution.   The price system makes a
> good servant but a bad master.[11]

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[11] Newman, _Theory of Exchange_, p. 124.

One of the features of the "new welfare economics"
was the attempt to get around this difficulty by means
of extending the Pareto principle in such a way as to
generate a complete ordering of social outcomes. The
attempt to rank points independently of distributional
judgments entailed the application of so-called compensa-
tion tests to situations where there would be losers
as well as gainers from some social change. The well-
known works of Kaldor and Hicks[12] adopted the line of
reasoning that if the gainers from a change could
hypothetically compensate the losers after a change had
been made, then, in some (supposedly) meaningful sense,
the gainers must benefit more than the losers sacrifice.
Thus, it was believed, the change resulted in increased
national income, and therefore, it should be made.
Scitovsky[13] soon showed that this reasoning could lead
to a contradiction--it is possible that in considering
a change from some situation A to some other, B, it
would be possible for gainers, with the distribution of
utilities after the change, to (hypothetically) compen-
sate the losers, but it might also be possible for the
losers from the change, before the change is made, to
profitably bribe the potential gainers from making the

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[12]Kaldor, "Welfare Propositions," and Hicks,
"Foundations of Welfare Economics."

[13]Scitovsky, "A Note on Welfare Propositions in
Economics."

change. A and B can both, by the Kaldor-Hicks criteria, be shown to be preferable to each other. Scitovsky proposed a double test for an increase in welfare: the gainers must be able to profitably compensate the losers after the change, and the would be losers must not be able to bribe the gainers from making the change.

Samuelson[14] then demonstrated that the Scitovsky criteria also were inadequate. Scitovsky was able to judge the two points in utility space only by the distribution prevailing before the change or by the distribution prevailing after. Samuelson showed that when considering a change in economic environment, all possible distributional arrangements before the change must be compared with all distributional arrangements after the change. In other words, in Fig. 3, if point A is to be compared with point B for individuals 1 and 2, it must
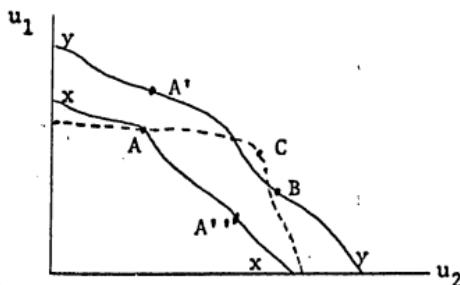


Fig. 3: Three Utility Possibility Frontiers

[14]Samuelson, Foundations of Economic Analysis, p. 251.

be that the utility possibility frontier of situation
B is everywhere outside that of situation A. It is not
proper to judge the two points only by the distributions
prevailing at the two points. For as Fig. 3 shows,
there may be some other point C which is preferable to
B which may be obtained by a simple redistribution of
income from the first situation if the utility possibili-
ty curve of situation A passes somewhere outside of that
of position B as shown by the dotted frontier.

Thus the compensation controversy led to the
development of Samuelson's utility possibility frontier.
This tool of analysis, in the absence of a social wel-
fare function, allows the economist to state only that
changes which push the utility possibility frontier
outward along all its length are "potentially" better;
in such a case, the economist may only state "it would
be better to make the change and compensate any losers,
if necessary." For whichever points on locus yy are
better than points on xx depends entirely upon the shape
of welfare contours derived from a social welfare func-
tion. We may not even say that an _actual_ increase in
welfare, such as would be obtained from moving from
point A to point A', would result in potential benefit.
The welfare function may rank some point like A'' above
A', and this requires only an alteration of distribution
within the original situation. The attempt to extend

the Pareto principle by means of compensation tests fails.

These matters are clearly exposed in the literature
concerned with a search for a community indifference
curve. If we possessed a community indifference mapping
we could superimpose it on goods-space and choose the
point where TT was tangent to the highest community
indifference curve. The problem of ideal-output would
be fully determined. Since we wish to locate community
preferences in some aggregation of the member's prefer-
ences, the group mapping must be derived from the
individual mappings.

An indifference curve incorporating only the Pareto
Principle as a value judgment was constructed by
Scitovsky.[15]  Select a point in good-space and construct
a box diagram such as point B and $AX_1BY_1$ in Fig. 4.
Set A's utility at indifference level $A_1$. $B_1$ is the
corresponding level of indifference of B for production
point B. Now construct through B a curve showing the
minimum amount of goods required to keep individual B
at $B_1$ level of indifference while at the same time
keeping A on $A_1$. For example, find a point B' which
has A on $A_1$ and B on B' (which is indifferent to $B_1$).

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[15]T. Scitovsky, "A Reconsideration of the Theory of
Tariffs," Review of Economic Studies, IX (1942). While
Scitovsky curves do not unambiguously indicate a socially
optimal production set, they are of value in other con-
texts. See, for example, Jaroslav Vanek, "A Rehabilita-
tion of 'Well-Behaved' Social Indifference Curves," Review
of Economic Studies, XXXI(1) (Jan., 1964).
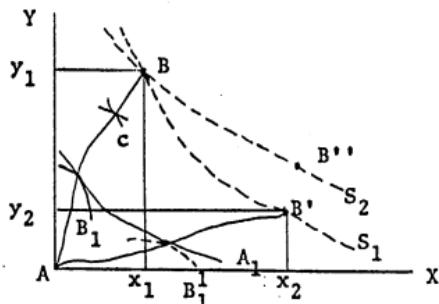
Fig. 4: Derivation of Scitovsky Curves

A curve $S_1$ is then traced out in goods-space (as we repeat the procedure) which has the property that A and B will be indifferent to all points on it as long as each bundle B of goods is allocated in such a way that the common PRS at each point has the slope equal to the slope of the aggregation curve.

$S_1$ was derived with prescribed levels of utility for A and B. If we start at point B once again, but select a different allocation of goods, say that at point C, we would trace out an entirely different Scitovsky curve, $S_2$. There is, in fact, a family of Scitovsky curves, one for each distribution, all passing through B. The intersection of these curves implies that this method will not yield a transitive (social) ordering of points in goods-space.

The intersection of these curves implies that any change in the combination of goods X and Y produced which

is allocated in such a way that A and B are both indifferent between the original position and the change, is indifferent to any other change which is accompanied by the appropriate different initial distribution of purchasing power. The Pareto Principle, incorporated in market choice, will sustain whatever initial distribution is given, whether it is "correct" or not. Changes from point B would leave no person worse off only if the change is to the northeast of $S_1$. Only such changes would be characterized by the property that a gainer from such a change could compensate the loser sufficiently to leave him at the original level of indifference. But because of the intersection of these curves, compensated changes are shown, in effect, to be indifferent to some uncompensated change which result in a different distribution of utilities. The Scitovsky curves summarize the conclusions of the compensation controversy and illustrate that the market mechanism, even when extended by the compensation principle, does not lead to unambiguously optimal social choice. Implicit social rankings in goods-space resulting from market choice are intransitive.

Bergson was the first to see that this problem could be avoided only by an explicit normative judgment concerning the income distribution which should obtain. One such normative axiom proffered by Bergson was "for

a given price-wage situation, and any i and k, if the
share of i is greater than that of k, a decrease in the
share of k would have to be accompanied by a larger
increase in the share of i, for economic welfare to be
unchanged.[16]   The group indifference curve based upon
this axiom bears Bergson's name,[17] although Samuelson's
later discussion of this judgment led to its role in the
construction of a community indifference curve.[18]

     The formation of the Bergson contour begins with
the recognition, obtained from the Scitovsky analysis
(and the compensation controversy), that an indifference
curve cannot be drawn in goods-space without a prior
examination of points in utility space.  In particular,
what we must know is what different distributions of
utilities between individuals will leave the welfare
of society unchanged.  A first step in answering this
question is to find what combinations of utility are
possible with given production constraints.  Fig. 5
shows the locus of all such combinations; it is Samuel-
son's "grand utility possibility frontier."  For each

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     [16]Bergson, "A Reformulation," reprinted in Abram
Bergson, Essays in Normative Economics (Cambridge,
Belknap  Press of Harvard University Press, 1966), p. 26.

     [17]Due to J. de V. Graaff's early discussion of it.
See Graaff, Theoretical Welfare Economics, p. 49.

     [18]Samuelson, "Social Indifference Curves."

point such as B on TT in Fig. 2 there is one point on
the efficiency locus where $PRS_1 = PRS_2 = MRT$. If we
find all such points and map them in utility space, we
obtain the locus $\overline{uu}$ in Fig. 5. It shows the maximum
level of (ordinal) utility of B, given specified levels
for A.
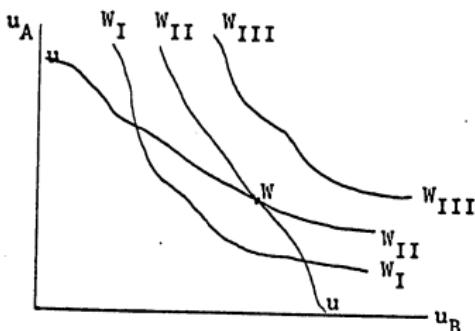


Fig. 5: Welfare Optimization Utilizing
Social Welfare Contours

In order to pick the appropriate point on $\overline{uu}$,
however, we must incorporate some ethical judgments into
the analysis which would have the effect of indicating
how social welfare may vary as a function of changes in
the individuals' welfare. Ignoring for the moment how
such judgments are arrived at, suppose we postulate the
existence of a social welfare function which is consis-
tent with the value proposition suggested by Bergson.
It may be shown diagrammatically as the family of con-
tours, $W_I$, $W_{II}$, $W_{III}$, in Fig. 5. The maximum level of
social welfare is given by $W_{II}$, where $\overline{uu}$ is tangent
to $W_{II}$.
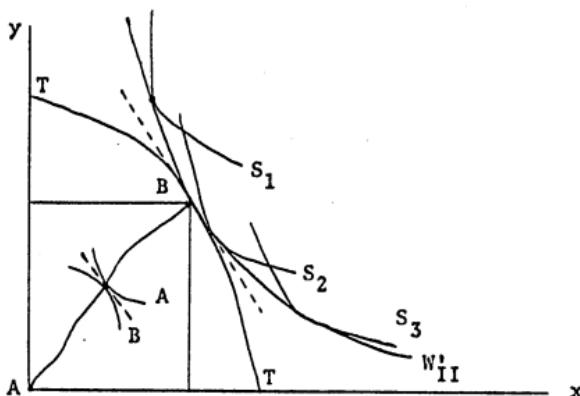
Fig. 6: Derivation of the Bergson contour

Because point W on $\overline{uu}$ corresponds to a unique arrangement in goods-space, the ideal output is immediately specified. This is point B in Fig. 6.

We may alternatively use the contours of the social welfare function to generate social indifference curves in goods-space which, unlike the Scitovsky curves are non-intersecting. Given the optimal level of welfare indicated by $W_{II}$ in Fig. 5, construct a curve in goods-space showing the minimum amount of x required to satisfy $W = W(u_A, u_B) = W_{II}$ for every fixed amount of y. Such a curve is a Bergson contour, and the contour corresponding to $W_{II}$ in utility-space will be $W_{II}'$ in goods-space and will be tangent to TT at point B. This Bergson contour is the inner envelope of the family of Scitovsky curves generated through points on $W_{II}'$ by preserving at each point the distribution of utilities read from

the corresponding points on $W_{II}$ in Fig. 5.

What must be known before it is possible to draw $W_{II}'$, however, is the optimal distribution of utilities corresponding to any combination of x and y. It is just this information which we have gained from examining points in utility space. The implication of this analysis is that in order to attain an optimum, optimal lump sum distributions of income must be made before each consumption period. If, and only if, such transfers are carried out, will ordinary competitive exchange and production processes move society to its optimum at point B in Fig. 6. Therefore the role of the social welfare function in a purely private goods model is clearly exposed: it must indicate by ethical judgment, the proper distribution of purchasing power. And that is all it needs to do--it need not specify the type and amount of goods to be produced, the method of production or anything else. To the competitive game it adds only one rule: the initial distribution must be given.

But how are such lump sum transfers continuously to be made in practice? And how are we to discover the precise shapes of the welfare contours in Fig. 5? Presumably the explicit value judgments used in drawing these contours must themselves derive from value-preferences of individuals. It is of little help to know that ethical judgments must be made concerning the

"worthiness" of each individual's consumption as it
contributes to total social welfare unless there is an
institutional mechanism for making and implementing such
judgments. If there is no such mechanism, then the soci-
al welfare function interpreted as the family of con-
tours in Fig. 5 constitutes merely a _deus ex machina_
to solve a theoretical problem while leaving the real
issue untouched. The social welfare function had to be
introduced because the market system by itself does not
imply a transitive social ranking of points in goods-
space. But it is still possible that the mechanisms by
which distributional decisions themselves are made
are equally incapable of yielding a social ordering of
points in utility space. Unless we discover a mechanism
that has such capability, the attempt to rank points in
utility space has no advantage over rendering value
judgments about the goods themselves.

The attempt to indicate the ideal output by a prior
examination of points in utility space was motivated by
the wish to allow the "worthiness" of goods to be loca-
ted exclusively in the preference orderings of indi-
viduals. This procedure avoids the difficulties involved
in trying to assess inherent goodness of output. But
since the Bergson-Samuelson analysis has "solved" the
problem of ideal output by resort to a scheme of optimal
lump sum transfers called for by judgments of "worthiness"

of individuals' consumption (which must somehow be revealed), it would appear logical to push the analysis a step further removed from output space, and allow the "worthiness" of individual contributions to social welfare to be located exclusively in the preference orderings of individuals. This step is implied by the form of Bergson's function but the implications of this procedure were not fully explored until Arrow's treatment of the problem. It must be shown that individual orderings of social states based upon attitudes toward distribution as well as own-consumption yield a social ordering of social states. Otherwise, we could not infer a process of formulation of social welfare judgments which was itself internally consistent. Until a method is found of aggregating individual orderings of social states based upon judgments of total welfare which has the appropriate group properties, we cannot infer that market aggregation of individual orderings of production sets will yield a social ordering in goods-space which has the appropriate group properties.[19]

It is necessary to inquire whether the existence of a social welfare function bears upon the desirability

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[19]If there is a social ordering of points in utility space, the corresponding social ordering in output space will yield normatively unambiguous results. See Negishi, "On Social Welfare Function."

of satisfying the necessary conditions for a welfare
optimum.  In the absence of a social welfare judgment,
the normative significance of the necessary marginal
equalities becomes somewhat ambiguous.  The locus of
states satisfying the necessary conditions is ethically
equivalent in terms of the Paretian value judgment.
Ethical distinction among these states requires addition-
al value axioms.  Suppose society does not wish to make
further explicit welfare judgments.  Then the partial
ranking of social states derived from the Pareto princi-
ple calls for any social change which moves society
towards the grand utility-possibility frontier.[20]  But,
as such movements must constitute _actual_ improvements
in terms of the Pareto criteria, all moves toward the
frontier would have to be within the range of utility
distributions dictated by the one prevailing at the
initial position.  Such a procedure imparts a special
significance to the original distribution;[21] by refus-
ing to judge explicitly between income distributions,

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[20]Note that in this case the _deus ex machina_ which
"solves" the problem of identification of ideal output
is the price system itself.

[21]It also requires that any proposed change which
would otherwise damage some individual's welfare must
be accompanied by the actual payment of compensation.
In a context of social choice which includes non-market
decision, those institutions of group decision must
arrange for the appropriate payment of compensation
for every change made.  It is not clear that it is possi-

society implicitly sanctions the one prevailing in the
status quo. And that distribution is arbitrary (unless,
perhaps, one can justify it in terms of some historical
argument). The implicit choice of the initial distribu-
tion is acceptable, however, for if society is restricted
to the Pareto axiom, the notion of optimum optimorum is
meaningless.

Suppose, on the other hand, that some additional
welfare judgments are implicit in the configuration of
individual attitudes about social states, but that there
is no method of revealing what they are. Our problem
is not that the social welfare function does not exist,
but that we do not have the acuity to determine what it
is. Under this view, the attainment of conditions
necessary for an optimum may not be desirable. The
optimum does exist, and it must lie on the grand utility
possibility frontier. But the social welfare function
which gives rise to the specification of the optimum
will rank many points on the possibility frontier lower
than many other points interior to it. Thus, many
efficient moves, even if they constitute improvements
in actual welfare may make society worse off in terms of

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ble to devise such institutions; the burden of patterns
of logrolling, and vote-trading in performing such a
function is heavy, but clearly desirable under the value
assumptions contained in the Pareto principle.

its (unrevealed) ethical judgments. This understanding seems to be generally accepted in theoretical welfare economics and I adopt it here.

We cannot alleviate this difficulty by attempting to separate the problems of production and distribution of the total product. The practical economist, perceiving that the optimum distribution is not known, and being unwilling to dictate to the rest of society his own opinion of which distribution should obtain, may wish to recommend all changes in potential welfare without qualification, thus leading society to the grand utility-possibility frontier. He may simply allow the political mechanisms that be to attain the general optimum by means of distributional policy. But this view cannot be maintained so long as it is recognized that there is no measure of the size of real income independent of its distribution. To argue otherwise, as Graaff has remarked, amounts to contradicting the Pareto principle which asserts that only individual evaluations of product are to count.[22] The only way we can weigh goods in any collection is by reference to individual valuations when a distribution of the goods is given. These individual evaluations then are considered only by reference to a particular welfare function which gives social welfare as a function of individual utility levels.

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[22]Graaff, _Theoretical Welfare Economics_, pp. 91-92.

The conclusion is that unless we assert that the only value of society is efficiency and the construct of a social welfare function is meaningless, the necessary criteria for a welfare maximum are of slight value, and therefore, the knowledge that competitive institutions which operate perfectly will achieve them is also of dubious worth. It is only the society with a social welfare function and a defined optimum which will find a particular set of marginal conditions normatively significant and which may attach particular significance to institutional forms which may aid in attaining them.

C.  Welfare Models with Interdependence.

The lesson to be drawn from the welfare economics model is that individuals maximizing their own utility functions will, in a system of decentralized decision-making, create social outcomes which do not correspond to any individual or social ordering of outcomes. Since the social outcomes of such a system do not conform to a definition of an ordering (they are not transitive), the social ordering, or welfare function, must be obtained in some way from outside the system. Until the ethical judgments are obtained externally there is no optimum position for the society and without such an optimum there are no institutions which may be desired for their effect on welfare. There can be no particularly expeditious way of choosing something which has not been shown

to exist. The reliability of choice procedures is a meaningless concept in the absence of internal consistency.

Several recent contributions to welfare economics have raised further doubt about whether it is possible for a society to choose an optimum, even if we assume that an optimum does exist. The contributions to be discussed here all turn on the presence of some kind of interdependence between economic agents.

Explicit inclusion of interdependence effects such as externality or public goods necessarily alters the nature of the conditions for a welfare optimum, and leads almost surely to the conclusion that the ordinary necessary marginal equivalences achieved in a context of a purely competitive system would not be desirable.

Even in the traditional private goods general equilibrium model, economic agents are related in a sense. Any simultaneous equation model indicates that the value of any variable depends upon the solution for all other variables. But the interdependence in this case is not functional, or direct, because the cross-relations do not influence behavior of economic agents. Consumption, for example, depends upon activities which are entirely those of the individual consumer. Although he might feel better, subjectively, if his neighbor's welfare were increased, ceteris paribus, a change in

his neighbor's welfare in no way alters his own behavior.[23] In this example, the interdependence is irrelevant because the Pareto conditions are achieved whether or not it exists. That is, the market correctly "factors" the proper social choice into individual patterns of action in spite of the existence of such "sympathy." As Samuelson writes:

> It is the "decomposibility," or "tree" or "factorability" property of individualistic consumption that makes possible efficient use of a market price system. The miracle is not that a pricing system works, but that the strong conditions for it to work are approximately present in nature.[24]

One strong condition which must be present in order for the Pareto conditions to be desirable, however, is that they be achieved simultaneously in every market. The correspondence between market institutions and efficiency conditions depends on the ability of the market to properly factor a social outcome into the correct decisions for all economic agents. If one or more markets are somehow insulated from competitive rules, then the normal optimal conditions for all other markets

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[23]This kind of interrelation occurs in Buchanan's and Stubblebine's taxonomy of external effects as an "infra-marginal irrelevant externality." See James M. Buchanan and W. C. Stubblebine, "Externality," _Economica_, n.s., XXIX (Nov., 1962), pp. 372-73.

[24]Samuelson, "Arrow's Mathematical Politics," New York University Institute of Philosophy, 8th, 1966, Sidney Hook, ed., _Human Values and Economic Policy_ (New York, N.Y. University Press, 1967), pp. 49-50.

will, in general, be modified, and the familiar marginal
conditions, if imposed upon other markets which are not
insulated from the competitive rules (or a central author-
ity) will no longer be optimal. This realization, cen-
tral to the "theory of the second-best," implies that
factorability, the "irrelevance" to behavior of inter-
dependence in the general equilibrium sense, is not a
virtue but a deficiency of market institutions unless
competitive conditions obtain everywhere. This fact leads
to the conclusion that use of the market as a decision-
making technique may be irrational on grounds of
unreliability. That is, it is not clear whether the
Pareto conditions should be established even given an
optimum distribution and a method of achieving optimal
continuous lump sum transfers.

The problem of the "second-best" was raised first
by Samuelson[25] but began to receive serious attention
only after the article by Lancaster and Lipsey[26] ap-
peared. Lancaster and Lipsey specify some objective
function

$$(1) \quad F(x_1,\ldots,x_n)$$

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[25]Samuelson, Foundations of Economic Analysis,
pp. 252-53.

[26]Kelvin Lancaster and Richard G. Lipsey, "The
General Theory of the Second-Best," Review of Economic
Studies, XXIV(1), (1956).

which is to be maximized subject to a constraint

$$(2) \quad G(x_1,\ldots,x_n) = 0 \ .$$

The solution yields the familiar conditions for a Pareto Optimum.

$$(3) \quad \frac{\frac{\partial F}{\partial x_i}}{\frac{\partial F}{\partial x_n}} = \frac{\frac{\partial G}{\partial x_i}}{\frac{\partial G}{\partial x_n}}$$

But suppose at least one of the producers or consumers violates the Pareto condition. Then there will be a new constraint on the objective function. If the constraint is of the simple form

$$(4) \quad \frac{\frac{\partial F}{\partial x_1}}{\frac{\partial F}{\partial x_n}} = K\frac{\frac{\partial G}{\partial x_1}}{\frac{\partial G}{\partial x_n}} \ , \ K \neq 1$$

the conditions for a maximum become extremely complicated expressions

$$(5) \quad \frac{\partial F}{\partial x_i} - \lambda\frac{\partial G}{\partial x_i} - \mu \left[ \frac{\frac{\partial F}{\partial x_n}\frac{\partial^2 F}{\partial x_1 \partial x_i} - \frac{\partial F}{\partial x_1}\frac{\partial^2 F}{\partial x_n \partial x_i}}{(\frac{\partial F}{\partial x_n})^2} \right.$$

$$\left. - K\frac{\frac{\partial G}{\partial x_n}\frac{\partial^2 G}{\partial x_1 \partial x_i} - \frac{\partial G}{\partial x_1}\frac{\partial^2 G}{\partial x_n \partial x_i}}{(\frac{\partial G}{\partial x_n})^2} \right] = 0$$

If the behavior of the "deviant" who does not conform to the simple Pareto condition in his behavior cannot be corrected (and by assumption it cannot) then

the satisfaction of the Pareto conditions elsewhere
is not necessarily desirable.

The ability of the policymaker to choose the best
set of institutional decision rules is, in this situation,
extremely limited.  For suppose that any particular
time there are several deviants.  Then if some of these
are insulated in some way from policy, while others may
have their behavior adjusted, there are no criteria by
which the policymaker can determine what conditions
the deviant who will respond to policy ought to satisfy.
It is in doubt whether partially enforcing the satis-
faction of the Pareto conditions will make society better
or worse off, judged by efficiency criteria alone.

This conclusion is of great practical importance
for policy, because due to market imperfections of
various sorts, externalities and so forth, there must be
a great many divergences from "first-best" pricing in
the real world.  If all these corrections are not to
be made at once, partial application of marginal cost
pricing rules will, in general, achieve a social state
less well off than a "second-best optimum"[27] defined

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[27]In fact it is possible that striving for "first-
best" may even result in a position worse than the
original position.  But the emphasis placed on this pos-
sibility by some writers is not truly justified given
the extremely general formulation of the problem given
by Lancaster and Lipsey.  See Baumol, Welfare Economics
and the Theory of the State, 2nd edition, pp. 19 and 30.

by much more complex relations between price and marginal cost. If it is true that policy must proceed in piece-meal fashion, the simple Pareto conditions may not constitute a useful guide to policy whether an optimum is given or not.

The problem is not quite so serious, however, as it might appear. The conditions which indicate whether or not a departure from the simple Pareto conditions is called for have been clarified by Davis and Whinston.[28] It is possible that the objective function and constraints are separable.[29] For example, if (1) is interpreted as a utility function and (2) as a transformation function, the separability of both is often assumed. That is, individual utilities have as arguments only amounts of goods consumed and production functions of firms have as arguments only amounts of factors used under the standard assumption of no externalities in consumption or production. When it is appropriate to assume that (1) and (2) are separable, the cross partial derivatives in (5) vanish for all $i \neq 1$ and n, and the simple Pareto conditions will hold for all commodities save the first

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[28] Otto Davis and Andrew Whinston, "Welfare Economics and the Theory of the Second Best," Review of Economic Studies, XXXII(1) (Jan., 1965) and "Piecemeal Policy in the Theory of the Second Best," Review of Economic Studies, XXXIV(3) (July, 1967).

[29] A function is separable if and only if $f(x_1,\ldots,x_n) = f_1(x_1)+\ldots+f_n(x_n)$.

and the nth which appear in the additional constraint.
Davis and Whinston generalize this consideration in a
revised model which illustrates that competitive maxi-
mizing behavior for all economic agents will lead to
satisfaction of the Pareto conditions and that these
will be desirable except for individuals who control
variables which are in the new constraints. In their
own words, they demonstrated that

> When the additional constraint(s) contained
> only variables subject to the choice of the de-
> viant, then except for the deviant, all
> Pareto conditions and behavioral rules were
> of the same form as those for the second best
> problem. When the additional constraint(s)
> contained variables whose values depended on
> the choices of other units, then only for
> the deviant and those units whose variables
> entered into the additional constraint(s)
> did the Pareto conditions and behavioral rules
> differ from those of the second best
> problem.[30]

Therefore, piecemeal application of the Pareto
conditions will be consistent with the optimum in many
cases. But in some cases, it is inappropriate to assume
the separability of (1) and (2). For example, (1) might
be interpreted as a welfare function. Then separability
would be present only under the assumption that the wel-
fare function was a Bergson function of the individualist
type, i.e., where individuals rank social states solely
in accordance with the utility derived from the bundle

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[30]"Welfare Economics and the Theory of the Second
Best," p. 12.

of commodities privately consumed by him in each state.
Then the level of welfare experienced by each would be
independent of that experienced by others.[31]  Under a
more general formulation of a welfare function such that
well-being may depend upon values of equity, justice,
and so on in addition to own consumption, the individual's
welfare would depend to some degree upon that of others',
and therefore, would depend upon others' behavior.  When
the agents of decision are functionally inter-connected
in this way, the conditions for an optimum may be extre-
mely difficult to state, for wide departure from the
Pareto conditions may be called for.  The type of inter-
dependence entering here would be difficult to observe.
The fact of interdependence does not lead consumers to
account for it if the price structure does not.  And that
the price structure will not do so is precisely the
problem.

In this context it is the existence of implicit
externality which causes the second-best pricing equa-
tions to deviate from the Pareto prices.  Pricing must
diverge from the Pareto conditions only for commodities
which are produced or consumed by an individual who is
sensitive to the production or consumption activities of

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[31]In terms of the terminology of Chapter 2, separate
decisions with respect to the level of welfare of each
individual are independent.

others. The proper framework for such analysis would appear to be to treat the problem of externality directly rather than in the context of general equilibrium second-best models. Let us consider such an approach next.

In the tradition of Pigovian welfare economics, it is customary to explicitly introduce degrees of non-separability into the welfare maximization problem and to deal with such problems as arise in a partial and not a general equilibrium setting.[32] Admission of such forms of non-separability into the model has long been recognized as a serious cause of market failure.

Externality exists whenever the utility function of an individual or the production function of a producer depends upon the consumption or production activities of someone other than himself. The relationship may take several forms. The level of utility of an individual may be related to the level of a certain activity of another individual, but not to incremental changes of that individual's activity. In such a case an "infra-marginal externality" is present.[33] Or the utility of

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[32]Several classifications of externalities have been suggested. See Scitovsky, "Two Concepts of External Economies," Journal of Political Economy, LXII (April, 1954); Bator, "The Anatomy of Market Failure," and Buchanan and Stubblebine, "Externality."

[33]The term is Buchanan's and Stubblebine's. See "Externality," p. 372. Traditional interest in economics has been in the case of marginal externality almost exclusively, however.

the first individual may depend upon small changes in
the activity of the second as well as upon totals, in
which case a "marginal externality" is present. If we
let $\partial u_A/\partial x_B$ represent the change in A's utility with
respect to the change in B's xth activity, and we know
that A's utility does, in fact, depend upon $x_B$, then

$$\frac{\partial u_A}{\partial x_B} \lesseqgtr 0$$

implies that B's activity confers an external diseconomy,
an infra-marginal externality, or an external economy
on A, respectively. The fact that private competitive
behavior will not achieve optimal results if such rela-
tions exist has received enormous attention. The exist-
ence of such effects causes there to be a divergence
between marginal social cost and marginal social benefit
of the activity in question because the acting party
does not consider the effect of his behavior upon others
when he achieves his private equilibrium.

Since prices alone will not account for externality
and thus fail to lead individuals to make choices
consistent with an optimum, it is usual to assume that
some system of subsidies and taxes may be imposed to
increase (decrease) the output of firms conferring ex-
ternal economies (diseconomies), thereby removing or
"internalizing" the externality. This common Pigovian
prescription has come under heavy criticism.

recently.[34] The main line of criticism deals with the reciprocal nature of the externality problem.[35] There are at least two parties involved in any externality. In the case of a diseconomy conferred by B upon A, to alter B's behavior would improve A's position but presumably would worsen B's position. Before knowing whether to correct for the diseconomy by placing some sort of unit tax on B, we should know whether the disutility of A without the tax is greater or less than the disutility suffered by B in the presence of the tax. To put the problem this way clearly shows that whether or not any change should be made depends upon an explicit welfare judgment. In the absence of such a judgment, it is still possible to state the marginal conditions for efficiency. Suppose that A derives utility from his own activities $(a_1,...,a_n)$ while B derives utility from his activities $(b_1,...,b_n)$. Suppose that the jth activity is available on equal terms to both and can be taken as a "numeraire" activity. Further suppose that the 1st activity of B confers an external diseconomy on A.

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[34]See Coase, "The Problem of Social Cost;" Buchanan and Kafoglis, "A Note on Public Goods Supply;" Baumol, "External Economies and Second Order Optimality Conditions;" Buchanan and Stubblebine, "Externality."

[35]See Coase, "Problem of Social Cost," reprinted in William Breit and Harold M. Hockman, eds., Readings in Microeconomics (New York, Holt, Rinehart and Winston, 1968), p. 424.

Then A's utility function is $U_a = F(a_1,\ldots,a_n,b_1)$. Let the marginal rate of substitution for A between the extent of B's activity $b_1$, and the numeraire activity $a_j$, be

$$\frac{\dfrac{\partial U_a}{\partial b_1}}{\dfrac{\partial U_a}{\partial a_j}}$$

Since we have a case of diseconomy, this expression is negative--A would be better off with less of $b_1$. Suppose further that B is in private equilibrium. Then for B

$$\frac{\dfrac{\partial U_B}{\partial b_1}}{\dfrac{\partial U_B}{\partial b_j}} \cdot \frac{\dfrac{\partial f_B}{\partial b_1}}{\dfrac{\partial f_B}{\partial b_j}} = 0$$

where $f(b_1,\ldots,b_n)$ is B's budget constraint. This cannot be Pareto optimal, for since the absolute value of A's PRS between $b_1$ and $a_j$ exceeds the net benefit of $b_1$ to B, at the margin, gains from trade may occur. A may desire to compensate B for reducing his level of $b_1$. The condition for a Pareto optimum would be [36]

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[36] Note that this condition indicates that the simple existence of externality does not constitute a prima facie case for government interference in the market. Since the disutility of $b_1$ to A is equal to the net utility of $b_1$ to B the question of whether anything should be done to alter the level of the activity $b_1$ depends upon judgments of a distributional nature.

$$(-) \ \frac{\frac{\partial U_B}{\partial b_1}}{\frac{\partial U_B}{\partial b_j}} = \left[ \frac{\frac{\partial U_B}{\partial b_1}}{\frac{\partial U_B}{\partial b_j}} - \frac{\frac{\partial f_B}{\partial b_1}}{\frac{\partial f_B}{\partial b_j}} \right]$$

If this point were attained, there is no possibili-
ty for trade for the disutility of $b_1$ to A at the margin
is just equal to the net utility of $b_1$ to B.

Suppose that we are not at a Pareto equilibrium
and that A's PRS is greater than B's net marginal bene-
fit from $b_1$. If we hold B responsible for damaging A,
and therefore institute a marginal tax on $b_1$, we may
improve A's position, but we will not remove the
externality. B's private equilibrium will occur when
the marginal benefit of $b_1$, net of direct cost and the
tax, is zero. But at that point, if A's PRS is not zero
(and there is no reason why it should be), there still
exists the possibility for gains from trade until A's
marginal disutility is equal to B's marginal net benefit.
The Pigovian taxes are not sufficient to achieve a
Pareto optimum.[37] To achieve a Pareto optimum, the

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[37]This analysis assumes that the $PRS_{b_1 a_j}$ of A does
not vary iwth the <u>level</u> of $b_1$ and that the imposition of
the tax can be secured without cost to A. The second
assumption is quite likely unreasonable and the first may
be as well. If the diseconomy is in the form of smoke
nuisance, for example, and the tax is some sort of ef-
fluent fee, B may try to avoid the tax by alternative
measures such as smoke abatement equipment which would
lower the amount of smoke produced per unit of $b_1$. At
the lower level of nuisance, A's PRS between $b_1$ and $a_j$

behavior of both A and B must be modified.  As

Buchanan and Stubblebine conlude[38]

> Not only must B's behavior be modified to
> insure that he will take the costs externally
> imposed on A into account, but A's behavior
> must be modified so as to insure that he
> will take the costs "internally" imposed
> on B into account . . . Pareto equilibrium
> cannot be obtained so long as marginal
> externalities remain, until and unless those
> benefiting from changes are required to pay
> some "price" for securing the benefits.

Recognition of this fact is of little help in

deciding what to do about externality, however.  It does

help to explain how Coase is able to reach the conclu-

sion that if trade is possible between affected parties,

the solution achieved by bargaining will be indepen-

dent of the legal structure of liability.[39]  But where

such trade is impossible due either to the attainment

of a Pareto optimum or due to the size of organizational

costs involved in bargaining between many parties af-

fected by the externality, it is of little guidance to

assert, as Coase repeatedly does, that

> The problem which we face in dealing with
> actions which have harmful effects is not
> simply one of restraining those responsible
> for them.  What has to be decided is
> whether the gain from preventing the harm
> is greater than the loss which would be

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might be diminished.  Buchanan and Stubblebine do not
consider such a possibility.

[38]"Externality," in Breit and Hochman, _Readings_,
p. 486.

[39]Coase, "Problem of Social Cost" in Breit and Hoch-
man, _Readings_, pp. 424-429.

suffered elsewhere as a result of stopping
the action which produces the harm.[40]

For we can make that decision only by reference to
interpersonal comparisons; one can weigh "good" against
"harm" only by reference to a well-defined social wel-
fare function. And, of course, this statement holds
true whether or not trade can be achieved. Since "trade"
in this case refers to the payment of compensation by
the gainer to the loser from a change, it need not be
consistent with the optimum. For an optimum to be achiev-
ed, externality must be accounted for in persons' behavi-
or, but how it is to be accounted for depends entirely
on society's welfare judgments. There is no simple
rule for liability which may be relied upon. In the
absence of an optimum, given the likelihood of second-
best problems, it is not clear that even the efficiency
conditions should be sought, even if they could be
adequately formulated for cases under externality.

A final type of interdependence may be included
before concluding this chapter; that introduced by
public goods.[41] A public good is usually defined as one

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[40]*Ibid.*, p. 443.

[41]There is some question whether public goods may
be dealt with as a generalization of the externalities
problem, or whether they constitute a phenomenon novel
enough to merit separate analysis. See Bator, "Anatomy
of Market Failure", Baumol, Welfare Economics and the
Theory of the State, 2nd ed., and Buchanan, The Demand
and Supply of Public Goods (Chicago, Rand-McNally, 1967).

which, if produced, is available in equal amounts to all
consumers. That is, the act of consumption by one con-
sumer does not diminish that available to another, and
it is not possible (or economically feasible) to exclude
any individual from its consumption. If such goods
exist, it is well-known that the MRT in production be-
tween this good and some other for Pareto optimality,
must equal the sum of PRS's for all individuals who
consume both goods. Since these PRS's will not be equal,
in general, but since each individual must (by definition)
consume the same quantity of the good, it is immediately
evident that for optimality, different marginal
prices must be charged to consumers with different mar-
ginal evaluations of the good. But this is extremely
difficult to arrange institutionally.

Persons in very small groups may be able to arrange
trades. For example, where two individuals are
independently producing the public good, private equili-
brium will occur where:

$$\frac{\frac{\partial u_i}{\partial P_i}}{\frac{\partial u_i}{\partial x_i}} = -\frac{\frac{\partial f_i}{\partial P_i}}{\frac{\partial f_i}{\partial x_i}}$$

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These authors consider public goods to be susceptible
to treatment under the generalized externalities analysis,
although the two concepts do differ somewhat.

when $P_i$ is the activity producing the public good, x is some "numeraire" activity and the subscript refers to the individual. However, in "private" equilibrium the level of A's activity P exerts a marginal external economy on individual B. In a manner similar to that in the case of externality, B may in some way bribe A to increase his level of activity until

$$\frac{\frac{\partial u_B}{\partial P_A}}{\frac{\partial u_B}{\partial x_B}} = (-) \left[ \frac{\frac{\partial u_A}{\partial P_A}}{\frac{\partial u_A}{\partial x_A}} - \frac{\frac{\partial f_A}{\partial P_A}}{\frac{\partial f_A}{\partial x_A}} \right]$$

and, therefore, Pareto optimality is achieved.[42]

One individual's production of a public good, however, confers an external economy which extends to everyone in the society. This fact distinguishes the case of public goods from ordinary external relations which may find a small number of persons affected by the interdependence. Because of the larger number of persons affected, it may be difficult or impossible to arrange for the appropriate private "trades" which will lead to the provision of an optimal amount of the good.

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[42]Assuming only A produced the public good. It is arbitrary whether A bribes B or B bribes A to increase level of collective good, unless their respective "production functions" indicate that specialization would be indicated. If the optimality condition corresponding to B producing the public good is written, and if both conditions hold, this analysis can handle cases where the good is not purely public. See Buchanan, Demand and Supply of Public Goods, pp. 20-21.

The organizational costs of arranging for the production
of public goods may be costly and rational behavior
will lead individuals to try to escape the costs of its
provision, since they will share in its benefits whether
or not they make contributions. Everyone may desire to
be a "free-rider," and the public good will not be pro-
vided at all, or it will be provided only in the (sub-
optimal) amounts which single individuals may desire
to provide independently.[43]

Since all persons may be presumed to recognize
that independent behavior in such a situation is self-
defeating, it is reasonable to expect that they will
attempt to implement some institutional arrangement which
supplements voluntary trading so as to coerce, as it
were, members of the collectivity to behave consistently
with their own preferences.[44]  Buchanan puts the matter

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[43]Unusual forms of strategic behavior may follow in
this case.  If the good is sufficiently divisible, it
would be provided by the individual with the highest mar-
ginal valuation for it.  This amount exceeds the amount
any other individual would provide independently.  Since
everyone enjoys the good equally, there is no private
incentive for further production.  The provider of the
collective good may not threaten to cut back on production
in order to force others to contribute, for they will
only point out that he would suffer most.  Olson has
emphasized this phenomena which he calls the coercion of
the strong by the weak.  See Mancur Olson, The Logic of
Collective Action (Cambridge, Harvard Univ. Press, 1965)
and Mancur Olson and Richard Zeckhauser, "An Economic
Theory of Alliances," Review of Economics and Statistics,
XLVIII (Aug., 1966)

[44]This is the underlying theme of Baumol's Welfare
Economics and the Theory of the State.

clearly:

> When the large number dilemma . . . exists,
> the natural processes of trade, the emergence
> of market-like institutions, cannot be
> predicted to generate shifts toward opti-
> mality.  Gains from n-person trade person
> or agreement are clearly present, and
> these potentialities may be universally
> recognized.  But market like arrangements
> cannot readily materialize due to the ab-
> sence of votable gains from two-person or
> small number exchanges.
>
> In such situations individuals will suggest
> n-person "rules" or "arrangements" aimed
> explicitly at reducing or eliminating the
> inefficiencies generated by independent
> behavior.[45]

But it is difficult to conceive of individuals who

are rational in the normal economic sense, of ever

reaching such arrangements.  In economic theory the

individual maximizes his utility in a given situation.

He knows that his effect on the environment will be so

slight as to be imperceivable--he therefore will not be

expected to invest resources in an attempt to alter it.

He accepts all the actions of others, including "free-

riding" as unalterable and responds accordingly.  This

view is obviously not totally realistic.  Large numbers

of individuals do reach agreements which have the effect

of altering the environment, that is, they attempt to

alter inducements to behavior in such a way that the

"free-rider" problem is eliminated.  And these agreements

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[45]_Demand and Supply of Public Goods_, pp. 86-87.

may be explained by a type of utility maximization.[46]
But until recently, economic literature has not examined
such phenomena because choices of this type are not
mediated through markets, where all events may be factored
into bilateral agreements; they are mediated through
political mechanisms.

The introduction of public goods into welfare models,
therefore, does not only give rise to one more instance
where a system of decentralized choice based upon self-
interested behavior will generate a sub-optimal outcome,
it also raises the issue of how individuals may rationally
choose between alternative environments. This problem
has seldom been treated in economics, but without such
an analysis an adequate theory of social choice is not
possible.

D. Conclusions.

It is possible to draw some tentative conclusions
from this survey of welfare economics. First, even in
the simplest welfare choice models, it is impossible
to reach socially rational decisions. The market, of
itself, cannot rank social outcomes. It is customary
to indicate this lack of internal consistency by reference
to the necessity of a social welfare function. Further-

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[46]An important attempt to base agreement at the
"constitutional" level upon own-utility considerations
is found in Buchanan and Tullock, The Calculus of
Consent.

more, if the optimum can be defined, the conditions necessary for the attainment of the optimum must be simultaneously satisfied in order for them to take on normative significance. If an optimum exists, but is not feasible because of incurable non-optimality in one or more markets, a partial achievement of the necessary conditions is no longer clearly desirable. Thus, "perfect" market behavior may be an unreliable method of social decision when the social welfare function ranks points of "feasible" rather than optimum welfare. However, competitive behavior may be consistent with an optimum when the welfare function and exchange-production constraints are separable; they are appropriate for markets which are entirely independent of variables determined by the incurable deviant. Unless information about the structure of interdependence induced by second-best problems is known in advance, no a priori rules for satisfying optimal decision rules in individual markets may be stated. If an optimum did exist, market procedures would not choose the optimum.

Problems associated with attainment of the optimum, even if it exists, grow worse as interdependence is explicitly taken into account in the form of externality and the existence of public goods. That such phenomena destroy the optimality characteristics of market behavior is obvious, but that the method of correcting for

externality, or for determining a method for properly
factoring an optimal social outcome into individual
choice, implies adjustment of the behavior of both (all)
parties to the external effect is not so widely recog-
nized.  Nor has it been emphasized that the method for
correcting for externality is largely a matter of pure
judgment of distributional alternatives.

    Given that market choice may not achieve an opti-
mum unaided by external forces, it appears that at
least part of the total social choice must be made by
a different process of mediating individual decisions.
This external process apparently must be charged with
at least three requirements.  It must be able to choose
among alternative distributions of income.  Within the
context of the chosen distribution it must properly
correct for external relations.  And it must allocate
the proper share of total resources to the production
of public goods.  Since it is presumed that there is no
social outcome which is not fundamentally a result of
individual choices, a theory of independent utility
maximization must be retained; thus, the importance of
pure market analysis is not diminished.  But the formida-
ble task of the external mediation process, in general,
is to devise an environment in which social choice will
be correctly partitioned into individual choices.

    These difficult tasks must be performed in some

way by the polity. It is suggestive to consider that
the political process must not function in a manner
analogous to the market system if it is to perform these
functions.[47] Something must be different about its
method of decisions, the scope of participants' utility
functions, or some other feature. If not, an analogous
system of choice is likely to make analogous errors.
In fact, it is possible to argue that the most important
results of welfare economics will be to suggest how poli-
tical choice must differ from economic choice in order
to permit citizens to choose the full complement of
goods and goals which are necessary for the attainment
of maximal social welfare.

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[47]The view that political processes and market pro-
cesses may be very similar, if not identical, in structure,
has been suggested perhaps most strongly by James
Coleman, "The Possibility of A Social Welfare Function,"
American Economic Review, 1966.

CHAPTER 4

Non-Market Choice and Collective Rationality

The greatest portion of recent literature which investigates the capability of political choice systems to accomplish the requirements set for it by economic welfare theory has been inspired by the path-breaking work of Kenneth Arrow.[1]  The importance of this contribution, if judged by the volume of literature following it, is staggering.  The approaches taken, primarily in cirtical reaction to Arrow's General Possibility Theorem, have been so varied in content and method that Strotz has remarked[2]

> One can only smile upon the efforts of so
> many to exorcise this theorem from welfare
> economics.  This reviewer is not impressed
> that any of these efforts has succeeded,
> though a great babel of interpretations has
> been proliferated.  The issues have been
> so semantically subtle, so involved with
> the elaboration of nuances of meaning and
> intent, that it is impossible to comment
> in any succinct way on any of the continu-
> ing discussion.

The aim of this chapter is to draw some of the recent elaborations on social choice into a more coherent framework than currently exists.  To reduce the "babel"

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[1]Kenneth Arrow, _Social Choice and Individual Values_.

[2]Robert Strotz, "Essays in Normative Economics,"
A Review, _American Economic Review_, LVIII (March, 1968),
p. 190.

of interpretations of the Possibility Theorem to an
orderly set of propositions is fundamental to the prob-
lem of social choice, for it is, as the previous chap-
ter suggests, rightly conceived as the cornerstone of
a theory of rational collective behavior. Until a con-
sensus among theorists concerning the impact of the
theorem is achieved, the normative body of welfare the-
ory is at an impasse.

The theme of this chapter is that the Arrow Theorem
is relevant to several conceptually distinct problems,
that the status of the theorem is slightly different
under different interpretations, and that much confusion
about the importance and meaning of the theorem is due
to the fact that many authors have discussed separate
applications of it in such a way that it appears they
are dealing with the same problem when, in fact, they
are not. First, an interpretation of the theorem and
its implications under each of several possible inter-
pretations will be presented. Then, in subsequent chap-
ters, several lines of argument for avoiding the stric-
tures of the theorem will be examined and evaluated.

A. The General Possibility Theorem.

Societies have at their disposal several mechanisms
by which individual behavior may be transformed into
social outcomes. We can generalize these mechanisms into
two types: market choice and political choice. Arrow's

book forges a link between these two broad classes of decision procedures. Its chief significance to the discipline of welfare economics is that it generalizes a well-known conclusion about market choice to methods of political choice, if certain "reasonable" restrictions are made upon those latter methods. It is widely believed that Arrow "proved" the non-existence of a means by which a society could select an appropriate social welfare function. An evaluation of this assertion requires the observation that Arrow's method consists largely in the application of the paradigm market model as an analogy to political (generalized voting) models of choice. It is appropriate, therefore, to risk repetition of the obvious by beginning discussion of the theorem in the context of the standard welfare model.

The last chapter established that in the welfare economics model a logical impasse was reached when trying to attach normative significance to the necessary criteria for an economic maximum. We may not infer an ordering of possible outcomes in goods space from market choice behavior. Given an output-exchange equilibrium with prescribed levels of indifference for all consumers, a locus of points in output space can be found which keep each consumer just as well off as he is in the prevailing situation. But there is a different curve of this type for each initial distribution of utility,

each of which intersects the others in the initial point.
It is useful to re-state precisely what this means.
Consider Fig. 7.
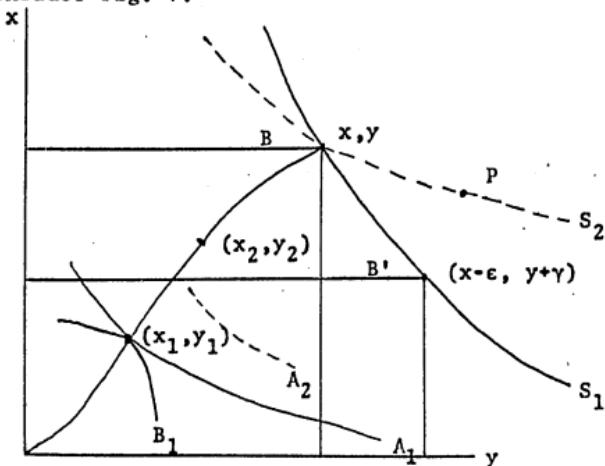


Fig. 7: Intransitivity of Market Ordering
Inferred from Compensated Changes

    A and B are individuals in equilibrium with alloca-
tion of goods X and Y initially established at $(x_1, y_1)$.
The totals of both goods are given by B's origin.
Suppose some change occurs which moves the production
configuration to B'. Less of X is produced and more Y.
Suppose that A were to suffer no loss of X due to this
change and gains the full amount of additional Y. A
has then gained from the change for his real income be-
comes that associated with indifference level $A_2$. B's
level of welfare, of course, falls accordingly. But if
goods collection B' is on a Scitovsky indifference curve,
A can just exactly compensate B for the change; if he

"pays" a combination of X and Y to B which restores B to his original level of well being, he (A) will also be left with his original level of indifference. Suppose a change to point P were proposed with A initially receiving the full benefit of extra Y while bearing none of the loss of X. Then if P were attained <u>and compensation paid</u> both A and B could be improved by the change. By the Pareto criterion, P is actually better than B. But there is a contradiction here; if the original allocation of goods was not $(x^1, y^1)$ but $(x^2, y^2)$, the Scitovsky curve $S_2$ would be relevant, and P is on $S_2$. Thus, P is socially indifferent to B (presuming the first distribution of income which yields initial allocation $(x_1, y_1)$) and b' is also indifferent to B (presuming the distribution yielding $(x_2, y_2)$): P cannot be unambiguously preferred socially to B. Thus, the Pareto partial ranking cannot be extended by the requirement of payment of compensation to yield a social ordering of points in output space.[3] It is precisely this realization which led Samuelson and Graaff into discussions of "potential welfare" in the economics

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[3]It is also important to note that, at least in this context, intransitivity is an all or nothing concept. Without transitivity, no social comparisons may be made in valid fashion. As will be discussed below, there are some writers who maintain that small ranges of inconsistent social choices are acceptable.

literature. This point is important because, as will
be seen below, one of the major lines of argument
intended to hurdle the possibility theorem amounts to
allowing for the payment of compensation.

Since the Pareto criteria are insufficient to order
points in goods-space, social welfare theorists have
attempted to locate an optimal point in utility space
by means of some explicit welfare judgment(s). The need
for a social welfare function which would rank points
in utility space in terms of some additional normative
axiom and, thus, fill out the welfare analysis was
discussed in the last chapter. Now we treat the matter
in greater detail.

A social welfare function is a rule for stating
levels of welfare for the society as a whole given
only information about the level of the individual's
welfare prevailing under various social arrangements.
Such a concept is extremely general. The possibilities
for interpretation of the content and nature of such a
rule are obviously great. Writers in the Bergson-
Samuelson tradition seem to have had only a peripheral
interest in specifying the content of the social welfare
function; their main intent has been to clarify the
methodological issue of the role of value judgments in
economic analysis.[4] The great number of possible inter-

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[4]See Samuelson, <u>Foundations of Economic Analysis</u>,
pp. 219-228.

pretations of the social welfare function, however, has led, in part, to the subtloty and nuance in the choice literature referred to by Strotz.

Since the social welfare function must yield an ordinal ranking of social states, the most general form of expression would be as a function of all conceivable variables characterizing a social state:

$$(1) \quad W = W[(z_1, z_2, \ldots, z_n), \ r, s, t, \ldots]$$

where the z's refer to the amounts of commodities con-sumed by each individual, the amount of labor expended by each consumer in each line of production, the amount of non-labor factors expended in each line, etc., and $r, s, t, \ldots$ indicate all other "non-economic" variables which may affect the welfare of the community. For any configuration of the variables, the function assumes a value which indicates the position of that social state relative to all others. Since welfare may be indicated by any index which preserves the order of social states, the value of the indicator has no cardinal significance.

The social welfare function thus reflects a social attitude toward various social states. But from where does this "attitude" emanate? It is usual to consider that individual preferences should "count" in the deter-mination of a social attitude. Suppose that each individual possesses a utility function which orders social states in terms of his level of consumption of

each of m goods under each state:

$$(2) \quad U_1 = f(z_1^1, \ldots, z_1^m)$$

If we make the value judgment that the social welfare function must reflect such individual orderings, then it can be written in the form:

$$(3) \quad W = W(U_1, U_2, \ldots, U_n)$$

This function is a "Bergson" function of the "individualist" type; the private utilities which are to be combined by the functional rule to an indicator of social welfare depend upon own-consumption only.

We may wish to allow individual attitudes to "count" but desire not to restrict individuals to consider only private consumption. Instead, let the individual utility functions vary in accordance with the level of well-being of others as well as self, and also in accordance with the values of the "non-economic" variables r,s,t,... Then the utility function of any one individual is essentially of the same form as (1):

$$(4) \quad W_1 = f[(z_1^1, \ldots, z_1^m), W_2, \ldots, W_n, r,s,t, \ldots]$$

If utility functions of this ethically broadened variety are allowed, the social welfare function has the form

$$(5) \quad W = W(W_1, W_2, \ldots, W_n) \quad [5]$$

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[5]W's are used as individual utility indicators merely to denote the departure from the individualistic

This welfare function is a general form of Bergson function. The $W_i$'s are utility indicators corresponding to individual orderings of social states and the function associates each configuration of individual utilities with a particular value of the social welfare indicator. Ethical judgments enter into this formulation at two stages. First, individual orderings may be based upon whatever standards of equity, justice, or other values the individual happens to employ. Second, the function presumes some judgment as to what weight each individual utility function is to be given in achieving a social valuation. Throughout the discussion which follows, this functional form will be deemed "the" social welfare function. But two ambiguities arise at this point. These must be discussed before examining the bearing of Arrow's contribution upon functions of type (5).

First, there has been considerable discussion concerning the differences in social ordering achieved by (3) as compared to (5). The social ordering presumed in (3) is derived from individual orderings according to the criterion of own-consumption only. The social ordering presumed in (5) is based upon individual rankings according to as wide a value structure as the

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assumptions employed in deriving (3). Note that by definition, the function admits of all forms of external relation.

individual chooses to use. Arrow originally raised the distinction between orderings based upon tastes only, and those based upon inclusive value structures.[6]

> In general, there will . . . be a difference between the ordering of social states according to the direct consumption of the individual and the ordering when the individual adds his general standards of equity (or perhaps his pecuniary emulation). We may refer to the former as reflecting the tastes of the individual and the latter as reflecting his values. The distinction between the two is by no means clear-cut . . . From a formal point of view, one cannot distinguish between an individual's dislike for having his grounds ruined by factory smoke and his extreme distaste for heathenism in Central Africa . . . it is the ordering according to values which takes into account all the desires of the individual, including the highly important socializing desires, and which is primarily relevant for the achievement of a social maximum.

Arrow thus implies that we really have a choice as to which individual orderings to admit, depending on the sort of criteria that we wish to allow individuals to resort to in achieving their orderings. Arrow does not pursue this point, however; he is content to assume that whichever set of orderings are taken as the basis for the aggregation to a social ordering must also be used by individuals as the basis for individual choice. Rothenberg has enlarged upon this issue and has concluded that since the "total personality" has but one set of attitudes

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[6]Arrow, Social Choice, p. 18.

> It makes no sense to say to an individual:
> first, order these alternatives on the
> basis of your "tastes," then on the
> basis of your "values." If he is
> asked to order alternatives in terms
> of their desirability to him, he has
> only one evaluative schema by which to
> do so.[7]

The apparent contradiction in these two views is
removed by the following consideration. Arrow wishes to
insist that the individual orderings to be amalgamated
by the social welfare function must also be the orderings
to which individuals refer in their choice behavior.
Suppose that some of the variables r,s,t,... appearing
in (4) are not subject to the exclusive control of
the individual; they are environmental characteristics
and must be provided collectively. We may ask the
individual to order all conceivable environments together
with his various possible positions within them. We
then have his ordering of social states in terms of (4).
But from an observation of the individual's pattern of
behavior, it may appear that his choices are taken with
reference to individualistic utility functions (3).
That is because if, for the purposes of choice behavior,
the individual assumes that he is confined to an
environment characterized by a particular set of values
r,s,t,...., he may not alter his behavior in such a

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[7]Jerome Rothenberg, The Measurement of Social Wel-
fare (Englewood Cliffs, N.J., Prentice-Hall, 1961), p. 33.

fashion as to contribute to a preferred environment.
He attempts to choose a pattern of behavior which will
give him his preferred position with the environment he
takes as given. In this case he may behave as if his
utility function were of the individualist type even
though his true preferences are comprehensive.[8] Of
course, nothing a priori can be said about whether the
individual will, in fact, behave in this manner. But if
he does, then, in effect, the choice orderings of indi-
viduals will not be identical with preference orderings
from which social welfare indicators are derived. This
is Arrow's real concern.

Rothenberg raises this issue when he writes:

> Noncontradictory sets of individual evaluations
> . . . can be disclosed only by examination
> of each individual's external relations--in
> effect, by individual orderings of all social
> states when external relations are admitted.[9]

If such interdependence is admitted, then the prob-

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[8]This is the case of interdependent decisions dis-
cussed in Chapter 2. Note that nothing in that example
suggests that the individual possesses different order-
ings of social states depending on whether he resorts
to his tastes or his values as criteria for judgment.
The inconsistency between individual choice and pre-
ference structures arises because, by assumption, the
individual does not recognize the relation between his
behavior with respect to one issue and the outcome of
some other interdependent issue. Whether or not he
possesses a comprehensive rather than an individualis-
tic utility function, his behavior will be the same.

[9]Measurement of Social Welfare, pp. 34-35. (Ital-
ics in original).

lem of reliability arises; the social optimum may not
be attained by private patterns of choice making. But
it is clear that that is not the issue at stake in
finding a rule of form (5). At present we are concerned
only with identifying an optimum. For this purpose it
is evident that the broadest notions of individual pre-
ference are the appropriate ones; there is no question
of choosing among various sorts of individual orderings.

The second ambiguity concerns the relation between
individual utility functions (4) and the social welfare
function (5). The function (5) reflects an attitude
toward various social states, but whose attitude is it?
Is (5) to be taken simply as some one of the $W_i$ in (4),
or is it some combination of the $W_i$ which may order
social states differently than any one individual?[10]
The latter view is adopted here. We are explicitly
seeking some concept of _social_ preference and this ap-
proach requires that (5) must be interpreted as some

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[10]Bergson has attempted to make clear that his
function was to represent orderings of social states by
individuals, based upon whatever ethical premises they
hold, as individuals. He insists that his interest
is not in a method for deriving a "master-ordering" (to
use Little's phrase) from the individual orderings.
Abram Bergson, "On the Concept of Social Welfare,"
Quarterly Journal of Economics, LXVIII (May, 1954),
p. 242. Yet he does require that there be some rule
for evaluating individual utility indicators resulting
under various social states. If this rule is not social-
ly chosen but emanates only from some one individual's
values, it is not likely to be of much interest. cf.
Samuelson, "Arrow's Mathematical Politics."

aggregation of all individual judgments (4).

Under this view, it is apparent that (5) may exist only if there is a rule for aggregating the individual welfare functions. That is, if some ordering of states to be taken as valid for the group as a whole is required, and if that ordering is to rest upon primary individual attitudes, it is logical to ask what kind of mechanism will operate, given only those attitudes as data, to generate the social ordering. This is the question Arrow investigated. He is interested in finding

> a process or rule which, for each set of individual orderings . . . for alternative social states (one ordering for each indivi- dual), states a corresponding social ordering of alternative social states . . .11

Arrow, in fact, chose to call this process itself the social welfare function. It is evident, however, that Arrow's process is not the same as (5). This is clear from the fact that some sets of $W_i$ might acceptably aggregate to a social ordering, W, while others might not. Arrow requires a process which will generate a rule (5) for <u>any</u> set of $W_i$.[12] Without such a process, however, we could not be sure that a rule (5) would exist for any configuration of individual preferences.

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[11]Arrow, Social Choice, p. 23.

[12]This matter will be considered in greater detail in the following chapter.

It is the possibility of passing from any set of individual utilities (4) to some rule (5) which we wish to consider presently. We will continue to assume that the content of all orderings are completely specified social states, although the following analysis is equally valid under some other interpretations of the alternatives.

Persons will not be indifferent to the form that an aggregation procedure such as we are considering will take. Someone whose tastes are different from Plato's would not be happy if the rule were to specify Plato as a philosopher-king and allow him to choose everything for everybody. Suppose that there are some values which are so pervasive in society that everyone would agree that the process should conform to those values. If there are such values, it must be shown that there is a process of combining individual attitudes toward social states which is consistent with them. Arrow's approach is to demonstrate the incompatibility of such a procedure with several supposedly non-controversial value judgments.

Arrow suggests five normative conditions as being particularly compelling or "reasonable"[13] in the sense

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[13]Arrow did not originally state his conditions in a form which made clear the extent to which these conditions underly market choice. The conditions stated here follow, with slight modification, the presentation of the conditions given in a modified proof of the the-

that virtually everyone would agree that the process
ought to be restricted so as to conform to them.

> 1) Condition of unrestricted domain--the
> process should allow all logically
> possible orderings of the alternative
> social states to be admissible.

The justification for this condition is straight-
forward. Since the process should fully account for
individual attitudes (tastes, values, whatever), it
should not restrict the rankings of individuals in
any way. Individuals must be sole judge of what is pro-
per to desire. Notice that it is presumed that indivi-
duals do have orderings; they are individually rational.
Arguments that social choice is irrational because
individuals themselves are inconsistent are ruled out.

> 2) The Pareto Principle--If all individuals
> prefer alternative x to alternative y,
> the social ordering must do so also.

This condition embodies two value judgments con-
cerning the aggregation procedure. First, it restricts
changes in the social ordering to reflect changes in
individual orderings positively. If from a fixed en-
vironment of alternatives and individual rankings, two
alternatives x and y are socially ranked $x R y$, then, if
one or more of the individuals raise x in their own

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orem in his 1963 edition so as to make clear the extent
to which these conditions do approximate assumptions
implicit in positive economic analysis. See _Social
Choice_, pp. 96-100.

rankings while no one lowers its position, the social ranking must still be xRy. Second, it implies that there is no social ranking of two alternatives which holds regardless of what the individual orderings are. The condition stipulates that citizens are to be sovereign; no rankings may be imposed by habit, custom, religious tradition, or other such arbitrary means.[14]

> 3) Condition of non-dictatorship--There must exist no individual i such that for all x and y, $xR_iy$ implies xRy regardless of the other rankings.

It must not be that the social ordering corresponds precisely to some one individual's ordering, regardless of the orderings of other individuals. This value judgment is obvious and requires no further comment.

> 4) Independence of Irrelevant Alternatives-- If an environment exists such that there are $(x_1,\ldots,x_m)$ feasible alternatives and $(x_{m+1},\ldots,x_n)$ infeasible alternatives, then if a change in individual orderings occurs for the $(x_{m+1},\ldots,x_n)$ alternatives which does not alter the individual orderings of the $(x_1,\ldots,x_m)$ alternatives, the social ordering of the $(x_1,\ldots,x_m)$ alternatives will not change.[15]

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[14]Arrow gives a proof that his original conditions of positive association and non-imposition together with his original third condition (Independence of Irrelevant Alternatives) imply this form of the Pareto Principle. _Ibid._, pp. 97, 52-53.

[15]This version of Arrow's original condition 3 is suggested by Rothenberg. See Rothenberg, Measurement of Social Welfare, p. 131.

This condition constitutes a stringent and controversial limitation on the process of aggregation. Its desirability depends not only upon an ethical premise, but also upon methodological issues. The signal role of condition (4) is to rule out the possibility of a social ordering which is sensitive to individual intensities of preference. Suppose that the social ordering between two feasible alternatives $(x_1, x_2)$ were to change in response to a change in an individual's ordering of alternatives $(x_{m+1}, x_{m+2})$. Initially, let the social ordering indicate $x_1 R x_2$. Suppose the individual's ordering of all alternatives initially is

$x_2 P_i x_1 P_i x_{m+1} P_i x_{m+2}$, and that his ordering changes to

$x_2 P_i x_{m+2} P_i x_{m+1} P_i x_1$. The new position of the infeasible alternatives in i's ordering may be taken as implying that the intensity of i's preference for $x_2$ over $x_1$ has increased. If the social ordering changed to $x_2 R x_1$ in response to i's change, the social ordering must take i's preference intensity into account. Condition (4) rules this possibility out and insures that only ordinal preference of individuals will count in the social ordering.[16]

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[16] Arrow originally gave a faulty example of the role of condition (4). He considers a case of individuals voting in an election of three candidates and states that condition (4) implies that if one of the candidates is removed from the election, the outcome as

Whether or not the social welfare function should
admit individual preferences weighted by intensity of
preference is a value judgment. But it is not clear that
such a procedure is possible, for that would require
means of observing individual preferences for infeasible
social outcomes. Arrow ultimately accepts the validity
of condition (4) in light of this methodological point.
He writes

> Only observable differences can be used as a
> basis for explanation. In the field of consumers'
> demand theory, . . . cardinal utility had no
> explanatory power above and beyond ordinal.
> It is the great merit of Bergson's 1938 paper
> to have carried the same principle into the
> analysis of social welfare. The welfare function
> was to depend only on indifference maps; in
> other words welfare judgments were to be
> based only upon interpersonally observable
> behavior. The condition of independence of
> irrelevant alternatives extends the require-
> ment of observability one step farther.
> Given the set of alternatives available for
> society to choose among, it could be expected
> that, ideally, one could observe all the
> preferences among the available alternatives, but
> there would be no way to observe preferences
> among alternatives not feasible for society[17]

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between the other two will not be affected. See _Social
Choice_, p. 26. Newman has pointed out that this
example violates the premise of a given environment.
When the environment changes (the field of candidates
is altered) statements about individual choice behavior
must be made in the context of uncertainty. See _Theory
of Exchange_, pp. 176-177

[17]Arrow, _Social Choice_, pp. 109-110. He also does
argue its legitimacy as a value judgment. But as his
1963 essay suggests, he is willing to sacrifice this
value judgment before any of the others.

In rendering the social ordering independent of all
unobservable individual preferences for infeasible alter-
natives, condition (4) insures that all individual
attitudes will be weighed equally by the social welfare
function.  The restriction of the process of aggrega-
tion to consideration only of orderings further implies
that all choice may be pairwise.  Since only orderings
matter, the social ranking may be generated by a series
of comparisons of alternatives taken two at a time.[18]
It is further guaranteed that in formulating the single
decision of identification of an optimal social state,
each individual will reveal his true preferences.
Only his ordinal ranking "counts" and, thus, there is no
incentive to dissemble.

Condition (4) is, therefore, a powerful restriction
on the aggregation procedure.  Further comment will be
made within a discussion of some of the literature which
has attempted to relax it.  For the purpose at hand,
the most important thing to notice is Arrow's explicit
extension of the concept of ordinality from market

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[18]Notice that any system of binary choice obeys
condition (4).  If all feasible alternatives are ranked
pairwise and each individual's vote counts equally on
each issue, the outcome between each pair will be inde-
pendent of preference intensity and rankings of other
alternatives besides the two in question.  Some proofs
of the theorem emphasize the role of this condition in
eliminating all but pairwise choices.  See, for example,
the proof by Inada, reproduced in Newman, Theory of
Exchange, pp. 181-184.

choice to a generalized system of social choice. It is
clear that the market does obey this condition. While
the independence of irrelevant alternatives does not exist
as an underlying value judgment as far as market choice
is concerned, it is incorporated into the model as a
reasonable approximation to reality. Consumer dollar-
votes count equally in determining the final composition
of output and equilibrium is independent of changes in
individual rankings of non-feasible configurations of
goods.

    5)  Condition of internal consistency--the
        aggregation of individual orderings must
        yield a social preference relation which is
        complete and transitive.

    The desirability of this condition is evident. The
purpose of deriving an aggregation procedure is to enable
society to select a social welfare function which may
be employed as a normative guide to social choice.
Social choices may be normatively meaningful only if
they correspond in some reasonable way to a social list-
ing of alternatives. That is, just as the theory of
rational individual choice is predicated upon the
existence of a preference ordering of the individual,
so too must a theory of rational collective choice be
predicated upon a social ordering. If actual social
choice bore no reference to a meaningful concept of
social preference, actual social outcomes would be dev-
oid of any normative significance. This condition has

met with some criticism, however. Completeness does not raise much controversy. As Arrow remarks:[19]

> Connectedness, when understood, can hardly
> be denied; it simply requires that some
> social choice be made from any environment.
> Abstention from a decision cannot exist;
> some social state will prevail.

The requirement of transitivity has been subject to more disagreement. Buchanan is chief among dissenters to the transitivity requirement. His objections will be considered in the following chapter. As was indicated in the first chapter, however, we accept condition 5 as a _sine qua non_ of collective rationality.

Given the five value judgments above which are to restrict the process of aggregating procedure for individual preferences, Arrow's theorem may be stated:

> There is no aggregation procedure for
> individual orderings of some set of more
> than two alternatives which satisfies
> conditions 1-4 and also satisfies condi-
> tion 5.

Proofs of this theorem (or very similar versions) may be found in several places[20] and I shall not repro-

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[19]Arrow, _Social Choice_, p. 118. It is well to recall at this point, however, that a complete social ordering is not a necessary condition for a existence of a choice set, as defined in Chapter 1. See Patanaik, "A Note on Democratic Decision and the Existence of Choice Sets." For the present, we accept the stronger requirement of the existence of a social ordering.

[20]See Arrow, _Social Choice_, pp. 98-100 for a proof which seems as straightforward and intuitively appealing as any other.

duce one here. Recall, however, that the method of proof is first to show that if one individual is decisive between any pair of alternatives, he is by implication, decisive between any pair of alternatives in the set to be ordered; he is a dictator. Therefore, no one individual may be decisive between any pair of alternatives. Then it can be shown by an application of voter's paradox that if individual orderings are sufficiently disparate, there is always at least one pair of alternatives from any triple upon which one individual will be decisive.

This result implies that there is no way of deriving a social ordering of completely defined social states from all logically possible individual orderings of social states. This, in turn, implies that for some possible configurations of individual welfare functions, $W_i$, no social welfare function of the form $W = W(W_1, \ldots, W_n)$ will exist. This is perhaps the most widely accepted interpretation of the theorem. But it is not the only possible interpretation. The proof of the Possibility Theorem is general enough so that the theorem will hold regardless of the content of the orderings. Four distinct interpretations will be given below. By distinguishing between these various interpretations, it is possible to clarify the extent to which Arrow's theorem is a generalization of the same problem of social choice

which lead to the introduction of a social welfare function in the first place: the intransitivity of market choice. The relation of the theorem to welfare economics is thereby brought out in greatest relief. The classification of various interpretations also provides a framework for the evaluation of several criticisms of Arrow's approach to be undertaken in the following chapter.

1. <u>Aggregation of Choice Structures.</u>

Suppose that individuals have utility functions but for the purpose of aggregation, we are not interested in their preferences but in the nature of the <u>decisions</u> each will make on various issues. In a certain world, we may derive individual orderings of alternative patterns of choices from the individual utility functions. For example, we might consider a consumer who ranks different patterns of purchasing given price ratios and his budget. We can define a choice ordering over alternative commodity bundles for each individual. Alternatively, we may postulate a rule which orders patterns of decisions with respect to all issues on which the individual is concerned (e.g., the amounts of consumption of each of m commodities). We map the individual utility functions defined over a commodity space, from which actual behavior is inferred,

$$U_1 = F(x_1, \ldots, x_m)$$

directly into the decision space itself:

$$D_1 = F(d_1^1, \ldots, d_1^m)$$

where $d_i^k$ represents a decision of the ith individual on the kth issue. For the moment, assume that the individual rankings of decision patterns depend only upon choices subject to the individual's control. Given such a set of orderings, we may inquire whether a rule exists for determining a social ordering of various configurations of individual choices which is also defined in the m-dimensional decision space. For n individuals, we seek a rule of the form:

$$D = F(D_1, \ldots, D_n)$$

In other words, individual orderings of decision patterns must aggregate to a social ordering of those patterns. Arrow's theorem implies immediately, however, that there is no method of aggregation of these orderings which is consistent with conditions 1-5.

It is important to note that this result is general corroboration of the finding of the "new welfare economics" that market choice does not imply a transitive social ordering of social outcomes. Conditions 1-4 are built into the market choice system. In the standard welfare economics model any conceivable individual rankings of bundles (and consequent behavior) are allowed, the Pareto judgment is made explicitly, only ordinal preferences are assumed, and the market responds to all

individual tastes, not merely those of some dictator.
But this system does not satisfy condition 5; this is
indicated by the intersection of Scitovsky indifference
curves. Market rules of aggregation of individual choices
will not yield an unambiguous social ordering of deci-
sions with respect to the m issues. When individual
orderings are of decision patterns, Arrow's theorem may
be interpreted as a proof that the values implicit in
market procedures are not sufficient to impart a
normative significance to social outcomes achieved by
such procedures.

Arrow's demonstration is general enough, however,
to permit a widened view of the types of choices to
be aggregated. The individual choices need not refer
only to private market choice. They may also include
rankings of social decisions to be made not by indepen-
dently acting individuals, but by some collective author-
ity. That is, alternatives to be ranked both by in-
dividuals and society may include "a vector whose
components are values of various particular decisions
actually made by the government such as tax rates,
expenditures, antimonopoly policy, and price policies
of socialized enterprise."[21]  If this vector is indicated

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[21]Ibid., p. 87. We are considering only attitudes
toward various decisions; no uncertainty has been intro-
duced by including decisions of central authority.

by $(g_1, \ldots, g_s)$ where there are s such collective deci-
sions to be made, the decision function of the indivi-
dual may be written

$$D_1 = F((b_1^1, \ldots, d_1^m), (g_1, \ldots, g_s))$$

Individual orderings of decision patterns defined over
this broadened field of decisions still may not be ag-
gregated without violating one of Arrow's conditions.
Aggregation of individual choice structures to a social
choice structure is impossible (in the sense of satis-
fying the 5 conditions) even if choice is to be mediated
through procedures other than the market. Some further
explicit second order value judgment must be made in
order to insure that the $D_i$'s will, in fact, be con-
strained to a configuration such that the aggregation
procedure will yield an acceptable social ordering.[22]

    2. <u>Aggregation of Value Structures</u>.

    The first interpretation, therefore, leads us to ask
how such a second order judgment may be made. We would
wish that the judgment be derived in some way from all
the value judgments of individuals and that all indi-
vidual attitudes "count" in the derivation of the
social judgment. Now we have precisely the interpre-
tation assumed above in the development of the theorem.

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[22]In terms of the economic welfare model, this
judgment takes the form of a determination of the
purchasing power distributed to each individual.

We consider that individuals possess utility functions of form (4) on page 115 above; preferences for social states are based upon equity, justice, and other ethical notions, in addition to own-utility considerations. Our problem is to see if individual orderings may aggregate to a social ordering of social states when all values of persons are taken into consideration, and if the social ordering will imply the second order social welfare judgment which was shown to be required under interpretation 1.

The essence of Arrow's contribution is to show that if we want a process of making such a social judgment to be consistent with the same 4 value judgments implicit in market choice, condition 5 must be violated; such a decision cannot be made. The problem is exactly analogous to that in the first interpretation, only now we are considering the aggregation of individual value judgments into a social judgment rather than aggregation of choice behavior to an optimal social choice directly. That there is no method consistent with the 4 conditions which can map individual orderings of social states based upon inclusive values into a social ordering implies that there is no distributional value which may be inferred from all individual orderings including such values.

The significance of this result for welfare economics

is apparent. Without a social ordering which implies
a distributional value, we cannot construct social
welfare contours in utility space thus completing the
welfare model. Once we recognize the impossibility of
generating social indifference functions taking only
individual choices as data (interpretation 1) it is of
no help to postulate a social welfare function based
upon individual value mappings and use the optimum
thereby implied to aid in the construction of community
indifference curves. If the underlying value conditions
of the market are carried over to the problem of
aggregating individual values, the realm of impossibili-
ty is merely pushed from goods-space to utility space.[23]
One way to put this conclusion is that just as there
is no way to assign weights to consumer utilities
under ordinalism, neither is there a way of distributing
"voting weights" to choosers of various distributioanl
alternatives if ordinalism is preserved in the aggrega-
tion of value structures by the imposition of condition

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[23]In other words, if the social welfare function
is considered as a rule which must meet the same con-
ditions as voluntary exchange as in free markets, the
General Possibility Theorem rules out the attempt to
construct a social indifference curve. See Samuelson,
"Social Indifference Curves." Samuelson's attempt
entails lump-sum transfers of purchasing power which vio-
late condition 4. See Arrow, Social Choice, p. 110, n.
Of course, it is not implied that a social indifference
relation will not exist for some configurations of
individual preference, but that it will not exist for
any conceivable configuration of individual preference.

4. This is the paradox of social welfare analysis:
the same reasoning which results in a demonstration of
the necessity of a social welfare function may be used
to show that such a mechanism cannot be found.

### 3. Aggregation of Decision Rules.

Since it is not possible to specify an optimum
social state based upon individual value orderings, we
might wish to attempt to push the social ordering to be
obtained one step further back from a straightforward
specification of ideal output. We have established that
a social choice structure may not be derived from
individual choice structures without a second-order
value judgment concerning distribution of utilities.
But the inference of such a judgment from individual
value orderings may not be made in the absence of a
further judgment concerning the distribution of voting
power. That is, we cannot call upon a "supplementary"
political mechanism to establish the correct distribu-
tion of utilities because, under the value conditions
to be met by such a mechanism, weights associated with
individual votes may not be specified. Suppose, then,
that we consider alternative ways in which the worth
of votes may be established together with alternative
rules by which votes may be counted in order to give
rise to a social outcome. We may call such specifica-
tions of precise ways in which individual orderings are

to be counted in passing to the social ordering <u>social</u> <u>decision</u> <u>rules</u>. Assume that all individuals have order-ings of alternative decision rules

$$C_1 = F(c_1^1,\ldots,c_m^k)$$

where the value of $C_1$ depends upon the particular decision rule $c_i$ employed for each of 1 through m deci-sions. The question now is whether a socially optimal configuration of decision rules may be generated by some method which takes only individual preferences for such rules as data. The Possibility Theorem implies that there is no such method.

The selection of rules by which choices are to be made, as distinct from the actual choices about ends, is the problem of the establishment of a <u>constitution</u>. Under this interpretation, therefore, Arrow's contri-bution is a non-existence theorem on constitution, as opposed to welfare, functions. But the same contra-diction arises in either case. A procedure concerned with preferences about methods of making choices con-sistent with conditions 1-5 is impossible just as a procedure concerned with aggregating preferences about social states is impossible. We have reached an infinite regress, apparently; no matter how far back we push the problem of establishing a social ordering which may serve as a standard by which to judge social outcomes, the derivation of that ordering may not be

achieved. Arrow touches upon this interpretation briefly in the last section of his book.[24]

From a logical point of view, some care has to be taken in defining the decision process since the choice of decision process in any given case is made by a decision process. There is no deep circularity here, however. If $x$ is the vector describing a possible social state, let $x_1$ be the components of that state which are not decision processes; let $x_2$ be the process of deciding among the alternative possible $x_1$'s; in general, let $x_n$ be the process of deciding among the alternative possible $x_{n-1}$'s. We may refer to $x_1$ as the first-order decision, $x_2$ as a second-order decision, etc.; then an nth-order decision is a process of choosing an (n-1)th-order decision method. Any particular social state is described in its entirety by a vector of the form $(x_1, x_2, \ldots, x_n, \ldots)$. In describing the United States Government, we might say that $x_1$ is a proposed bill or, more precisely, the proposed bill taken into conjunction with all the legislation now on the books; $x_2$ is the process by which bills are enacted into law by Congress and the President; $x_3$ is the process of choosing a Congress and the President, set down by the constitution, and $x_4$ is the process of constitutional amendment.

Suppose that for some value of n there is one possible $x_n$ which is so strongly desired by all individuals that they prefer any social state which involves accepting that particular $x_n$ to any which does not. For example, the belief in democracy may be so strong that any decision on the distribution of goods arrived at democratically may be preferred to such a decision arrived at in other ways, even though all individuals might have preferred the second distribution of goods to the first if it had been arrived at democratically. Similarly, the desire for a dictatorship or for a particular dictator may be overwhelming under certain conditions. In such a case, again, our social welfare problem may be regarded as solved

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[24]*Ibid.*, p. 90.

since the unanimous agreement on the decision
process may resolve the conflicts as to the
decisions themselves.

But since we have no reason to be assured that
unanimity is present at any of these levels, the problem
is not solved. Arrow's theorem implies that there will
be no suitable aggregation of individual preferences
at any level if the individual preferences are permitted
to assume any conceivable configuration. It must be
added also that it is far from clear that unanimous
agreement on the decision process would, in fact,
"resolve the conflicts as to the decisions themselves."
This result would have to be demonstrated in some par-
ticular case.

4. The Consistency of a Group Decision Procedure.

Under the three interpretations above, the theorem
concerns the existence of an aggregation procedure
for individual orderings defined by some particular
operator which related objects in some particular space
of alternatives. There is, as yet, no reason to expect
the theorem to apply to situations where a change in
the space of alternatives or the nature of the operator
is involved.[25] We do not have proof, for example, of

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[25] Newman recognizes this problem clearly, but does
not involve himself in a discussion of its implications
for welfare economics. See Theory of Exchange, pp. 175-
176. Little recognizes the distinction but is content
to conclude with its aid only that Arrow's "social
welfare function" cannot be identical with Bergson's.

the non-existence of a procedure of passing from individual value orderings of social states to a social ordering of various decisions to be taken with respect to each issue. In other words, the impossibility of passing from individual values to social choices has not been demonstrated. It is this transition which we wish eventually to make, as the title of Arrow's book implies.

The impossibility of passing from individual values to social choices could only be inferred, at this point, if the assumption is introduced that individual values are fully reflected by individual choices. Then, if individual value structures are such that they do not aggregate to a social ordering, it is implied that individual choice structures will not aggregate either. But the examples given in Chapter 2 indicate the inadvisability of making such an assumption. Since many of the elements of social states which are ordered by individuals will be of such a nature that individual choices with respect to them will affect other indivi-

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I. M. D. Little, "Social Choice and Individual Values," Journal of Political Economy, LX (Oct., 1952). Buchanan hints at the distinction in an early article, but veers away to argue that the very concept of rational collective choice is meaningless. James M. Buchanan, "Social Choice Democracy and Free Markets," Journal of Political Economy, LXII (April, 1954). With these exceptions, the distinction between these problems has seemingly gone unnoticed.

duals' welfare, and since this implies that some of
the decisions to be made by the individuals are not
independent, it may be that individual orderings of
various decision configurations will not be consistent
with their corresponding orderings of social states.

We may consider, then, the possibility of passing
directly from individual value structures to social
choices by means of a group decision procedure which
specifies a set of rules which determine a social out-
come from the actual voting behavior of individuals.
While this is not the procedure which the Possibility
Theorem refers to, it is possible to modify Arrow's
conditions so that the non-existence of this type of
group decision procedure may be "proved." To achieve this
result, it is necessary to further restrict the way in
which individual orderings may vary in response to
changes in the set of alternatives ordered. Since
condition (4) is the only one which restricts orderings
to correspond to changes in the field of alternatives
in a particular fashion, we may strengthen it to read

4') If a set of feasible alternatives
$(x_1,\ldots,x_m)$ exists, then if some $x$,
say $x_m$, is changed in the individuals'
orderings, the individual orderings of the
remaining $(x_1,\ldots,x_{m-1})$ alternatives
must not change, and the social ordering
of the $(x_1,\ldots,x_{m-1})$ alternatives must be
the same after the change as it was before.

An example may indicate the effect of this condition
on social decision procedures. Suppose over some triple

of alternatives in X individual orderings are

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A:   x   y   z
B:   y   z   x
C:   z   x   y
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We consider the single choice between some pair in this triple. The social choice between y and z will be y. Now suppose that x goes down in B's ordering, so that he desperately prefers both y and z to x, but barely prefers y to z. In that event, B might wish to vote for z over y in a decision between those two if he thought that the others' preferences were such that z would more likely be chosen over x than would y. But 4' explicitly rules out this possibility, the social choice between y and z must still be y after the change in B's ordering. It is clear that 4' stipulates that all individuals must vote according to their true preferences when choosing between any pair of alternatives. The decision procedure must not admit notions of preference intensity. The role of this condition is similar to that of condition 4, but now we are considering the consistency of actual social choices made by means of the group decision procedure.

With this modified condition, Arrow's theorem implies that there is no group decision procedure which is consistent with conditions 1-3, 4' and which is also transitive. Transitivity in this context must be interpreted somewhat differently than in other contexts, for the group decision procedure does not create an ordering

of social choices. It merely gives rise to a particular
decision. Transitivity here means that if the environ-
ment is allowed to change, the same social decision will
result in all instances where the change in environment
is the same.[26] If we interpret "change in environment"
to mean the new possibility of attaining one of some
triple of alternatives (heretofore unattainable) the
choice of that one alternative may depend only upon the
order in which the three alternatives are voted upon
(if the individual choice structures are sufficiently
disparate) and this means that the same social decision
will not necessarily result in any instance where similar
changes in opportunities occur. Thus we may say the
group decision procedure is intransitive.

The appropriateness of placing restriction 4' on
the group decision procedure may be questioned, however.
Individuals realize that the social outcome generated
by a group decision procedure will depend upon how all
persons vote. One's utility maximizing strategy will
depend upon beliefs concerning how others' preferences
are structured, as was discussed in Chapter 2. Choice
via a group decision procedure is, thus, characterized
by uncertainty, and in such a context it may be unrealis-

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[26]This interpretation has previously been suggested
by Buchanan. See "Social Choice, Democracy and Free
Markets," p. 122.

tic to assume individual behavior which reflects true preferences. It is also true that if the decision procedure treats interdependent issues separately, individuals may fail even to recognize what kinds of voting choices are consistent with his value ordering.

But if we do not restrict the decision process by condition 4', then social decisions actually made may not conform to the social ordering of social states, even if this latter ordering did exist. Without condition 4' the group decision procedure may generate consistent choices. But since the absence of 4' permits individuals to vote in some pairwise comparisons in ways not consistent with their true preferences, the social choice may not be the optimum indicated by the aggregation of individual preferences; the problem of reliability of the decision procedure arises. Merely finding a method of making consistent social choices, then, does not solve the problem of internal consistency as defined in Chapter 2. Such a mechanism does not insure collective rationality. Its existence does not mitigate the need for an acceptable aggregation procedure for transforming individual values into a social ordering of social states.

It has often been assumed that Arrow intended his theorem to be applicable to the existence of a consistent decision procedure. That literature which does so has

been concerned mainly with the reasonability of condition 4'. This literature is considered in the next chapter.

B. Conclusions.

This chapter may be concluded with the following remarks. The conditions Arrow placed upon procedures for aggregating individual orderings are those implicit in market choice. As long as those same value judgments are maintained for methods of non-market choice also, we may not be sure that there is a process for aggregating the individual orderings (in any space of alternatives) which actually obtain in society into a social ordering. Two approaches for solution immediately suggest themselves. Were condition 4 relaxed or eliminated, then at some level of aggregation a judgment with respect to distribution (based on some concept of cardinality of the orderings) might be made. The problem of infinite regress disclosed by interpretations 1-3 would be eliminated. Unfortunately, most of the literature which is purportedly concerned with this approach has confused it with the consistency of a group decision procedure.

Alternatively, we might undertake to determine whether or not the set of individual preferences actually obtaining in society are of a sort that will aggregate. Arrow, of course, requires a procedure which will

aggregate any conceivable set. If actual orderings do aggregate, at some level of choice, then it is a trivial matter to relax condition 1.

Further, since there are several different aggregation problems to which Arrow's theorem is applicable, the proper strategy for obtaining an acceptable aggregation procedure may be different in each case. It is not necessarily so that it would be proper to relax the same condition under all interpretations. In particular, since whatever interpretations are to be considered, the procedure for collectively rational decision must be in two stages, the identification of the optimum and then its selection, one might speculate that one condition could be relaxed for the purpose of identification of the optimum but could be maintained while another condition was relaxed for the purpose of choosing the optimum, thus resulting in a situation where all 5 conditions are satisfied by the two processes taken together.

CHAPTER 5

Arrow's Theorem, Social Welfare Functions And The
Independence of Irrelevant Alternatives

The importance of Arrow's theorem to the problem
of rational social choice is clearly indicated, by the
interpretations suggested in the previous chapter. Under
interpretations 1-3, the theorem implies that it is im-
possible to denote any pattern of choices, any social
state, or any particular configuration of decision rules
as optimal given only individual orderings as data,
if we wish to maintain a process of aggregating those
orderings which satisfies the five value conditions.
Under interpretation 4, the theorem suggests that even
if there were an optimum, there is no group decision
procedure which would choose it consistently, if that
procedure is also to satisfy the five (slightly modified)
conditions. The theorem is related to both the prob-
lems of internal consistency and reliability.

The severity of these conclusions, of course, calls
for a cautious evaluation. Much literature has appeared
which has argued that the strictures of the theorem are
not as serious as they appear. Early assessments of
the General Possibility Theorem contended that Arrow
wrongly conceived the entire problem of social choice.
This argument is essentially that while Arrow proved

the non-existence of a class of processes for aggregating individual values, this process is neither the same as nor does it preclude the social welfare function of the economics literature. Therefore, his results are not applicable to welfare economics although they may be of some interest in political science.

To other writers, the non-existence of a method of passing from individual values to implied social choice consistent with Arrow's conditions indicated simply that these conditions are too strong. The condition which has come under strongest attack is (4): the independence of irrelevant alternatives. The fundamental notion underlying this literature is that while the market process obeys condition 4, other mechanisms of choice do have the ability to make interpersonal utility comparisons and so it is unreasonable to restrict them not to do so. It is just this feature which is inherent in political processes of choice and which differentiates them from market processes.

Still a third possibility is that while Arrow's analysis is correct, it is unlikely that citizen tastes or values would truly be so disparate as to yield the type of intransitivity Arrow finds. Therefore, it is appropriate to search for minimal restrictions on the orderings of individuals such that intransitivity of social orderings would not arise. If restricted

individual orderings seem "reasonable" under real world
conditions, then Arrow's proof would be of no consequence
to the possibility of collectively rational decision.

I review here some of the recent literature which
considers the first two of these three cases. The third
will be treated in the next chapter. No attempt is made
to present an exhaustive review. The point to be
developed is that each of these approaches is legitimate
under some interpretations of the theorem given in the
last chapter, but none is appropriate in every case.

A. The Possibility of a Bergson Social Welfare Function.

The earliest reaction to Social Choice and Indivi-
dual Values was directed almost entirely to showing
that Arrow's work did not prove the impossibility of
a Bergson social welfare function and, therefore, bore
no relevance to formal welfare economics. Of course,
even if this contention were true, there is no reason to
assume that Bergson's formulation established a unique
claim on the welfare economics problem; Arrow's contri-
bution is an important one in social choice theory even
if his formulation is different from Bergson's. Some
important insights arise in this literature, however.
Little set the stage for debate in his brief, but broadly
based, review essay.[1] Little's criticism is in three

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[1]Little, "Social Choice and Individual Values."

stages. First, he interprets the Bergson social welfare function as

> "a process or rule" which would indicate the best economic state as a function of changing environment (i.e., changing sets of possibilities defined by different economic transformation functions), the individual tastes being given.[2]

Arrow's process, however, defines the optimal social state as a function of tastes of all individuals, as is clearly implied by conditions 2 and 4 above. It considers changes in the social ordering in response to changes in tastes of individuals for alternatives. Little, therefore finds Arrow's rule to be different from Bergson's. A second difference is that Bergson's function

> requires only that there should be an order (of social states). It does not require that it should be an order such that anyone would want to say of it that it represented choices of society. To call an ordering a social ordering at all implies that one approves of the order, or of the mechanism (if any), through which it is determined by the individual choices or preferences . . . the so-called "social welfare function", postulated by welfare economists, should be regarded as a social ordering only in the sense that it orders states of society . . . none of the advantages claimed for theoretical welfare economics, as a result of introducing such a function, depends in the least on the ordering of economic states being an ordering by society.[3]

In other words the Bergson function is only a

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[2]Ibid., p. 423.

[3]Ibid., p. 424.

"social" welfare function because it orders social
states. It does so, however, only by reference to one
individual's tastes or values. Since Arrow's process
produces a social ordering as a function of the tastes
of all individuals, his analysis has nothing to do with
"what is commonly thought of as welfare economics."[4]

Even if Arrow's process is not the traditional
rule of welfare economics drawn from one individual's
value structure, the question remains whether Arrow's
function may be rightly considered a social welfare
function. Little argues that it cannot. He envisions
Arrow's process as a "machine" which has value orderings
of individuals fed into it from which it generates a
"master" order. But the "master order" does not consist
of value judgments, for such judgments require that
some one person has made them. The "machine" merely
rules between conflicting judgments; it is a decision-
making process.[5] It is worth quoting Little at length
on the implications of this view:

> But what would it mean to call the machine a
> social welfare function? One would be asserting,

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[4]*Ibid.*, p. 424.

[5]By which Little apparently means what has been
termed above, a "group decision procedure," although
he might equally well mean "an aggregation procedure
for choice structures" under conditions of perfect cer-
tainty. Little does not draw the important distinction
between these two interpretations.

in effect, that if the machine decided in favor
of x rather than in favor of y, then x would
produce more social welfare than y or simply be
more desirable than y. This is clearly a value
judgment, but it is, of course, a value judgment
made by the person who calls the machine a social
welfare function. Thus, in general, to call
the machine a social welfare function is to assert
that x is better than y whenever the machine writes
the sentence "x is better than y." Now we may
suppose that the individual who calls the machine
a social welfare function is one of those who
has fed his own value order into it. It is clear
that this person must be contradicting himself
unless the "master"-order coincides with his
own value ordering. It follows that if the
machine is to be called a social welfare function,
then anyone who is called upon to accept or
reject the principles on which it is built
(i.e., the conditions of correspondence) must
refuse to accept any principle which insures that
the "master"-order will necessarily not coincide
with his own. This is because the conditions
of correspondence determine the "master"-order
and because by calling the machine a social
welfare function the person in question has
accepted the "master"-order. (It should be noted
that accepting or rejecting a value sentence
entails agreement or disagreement with the cor-
responding value judgment.) In other words, it is
inconsistent both to call the machine a social
welfare function and to accept the condition of
nondictatorship.[6]

Thus, it would not be proper to interpret Arrow's

rule as a social welfare function:

> We must . . . make an important distinction which
> Arrow fails to draw. He calls his function both
> a social welfare function and a decision-making
> process. He believes that "one of the great
> advantages of abstract postulational methods is
> the fact that the same system may be given
> different interpretations permitting a considerable
> saving of time." . . . but we must be careful
> not to give such a system a nonsensical interpreta-

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[6]Ibid., p. 427.

tion, and . . . to interpret it as a social
welfare function is to give a nonsensical inter-
pretation.[7]

Arrow's theorem, in Little's view, then, may be,
appropriate under interpretation (4) above, but is not
appropriate under interpretation (2).

Kemp and Asimakopulos reach the same conclusion.[8]
Since Arrow's function is a rule for choosing among
conflicting individual welfare judgments, and is not
itself a welfare judgment, these authors suggest that
he should call his function a "constitution function"
rather than a social welfare function.

Now correctly interpreted, the problem of the exist-
ence of a constitution arises under interpretation (3)
above. A constitution would arise if there were a
process of aggregating orderings of decision rules for
individuals to a social ordering consistent with Arrow's
conditions. The problem is quite the same as that under
interpretation (2); only the content of the orderings
is changed. Therefore, Arrow is able to say that the
difference between social welfare functions and consti-

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[7]Ibid., p. 427. The quotation from Arrow is drawn
from a section where aggregation of orderings of deci-
sion rules is under discussion, not from where "group
decision procedures" are directly at issue.

[8]M. C. Kemp and A. Asimakopulos, "A Note on 'Social
Welfare Functions' and Cardinal Utility," Canadian
Journal of Economics and Political Science, 18 (May, 1952)
p. 195 n.

tution functions is largely terminological.[9]  If there is no constitution in this sense, there is no way of determining a welfare function of the Bergson form. Without a method of selecting one Bergson social welfare function as operative for society, the very concept of a Bergson function is operationally sterile.  Arrow has stated, however, that "it is indeed a social decision making process with which I am concerned and not, strictly speaking, a welfare judgment by an individual,"[10] and since he is concerned with the rule for achieving a social ordering rather than with the ordering itself, he has agreed to the usage "constitution function" rather than "welfare function" for his process.

But Arrow does not seem to see that the term "constitution" may be interpreted as the group decision procedure which actually generates social choices from individual preference orderings.  The difference between this view of "constitution" and the social welfare function is not merely terminological, as the argument of interpretation (4) illustrated.  The group decision procedure passes from individual orderings to social action directly; it is not concerned with the problem of ordering decision rules optimally for society given

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[9]Social Choice, p. 105.

[10]Ibid., p. 106.

individual orderings of rules.  That Arrow does not
see this problem is evident from the following statement
in which he assents to the term "constitution function"
for his process:[11]

> the process of formulation of welfare judgments
> is logically equivalent to a social decision pro-
> cess or constitution.  Specifically, a constitution
> is a rule that associates to each possible set of
> individual orderings a social choice function;
> i.e., a rule for selecting a preferred action
> out of every possible environment.

But neither the choosing of a welfare judgment nor
the choosing of decision-rules is the same as the choos-
ing of a social action.  Arrow here slips almost imper-
ceptively from the space defined over value orderings
to the space defined over choices.  It is the distinc-
tion between these problems that caused critics such as
Little, Kemp and Asimakopulos to insist that he call
his function something other than a social welfare
function in the first place.  The existence of a social
welfare function is contingent upon the existence of a
constitution in the sense of the paragraph above; its
existence is not contingent upon the existence of a
group-decision procedure which can choose consistently
among alternative social actions.  One can hardly believe
that Arrow meant the words in the quotation above.  If

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[11]Arrow, "Public and Private Values," _Human Values
and Economic Policy_, p. 13.  Emphasis added.

he did, then he should have little disagreement with
Little on the relevance of the theorem. On the other
hand, since Arrow has appeared to agree with the view
of his critics, one may appreciate their confusion at his
maintaining that the theorem yet has relevance to social
welfare functions of the ordinary type.[12]

It is apparent from the discussion above that
Arrow's function is not the same as the Bergson social
welfare function. The Bergson function is a rule for
choosing a social state based upon the given tastes and
values of one individual. The Arrow function is a rule
for determining the value of each element of welfare
from opinions of all individuals as to what the value
of each element should be; it is a rule for determining
which Bergson function should obtain for society. If
one agrees with Arrow that the Bergson function is not
a useful concept unless it is possible to show that such

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[12]Samuelson, for example, states that he does not
find Arrow's rebuttal to Little's arguments convincing.
See Samuelson, "Arrow's Mathematical Politics," Human
Values and Economic Policy, p. 48. The reason must be
that he does not see that there is a definition of "con-
stitution" which if non-existent, would preclude the
existence of social welfare functions. Arrow, however,
does not see that the non-existence of a group decision
procedure does not preclude a procedure for aggregating
preferences. My own view is that if "social welfare
function" is not the proper term for Arrow's function,
the term "constitution function" is at least equally mis-
leading. Such a term tends to indicate that the theorem
is relevant only under interpretations (3) or (4) above.
These interpretations lead one to believe that the impor-
tance of Arrow's work lies within political theory, to

a function may actually be selected, then, since the non-existence of the Arrow aggregation procedure for either values or decision rules implies that a function of the Bergson type cannot be achieved for society, Little's second criticism, that the Bergson function is aranking of social states by any one individual, and not a combinational rule for achieving a social ranking of social states, is answered; Arrow's finding certainly is relevant to pure welfare economics.

But if it is considered that the problem of social welfare is one of achieving a social ordering for social states from individual functions of the Bergson type, then Little's first criticism, that welfare functions presume given tastes and do not specify responses of the social welfare function to changes in individual tastes, is also of dubious merit. Rothenberg makes this point in the following way.[13] Let R,

$$(1) \quad [R] = \begin{bmatrix} R_{A_1} \\ R_{A_2} \\ \vdots \\ R_{A_n} \end{bmatrix} = R(U^1, U^2, \ldots, U^m)$$

the exclusion of economic theory. But the relevance of the theorem to economic theory under interpretations 1 and 2 is clear.

[13]Rothenberg, The Measurement of Social Welfare, pp. 37-41.

be a social ordering of h social states implied from some
Bergson Social Welfare Function of the form

$$E = E[u'(x_1^1,\ldots,x_n^m),\ldots,u^m(x_1^1,\ldots,x_n^m)]$$

where there are m individuals and n goods. Each indivi-
dual also has an ordering of h social states,

$$(2) \quad [R^i] = \begin{bmatrix} R_{A_1}^{\;i} \\ R_{A_2}^{\;i} \\ \vdots \\ R_{A_h}^{\;i} \end{bmatrix} = [R^i][u^i(x_1^{\;i},\ldots,x_n^{\;i};$$

$$x_1^1,\ldots,x_n^1;\ldots;x_1^m,\ldots,x_n^m)]$$

Then

$$(3) \quad R(u^1,u^2,\ldots,u^m) = [R] = f(R^1,R^2,\ldots,R^m)$$

It is clear from (3) that, where the central problem
is to develop s social ordering of social states, the
social ordering must respond to changes in the $R^i$. The
view that "we do not require that the difference between
the new and the old [social] ordering should bear any
particular relation to the changes of taste of which have
occured,"[14] is only true, when the problem is viewed
this way, <u>if the rule $R = f(R^1,\ldots,R^m)$ itself</u> is a func-
tion of tastes, and if every change of tastes implies

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[14]Little, "Social Choice and Individual Values,"
pp. 423-424.

a change in the rule for socially ordering social
states.[15] If this were the case, it <u>would</u> be true that
we could not compare two social states evaluated in two
different ways and Little would be correct. But if
$R = f(R^1, \ldots, R^m)$ is itself a function of tastes, then
it is, in the terms employed above, a constitution func-
tion. We do not require[16] a change in a constitution
function in response to a change in tastes for <u>social
states</u>. But to recognize this fact is to reach precisely
the opposite conclusion of Little concerning the admissi-
bility of changing tastes in welfare economics. For if
the <u>rule for achieving</u> a social ordering of social states
is to be invariant with respect to changes in tastes
which affect only individual ordering of social states,
then the social ordering of the social states ought
indeed reflect those changes in taste in some restricted
way.

Little's criticism of Arrow on the grounds that he
(1) has not proved the non-existence of Bergson welfare
functions as formulated by individuals, and (2) has
incorrectly introduced changes in tastes which are inad-
missible in welfare economics, are insubstantial, if

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[15]Rothenberg, <u>Measurement of Social Welfare</u>, p. 40.

[16]Although it is not impossible that there may be
a change in ordering of decision rules corresponding to
changes in orderings of social states.

the view is adopted that the social welfare function must
imply _social_ rankings of social states, even though it
may take the same general form as a Bergson function
held by some particular individual.

What of Little's third criticism that Arrow's pro-
cess cannot sensibly be called a social welfare function
in its own right?  Reflection upon the quotation on
page 152-3 above indicates that this is not a criticism
but a particularly clear statement of the connundrum
Arrow's analysis exposes.  It is true that no one would
call Arrow's process a social welfare function unless
his own ordering corresponds to the social ordering.
It is true that it is illogical for one who accepts
the process as a welfare function to also accept the
condition of non-dictatorship.  It is also true that
without such consensus in society adequate to render
the non-dictatorship condition trivial, such a process
cannot exist; that is precisely what Arrow demonstrated.

Where Little argues that Arrow's theorem may be
relevant to actual decision processes, but not to the
matter of social welfare functions, Buchanan reaches
just the opposite conclusion.

> (Arrow) fails to see that his _conditions, properly_
> _interpreted, apply only to the derivation_
> _of the (social welfare) function and do not_
> _apply directly to the choice processes._[17]

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[17]James Buchanan, "Social Choice, Democracy, and
Free Markets," p. 115.  Italics in original.

Buchanan's view is that the application of Arrow's
analysis of social welfare functions, however, is unin-
teresting, for such functions are based upon organic no-
tions of society which are meaningless. And he holds
that while application of his analysis to group decision
procedures is meaningful, it is in error. He believes
that Arrow's conditions are weightless in the context of
derivation of a social welfare function. He holds that

> The proper approach to social welfare functions
> appears to begin with the frank admission that
> they are social, not individual, and therefore
> are of a fundamentally different philosophical
> dimension from individual values or from
> individualistically oriented decision-making pro-
> cesses. It seems meaningless to attempt to
> test such choice processes for social rationali-
> ty.[18]

Buchanan, thus, believes that Arrow's analysis is
not relevant to a search for a normative standard to be
employed by society as a whole. It should be considered,
in his view, as concerned entirely with the matter of
the consistency of various decision procedures such as
voting and the free market.[19] But Buchanan argues that
it is improper to require that decision procedures be
transitive. He argues that, while majority voting may
lead to consistent decision, if consensus is high in

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[18]Ibid., p. 118.

[19]In the absence of an optimum, however, the con-
sistency of the process of choice has no normative
significance.

society, it also may not; when it does not, it is desirable that it does not, for only in such a case would individuals accept majority rule as a decision making procedure at all:

> A decision reached through the approval
> of a majority with minority dissent has never
> been, and should never be, correctly interpreted as anything other than a provisional or
> experimental choice of the whole group. As
> a tentative choice, the majority-determined
> policy is held to be preferred to inaction, but
> is not to be considered as irrevocable. The
> fact that such decisions may be formally
> inconsistent provides one of the most important safeguards against abuse through this
> form of the voting process. If consistency
> were a required property of decision, majority rule
> would not prove acceptable even as a means
> of reaching provisional choices at the margins
> of the social decision surface . . . certainly,
> majority rule is acceptable in a free society
> precisely because it allows a sort of
> jockeying back and forth among alternatives,
> upon none of which relative unanimity can
> be obtained.[20]

Buchanan argues that free markets, in contrast to voting, are a consistent decision-making process not in the sense that a complete social ordering of social states may be inferred from market behavior, but in the sense that in two situations, given identical initial conditions and identical sets of alternatives, identical social choices will result. But there is no reason to

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[20]Buchanan, "Social Choice, Democracy and Free Markets," pp. 118-119. cf. Robert Dahl, A Preface to Democratic Theory (Chicago, University of Chicago Press, 1956), Chapter 1. Buchanan comes close to suggesting that if we are not all Madisonian democrats, we ought to be.

require voting to satisfy this consistency requirement.
Therefore, the difficulties raised by Arrow's theorem
under interpretation (4) are avoided by dispensing with
condition 5. Buchanan concludes:

> Thus the voting process serves neither as a
> basis for deriving a social welfare function in
> the Arrow sense nor as a means for producing con-
> sistent choices if tested by the Arrow condi-
> tions.[21]

Buchanan appears then to agree fully with Arrow's
analysis under interpretations (1) and (2), but judges
that such interpretations employ an organic concept of
society and therefore are meaningless. He believes
that interpretation (4) is without weight because of the
imposition of the transitivity requirement. It is
difficult to take these arguments too seriously, however.
In the first place, Buchanan himself seems to employ
a concept of a social optimum at the level of choice of
decision rules. To judge that only rules which do per-
mit inconsistent social choice are acceptable raises
the question of who it is that they are acceptable to.
If Buchanan means that such rules would be preferred to
others by such a large number of individuals that only
such rules should be employed, he has adopted precisely
the same notion of social optimum as Arrow; there is
nothing "organic" about this concept of social optimum.

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[21]"Social Choice, Democracy and Free Markets,"
p. 171.

It seems that Buchanan's argument requires some social ordering of decision rules; he has not avoided the difficulties depicted under interpretation (3). Further, if one accepted Buchanan's analysis literally, there would seem to be no reason why society should ever employ a voting decision procedure. Suppose some social state results from the pattern of market choice of individuals only. Individuals would wish to employ a voting mechanism if there were some other social state, not attainable by market choice alone, which is preferred to the first by a large enough number of persons such that it would be chosen over it if the two alternatives were put to a vote. But since the voting rule may not be consistent, it is possible for some third state, less preferred by individuals to the market choice, may be chosen over the one which can defeat the market solution. It seems obvious that citizens would not wish to employ such a rule.

The main remaining literature which attempts to demonstrate the irrelevance of Arrow's theorem to welfare economics is that of Bergson and Samuelson, although they largely follow in Little's view and therefore require less comment. Bergson agrees with Little in the view that Arrow's theorem is irrelevant to welfare economics, but wishes to show that this conclusion follows "from the very nature of this discipline."[22] The

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[22]Bergson, Essays in Normative Economics, p. 43.

"nature" of the discipline is counseling. The welfare
economist supposedly wishes to inquire of each indi-
vidaual by what ethical standards he wishes to judge
social states; on the basis of those values, the econo-
mist attempts to deduce which actions are consistent with
those values. It does not make sense to presuppose that
the economist is to counsel an official who is ethically
neutral as to what sort of values are held by society
as a whole.[23] This seems non-sensical to Bergson, for
if there were a rule of collective decision making which
was acceptable as a source of valid values, the proper
collective choice would be established directly from
the uncounseled votes of individuals.[24] In other words,
the welfare economist would have nothing to do at all,
except perhaps to count ballots. The answer to this
view is that if there is a decision procedure which may
choose valid values, citizens generally, and the public
official as well, must be counseled to choose that
decision procedure. Bergson is led to the interpretation
of Arrow's theorem for the case of an aggregation of
preferences for decision-rules--the constitutional
level of choice--but he veers away due to a belief that

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[23]Although Arrow has asserted that this view pre-
cisely is his own. See Social Choice, p. 107.

[24]Bergson, Essays in Normative Economics, p. 37.

what is at issue is not an ordering of decision rules, but an actual decision making process:

> Conceivably, in counseling the official the problem might be to discover some rules of collective decision-making which would be ethically appealing to him. If one wishes to call this welfare economics, he is welcome to do so, but he should be clear that he is back in the realm of political theory . . . The problem is one of providing counsel on political institutions rather than on the economic aspects of social states.[25]

It is clear from this passage that Bergson does not fully appreciate the nature of the difficulty. In counseling about political institutions, just as in counseling about "social" preferences, the welfare economist would be suggesting to the official what the "socially" desired set of political institutions would be. The derivation of a social ordering from individual preferences is required in either case. This, at any rate, is Bergson's leading criticism of the relevance of the Possibility Theorem to welfare economics. It is the same as Little's argument that the social welfare function does not refer to a social ordering of social states.

Bergson is more doubtful than Little on the matter of the theorem's relevance to political theory. He comes very close to distinguishing between interpretations (3) and (4) above.

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[25]*Ibid.*, p. 38.

> [Arrow's] rule may be envisaged as aggregating
> values of all . . . elements in the social
> state . . . if the rule itself should be valued
> ethically, this might be done in one or the
> other or both of two ways. The rule might be
> appraised as a political process for its own
> sake; for example, majority rule is considered a
> good thing in itself. Alternatively, the
> rule might be appraised for its consequences, that
> is, in terms of the nature of the social states,
> exclusive of the political process that the rule
> is likely to establish.[26]

Desiring decision rules "for their own sake"

implies orderings of decision rules--it is the view held

under interpretation (3). Desiring decision processes

because of their "likely consequences" implies the view

under interpretation (4). Bergson argues that consis-

tency is not desirable under the case of interpretation

(3).

> If collective consistency is of ethical value,
> this is only because . . . one values the rule
> in terms of its consequences.[27]

His reasoning is precisely the same as Buchanan's;

consistency of decision is not intrinsically desirable

because minorities would fear majority tyranny if cycli-

cal decisions were ruled out. But we have the same con-

nundrum raised by Buchanan's arguments. To assert that

actual social choices need not be consistent does not

imply that there should not be a consistent method of

passing from individual orderings of decision rules to

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[26] Ibid., pp. 30-31.

[27] Ibid., p. 31.

a social ordering of rules. Quite the contrary, if the possibility of cyclical decision is desired by everyone, only a consistent procedure of aggregating individual rankings of constitutions will insure the possibility that actual decisions may be cyclical. That is what consistency under interpretation (3) would require.

Bergson goes on to argue that, while Arrow's condition 5 is impelling only under the case where individuals order decisions according to their consequences, the conditions of non-dictatorship and independence of irrelevant alternatives are impelling only when orderings pertain to the inherent value of the rules. Here his argument is identical to Little's; the individual, assessing consequences by his own values would be inconsistent if he were to accept judgments of some ordering other than his own; each would insist on being a dictator.[28]  Neither would anyone consent to equal weighting of everyone's votes. Thus conditions (3) and (4) would be impelling under interpretation (3) but not under interpretation (4)[29]  But this position makes sense only if we are appealing to some one individual's

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[28]*Ibid.*, p. 32.

[29]Bergson's language would also indicate that conditions (3) and (4) would be inappropriate for interpretation (1) as well. Neither Bergson nor other critics distinguish between interpretation (1) and (4) as indicated earlier.

values for the purpose of assessing the "goodness" of
the group decision procedure. If we recognize that Arrow
intends to evaluate such a procedure by reference to
some notion of group values, non-dictatorship has not
lost its ethical validity. No one would wish that any
individual other than himself should retain the power
to veto any action. Here Bergson has simply neglected
the nature of the problem Arrow has investiaged; the
aggregation of individual evaluations. Bergson, of course,
does not believe that is the proper way to view the
problem, but in his criticism of Arrow he must, nonethe-
less, accept Arrow's view and not substitute his own
where convenient.

> It is not proper to conclude, then, that
> Arrow's requirement of logical consistency is
> without force if the rule of collective
> decision making is valued only for its own
> sake. On the other hand, several of his ethical
> conditions are likewise footless óf the rule
> is valued only for its consequences. If . . .
> one focuses exclusively on either of these
> approaches to the complete neglect of the
> other, it follows that Arrow's theorem col-
> lapses altogether.[30]

Once one recognizes that Arrow's difficulty does
arise because, and only because, he is attempting to
derive a method of combining individual valuations into
a social valuation valid for all, then it is seen that
the same difficulty arises regardless of the content of

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[30]Ibid., p. 33.

the orderings. Since Bergson admittedly does not wish
to go further than to explore the welfare implications
of individual welfare functions, and to counsel them as
to behavior consistent with their values, he need not
involve himself in criticisms of restrictions Arrow
would place upon a rule for combining welfare functions.
Bergson's argument is instructive, insofar as he suggests
that the impellingness of each condition may vary with
respect to the content of orderings to be combined.
But Bergson has not adequately demonstrated which condi-
tions are least impelling under various interpretations.
His failure to do so is largely due to an inadequate
understanding of the distinction between the various
interpretations.

Finally the views of Little and Bergson may be
contrasted with those of Samuelson.[31] Samuelson appar-
ently accepts the substance of Arrow's argument. His
criticism is not concerned with Arrow's method or the
validity of his conditions. In fact, he is more reluc-
tant than any other writer to sacrifice any of the con-
ditions. His only objection is that since Arrow requires
a process of aggregating any conceivable set of indivi-
dual orderings to a social ordering, he has not proved

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[31]Samuelson, "Arrow's Mathematical Politics," Human
Values and Economic Policy.

the impossibility of the Bergson social welfare function.
For the Bergson function requires a rule for aggrega-
ting only the one set of individual orderings which ac-
tually obtains. We may follow Bergson in interpreting
the social welfare function simply as any one of the
individual welfare functions. Or we may search for a
social ordering which is drawn from the individual or-
derings but is identical with no one of them. But in
either case, the social ordering is derived from only
one pattern of the individual orderings, not all con-
ceivable ones, and Arrow has not shown that this deriva-
tion is impossible. The particular set of orderings
held by individuals may aggregate. Therefore, Samuelson,
states that

> The Arrow result is much more a contribution
> to the infant discipline of mathematical politics
> than to the traditional mathematical theory of
> welfare economics . . . I do not believe he has
> proved the impossibility of the traditional
> Bergson welfare function of economics, even though
> many of his less expert readers seem inevitably
> drawn into thinking so.[32]

That Arrow has not strictly proven the impossibility
of the Bergson function is true. But to assert that
Arrow's work is an inquiry into "welfare politics" and
not "welfare economics" is no more than to draw an
arbitrary line. If we wish the concept of Bergson

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[32]Ibid., p. 67.

functions to have operational significance, there must
be some method of selecting which function of the Bergson
type is to be selected for society. Although Arrow's
theorem does not prove that no Bergson function exists
for the configuration of individual value orderings
obtaining, neither are we sure that those orderings do
yield such a function. We evidently must inquire what
sorts of individual orderings do aggregate and deter-
mine whether orderings of such types are present, in
fact.

B. Preference Intensity and Vote-Trading.

Evidently few economists have been persuaded by
Little, Bergson, and Samuelson that Arrow's work has
nothing to do with welfare economics, for there has
been much literature aimed at exorcising the theorem on
other grounds. One of the leading attempts to accom-
plish this feat has been to argue that the formulation
of social welfare judgments should weight the alterna-
tives in individual orderings by the intensity with
which individual preferences are held. This view implies
that condition (4) is not a valid restriction to place
upon the aggregation procedure; something in addition
to the mere ordering of alternatives of individuals
should be taken into account in order to achieve a
social ordering.

An early appeal to the role of preference intensity in problems of social choice was made by Hildreth[33] who first formulated the now familiar argument that if the majority holds a "bare preference" for alternative x over some other alternative y, and the minority hold an intense preference for y over x, the social ordering may quite properly order y over x, thus violating majority rule. This notion would apparently deserve serious consideration due to the fact that, under uncertainty, utility may be measurable up to a linear transformation. If social decisions should reflect intensities of preferences, Arrow's condition (4) is not binding, and we are freed from the strictures of the theorem.

To be operational, however, such a concept requires that over a series of votes comparing pairs of alternatives, some method must be given for determining how to change the weights of individual votes for each separate comparison. It certainly is not obvious that there is any method of achieving optimal voting weights for different decisions. As was argued above in discussion of condition (4), the lack of such an institutional mechanism seemed sufficient reason to Arrow for requiring equal voting weights for each decision. Even

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[33]Clifford Hildreth, "Alternative Conditions for Social Orderings," Econometrica, 21 (Jan., 1953).

if there were such a mechanism, however, we are caught
in a quandry.  We are assuming that individuals have
different intensities of preference for alternative
social decisions or actions.  Actual social choice must
reflect these intensities, and thus it is required to
allow voting weights to change appropriately for separate
decisions.  But, if such a method exists, it presumes a
constitutional structure which makes provision for varia-
tions in weighting.  How is the constitution to be
selected?  Must the manner of its selection also reflect
intensities of preference in individual orderings of
decision rules?  If so there must be a level of choice
which allows for variations of voting weights assigned
individuals in the process of forming a constitution.
And how are preference intensities to be reflected in
voting weights at that preconstitutional level of choice?
Sooner or later we must reach a level of choice where
voting weights are (perhaps arbitrarily) assumed to
be equal.  At this level, only orderings matter, Arrow's
condition (4) holds, and there is no process of aggrega-
ting individual orderings into social ordering.

It is very important to keep in mind the distinction
between votes interpreted as an element of actual social
decisions and "votes" interpreted as the relative weights
of individual preferences to be aggregated into the men-
tal construct of a social ordering.  Preference intensity

can be accounted for, at least to some degree, by voting models interpreted as group decision procedures, as we shall see. But such models cannot generate social outcomes with the normative significance which some writers would apparently attach to the outcomes. When voting models are interpreted as a method for hypothetically generating a social ordering of sets of decisions, sets of decision rules, or social states, on the other hand, it is impossible to observe preference intensity, and it is therefore inappropriate to consider such information for the purpose of defining the optimum. This last point will become clear after investigating how group decision procedures may account for preference intensity.

That certain common voting procedures do, in fact, allow for different effective distributions of voting weights for different issues was first argued by Buchanan and Tullock.[34] If there are many issues to be decided, and if the intensity of voter's preferences among alternative solutions to each issue vary, then voters may engage in logrolling; they may trade their votes over issues of slight concern to them in exchange for favorable votes of others on issues of great concern. The political process itself may reveal intensities of preferences held by the participants, therefore, and these

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[34]Buchanan and Tullock, The Calculus of Consent, especially pp. 131-145 and pp. 270-281.

will be reflected in the social outcome. Buchanan and Tullock are not particularly explicit in demonstrating the implications of this consideration for Arrow's theorem, but it is apparent from some remarks of Tullock that they believe the existence of logrolling renders Arrow's results quite weightless. Tullock writes:

> Due to the predominance of processes in which votes are traded, where the particular type of irrationality described by Arrow is impossible, the basic irrationality of governmental decision-making becomes less important . . .[35]

> In a note expanding on the above statement, he

goes on to say:

> If votes are traded, then the order of preference of the individual voter becomes less important than the strength of his preferences. The cyclical majority, vital to Arrow's proof, results from the likelihood of certain orderings of preferences, together with the apparently obvious assumption that voters vote according to their preferences on each issue. Logrolling, which results in many voters voting against their own preferences on many issues, simply is not covered by Arrow's book.[36]

In a recent article, Coleman[37] has elaborated on the implications of vote trading for Arrow's work specifically. Since Coleman's is the most complete argument of this type, I will treat it in some detail. Coleman agrees with Buchanan and Tullock that Arrow's

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[35]Ibid., p. 332.

[36]Ibid., p. 359.

[37]James Coleman, "The Possibility of a Social Welfare Function."

aggregation process rules out the possibility of expression of preference of intensity:

> [Arrow's] approach to a social welfare function
> leaves out of consideration precisely those
> elements which are most crucial in empirical
> cases of social choice, and it is this omission
> which creates the apparent paradox . . . Arrow's
> impossibility theorem is relevant only to those
> social choice mechanisms in which it is impossible
> to express relative intensities, and . . . when
> it is possible to express such intensities,
> (condition 4) is inconsistent not only with
> "collective rationality" but with individual
> rationality.[38]

Coleman suggests that the "proper" approach to social welfare functions is under conditions of uncertainty, for under such an assumption, persons may actually express preference intensity. He is, therefore, concerned explicitly with interpretation (4) above. While he apparently recognizes that here may be other interpretations, he defends interpretation (4) as the only proper one, for Arrow's analysis presumes that persons must be able to express their preferences, and this expression may be manifested only in actual voting behavior.

> It could be said that this approach violates the
> spirit of Arrow's approach, because Arrow was
> concerned merely with the ordering of tastes in
> the community, and showed that no aggregation of
> these ordered tastes could be consistent, in
> the sense of meeting his conditions for
> reasonableness or social rationality. The flaw
> in this argument is that citizen sovereignty,
> which is one of Arrow's conditions, implies

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[38]*Ibid.*, p. 1106.

> the expression of these tastes or preferences,
> the expression that is normally embodied in
> voting, with the voting rule serving as
> the aggregation device for the society. Once
> this is allowed . . . it becomes irrational for
> an individual to express only his preferences,
> and not something more which would maximize his
> expected utility.[39]

Coleman then considers three cases of individual

utility maximization in voting procedures under uncer-

tainty. First, he supposes that voting is on a single

issue, and individuals have no information about

the preferences of others. Under such a situation, all

possible orderings of others are equi-probable and it

is evident that the orderings of expected utilities of

outcomes, for some individual, are the same as his

orderings of the outcomes based on their actual utility

for him. Thus, in such a case, we only need to know

the ordering of outcomes for an individual in order to

predict his voting behavior.

Second, he supposes that voting is on a single

issue, but that individuals have information about

others' preferences. Suppose there are three alterna-

tive actions, A, B, and C, and that individual Y is known

to desire B, while individual Z desires C. Then indi-

vidual X knows that he can obtain B or C by voting for

either; he also knows that if he votes for A, the actual

outcome may be either A, B, or C, depending on the order

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[39]Ibid., p. 107.

in which the alternatives are voted upon. If he votes for A, the probabilities of A, B, or C resulting are 1/3. Then it is obvious that we may not predict his behavior on the basis of his ordering of A, B, and C. Suppose X's ordering is ABC. If the expected utility of a vote for A, $1/3 \, U_{xa} + 1/3 \, U_{xb} + 1/3 \, U_{xc}$, is less than the expected utility of a vote for B, $U_{xb}$, X will vote for B rather than his true preference, A. This implies, of course, that B is closer to A than B is to C in terms of the individual's scale of evaluation. It is just this crucial difference between behavior under uncertainty and behavior under certainty which led us to accept Newman's argument that Arrow's theorem pertained only to behavior under <u>certainty</u>. In order for the theorem to apply to the problem of the existence of a group decision procedure, original condition (4) must be reformulated as (4'). This condition then requires that persons must vote according to their true preferences. But this condition is unreasonable in the present case, for, as Coleman writes:[40]

> . . . because the individual must calculate
> expected utilities of the outcome, no outcome
> is "irrelevant," so long as there is some
> subjective probability of its occurrence.
> In fact, in this case, the utility of C for x plays
> the crucial role for A or B. Thus the elimination
> of C might have--and legitimately so from
> the point of view of individual rationality as

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[40]<u>Ibid.</u>, p. 1111.

well as social welfare--an effect upon the outcome.

Finally, Coleman supposes that individuals have information about others' preferences but are to vote on more than one issue. In such a case, logrolling may be possible. If there are two issues, 1 and 2, and three alternative actions for each issue they may be ordered among three individuals x, y, and z as follows:

$$x \quad A_1, A_2 \quad B_1, B_2 \quad C_1, C_2$$

$$y \quad B_1, B_2 \quad C_1, C_2 \quad A_1, A_2$$

$$z \quad C_1, C_2 \quad A_1, A_2 \quad B_1, B_2$$

If x's preference for $A_1$ is intense relative to $B_1$ and $C_1$, but he knows y and z will vote for $B_1$ and $C_1$, respectively, he is not forced to vote for $B_1$ to maximize his expected utility. For now he may directly influence the behavior of one or the other of y and z. Suppose his preference for $A_2$ over $C_2$ is relatively weak, but z's preference for $C_2$ over $A_2$ is strong in his scale of evaluation. Additionally, z's preference for C, over A, is relatively weak. Then x and z may arrange a trade such that both agree to vote for $A_1$, $C_2$. It may well be that individuals will vote for their least preferred choice on one issue if they are compensated by obtaining their most preferred choice on an issue in which they are more intensely concerned. Of course, such a result is not necessarily stable, for x may find

he could arrange an agreement with y, instead of z, such that they would both vote for combination $A_1$, $B_2$, which would improve x's position. Other coalitions could alternatively be formed, all of which would depend upon the differing intensities of preference over the alternatives for both issues.

As the number of decisions and number of individuals are increased, the flexibility and number of potential trades also increases. Each voter has a large number of votes which he may "spend" as he chooses in order to gain maximal control over the issues in which he is most interested. Thus, each person can "pay out various quantities of resources in order to get those things he wants most. In terms of political preferences, this means that those decisions for which the alternatives differ most for him in utility are the ones over which he can gain control through giving up his partial control over those of alternatives which differ least. It allows, in short, an expression of 'intensity' such that the outcome of the decision will tend to be one which maximizes aggregate utility."[41]

Coleman believes that this approach frees us entirely from the Possibility theorem.

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[41]Ibid., pp. 1117-1118.

The above discussion should be sufficient to show
that when individuals are released from res-
trictive conditions upon behavior and information
(i.e., voting on only one issue and complete
absence of information about others' behavior)
then rational behavior demands that every
alternative which has some subjective probability
of occurrence can affect their behavior, though
it may be least preferred. As a consequence,
when such behavior is allowed, the condition
which Arrow states, independence of outcomes from
"irrelevant" alternatives, can hardly be imposed
. . . Given, then, the freedom from Arrow's
impossibility theorem, we can proceed to investi-
gate the structural conditions that will allow
expression of more than simple preferences than
voting on a simple issue implies.[42]

Coleman has demonstrated persuasively the inadvisa-

bility of imposing condition (4') upon procedures of

group decision. Nothing in Arrow's results, then, leads

us to expect that decisions must be inconsistent; of

course, the stability of various voting rules still

must be shown. But Coleman definitely has not demon-

strated the possibility of a social welfare function

in Arrow's sense. That Coleman should have recognized

this fact is suggested by the analogy he wishes to draw

between his vote trading political model and a competi-

tive market model. The market model is clearly incon-

sistent as a social choice mechanism, as was shown in

Chapter 3. The only advantage the logrolling political

model has is that it can effectively arrange payment of

compensation, as the economic market cannot, for changes

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[42]*Ibid.*, p. 1115.

which worsen at least one person's position. Such a
mechanism may only make "optimal" changes in the
ambiguous sense of the Pareto criteria.

To make the logrolling model analogous to the
market, and to make the point above more clear, we must
drop the assumption of majority rule. Decisions made
by majority rule will not, in general, satisfy necessary
conditions for an optimum because external costs will
tend to be imposed upon those in the minority.[43]
Changes made by market techniques are implicitly unani-
mous; persons who are not affected by advantageous trades
between any two individuals endorse the right of indi-
viduals to improve their positions by such trades.
Under unanimity rule, the political mechanism will make
moves to Pareto superior points.[44] If each person is
given an effective veto over proposed changes, no change
may leave any one person worse off than before the
change. If the proposed change constitutes a package of
various decisions and if vote trading is permitted, any

[43]As has been argued extensively by Buchanan and
Tullock. See Calculus of Consent, pp. 171-190 and passim.

[44]A point A is Pareto superior to point B if at
least one person is better off, and no person worse off,
under A than B. Moves to Pareto superior points need
not be Pareto optimal in the sense that from the superi-
or point there is no further move which may improve one
person's welfare while leaving that of others unchanged,
but given enough time under a given environment, we
would expect that all such superior moves would be made
and result in a Pareto optimum.

decision which would worsen some set of individuals must
be compensated by other decisions which are so favorable
to those individuals that the net utility of the whole
package is positive for them, as for everyone else.
Further, the outcome is consistent in the limited sense
that given identical initial conditions and an identical
change in environment, the same social outcome will re-
sult.

But this is not the sense of inconsistency which
is involved in Arrow's analysis. What we require of a
social welfare function is a complete ordering of points
in output or "decision" space. The political vote-
trading model cannot give us such an ordering. It may
only compare points with the status quo and then only
in terms of the distribution of wealth (goods, income,
political power) prevailing at the status quo. It is
subject to the same ambiguity as ordinary market choice;
there is no way of assessing the desirability of a change
independently of the distribution of claims. Just as
the Scitovsky curve analysis illustrates that we may not
extend the Pareto criteria to yield a complete ordering
of alternative outputs, so we may not obtain an optimal
set of decisions by introduction of vote trading which
provides for the "payment" of compensation.

That the Arrow paradox remains in the presence of
logrolling is evident if one considers that the type of

ordering of social states required may not be achieved by voting upon various decisions separately. If the social states which individuals and society are to order are conceived as bundles of decisions, such an ordering includes all possible combinations of the decisions. The bundles which may be obtained by vote-trading are among those to be ordered. But there is no way to generate a social ordering of such bundles from individual orderings consistent with Arrow's conditions. And condition (4) is binding, for the vote over bundles for society is, in effect, a single decision, that of selecting the optimal set of decisions considered as a whole. Since there is but one decision to make at this level of analysis, there are no vote trades to be made; thus, there is no way of expressing intensity of preference. The problem of defining a social optimum is not solved by the introduction of vote-trading.

Under majority rule rather than unanimity rule, the vote trading models raise additional problems. The arrangement of compensation by such a group decision process would be incomplete. Only those in the majority coalition need experience utility increases. In this case cyclical decisions may occur as new bargains are struck between persons in the minority and other persons in the prevailing majority. Plott[45] has demonstrated

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[45]Charles R. Plott, "A Notion of Equilibrium and its Possibility Under Majority Rule," American Economic

that an equilibrium situation may be obtained by majori-
ty rule if and only if persons indifferent to proposed
changes do not vote "yes," the equilibrium point is that
of maximum utility for one voter, and finally, that all
other voters may be divided into pairs whose interests
are mutually opposed. This last condition has the
function of ruling out bargains which might result in a
new coalition thus upsetting the equilibrium point. Such
a condition is obviously very strong and could occur
only fortuitously.

The fundamental point of this section, however, is
that even if majority rule did lead to a stable solution,
it would not mean much from a normative point of view.
The point chosen is not necessarily optimal. The group
decision procedure cannot yield a social ordering of
alternatives which may be used as a standard of norma-
tive comparison. And we still do require such a stan-
dard; we may not infer that the actual social choice is
optimal for it has been achieved by individual actions
which do not necessarily reflect the true preferences

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Review, LVII (Sept., 1967). Plott, incidentally, states
that "any majority rule equilibrium is obviously Pareto
optimal." (p. 791) This is true in the sense that no
alternative exists which may be obtained which will
not worsen anyone's welfare while improving one person's
welfare. But if the equilibrium exists, it would
not be reached in a Pareto optimal manner. The majority
decision would impose costs on the persons in the
minority.

of the individuals. The discovery of a voting method for actual social decisions which is not subject to the problem of cyclical majorities does not avoid the difficulties of defining a social optimum; it does not solve the Arrow problem under interpretations 1, 2 and 3.

Of course, even though the existence of a consistent group decision procedure does not guarantee the possibility of collectively rational choice, the investigation of properties of such procedures is important in its own right. If it is possible to demonstrate that an optimum for a given society does exist, the attainment of that optimum requires an understanding of the operation of various voting methods. A reliable group decision procedure is necessarily consistent. But any consistent procedure is not necessarily reliable. If consistency is to be obtained by sacrificing condition 4', thereby "permitting" individuals to fail to reveal their actual preferences in their behavior, it would seem necessary that reliability of the procedure must be achieved by imposing some set of external interferences on the mechanism of choice. Such an attempt requires a positive theory of political choice processes just as it requires a positive theory of economic markets.

Since the analysis of vote-trading models cannot solve the problem of internal consistency of the social

preference ordering, we consider an alternative route:
restrictions on the individual orderings which are
admissible.

CHAPTER 6

Similarity In Orderings As A Basis Of A
Social Welfare Function

For the purpose of identifying a social optimum,
it is required that individual orderings of social
states aggregate to a social ordering. The last chap-
ter argues that it is inappropriate to relax condition
(4) in order to permit the existence of such an aggre-
gation procedure. In this chapter, the alternative
approach of restricting admissible individual orderings
to forms which will aggregate is examined.

Arrow's first condition allows individual orderings
to assume any form whatever. But for some configuration
of orderings, a social ordering may be derived. If such
a configuration is present in actuality, then Arrow's
first condition is inconsequential and a method for
aggregating preferences consistent with Arrow's other
conditions may exist. For example, suppose that the
preferences of all individuals for social states were
identical. One social state is unanimously accepted as
the optimum. If this were the case, any rule of aggre-
gation satisfying conditions 2-5 would yield a social
ordering; condition (1) would be violated trivially.

In fact, it has not been traditional in economics
to allow completely unrestricted individual orderings of

social states. A common restriction is the specification
that the ordering over all conceptual social states by
individuals should be compatible with the partial or-
dering suggested by the "individualistic" assumptions
of economic theory. That is, it is often presumed that
individuals order social states in accordance with the
level of direct consumption they receive under each
state. States which yield a particular level of con-
sumption to an individual may then be further ordered
in accordance with some other criteria based upon values
other than own-utility. Arrow, however, has proved
that there is no aggregation procedure for individual
orderings which are restricted to be compatible with
the quasi-ordering given by the individualistic assump-
tions.[1] This result is hardly surprising; the individ-
ualistic assumptions insure that individual attitudes
toward alternative social states will be markedly
different among individuals.[2] One would expect that the
possibility of defining a social optimum must rest upon
the similarity of individual rankings.

    It is, then, appropriate to search for other res-

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[1]Arrow, Social Choice, pp. 62-64.

[2]Ibid., p. 69. Note that if individual tastes are
very similar, attitudes toward social states, under the
individualistic assumptions, will be very disparate; if
tastes for objects of consumption are less similar,
attitudes toward social states would be less disparate.

trictions on the individual orderings which are suf-
ficient to guarantee that the orderings will aggregate
and thus imply a social welfare function. If it can
be shown that such restrictions are "reasonable" in
terms of their apparent conformity with reality, the
problem of internal inconsistency may be solved. We
turn to an evaluation of some of the literature which
has adopted this approach.

A.  Aggregation of Value-Restricted Preferences.

There are two lines of analysis in current litera-
ture which are intended to rely upon similarity of indi-
vidual orderings in order to avoid the Arrow difficulty.
One attempt is to find a set of restrictions on admis-
sible individual orderings which are sufficient to assure
the existence of an aggregation procedure consistent
with Arrow's conditions 2-5. The other is to find a
set of restrictions on orderings such that an aggregation
procedure consistent with conditions 2-4 will yield
consistent decisions with a tolerably high level of pro-
bability.

An early example of the first approach is to res-
trict individual orderings to "single-peakedness."[3]

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[3]See primarily Duncan Black, "On The Rationale of
Group Decision-Making," Journal of Political Economy,
LVI (Feb., 1948); Black, "The Decisions of a Committee
Using a Special Majority," Econometrica, 16 (July, 1948);
and Arrow, Social Choice, pp. 75-80.

Singlepeakedness requires that alternatives may be
arrayed along a single axis in such a way that the
utility curves for all individuals corresponding to the
alternatives are singlepeaked. The criteria for
individual judgment are, as it were, compressed into a
single variable and all individuals order the alterna-
tives by reference to this single variable. For example,
in an election between several candidates, it may be
that voters agree as to how the candidates may be ordered
along a left-right spectrum where a movement to the
right along the spectrum represents decreasing control
of a central authority or some other similar ideological
concept. If each individual's "utility" for each can-
didate may be expressed as a function of the candidates'
position along this spectrum, and if each individual
most prefers some particular point along the spectrum,
and if the utility of each individual falls monotonical-
ly as movements are made in either direction from his
optimum point, and if all voters vote in accordance
with their preferences, then majority rule will yield
a consistent social choice. It is easy to see why this
must be so. In any pairwise comparison between alter-
natives, each individual casts his vote for the alter-
native closest to his optimum point. All alternatives
to the right of the preference of the median voter
would lose to the alternative optimal for the median

voter. All alternatives to the left of the median
voters' optimum would similarly be defeated by it. Thus
the optimum point of the median voter is the only point
which cannot be defeated by any other alternative under
majority rule.

Black apparently intended his analysis of single-
peakedness to be relevant to group decision processes.
As the discussion of the previous chapter indicates,
however, this analysis is not adequate for the case of
reaching actual social decisions. The difficulty is that
Black assumes that individuals will vote in accordance
with their preference orderings for every comparison.
Even if all decisions are considered separately, this
assumption is unreasonable if voters have information
about others' preferences, as was shown by Coleman.[4]
If the assumption of a single issue election is dropped
vote trading may occur, and individuals will disregard
their orderings for many decisions.

Although the assumption of singlepeakedness may be
of little value in application to actual group choice,

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[4]Arrow also notes this fact, but does not explore
its implications. He cites an example to show that
individuals will have an incentive to misrepresent their
preferences. Suppose the orderings of individuals 1,
2, and 3 are x,y,z; y,x,z; and z,y,x, respectively,
and that the motions are raised in order y, z, x. Then,
in voting on y and z, 1 would disobey his ordering and
vote for z, as he may know that his most preferred
choice, x, can defeat z but not y. See <u>Social Choice</u>,
p. 80 n.

it is relevant to other interpretations of Arrow's the-
orem. If Arrow's range condition is restricted such
that only orderings satisfying the assumption of single-
peakedness are admissible, majority rule does offer a
procedure for arriving at a social ordering consistent
with Arrow's conditions 2-5. Condition 4 is satisfied
by the restriction that individuals do vote in accordance
with their orderings; non-dictatorship and the Pareto
principle obviously are satisfied. It may be seen in-
tuitively that a complete transitive social ordering
must result from majority rule in such a case since if
all the alternatives are compared and one is selected,
the selected alternative may be eliminated and a vote
among remaining alternatives will select the next best;
it may be eliminated and the third best will be selected,
and so on.[5] Preference orderings restricted to single-
peakedness will aggregate to a social ordering in a
fashion consistent with Arrow's other conditions.[6]

Two attempts to generalize Black's analysis have
occurred in the recent literature. Tullock[7] investigates

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[5]Of course, the number of alternatives must be finite.

[6]It is additionally necessary that Black's analysis
require the number of voters to be odd and that all or-
derings of individuals over any triple must be strong
in order for this result to be mathematically strict.

[7]Gordon Tullock, "The General Irrelevance of the
General Impossibility Theorem," Quarterly Journal of
Economics, LXXXI (May, 1967).

the implications of singlepeakedness if there is more than one dimension of choice. Sen[8] has shown in a particularly masterful paper, that singlepeakedness is only one case of a more general theorem on sufficiency conditions for the existence of a complete social ordering. I shall present Sen's results and an extension by Pattanaik[9] before dealing with Tullock.

Sen's theorem can best be grasped intuitively by first considering precisely how singlepeakedness restricts preference orderings. Suppose that individuals A, B and C are to order alternatives x, y, and z and that their preferences are singlepeaked. Their preferences might be indicated graphically as in Fig. 8.



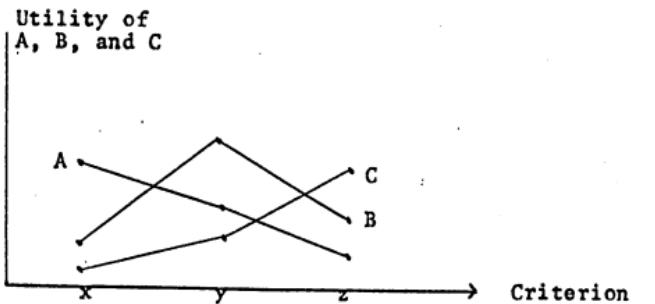Fig. 8: Singlepeaked Preferences For Three Individuals

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[8] Amarta K. Sen, "A Possibility Theorem on Majority Decision," *Econometrica*, 34 (April, 1966).

[9] Presenta K. Pattanaik, "A Note on Democratic Decision and The Existence of Choice Sets."

Individual B has the ordering $yP_B zP_B x$. Given B's ordering, it is evident that a non-transitive social preference would be generated only if $xP_A yP_A z$ and $zP_C xP_C y$ or $xP_C yP_C z$ and $zP_A xP_A y$. In that case, the preferences of the three over the three alternatives would form a "Latin-square" (each alternative is ranked first by one voter, second by another and third by the other):

$$
\begin{array}{ccc}
x & y & z \\
y & z & x \\
z & x & y
\end{array}
$$

But this array of preferences is ruled out by the assumption of singlepeakedness which insures that y cannot be ranked last by any of the individuals. In other words, the median alternative is restricted to be "not worst" in any of the individual orderings. The set of orderings cannot form a Latin-square and majority rule will therefore yield a transitive social ranking. The social ordering in the example must be $yPzPx$. It follows that any triple of alternatives which possesses one alternative which is never ranked "worst" must yield a consistent solution if majority rule is the aggregation procedure. Such a triple can be said to obey the "not worst" value restriction.

It is evident, however, that "Latin squarelessness" may be assured by other value restrictions. If among a triple there is one alternative which is "not best" or "not medium" a consistent solution is also reached by

majority rule.[10] Sen's paper formalizes this observation. In each case the individuals must have strong orderings. To account for individuals who may be indifferent between alternatives, Sen wishes to consider the rankings only of "concerned individuals" who are not indifferent between all alternatives. Sen's theorem may then be stated:

> Sen's Theorem: The method of majority decision is a social welfare function satisfying Arrow's conditions 2-5 and the consistency condition for any number of alternatives, providing the preferences of concerned individuals over every triple of alternatives is value restricted and the number of concerned individuals for every triple is odd.[11]

This theorem constitutes a significant weakening of the singlepeakedness restriction. The most dramatic result is that each triple among n alternatives need not obey the same value restriction. For consistency to result from majority rule, it is only required that each triple obey one of the three value restrictions. Additionally, where singlepeakedness requires strong orderings over all alternatives for all individuals (the

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[10]Sen shows that the "not best" value restriction corresponds to Inada's concept of "single caved preferences" and that "not medium" restriction corresponds to Inada's Possibility Theorem for alternatives separable into two groups. Sen, "A Possibility Theorem," p. 496. See also K. Inada, "A Note on the Single Majority Decision Rule," Econometrica, 32 (Oct., 1964).

[11]Sen, "A Possibility Theorem," p. 493. See pp. 493-495 for proof of this theorem.

number of which must be odd), Sen's value restrictions require strong orderings of triples only for concerned individuals over a triple, and that the number of concerned individuals be odd.

The usefulness of Sen's theorem, however, depends upon the likelihood that individual orderings actually satisfy his value-restrictions, Sen takes an optimistic view:

> It would seem that Value-Restricted Preferences will cover a variety of practical cases. A comparatively limited measure of agreement seems to be sufficient to guarantee consistent majority decisions, and to get from it a Social Welfare Function with the other properties specified by Arrow.[12]

But the degree of consensus implicit in Sen's value restrictions is not so limited as it appears. Consider the case of the "not worst" restriction which is the same as singlepeakedness. Individual preferences over some triple may satisfy this restriction accidentally. But two assumptions must be made in order to contribute to the realistic expectation that this restriction will be satisfied by individual rankings. First, there must exist some criterion variable by which all individuals may rank alternatives. Second, there must be unanimous agreement among the individuals as to the ordering of alternatives along the criterion variable. Only under

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[12]Ibid., p. 499.

such assumptions is it meaningful to assert that individual preference functions may decrease monotonically for movements along the "criterion vector" in either direction away from their respective optima. The amount of consensus underlying this value restriction is clearly very great, although the area of agreement concerns not the similarity of preference functions themselves, but the manner by which each individual ranks the alternatives. A similar argument could be made for the two other value-restrictions.

It is possible that these assumptions which underly value-restrictedness are satisfied in "a variety of practical cases." For example, if a committee is to make a single decision as to the level of appropriation for some project, it is not unrealistic to assume that each committee member has a preference curve peaked at his preferred level of appropriation. The level of expenditure is the single criterion by which alternatives are ranked. But such a situation involves decision under uncertainty, and it is in precisely such situations that it is unwarranted to assume that individuals will express their true preferences in voting. The problem is obviously compounded when the committee is assumed to make many decisions, for then the possibility for vote trading arises. If members do not vote according to their true preference orderings, the consistency of the

outcome will not depend on the value-restrictions satisfied by the orderings; nor will it possess normative significance.

On the other hand, we may apply value-restrictedness to the case of a large number of individuals who are to vote (hypothetically) on the multi-dimensional issue of selection of a social state. The wording of Sen's theorem indicates that this is the fundamental situation he has in mind; only under such an application is majority rule a social welfare function.[13] In this interpretation we may expect all individuals to express their true preferences in "voting." But now value-restricted preferences require that all individuals rank all the multidimensional social states in accordance with one universally recognized criterion variable. Furthermore, everyone must agree as to the "natural" arrangement of the alternatives along the criterion vector. This possibility seems most unlikely.[14]

It cannot be argued, of course, that the criterion

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[13]But note that this terminology is inconsistent with that in quotation 12 where Sen wishes to derive a social welfare function from apparently separate decisions achieved by majority rule under value-restricted preferences.

[14]It is interesting to observe that political ideologies do attempt to determine simple criteria according to which complex social states may be judged, e.g., the extent of government control, degree of class differentiation, etc.

variable itself can be selected by majority rule;
majority decision can be consistent only under the
assumption of the prior existence of the criterion.[15]

The remarks above indicate that Sen's value restric-
tions constitute severe limitation on the admissibility
of individual orderings. The point is not that value-
restrictedness cannot occur in the individual rankings
which actually obtain in society; it is only that it
does not appear that society, at least at present, has
determined a criterion with reference to which social
states possess a "natural" order. Without such a criter-
ion, there does not appear to be any empirical basis for
expecting the value restrictions to be satisfied.

An interesting extension of Sen's argument which
weakens the value-restrictions sufficient for the defini-
tion of an optimum has been made by Pattanaik.[16] Patta-
naik argues that the existence of a complete social order-
ing of alternatives is sufficient, but not necessary, to

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[15]Note that Bergson functions, under individualistic
assumptions, propose a single criterion variable: the
individual's own-utility. But since interpersonal
comparisons are ruled out, no one criterion for society
as a whole is possible. The problem of finding a social
welfare function might be viewed as the selection of a
criterion valid for everyone. While the purpose of
restricting condition (1) is to avoid the necessity of
unanimity in preference orderings, such restrictions
implicitly introduce unanimity by requiring a single
criterion for judgment.

[16]Pattanaik, "Democratic Decision and Existence of
Choice Sets."

denote an optimum. From a set of alternatives, there may be at least one which may be used for the basis of policy recommendation even though the social ordering of the remaining alternatives may be intransitive. He terms the set of all alternatives at least as good as any other the society's "choice set."[17] The problem of social choice is then to find a non-empty choice set. Pattanaik proceeds to develop sufficiency conditions for a non-empty choice set and shows that these are weaker than Sen's sufficiency conditions for a complete social ordering.

To reach his result, Pattanaik defines three ordering relations.[18] The first is the Pareto relation which yields a partial ordering, Q, over the alternatives. Q is defined formally as

$$xQy \leftrightarrow [\forall^i (xR_i y) \, \& \, \exists_i (xP_i y)]$$

According to the Q relation, all alternatives may be partitioned into two sets; that of Pareto optimal alternatives and that of Pareto inoptimal alternatives.

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[17]Although he does not mean by this that such alternatives are actually chosen by society, but merely that they occur "at the top" of the social ordering and may therefore be taken as the basis for actual implementation. If there is more than one element in the choice set, by definition, society is indifferent among them, and any one may be picked for the basis of policy recommendation.

[18]All the definitions which follow are from Pattanaik, "Democratic Decision and Existence of Choice Sets," p. 2.

The second ordering is the weak social ordering R defined as

$$xRy \text{ if and only if } N(xR_iy) \geq N(yR_ix),$$

where $N(xR_iy)$ represents the number of voters weakly preferring x to y. Pattanaik calls this ordering relation "the method of majority decisions." This notion of majority rule is contrasted with that embodied in the concept of "non-minority rule." The social ordering, $R_{maj}$, defined by non-minority rule is

$$xRy \text{ if and only if } N(xR_iy) \geq \frac{n}{2},$$

where n is the total number of voters.

Pattanaik then cites examples which illustrate that non-minority rule does not lead to the existence of a choice set. He proves, however, that the method of majority decision will lead to a non-empty choice set if every triple of Pareto optimal alternatives satisfies one of Sen's value restrictions.[19]

What this means, in effect, is that society does not require a decision rule which orders all alternatives consistently. For a guide to public policy, all we

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[19]Ibid., pp. 5-8. Note that in this context, the Pareto optimality of a point only implies that no other point dominates it in terms of Q. For every alternative, there must be at least one point which is Pareto superior to it, or the alternative itself is Pareto optimal. It is not implied that any Pareto point is better than every Pareto inoptimal point, although the choice will be from Pareto optimal points only.

require is that the number of Pareto optimal alternatives
be ordered consistently. And the "method of majority
decision" will yield such an ordering if triples among
Pareto optimal alternatives are value restricted. It
follows, then, that in order to define a social optimum,
all triples which contain at least one Pareto inoptimal
alternative need not be restricted; it does not matter
that such triples cannot be ordered transitively by
society. This is a significant weakening of Sen's range
restrictions.[20]

Even so, it is apparent that the range of alterna-
tive social states available to society is very great,
although perhaps finite, and there must be a great num-
ber of those which are Pareto optimal. The existence
of a criterion unanimously adopted by the individuals
is still a necessary condition for value restrictedness.
Pattanaik's weakening of Sen's value-restrictedness
assumption does nothing to avoid this fundamental ob-
jection to the reasonability of such restrictions. And,
of course, it remains true that even if value-restricted-
ness did hold, thus allowing the "method of majority
decision" to indicate a social optimum, it would still
be necessary to implement the decision. It is still

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[20]It also appears that Pattanaik's proof does not
require the number of concerned individuals over any
triple to be odd. The number of alternatives, however,
must be finite.

necessary that individual action in a group decision process correctly reflect simple individual orderings.

**B. Consistency of Choice Under Value-Restricted Preferences.**

Tullock,[21] however, has attempted to extend the application of singlepeakedness to cases of group decision with more than one issue for choice. To begin with, he assumes that there are two issues to be decided and that the individuals who are to vote have singlepeaked preferences over combinations of decisions on the two issues. This permits us to represent "issue space" by a two dimensional graph and to locate an optimal point for each voter on the graph. Tullock uses an example of determining the level of appropriations for the army and navy. Given that even the most avid supporters of each military branch wish to spend finite amounts of money, a box diagram may be formed within which all of the voters' optima lie.[22] Fig. 9 is such a box.

Tullock assumes further that the distribution of optima over the issue space is uniform and that indifference curves of individuals are circles centered on their optimum point. The geometry of this situation

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[21]Tullock, "General Irrelevance."
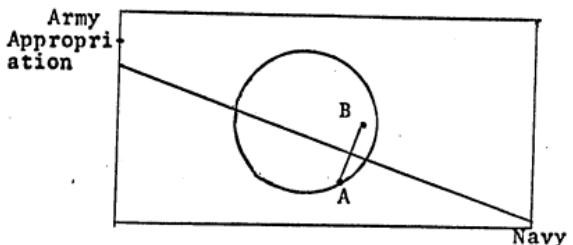
[22]Ibid., p. 258.

Fig. 9: Consistency of Majority Rule With Uniformly Distributed Optima and Singlepeaked Preferences

indicates that voting cycles cannot occur; majority rule will lead to consistent decisions as the alternatives are considered pairwise. Consider a vote between any two points A and B. If a line is drawn between A and B the perpendicular bisector of that line partitions the voters' optima into two sets. The point which lies on the same side of the bisector as the larger number of voters' optima will obviously be selected. A circle constructed around the center of the issue space passing through one of the points, say A, will be the locus of all points indifferent to A (perpendicular bisectors of chords of circles pass through the center). It is clear that any point closer to the center of issue space will defeat any point further from it, and that the center will be an equilibrium solution selected by majority rule.[23]

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[23]This procedure assumes implicitly that the number of voters is odd.

Tullock argues, however, that since the number of voters is finite and the number of optima therefore will not fill the issue space, it is possible that the perpendicular bisector of the line segment connecting any two points equally distant from the center will not pass through anyone's optimum. Then one of the two points must be preferred to the other, if the number of voters is odd. Additionally, points within a small neighborhood of the preferred point will also be preferred to the other. And some of the points in the neighborhood of the preferred point will be further from the center of the issue space than the less preferred point. Therefore, it is possible for majority rule to move society away from the center as well as toward it,[24] and this tendency would become more serious as the number of voters decreases. It is cycles created by this difficulty to which Tullock wishes to direct his attention.

The nature of the problem which arises when lines through the center do not partition the optima into two equal groups may be illustrated with what Tullock calls "median lines." A median line is constructed so that it

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[24] cf. Plott, "A Notion of Equilibrium and Its Possibility Under Majority Rule." Note that one of Plott's conditions for existence of majority rule equilibrium is that the equilibrium point must be one person's optimum. Tullock's geometrical model also obeys Plott's other conditions, i.e., that the interests of other individuals are mutually opposed and that indifferent voters do not vote "yes."

passes through two individuals' optima and partitions
the remaining optima into two equal groups (or if the
number of voters is odd, into two groups differing in
size by one voter). These median lines do not pass
through the center, by assumption, but it is probable
that they will all intersect in a small area close to
the center. In Fig. 10 below, several such median lines
are drawn.[25]
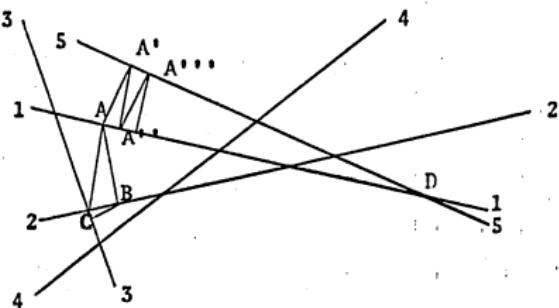


Fig. 10: Intersection of Median Lines.

If we begin at point A and drop a perpendicular
to point B, B can defeat A if the two are put to a vote.
This is true, for, by construction, there cannot be more
than one voter above line $\overline{II}$ than there is below line
$\overline{II}$. Since there also cannot be a greater difference
than one between the areas above and below $\overline{ZZ}$, and since
two optima are on $\overline{ZZ}$ and therefore closer to B than A,

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[25]See Tullock, "General Irrelevance," pp. 262-263.
The figure is based on that of Tullock.

B must defeat A by majority rule. Similarly, C would defeat B and A would defeat C; majority rule is inconsistent. Also, if two median lines did intersect at a point distant from the center, such as at point D, the proper scheduling of motions could move society to point D. If A were compared first to A', then A' to A'', then A'' to A''', and so on, majority rule would result eventually in the selection of D.

Tullock believes that Arrow's theorem holds because of the existence of cycles of this type. He argues that the possibility of such cycles is not important, however. We need not worry about movements to points such as D because that would require a long series of votes on an issue, the outcome of each being decided by a one vote majority. Since we never observe such a process in actual decision-making, we may be sure such movements do not occur.[26] We need not be concerned about cycling near the center of the issue space either, for common rules of order will not allow a long series of votes on alternatives which differ only by insignificant degrees. Thus voting will be brought to an end at some point which is not a unique solution but which is so close to the optimal solution that cycles are of no practical

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[26]Ibid., p. 263.

concern.[27] Thus

> majority voting will, indeed, always be subject
> to the paradox of voting, but . . . this is of
> very little importance. Majority voting will
> not produce a "perfect" answer, but the
> answer it does produce will not be significantly
> "worse" than if the paradox of voting did not
> exist. Any choice process involving large numbers
> of people will surely be subject to innumerable
> minor defects with the result that the outcome,
> if considered in sufficient detail, will always
> deviate from Arrow's conditions. The devia-
> tion may, however, be so small that it makes
> no practical difference.28

Tullock is in error in drawing such an optimistic

conclusion from his analysis. Not only has he not

demonstrated the "irrelevance" of the Possibility Theorem,

he has, in actuality, demonstrated that it remains even

under very severe restrictions upon individual orderings.

To see why this is so, it is necessary to clear up a

fundamental ambiguity in his argument. Tullock's

statement that his analysis is a generalization of single-

peakedness to "more than one dimension"[29] leaves vague

just what this "dimension" is. From his example of

determination of appropriations for two groups, and from

his repeated observations of real world decision pro-

cesses, one would infer that this extra dimension is

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[27]*Ibid.*, pp. 263-265.

[28]*Ibid.*, p. 267. Tullock is so sure of the practi-
cal unimportance of the theorem, he goes so far as to
apologize to his students for "inflicting it upon them."
See p. 256 n.

[29]*Ibid.*, p. 256.

that of more than one issue. But this cannot be.
Tullock assumes throughout his paper that individuals
vote strictly in accord with their preferences. Under
actual decision processes, individual "optima" refer not
to most preferred points but to points of maximum ex-
pected utility. If Tullock wishes to have points in
issue space interpreted in this fashion, he is not
talking about preference aggregation or social welfare
functions at all, and the stability of majority voting
is neither here nor there. And, of course, there is no
reason why expected utility curves should be single-
peaked,[30] even if underlying preferences are.

If, on the other hand, Tullock wishes his model to
be interpreted as pertaining to the problem of aggre-
gation of preferences over multi-dimensional social
states, his "extra dimension" must be that of more than
one criterion for evaluation.[31] The distinction between
several issues as averse to several criteria is an

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[30]Seen in this light, Tullock's analysis seems to
be a geometrical presentation of Plott's more general
algebraic result. (See Plott, "A Notion of Equilibrium")
That is, Tullock shows, in effect, that for stability of
majority rule, (1) the selected point must be one per-
son's optimum, and (2) for each individual there must
be some other individual such that their interests are
mutually opposed. Tullock obtains his result that the
range of the cycle will tend to be small only because of
his additional assumption of singlepeakedness.

[31]Tullock also offers this interpretation, but does
not draw the distinction between it and the previous one.
See *Ibid.*, p. 267.

important one. The first implies the problem of making
a set of decisions. The second implies that there is a
single decision to be made where alternatives may be
ranked according to a number of different characteris-
tics. Under the latter interpretation, Tullock's model
requires that there exist a criterion variable for each
separate characteristic, that all individuals are in
agreement as to what the criterion variables are, and
that all individuals are further agreed as to how alter-
natives correspond to points along each of the criterion
vectors. If these assumptions are satisfied for every
characteristic, then Tullock[32] shows that even if utility
curves are singlepeaked for all individuals along each
criterion axis, majority rule still need not lead to a
transitive ordering of the alternatives. Intransitive
triples are likely to occur only over a small number of
possible triples, however, and these will all be of al-
ternatives very similar to one another. Tullock's argu-
ment that common rules of order will not allow us to
move too far from the center of issue space does not
obtain, however; the hypothetical act of preference
aggregation knows no rules of order.

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[32]This is assured by Tullock's assumption that
indifference curves of each individual are perfect cir-
cles. Of course, circular curves are not required;
they need only be strictly concave; utility must fall
for any movement away from the optimum in any direction.

What Tullock has actually shown appears to be the following. If there are several criteria by which social states may be judged and if all preferences may be singlepeaked along each criterion axis, then majority rule will lead to a transitive social ordering of social states only if the voter with the median optimum according to one criterion also has the median optimum according to all other criteria. Only then is it possible that all lines drawn through "criteria-space" which partition optima into equal portions and upon which two optima lie may all intersect in a single point. In such a case, cycles are impossible. Under the assumption that the distribution of optima throughout the space is uniform, this means that some one person's optimum must be at the center. That one person will be the median voter for all possible triples which may be formed. Thus, the median voter, in effect, becomes a dictator; the social ordering between all pairs of alternatives coincides with his ordering.

If, on the other hand, the voter whose optimum is median with respect to one criterion is not the person whose optimum is median with respect to the other criterion, the center of the space coincides with no one's optimum and the problem of cycles occurs. This result is apparently of extremely limited applicability. The restrictions on individual orderings are very severe.

It is true that in Tullock's analysis, unlike Black's, entire social states do not need to be ranked by a single dimensional criterion. But this does not weaken the objections to singlepeakedness. Individuals must unanimously agree upon what criteria are to be employed, must agree as to what the "natural" ordering of states is along each of the criterion vectors, and must reveal their true preferences in voting and given all this, the assumption of singlepeakedness must, of course, still be satisfied. We simply have no reason to guess that the real world conforms to this picture.

Nor are these the only difficulties. All of the arguments above are relevant to the case of uniform distribution of optima throughout the space of alterna- tives. A more likely assumption is that optima are bunched, or clustered. As Tullock himself notes, if the distribution is multipeaked with three concentrations of optima cycles of significant size may occur.[33] It is no valid argument to suggest, as Tullock does,[34] that the difficulties of a triple peaked distribution do not arise if there are four peaks. It does not seem unreasonable that attitudes in the real world may be clustered into three rather distinct groups. If that

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[33]Ibid., p. 268.

[34]Ibid., p. 268.

happens to be the case, so much the worse for "generalized singlepeakedness." The validity of Tullock's argument clearly does depend on the distribution of optima and we require empirical knowledge of that distribution before we may assess even the limited applicability of his argument. Suffice it to say that Tullock has not given as argument sufficient to persuade "practical" men to consider the possibility theorem irrelevant.

All of the literature in this section has been directed (although sometimes implicitly) at finding restrictions on individual orderings necessary and sufficient for obtaining a social optimum. All of these attempts culminated in a particular conception of "similarity" of voter attitudes, or "consensus." This conception is involved in the underlying assumptions of singlepeakedness--existence and universal recognition of criterion variables, and so on.

There is another strand of analysis which rests on a completely different notion of "similarity." I refer to the study by Williamson and Sargent.[35] The point of view of these authors is that rather than searching for sufficiency conditions for a complete social ordering,

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[35] Oliver E. Williamson and Thomas Sargent, "Social Choice: A Probabilistic Approach," The Economic Journal, LXXVII (Dec. 1967).

which entail severe restrictions on orderings, it is expedient to look for restrictions on individual orderings which merely lead to consistent decision with a high degree of probability.

Other authors have also taken the probabilistic approach to social choice. Most notably, Campbell and Tullock[36] have calculated the probabilities of the occurrence of cyclical majorities for small groups of varying size over a varying number of alternatives, where all possible orderings are allowed and are assumed equiprobable. The probability of inconsistency is shown to be relatively high, and it increases as the number of individuals and alternatives increases. The main point of Williamson's and Sargent's paper is to show that such pessimistic results can be avoided if small departures from the uniform distribution of preferences are made. There is, of course, no a priori argument in favor of each possible ordering occurring with the same frequency. Williamson and Sargent, therefore, stipulate that one of the preference profiles occurs with slightly greater frequency than any of the others, while the others are assumed to be equi-probable. With this simple modification of the underlying distribution of preferences,

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[36]C. D. Campbell and Gordon Tullock, "A Measure of the Importance of Cyclical Majorities," Economic Journal, LXXV (Dec., 1965), pp. 853-57.

transitivity is secured with a very high level of probability even for small values of the "epsilon-advantage" enjoyed by one of the alternatives.[37]   The similarity which leads to transitivity in the social ordering is represented by the fact that slightly greater number of persons have one particular ordering than any other. Williamson and Sargent do not suggest what might happen if several orderings each have an epsilon-advantage over the remaining orderings.  It seems as reasonable that the distribution of preferences would be multi-modal as uni-modal.  If there were, for example, three different profiles which occurred with greater regularity than the others, it would seem likely that significant numbers of intransitive sets of orderings would again occur.  It is difficult to imagine, however, how the actual distribution of orderings may be inferred empirically.  Nevertheless, it is possible that important instances might exist where a relatively small degree of consensus is sufficient to guarantee consistent results with high probability.  The implications of this approach to the possibility of socail welfare functions, though, are by no means conclusive.

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[37]See Williamson and Sargent, "Social Choice," pp. 806-809.

## C. Conclusions.

We may draw the following conclusions from the discussion in this chapter. First, none of the restrictions on individual orderings proposed render Arrow's theorem "irrelevant." The restrictions are severe enough so that it is somewhat implausible to expect that they are met by actual individual orderings. Sufficient conditions for a social orderings of social states are far weaker than the unanimity condition on individual orderings, however, and the problem of internal consistency is solved for triples of alternatives in the event that individual preferences for these alternatives are value-restricted.

Second, while value-restricted preferences may yield a social ordering of alternatives, they are not sufficient to guarantee the reliability of the actual procedure of group choice. There is nothing inherent in restricted preferences which would lead the optimum to be chosen by group decision processes which do not necessarily reflect true individual preferences.

## CHAPTER 7

### Collective Rationality, Interdependent Decisions, And Orderings of Decision Rules

The discussion of the previous chapters does not engender optimism concerning the possibility that social choice may satisfy the two rationality conditions adopted in Chapter 2. Let us summarize and review the main conclusions of the entire argument so far.

A. <u>A Summary of Conclusions From Previous Chapters</u>.

The first condition requires that a social ordering of social states exist in order to serve as an unambiguous normative standard. No method of achieving such an ordering exists which is consistent with Arrow's five conditions; one of the conditions must be relaxed or eliminated. It is inappropriate to relax condition 4, however. In the context of the aggregation of individaul preferences over the single decision of selection of a social state, there is no means by which preference intensity may be inferred, thus allowing for interpersonal comparisons. The relaxation of condition 4 is appropriate only in the context of voting procedures which may account for preference intensity through patterns of vote-trading over many issues considered serially.

Apparently, then, a social ordering may be achieved only if the degree of consensus is great enough to allow the relaxation of condition 1. But the degree of consensus among individuals necessary to yield a social ordering of states appears to be very great. While such strong consensus might occur fortuitously, theoretical or empirical grounds for assessment of the degree of consensus seem inadequate, at least at present.

A basis for such assessment might emerge from a detailed study of the formation of political ideologies. One of the apparent functions of ideology is to devise a set of simplified criteria by which complex social states may be judged. If a systematic survey of individual attitudes (perhaps by questionnaire) were to demonstrate that the ideologically prescribed criteria were widely adopted, then there would be at least a presumption that preferences for diverse social states might be value-restricted. A social ordering might then be inferred even though substantial disagreement is manifest in individual orderings of social states. Since the validity of Arrow's first condition may turn on an investigation of this type, such study is of immediate concern to the welfare economist.

Of course, the discovery of a widely adopted set of criteria for judgment (and concomitant consensus among individuals as to the "natural" ranking of states

in terms of the criteria) would not insure value-restricted preferences. Value-restriction must ultimately be inferred from a direct expression of individual preferences. The previous discussion has indicated the difficulties involved in attempting to infer individual preferences from overt behavior in processes of collective decision. The alternative remains that preferences for social states might be discovered by some sort of special inquiry. This alternative hardly seems realistic. The upshot is that even if individual preferences did aggregate to a social ordering, it is hard to imagine how we might discover what the social ordering is.

The second rationality condition is that society's social decision procedure must be reliable; social choice must accurately reflect the social ordering. But the existence of a social preference ordering does not imply reliability; neither does the consistency of actual social choices. Of course, if an optimum did exist, it would be chosen by majority vote if all individuals voted sincerely in accord with their (value-restricted) preferences in the single utopian decision of one completely defined social state among (a finite number of) feasible alternatives. Balloting over such utopian alternatives would hardly seem possible. Rather, at least in democratic societies, it is the case that elements of the social state are chosen separately over

a series of decisions taken with respect to each element. Indeed, the actual decision procedure often specifies different rules for determining different elements, e.g., commodity consumption by individuals may be determined by market rules, allocation of funds to public projects may be determined by majority vote, other decisions may require extraordinary majorities. Actual social decision procedures are, of course, extremely complex. But we have no means, at present, for determining the extent to which decision procedures may reveal social preferences even under the minimal complexities introduced by allowing consideration of issues separately.

Choice structures which are aggregated by market means will reliably reflect individual preferences (of social states) only if none of the decisions with respect to elements in the social state so made are interdependent. Even under the individualistic assumptions, however, the distribution of income is contingent upon the aggregation of individual choices derived by (perfectly operating) market techniques. Individual decision with respect to consumption, type of skills to acquire, amount and type of labor to supply, etc., all affect the distribution; the optimal outcome could be achieved only if each individual knew precisely what the impact of his behavior on the distribution was and chose

accordingly.

As both Arrow and Rothenberg have argued, however, the similarity of attitudes toward social states must rest upon the assumption of a wider set of values than those implicit in extreme individualism. If agreement in orderings of social states exists, the basis for it will likely be found in preferences concerning elements which will not be determined optimally by a process of independent own-utility maximizing; that is, when those elements concern the treatment of external relations, the provision of public goods, and the determination of other non-economic qualities of the environment. Decisions with respect to such variables are obviously (often by definition) interdependent with market choices. The existence of an optimum based upon similarity of individual preferences then, implies that many decisions will be interdependent with market choices. For market-aggregated choices to achieve an optimum in such a context, each individual must order his behavior with respect to separate elements of choice so as to be consistent with his comprehensive preferences. But this entails that he knows the full effect of his isolated choices upon the social outcome in the large. While it is possible that individuals may attempt to behave in this fashion[1] it seems unreasonable to expect

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[1]The relation of this discussion to notions of the categorical imperative is obvious. Of course, one of

individuals to possess such a synoptic view of the
social system. In the presence of interdependent deci-
sions, we cannot reasonably expect individual choices
to reflect true preferences.

Some supplementary mechanism of choice is then
required to correct for errors made by market aggregation.
Alternative mechanisms usually take the form of voting
processes. But individual choices made in this context
may not reflect true preferences either, for various
forms of strategic behavior are possible so long as
many separate decisions are to be made. In fact, even
if there is sufficient consensus of attitudes toward
social states to generate a social ordering, there may
not be such consensus on each separate issue necessary
to generate consistent social choice in every case. Cyc-
lical patterns of voting on some issues may be avoided
only if vote-trading does occur. But even if vote-
trading does occur, thus permitting (but not necessarily
insuring) consistent social choice, we could be sure
that the choices were optimal only if the initial distri-
bution of voting power were optimal. There are

the leading advantages of market choice is the limited
information necessary in order to properly organize
decentralized decisions. This "virtue" requires that
the assumption may be legitimately made that each
individual's effect upon the social outcome in the large
is so small as to be negligible. It seems fairly safe
to assume that the typical individual makes such an
assumption with respect to most choices.

ambiguities in political choice which parallel those of the market process. There is no basis for expecting that decision procedures will be reliable, even if a social ordering exists.

These remarks serve to motivate the question of whether the process of social decision might be constrained in some way such that it would reveal the true preferences of voters. If conditions for reliability of collective decision existed, then if the proper decision rules were employed, consistent decision would imply optimal decision. If, on the other hand, cyclical decisions resulted from such a procedure, it would be implied that consensus among voters was not sufficient to yield a social ordering.

If it is possible to deduce a priori constraints on the SDP such that it would lead to the optimum social state if a social ordering existed, it would be possible to alter the concept of collective rationality described in Chapter 2 so as to avoid requiring a social ordering. We might assume simply that a collectively rational response to the difficulties raised in preceding chapters would be to employ a decision procedure which would lead to the optimum if one exists. In other words, we might interpret collective rationality to refer to the means by which choices are made rather than to the ends of decisions themselves. In the presence of

conditions for reliability, collective rationality would
entail the use of decision rules which meet those con-
ditions regardless of whether individual preferences
aggregate to yield an ordering.  This essay will be
concluded with a brief examination of the implications
of this possibility and a review of some literature which
has focused upon the properties of particular decision
techniques.

B.  A Basis For Evaluation of Decision Rules.

A demonstration that particular rules of choice
would lead to the optimum social state if a social
ordering exists would serve as a basis for valuation of
decision rules for their own sake.  On the basis of
this information, individuals may order decision rules
in accordance with their ability to select a social
optimum even though they do not know, in fact, whether
the similarity in individual orderings of states is
sufficient to guarantee an optimum.  If the conditions
for reliability of the SDP are widely accepted, there
is a basis for expecting that consensus among individuals
concerning orderings of decision rules will be great
enough to yield a social ordering of sets of rules.[2]

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[2]Arrow considers the possibility that agreement
might be attained over the space of alternative decision
rules in his final paragraphs.  Social Choice, pp. 89-91.
But he considers it unrealistic to assume that individuals

In this event, condition (1) may be relaxed at the constitutional level of choice and the Arrow problem would be solved even though no social ordering of social states exists. The search for reliable decision procedures is, therefore, related to the Arrow difficulty. This reasoning must be considered cautiously, of course; for the Arrow porblem to be solved at the constitutional level, persons would have to make the value judgment that the possibility of attaining a socially optimal outcome is so desirable that they prefer rules allowing this possibility to other rules which might yield greater direct benefits (in terms of consequences) to themselves.

Nevertheless an investigation into the ways in which decision procedures might be made reliable is of obvious interest even in the absence of a social ordering of social states. This problem has not received emphasis in the choice literature largely because of the preoccupation with the search for sufficient conditions for a social ordering. The conditions under which complex decision procedures may reveal true social

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value decision rules for their own sake rather than for their effects upon individual welfare. A study of the conditions under which particular rules will maximize group weflare, however, might inform the individual's valuation of rules and yield a wider basis for judgment. Indeed, in this context, the distinction between valuation of rules for their own sake and for their consequences does not appear to be useful.

preferences need much attention. To treat the matter in detail is beyond the scope of this paper; however, the discussion of previous chapters leads to some preliminary considerations with respect to reliable means of decision. I attempt to raise some of these considerations in the following example.

Suppose that each individual possesses an individual welfare function which ranks alternative social states. It is unknown whether there is sufficient similarity in these orderings to yield acceptably a social ordering. Suppose further that it is impossible to determine the social preference by voting on alternative social states. Rather, the SDP requires that elements of the social state be chosen separately. We wish to place some restrictions on the SDP such that the choice over each single issue will be consistent with the socially optimum state, if one exists. Any particular pattern of choices over each of the elements of decision will define a unique social state.

Let us divide the issues for choice into several classes. Suppose the SDP is of the general form:

$$D = D(A, B, C)$$

where A, B, and C represent vectors of issues $a_i$, $b_i$, and $c_i$. Suppose that issues in A, B, and C are distinguished, generally, by the extent of spillover effects which result from each type, that is, by the number of

persons whose interests will be significantly affected by the outcome of each issue. The $c_i$ refer to issues of overriding national concern, the $b_i$ refer to issues of concern to various sub-groupings of persons, and the $a_i$ refer to issues whose solution affects only particular individuals. More specifically, let the $a_i$ represent the various "economic" variables of individual consumption, output and technologies of firms, types and amounts of factors supplied, etc. Let market techniques aggregate individual choices over this set of variables. Unconstrained market behavior, however, will not, in general, lead to solution for this set of variables which is consistent with the optimum (if one exists). Market decisions will be interdependent with other issues. Market error involving neighborhood effects, sub-optimal supply of public and quasi-public goods, and income distribution are overwhelmingly likely in a world of imperfect knowledge, no matter now comprehensive the values employed by individuals in the determination of their private choices. It will simplify matters to assume that individual market behavior is motivated by own-utility considerations only; given an "environment" of prices and income distribution, each individual acts so as to maximize his personal advantage.

Let the $b_i$'s, then, refer to issues which are intended to correct for market error and otherwise

supplement market outcomes. In cases where decisions in A are interdependent with decisions in B, some $b_i$ may be of the form of constraints on individual choice concerning some elements in A which enforces optimal market choice concerning those interdependent issues. For example, if the unregulated market behavior results in the pollution of a lake which is designated by a choice in B to be used for recreational purposes, some other decision in B must be made to effect a system of taxation, subsidy or other means to induce the polluters to desist. Other choices in B might allocate funds to various public works projects which would not be produced by market decision processes and from which specific groups of individuals would derive benefit. The total of all such decisions will determine the distribution of welfare among individuals. Assume that this set of decisions will be made by a majority rule voting procedure.[3]

Let C, in turn, be a vector of issues with which the choice of $b_i$'s may be interdependent. Suppose issues in C are "fundamental environmental characteris-

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[3] Define majority rule as $(xRy) \Longleftrightarrow N(xR_iy) > N(yR_ix)$: x is socially as good as y if the number of persons considering x at least as good as y exceeds the number who consider y at least as good as x. Under this rule, the behavior of indifferent voters cannot affect the outcome. A partial justification for the acceptance of majority rule will be given below.

tics," pure public goods which all persons in society have equal opportunity to enjoy. These issues, in other words, are of the type which are clearly matters of concern to the total society. Maintenance of the physical environment, provision for national defense, provision for equal justice under the law, and so on. Whenever decisions in B would otherwise be inconsistent with those in C, additional choices in C must specify procedural rules, distribution of voting power and other constraints upon the behavior of individuals with respect to issues in B in order to assure consistency. Let decisions C also be made by majority rule.

We have, then, a type of hierarchy of decisions based essentially upon the extensiveness of the spillover effects to be expected from each.[4] To take an extreme example, suppose a decision were made in the set C which stipulated that the total level of air pollution would not be so great as to result either in the melting of the polar ice-cap or the blocking out of the sun.

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[4]It is obvious that it would be extremely difficult to separate decisions into such categories. Legislatures, for example, usually determine the means for achieving some goal simultaneously with the specification of the goal. Further, how could one decide whether or not some decision referred to a "fundamental environmental characteristic"? Does the quality of education qualify? Does support of literature and the arts? These issues themselves are matters of taste. Since the example is intended to be only suggestive, I will not dwell on this problem, although it clearly is crucial to the argument.

Then the particular projects enacted in set B must not lead to pollution in excess of that level, and further, if the pattern of individual choices in set A would lead to excessive pollution, decisions taken in set B must constrain those individual choices in the appropriate manner. Similarly, if a decision in C specified that price stability should be attained, decisions in B with respect to public spending measures and with respect to constraining market behavior to the appropriate level of aggregate demand must be made accordingly. Our problem is to postulate some further conditions on the procedure of decision such that it will lead to the socially optimum state, if one exists.

Consider first the issues referred to as "fundamental environmental characteristics" in set C. The type of agreement necessary to yield a social ordering of alternatives concerning each element is most likely to be present in this subset of issues. Each individual must choose among these alternatives, in effect, in the absence of knowledge of what position he will occupy in the social state to be chosen. This feature results from the way in which the choice process has been specified as well as from the nature of the alternatives. Disagreement on grounds of the distribution of benefits arising from these decisions is removed from consideration. Indeed, if there actually is sufficient similarity

in individual orderings of completely defined social states, it must be because attitudes toward this subset of issues are predominate over purely distributional matters in the individual orderings. We have grounds, then, for requiring this set of decisions to be primary; once made, no pattern of decisions with respect to other issues should be allowed to be inconsistent with them.

We would wish further that only feasible configurations of choice among issues in C would be considered. That is, the separate choices among issues in C must not be contradictory. For example, (given the state of the art of economic policy formulation), the simultaneous choice of full employment and price stability may not be feasible. To insure that individuals choose in the presence of knowledge of the relationship between various issues in C, we might require that all interdependent choices within the set of issues C must be considered as a single decision. Although there may be many issues in C, it is obvious that they are a small number relative to the entire set of choices to be made, and therefore the interdependence among issues should be recognized with comparative ease.

More important, since we wish that the final outcome of the SDP should be consistent with the social ordering (if it exists) we must require that choices in C (where agreement is likely to be greatest) reveal

the true (ordinal) preferences of individuals. Vote-trading among individuals and other forms of strategic behavior must then be disallowed over this set of choices. For these issues, then, Arrow's condition 4 must be retained. However, if a social ordering of social states does exist, the similarity of individual attitudes toward this set of issues will yield consistent choices among alternatives for each issue. In fact, if voting in accord with true preferences does not yield consistent decisions over this set of issues, the presumption is obviously strong that no social ordering of social states does exist.

Consider the set of issues B. The presumption that choices in B should be achieved by majority rule may be justified by the following considerations. The choices among fundamental environmental characteristics must specify some distribution of voting power and additional procedural rules in order to assure optimal choices among issues in B. We know from theorems of May[5] and Mura-kami[6] that majority rule is the only decisive voting rule which is positively responsive to changes in individual tastes, neutral with respect to the labelling of

[5]Kenneth O. May, "A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision," *Econometrica*, 20 (Oct., 1952), pp. 680-684.

[6]Y. Murakami, *Logic and Social Choice* (New York, Dover, 1968), pp. 45-46 and *passim*.

alternatives, and egalitarian. Since in choosing among various possible voting rules, no individual knows what position he will occupy within the social state eventually chosen, these properties of majority rule would be appealing. That is, one would not likely choose an unequal distribution of voting power if he faced the porspect himself of having less power rather than more. There is some basis for expecting that majority rule would be sanctioned by its selection at the level of choices in C.[7] If majority rule is chosen in C, however, then the distribution of voting power (one man-one vote) thus selected does have normative significance; the distribution of power can no longer be taken as arbitrary.

If majority decision is adopted on such grounds, then decisions achieved by majority rule will also have normative significance if they are consistent (non-cyclical) provided that they do not contradict other decisions with respect to "fundamental environmental characteristics." In other words, so long as decisions in B are compatible with those in C (upon which there is consensus) they may be taken as optimal if they are

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[7]There are other arguments supporting the hypothesis that majority rule would be selected at the constitutional level of choice. The most extensive treatment of this problem has been given in Buchanan and Tullock, The Calculus of Consent, which will be discussed briefly below.

stable, for the initial distribution of voting power is initially correct for choice over each issue.

Similarity of preference orderings over each issue is, then, not necessary to provide optimal choices at level B. Consistency of choice may be obtained by relaxing Arrow's condition 4, thus permitting vote-trading.[8]

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[8]But it is not known whether vote-trading strategies guarantee consistent majority decision regardless of similarity in preferences. The types of strategy possible in vote-trading models are highly complex. One sort of trade, which is associated with the concept of "logrolling" deals with a situation where one voter actually yields his vote on one issue in exchange for a vote on another. In this situation, it can be inferred from the work of Plott that a majority coalition which itself satisfies the Pareto principle (no further exchange of votes among the coalition's members may increase one person's utility without decreasing that of another) will achieve an equilibrium outcome. See Plott, "A Notion of Equilibrium," p. 795. Apparently in this situation, vote trading must lead to majority coalitions. However, another short of trade deals not with exchange of votes, per se, but with a mutual alternation of positions on both issues (assuming there are only two issues involved). This type of behavior is associated with the concept of compromise. For example, one voter may agree with another to support a combination of appropriations for two public agencies which is intermediate between the two combinations most preferred by each. Coalitions formed by such compromise would not have to include a majority of voters. Plott has demonstrated that the conditions for equilibrium are (1) indifferent voters must not count yes, (2) the equilibrium must be at least one voter's optimum, and (3) the remaining voters must be partitionable into pairs whose interests are diametrically opposed. The definition of majority rule in note 3 satisfies (1). The agreement on the compromise solution guarantees (2). Then, if the compromise solution is chosen skillfully so that it divides the others into mutually opposed pairs, the compromise will be an equilibrium. While

It should not be necessary, then, to dispense with vote trading on all issues in order to achieve an optimal social state. If there are special interest groups which desire particular quasi-collective goods which may be most efficiently provided by collective decision, there is no reason why the voting process should not respond to them within the limits specified by decisions taken which are of national concern.[9] When interests of parties to governmental decision conflict, it is no doubt only through processes of vote-exchange that actions may be made which increase the net benefits of all concerned.[10] The failure of some voters to reveal

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much careful work is required to solve this problem, it is clear that vote-trading will greatly enhance the probability of gaining a majority equilibrium if preferences are not value-restricted.

[9]The leading criticism of logrolling is that in serving special interests, its general effect if often to prejudice decisions with respect to the public interest. For example, a public works bill resulting from vote exchanges may result in greater than optimal expenditures for public projects. See Buchanan and Tullock, Calculus of Consent, pp. 135-140. Similarly, import restrictions enacted through logrolling may interfere with national trade policy, defense bills enacted by legislators who evaluate too heavily the proposed military installations in their own districts interfere with national foreign policy, and so on. This criticism vanishes if vote trading outcomes are constrained to be consistent with the "national interest."

[10]As Buchanan and Tullock also emphasize. However, in order for compensation to be completely arranged by vote-trading, a unanimity voting rule would be required, if the costs of reaching decisions were assumed to be zero. The use of majority rule would not seem to

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their true preferences on some issues could have results
inimical to group welfare only if the consequent out-
comes on those issues were inconsistent with decisions
at level C.

If decisions in B are stable, then they may be taken
as optimal.[11]  Some (perhaps a great many) of these
decisions will arrange for constraints on the market
solution of some decisions in A.  For example, tax-sub-
sidy interferences may be imposed in appropriate fashion
upon markets affected by externality, decreasing cost
industries, and so on.  Or, if price stability and
full-employment are selected as goals by decisions in C,
decisions in B will have to insure that the appropriate
level of aggregate demand is attained in some way.
Within the limits set by these constraints, however, the
price structure established by own-utility maximizing

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be a serious restriction, however; if there are many
issues, each voter would likely find himself in the maj-
ority on some and therefore would be compensated for
his other minority positions.  And, of course, costs
of achieving decision are not zero; the net benefits to
voters would likely be higher under a voting rule which
permitted some action rather than under one which effect-
ively prevented any action at all.

[11]Any stable majority outcome is obviously Pareto
optimal.  But it may not be achieved in a Pareto opti-
mal fashion, the majority may impose external costs
on the minority on any particular decision.  If these
external costs balance out so that they are borne
appropriately by all members of the voting group, how-
ever, the final outcome may be viewed as unanimously en-
dorsed.

behavior will call forth optimal decisions in the market setting. The particular Pareto optimum emerging from competitive behavior will, in other words, be consistent with the decisions made by voting and therefore, may be taken as revealing the true preferences of consumers. The problem of market decisions interfering with other related collective decisions is removed.

Although this example is too simple and general to be of practical importance, it does suggest some conditions for reliability which have intuitive plausibility. If a social optimum is to be attained, those issues upon which there is greatest consensus must be determined in such a way that they will not be contradicted by outcomes achieved through a process of mediating differences. They also must be determined by voters who reveal their true preferences in voting. These requirements lead society to honor such consensus as it does have, irrespective of whether that consensus is great enough to specify as optimal a completely defined social state. These decisions result, basically, in the choice of an environment, a determination of the nature of the social state in the absence of consideration of the relative positions particular individuals will come to occupy in that state. Once that environment is determined, the exchange behavior of groups and individuals, both in the political and market setting

may determine a Pareto optimum feasible within that environment.[12]  If the environment is optimal, then the outcome of political bargaining and self-interested market behavior which is constrained to be consistent with the outcome of decisions made on the basis of consensus will be optimal also, and the preferred social state will result.

These suggestions at present are little more than speculation, of course.  It is hardly necessary to list the deficiencies of the example.  To begin, what a priori rule could be found to determine which decisions were of the type C and which were type B?  In the real world, this distinction would be difficult to draw.  However, an important empirical question is raised at this point. It would be interesting to know if legislators actually do distinguish between issues upon which vote-trading is permissible and those upon which it is not.  If they do so distinguish them, is vote-trading limited by the preferences of the legislator's constituents, by the legislator's obligation to serve the national interest, or by some other factors?  Are there general rules which the legislator follows in deciding whether to vote in accord with his regional interests and when to vote in

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[12]cf. Trygve Haavelmo, "The Notion of Involuntary Economic Decisions," _Econometrica_, 18 (Jan., 1950).

the national interest?  These questions, in fact, may be crucial to a solution of Arrow's paradox.

But aside from the matter of distinguishing between various types of issues, there are many other shortcomings of the example.  It is generally impossible to tell a priori whether quasi-public goods are provided more efficiently by private or public techniques.[13]  While it is true that private behavior often imposes external costs on others, it is also true that attempts by government to correct for those errors also impose external costs on some individuals.  It is unreasonable to assume, as a practical matter, that public decision may correct for every market error.  Whether or not some activity is to fall within the domain of the public sector must ultimately rest upon a comparison of the costs and benefits of public and private means for dealing with such matters.  The public authority simply may not have sufficient knowledge to enable it to constrain private behavior appropriately.

Furthermore, it is doubtless not extremely useful to discuss properties of public decision while not accounting for political parties, representative government, multi-layered government, processes of bureaucratic

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[13]See, for example, Buchanan and Kafoglis, "A Note on Public Goods Supply."

decision and other salient features of actual collective
choice.  Finally, any adequate theory of the social
decision process must account for dynamics:  as social
changes are enacted at the margins of public decision,
what kind of adjustment mechanism operates so as to
bring all other issues affected up for re-evaluation
and possible action?

In spite of all these objections, the example still
has some merit insofar as it suggests that every sort
of decision to be made publicly need not satisfy all
give of Arrow's value-conditions.  We must accept Arrow's
result that no single and universally applicable deci-
sion rule may acceptably satisfy his conditions.  But
rather than to try to decide which condition may be
dropped for all types of decision, it would seem sensi-
ble to attempt to disaggregate the decision procedure
and inquire what normative consequences result from
dropping alternative conditions for different types of
social decision.  The possibility is open that a social
decision procedure may be devised such that at each
stage and for each type of decision, at least four of
Arrow's conditions may be satisfied, but for the proce-
dure taken as a whole, all five are acceptably satisfied.
For example, choices with respect to environment might
satisfy all but the range condition (which is acceptable
if there is high consensus on such issues) while all

choices with respect to maximizing advantage within a
given environment might satisfy all but the indepen-
dence of irrelevant alternatives (which is acceptable
if there are means of expressing preference intensity,
if the initial distribution of voting power is correct,
and if none of the decision so made actually alter the
environment chosen).

Furthermore, if such a procedure could be discovered,
since the outcomes it generates, if stable, would be
optimal, there is some basis for expecting that the set
of rules it embodies would be preferred by many citizens.
If citizens were to make the value judgment that the
decision procedure should maximize group welfare, then
these rules would enjoy wide support, and Arrow's prob-
lem would be solved at the constitutional stage as well
as at the action level. At least in this context, the
notion of choosing between personal and group welfare
would have empirical meaning.

I know of no writer who has adopted this approach
to the study of properties of group decision. But such
an approach would appear to me to be fruitful; if the
difficulties raised by Arrow cannot be solved by this
approach, I fail to see how they can ever admit of
solution.

C. Some Preliminary Investigations of Properties of
   Decision Techniques.

While no writer has attempted to relate the prop-
erties of various means of social decision specifically
to the problem of acceptable aggregation of individual
preferences, several writers have explored aspects of
choice in other contexts. Since these contributions
bear heavily on the sort of investigation called for
in the previous paragraphs, and since I have referred
to several of them periodically on specific points, it
is appropriate to conclude this essay by commenting
briefly, but more systematically, upon the general
thrust of their work. Each contribution may be viewed
as a type of investigation of specific properties of
various mechanisms of group choice. Review of some
highlights of this literature may illustrate this (often
implicit) theme and help to relate it to arguments made
above.

One of the earliest attempts to relate political
behavior to the outcome of voluntary market behavior is
found in the work of Baumol.[14]  Baumol attempts to
extend the Pigovian externalities argument into a logi-
cal basis for the existence of institutions of govern-

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[14]Baumol, Welfare Economics and the Theory of the
State.

ment.  Because of the various forms of economic inter-
dependence, the marginal social cost of certain activi-
ties will differ from marginal social benefit.  Although
individuals may be aware of such interdependence, there
will be no change in the purely self-interested behavior
of individuals unless there is some change in the
economic environment.  Prices obviously will not change
so as to promote optimal behavior.  So in cases where
there is interdependence of the activities of individuals,
it will be rational for them to agree jointly to estab-
lish a central authority whose function will be to
directly coerce the parties of the agreement to follow
optimal patterns of behavior.  Baumol suggests, for
one example, that this is precisely the justification
for economic stabilization policy which entails a form
of central coercion which "forces" individuals into the
correct savings-expenditures decisions.  Thus, although
unrestricted individuals will not voluntarily constrain
themselves to optimal behavior patterns, even if they
know what behavior is, in fact, optimal, they will ex-
press a desire for institutions which will universally
impose those restrictions on all individuals.  Each
citizen views himself as in an n-person prisoner's
dilemma, and he will choose optimally only if assured
that the behavior of everyone else will be similarly res-
tricted.

Baumol does not explore the normative implications of such phenomena; he is content merely to give a rationale for public action in terms of an economic calculus. He does not delve into optimal means of making public decision, nor does he address the problem of whether market choice supplemented by public decision to correct for market failures can result in a social optimum. Thus, he does not confront the difficult problems raised by Arrow. Baumol's contribution is primarily directed at demonstrating that sole reliance on market techniques of choice will not lead to acceptable social outcomes. Baumol's work has been extended by the work of Olson[15] and Buchanan and Tullock.[16]

Olson also wishes to ground a theory of group behavior in the generalized externalities nexus, but his main purpose is to attempt to demonstrate that, in general, no group will voluntarily organize for the purpose of collectively attaining some common objective. His argument is derived from a straightforward application of the theory of public goods. Olson considers first the case of a small group which is defined in terms of some common goal which is shared by all members of the group. For members of the group, this common goal assumes

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[15] Mancur Olson, The Logic of Collective Action.

[16] Buchanan and Tullock, The Calculus of Consent.

the characteristics of a public good. If the group is small, the share of the total gain to be realized which is enjoyed by any one individual may be greater than the total costs of providing some amount of the public good. The individual with the highest subjective evaluation of the benefits would then be expected to provide as much of the good as would be justified by the equation of his marginal private benefits and cost. This amount of the good would then be enjoyed by all members of the group. While in the small group setting, there is an expectation that some of the public good will be provided, the amount obtained is likely to be sub-optimal. According to Olson, the sub-optimality will result because there is no way for the provider of the good to induce others to contribute to increasing the activity in question.[17] Only in very small groups,

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[17]If the provider threatened to reduce his outlay unless others contributed, they would simply point out that he would damage his own welfare more than their own. There is, thus, an "exploitation" of the person with the strongest effective preference by those with weaker. This result does not necessarily follow from the nature of public goods, however. Given that one member of a small group agrees to provide some amount of the public good, others may find it in their interest to supply some additional amount of the good or to subsidize the present provider to provide more of it. Each person responds to implicit differential marginal prices. For the two-person case, see James Buchanan, The Demand and Supply of Public Goods, pp. 29-33. It is possible that in the small group, an optimal amount of the good can be provided. Whether Buchanan's or Olson's result is actually achieved depends upon the divisibility of the good and upon whether the interdependence is of totals (separable) or marginal (non-separable).

where face-to-face contact between members occurs, is
it possible to arrange various social sanctions which
induce members to provide their "fair share" to the
collective effort.[18]  But Olson does not elaborate on
the function of such social pressure in group situations,
for his main interest is in groups large enough so that
face-to-face contact is impossible.

In the large group setting, Olson contends, some
system of side payments to members must be arranged
which serve the purpose of "internalizing" the benefits
of membership.  hhat is, since members will not volun-
tarily contribute to the fostering of the common goal,
they must receive direct benefits which compensate
them for the cost of their association.  For example,
physicians participate in the American Medical Associa-
tion not in order to pursue professional goals but in
order to receive an essential medical journal, to receive
referrals from other physicians, and so on.  These side
payments, whether positive rewards or negative sanctions,
coerce, as it were, each member to bear his share of
the burden in attaining the public good.  The existence
of the interest group, then, depends not upon the public

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[18]Olson, Logic of Collective Action, pp. 60-65.  For
an interesting discussion of the role of social rewards
and punishments in motivating behavior, see Peter Blau,
Exchange and Power in Social Life, (New York, Wiley,
1964).

good which the group provides, but upon the other private goods which the group may dispense to its members.

Thus Olson reaches the conclusion that "unless the number of individuals is quite small, or unless there is coercion or some other special device to make individuals act in their common interest, <u>rational, self-interested individuals will not act to achieve their common or group interest.</u>[19] Since individuals will not voluntarily act to foster the goals they have in common with others, Olson believes it is unrealistic to assume that voluntary cooperation in spontaneous private behavior would be preferred to centralized decision in any situation characterized by externality.

Two points emerge from Olson's argument which are of particular relevance to the approach to Arrow's problem suggested in this chapter. First, he reaches the conclusion that individuals will not adjust their private decisions so as to optimally choose on collective issues which are interdependent with their private behavior, even if all individuals recognize the interdependence. He assumes, then, that individuals never choose in accord with what I have termed their comprehensive preferences. This extreme assumption may be at least partially justified by the wide prevalence of the

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[19]Olson, <u>Logic of Collective Action</u>, p. 2. Italics in original.

dispensation of direct benefits to members of many interest groups. It is further justified, however, in situations where individuals are, in fact, unaware of the interdependence of their choices on some issues with the outcome of others. More importantly, it is implicit in Olson's argument that the fundamental purpose of constitutional choice is to provide an appropriate system of coercion so that individuals are encouraged to adjust their private decisions optimally. Insofar as this is true, optimal rules may be highly unpopular from the standpoint of individuals, at least when they are in the process of considering their preferred course of action on isolated decisions. This possibility forms the basis for requiring individuals to ignore questions of personal advantage when choosing among decision rules and other fundamental characteristics of the environment.

Olson does not investigate, however, who it is that is to make decisions with respect to optimal forms of coercion or how such decisions can be made in the correct fashion. The work by Buchanan and Tullock attempts to extend Baumol's basic argument in the direction of answers to these questions.

These authors distinguish sharply between collective activity pursuant to the choice of goods and collective activity pursuant to the choice of institutional rules to be followed in choosing various goods. In both cases,

Buchanan and Tullock believe that the social outcome can be derived from an individual calculus of alternative actions presented in situations marked by externality, but it is with the constitutional level of choice that they are most concerned. Their focus is upon the attempt to derive from individual maximizing behavior the selection of decision rules which specify which activities are to be collectivized and how decisive sets for each collective activity will be determined. The main burden of their argument is that, in contemplating a collective decision rule, the individual sees two types of potential cost. First, the fewer the number of individuals required to be decisive in making a decision, the more likely it is that choices will be made for the whole group of which any individual does not approve. The smaller the decisive set, the lower will be the probability that the ith individual will be in it; therefore, the greater will be the expected costs of the decision to i. This element of cost is the "expected external cost" and is a decreasing function of the size of the decisive set. Second, as the number of individuals in the decisive set increases, so too will increase the direct costs of decision-making, that is, the time, effort, informational processing and other organizational costs necessary to secure agreement within the group. This element of cost is an increasing function

of the size of the decisive set. The total cost of
decision (to any one individual) then is the sum of
these two elements, which is a U-shaped function of the
number in the decisive set. The individual may find
the lowest point on his expected cost curve and this
point will indicate the size of the "efficient" decisive
set in terms of this individual's calculus, say k% of the
group. But there must be some more basic (constitutional)
rule which indicates how a decisive set is to be chosen
from all the individual preferences concerning different
size sets for each activity. Buchanan and Tullock lay
particular stress on the unanimity voting rule at this
stage of decision. They argue essentially that only
if unanimity is required will the logrolling process
achieve full payment of "compensation" for all voters
whose preferences are such that they will not acquire
their optimal decisive set on each issue. Only the
unanimity rule, in other words, assures a Pareto optimal
constitution.[20] The fundamental point of the analysis
is that a constitution which specifies different voting
rules for different sorts of collective decisions is a
rational social construct, in terms of individual
preferences.

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[20]As noted previously, the authors believe that the
presence of vote-trading renders Arrow's theorem irrele-
vant. See Calculus of Consent, pp. 333-334 and 359, n.
14.

It is impossible to do justice to the wealth of analysis of mechanisms of social choice which Buchanan and Tullock have provided in such a brief characterization of their basic approach. But several points stand out in the context of the first part of this chapter. First, these authors do not attempt to relate their study to the problem of social welfare functions.[21] They make no investigation of whether the constitutional arrangements arising out of an individual calculus based entirely on self-interest will, in fact, yield a set of decisions at the "action" level which are mutually consistent, let alone optimal (except in the Pareto sense). Furthermore, it is explicit in their analysis that individuals will value particular decision rules solely in terms of the private consequences which will befall them under any alternative set; no account is taken of the possibility that individuals might desire particular rules for the sake of their intrinsic ability to choose that which is in the social, rather than the individual, interest. Of course, it is one of the oldest questions in political theory whether there is some way of organizing the types and sequence of decision so that the apparent conflict between these two evalua-

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[21] Both authors believe that since choices may only be made by individuals, the very notion of social choice and welfare is inappropriate.

tions may vanish. It is tempting to speculate whether a sequence of decisions such as that proposed above might lead to similar choices whether individuals were purely self-interested or more "socially concerned."

The basic contribution of Buchanan and Tullock is entirely consistent with the strategy of decision suggested above, however; there is a rational basis for constitutionally providing for different decision rules for different issues. Their view is simply that activities will be "collectivized" if a decisive set of individuals believe that their utilities, individually, will be higher with collectivization of decisions than without. The additional question posed above is if it is not also possible that a decisive set of individuals might select particular rules of collective decision which will foster some common notion of group welfare.

Another noteworthy attempt to achieve a positive theory of political choice behavior has been made by Downs.[22] Unlike Baumol, Olson, and Buchanan and Tullock, Downs does not seek to ground a theory of government in the generalized externalities nexus. He does not try to determine the scope of situations in which governmental

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[22]Anthony Downs, An Economic Theory of Democracy, (New York, Harper and Row, 1957).

activity may improve the welfare (as given by a welfare function) of a totally decentralized decision-making system. Neither does he wish to speak to the way in which governmental institutions or decision rules may be derived from a rational individual political calculus. According to Downs, the first approach presupposes a mythical entity: a state apart from individual men; the second is an "over individualistic" approach because it does not account for coalitions.[23] Downs' chief contribution is to introduce political parties into the "economic" analysis of government. The party is an institutional manifestation of political coalition which intervenes between the state and the individual. Downs examines governmental decision at the level of the coalition and seeks to discover "a relationship between the ends of individuals at large and the ends of the coalition. . ."[24]

Downs finds it difficult indeed to forge an accepta-ble link between individual goals and party behavior. His chief working hypothesis is that parties, or candi-dates, will seek to maximize votes, much as a firm maximizes profits or an individual, utility. The goal of the politician is to attain, and maintain, office.

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[23]Ibid., p. 17.

[24]Ibid., p. 17.

Voters, in turn, cast their votes strictly on the basis
of self-interest. The voter calculates expected utility
streams from government activity under each alterna-
tive party for the forthcoming election period and casts
his vote for the highest ranking party.[25] Thus, the
rational politicians carry out spending programs which
gain the most votes and attempt to finance them in ways
which lose the fewest. The decision rule for determining
elections is majority rule.

By exploring the implications of these simple
assumptions, Downs is able to reach some rather spectacu-
lar conclusions: for example, under a two-party system,
rational party behavior will be to encourage voter
irrationality by making its platform ambiguous;[26] the
rational voter will remain uninformed about elections.[27]

What is important for purposes here, however, is
the pervasive cast of Downs' whole book. The fundamental
point Downs wishes to make is that it is often rational
for government to act in ways which are ambiguous with
respect to, if not contrary to, any sensible concept

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[25]Under a multi-party system, he will either vote
for the highest utility stream he believes has a chance
of winning, or vote for an expected loser if he expects
that strategy to improve future alternative choices.
Ibid., p. 50.

[26]Ibid., p. 115.

[27]Ibid., pp. 238-247.

of public interest. In other words, even if policies
which lead to some sort of social optimum are technically
feasible, they may be politically impossible. Downs
cites the following kinds of examples. Suppose that the
electorate is voting on two issues. A majority favors
each issue. But on each issue there may be a passionate
minority which would gladly forego the adoption of a
preferred outcome on one issue in order to prevent the
adoption of a policy they strongly oppose. In this case,
a party may form a coalition of minorities by opposing
each of the issues.[28] Whether the adoption of each
issue would be socially preferred, of course, is a value
judgment; one might wish to hold that the intense prefer-
ences of the minority evinged in voting should "count"
in the outcome. But one may ponder the justification
for making interpersonal comparisons of utility in
this way. For suppose some position x is Pareto optimal;
everyone would be better or just as well off in x as in
the status quo. Then a move to x would be supported
unanimously. But it is not necessarily true that the
Downs model of the political process would achieve x.

Downs reaches this conclusion from an application
of voters' paradox. Suppose there are three alternatives,
X, Y, and Z and that each one of these alternatives is

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[28]Ibid., pp. 55-59.

dominated by a Pareto superior outcome, respectively, X',
Y', and Z'. Each citizen prefers the optimum to its
corresponding sub-optimum, but suppose the rankings of
the optimum positions among citizens are highly disparate.
Suppose that for three citizens, the rankings are

| Ranking: | A | B | C |
|----------|----|----|----|
| 1st | X' | Y' | Z' |
| 2nd | X | Y | Z |
| 3rd | Y' | Z' | X' |
| 4th | Y | Z | X |
| 5th | Z' | X' | Y' |
| 6th | Z | X | Y |

Then every Pareto optimum is dominated by a
sub-optimum position which would win in a comparison
by majority rule. No matter what stand an incumbent
party were to adopt on the issue, it would be possible
for an opposition party to defeat it by proposing
a sub-optimal position.[29]

One of Downs' central points, then, is that if
there is sufficient disparity in voters' tastes, compe-
tition among political parties may not cause social
movements to points of Pareto optimum. He raises the
fundamental question of whether the political system can
be expected to correct for market error, thus allowing
the attainment of an optimum optimorum, or, on the

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[29]Ibid., pp. 177-182. Of course, it is also possi-
ble that the opposition party could win by proposing
one of the Pareto optimal positions. But it has no in-
centive to propose an optimum rather than a sub-optimum.

contrary, if the dynamics of party behavior introduces
a host of new problems so complex that it makes unlikely
the possibility that the political process will attain
even allocative efficiency regardless of distributional
arrangements. Downs' leaning is obviously toward the
latter view. Whether or not this pessimism is justified,
it is apparent that the effects of complex institutions
such as political parties must be accounted for in
theories of social choice. This omission is one of the
chief weaknesses of the work so far accomplished in the
field of welfare and social choice.

Dahl and Lindblom[30] have contributed the analysis
of various social choice mechanisms which is most rich
in institutional description. Because the scope of
their work is so broad, and rather outside the traditional
approaches to social choice theory, it cannot be categor-
ized to fit into a single line of attack or unique body
of insights. Because of their wide perspective and
their extensive investigation of actual institutions of
social decision, they suggest the awesome nature of the
problem of rational social choice more fully, perhaps,
than any other single source. Their work requires
some comment here, particularly because their conclusions
are supportive of the notion that the achievement of

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[30] Robert Dahl and Charles E. Lindblom, Politics,
Economics and Welfare, (New York, Harper and Row, 1953).

optimum social welfare depends upon the expeditious choice
of particular techniques and institutions of decision
for different classes of decisions.

Dahl and Lindblom are not interested in the prob-
lem of preference aggregation. Rather, they postulate
several goals[31] and suggest that social rationality
consists in the establishment of processes of social
choice which "maximize" these ends. They analyze the
price-system, hierarchy (control by leaders), polyarchy
(control of leaders), and bargaining as techniques of
choice and attempt to evaluate each of these in terms
of the degree to which each fosters the specified ends.

Each decision process is assessed according to the
extent to which it allows social actors to calculate the
effects of alternative choices and how acceptably the
process controls the actors' behavior. Some mechanism
of control is implicit in any decision process; its
function is to induce "proper" responses on the part
of those whose behavior must be "regulated" in order to
promote the desired ends. The authors discuss several

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[31]The goals are rationality, freedom, subjective
equality, security, progress, democracy, and appropriate
inclusion, the last of which entails that individuals
should bear in mind the appropriate group for which
their actions have relevance while calculating the
benefits of particular behavior. Ibid., pp. 51-54.
Problems of definition and operationality of each goal
are irrelevant to remarks to be made here.

alternative mechanisms of control[32] and consider the effects of each on the presupposed social values. This analysis serves as a basis for evaluation of decision processes which employ different means of control. Each process may be further evaluated in terms of its relative ability to organize decisions of different sorts. Dahl and Lindblom differentiate between the basic decisions with respect to distribution of claims, resource allocation, resource development, and attainment of stability (the reconciliation of total claims with total resources) and find that different processes of choice are appropriate for different types of decision.

A brief summary of the arguments of these authors cannot do justice to the richness of their insights into the four processes of decision. Some examples of the advantages and disadvantages of each technique for several types of decision may indicate at least a flavor of their analysis.

The price system regulates behavior by means of spontaneous field control. There is extensive freedom

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[32]The mechanisms of control are command, "spontaneous field control" (the constraints on individual behavior arise from the process of interaction of individuals, e.g., a set of market prices emerges from the interaction of individuals in market behavior), "manipulated field control" (some individuals have the ability to alter the field of choice open to others), and reciprocity (an individual may induce another to alter his behavior in exchange for an alternation in his own behavior).

of choice for individuals within the limits determined by market prices but individual freedom is limited to the extent that the deprivations which result from the disregard of market incentives tend to be severe. The price system is an efficient mechanism for ordering the production and distribution of private goods, but is deficient as a means of satisfying collective needs and for determining the distribution of economic and political power.

Hierarchy relies upon the efficient control technique of command and may aid the attempt of a small group of people to achieve a rationally planned social state of affairs. It permits the use of specialized knowledge through the use of bureaucracy. There are costs of bureaucracy, however, such as inflexibility, impersonality, the resultant inequality of status, income, and power which is implicit in such a technique, and the difficulties of gathering requisite information for decision and enforcement of decision.

Polyarchy (control of leaders by non-leaders) forces leadership to be sensitive to citizens' desires. The reciprocal dependence of leaders and non-leaders upon each other weakens unilateral control and, _inter alia_, fosters equality. Polyarchy, however, requires wide consensus on issues and methods. For example, leaders and non-leaders must agree that excessive use of uni-

lateral control is undesirable, that it is acceptable for non-leaders to freely criticize governmental policy, to switch their support to rival leaders, and so on. Polyarchy, like hierarchy, lacks a mechanism for insuring that the marginal costs of various actions are estimated. Except where public goods or externality exist, and the voters' subjective estimates of marginal cost and gain may be more accurate than those suggested by the price system, polyarchy is less rational than the market system which does account for marginal (opportunity) cost in all situations.

Bargaining, or reciprocal control among leaders, increases consensus within the decision taking group thus making action possible, but also increases the likelihood of irrational outcomes due to logrolling effects. Its use will also serve to shift power and income distributions in favor of the highly organized at the expense of those who are not.

Dahl and Lindblom derive the conclusion from their extended discussion that the maximum attainment of the basic goals will be reached only through an expeditious use of all four decision processes. The implication is that some blending of the techniques is optimal; the rational society is the one which chooses the proper instruments of decision. Perhaps their most significant contribution is their demonstration that in attaining

an optimal social state, it is not enough to ask "which
goods shall be selected and provided by the market
and which by voting?" The whole network of choice-making
involves many overlapping systems of coordination of
activities and control of some individuals and groups
by others. This realization serves to underscore the
extreme simplicity of the choice models discussed in
this paper, and suggests the tentative nature of any
conclusions based upon them.

But Dahl and Lindblom do not shed any light at
all upon how the proper mix of choice-institutions is to
be made. They do not include in their study an analysis
of how individuals, through a utility calculus, or in
any other way, may actually choose this happy state of
institutional affairs. Although these authors define
rationality in terms of achieving the maximum of given
ends, and although they see these ends as being provided
by some optimal mixture of social decision processes,
they do not specify moves that should be made in order
to approach that optimal mixture. This is a fundamental
flaw in the logic of their approach and no doubt
explains why few choice theorists have attempted to draw
heavily on their work.[33] Yet Dahl and Lindblom have

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[33]Together with the fact that their work is rather
more descriptive and institutionalist in tone than
customarily suits the economist's taste.

offered perhaps the most extensive analysis of the interplay between technique of choice and various social goals. It is unfortunate that this sort of analysis has not received greater attention.

Lindblom has partially remedied this deficiency in a subsequent book.[34] Here, Lindblom tries to compare the results of centrally coordinated activity with voluntary activity coordinated by bargaining. His arguments support the contention that bargaining is generally a preferable mechanism of social choice than hierarchy. Lindblom's effort constitutes a descriptive and institutional elaboration of the idea that interference by central authority may not be an expedient method for handling the problem of externality.

His argument is in two stages. First, Lindblom attempts to show that centralized activity is less coordinated and rational than is commonly believed. It requires a synoptic view of the complexities of the problem to be solved and such a view is often impossible due to lack of information and the cost of information gathering. The overview of relations of interdependent decisions, which is often presupposed on behalf of centralization does not exist, according to Lindblom. The

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[34]Charles E. Lindblom, The Intelligence of Democracy (New York, Free Press, 1965).

central coordinator seldom has motivation to seek such
an overview. There is often insufficient agreement upon
the criteria for decision-making. Furthermore, much
empirical evidence suggests that there is often marked
inconsistency in decisions made by central coordinators.

On the other hand, voluntary bargaining behavior
(which Lindblom calls "partisan mutual adjustment") has
several advantages as a means of coordinating activity.
Most of these have received discussion above: voluntary
adjustment may proceed without consensus on values,
partisans are strongly motivated to find a mutually
acceptable outcome, bargained outcomes result in the
payment of compensation, and so on.

The particulars of Lindblom's argument are not
crucial here. What is important is his suggestion that
preferences for particular institutions and rules of
choice can be informed by an investigation of the
properties of various techniques of choice. It does not
seem unlikely that consensus concerning appropriate
techniques might increase as more is discovered about
them. It is obvious that choice theory has not made
adequate consideration of properties of various alterna-
tive techniques. For example, it is often assumed by
writers interested in preference aggregation that a
central authority could, in fact, implement whatever
action might be deemed optimal. But this assumption

simply presumes a theory of administrative behavior which does not exist.

While it is true that the properties of various techniques of social choice have not been well accounted for in the theoretical welfare literature, it is also true that writers concerned with positive properties of techniques have ignored the relevance of their study to problems of preference aggregation. A blending of these two approaches into a comprehensive and general theory of social choice is called for.

## D. Concluding Statement.

The essential argument of this paper has been summarized at the beginning of this chapter. A lengthy concluding statement is therefore unnecessary, but perhaps the following remarks deserve some final emphasis.

The search for the meaning of rationality of social choice is motivated ultimately by the belief that social policy must be justifiable in terms of an internally logical body of normative theory as well as in terms or pragmatic rules of thumb. Such justification requires a precise notion of what collective ends are to be attained and what constraints operate to limit their achievement.

The theory of social choice has failed to reach an adequate understanding of collective rationality

because of inadequate knowledge of the constraints as well as because of the failure to specify a means of determining goals. The interpretation of the concept of rational collective choice may be expected to vary widely depending upon the constraints which obtain in a given situation. For example, if citizens possess sufficient similarity in their preference orderings of social states to generate a social ordering, then collective rationality may be interpreted as the implementation of the optimal social state. Value statements may be made concerning outcomes, and the optimum may be achieved by any convenient means. If, on the other hand, insufficient consensus exists to generate a social ordering of outcomes, then collective rationality must be interpreted without reference to internal consistency of social preference defined on outcomes. An alternative course is to interpret collective rationality in terms of decision procedures. In this case, value statements may be made concerning rules of decision, the processes by which differences are to be mediated, rather than concerning outcomes.

Whether sufficient consensus exists to generate a social ordering is an empirical question. The appropriate interpretation of rational collective choice cannot finally be made until such empirical problems are solved. The point is that, in retrospect, one could not expect

purely a deductive body of social choice theory to select from among alternative interpretations. Such selection requires knowledge of whether social choice is to proceed constrained by substantial lack of consensus.

The measurement of degree of consensus, then, would appear to be as necessary as it is difficult. The choice literature offers an important contribution to this inquiry, however. Present knowledge of sufficiency conditions for a social ordering shows that agreement concerning criteria for judgment of outcomes may be as important as agreement on the ends themselves. Even if there is substantial disagreement about ends, if citizens evaluate ends by means of an ideological frame of reference held in common, majority rule may yield consistent decisions which also possess normative significance, at least if true preferences are revealed in voting. The social choice literature has, thus, opened up an approach to the study of ideology which is somewhat novel.

In the final analysis, however, it would be reassuring to determine a process of choice such that if an optimum existed, it would be revealed by actual choice behavior of citizens. To this end, much further study of properties of various decision techniques is required. The fundamental question is how individuals may reliably

reveal their preferences for well-defined social states through a series of seemingly isolated decisions taken on separate issues in both private market behavior and voting. Here again the choice literature is plagued by lack of knowledge of constraints. On the one hand, too little is known about the ability of democratic process to reveal the comprehensive values of citizens; on the other, too little is known of the sort of constraints which would ideally have to be placed on any decentralized system of choice in order to insure that individual choices are consistent with the optimum. In short, it is not known whether there is a reliable decision procedure within the set of feasible procedures. At the very least, one may judge that the only known potentially reliable procedure, that of balloting periodically upon completely defined social states, is infeasible.

The search for reliable procedures, of course, must proceed in small steps. The discussion of this chapter strongly suggests that one of the most helpful steps would be to attempt to determine precisely when logrolling and consequent payment of compensation are consistent with the attainment of an optimum and when they are not. It is disconcerting to view logrolling both as a means of approaching the locus of Pareto optimal states and of causing the political process to fail to reveal true social preferences. The paradox must be

removed by the discovery of conditions for optimal vote exchanging.

Two further general points stand out in the discussion above. It is essential to future progress of social choice theory that investigators clearly differentiate between problems of aggregation of preference and properties of choice processes. To fail to do so is to create the same confusion between normative and positive analysis which has plagued social science in the past. It is also necessary that the theory of social choice develop a more full awareness of the articulation between systems of choice making employed for different issues. The state of affairs obtaining at a given time results from the totality of choice-behavior of individuals in economic, political and social settings. If choices made in economic, political, and social contexts are to be consistent with an optimal social outcome in the large, they must be consistent with each other. An understanding of social choice must entail a more adequate understanding of what the social science disciplines owe to each other.

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Vita

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