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PRICE-LEVEL CHANGES AND THE INVENTORY POLICY OF THE FIRM

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FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

By

Claude Machline

June 1971

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

(Principal Adviser)

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Approved for the University Committee
on Graduate Studies:

Dean of Graduate Studies

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The topic discussed in this dissertation was initially suggested by the inflationary conditions prevailing for so many years in Brazil and the need for Brazilian businessmen to adapt to an inflationary environment. The main objective of this study is to contribute to arrival at more rational decisions concerning inventory policy in times of price-level changes. The recent spreading of inflation throughout most advanced countries in the world has encouraged persistence in the project.

Final thanks are due to the many businessmen, here and abroad, who gave of their time and provided much of the data necessary to the completion of this study; as well as to the following publishers, who kindly gave permission to use direct quotations reproduced in Chapters II and V of this dissertation: Longmans, Green & Co., Ltd; The University of Chicago Press; Harper and Row; George Allen & Unwin Ltd.; Princeton University Press; St. Martin's Press Inc.; and National Industrial Conference Board, Inc.

PREFACE

The purpose of this dissertation is to study the influence of anticipated price-level changes on the inventory policy of the industrial firm. It will be determined whether or not the reordering and scheduling procedures, such as economic lot sizes of purchasing and manufacturing, or replenishment and safety levels, must be modified, and are indeed modified in actual practice, in order to take into account anticipated changes in material, labor, and holding costs.

In many countries, continuously rising costs create a challenge to the administrator who desires to use quantitative tools of operations management. He observes that the mathematical models suggested in the literature for the purpose of optimizing the inventory policy of the firm postulate constant costs during the period studied. He wonders whether these methods can still be used in a situation of upward moving costs, and, if this is the case, how they should be modified.

It is not obvious that the prospect of rising costs should lead the rational manager to increase or decrease inventories. On the one hand, the foreseen rise in the costs of inputs such as materials and labor seems to favor an increase in the amount of purchases and production which should be made in earlier periods, so as to avoid the effect of later increases in costs. On the other hand, an upward trend in prices is often accompanied by an increase in interest rate and storage costs that makes it more expensive to stock commodities. Executives are aware that they should turn away from cash and hold physical goods instead when money loses its purchasing power day by day. At the same time, the accumulation of inventories reduces the inventory turnover, and the manager knows that such a decrease is a sign of inefficiency. Caught between these conflicting forces, the inventory manager might be doubtful as to the proper action to take with respect to his inventories in periods of price instability.

In summary, the study will pursue the following plan: the Introduction, Chapter I, makes a general description of an inflationary economic environment. Chapter II contains an examination in some detail of the influence that changes in costs of materials and in the interest rate could have on the inventory policy of the firm. It includes characterization of patterns of price-level changes which are of particular importance for inventory management and which can be designated for convenience as uncontrolled and controlled inflations.

Chapter III is taken up by an investigation of how some traditional methods and formulas of inventory management must be adapted in order to take into account the effect of cost changes forecasted for the perceivable future.

Chapter IV reports the results of a field survey involving 34 firms, 17 in Brazil and 17 in the United States, which was directed at revealing the actual behavior of managers in regard to their inventory policy in periods of rising costs.

Chapter V presents the results of regression studies of aggregate manufacturers' stocks in the United States in the years 1955-1970, undertaken in order to verify whether or not the managers conform to the rules suggested by the normative conclusions that were reached. A regression study of aggregate inventories in Brazil, in the years 1948-1967, is also undertaken.

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CHAPTER I

INTRODUCTION

AN INFLATIONARY ECONOMIC ENVIRONMENT

The rational manager tries to minimize the cost of procuring and holding stocks. A large component of the holding cost is the interest rate. In investigating adaptations of the firm's inventory policy to an economic environment characterized by cost increases, it becomes necessary to examine the changes in both price-level indexes and the interest rate in an inflationary situation.

This chapter includes an initial description of an inflationary economy by reference to price-level changes. As the relationship between price-level changes and the interest rate depends on monetary conditions, as well as on the aggregate demand and supply situation, indication is made of how these economic variables are tied together in the inflationary situations under scrutiny. The chapter concludes with an examination of the inflationary interest rate and its implications with respect to inventory holding cost.

1.1. Definitions of Inflation

The term inflation is used in this text in the sense widely acknowledged by most economists: namely, a general and persistent rise in price levels. It is precisely the effect of this situation upon inventory management that serves as the principal object of this study. In other words, the term inflation is used here to signify upward price-level changes.[1]

An inflationary situation is characterized by a rise in most prices and by a tendency toward continuation of increases. Not all prices go up at the same rate, time, or speed, and the differences between the increases in several sectors of the economy and in the different components of costs are of pivotal importance in a study of the effects of inflation on inventory management. Inflation, as used here, must also be differentiated from price-level fluctuations in one commodity due to conditions peculiar to this product such as shortage or glut; only situations of upward going prices will be discussed.

1.2. Theories of Inflation: An Overview

In order to clarify the kind of setting which the management of the firm faces in an inflationary situation, a summary description of some well-known explanations of the dynamics of inflation seems appropriate.

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[1]This definition of inflation tends to supplant former ones, such as: increase in the money supply (monetary inflation), increase in bank credit (credit inflation), or devaluation of the currency in relation to foreign exchange or gold (exchange inflation or gold inflation). "A rise in the price level of all currently produced goods and services--that is, a rise in the average price of all goods and services in the gross national product . . ." is the definition of inflation employed by George Leland Bach, in Economics: An Introduction to Analysis and Policy (6th ed.; Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1968), p. 124.

The major theories of inflation are three in number.[2] The inflationary mechanisms depicted in these theories are not mutually exclusive, and they can be combined to create more complex, but more likely, models.

A classical explanation of inflation is the demand-pull theory. It has two different formulations. The first one, favored by monetary economists, puts the primal responsibility for inflation upon an increase in the supply of money. A look at the "quantity equation of money," or "equation of exchange,"

$$PY = MV \tag{1}$$

shows that, assuming the velocity V of money circulation to be constant, an increase in the quantity of money, M, will result in an increase in the product PY, where P stands for aggregate price level and Y for aggregate production. If the production Y does not increase enough to absorb the volume M of money, the price level P must increase, for the identify (1) to remain valid.

The second version of the demand-pull theory, due to John Maynard Keynes, stresses a shift in the aggregate demand curve to the right, as illustrated in Figure 1-1; an increase in demand is represented by the succession of demand curves $D_1, D_2, D_3, \ldots$. When close to point F, corresponding to the full employment of economic

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[2] Martin Bronfenbrenner and Franklyn D. Holzman, "Survey of Inflation Theory," American Economic Review, LIII (September, 1963), 593-661.

resources, the excess of demand is all translated in price increases, along the path P_3, P_4, P_5, . . . corresponding to the intersection of the demand curves with supply curve S.

Figure 1-2 shows what has been called the "inflationary gap," namely, the difference between demand and production. This difference is the source of the inflationary pressure, and it justifies applying the name of "buyer's inflation" to the demand-pull inflation.

Figure 1-1 illustrates that, in order to proceed, the inflationary trend requires an increase in the quantity of money, corresponding to the increases in the areas of the rectangles, which represent the products PY = MV. The rise in demand may be due to a government deficit, caused by war, a policy of economic development, agricultural subsidies, public works, etc., or it may be due to an increase in population, or it may be due to rising demand from abroad (imported inflation). Whatever its origin, this kind of inflation can only happen under the two following conditions: too much money "chasing" too little goods and an excess of demand over supply.

The second traditional explanation of inflation is the cost-push theory. Although "rediscovered" in recent decades under the name of "new inflation," it has been known for a long time, as Bronfenbrenner and Holzman point out:

> Cost inflation has been the layman's instinctive explanation of general prices increases since the dawn of the monetary system. We know of no inflationary movement that has not been blamed by some people on "profiteers," "speculators," "hoarders," . . . .[3]

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[3] *Ibid.*, p. 613.

Just as the demand inflation operates through a shift of the demand curve, the supply inflation or "seller's inflation" can be represented by a shift of the aggregate supply curve along a fixed demand curve. The points P_1, P_2, P_3, . . . represent the equilibrium path (Figure 1-3).

Such an inflationary episode could occur in consequence of a decrease in the production facilities of a community, but it is usually associated with cost increases in inputs--with wage increases, for instance. Invoking the marginal cost concepts of the production theory of the firm indicates that a cost increase will result in a decrease in the quantity offered, in both competitive and monopolistic situations, and therefore to a shift of the supply curve to the left. This kind of recessive inflation could also be caused by pessimistic expectations of the business community.

The combination of demand-pull and cost-push effects will cause an inflationary path which has become quite frequent in modern economies, when the government is committed to a policy of maximum employment and development, the so-called wage-price spiral. After the first "innings" (Keynes) of wage increase, which brings the equilibrium point from P_1 to P_1' (Figure 1-4), the induced increase in income resulting from this wage increase will shift the demand from D_1 to D_2, and the new equilibrium point will be P_2, if the government issues the amount of money necessary to fuel this shift. The wage-price spiral continues in the same way along the path P_2', P_3, P_3', P_4, ...

Another inflationary path is indicated by the points P_1, P_2'', P_2, P_3'', P_3, P_4'', . . . in Figure 1-4. It would result from

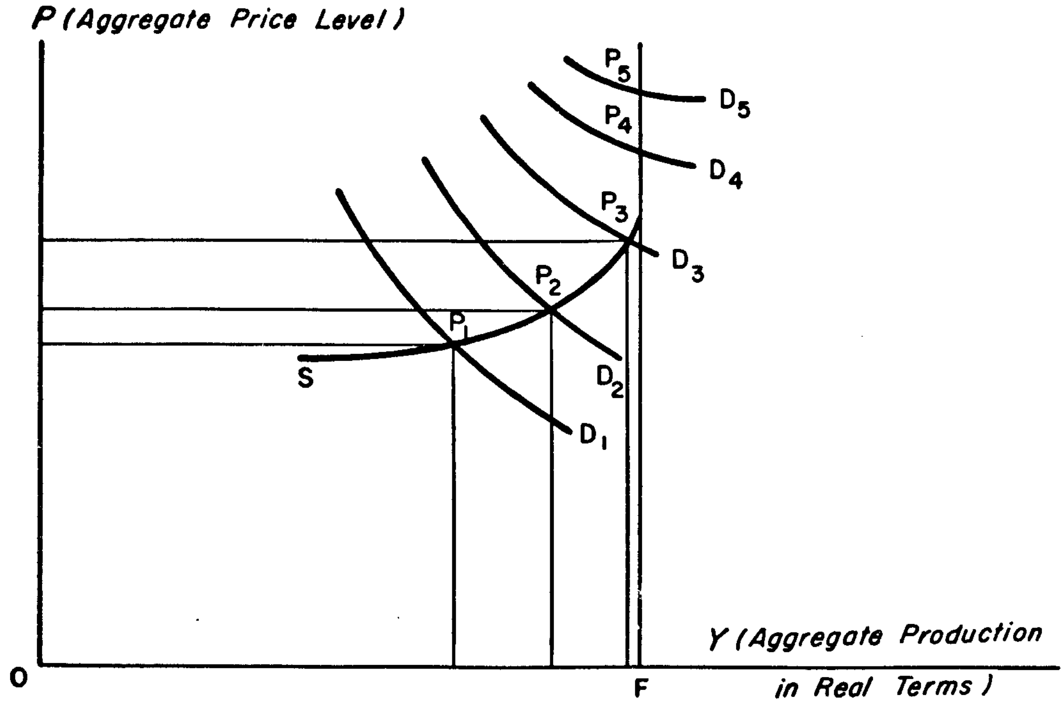

Figure 1-1.--Demand-Pull Inflation

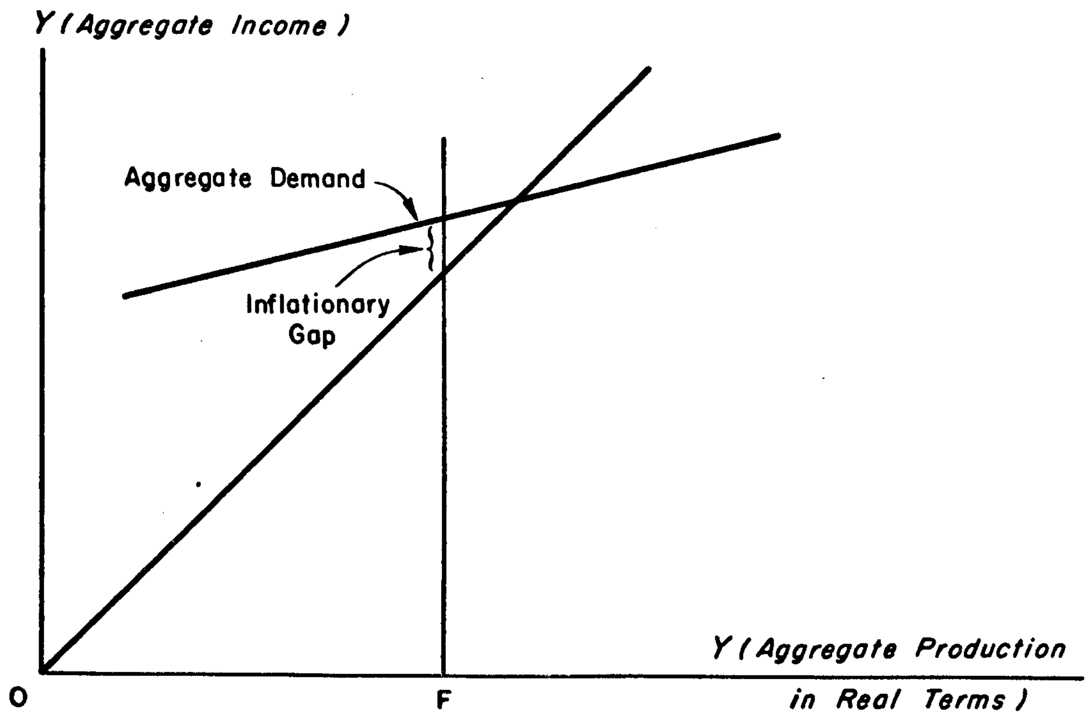

Figure 1-2. The Inflationary Gap

an initial increase in demand, followed by a "speculative" shift in the supply curve. Its final effects are the same as the wage-price spiral, and it could not proceed without the issuance of money or increases in nominal wages.

The "administered inflation" is an explanation of some inflationary movements like the one which occurred in the United States between 1953 and 1958. It says that the shifts of the supply curve shown in Figure 1-3 occur spontaneously due to the monopolistic power of some sectors.[4] Eckstein and Fromm have tried to validate this hypothesis with statistical data and succeeded in showing that steel increased in price more than the rest of the economy during that period.[5] The issue has led to an extensive controversy among economists.[6]

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[4]Gardiner C. Means, Administered Inflation and Price Policy (Washington, D.C.: Anderson Kramer Associates, 1959); Gardiner C. Means, Pricing Power and the Public Interest: A Study Based on Steel (New York: Harper & Brothers, 1962).

[5]U.S. Congress, Joint Economic Committee, Steel and the Postwar Inflation, by Otto Eckstein and Gary Fromm, Joint Committee Print, Study Paper 2 (Washington, D.C.: Government Printing Office, 1959).

[6]The main theoretical rebuttal of the administered price inflation theory is that even monopolies have an optimal price-quantity equilibrium point, as is well known in economic theory. Monopolists have no means to increase their prices beyond that point spontaneously, because to do so would lessen their returns. Only if they have not previously exercised their monopolistic power, for some extraneous reason, would such a price increase make sense. But, starting from equilibrium, the monopolistic firm would be unlikely to originate the first round of increases. See: Henry W. Briefs, Pricing Power and "Administrative" Inflation: Concepts, Facts, and Policy Implications (Washington, D.C.: American Enterprise Institute for Public Policy Research, 1962).

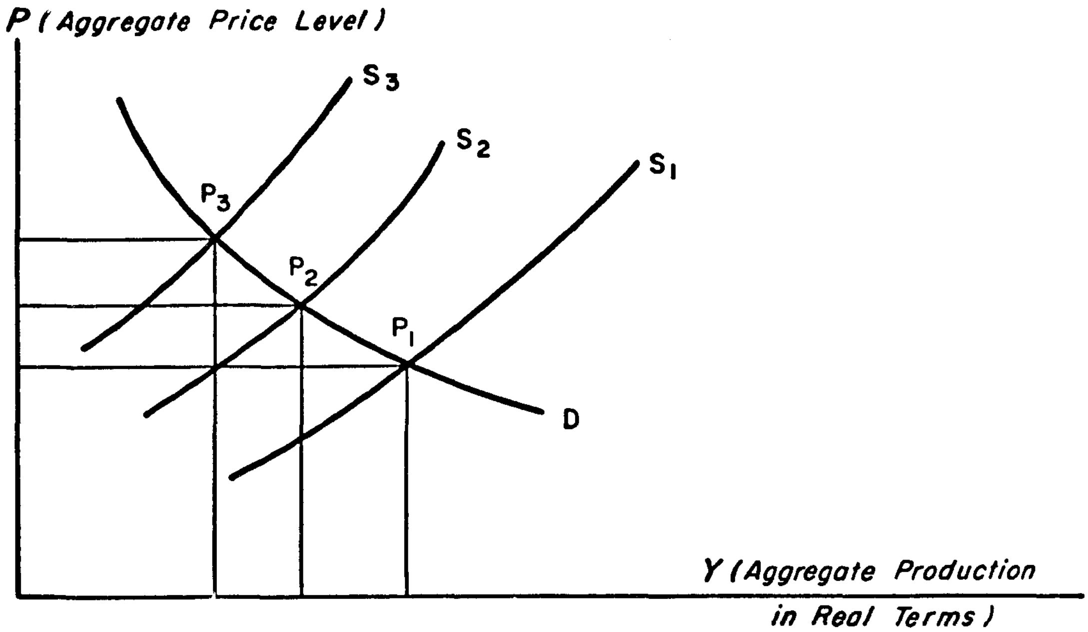

Figure 1-3.--Cost-Push Inflation

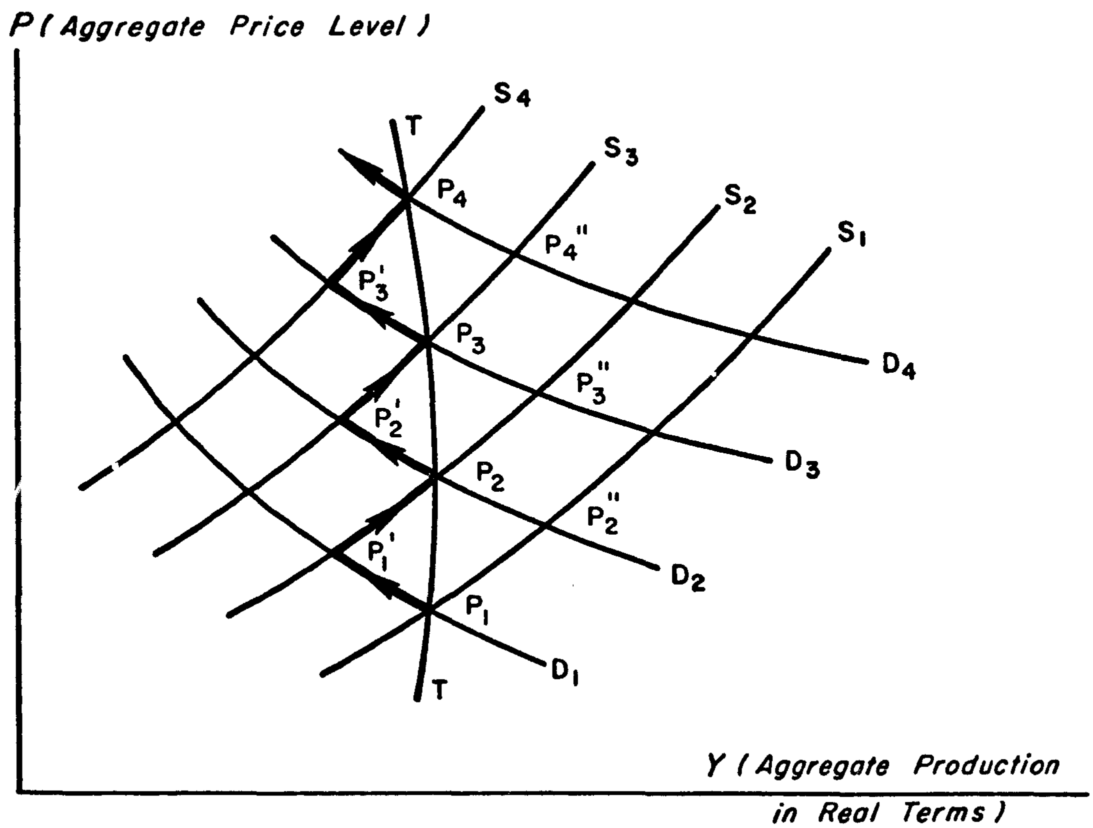

Figure 1-4. The Wage-Price Spiral

The third group of inflation theories essentially comprises the structuralist views. As known in the United States, the structuralist theory really builds on the foundation of the demand-pull and the cost-push theories. But it adds two mechanisms: one is the sectoral transfer of inflation; the other is the irreversibility of the increase in prices. It has also been called "sectoral" or "demand-shift" inflation. To quote Bronfenbrenner and Holzman:

> A form of structuralism native to the United States, with roots going back to price-inflexibility theorists of the Great Depression, relates to the prices of capital goods and to wage rates in strongly organized sectors of the economy. Such prices, it is said, rise in consequence of increased demand or cost but do not fall in response to decreased demand or cost. Wages likewise rise in response to increases in living costs or in business profits but do not fall when the latter decline. The results of this one-way flexibility is that shifts in demand have inflationary effects, raising prices and wages in sectors to which demand shifts, but leaving them substantially unchanged in sectors from which demand shifts.[7]

Estruturalismo, as it is known in Latin America, holds that inflation promotes the development of industry and the growth of the most active sectors of the economy, namely, industrial entrepreneurs, through the expropriation of landowners and *rentiers*, as well as through lowering the real interest rate. For the structuralists, the curve TT of Figure 1-4 has a positive rightward slope. Therefore, real output and employment increase with an increase in prices.

According to Mario H. Simonsen:

> If I have well understood the structuralist theory, its main issue is that underdeveloped countries facing stagnant . . . exports cannot grow without some degree of inflation. . . . Economic development under these conditions requires

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[7]Bronfenbrenner and Holzman, loc. cit., p. 612.

> structural changes. Because of the rigidity of certain supplies, because of the very limited capacity to import, and because of the need for import substitution, these structural changes are likely to require some modifications in the relative price system. Since money prices hardly could be expected to fall, relative price falls can only come about as required if the general price level goes up. A blind stabilization program, with no regard to this peculiar feature, could only stop prices from rising by slowing down the process of structural change and, therefore, by hindering economic growth.[8]

Simonsen finds the lack of quantitative reasoning to be the main weakness of structuralist thought. He also points out that it is based upon insufficient statistical evidence in view of the "many kinds of inflation and several types of growth."

1.3. Rate of Inflation

A concept which is to be used here is the notion of "inflation rate." The rate of inflation for a period can be defined as the percent change in the implicit price deflator for the Gross National Product. Calling the rate of inflation d, for a period--such as a year, for instance--and P_1 and P_2 the implicit price deflators for GNP at the start and end of the period, respectively, we have:

$$d = \frac{P_2 - P_1}{P_1}$$

Employment of the Consumer Price Index, or the Wholesale Price

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[8]Mario Henrique Simonsen, "Comment," in Inflation and Growth in Latin America, ed. by Werner Baer and Isaac Kerstenetzky (Homewood, Ill.: Richard D. Irwin, Inc., 1964), p. 109.

Index, or some other index reflecting price movements in a particular sector in place of the GNP deflator in the formula above would yield sectoral rates of inflation, such as the rates of increase in the cost of living, in wholesale commodities, in industrial commodities, etc.

The rate of depreciation of money, D , also called the rate of devaluation or loss of purchasing power of money for a period, is defined as:

$$D = \frac{P_2 - P_1}{P_2} = 1 - \frac{P_1}{P_2}$$

The ratio $\frac{P_1}{P_2}$ represents the value of money, or purchasing power of money, in reference to a base period in which it was 1. The rate of devaluation is then the original value of money minus the value of money at the present time.[9]

The rate of inflation can be seen as the velocity of price-level changes; it is the speed at which prices change. In view of the inflation rate's display of strong arithmetic similarity to an interest rate, all of the operations of discounting and compounding used with interest rates can be applied.[10] It is necessary to emphasize that many different rates of inflation occur simultaneously in an economy according to the specific measures of price changes used.

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[9] For instance, if $P_1 = 130$ and $P_2 = 140$, $d = 7.7\%$ and $D = 7.0\%$ The following relationship holds: $D = \frac{d}{1+d}$.

[10] Like the conversion of monthly to annual rates and vice versa.

1.4 Real and Apparent Interest Rates

If an inflation rate of d percent per year is anticipated, the amount of currency $C = C_o(1+d)^n$ at the end of n years[11] will have the same purchasing power that the amount C_o had at the start of the period. Therefore, in the absence of any interest rate charged for use of the money, an increase of d percent would have to be asked by a lender in the amount he gets back after one year just in order to maintain the purchasing power of his capital.

In times of inflation, it is necessary to distinguish between the real interest rate--corrected for inflation--and the apparent interest rate, which is the nominal or stated interest rate, also called "money" interest rate.[12] If the lender were able to anticipate the inflation rate d exactly and if the real interest rate were i , he would have to charge the apparent interest rate r , such that:

$$r = i+d \tag{1}$$

in order to protect himself against the erosion of the currency's

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[11] Or $C = C_o e^{dn}$, if continuous compounding is utilized.

[12] Sometimes "apparent interest rate" is referred to in a different context. Some lenders charge a stated interest rate on the initial value of the loan, instead of charging it, as it should be, on the unamortized portion of the loan. In that case, the stated interest rate is an "apparent" one, approximately equal to half the real rate paid by the borrower. Also, the term "true interest rate" means the rate really paid when all of the financial charges and effects of compensating balances have been taken into consideration.

purchasing power.

The rate r is an expected, an *ex ante* rate. Conversely, looking back at a period in which the nominal interest rate has been r, and the inflation rate d, the *ex post* real interest rate will be found to have been:[13]

$$i = r-d \quad .$$

In a period of inflation, the lenders will charge more than the real interest rate i, but it is not sure that they will be able to forecast perfectly the inflation rate d and charge, in consequence, the theoretical apparent interest rate r such that $r = i+d$. As the desired real interest rate of the lenders, i, is also not known, it

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[13] The relationship between real and apparent interest rates is demonstrated as follows: if no inflation existed, the amount C_o would become $C_o(1+i)^n$ when compounded at the real rate i for n years. If inflation obtains, the amount $C_o(1+i)^n$ must be transformed into $C_o(1+i)^n\ (1+d)^n$ in order to preserve its purchasing power. The lender will want, in other words, to recover $C_o(1+i)(1+d)^n$ at the end of n periods. The lender has, therefore, to charge an apparent interest rate r such that $1+r = (1+i)(1+d)$. And $r = i+d+id \cong i+d$. Therefore, $i = \frac{r-d}{1+d} \cong r-d$. The element *id* represents the interest charged over inflation, or the inflation charged over the interest term. The instantaneous apparent interest rate r *equivalent* to a periodic real interest rate i and a periodic inflation rate d is such that $e^r = (1+i)(1+d)$.

cannot be "proved" that the formula $r = i+d$ holds in the real world.[14] But it can be observed that, in general, in an economy, the nominal interest rate r and the inflation rate d are positively correlated in the long run, although they display lags in the short run.

Figure 1-5 shows the evolution of the prime commercial interest rate and inflation rate, measured by several indexes, in the United States from 1920 to 1970. It may be seen that, during most of the 1920's, a period of stable prices, the stated interest rate was 5% p.a.; this was also, therefore, the real interest rate. During the middle 1930's, the apparent interest rate dropped to 2.5-3%, and the real rate to 1% or even less; the war and postwar periods must be ignored, as product rationing, price controls, and reconversion to peace distort the picture. In the 50's and 60's, the stated interest rate and the inflation rate showed a tendency to follow roughly parallel paths.

It seems, in conclusion, that whenever the inflation rate grows, it is accompanied by growth in the stated interest rate, providing some support to formula (1). The next chapter will return to this relation and bring empirical and behavioral evidence to the hypothesis that the

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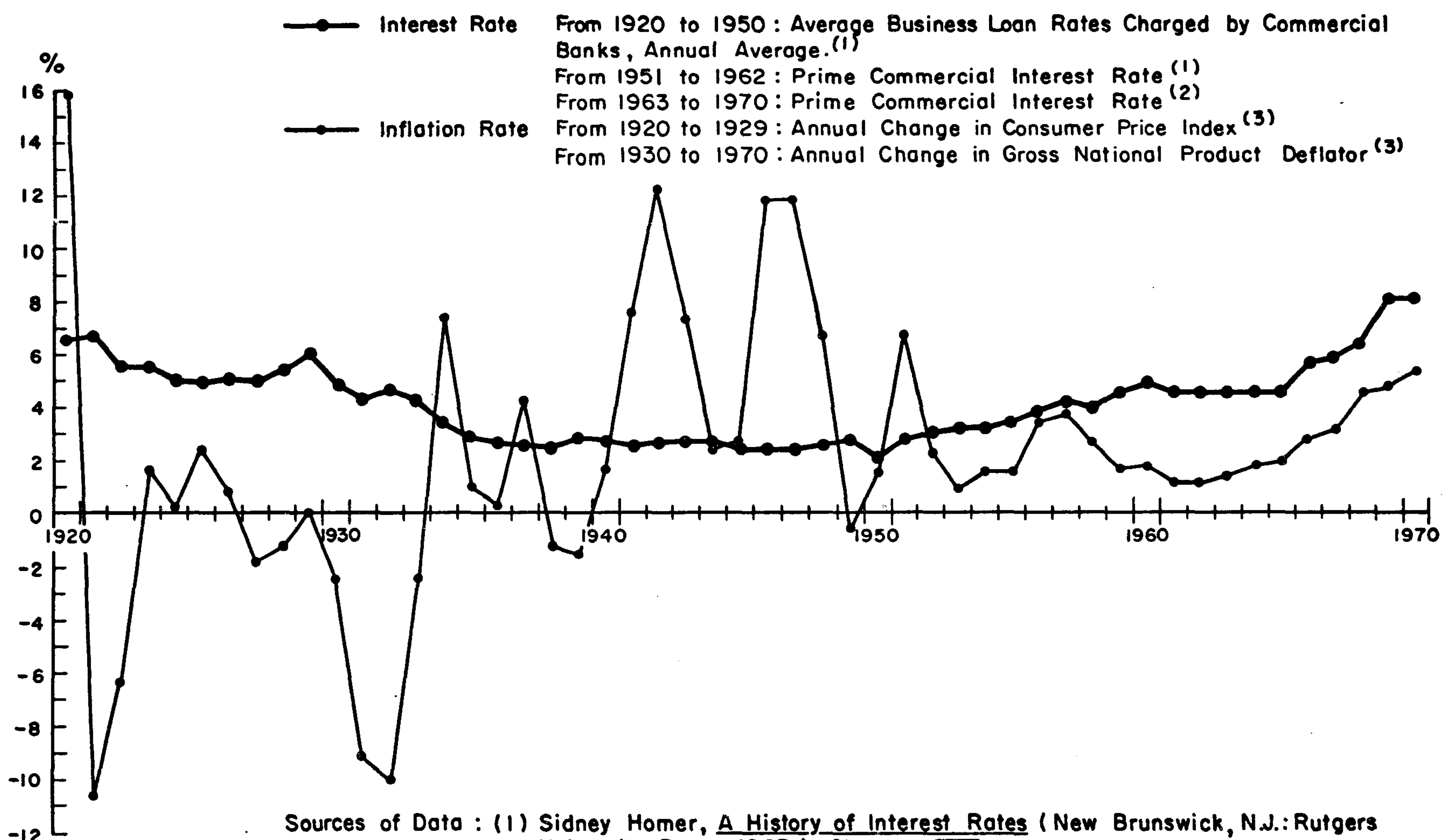
[14]As Irving Fisher points out in The Rate of Interest (New York: The Macmillan Company, 1907), p. 356, the relationship between inflation and interest rates was known centuries ago. In "A Discourse Concerning the Currencies of the British Planatations in America," Boston, 1740 (reprinted in the Economic Studies of the American Economic Association, 1897), Fisher reports that an anonymous author wrote: ". . . Large Emissions of Paper Money does naturally rise the Interest to make good the sinking Principal . . . the larger the Emissions, natural Interest becomes the higher . . . ." Irving Fisher attempted to validate this relationship by showing that changes in price level were accompanied by changes in the nominal interest rate in the same direction. (Irving Fisher, The Theory of Interest (New York: The Macmillan Company, 1930), Chapter XIX).



Sources of Data : (1) Sidney Homer, <u>A History of Interest Rates</u> (New Brunswick, N.J.: Rutgers University Press, 1963), Chapter <u>XVII</u>.
(2) Data Computed from Standard & Poor's, <u>Trade and Securities Statistics</u> (N.Y.: Standard & Poor's Corporation, 1970)
(3) Data Computed from <u>Survey of Current Business, passim</u>

Figure 1-5.--Interest Rate and Inflation Rate in the United States, 1920-1970 stated interest rate generally lags behind fluctuations in price-level changes. Figure 1-5 also reveals that large fluctuations in price level are not accompanied by corresponding fluctuations in interest rate.

With appropriate modifications, formula (1) applies also to rates of return, both before and after income tax. If a 10% real rate of return is desired, before or after taxes, and if the foreseen inflation rate is 20%, the stated rate of return, before or after taxes, should be 30%.

Even in a noninflationary economy, a difficult and unsolved problem is the choice of the proper rate to be selected as the financial component of the cost of holding inventories. A consensus obtains among most theoreticians and practitioners that the lowest limit of a possible range of candidate figures is the effective bank rate (after the effect of compensating balances) charged to the firm for its short-term loans. This is actually the rate which most companies adopt; it has the advantage of being a very concrete datum. Theoretically, however, a case could be made in favor of some opportunity cost--for instance, the rate of return (before taxes, presumably) that the best alternative investment would yield. This solution might be conceptually superior from an economic standpoint but requires making a subjective decision, a procedure apt to be questioned by many a manager. Other possibilities would be: last year's rate of return on investment (before taxes, presumably); the last five years' rate of return; some arbitrary rate of return;

or the industry's average rate of return.[15]

In times of inflation, it is not the real interest rate i that should be used in computing inventories holding costs but, instead, the apparent interest rate $r = i+d$. Indeed, assume that the firm borrows from a bank in order to finance its inventory. The bank, careful to protect the purchasing power of its principal, charges the apparent interest rate. After completion of the manufacturing cycle, the goods leave the production department. The holding costs to be debited to the operation department are the interest charges incurred during the manufacturing cycle. This is an out-of-pocket expenditure.

If the firm were able to readjust its sales prices during the period, it could be argued that the sum received as readjustment parcel compensates for the sum paid as inflated interest. But the loan is contracted at a time that the firm ignores its ability to readjust its prices. From a conservative viewpoint, and especially in periods in which the firm has no certainty of being able to readjust its own prices and "pass the increase to the customer," the apparent interest rate is, therefore, the proper cost of money to be adopted.[16]

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[15]In Chapter IV, it is reported that, among those firms--in the sample of 34--which compute the cost of inventory holding, a small minority predominantly employ the bank (prime) interest rate. Only one company settled upon a subjective rate of return, being 50% p.a. for purchased items (in the purchasing division) and 25% in the manufacturing divisions for the fabricated parts. The choice seems to be partially dictated by the borrowing situation of the firm: if it borrows for financing inventories, it will be likely to adopt the bank interest rate as its holding cost.

[16]The survey reported in Chapter IV indicates that it is, indeed, the apparent interest rate which most firms use. But, as it coincides with the nominal rate, which is the only visible one, the adoption of

During an inflationary period, it becomes even more difficult than in noninflationary times to decide what specific opportunity cost to attribute to holding inventories. The effect of the inflation on the firm's opportunity profits becomes a new factor to be reckoned with. If the forecast is that the inflation will not affect business profits, the opportunity costs debited to inventory should not be modified. If the inflation hurts profits to the extent that the rate of return drops below the apparent interest rate, the apparent interest rate or some reasonable desired rate of return should be used--whichever is higher.

In periods of rising costs, the relative increases in materials, labor, and other prices and in the interest rates chargeable to inventories play a decisive role in the determination of the inventory policy of the firm. In Chapter II, the possible inflationary situations are classified according to this relationship.

Before closing this chapter, it seems appropriate to remind the reader of the specific focus of this work. The problem examined in this dissertation is restricted to the viewpoint of operations management with respect to the effect of changes in costs. Inflationary changes have further effects on investments and sales policies, which will not be discussed here.

In other words, the attention of this monograph will be confined to the transaction motive for holding stocks. The purely speculative

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this rate is possibly done mechanically, without deliberate purpose.

motive for investing in inventories is not considered. Nor is speculation in finished goods, which consists in manipulating price and output in order to maximize profits. The effect of price changes on the finished goods inventory, output, and price policies has been the object of a number of investigations in the literature of economics, as described in the following paragraphs.

Edward S. Shaw draws attention to the parallelism between the theory of inventory accumulation over time and the theory of monopolistic sales discrimination among markets.[17] For the case of two periods, he utilizes a graphical method in order to determine how much must be sold in each period, when a cost increase is expected from one period to the other by the imperfectly competitive firm. The perfectly competitive firm will produce ahead of current sales requirements so as to exploit an anticipated increase in selling price.

Frederick Lutz and Vera Lutz[18] also resort to a graphical method, similar to the price-discrimination approach utilized by monopolies in spatial markets, in order to find out the optimal amounts to be produced in each period, in a two-period problem, when prices increase over time.

Making use of calculus, Arthur Smithies studies a two-period problem, in which production costs and prices of goods sold increase

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[17] Edward S. Shaw, "Elements of a Theory of Inventory," Journal of Political Economy, XLVIII (August, 1940), 465-484.

[18] Frederick Lutz and Vera Lutz, The Theory of Investment of the Firm (Princeton, N. J.: Princeton University Press, 1951).

exponentially over time; the costs increase more than the sales prices.[19] The best scheduling policy consists in manufacturing relatively more at the start of the period and less at the end. Sales should follow the inverse pattern, so that finished goods inventories accumulate over the first part of the period and decrease during the second part.

The problem that Edwin S. Mills examines is exactly the marketing counterpart of the central problem of this dissertation; he investigates the production and inventory policy of a firm when "prices may be expected to change in the future, [but] the firm does not expect its production and inventory cost functions to change over time."[20] Both cases of perfect and imperfect competition are considered.

A summary of the economic literature concerning the speculative motive for holding finished goods inventories when prices are expected to rise can be found in a paper by Kenneth J. Arrow.[21] Harvey M. Wagner and Thomson M. Whitin present a numerical example of ". . . the dynamic problems of a firm faced with varying demands and production

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[19] Arthur Smithies, "The Maximization of Profits Over Time," Econometrica, VII (October, 1939), 312-318.

[20] Edwin S. Mills, Price, Output, and Inventory Policy: A Study in the Economics of the Firm and Industry (New York: John Wiley & Sons, Inc., 1962), p. 51.

[21] Kenneth J. Arrow, "Historical Background," in Studies in the Mathematical Theory of Inventory and Production, ed. by Kenneth J. Arrow, Samuel Karlin, and Herbert Scarf (Stanford, Cal.: Stanford University Press, 1958), 3-15.

costs over time."[22] They utilize a recursive-enumerative procedure to obtain the optimal scheduling pattern.

The problems treated by most of these authors--involving speculation with both volume and price of finished goods' inventories--are different from the cost minimization inventory problems studied in this dissertation. Their methods are based generally upon marginal economic analysis and their solutions are theoretical, while we use mathematical analysis and drive at the obtention of operational solutions.

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[22] Harvey M. Wagner and Thomson M. Whitin, "Dynamic Problems in the Theory of the Firm," in The Theory of Inventory Management, ed. by Thomson M. Whitin (2d ed.: Princeton, N.J.: Princeton University Press, 1957), p. 327.

CHAPTER II

UNCONTROLLED AND CONTROLLED INFLATIONS

In the chapter, the analysis of the inflationary environment that was started in Chapter I is treated in greater detail. Inasmuch as the price-level changes and the cost of money are two major determinants of how to modify the inventory policy in an inflationary economy, it must be carefully resolved as to whether the price advances are larger or smaller than the changes in the nominal interest rate. The first situation will be called "uncontrolled inflation" and the second one "controlled inflation." In the next chapter, it will be shown that these two situations lead to completely opposite policies in inventory management. The changes in price-level and interest rate are normally tied to other elements of the economic cycle, such as the general level of activity. A distinction must be made, therefore, between the direct effects of price and interest rate changes, on the one hand, and the direct effects of the aggregate demand, on the other hand, upon inventory management.

2.1. Price Increases and Inventory Accumulation

Abstracting from any accompanying rise in the interest rate makes it seem likely that an anticipated increase in prices will favor an increase in inventories. The economic literature has provided both normative support and empirical evidence to this effect.

An early reference to the topic was made by John M. Clark:

> One other factor which may make merchants more willing to invest in considerable stocks is that a time of growing demand for some one commodity, or a time of general increase in activity, are both times of rising prices for the intermediate products called for in the business affected. This makes these commodities a profitable investment so long as credit can be had on easy terms with which to enlarge one's holdings. Merchants tend to assure their future supplies by buying either outright or for future delivery.[1]

Irving Fisher gives two reasons for purchasing more in periods of rising prices: flight from money and the appreciation of inventories. As he puts it:

> We all hasten to get rid of any commodity which, like ripe fruit, is spoiling on our hands. Money is no exception; when it is depreciated, holders will get rid of it as fast as possible. As they view it, their motive is to buy goods which appreciate in terms of money in order to profit by the rise in their value.[2]

Ralph G. Hawtrey presents two simultaneous reasons for the accumulation of stock--growing productive activity and the rise of prices:

> If . . . the purchasing traders are led by the growing activity of the producers to expect a rise of price, they will give further orders, in excess of those required by their stocks on the basis of actual current sales, in order to take advantage of the market while the price is still low. In effect the traders will be acquiring an additional stock by way of speculation.[3]

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[1] John Maurice Clark, "Business Acceleration and the Law of Demand, a Technical Factor in Economic Cycles," *Journal of Political Economy*, XXV (March, 1917), 232-233.

[2] Irving Fisher, *The Purchasing Power of Money* (Revised ed.; New York: The Macmillan Company, 1913), p. 63.

[3] Ralph G. Hawtrey, *A Century of Bank Rate* (London: Longmans, Green and Co., Ltd., 1938), p. 211.

In 1919-20, Hawtrey goes on to say the following events occurred:

> In every trade people anticipated a further expansion of demand and a further rise of price. And traders proceeded accordingly to order additional supplies of goods with a view to adding to their stocks before the rise of price materialised. . . . . The vicious circle of inflation revolved merrily. Prices were rising at the rate of 3 to 4 per cent. a month. The forward buyer or the trader who bought commodities with borrowed money made an extra profit at that rate. It appeared to be worth while to borrow at any rate of interest not exceeding the expected rate of the rise of prices.[4]

The following description of the piling up of inventory in 1919-20 indicates the part played by expectation of price advances:

> As the revival progressed, new factors came into play. The forces which produced the upturn would not in themselves have precipitated the frenzied buying and the hectic increase of prices which is now regarded as the most conspicuous characteristic of the 1919-20 boom. Several factors, however, conspired to turn the revival into a runaway.
>
> Inventory buying in the first few months of 1919 had been increasing at a relatively moderate rate. Once prices began to rise, however, . . . business broke into a wild scramble for stocks. . . . The price rise itself being regarded as a certain harbinger of a continued rise. One other factor, however, may help to explain the sudden skyrocketing of inventory demand. A transportation tieup and strikes had made deliveries uncertain. Anxious to secure goods, distributors began duplicating orders with different suppliers. The latter, confronted with a rapidly increasing backlog of demand, likewise duplicated orders with their own suppliers. Each was thus under the "competitive illusion" that the demand for his product was virtually insatiable, and each, proceeding under this illusion, magnified the demand to his own source of supply. Prices rose rapidly, encouraging forward buying and forcing prices up further. Rising prices and stock accumulation fed one another until buying was whipped into a feverish pace. [In a footnote, Professor Rotwein quotes the following from After the War 1918-20, published by the National Resources Planning Board in 1943: "During 1919, according to Kuznets' estimate, the value of business inventories increased by $6,000,000, an increase almost twice that of any other year of the decade to follow and

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[4] Ibid., p. 212.

> almost four times that of any other year except 1923. Two thirds of the change, Mr. Kuznets calculates, was due to the increase in the physical volume of goods held, one third to the increase in prices."] An additional inflationary force was now also operating. The cost of living . . . rose continuously throughout the recession period in early 1919. Although labor managed to improve its position by securing increased hourly earnings, the continual expectation of further price increases engendered anticipatory demands for additional wage increases. This increased costs of production and drove prices and living costs up again. Thus, enmeshed with the inventory-accumulation inventory-price spiral was a potent wage-cost-of-production spiral.[5]
>
> Inventory hoarding and speculation were rife. Dealers, expecting further price increases, were ordering vast quantities of goods far in advance. Acting as a stimulus to production, this resulted in a great increase of output, most of which ended up in storage, where it was held by speculators. Artificial shortages were created and prices boosted still higher.[6]

Rotwein reports further: "Between August, 1919, and June, 1920, food prices rose 20 per cent, clothing 21 per cent, rent 15 per cent, and house furnishings 24 per cent."[7]

The period of 1936-37 has also been cited as a period of inventory speculative activity in the United States, although to a lesser extent than during the 1919-20 episode.[8]

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[5] Eugene Rotwein, "Post-World War I Price Movements and Price Policy," Journal of Political Economy, LIII (September, 1945), 246-247.

[6] Ibid., p. 251.

[7] Ibid., p. 249.

[8] Clarence L. Barber, Inventories and the Business Cycle: With Special Reference to Canada (Toronto: University of Toronto Press, 1958), Chapter 2.

Turning to situations of chronic inflation, such as exist in some less developed countries, many observers believe that a constant rise of prices favors inventory accumulation, but empirical evidence is lacking.

> Some of the members of the panel [at the Conference on Inflation and Economic Growth in Latin America, held in Rio de Janeiro in 1963] felt that the distorting effects should cause increases in inventories. . . . Other panel members indicated that, in some countries, this type of distortion has not taken place despite considerable inflation.[9]

Another paper presented at the same conference suggests that:

> If money, and financial assets denominated in money, cease to provide satisfactorily protected liquidity, other sources of this protection will be sought. The accumulation of salable inventories is one means of obtaining realizable assets whose real value is likely to be maintained in the face of rising prices. Consequently, inflation may be expected to encourage investors to forego the purchase of financial assets which could have financed long-term physical investment, and to accumulate inventories directly. As a result, the available resources will be devoted to inventory stockpiling rather to long-term investment.[10]
>
> In Brazil, when the rate of inflation rose, inventories were increased sharply. Thereafter, even though inflation might be rapid, the rate of inventory investment reverted to a more normal level; when the rate of inflation was reduced, there was a temporary decline in the rate of inventory investment.[11]

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[9] Richard Ruggles, "Summary of the Conference on Inflation and Economic Growth in Latin America," in Inflation and Growth in Latin America, ed. by Werner Baer and Isaac Kerstenetzky (Homewood, Ill.: Richard D. Irwin, Inc., 1964), p. 15.

[10] Graeme S. Dorrance, "The Effect of Inflation on Economic Development," ibid., p. 53.

[11] Ibid., p. 54.

Dorrance presents the following data concerning Brazil:

TABLE 2.1

RATE OF INFLATION AND INVENTORY INVESTMENT, 1950-60[12]
(Based on data in current prices)

Year	Brazil	
	A[a]	B[b,c]
1950	7	- 3
1951	5	10
1952	22	25
1953	22	15
1954	18	19
1955	19	8
1956	22	8
1957	20	8
1958	16	--
1959	37	--
1960	...	...
Average	19	10

Sources: Based on data in United Nations, Yearbook of National Accounts Statistics and Statistics of National Income and Expenditure; International Financial Statistics.

[a]Rate of inflation, i.e., percentage change in annual average of cost of living index.

[b]Inventory investment as percentage of gross domestic investment.

[c]Excluding stockpiling of coffee and cotton by the government.

Another economist, Werner Baer, also writing about Brazilian experiences, states that:

> The action of investors during an inflation leading to distortions could consist of a bias in favor of inventory investment and/or direct investment in projects with relatively short gestation periods. . . . . .

[12]*Ibid.*, p. 82.

. . . .

> At the moment, there are no yearly sectoral investment breakdowns available. It is therefore necessary to take into account some indirect evidence about the degree of efficiency of investment allocation. The one breakdown which does exist is the division of total investment between fixed capital formation and inventory investment. This is reproduced in Table 2 [i.e., 2.2]. The early postwar years were characterized mainly by inventory decumulations, while the 1950's were dominated by inventory accumulations, though the rate does not seem to have a close connection with changes in the rate of inflation. Thus, for example, in 1952, when there was a noticeable slackening in the rate of inflation, the proportion of inventory accumulation rose to 24 percent, while in 1956, when the rate of inflation jumped substantially, the rate of inventory accumulation remained fairly steady. The latter part of the 1950's were, of course, dominated by government accumulation of coffee inventories. Although the decade of the 1950's was thus dominated by inventory accumulation, it is difficult to say to what extent this was influenced by the inflationary climate and to what extent by the necessity of an industrializing and urbanizing country to rely on a higher proportion of accumulated stock. It should also be mentioned that due to the periodic balance of payments crises which produced different kinds of direct government controls, inventory accumulation was often influenced by expected changes in foreign exchange and other import policies.[13]

Data of Table 2.2 indicate that the bulk of inventory investment after 1954 was in the agricultural and governmental sectors. Werner Baer produces additional evidence which suggests that the commerce and industry sectors contain ". . . not only . . . a relatively small proportion of the funds being channeled into inventory accumulation, but this proportion tended to decrease during the late 1950's."[14] In conclusion, for this writer, the data support the likelihood that inflation does not affect the inventory decisions of the firm.

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[13] Werner Baer, "Brazil: Inflation and Economic Efficiency," Economic Development and Cultural Change, XI (July, 1963), 396, 398.

[14] Ibid., p. 400.

TABLE 2.2[15]

BRAZILIAN INVENTORY ACCUMULATION

Year	Rate of Inflation[c]	Rate of Growth of Real GNP	Inventory as a % of Total Capital Formation	Distribution of Inventory Accumulation				
				Private			Government	
				Agri-culture	Urban	Total	Of Which Coffee	Total
1947	----	1.8	-11[a]	--	--	--	--	---
1948	3.7	9.5	- 2[a]	--	--	--	--	---
1949	9.3	5.6	-14[a]	--	--	--	--	---
1950	11.9	5.0	-16[a]	--	--	--	--	---
1951	14.9	5.1	15	24	76	--	--	100
1952	8.6	5.6	24	27	42	31	31	100
1953	18.6	3.2	5	167	-30	-37	-37	100
1954	20.2	7.7	20	36	54	10	15[b]	100
1955	16.7	6.8	12	70	6	24	44[b]	100
1956	25.5	1.9	12	83	2	15	7.5	100
1957	11.8	6.9	19	42	18	40	30	100
1958	16.3	6.6	12	36	-36	100	97	100
1959	27.2	7.3	19	16	3	81	74	100
1960	25.2	6.3	16	18	2	80	71	100
1961	37.1	7.7	--	--	--	--	--	---

[a]From 1947 to 1950, the decrease of inventory was almost entirely concentrated in the private sector.

[b]These numbers are higher than the proportion attributed to the government sector because the decrease took place in the noncoffee government sector.

[c]The rate used is the implicit GNP deflator.

Source: Computed from data furnished by the Fundaçáo Getúlio Vargas and calculations made by Professor Isaac Kerstenetzky.

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[15] Ibid., pp. 397-398.

The rather confusing empirical evidence concerning accumulation of aggregate inventories in Brazil during the 1950's is reviewed by A. S. Shaalan who also observes that the length of the inflation seems to play a role:

> . . . In Brazil [when] . . . there was relative price stability (relative for Brazil) in the late 1940's and up to 1951, inventory accumulation seems to have proceeded at a slow pace. A sharp rise in the rate of inflation in 1952 was associated with a sharp increase in the proportion of investment in inventories. Thereafter, and up to 1958, the rate of inflation was held at about 20 per cent per annum and inventory investments proceeded at a much slower rate.[16]

A similar correspondence between periods of high inflation and high inventory investment is reported to have been found by the same author in Chile, Mexico, and Colombia. He concluded that: ". . . Long periods of continuous inflation were not associated with unduly large investment in inventories, but short periods of inflation tended to bias the composition of investment toward inventories."[17]

Aggregate studies fail to separate individual movements occurring in very different sectors, such as agriculture, industry, and trade. No distinction is made between raw materials, work in process, and finished goods. Nor do they reveal whether the increase in inventory is brought about through the purchase of larger lots or through a "once-and-for-all" extra reserve purchase. The procedural details, the how, when, and how much are not described.

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[16] A. S. Shaalan, "The Impact of Inflation on the Composition of Private Domestic Investment," Staff Papers: International Monetary Fund, IX (July, 1962), 254-255.

[17] Ibid., p. 255.

Concentrating on a selected sector of the economy, the leather-hide-shoe sequence, Mrs. Ruth Mack analyzed the effects of anticipated price increases on the purchases of hides by tanners and leather by shoemakers.[18] In the equations depicting the desired inventory levels, she includes a term which accounts for advance purchasing in anticipation of price increases in raw materials. Similar assumptions were made by Kalman J. Cohen in a simulation model of the hide-leather-shoe sector, based upon Mrs. Mack's data and behavioral postulates, concerning the purchasing, inventory, production, and pricing mechanisms in that industry, from 1920 to 1940, in the United States.[19]

In a later work, Mrs. Mack built an "ecological model of price-timed buying." She says:

> Expectations that materials prices will rise could be another reason for extending materials ownership.
>
> . . . .
>
> [The model] . . . focuses on one aspect of market-oriented buying (by no means the most important one), price; it . . . describes how the expectation that materials prices will rise is capable (under appropriate circumstances) of causing a wave of price-timed buying with rising and falling phases of reasonable duration and containing intrinsic reversing mechanisms.[20]

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[18] Ruth P. Mack, Consumption and Business Fluctuations: A Case Study of the Shoe, Leather, Hide Sequence (New York: National Bureau of Economic Research, 1956).

[19] Kalman J. Cohen, Computer Models of the Shoe, Leather, Hide Sequence (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1960).

[20] Ruth P. Mack, Information, Expectations and Inventory Fluctuation: A Study of Materials Stock on Hand and on Order (New York: National Bureau of Economic Research, Columbia University Press, 1967), pp. 10, 14.

This section has stressed the influence of price increases on inventory; but inflation affects the interest rate and, therefore, the cost of carrying inventories. It is to this countervailing effect that attention will now be turned.

2.2. Interest Rate and Inventory Accumulations

Prior to the Great Depression, it was generally held in the literature of economics that variations in the rate of interest exerted great influence on the responsiveness of all forms of borrowing including inventories. R. G. Hawtrey declares that changes in the rate of interest govern the fluctuations of business activity to a considerable extent:

> . . . When a trader borrows from his bank in order to purchase goods, either for resale or for use in manufacture, his outstanding indebtedness will depend on the quantity he chooses to get delivered at a time. He will not let his stock fall below the minimum which he judges necessary to guard against its being depleted by unexpected demands, but when the minimum is reached, and he has to replenish the stock, he is free to buy a week's supply or a month's or three months'. If he suddenly finds that he is charged 8 per cent. for an overdraft instead of 4, he can reduce the amount he has to pay in interest at very little cost of inconvenience by obtaining the goods at shorter intervals and in smaller quantities. That does not necessarily mean buying smaller supplies; he can order the goods far ahead, if he thinks the moment favorable for a forward purchase, and can nevertheless stipulate for a series of deliveries at short intervals, so that at no time during the period over which his order extends will his stock be inconveniently large.[21]

Hawtrey continues: "A rise of Bank rate makes traders 'less disposed to buy and more disposed to sell' and a fall has the contrary effect."[22]

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[21]Hawtrey, op. cit., p. 190.

[22]Ibid., p. 192.

In the previous quotation, Hawtrey goes into operational details concerning the manner in which inventories are actually purchased. He recognizes that the cost of money is small in comparison with stockout costs; therefore, it is mainly to the "merchant" that he attributes sensitiveness to the interest rate:

> The direct increase in a manufacturer's costs on account of a rise in the rate of discount was trifling. If he had to calculate the price he would ask when offered an order, the difference between 8 per cent. and 4 per cent. on a portion of his working capital during the currency of a three months' bill would be only 1 per cent. on that portion and much less than 1 per cent. either on his turn-over or on his total capital. The item is negligible in comparison with the loss involved in leaving his works idle.
>
> On the other hand the merchant can regard each purchase of goods as a separate venture; practically the whole capital involved is working capital, the cost of the goods themselves. And by <u>postponing</u> his purchases he can reduce the amount of capital employed without curtailing the scale of his operation at all. The additional charge imposed upon him by a rise in the rate of discount affects him like a tax on the holding of goods. The tax may be a light one, but the subject of the tax, the amount of goods held in stock, can be varied so easily that it will be responsive even to a light tax.[23]

Contrary to Hawtrey, Keynes believed that the borrower for the purchase of commodities is insensitive to the short-term rate of interest that he has to pay for holding goods in stock. Keynes considered the fluctuations in interest rate to be insignificant in comparison with other costs of carrying stocks, namely, obsolescence risks, warehouse and insurance charges, and price-level fluctuations:[24]

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[23] *Ibid.*, p. 34.

[24] John Maynard Keynes, *A Treatise on Money* (London: Macmillan & Co., Ltd., 1930), Chapter 29.

> . . . It is evident that a fluctuation of 1 or 2 per cent in bank-rate will represent so small a part of the total carrying charges that it is not reasonable to assign to the expense of high bank-rate a preponderating influence on the minds of dealers in stocks. Insofar as low and high bank-rates are regarded by dealers as symptoms of impending rising or falling prices, it is another matter.[25]

Keynes does not look upon interest-rate fluctuations as the fundamental cause of changes in the level of business activity, except so far as capital equipment is concerned through fixed investment and the multiplier effect.

Edward S. Shaw denies the importance of the interest rate as a determinant of inventory investment and endorses the Keynesian position that price fluctuations and, to a lesser extent, cost fluctuations are more relevant to an explanation of inventory changes.[26]

A similar position is taken by Michael K. Evans, who contends that:

> Other possible influences on inventory investment . . . are the possible cost and availability of funds as measured by interest rates and cash flow. Only a few studies have examined the importance of these variables on inventory investment; most of these show no significant relationships. There are several theoretical reasons for this. As is well known, a large percentage of the total cost of fixed business investment may represent interest payments; this is almost never true for inventories. The internal risk factor on fixed investment is virtually absent for inventory investment, because firms are not committed to any long-term debt service. There is more of a short-term technical relationship between inventories and sales than between fixed business investment and production. A firm may work existing plant and equipment overtime or add additional man-hours in the short run; however, it often cannot produce its product without closely specified proportions of raw materials

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[25] Ibid., p. 145.

[26] Shaw, loc. cit.

and goods in process. For all these reasons, it is not surprising that the interest rate and cash flow are not relevant determinants of inventory investment.[27]

Other economists do not believe that the influence of monetary factors on inventory investment can be dismissed. Thomas F. Dernburg and Duncan M. McDougall, for example, write:

> The demand for money is but one aspect of the general inventory problem. Weighed against the desirability of holding inventories is the cost of holding inventories--a cost that is partly dependent on the rate of interest. A high rate of interest raises carrying costs and is apt to lead merchants to try to get by on a smaller margin of inventory, while a low interest rate will have the opposite effect. It is in the relationship between inventory costs and interest rates and in the behavior of the banking system that Hawtrey found the source of industrial fluctuations.[28]

Gottfried Haberler reviews Hawtrey's position at length and concludes:

> Mr. Hawtrey is aware of the objection, which has been raised very frequently, that a reduction of 1 to 2% in the interest on bank advances is too unimportant an item in the profit-and-loss account of the average business-man to induce him to expand his business and to borrow more. His answer to this objection is that there exists one class of business-men which is very sensitive even to small changes of the rate of interest--namely, the merchants. The merchant buys and sells large quantities of goods compared with his own capital, and he adds to what he buys the relatively small value which represents the dealer's profit. To him, a change in interest charges of 1 to 2% is not negligible, as it is perhaps to the manufacturer.
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> . . . .
>
> Rising prices operate in the same way as falling interest charges: profits are increased and traders stimulated to hold larger stocks in order to gain from a further rise in prices.

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[27]Michael K. Evans, Macroeconomic Activity: Theory, Forecasting and Control (New York: Harper and Row, 1969), pp. 213-214.

[28]Thomas F. Dernburg and Duncan M. McDougall, Macroeconomics: The Measurement, Analysis, and Control of Aggregate Economic Activity (3rd ed.; New York: McGraw-Hill Book Company, 1968), p. 330.

> In the same way, the producer is stimulated to expand production and to borrow more freely in order to finance the increased production.[29]

Thomson M. Whitin summarizes Hawtrey's theory in the following terms:

> R. G. Hawtrey, although addressing himself principally to business cycle theory, wrote much that is pertinent to the theory of the firm.
>
> . . . .
>
> How much effect interest rate variations actually have on inventories is an empirical question. Enough data have not yet been compiled to provide an answer which will either prove or disprove Hawtrey's theory. . . . Changes in the interest rate should influence the rational entrepreneur to change his lot size, but two factors are present that help reduce the effects of changes in the interest rate. The first of these is the economic purchase quantity which varies inversely with the square root of carrying charges. The second factor is the set of components other than interest included in the carrying charge. In order to avoid this difficulty Hawtrey assumed that carrying charges other than interest were small and constant, an assumption that is ordinarily not realistic.[30]

Several surveys concerning investment practices of U.S. and British firms throw some light on the influence of interest rates on inventory accumulation. Frederick Stevenson reports the results of a survey undertaken by the National Industrial Conference Board regarding inventory policies and methods of a sample of manufacturing firms from

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[29] Gottfried Haberler, Prosperity and Depression: A Theoretical Analysis of Cyclical Movements (5th ed.; London: George Allen & Unwin Ltd., 1964), pp. 18, 19.

[30] Thomson M. Whitin, ed., The Theory of Inventory Management (2d ed.; Princeton, N.J.: Princeton University Press, 1957), p. 81.

1952 to 1961.[31] The companies were asked to rank in order of importance various types of operating data considered by them in a decision to change purchased material stocks and finished goods stocks. For both kinds of inventories, and for all the industrial sectors investigated, economic considerations such as expected changes in prices of materials and in labor costs, availability of working capital, and cost of borrowed funds were ranked behind physical considerations such as forecasted or planned rate of production, current production rate, backlog of unfilled orders, and sales-inventory turnover.

In a survey covering 37 firms in Great Britain in 1938, Meade and Andrews found that ". . . short-term rates of interest do not directly affect investment either in stocks or fixed capital."[32] Of course, it must be remembered that depression conditions had prevailed since 1929, the period being characterized by low demand for investments.

A mail survey involving 1,308 industrial firms was undertaken in Great Britain by the Oxford Economic Institute in 1939. This research uncovered some effect of credit cost and availability on investment. Fifteen percent of respondents answered affirmatively to a question asking them: if interest rates, the yield on government securities, and/or the facility with which they could raise new capital from the public

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[31] U. S., Congress, Joint Economic Committee, "Experience in Inventory Management: A Survey of Large Manufacturing Firms," by Frederick Stevenson, Inventory Fluctuations and Economic Stabilization, Joint Committee Print, Part IV, Supplementary Study Papers (Washington, D. C.: Government Printing Office, 1962), pp. 1-33.

[32] James E. Meade and P. W. S. Andrews, "Summary of Replies to Questions on Effects of Interest Rates," Oxford Economic Papers, No. 1 (October, 1938), p. 28.

ever affected the size of their holdings of stocks?[33]

Another British survey was undertaken in 1957 by the Federation of British Industries for the Radcliffe Committee. The period covered was 1951-mid-1957; a total of 1595 firms answered the mail questionnaire. One question was: "In any of the periods indicated (1952-53, 1954-55, 1956-57), did the difficulty of raising extra money outside the firm (including both the cost of borrowing and the administrative difficulties involved) cause you to: a) reduce your stocks? b) decide against raising your stocks? . . ."[34] The proportion of affirmative answers grew from approximately 4% in the earliest period to approximately 11% in the latest.

In March, 1958, still another survey was conducted in the United Kingdom for the Radcliffe Committee by the Association of British Chambers of Commerce.[35] Replies from 3,404 companies were obtained. Approximately 5% of all respondents indicated that "the increased cost of borrowing," or "the greater difficulty in obtaining finance," was the principal reason for reduction in turnover, stocks, or fixed investment.

It is clear from the previous results that, as Shapiro, Solomon, and White put it, " . . . our present knowledge concerning the degree of

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[33] P. W. S. Andrews, "A Further Inquiry into the Effects of Rates of Interest," Ibid., No. 3 (March, 1940), pp. 32, 35.

[34] Great Britain, Committee on the Working of the Monetary System, Principal Memoranda of Evidence, Vol. II (London: H. M. Stationery Office, 1960), p. 119.

[35] W. H. White, "Bank Rate Vindicated? Evidence before the Radcliffe Committee." The Bankers' Magazine (London, CLXXXVIII (August, 1959), 98-104.

sensitivity of business investment to credit terms is incomplete. . . ."[36] It is safe to conclude that interest rates ". . . may not affect the majority of businesses or the full amount of many loans, but they will affect some borrowers and the amounts involved in some loans."[37] "Forward inventory commitments, and later the actual purchases, may be curbed. . . ."[37] A clear awareness prevails among some authors that the influence of interest rate is ultimately a question of degree, depending on the situation of the economy, the particular firm analyzed, and the size of the interest rate changes.[38]

2.3. The Money Illusion

As has been seen, many modern economists attribute more importance to "real" phenomena such as desired sales-ratio and physical turnover than to "money" phenomena such as price-level changes or interest rates in managerial planning of inventories. The survey of 34 firms reported in Chapter IV indicates, indeed, that a majority of the executives interviewed were thinking more in terms of physical units than of cost concepts. Nevertheless, while much of the current literature concerning economics downgrades the effect of economic factors in inventory

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[36] Eli Shapiro, Ezra Solomon, and William L. White, Money and Banking (5th ed.; New York: Holt, Rinehart and Winston, Inc., 1968), p. 534.

[37] The Federal Reserve System: Purposes and Functions (5th ed.; Washington, D. C., 1963), p. 139.

[38] Thomas Wilson, Inflation (Cambridge, Mass.: Harvard University Press, 1961), Chapter 12.

planning, a no smaller amount of literature in engineering and operations research emphasizes the preponderance of economic considerations in the normative planning of inventories.

If managers seem to be somewhat reluctant to pierce the "physical veil" and go to the money heart of the problem, even when prices are stable, they will have even more trouble in periods of inflation. The habit of thinking in terms of a stable currency creates a "money veil," which makes it even more difficult to reach the ideal goal of cost minimization.

Irving Fisher introduced the expression "money illusion" in recognition that people do not immediately perceive the loss of purchasing power of money in periods of inflation.[39] This "illusion" can be documented in everyday experience, in time of inflation, although as yet no controlled experiment has been performed to demonstrate its existence.

The "money illusion" might be due to any of the following:

1. First, it may be an inertia effect. One becomes conditioned to the previous purchasing power of money and converts instinctively the amount received today into the volume of physical goods which it could have purchased yesterday. The difficulty of a foreigner's getting acquainted with the purchasing power of the currency of the country in which he has recently disembarked is well known. In the land of inflation, all are naive tourists continually surprised by a persistently protean currency.

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[39] Irving Fisher, The Money Illusion (New York: Adelphi Company, Publishers, 1928).

2. A second explanation is that the sheer volume of money produces a "halo" effect and seems to be more than the previously smaller volume of money to which it corresponds, even if it has less unit-purchasing power.

3. The third possibility is that the very notions of purchasing power and price index are difficult to grasp, and their mechanics are not easy to utilize. In any event, which year should serve as a base year during prolonged inflation?

4. Finally, the "money illusion" may be in part ascribed to what might be called "lack of perception." Very small, creeping inflations--diminutive increases in costs--are often overlooked, although their cumulative effects are far from negligible.

The "money illusion" takes many forms, of which only a few will be mentioned here. A most important variety of the "illusion" arises from the failure to distinguish between real and nominal interest rates, as defined in the section entitled "Real and Apparent Interest Rates" in Chapter I above. It takes years of practice to perceive that, e.g., a $5.00 annual interest payment upon a $100.00 principal is really no interest at all if inflation is proceeding at a 5% annual rate.

The "replacement cost illusion" is also very familiar. It consists in using historical, or even present-day cost figures, in lieu of future costs, when pricing goods for future delivery. The system of historical accounting prevailing in most countries creates a host of "money illusions" of different kinds, which have been accurately labelled "profit illusion," "salvage value illusion," "depreciation illusion."[40]

2.4. The Flight from Money

The "money illusion" does not last forever. In countries which have witnessed sustained and strong inflations, year after year, the businessmen and the public at large learn to adjust to changing price levels. If the inflation is violent, the learning is rapid.

The converse of the "money illusion" seems then to occur. Inflation is perceived and anticipated. Businessmen and consumers alike develop an acute awareness of the declining value of money, and they purchase goods in excess of immediate needs to protect themselves against the declining purchasing power of the currency. In its initial phase, this flight from money, as far as merchants and industrialists are concerned, is likely to be accompanied by some elation because the goods on the shelves and in the stockrooms advance in selling value; at a still later phase, however, the businessmen observe that the replacements also advance in costs and that the "windfall profit"[41] due to inflation is nothing but a mirage.

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[40]"How Inflation Warps Accounts," The Economist, January 16-22, 1971, pp. 58-59.

[41]John Maynard Keynes, "Inflation and Deflation," in Essays in Persuasion (New York: Harcourt, Brace and Co., 1932), p. 93. . . . While prices are rising month by month, the business man has a further and greater source of windfall. . . . Merchant or . . . manufacturer, . . . his stock appreciates on his hands, he is always selling at a better price than he expected and securing a windfall profit. . . .

2.5. The Flight from Stock

At a still later stage in the inflation, the effect of price-level changes upon interest rates becomes visible to the businessman. The apparent interest rate soars and is maintained high in times of "controlled" infaltion as will be seen in succeeding sections of this study. The apparent costs of keeping inventories loom large. A third distortion enters into the picture which often leads the manufacturer to avoid maintaining any inventory; this phenomenon can be called the "flight from stock."

Only when the understanding of the interplay between the inflationary distortions has been completed, and when the operations management of the firm has learned to shun the "money illusion," the flight from cash, and the flight from stock--only then can a rational inventory planning be adopted.

It is worthwhile to stress that all of the effects mentioned above apply to the purchasing pattern and <u>not</u> to the consumption pattern. The consumption of goods by the firm is not altered, only its purchasing rhythm. The distinction is essential, because similar effects have been invoked in order to explain the consumption behavior of household customers.

The "money illusion" and the "flight-from-money illusion" have been ascribed to households and to firms alike. But the so-called "Keynes effect," "Pigou effect," "Pigou effect in reverse," "Duesenberry and Modigliani effect," and "Friedman permanent income effect"--all refer to consumption behavior, although an increase in demand for consumer semidurables and durables is also an inventory purchasing. These last-named effects are restricted to households, which are not being dealt with in this study.[42]

2.6. The Lag in Interest Rate

The changes in interest rate lag behind the changes in price level. The collected historical evidence confirming the existence of such a lag is considerable.

According to Arthur J. Brown, the United Nations' report on Inflationary and Deflationary Tendencies, 1946-48 (Lake Success, N.Y.: U. N., Dept. of Economic Affairs, 1949, p. 48) points out that in China the black market rate of interest did in fact rise to rough equality with the actual rate of increase of prices. The data quoted in that study are as follows:

	Rate of Increase of Wholesale Prices	Black Market Rate of Interest
	(Percent per annum)	
1940	158	22
1941	177	32
1942	180	30
1943	202	136
1944	224	197
1945	253	270
1946	151	185

[42] See Joseph P. McKenna, Aggregate Economic Analysis (Revised ed.; New York: Holt, Rinehart and Winston, Inc., 1965), Chapters 6 and 12; Wallace C. Peterson, Income, Employment, and Economic Growth (Revised ed.; New York: W. W. Norton & Company, Inc., 1967), Chapter 6.

Brown goes on to say:

> From this it appears that it required four years or more of price increase amounting to a doubling or trebling of prices each year to make the conviction that this process would continue so strong as to be fully reflected in interest rates. . . . . . In Hungary, also, it required much experience of price inflation to raise interest rates abnormally. . . . . The rate for day-to-day money rose to 5 or 6 per cent. a day by early April 1946. At that time, however, the cost of living index was rising at some 10 per cent. a day. By mid-July, . . . this interest rate was 35-40 per cent. a day, but cost of living was then rising at about 140 per cent. a day. . . . . [In Greece] the commercial banks during 1945 . . . lent at an effective rate (including commission) of only 15-20 per cent., while prices were rising at some 900 per cent. per annum.[43]

The German hyperinflation of 1920-1923 did not differ in this respect:

> The intensity of the stimulus toward borrowing from the Reichsbank may be gauged from a comparison of the sluggish rise in the discount rate at the central bank with the ever-accelerated pace of currency depreciation. From the early days of the war till the end of June 1922 the Reichsbank rate remained unchanged at 5%; it was raised to 6% in July, to 7% in August, 8% in September and 10% in November 1922, to 12% in January 1923, 18% in April, 30% in August and 90% in September. But these increases were as nothing when measured alongside the progressive lightening in the burden of a loan during the time for which it ran. Though after September 1923, a bank or private individual had to pay at the rate of 900% per annum for a loan from the Reichsbank, this was no deterrent to borrowing. It would have been profitable to pay a so-called interest, in reality an insurance, charge, of thousands or even millions of percents per annum, since the money in which the loan would be repaid was depreciating at a speed which would have left even rates like these far in the rear.[44]

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[43] Arthur J. Brown, The Great Inflation: 1939-1951 (London: Oxford University Press, 1955), pp. 202-203.

[44] Frank D. Graham, Exchanges, Prices, and Production in Hyper-Inflation: Germany 1920-1923 (Princeton, N.J.: Princeton University Press, 1930), p. 65.

The inflationary boom in the United States in 1919-1920 provides another illustration of the lag of interest rates behind price-level movements. In the words of Hawtrey, writing about a similar inflationary surge in Great Britain in the same years:

> Bank rate was put up to 6 per cent. in November, 1919, and to 7 in April, 1920. The predominant rate for advances and overdrafts was perhaps 1 per cent. higher. Even 8 per cent. seems moderate in comparison with the rate at which prices had been rising.[45]

The prolonged Latin American inflationary experience indicates a similar dephasing between price changes and interest rate. Eugenio Gudin observes:

> Under present inflationary conditions, it is remarkable how people have for so long, by force of habit or respect for institutional prescriptions, admitted the persistence of rates of interest which do not compensate the lender even for the depreciation of his money during the period of the loan. In Chile, the country that, after Bolivia, retains the inflationary record, the rates of interest charged by the Central Bank do not exceed 14 for the public and 11 per cent for rediscount to banks. In Mexico, where inflation has been running at a milder rate of some 10 per cent per annum, the market discount rate has not reached 11 per cent while the rediscount remains at 4.5 per cent.
>
> . . . .
>
> Only after several years of serious inflation did people begin to react against the negative real rates of interest, by resorting to practices of doubtful legality.
>
> . . . .
>
> These contrivances, although rapidly developing, are for the time being restricted to people conversant with business and financial affairs. The poorer classes continue to keep their money in the commercial banks, and especially with the National Savings Bank, at the ordinary legal rates of interest.[46]

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[45]Hawtrey, op. cit., p. 213.

[46]Eugenio Gudin, "Inflation in Latin America," in Inflation, ed. by Douglas C. Hague (New York: St. Martin's Press, Inc., 1962), p. 356.

Summarizing his extensive studies regarding the connection between interest rates and price changes, Irving Fisher reaches the following conclusions, based upon British data from 1820 to 1924: (1) The rate of interest tends generally to be high during a rising price level and low during a falling price level; (2) the rate of interest lags behind the rate of price change, so that often the relationship (apparent interest rate = real interest rate + rate of price change) is obscured when direct comparison is made; (3) the rate of interest correlates very markedly with a weighted average of successive rates of price changes, representing the distributed effect of lag.[47] And Fisher observes that the computed real rate is exceedingly erratic during a serious inflation or deflation and that men are unable or unwilling to adjust accurately and promptly the money interest rates to changed price levels. He points out: "[These] results and other evidence, indicate that, over long periods at least, interest rates follow price movements."[48]

The following reasons can be offered in order to explain the lag of the interest rate behind the price changes: (1) The price changes, if small, are not perceived; (2) the future price changes are not accurately forecast; (3) the relationship between inflation and interest rate is not understood; and (4) legal ceilings exist on the allowed interest rate.

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[47] Irving Fisher, Theory of Interest, op. cit., Chapter XIX.

[48] Ibid., p. 425.

A further and very important reasons for the lag of interest rate behind price changes, in "disinflationary" periods, is the adoption of a restrictive money and credit policy by the government in order to curb inflation. The bank interest rate is maintained high, either through a high rediscount rate, or through money supply restrictions, so that, in this phase of the inflation cycle, the apparent interest rate becomes higher than the inflation rate could justify, and the real interest rate becomes higher than the "natural" interest rate.[49] Such a high interest rate will make inventory holding more expensive.

Because cost changes in the inputs, on the one hand, and holding costs, on the other hand, would be decisive in any alteration of the rational inventory policy of the firm in an inflationary situation, it is convenient to distinguish carefully between two opposite inflationary environments: (1) In the first inflationary condition, the price-level change is larger than the increase in nominal interest rate; (2) in the second inflationary condition, the increase in nominal interest rate is larger than the price-level change. Attention will now be turned to the

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[49] The "natural" interest rate, a concept which goes back to Ricardo, is defined by Wicksell as a rate which is "neutral" in its effects on the prices of goods, tending neither to raise nor to lower them; it is a commodity rate, prevailing in a nonmonetary economy and determined by the technological system of production as if all lending was in the form of physical materials. It has been equated by Keynes to the ". . . rate at which saving and the value of investment are exactly balanced, so that the price-level of output as a whole . . . exactly corresponds to the money-rate of the efficiency-earnings of the Factors of Production." (Keynes, A Treatise on Money, op. cit., p. 155). For Don Patinkin, "natural rate" means ". . . not a rate quoted upon a market, but the investors' rate of return on capital in the commodity market." See Don Patinkin, Money, Interest, and Prices: An Integration of Monetary and Value Theory (2d ed.; New York: Harper & Row, Publishers, 1965), p. 368.

study of these two different situations, which will be called, respectively, "uncontrolled" and "controlled" inflations.

2.7. Uncontrolled and Controlled Inflation

Two inflationary situations are of particular importance to the inventory manager. In the first, costs rise frequently and freely, either by jumps or through small continuous increments; the costs of holding inventories, namely, the cost of money, the storage, insurance, and tax costs, are slow to react and lag behind the increases in material and direct labor. For brevity's sake, it is convenient to call this situation: uncontrolled inflation. This name is actually traditional in the literature of economics as a designation of a situation in which the government does not interfere actively in the control of wages and prices. As defined here, uncontrolled inflation coincides roughly with the habitual economic definition.[50]

The second situation will be called controlled inflation; in brief, it is characterized by small cost increases and by high interest rate and other inventory holding costs. This term, controlled inflation, has been used in economics to mean a situation in which the government tries to curb inflation through monetary, credit, or fiscal policy, or through price control.[51]

In this dissertation, the effect of inflation on the inventory

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[50] Uncontrolled inflation is also called open inflation.

[51] Controlled inflation has also been called repressed inflation. The expression disinflation has the same meaning.

policy of the firm is being investigated. Uncontrolled and controlled inflations have opposite effects. In general, uncontrolled inflation will be shown to lead the rational manager to an increase of its inventories, although, except in some extreme cases, this increase will be relatively small. Indeed, in practice, the data gathered in the survey and the macroeconomic regression studies to be described in Chapters IV and V showed that such an increase usually takes place. Controlled inflation, as will be shown, has the opposite effect; the empirical evidence gathered precisely suggests that this happens.

In the models which are going to be studied in Chapter III, it is possible to create mathematical inflations and to examine their effect upon the optimal inventory policy. In practice, however, both uncontrolled and controlled inflations are accompanied by a host of other changes in the economic situation. The demand, an essential determinant of the levels of inventories, increases in the first case and decreases in the second. Expectations change. Financial constraints are stronger in controlled inflation. In Chapter III, most of these conditions will be abstracted, and the optimal inventory will be studied as if the demand had not changed due to inflation.

Many economists and businessmen alike believe that these two states of affairs--uncontrolled and controlled inflations--seem destined to become the normal in the future. The achievement of growth with a reasonable degree of monetary stability represents one of the major challenges faced by most countries. Governments will oscillate successively between these two objectives. In Figure 2.1, a schematic representation of the four main parameters is drawn as a characterization of

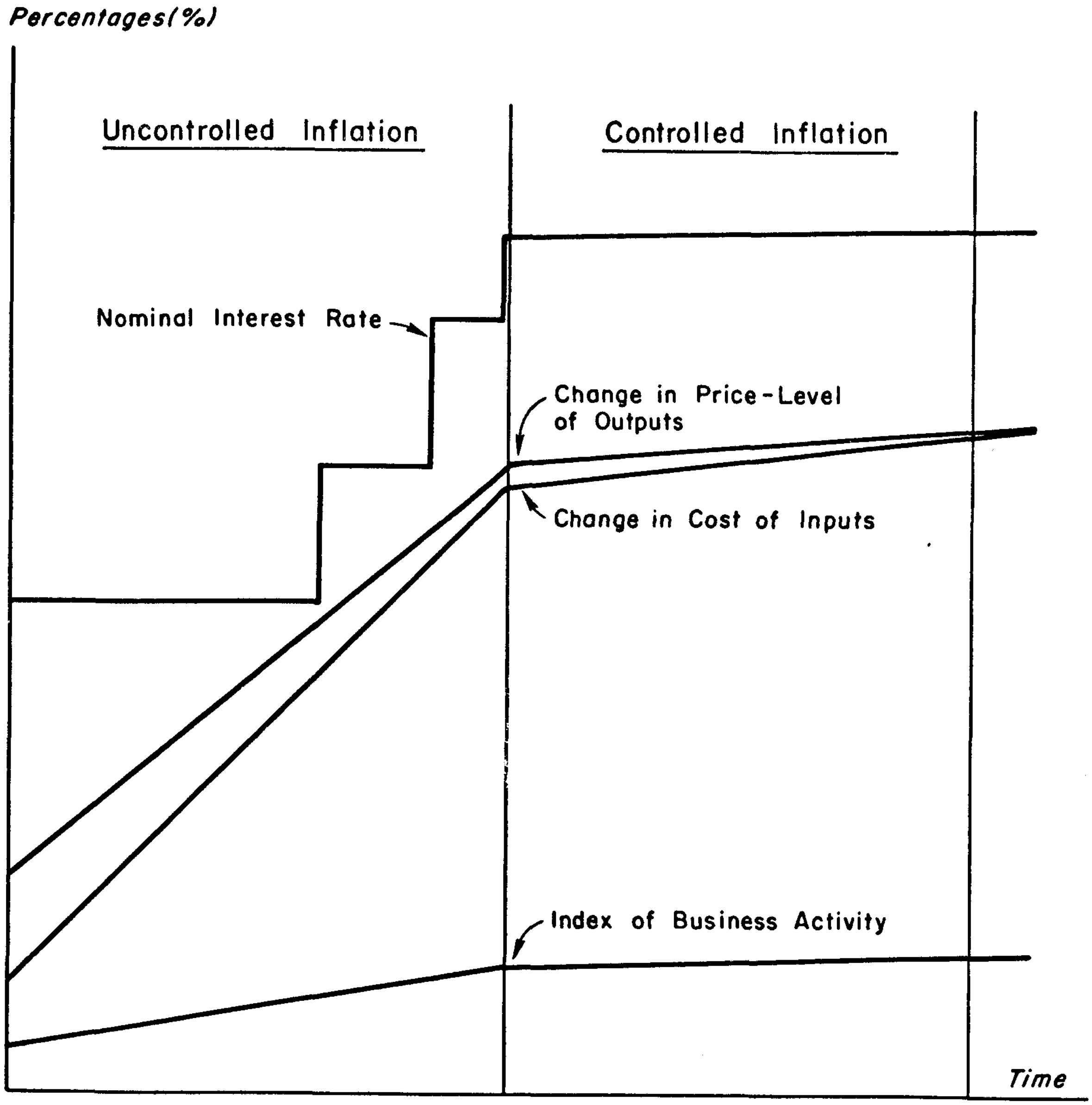

Figure 2-1.--Uncontrolled and Controlled Inflations

these economic situations.

The left part of the figure shows the rising change in costs of the inputs for a situation of uncontrolled inflation, whereas the nominal interest rate is shown lagging behind at a lower level; the price level of the products sold by the firm is indicated as increasing without much lag and so is the level of business activity. The right part of the figure illustrates the case of controlled inflation. The costs of inputs rise less steeply than before; the nominal interest rate has caught up with the cost changes and reached a high level; the firm has lost much of its power to pass on cost increases to its customers, as the changes in price level suggest; the business activity has slowed down.

The concepts of uncontrolled and controlled inflations help us to relate the influence of price-level changes and of the interest rate on inventories sizes. These influences have been studied separately in the literature, as was seen in this chapter. Some authors look exclusively at the influence of interest rate, while others concentrate on the influence of price-level changes. In this dissertation, these two variables are studied for their joint influence on inventories.

CHAPTER III

PRICE-LEVEL CHANGES AND INVENTORY MODELS

In this chapter, a systematic investigation is made of the effect of price-level changes on the optimal solution of the best-known mathematical inventory models. After first examining the constant review policies, attention is turned to the periodic review policies. Some miscellaneous problems are considered at the end of the chapter.

In each case, an analysis is made of deterministic and probabilistic demand situations as well as of conditions of deterministic and probabilistic price-level changes. The optimality criterion is cost minimization in all of the models studied.

3.1. Constant Review Inventory Models

The well-known economic order quantity or economic lot size formula typifies the constant review inventory models. The assumptions underlying these models are the following:

- the inventory level is examined continually, i.e., whenever a withdrawal is made;
- if the inventory level falls below a "minimum level" (a misnomer, inasmuch as the inventory level will actually fall below this so-called minimum), which properly should be called "reorder point," an order is placed;

- the size of the orders is constant; the order size is such that it minimizes the total cost of the inventory;

- the total cost of the inventory is the sum of the following costs:

  - the acquisition costs of the goods; for a planning period of one year, if the annual demand is D and the unit cost is C, the cost of purchasing the goods will be CD;

  - the setup cost; this is the cost of placing an order, or of adjusting the equipment for a batch run; the setup cost is represented by the symbol P;

  - the cost of holding the inventory; calling i the annual interest rate, and a the annual rate of the other holding costs (i.e., the storage cost, insurance, tax, obsolescence, etc.), the cost of holding Q units during a period t (measured in fractions of a year) is: $QC(i+a)t$. If a batch Q is instantaneously purchased or produced and consumed progressively during time t is: $\frac{QC}{2}(i+a)t$. The symbol j will stand for $(i+a)$.

The economic order quantity, EOQ, is the lot size Q which minimizes the total yearly cost:

$$\text{Total Cost} = DC + \frac{D}{Q}P + \frac{QC}{2}(i+a) \qquad (1)$$

By deriving in relationship to Q and equalling to 0, one finds that EOQ is given by the formula:

$$EOQ = \sqrt{\frac{2DP}{C(i+a)}} \qquad (2)$$

- the time elapsing between the reorder point and the replenishment of the inventory is called the "lead time";
- a reserve inventory is established in order to forestall the eventuality of stockouts, which might occur either due to excess demand during the lead time or on account of an excessively long lead time. Figure 3-1 represents the constant review inventory system for deterministic demand.

For the completely deterministic models, no reserve inventory is needed because the exact demand and lead time values are known. In probabilistic models, both demand during lead time and lead time have frequency distributions, and, therefore, some reserve is required, so as to give protection against the eventuality of excessive demand, excessive lead time, or both.

The reserve level may be computed in essentially two ways. The first is based exclusively upon physical considerations. After initially establishing an allowed probability of stockout occurrence, say a 2% probability, the reserve level is then set so as to give a 98% protection against stockouts for known probability distributions of the demand during lead time and lead time itself.

The second method of computing the reserve level is based upon

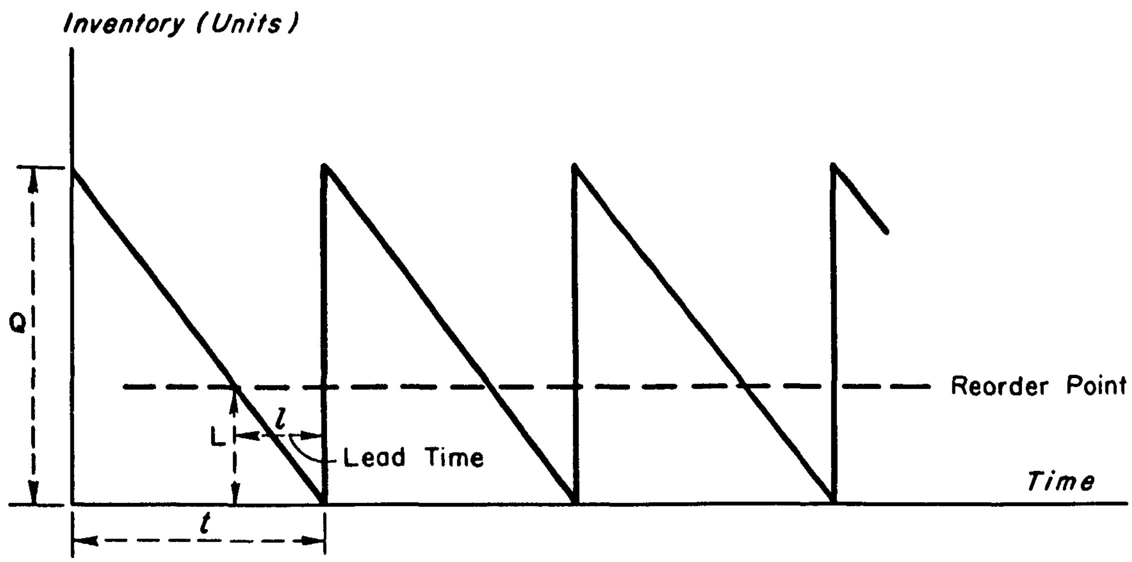

Figure 3-1.--Deterministic Demand Constant Review Inventory System

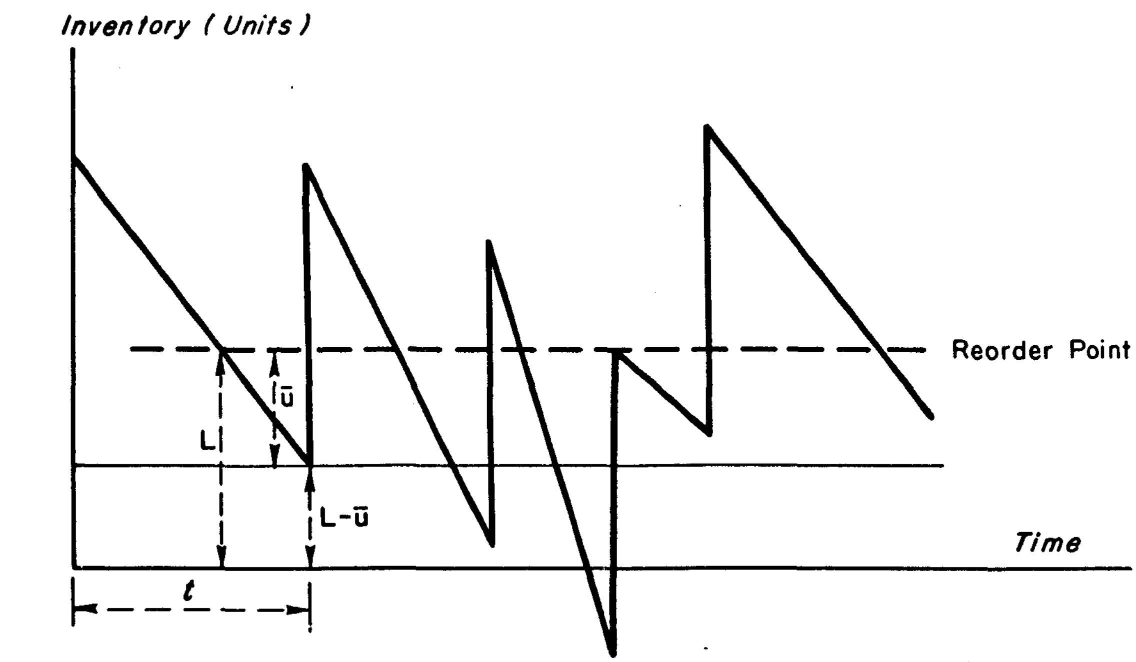

Figure 3-2.--Probabilistic Demand Constant Review Inventory System

economic considerations; the total cost of carrying inventory reserve and of incurring stockouts is minimized for a certain optimal reserve level. The probability distributions of lead time and demand during lead time must be known; the cost of a stockout, per unit of missing good, must be established. Only the second method will be considered here.

In this section, the following designations will be utilized:

- u, the demand during the lead time, a variable, with mean $\bar{u}$;
- S, the cost of a stockout, per unit;
- L, the reorder point, in number of units;
- D, the expected annual demand;
- t, the expected cycle time, in fraction of year;
- ℓ, the lead time.

As can be seen in Figure 3-2, the total cost per cycle is:

$$\text{T.C.p.c.} = P + \frac{CQ}{2}(i + a)t + C(L - \bar{u})(i + a)t + E(u>L).S \tag{3}$$

The terms in (3) are, respectively, the reordering cost, the holding cost of the decreasing part of the stock, the holding cost of the reserve part of the stock, and the stockout cost. Since $\bar{u}$ is the average demand during the lead time, $L - \bar{u}$ will, indeed, be the average reserve stock. The term $E(u>L)$ represents the expected demand during lead time in excess of the reorder point; stockouts occur when $u>L$; the term $E(u>L)$ can also be written as:

$$E(u>L) = \int_L^\infty (u - L)f(u)du \tag{4}$$

where $f(u)$ is the probability distribution of demand (density function) during the lead time.

If it is assumed that the stockouts do not cause loss of demand and, instead, that the parts out of stock will be backlogged, the total cost for the period, say, one year, will be:

$$T.C. = DC + \frac{D}{Q} P + \frac{CQ}{2} (i + a) + C(L - \bar{u})(i + a) + \frac{D}{Q} E(u>L).S \qquad (5)$$

Differentiating with respect to Q, then to L, and equalling the derivates to zero, we find, as optimal values for Q and L:

$$Q^o = \sqrt{\frac{2D[P + E(u>L)S]}{C(i + a)}} \qquad (6)$$

$$\int_L^\infty f(u)du = \frac{QC(i + a)}{DS} \qquad (7)$$

In (7), the left-hand term represents the complementary cumulative probability distribution of the demand during the lead time (see Figure 3-3). The system of equations (6) and (7) must be solved to find the optimal values of Q and L.

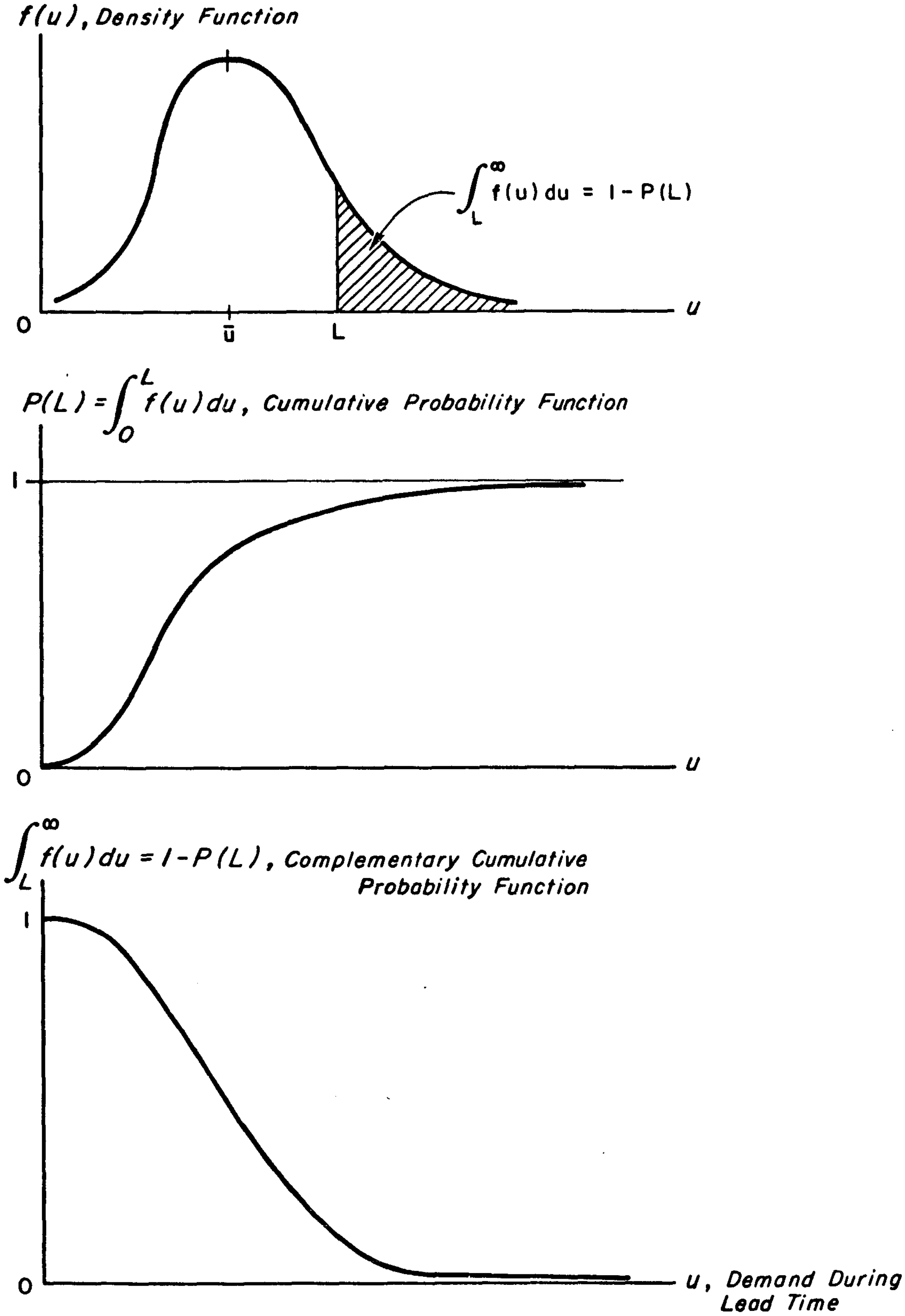

Figure 3-3.

Density Function, Cumulative Probability Function, and Complementary Cumulative Probability Function of Demand during Lead Time

3.1.1. Deterministic Demand Models

Initially, the effect of cost changes on constant review models will be investigated in relation to deterministic demand.

3.1.1.1. Deterministic Step Increase in One Input

A step increase in a purchased part, that is, a one-shot increase in one input, occurs frequently as a consequence of a sectoral scarcity or of an increase in demand for this commodity. Many inflationary situations are marked, precisely, by relatively large increases spaced at large intervals.

The size and timing of the price increase are often announced in advance by the vendor; it can then be said that the case is one of deterministic increase. Or the firm will only be able to forecast the size and timing of the increase with some degree of precision; the last situation will be called a case of probabilistic increase.

When the increase in the purchased part is known in advance, the firm has the option to purchase a larger lot than usual, before the increase enters in effect. By so doing, it incurs an excess holding cost. The question is: what is the optimal amount that the firm ought to purchase in advance so as to minimize the total cost of purchasing and holding the extra inventory?

This problem has been the object of considerable examination in the literature, under the headings of "speculative purchasing," "precautionary purchasing," or "advance purchasing."[1] When it is known for sure

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[1]See, for instance, Robert C. Swanton, "Forward Buying," in

that a price increase will take place, as often happens in prolonged inflations, the element of risk that enters in an advance purchasing decision is small, and the acquisition of an extra amount of material can properly be considered to belong to a policy of protective purchasing, rather than of speculation.

Thomson M. Whitin has developed a formula[2] that gives the economic purchase quantity after cost anticipations in the input have been considered, under the following simplifying assumptions:

- It is expected that a cost rise will occur immediately after the purchase is made;

- Annual dollar sales (in terms of "cost prices") are the same before and after the cost rise.

Using assumptions less restrictive than those adopted by Whitin, Eliezer Naddor has also computed the size of the optimal quantity that should be purchased, in anticipation of a cost increase, in lieu of the habitual economic order quantity.[3] He assumes that the increase is announced by the vendor and takes place at the eve of a normal replenishment time, so that no inventory is left at the time of deciding how much to purchase in advance; furthermore, there is no delivery lead time.

Purchasing Handbook, ed. by George W. Aljian (New York: McGraw-Hill, 1958), pp. 12-1 to 12-15.

[2]Whitin, op. cit., p. 38.

[3]Eliezer Naddor, Inventory Systems (New York: Wiley and Sons, 1966), pp. 96-102. A similar formula is to be found in: Robert G. Bunch and George Allen, "Be Sure It Pays to Stock Up," Purchasing, LIX (November 18, 1965), 85-89. See also: Richard G. Newman, "Analysis of Forward Buying," Production and Inventory Management, VIII (April, 1967), 64-70.

A formula very similar to Naddor's will be derived for a more general case when the increase is announced and will become effective <u>at any time</u> during the inventory cycle, so that some existing inventory will usually be available when the surplus replenishment arrives. It is assumed that the firm follows an EOQ policy; the lead time is ℓ; the economic lot size is given by formula (2). At time t_1 the vendor announces that an increase of K dollars will take place in the cost of the input, entering in effect for all purchases made from the date t_2 on.

In order to avoid the effect of the cost increase, the purchasing agent can buy a larger quantity than usual, q_o, at time t_2, to be delivered at time $t_2 + \ell$. We see in Figure 3-4 that the inventory level will be $q_o + q_1$ at time $t_2 + \ell$. At an annual rate of usage D, this amount of inventory will last during time T. The total cost $T.C._1$ incurred by following this policy will be the sum of the setup cost, P, of the purchased units' cost, q_oC, and of the holding cost, $(q_o + q_1)\frac{Cj}{2}T$. Calling z the period that the inventory q_1 will last, we have:

$$q_o = (T - z)D = TD - q_1 \quad \text{and} \quad q_1 = zD .$$

Then:

$$T.C._1 = P + D(T - z)C + [D(T - z) + zD]\frac{Cj}{2}T = P + D(T - z)C + D\frac{Cj}{2}T^2 \quad (8)$$

If the firm follows the habitual EOQ policy, the cost $T.C._2$ for period T is seen to be:

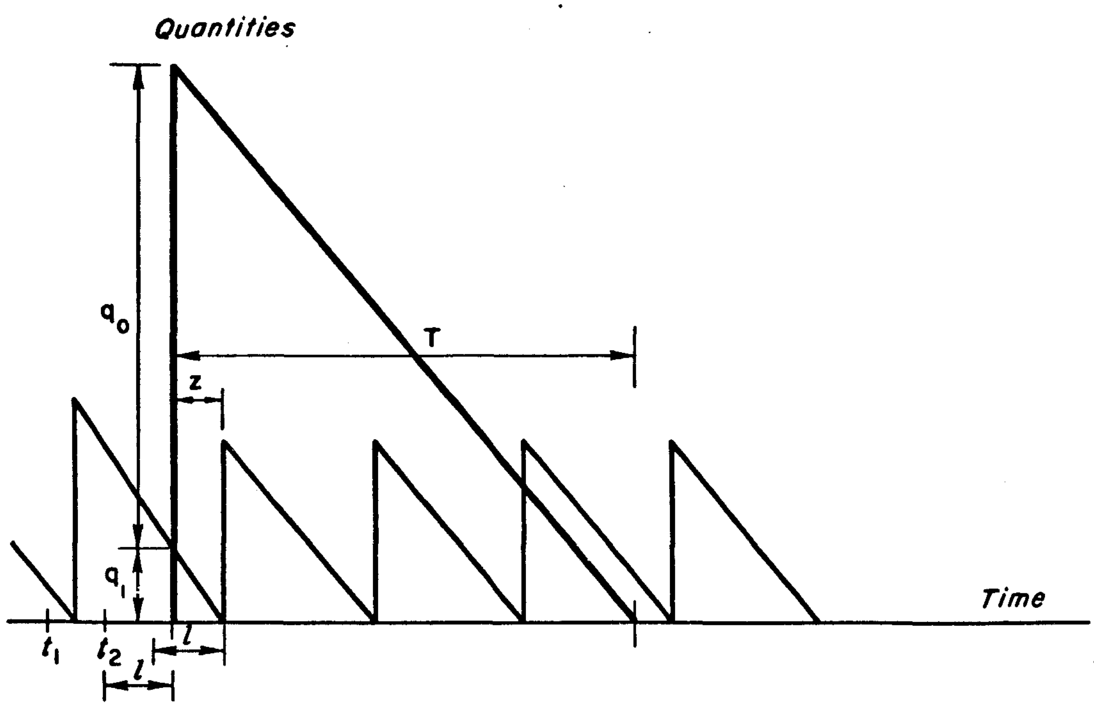

Figure 3-4.--Advance Purchasing for a Step Increase in Cost

$$T.C._2 = (T - z)D(C + K) + (T - z)\frac{D}{Q}P + Q\frac{(C + K)j}{2}(T - z) + q_1\frac{Cj}{2}z, \quad (9)$$

where Q is the economic lot size, corresponding to the increased cost of the input, $C + K$:

$$Q = \sqrt{\frac{2DP}{(C + K)j}}$$

The savings obtained by following the advance purchase policy are given by the difference between $T.C._2$ and $T.C._1$. Differentiating this difference in relationship to T and equating to zero, we obtain, for the optimal T:

$$T_o = \frac{K}{Cj} + \frac{1}{C}\sqrt{\frac{2(C + K)P}{Dj}} \quad (10)$$

and the optimal q_o is given by the expression:

$$q_o^o = \frac{KD}{Cj} + \frac{1}{C}\sqrt{\frac{2(C + K)PD}{j}} - q_1 \quad (11)$$

Calling $k = \frac{K}{C}$ the percentual cost increase, we can rewrite expression (11) of the optimal q_o as follows:

$$q_o^o = (1 + k)Q + k\frac{D}{j} - q_1 \quad (12)$$

Formula (12) shows that the increased economic lot, q_o^o, is a growing function of the percentual cost increase, k. Substituting the optimal q_o value, obtained from (12), into the expression that gives the savings resulting from an advance purchase, one can compute the maximum savings, G^o :

Maximum Savings: $\frac{DK^2}{2Cj} + \frac{K}{C}\sqrt{\frac{2DP(C + K)}{j}} + \frac{KP}{C} - q_1 K -$

$$- q_1 \sqrt{\frac{2(C + K)Pj}{D}} + \frac{Cq_1^2 j}{2D} \qquad (13)$$

Expression (13) reduces to the three first terms when q_1 is zero.

Maximum Savings: $\frac{DK^2}{2Cj} + \frac{K}{C}\sqrt{\frac{2DP(C + K)}{j}} + \frac{KP}{C}$ (13')

We see, by inspection of formula (12), that, if the percent cost increase k is large enough, then q_o^o, the augmented economic lot, becomes equal to the annual demand D. It will happen whenever $k = j$, a rather unusually large increase. We can therefore state that, except for very large cost increases, the constraint $q_o < D$ will be satisfied.

We have seen that the total cost of advance purchasing is:

$$T.C._1 = P + D(T - z)C + D \frac{Cj}{2} T^2 \qquad (8)$$

while the total cost of the economic lot size policy if no cost increase had happened would have been:

$$(DC + \sqrt{2DCPj})T - zDC \qquad (14)$$

where T is given by (10).

Therefore, if the advance purchasing policy is followed, the loss due to the cost increase is limited to the following amount: (8) - (14), which, after computations, reduces to:

$$\frac{DK^2}{2Cj} + K\sqrt{\frac{PD}{j}} \left[\frac{1}{C} \sqrt{\frac{3}{2}(C + K)} - \frac{1}{2(C + K)} \right] \qquad (15)$$

If the outlook is for two increases per year of sizes K_1 and K_2, for a one-year planning horizon, we have two unknown, q_o and q_2, the amounts purchased in advance, respectively, at the time of the first and second cost increases. For instance, Figure 3-5 illustrates the situation of two cost increases spaced at a six-month interval.

The cost of the advance purchase policy is:

$$2P + D(\frac{1}{2} + z_3)C + (T_2 - z_3)D(C + K_1) + \frac{jCD}{4}(\frac{1}{2} + 2z_3) + \frac{jCD}{2}T_2^2 + + \frac{1}{2}K_1 jT_2(T_2 - z_3)D ,$$

while the cost of the habitual EOQ policy is:

$$\frac{1}{2}D(C + K_1) + D(C + K_1 + K_2)T_2 + \sqrt{\frac{(C + K_1)jDP}{2}} + \sqrt{2(C + K_1 + K_2)jDPT_2} .$$

After taking the derivatives of the difference between these costs in relationship to T_2 and z_3, and equating to zero, we obtain a system of equations in T_2 and z_3, which maximizes the savings that can be made with advance purchasing. From these values, we obtain, for the optimal q_o and q_2 :

$$q_o^o = \frac{D}{2} + 2\frac{C}{K_1}D + 2\frac{C^2}{K_1^2}D - \frac{4D}{j} - 2\frac{K_2 D}{K_1 j} - \frac{4CD}{K_1 j} - \frac{2}{K}\sqrt{\frac{2(C + K_1 + K_2)PD}{j}}$$

$$q_2^o = \frac{2D}{j} + \frac{2K_2 D}{K_1 j} + \frac{4CD}{K_1 j} + \frac{2}{K_1}\sqrt{\frac{2(C + K_1 + K_2)PD}{j}} - \frac{CD}{K_1} - \frac{2C^2 D}{K_1^2}$$

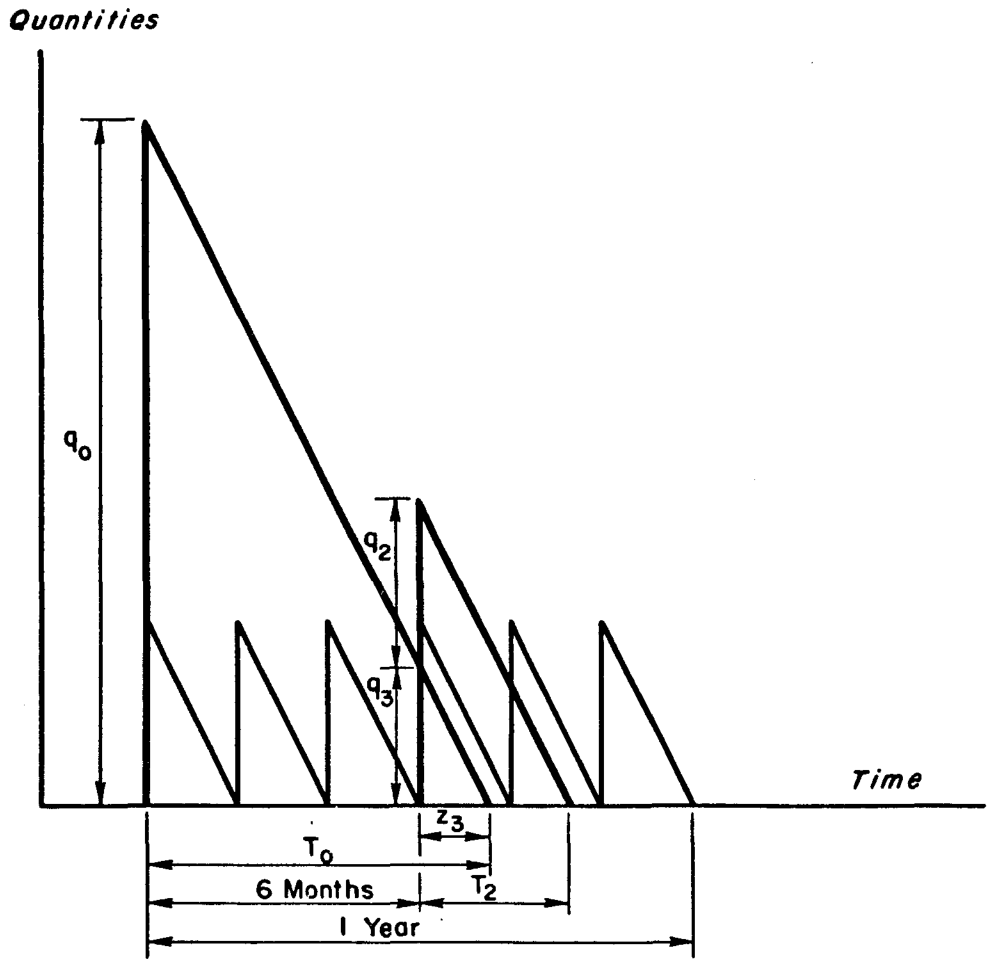

Figure 3-5.

Advance Purchasing: Two Increases per Year
Spaced at a Six-Month Interval

3.1.1.2. Probabilistic Step Cost Increase in One Input

When the size of the cost increase of the input is not known for certain in advance but has a probability distribution, Naddor has shown that the optimal advance purchasing quantity depends only on the expected size of the cost increase of the input, whichever the shape of the probability distribution.[4] Naddor's demonstration holds for the case where no inventory is left at the time of the advance purchase. This result can be generalized for the case where some inventory is left at the time of the advance purchase. The demonstration runs as follows (see Figure 3-4 and the previous section for the meaning of the symbols): Cost of habitual EOQ policy with no advance purchasing:

$$q_o(C + K) + q_o\sqrt{\frac{2(C + K)Pj}{D}} + \frac{q_1^2 Cj}{2D}$$

Cost of advance purchasing policy:

$$P + q_oC + \frac{(q_o + q_1)^2 Cj}{2D}$$

The savings resulting from the adoption of the advance purchase policy are equal to the difference between these two costs:

$$G = q_oK + q_o\sqrt{\frac{2(C + K)Pj}{D}} - P - \frac{(q_o + q_1)^2 Cj}{2D} + \frac{q_1^2 Cj}{2D} \qquad (1)$$

But $\sqrt{(C + K)} \cong \sqrt{C}\,(1 + \frac{K}{2C})$. Therefore:

---

[4]Eliezer Naddor, op. cit., pp. 96-102.

$$G = q_o K + q_o \sqrt{\frac{2PCj}{D}} \left(1 + \frac{K}{2C}\right) - P - \frac{q_o^2 Cj}{2D} - \frac{q_o q_1}{D} Cj$$

Calling $f(K)$ the density function of the size K of the cost increase, the expected savings, $\bar{G}$, are seen to be:

$$\bar{G} = \int_o^\infty Gf(K)dK = q_o\bar{K} + q_o \sqrt{\frac{2PCj}{D}} \left(1 + \frac{\bar{K}}{2C}\right) - P - \frac{q_o^2 Cj}{2D} - \frac{q_o q_1}{D} Cj \qquad (2)$$

In order to compute the q_o which maximizes the expected savings, we differentiate (2) in relationship to q_o and equate the resulting expression to zero. The optimal q_o is given by (3):

$$q_o^o = \frac{D\bar{K}}{Cj} + \frac{1}{C} \sqrt{\frac{2PD(C + \bar{K})}{j}} - q_1 \quad , \qquad (3)$$

precisely the same value as we had found in (11) for the deterministic case with the expected increase $\bar{K}$ substituting the deterministic increase K.

When we use the optimal value q_o, given by (3), in the expression (2) of optimal savings, we obtain, after some computations, the following value for the maximal savings:

$$\bar{G}^o = \frac{D\bar{K}^2}{2Cj} + \frac{\bar{K}}{C} \sqrt{\frac{2PD(C + \bar{K})}{j}} + \frac{\bar{K}P}{C} - q_1\bar{K} - q_1 \sqrt{\frac{2Pj(C + \bar{K})}{D}} + \frac{Cq_1^2 j}{2D} , \qquad (4)$$

precisely the same value that we had found in (13) for the deterministic case with the expected increase $\bar{K}$ substituting the deterministic increase K.

Up to now, no uncertainty existed concerning the time at which the cost increase would occur: the perfect information to this effect was supplied by the vendor. Another problem arises when the uncertainty lies, not in the size of the increase, but in its timing. We assume now that the size of the cost increase is known and that its timing is subject to a probability distribution.

Several formulations of this problem are possible. Let us assume initially that the time horizon is one year; that the inventory manager will appraise only once, at the start of the year, the probability distribution of the cost increase; and that it is certainly known that one increase will occur during the year--the only doubt concerning its timing, not its size.

We call t_b (Figure 3-6) the time at which the cost increase will in fact occur, and t_a the optimal time at which the advance purchase should be made. We want to find t_a, as a function of the parameters D, C, K, P, j and as a function of the probability distribution $p(t_b)$ that an increase occurs at time t_b. The inventory manager establishes a $p(t_b)$ density function at the start of the one-year period and makes no revision of this density function during the year, either because no additional information is supplied to him as time elapses or because the cost of reappraisal is high.

The inventory manager makes, then, at the start of the period, the decision to purchase in advance the amount q_o at time t_a. The actual timing of the cost increase will be t_b. The costs of performing the advance purchase at time t_a will be compared with the cost of performing the advance purchase at the optimal time t_b.

Consider, initially, the case $t_b<t_a$. The opportunity of making savings through advance purchasing will be lost, due to a wrong guessing

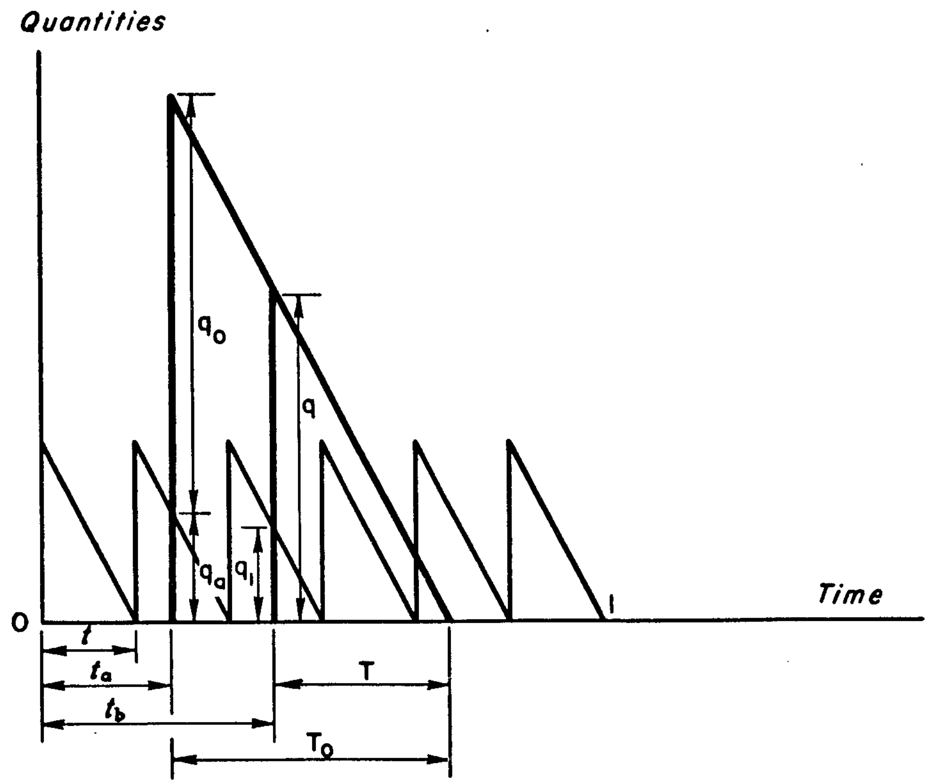

Figure 3-6.

Advance Purchasing: Uncertainty in the Timing of the Increase in Cost of the unknown variable t_b. The cost of this event is equal to the maximum savings of the advance purchasing policy, equation (13').

Consider, in second place, the case $t_a < t_b$, i.e., one buys ahead of the time at which the increase will occur. The consequence of a premature advance purchasing is that one will fail to buy the amount $(t_b - t_a)D$ at the earlier price C, so one fails to save $(t_b - t_a)DK$. If $t_b - t_a \geqslant T_o$, the excess cost of this event, compared to the optimal possible savings, is T_oDK.

The average excess cost of advance purchasing, when the time of the occurrence of the cost increase is subject to a probability distribution of density $p(t_b)$, is, therefore, given by the expression:

$$\overline{EC} = \int_{t_b=0}^{t_a} \left[\frac{DK^2}{2Cj} + \frac{K}{C}\sqrt{\frac{2DP(C + K)}{j}} + \frac{KP}{C}\right] p(t_b)dt_b +$$

$$+ \int_{t_b=t_a}^{t_a+T_o} (t_b - t_a)DKp(t_b)dt_b + \int_{t_b=t_a+T_o}^{1} T_oDKp(t_b)dt_b \qquad (5)$$

If we know $p(t_b)$, the density function representing the probability that the increase will occur at time t_b, we can compute $\overline{EC}$. In order to minimize the excess cost $\overline{EC}$, we determine the value of t_a which minimizes the integral (5). This will give the optimal time t_a^o to purchase the amount q_o in advance.

Suppose t_b has a uniform distribution of density $p(t_b) = 1$; computing (5), we find:

$$\overline{EC} = T_o DK - \frac{T_o^2 DK}{2} + \left[\frac{DK^2}{2Cj} + \frac{K}{C}\sqrt{\frac{2DP(C + K)}{j}} + \frac{KP}{C} - T_o DK \right] t_a \qquad (6)$$

If the coefficient of t_a in (6) is ≥ 0, the value of t_a that minimizes the excess cost $\overline{EC}$ is $t_a^o = 0$. The optimal policy is to purchase q_o in advance at the year's start. If the coefficient of t_a in (6) is < 0, then the value of t_a that minimizes the excess cost $\overline{EC}$ is $t_a^o = 1$. These conditions can be expressed as follows: if $KD > 2Pj$, we postpone the advance purchase up to the end of the period; if $KD \leq 2Pj$, it is worthwhile to "speculate" and to purchase in advance at the start of the period.

Assume now that t_b has a distribution function given by the density: $p(t_b) = a + bt_b$, where a and b are constants, related by the expression: $a + \frac{b}{2} = 1$. Differentiating (5) in relationship to t_a and equating the result to zero, we obtain, after some computation, the value of t_a^o, the optimal time to purchase the quantity q_o in advance:

$$t_a^o = \frac{a\left[\frac{DK}{2Cj} + \frac{1}{C}\sqrt{\frac{2(C + K)PD}{j}} + \frac{P}{C} - \frac{DbT_o^2}{2a} - DT_o \right]}{\frac{b}{2}\left[\frac{DK}{2Cj} + \frac{1}{C}\sqrt{\frac{2(C + K)PD}{j}} + \frac{P}{C} - DT_o \right]} \qquad (7)$$

Recalling that $DT_o = q_o$ is given by:

$$DT_o = \frac{1}{C}\sqrt{\frac{2(C + K)PD}{j}} + \frac{KD}{Cj} \quad ,$$

we obtain:

$$t_a^o = \frac{a}{b} - \frac{CDT_o^2 j}{2Pj - DK} = \frac{a}{b} + \frac{CDT_o^2 j}{DK - 2Pj} \qquad (8)$$

Substituting the value of T_o in (8), we have, for $a = 0$:

$$t_a^o \cong \frac{2(C + K)P}{CDK} + \frac{K}{Cj} + \frac{2}{C}\sqrt{\frac{2(C + K)P}{Dj}} \quad , \text{ for } KD > 2Pj \ , \qquad (8')$$

and $t_a^o = 0$, for $KD \leqslant 2Pj$.

The results which were just presented can be explained as follows: no provision was made for the possibility that T_o might be larger than one year. The horizon is not well defined and can extend actually over much more than one year. Therefore, it is conceivable that postponement of advance purchasing is optimal.

We could reformulate the problem establishing the following conditions: $T_o < 1$, and, in (5), the third integral is eliminated, and the upper limit of the second integral is $1 - T_o$, so that the horizon is strictly limited to one year; eventually, we assume that the increase can occur only in the period $1 - T_o$ with constant probability distribution.

Still another, and less restrictive, way to formulate the problem is to accept a T_o of any size; assume that it is large compared to the habitual EOQ; and assume that the probability of occurrence of an increase is restricted to a period Z , starting at time 0 ; besides, $Z + T_o > 1$. Under such conditions, (5) becomes:

$$\overline{EC} = \int_{t_b=0}^{t_a} \left[\frac{DK^2}{2Cj} + \frac{K}{C}\sqrt{\frac{2DP(C + K)}{j}} + \frac{PK}{C} \right] p(t_b)dt_b +$$

$$+ \int_{t_b=t_a}^{Z} (t_b - t_a)DKp(t_b)dt_b \tag{7}$$

If $p(t_b)$ is constant, one finds, from (7):

$$\frac{d\overline{EC}}{dt_a} = \left[\frac{DK^2}{2Cj} + \frac{K}{C}\sqrt{\frac{2DP(C + K)}{j}} + \frac{PK}{C} \right] - (Z - t_a)DK = 0$$

Usually, the term $\frac{DK^2}{2Cj}$ will be large compared to the two other terms in the first parenthesis of the preceding expression. Eliminating these, we find for the optimal time t_a for advance purchasing, the value:

$$t_a^o = Z - \frac{k}{2j} \tag{8}$$

As an example, if $C = \$1$, $k = .2$, $P = \$5$, $j = .16$, and $D = 100{,}000$ units, then EOQ is 2,500 units and $T_o = 1.3$ (in years); if $Z = .5$, that is, if the cost increase can occur only during the first semester with constant probability then the optimal time to purchase in advance is

$$t_a^o = .5 - \frac{.2}{2(.16)} = - .12$$

One should "speculate" at the start of the year. With the same data and $k = .05$, one finds $t_a^o = .34$, and, upon completion of the first third of the year, one should purchase in advance the amount corresponding to T_o .

As was said above, this probabilistic problem, involving time uncertainty, can be formulated in different ways. For instance, considering again a one-year horizon, and when it is known for certain that an increase of given size K will occur during the year, we can assume, now, that the inventory manager will reappraise several times during the year the probability that an increase will be forthcoming. If this reappraisal is done at each reordering point, the same formula as before, (5), holds for the value $\overline{EC}$, the only difference being that the times must all be counted from the start of each EOQ cycle. The previous conclusions hold for a moving one-year horizon. Of course, at each reappraisal point, one can decide to change the density function of the probability of the cost increase as well as to revise the size K of the increase based upon more recent information.

If, for instance, the distribution function of t_b is linear, and we use formula (8), then, whenever t_a^o is smaller than the EOQ cycle time t , we ought to purchase in advance; otherwise, we postpone the advance purchasing until a later decision. The same principles apply if the revision of the probability of a cost increase is done at even shorter time intervals, such as monthly or weekly.

If we assume, now, that the probability of an increase is concentrated in a span of time m , situated somewhere in the middle of the year; that the probability density is uniform; that $m < T_o$; and that the economic lot size is relatively small in relationship to q_o ; then (Figure 3-7), if the advance purchase is made at time t_a , and the cost increase occurs at time t_b , the average excess cost resulting from a premature or tardy purchase is given by:

$$\overline{EC} = \int_{t_b=m_1}^{t_a} \left[\frac{DK^2}{2Cj} + \frac{K}{C} \sqrt{\frac{2(C + K)DP}{j}} + \frac{KP}{C} \right] dt_b + \int_{t_a}^{m_2} (t_b - t_a) DK dt_b \qquad (9)$$

The optimal purchase time t_a^o , that minimizes the previous expression, is given by:

$$t_a^o = m_2 - \frac{K}{2Cj} - \frac{1}{C} \sqrt{\frac{2(C + K)P}{Dj}} - \frac{P}{CD} \cong m_2 - \frac{k}{2j} \qquad (10)$$

The optimal time for advance purchasing is situated somewhere at the end of the span during which the cost increase is certain to occur, if k is small, or at the very start, if k is large. Formula (10) is analogous to formula (8) found previously.

3.1.1.3. Continuous Cost Increase in One Input

In the previous sections, the effect of step increases in the cost of an input on the inventory policy of a cost-minded manager was analyzed. In the present section, the influence of a continuous cost increase will be investigated.

Relatively small, but frequent, monthly or quarterly increases, are characteristic of persistent, uncontrolled inflations. First, the

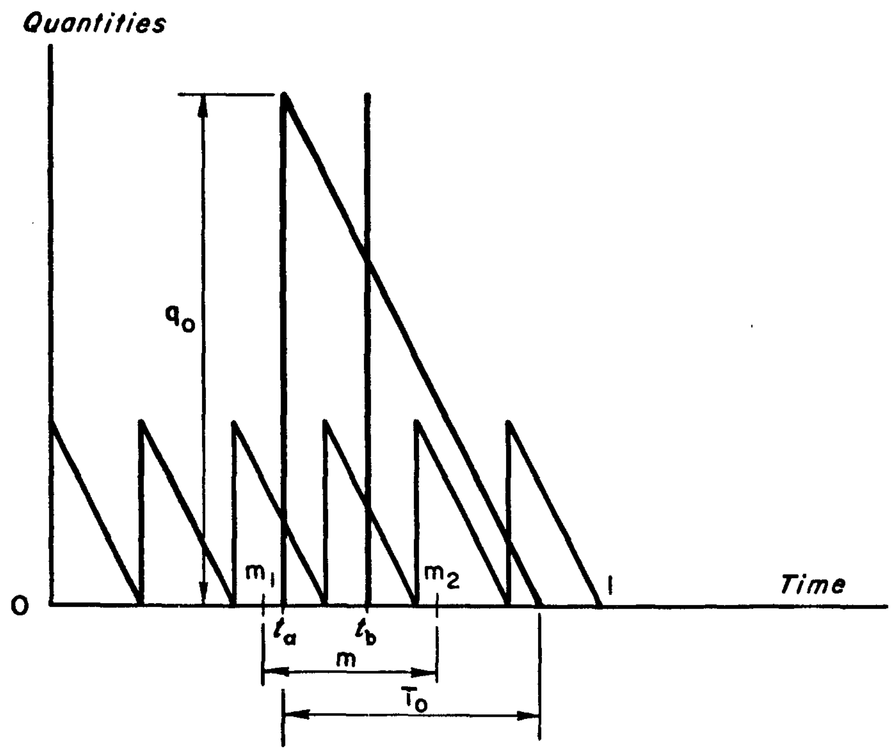

Figure 3-7.

Advance Purchasing: Uncertainty in the Timing of the Increase in Cost. Step Increase Restricted to the Time Interval m case of continuous cost increases in one input only will be examined, and, in the next section, the case of cost increases in all inputs will be taken up.

Let it be assumed that the inventory manager is forecasting monthly increases in cost of the input to be purchased. The symbol I will be used to represent the annual percentual increase, and the symbol $d = \frac{I}{12}$ to represent the monthly percentual increase in cost. The other symbols to be used were defined earlier.

We look first for a formula which gives the total cost or purchasing D units per year, through batches of size Q, when the cost of purchasing a unit is C during the first month, $C(1 + d)$ during the second month, $C(1 + 2d)$ during the third month, and $C(1 + 11d)$ during the last month of the one-year horizon.

If one buy is made, the cost of purchasing will just be:

$$\text{Cost} = QC \quad ,$$

since, obviously, this purchase is made at the year's start.

If two purchases are made per year, the cost of purchasing will be:

$$\text{Cost} = QC + QC(1 + 6d) \quad ,$$

the second parcel being purchased at a six-month interval after the first one.

With three purchases equally spaced per year, we have:

$$\text{Cost} = QC + QC(1 + 4d) + QC(1 + 8d) \quad .$$

With four, six, and twelve purchases per year, at regular intervals, we have, respectively:

$$\text{Cost} = QC + QC(1 + 3d) + QC(1 + 6d) + QC(1 + 9d) \ ,$$

$$\text{Cost} = QC + QC(1 + 2d) + QC(1 + 4d) + \ldots + QC(1 + 10d) \ ,$$

$$\text{Cost} = QC + QC(1 + d) + QC(1 + 2d) + \ldots + QC(1 + 11d) \ .$$

Calling n the number of purchases per year, the preceding expressions can be written:

$$\text{Cost} = QC + QC(1 + \frac{12}{n} d) + QC(1 + \frac{2 \times 12}{n} d) + \\ + \ldots + QC \left[1 + (n - 1) \frac{12}{n} d \right] \qquad (1)$$

The terms of (1) are the members of an arithmetic progression of ratio: $QC \frac{12}{n} d$. Expression (1) can therefore be written:

$$\text{Cost} = QC \left[1 + 1 + (n - 1) \frac{12}{n} d \right] \frac{n}{2} \qquad (2)$$

Let us write $12d = I$, the annual percentual increase, and substitute n in (2) by $\frac{D}{Q}$. We have:

$$\text{Cost} = QC \left[\frac{D}{Q} + \frac{D}{Q} \frac{I}{2} - \frac{I}{2} \right] = DC \left[1 + \frac{I}{2} - \frac{QI}{2D} \right] \qquad (3)$$

The factor $f = 1 + \frac{I}{2} - \frac{QI}{2D}$ reflects the influence of a continuous monthly increase of percentual value $\frac{I}{12}$ on the annual cost of purchasing. We observe that the factor f is 1 when the cost increase I is zero, or when all of the purchase is made at the year's start, that is, when $Q = D$. Factor f is a linear function of Q , which, under the assumptions made, reaches its maximum when Q is as small as possible, namely, for $Q = \frac{D}{12}$, when f becomes equal to

$$1 + \frac{I}{2} - \frac{I}{24} \cong 1 + \frac{I}{2} \quad .$$

The total cost of inventory for one year becomes:

$$T.C. = DC\left[1 + \frac{I}{2} - \frac{QI}{2D}\right] + \frac{DP}{Q} + QC\left[1 + \frac{I}{2} - \frac{QI}{2D}\right] \cdot \frac{i + a}{2} \tag{4}$$

The last term in (4) reflects the fact that the average unit cost during the year is Cf . Whatever the value of Q , the last term of (4) can be approximated with little error by $QC\,\frac{i + a}{2}$, so that (4) becomes:

$$T.\ C. = DC\left[1 + \frac{I}{2} - \frac{QI}{2D}\right] + \frac{DP}{Q} + QC\,\frac{i + a}{2} \tag{5}$$ [5]

Differentiating (5) in relationship to Q , and equalling the result to zero, we obtain, for the optimal lot size, the value:

$$Q^{o} = \sqrt{\frac{2DP}{C(i + a - I)}} \tag{6}$$

In terms of the habitual EOQ, (6) can be written:

$$Q^{o} = EOQ\sqrt{\frac{i + a}{i + a - I}} \tag{7}$$

We see that, for a continuous cost increase in one input, the economic lot size becomes larger than it does without an increase.

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[5]Claude Machline, "Inflacão e lote econômico de compra," Revista de administração de emprêsas (São Paulo), I (May, 1961), 17-33. Sergio B. Zaccarelli, in Programacão e contrôle da produção (São Paulo: Livraria Pioneira Editôra, 1967), pp. 273-278; and L. Pack, in "Economic Order Sizes and Lot Sizes as an Aid to Efficient Management," Management International, 1965, no. 1, pp. 82-99, have also obtained formulas for

Clearly, when $I = i + a$, one purchases in advance the whole yearly requirement D at the start of the year.

As an example, if the interest rate i is 5% per year, the storage cost a is 10%, and the percentual cost increase I is 5%, the economic lot size increases by 22%. If the percentual annual cost increase is 10%, the economic lot size increases by 73%.

Examining (5), we see that, if I were of a probabilistic nature, solution (7) could still be applied, with the average value $\bar{I}$ substituting the deterministic value I.

Expression (7) is approximate, not only on account of the approximation made in the last term of (4), but also because cost increases are assumed to occur at discrete intervals of one month and the only EOQ solutions considered are those for which n is a divisor of 12. We can see that the formula of factor f does not hold for $n = 8$, for instance, nor for $n = 24$ or 48.

If EOQ in formula (7) is, say, a one-month supply, and the factor under the root in (7) is 22%, for instance, then, actually, no change in the economic lot size should be made, because an increase of 22% is too small in relationship to the precision of the formula. But if the EOQ with stable cost is, say, a two-month supply, and the factor under the root is 73%, then one should shift to a three-month economic lot size.

If we assume daily increases of percentual size $d = \frac{I}{365}$, then the difficulties arising from discontinuities in (5) are removed. Expression (5) applies even for small increases in the economic lot size.

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this problem.

Indeed, with daily increases of size $d = \frac{I}{365}$, we have the following purchasing costs:

- One purchase per year, at the year's start:

  Cost = QC

- Two purchases per year, spaced at six-month intervals:

  Cost = QC + QC(1 + 182.5d)

- Three purchases per year, spaced at four-month intervals:

  Cost = QC + QC(1 + 121.7d) + QC(1 + 243.4d)

- Four purchases per year, spaced at three-month intervals:

  Cost = QC + QC(1 + 91d) + QC(1 + 182d) + QC(1 + 273d)

- Five purchases per year, spaced at 73-day intervals:

  Cost = QC + QC(1 + 73d) + QC(1 + 146d) + QC(1 + 219d)

  + (QC(1 + 292d)

- Six purchases per year, spaced at 61-day intervals: Cost =

  QC + QC(1 + 61d) + QC(1 + 122d) + QC(1 + 183d) + QC(1 + 244d)

  + QC(1 + 305d)

And so on for any number of purchases per year, up to, say, 52 purchases, i.e., a purchase per week, at a cost of:

Cost = QC + QC(1 + 7d) + QC(1 + 14d) + ... + QC(1 + 357d).

Then we would have for 2 purchases per week, spaced at 3.5-day intervals:

Cost = QC + QC(1 + 3.5d) + QC(1 + 7d) + ... + QC(1 + 360.5d)

And for a purchase a day:

Cost = QC + QC(1 + d) + QC(1 + 2d) + ... + QC(1 + 364d)

In all cases, calling n the number of yearly purchases, we have:

$$\text{Cost} = QC + QC(1 + \frac{365}{n} d) + QC(1 + 2\,\frac{365}{n} d) + \ldots +$$
$$+ QC\,[1 + (n - 1)\,\frac{365}{n}]\,d =$$
$$= QC + QC(1 + \frac{I}{n}) + QC(1 + 2\,\frac{I}{n}) + \ldots + QC\,[1 + (n - 1)\,\frac{I}{n}]$$

This is an arithmetic progression, the sum of which is:

$$\text{Cost} = QC\left[1 + 1 + \frac{(n-1)I}{n}\right] . \frac{n}{2} \quad ,$$

and, after substituting n by $\frac{D}{Q}$, one has:

$$\text{Cost} = DC(1 + \frac{I}{2} - \frac{QI}{2D}) \; ,$$

as in (3).

We also obtain formula (4) for the total cost of the yearly inventory, and, now, since Q is practically continuous, we are justified in differentiating expression (4) and in obtaining expression (6) for the optimal lot size Q^o .

Figure 3-8 shows the influence of the linear cost increase in one input over the economic lot size, assuming continuity in the cost increase and in the size of the purchased lot.

3.1.1.4. Continuous Cost Increases in All Inputs

Due to the large variety of possible combinations in the timing and size of costs increases, and due to the complexity of their effect, attention is necessarily limited to the simplest cases. In particular, regularity of increases in costs allows us to obtain operational expressions for economic lots.

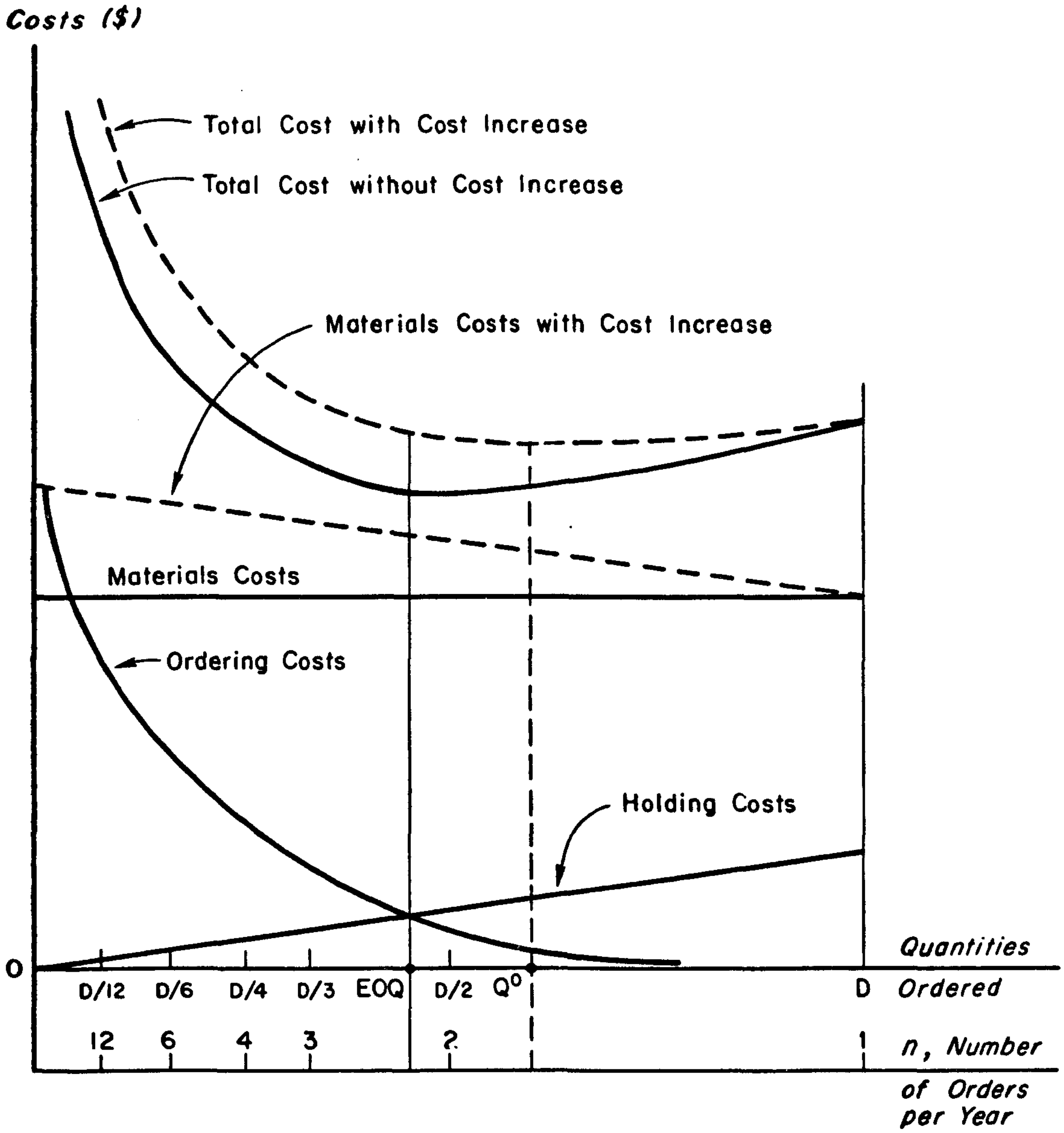

Figure 3-8.

Linear Increase in Cost of Material and Its Effect upon the Economic Lot Size

In the previous section, the effect of a continuous cost increase in one input on the inventory policy of the firm was investigated. Namely, we examined the effect of a cost increase on the purchased price of the item. In many inflationary situations, though, the increases are not restricted to one item but extend also to administrative costs, interest rate, and storage costs.

In this section, we will have different situations, which we will describe in a moment. In all of them, the postulated horizon for which one tries to minimize costs is one year.

To make the problem more realistic as well as more manageable, it will be assumed, in all situations, that the cost of holding inventories--that is, the interest rate and storage costs--do <u>not</u> change <u>during</u> the year. In other words, j is a constant during the one-year horizon considered. Both the interest rate and storage costs are determined by the market at the start of the period and are not changed afterwards. If the market had perfect foresight of future inflation, the market or apparent interest rate would be, as discussed in Chapter I.

$$r = i + I \tag{1}$$

where r is the nominal interest rate, i the real interest rate, and I the foreseen inflation rate--all annual.

And the total holding cost, expressed as an annual rate, would be:

$$j = i + a + I \tag{2}$$

These assumptions are plausible, because we have already seen in Chapters I and II that the interest rate lags behind the price-level changes. Usually, interest rates cannot be changed at short intervals. The same applies to rental charges for space, to insurance rates, and to taxes. It is therefore reasonable to postulate that the rate j will stay stable for the one-year horizon considered.

It is now necessary to characterize the several situations mentioned above. First of all, we shall distinguish between the cases of uncontrolled and controlled inflations. In the case of controlled inflation, attention will be confined to an economy in which the firm is forbidden to readjust its sales prices in any way, or, at least, it will be limited to an economy in which the firm is uncertain as to whether or not it will be able to readjust its selling prices during the next year; the firm, therefore, for conservative reasons, feels that it must follow an inventory policy based upon a perspective of cost increases but of stable selling prices. In the case of uncontrolled inflation, the possibilities will be subdivided further: we can take the standpoint of the operations manager, who only takes the cost rises in the inputs into consideration, or we can adopt the viewpoint of the financial manager, who looks at the total system costs.

Looking first at the case of controlled inflation, we note that the cost of holding inventories is the apparent interest rate, plus the storage costs. If the increase in cost is linear, that is, if the rate of cost increase is $\frac{I}{365}$ per day, or $\frac{I}{12}$ per month, and is the same for all the inputs, then, referring to what was seen in the previous section, 3.1.1.3, expression (4) becomes:

$$T.\ C. = (DC + \frac{DP}{Q} + QC\,\frac{i + I + a}{2})\ (1 + \frac{I}{2} - \frac{QI}{2D}) \tag{3}$$

The first parenthesis represents the well-known expression of total annual costs in stationary economic conditions; the second parenthesis is a factor which reflects the influence of the cost changes.

The differentiation of expression (3), in order to compute the optimal Q, gives a third-order equation. The best way to calculate the economic lot size would then be to use a tabular method, computing TC through (3) and selecting the Q which minimizes the total cost TC.

The problem of continuous cost increases in all inputs can still be treated with more generality: assume now that the cost increases proceed at different rates for the several inputs and that they are linear. Let I_1 be the rate of increase in the materials cost, I_2 the rate of increase in the setup cost, and I_3 the rate of increase in the interest rate. We have, instead of (3), the following expression for the total annual costs:

$$T.C. = DC(1 + \frac{I_1}{2} - \frac{QI_1}{2D}) + \frac{PD}{Q}\,(1 + \frac{I_2}{2} - \frac{QI_2}{2D}) + $$

$$+ QC\,\frac{i + a + I_3}{2}\,(1 + \frac{I_1}{2} - \frac{QI_1}{2D}) \tag{4}$$

As before, we neglect the term $QC\,\frac{i + a + I_3}{2}\,(\frac{I_1}{2} - \frac{QI_1}{2D})$ and get:

$$\frac{d(T.C.)}{dQ} = -\frac{CI_1}{2} - \frac{PD}{Q^2}\,(1 + \frac{I_2}{2}) + C\,\frac{i + a + I_3}{2} = 0\ ,$$

from where:

$$Q^o = \sqrt{\frac{2DP}{c(i + a + I_3 - I_1)} \left(1 + \frac{I_2}{2}\right)} \tag{5}$$

The increase in P , the administrative cost of purchasing, tends to increase the economic order quantity, but rather slightly, inasmuch as the factor $\sqrt{1 + \frac{I_2}{2}}$ is small for moderate rate increases I_2 . If the foreseen increase in the cost of money, I_3 , is larger than the foreseen increases in the cost of materials, I_1 , i.e., if $I_1 < I_3$, a situation characteristic of controlled inflation, in which the interest rate is expensive, then Q^o is smaller than in noninflationary times. Controlled inflation tends to decrease the size of stocks.

If $I_1 = I_3$, in other words, if the increase in materials is equal to the increase in interest rate, a situation characteristic of uncontrolled, general, and balanced inflation, the economic lot size formula becomes the same as without inflation. Uncontrolled, general inflation leaves the size of stocks unchanged.

If $I_1 > I_3$, that is, if the anticipated increase in materials is larger than the anticipated increase in interest rate, as occurs in uncontrolled, sectoral inflations, then the economic lot size becomes larger than in noninflationary periods. Uncontrolled, sectoral inflation increases the size of stocks.

These conclusions apply to the case of firms which consider the money cost of holding inventories to be dictated by the apparent interest rate $i + I_3$, a rate justified when uncertainty exists concerning the possibility of readjusting one's own selling price, or when one adopts the position of the operations manager, who works with a standard cost determined at the start of the year and independent of any price readjustment. If the firm is confident that it will be able to readjust its selling price, and if it wants to adopt the total system viewpoint in its inventory policy, then the money cost of holding inventories is governed by the real interest rate i, and the economic lot size formula becomes:

$$Q^o = \sqrt{\frac{2DP}{C(i + a - I_1)} \left(1 + \frac{I_2}{2}\right)} \tag{6}$$

Except for the factor involving I_2, formula (6) is identical with formula (6) of section 3.1.1.3. Under the assumptions made, the stock size increases, in comparison to non inflationary conditions.

If I_3 is negative, that is, if the firm not only is able to readjust its prices at the inflation rate but is even able to anticipate its price increases ahead of the general inflation, then equation (5) indicates that, theoretically, the stocks should increase for cost optimality.

Consider the following expression for economic lot size:

$$Q^o = \sqrt{\frac{2DP}{C(i + a + I_3)}} \tag{7}$$

This expression is the habitual EOQ formula, as we see, with the apparent interest rate substituting for the real interest rate. Formula (7) makes abstraction of the increases in administrative costs, I_2, a justifiable assumption in most cases, but abstracts also of the increases in materials costs, a much less justified assumption. Formula (7) tends to be a merely static formula, which is correct if no costs increases in materials are anticipated. If such an increase is expected, formula (7) is not optimal.

If the sizes I_1, I_2, and I_3 of the yearly cost increases are not known with certainty but depend on probability distributions $f_1(I_1)$, $f_2(I_2)$, and $f_3(I_3)$, which are assumed to be independent, then, from (4), after eliminating the less important terms, we have, for the total expected inventory costs:

$$\overline{T.C.} = \int DC(1 + \frac{I_1}{2} - \frac{QI_1}{2D}) f_1(I_1)dI_1 + \int \frac{PD}{Q} (1 + \frac{I_2}{2} - \frac{QI_2}{2D}) f_2(I_2)dI_2 + \int QC \frac{i + a + I_3}{2} f_3(I_3)dI_3 , \qquad (8)$$

where the integrals are taken over the ranges of I_1, I_2, and I_3.

By integration of (8), it is readily seen that one obtains:

$$\overline{T.C.} = DC(1 + \frac{\overline{I_1}}{2} - \frac{Q\overline{I_1}}{2D}) + \frac{PD}{Q} (1 + \frac{\overline{I_2}}{2} - \frac{Q\overline{I_2}}{2D}) + QC \frac{i + a + \overline{I_3}}{2} \qquad (8')$$

Differentiating (8') in relationship to Q, we obtain for the EOQ a formula identical to (5), in which the average values $\overline{I_1}$, $\overline{I_2}$, and $\overline{I_3}$ substitute the deterministic values I_1, I_2, and I_3.

If the probabilities of the cost increases are not independent, and, instead, given by a joint probability distribution $f(I_1, I_2, I_3)$, one has, for total average costs:

$$\overline{T.C.} = \iiint \left[DC(1 + \frac{I_1}{2} - \frac{QI_1}{2D}) + \frac{PD}{Q}(1 + \frac{I_2}{2} - \frac{QI_2}{2D}) + \right.$$

$$\left. + QC\,\frac{i + a + I_3}{2} \right] f(I_1,I_2,I_3)\cdot dI_1 dI_2 dI_3$$

In order to compute the optimal EOQ, a tabular approach would be used. As an illustration, purposely kept simple, assume that I_2 is of negligible size and that I_1 and I_3 are given by the following joint discrete probability distribution $P(I_1,I_3)$:

I_1 \ I_3	5%	10%	Total
10%	25%	15%	40%
20%	5%	55%	60%
Total	30%	70%	100%

We have:

$$\overline{T.C.} = \sum_{I_1}\sum_{I_3} \left[DC(1 + \frac{I_1}{2} - \frac{QI_1}{2D}) + \frac{PD}{Q} + QC\,\frac{i + a + 1_3}{2} \right] P(I_1,I_3) \quad ,$$

Assume $D = 1{,}000$, $C = \$1.00$, $P = \$5.00$, $i + a = 16\%$ per year. Performing the computations, one finds that $\overline{T.C.}$ is: $1,127.5 for $Q = D$, 1,111.2 for $Q = \frac{D}{2}$, 1,110.6 for $Q = \frac{D}{4}$, 1,134.2 for $Q = \frac{D}{10}$. The EOQ is therefore 250.

We shall now illustrate formulas (6) of the previous section and (5) of this section. As a numerical example, consider the following values:

$C = \$1$

$P = \$5$

$i = .06$ per year

$a = .10$ per year

$D = 100{,}000$

Consider first the case of a sectoral inflation affecting materials costs, at the annual rates $I_1 = 10\%, 20\%$, and 50%, successively; I_2 and I_3 are zero. We have: $Q^o = 4{,}080$, for $I_1 = 10\%$; $Q^o = 100{,}000$ for $I_1 = 20\%$ and 50%, from:

$$Q^o = \sqrt{\frac{2DP}{C(i + a - I_1)}} \tag{9}$$

The following figures, giving the value of T.C. in (4), confirm that the optimal T.C. value is obtained by Q^o of (9).

Q, Lot Size (units)	T.C., Total Cost ($)		
	I = 10%	I = 20%	I = 50%
2,500	105,284	110,170	124,824
4,000	105,260	110,076	124,522
5,000	105,269	110,038	124,345
10,000	105,386	109,912	123,530
25,000	105,845	109,670	121,145
50,000	106,610	109,210	117,010
100,000	108,005	108,005	108,005

Consider now, with the same parameters, the case of a general inflation: $I = I_1 = I_2 = I_3 = 10\%, 20\%$, and 50%, successively. We have, from (5):

$$Q^o = \sqrt{\frac{2DP}{C(i + a)} \left(1 + \frac{I_2}{2}\right)} \qquad (10)$$

The value of Q^o is approximately 2,500 for the three cases.

The following figures, giving the value of T.C. in (4), confirm that the optimal T.C. value is obtained by Q^o of (10).

Q, Lot Size (units)	T. C., Total Cost ($)		
	I = 10%	I = 20%	I = 50%
1,250	105,527	110,561	125,702
2,000	105,435	110,470	125,632
2,500	105,426	110,463	125,650
5,000	105,535	110,595	127,060
50,000	109,173	114,460	131,073
100,000	113,005	118,005	133,005

3.1.1.5. Irregular Cost Increases

We have examined the case of one cost increase per year, on the one hand, and the case of a continuous series of increases, on the other. These situations represent the extremes of a whole spectrum of possible cost increases. We could have a situation in which costs are expected to increase twice, three times, ... , a year. We examined the case of advance purchasing for two successive increases and presented a system of two equations which would solve this situation.

In the case of three, four, six, or any number of cost increases of this order, it would be impractical to establish a system of three, four, six, etc., ... equations. It seems simpler to use a tabular, trial and error method, in which several possible alternatives are explored and compared.

As an example, suppose we have the following data: C = \$1.00, D = 100,000 units, i + a = 16%, P = \$5.00. Assume three increases in costs have occurred in materials input during the year according to the following pattern:

- at the end of the second month: a \$ .05 increase,
- at the end of the seventh month: a \$ .10 further increase,
- at the end of the ninth month: a \$ .05 further increase.

Some of the alternatives are the following:

1. Follow the habitual EOQ policy. The EOQ is 2,500 units; we buy 40 lots, at an interval of approximately 9 days. We buy:

7 times at \$1.00, for a total of	7 x 2,500 x 1.00	= \$ 17,500
16 times at \$1.05, for a total of	16 x 2,500 x 1.05	= \$ 42,000
7 times at \$1.15, for a total of	7 x 2,500 x 1.15	= \$ 20,125
10 times at \$1.20, for a total of	10 x 2,500 x 1.20	= \$ 30,000
40 times	40 x 2,500	\$109,625

We also incur holding costs of about $\frac{2,500}{2}$ x 16%	= \$ 200
and setup costs equal to 40 x \$5.00	= \$ 200
Total costs of this alternative:	\$110,025

2. Purchase the yearly demand in advance. We have:

Purchasing costs:	100,000 x 1.00	= \$100,000
Holding costs:	$\frac{100,000 \times 16\%}{2}$	= \$ 8,000
Setup costs:		= \$ 5
Total costs of this alternative:		\$108,005

3. Purchase 3 lots, each of them on the eve of the cost increase, after the initial 2 months, during which we follow the EOQ policy. We purchase:

7 lots of 2,500 units, or 17,500 units, at $1.00	=	$ 17,500
1 lot of 2,500 units, or 40,833 units, at $1.00	=	$ 40,833
1 lot of 2,500 units, or 16,667 units, at $1.05	=	$ 17,500
1 lot of 2,500 units, or 25,000 units, at $1.15	=	$ 28,750
Total: 100,000 units	=	$104,583

The holding costs incurred amount to:

$$\left(\frac{2,500}{2} \times \frac{2}{12} + \frac{40,833}{2} \times \frac{5}{12} + \frac{16,667}{2} \times \frac{2}{12} + \frac{25,000}{2} \times \frac{3}{12} \right) 16\% = \\ = \$ \ 2,117$$

The setup costs are: 10 x 5.00 = $ 50

The total costs of this policy amount to: $106,750

By what we have seen in the previous sections concerning the size of the optimal advance purchase lots, this last alternative must be the optimal solution.

3.1.2. Probabilistic Demand Models

Up to now, we have considered the demand deterministic. If the demand is probabilistic, then, referring to (5), Section 3.1, and considering only the case where the firm is unable to readjust its selling price, we have, for anticipated increases I_1, I_2, and I_3 :

$$T.C. = DC(1 + \frac{I_1}{2} - \frac{QI_1}{2D}) + \frac{D}{Q} P(1 + \frac{I_2}{2} - \frac{QI_2}{2D}) +$$

$$+ QC \frac{i + a + I_3}{2} (1 + \frac{I_1}{2} - \frac{QI_1}{2D}) +$$

$$+ C(L - \bar{u})(i + a + I_3)(1 + \frac{I_1}{2} - \frac{QI_1}{2D}) + \frac{D}{Q} E(u > L) \cdot S$$

As before, some second-order terms will be eliminated. Differentiating in relationship to Q and L and equating the derivatives to zero, we have the system:

$$Q^o = \sqrt{\frac{2D \left[P(1 + \frac{I_2}{2}) + E(u > L^o)S \right]}{C(i + a + I_3 - I_1)}} \tag{11}$$

$$P(u > L^o) = \int_L^\infty f(u)du = \frac{Q^o C(i + a + I_3)(1 + \frac{I_1}{2} - \frac{QI_1}{2D})}{DS} \tag{12}$$

These formulas are correct as long as the demand fluctuations are not excessive. The system of equations (11) and (12) can be solved to find the optimal values of Q and L, Q^o, and L^o. Similar formulas can be developed for the case where the firm is able to readjust its selling price.

Looking at (11) and (12), we see that the conclusions regarding Q^o, obtained for the deterministic case, do still apply to the probabilistic case, as long as $E(u > L^o)$ is small. With respect to L^o, it should be recalled that the left-hand terms of (12) represent the complementary cumulative probability of demand distribution during the lead time, $P(u > L)$. It must also be recalled that as $P(u > L)$ increases, L decreases.

If $I_1 < I_3$, we have already seen that Q^o is smaller than in noninflationary times. But the numerator of (12) contains factors which compensate for the decrease in Q^o. Therefore, $P(u > L)$ increases, and the reserve level L will tend to decrease. Controlled inflation decreases the inventory reserve level.

If $I_1 = I_3$, the numerator of (12) is larger than in noninflationary times. Therefore, $P(u > L)$ increases, and L^o decreases. The safety allowance level should decrease during a general, uncontrolled inflation. Of course, if the firm can readjust its selling price, then the reserve level will also be unchanged.

If $I_1 > I_3$, both Q^o and the parentheses tend to increase the numerator in (12). Sectoral inflation decreases the reserve level.

In all cases, the changes in the values of $P(u > L)$ will occur in the very flat portion of the curve representing the complementary cumulative probability distribution of demand during the lead time, because the high cost of a stockout usually results in the reserve level being set at a high value. In the flat portion of the curve, any small change in $P(u > L)$ will cause a large change in L. Therefore, the changes in reserve level could easily be as large as the ones in the EOQ itself.

In the case of controlled inflation, the decrease in reserve is identical in effect with the change in EOQ. In the case of balanced, general inflation, only the reserve level decreases, but the EOQ does not change. In the case of sectoral inflation, the decrease in reserve runs opposite in effect to the increase in EOQ.

3.2. Periodic Review Inventory Models

In constant review inventory models, studied above, the status of inventories was reviewed whenever a withdrawal was made. In periodic review inventory models, which will be examined now, the status of inventories is studied at periodic predetermined intervals, such as, for instance, every month, every week, or even, as is becoming increasingly common with the rapidly growing use of computers in inventory systems, every day.

First, the effect of anticipated cost increases on periodic review inventory models with deterministic demand will be examined; probabilistic demand models will be dealt with later.

3.2.1. Periodic Review Inventory Models with Deterministic Demand: The Hand-to-Mouth Purchasing Policy

The only policy which will be studied is the "hand-to-mouth" purchasing policy. It is not cost-minimizing. It consists in examining the inventory position at periodic intervals, and reordering, after each review, the minimum amount necessary to satisfy demand. Typically, a scheduling meeting would take place every week, in which the requirements for the next week would be reviewed and the amount estimated to be necessary for that week's demand would be ordered.

This policy can be considered "myopic," inasmuch as it looks ahead at a very short horizon, if applied strictly. Of course, it can be rendered less myopic by using a flexible approach, for instance, by considering future increases in costs and purchasing larger quantities in advance of these cost increases. This is the problem which will be investigated.

In this section, Q will designate the amount normally purchased per period, that is, the expected demand per period. The other symbols' meanings have already been explained. If n refers to the number of periods for which we shall purchase in advance, then the extra holding costs resulting from advance purchasing can be seen to be:

$$[n + (n - 1) + (n - 2) + \ldots + 2 + 1]\ QCj = \frac{n(n + 1)}{2}\ QCj$$

The advance purchasing is assumed to have been done just before the cost increase. The savings resulting from advance purchasing are: $nQK = nQCk$. In this expression, as we remember, K is the step cost increase, in \$, and k is the percentual cost increase. It must be kept in mind that j , here, is the holding cost rate <u>per period</u> (<u>not</u> per year).

The net savings resulting from advance purchasing are:

$$nQCk - \frac{n(n + 1)}{2}\ QCj \tag{1}$$

Differentiating this expression in relationship to n , we compute the optimal n , n^o , that is, the optimal number of periods for which one ought to purchase in advance:

$$n^o = \frac{k}{j} - \frac{1}{2} \cong \frac{k}{j} \tag{2}$$

The net maximum savings which can be realized with advance purchasing are obtained by substituting the optimal n^o value derived from

(2) in (1). One has:

$$\text{maximum savings} = \frac{QC}{2}\left(\frac{k^2}{j} - k\right) + \frac{QCj}{8} \qquad (3)$$

If the cost increase is of a probabilistic nature in respect to its size, the preceding formulas hold, with the average percentual increase $\bar{k}$ substituting for k. Instead of the point of maximum savings, a break-even point can also be computed which is the maximum amount one could afford to purchase in advance; it is the point at which the net savings resulting from equation (1) are zero. The break-even point is equal to $2n^o$.

We observe that formula (2) is similar to the first term of expression (10) of section 3.1.1.1.,

$$T_o = \frac{K}{Cj} + \frac{1}{C}\sqrt{\frac{2(C + K)P}{Dj}} \quad ,$$

which means that the optimal amounts to purchase in advance are the same under policies of constant review and periodic review.

When the time at which the cost increase will occur is not known with certainty but is probabilistic, under certain conditions, simple formulas can be derived for the optimal advance purchase time, as was done in section 3.1.1.2. for the continuous review inventory model. Our method will be slightly different from the one used there, inasmuch as we shall compare the net extra costs resulting from advance purchasing at different points of time with the usual purchasing policy, instead of comparing them with the optimally timed advance-purchasing policy.

We shall restrict the possibility of occurrence of an increase to region OM (See Figure 3-9). As the size of the increase is determined, the quantity that will be purchased in advance is determined: we call it n, expressed in number of periods for which one purchases in advance. We assume that n is large in relationship to M and to Q, the period demand. We call x the time at which the advance purchase is made, and N the unknown time at which the cost increase will take place.

Either the increase in cost takes place before the scheduled time to purchase in advance, or it takes place afterward. In the first case, we lose the opportunity to perform any savings through advance purchasing and incur the usual costs of the hand-to-mouth purchasing policy (at the higher costs). Therefore, the net extra costs, as compared with the usual policy, are 0. In the second case, the purchasing is done before the cost increase incurs; the cost savings are restricted to the amount existing when the increase occurs, $n - (N - x)$, and the extra holding costs, always in comparison to the usual strategy, are $\frac{n(n+1)}{2} QCj$. The net extra costs resulting from advance purchasing are:

$$NEC = \frac{n(n+1)}{2} QCj - [n - (N - x)] QCk$$

The average net extra costs resulting from advance purchasing in the time probabilistic case, are, therefore:

$$\overline{NEC} = \sum_{N=x}^{M} \left\{ \frac{n(n+1)}{2} QCj - [n - (N - x) QCk] \right\} P(N) \qquad (4)$$

If $P(N)$, the probability that the cost increase will occur in

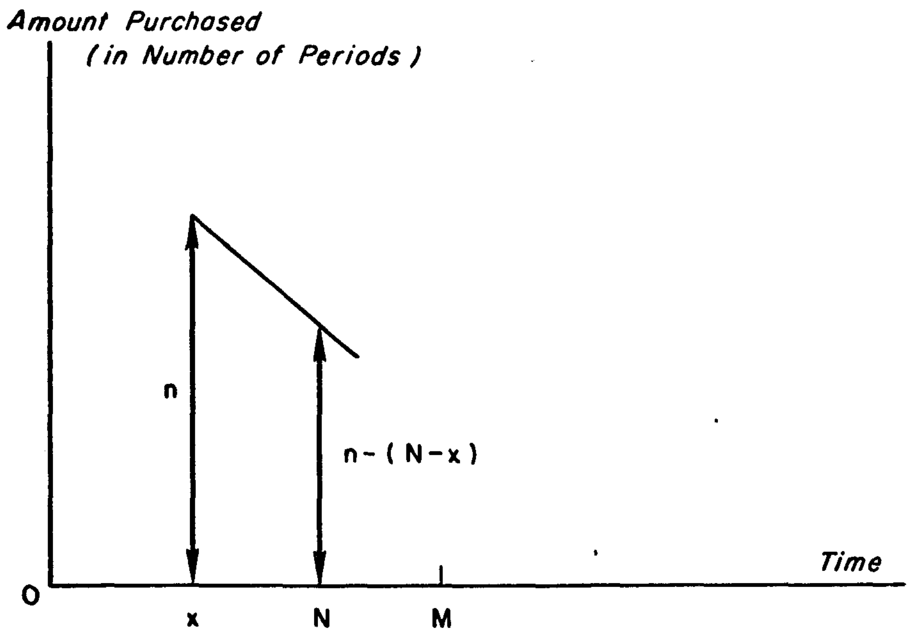

Figure 3-9

Step Cost Increase for Hand-to-Mouth Purchasing Policy, Probabilistic Time Case period N, is a constant b, expression (4) becomes:

$$\overline{NEC} = \left[\frac{n(n+1)}{2} QCj - nQCk - xQCk\right] . (M - x + 1)b + \sum_{N=x}^{M} NQCkb =$$

$$= \left[\frac{n(n+1)}{2} j - nk - xk(M - x + 1)\right] . QCb + \frac{x + M}{2}(M - x + 1)QCkb$$

We differentiate in relationship to x in order to find out the optimal time of advance purchasing which will minimize the expected net excess cost; equalling to zero, we have:

$$\frac{d\overline{NEC}}{dx} = - kM - \frac{n(n+1)}{2} j + nk + 2kx - k + \frac{M}{2} k - xk + \frac{k}{2} - \frac{Mk}{2} = 0$$

Replacing n by $\frac{k}{j} - \frac{1}{2}$ and simplifying, we have:

$$x^{o} = M + 1 - \frac{k}{2j} - \frac{j}{8k} \cong M + 1 - \frac{k}{2j} \tag{5}$$

For instance, if the base period is one month, $k = 20\%$, $j = \frac{16}{12}\%$ per month, and $M = 6$ months, we have: $x^{o} = - .5$ months, which means that one should purchase in advance at the start of the horizon. If $k = 10\%$, the advance purchase ought to be scheduled for the start of the third month. We emphasize that formula (5) applies only when the condition $\frac{k}{j} > M$ holds.

When both the size and the time of the cost increase are probabilistic, we have, from (4):

$$\overline{NEC} = \int_{k=k_1}^{k_2} \left[\sum_{N=x}^{M} \left\{\frac{n(n+1)}{2} QCj - [n - (N - x)]\ QCk\right\} P(N)\right] f(k)dk \tag{6}$$

Limits k_1 and k_2 are set, so that the size of the cost increase satisfies the condition: $\frac{k}{j} > M$. Assuming first that the time density of the cost increase is a constant, we have:

$$\overline{NEC} = \int_{k=k_1}^{k_2} \left\{ \left[\frac{n(n+1)}{2} j - nk - xk \right] (M - x + 1) + \right.$$

$$\left. + \frac{x + M}{2} (M - x + 1)k \right\} QCb \; f(k)dk \qquad (7)$$

In (7), there are two unknowns, n and x. We differentiate first (7) in relationship to n; the derivation can be carried on under the integral sign:

$$\frac{d\overline{NEC}}{dn} = QCb \int_{k=k_1}^{k_2} \left[(n^o + \frac{1}{2})j - k \right] (M - x + 1) \; f(k)dk = 0 \qquad (8)$$

From (8), one gets:

$$\int_{k=k_1}^{k_2} n^o \; jf(k)dk = \int_{k=k_1}^{k_2} (k - \frac{j}{2}) \; f(k)dk = \bar{k} - \frac{1}{2} j \qquad (9)$$

or:

$$\bar{n}^o = \frac{\bar{k}}{j} - \frac{1}{2} \qquad (10)$$

If the density of the cost increase distribution is constant, (10) becomes:

$$n^o = \frac{\bar{k}}{j} - \frac{1}{2} \tag{11}$$

Formula (10) shows that, under the restrictions made, the size of the optimal advance purchase amount, n^o, is independent of the time of occurrence of the cost increase. We differentiate now (7) in relationship to x :

$$\frac{d\overline{NEC}}{dx} = QCb \int_{k=k_1}^{k_2} \left[- kM - \frac{n(n + 1)}{2} j + nk + x^o k - \frac{k}{2} \right] f(k)dk = 0$$

or:

$$\int_{k=k_1}^{k_2} x^o k f(k)dk = \int_{k=k_1}^{k_2} \left[kM + \frac{n(n + 1)}{2} j - nk + \frac{k}{2} \right] f(k)dk \tag{12}$$

Substituting n by its optimal value n^o, given by (11), we have, after simplifying:

$$\int_{k=k_1}^{k_2} x^o k f(k)dk = \bar{k}M + \bar{k} - \frac{j}{8} - \frac{\bar{k}^2}{2j} \tag{13}$$

If we assume that x^o is a function of $\bar{k}$ only, we obtain, from (13):

$$x^o = M + 1 - \frac{\bar{k}}{2j} - \frac{j}{8k} \tag{14}$$

In conclusion, under the rather severe restrictions made in the formulation of this problem, the optimal solution is to purchase in advance the amount n^o given by formula (11), at time x^o, given by formula (14). Both size and amount are functions of the average percentual increase.

3.2.2. Periodic Review Models with Probabilistic Demand

The inherent structure of standard dynamic programming procedures[6] permits us to obtain solutions for periodic review deterministic demand inventory problems containing nonstationary costs as easily as for those which only involve stationary costs.[7] Though each specific problem can be solved with these dynamic programming techniques, they do not provide a formula, like the economic lot size formula, for instance, in which could be incorporated the influence of anticipated costs changes in the inputs in order to compute how much more should be stocked in times of rising costs. The most one can hope to do in this respect is to establish qualitative theorems indicating the directions of the changes in optimal inventory sizes caused by changes in costs.

The difficulty of studying, in a most general way, the effect of

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[6]The method of obtaining optimal reordering levels for dynamic inventory problems is well explained in Frederick S. Hillier and Gerald J. Lieberman, Introduction to Operations Research (San Francisco: Holden-Day, Inc., 1967), pp. 370-396. See also Harvey M. Wagner, Principles of Operations Research (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1969), pp. A19-A50.

[7]Herbert E. Scarf, "A Survey of Analytic Techniques in Inventory Theory," in Multistage Inventory Models and Techniques, ed. by Herbert E. Scarf, D. M. Gilford, and M. W. Shelly (Stanford, Cal.: Stanford University Press, 1963), pp. 185-225.

rising costs on the reorder point and reorder level of periodic review inventory systems has been indicated by several authors.[8] A full treatment of this problem would require considerable space. Our presentation of this subject will be limited to a few numerical illustrations.

Initially, we use the example solved by Hillier and Lieberman[9] for stationary costs and two periods. The cost c of producing an item is \$10 per item. If any excess inventory remains at the end of the period, it is charged a holding cost h of \$10 per item. If a shortage occurs within a period, a penalty or stockout cost of $S = \$15$ is exacted. The density function of demand is uniform and equal to $f(D) = \frac{1}{10}$, and the demand D ranges between 0 and a maximum of 10 units per period.

If the costs are stable over the two periods, the optimal reordering levels are, respectively:

$$y_2^o = 5.42 \quad \text{and} \quad y_1^o = 2 ,$$

for the first period in time (subscript 2), and the second, (and last) period in time (subscript 1).

Reworking the same problem under the assumption that cost c will change from $c_2 = 10$ to $c_1 = 12$, i.e., anticipating a cost

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[8]Arthur F. Veinott, Jr., "On The Optimality of (s,S) Inventory Policies: New Conditions and a New Proof," Journal of the Society for Industrial and Applied Mathematics, XIV (September, 1966), 1067-1083.

[9]Hillier and Lieberman, op. cit., p. 383. We refer to these authors for a presentation of the dynamic programming techniques necessary for the solution of periodic review inventory problems. Some of their symbols have been changed in order to maintain uniformity in our text.

increase of 20% in materials from one period to the other and letting all other costs constant, one finds:

$$y_2^{o'} = 5.6 \quad \text{and} \quad y_1^{o'} = 1.1 \quad .$$

We see that one ought to stock a larger amount in advance of a cost increase in materials and a smaller amount after the cost increase has taken effect than when costs are stable.

For the case of 3 periods and stable costs, one would find:

$$y_3^o = 6 \ , \ y_2^o = 5.42 \ , \ \text{and} \ y_1^o = 2 \quad ,$$

and one can show that, for an infinitely large number of periods, with stationary costs, the value of the optimal reordering level y_n^o tends toward an asymptotic value, which for this example, is equal to 6; besides, the sequence of optimal inventory levels is monotonic.[10]

For the case of 3 periods, in which costs are assumed to increase successively according to the values:

$$c_3 = 10 \ , \quad c_2 = 12 \ , \quad c_1 = 14 \ ,$$

one finds:

$$y_3^{o'} = 6.4 \ , \quad y_2^{o'} = 5.6 \ , \quad y_1^{o'} = .4 \ ,$$

and we see again that one should increase the stock level in advance of

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[10]Kenneth J. Arrow, Samuel Karlin, and Herbert Scarf, Studies in the Mathematical Theory of Inventory and Production (Stanford, Cal.: Stanford University Press, 1958), Chapter IX. See also Hillier and Lieberman, op. cit., pp. 386-387.

increases of costs of materials.

We take now another example, with a step increase in materials costs and 4 periods. The data are: $c = \$10$; $D = 1{,}000$ units; $f(D)$ is uniform between 0 and 1,000; $S = \$30$; and $h = \$5$. Initially, costs are supposed stable, and one finds:

$$y_4^o = 857 \ , \quad y_3^o = 854 \ , \quad y_2^o = 844 \ , \quad y_1^o = 571$$

Without changing any of the preceding figures, except the cost in the last period in time, which is anticipated to become $c_1 = \$20$, we have the following optimal reordering levels of inventory:

$$y_4^{o\prime} = 868 \ , \quad y_3^{o\prime} = 903 \ , \quad y_2^{o\prime} = 931 \ , \quad y_1^{o\prime} = 286$$

One can indeed show theoretically that y_2^o is a growing function of c_1, so that the inventory level y_2^o, to be maintained before a cost increase in materials to c_1, might become higher than the asymptotic value of the stable costs situation. When material costs are increasing over time, the sequence of optimal inventory levels does not tend always monotonically toward an asymptotic value but reaches a peak, corresponding to an advance purchasing or scheduling, before it tends to its asymptotic limit. Figure 3-10 represents this.

The preceding examples referred to situations where the costs for reordering or rescheduling in each period were zero. When there is a setup cost K, assumed constant over time, the optimal periodic review inventory policies are of the (s, S) type, the symbol s meaning the reordering point, and the symbol S standing for the reordering level. (We have previously used the symbol S to mean stockout cost. S, with

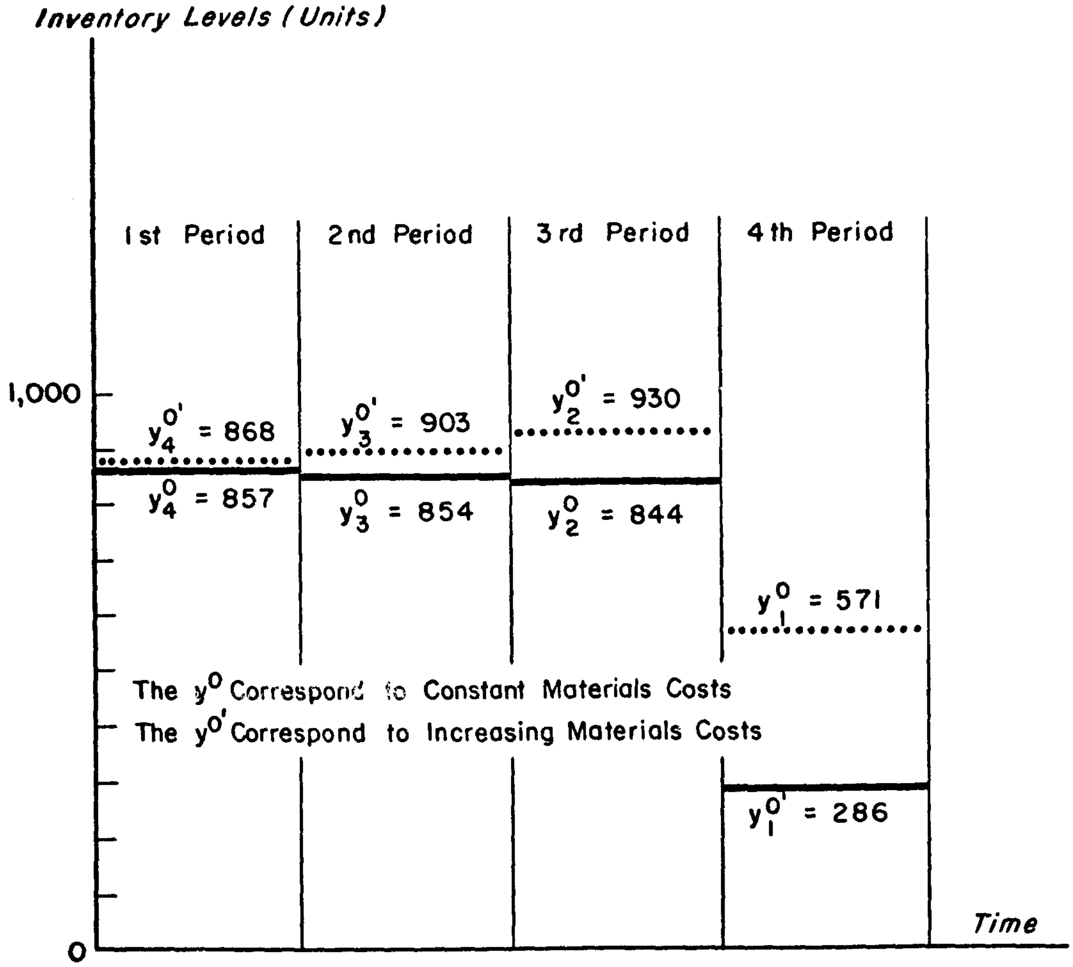

Figure 3-10.

Reordering Levels of Periodic Review Inventory Models, with Constant and Increasing Costs of Material the meaning of reordering level, is only used here within the parenthesis, together with s , and should not be mistaken for stockout cost. In the text, y^o has been used to mean reordering levels.)

In order to study the changes in the reordering point, occurring as a consequence of anticipated costs increases in materials, we must compute the expected costs resulting from the selection of all possible reordering levels in each period. This is shown in Figures 3-11 and 3-12 for the third (in time) and the last (in time) periods of the previous numerical example, respectively.

The curve shown in full line in Figure 3-11 represents the cost $G_2(x_2)$ incurred when one adopts the reordering level stated in the abscissa. For instance, an inventory level of 844 units corresponds to the optimal cost of managing inventories, which includes materials, holding and stockout costs for the two last (in time) periods and which amounts to $6,651. This, for a x_2 of 500 units, i.e., for a starting inventory level, at the beginning of the third period (in time) of 500 units. Another inventory level for this third period would give us another total inventory cost: for instance, if y_2 were 500, we read from Figure 3-11 that the total inventory cost would be $8,773.

The dashed line represents the cost $G_2(x_2)$ incurred when one adopts the reordering level stated in the abscissa, for the inflationary assumption that material cost would rise to $20 in the last period in time. For instance, when the inventory level in the third period in time is 931 units, the total inventory costs for the two last periods are $12,939, corresponding to minimum costs, for an entering inventory level of x_2 = 500 units. Each entering inventory x_2 would give a

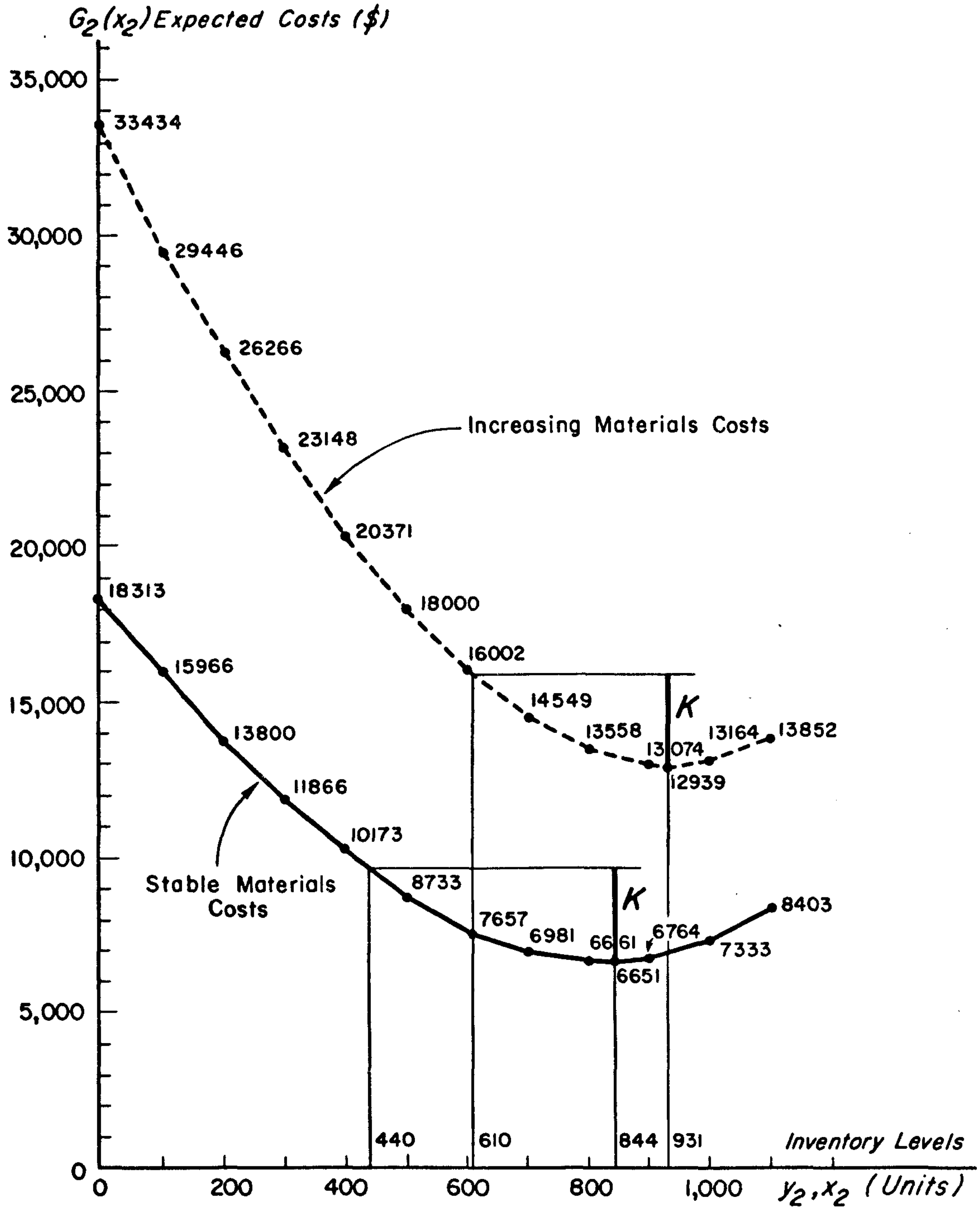

Figure 3-11.

Expected Costs of Reordering Inventory Levels for the Third Period of Example

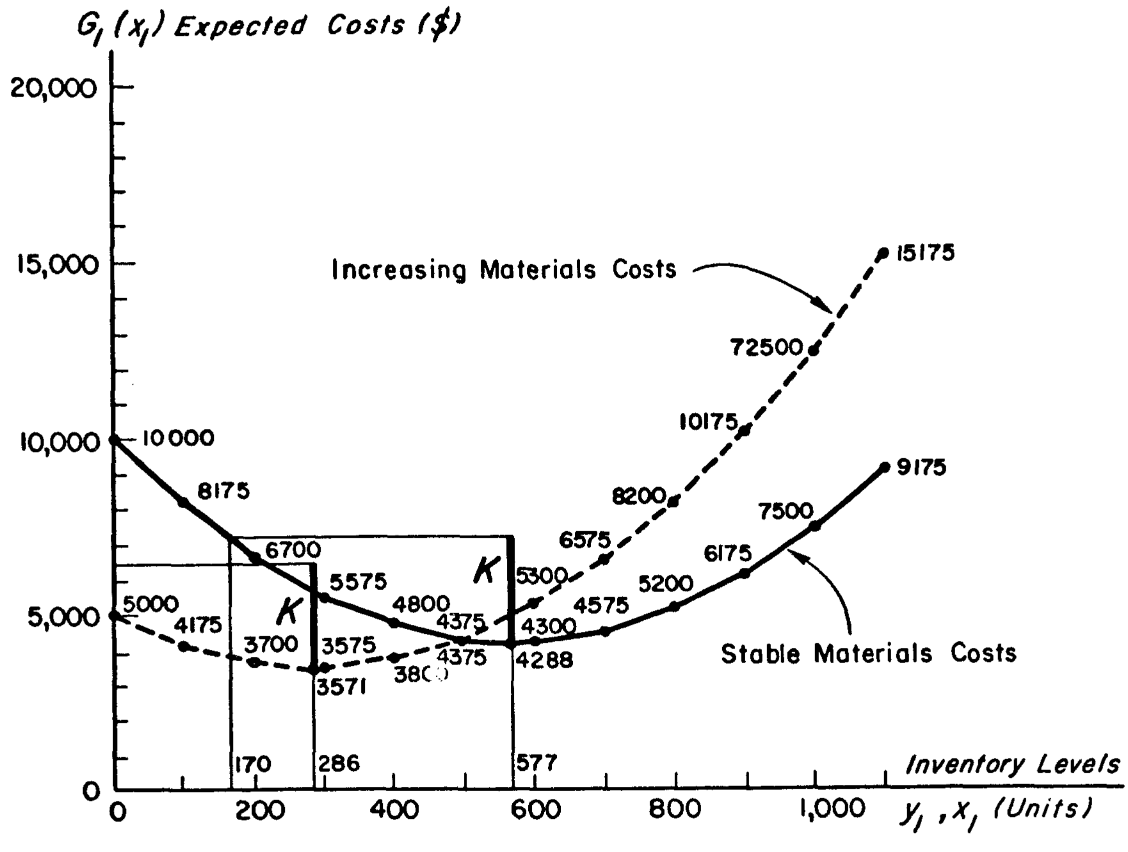

Figure 3-12.

Expected Costs of Reordering Inventory Levels for the Fourth Period of Example

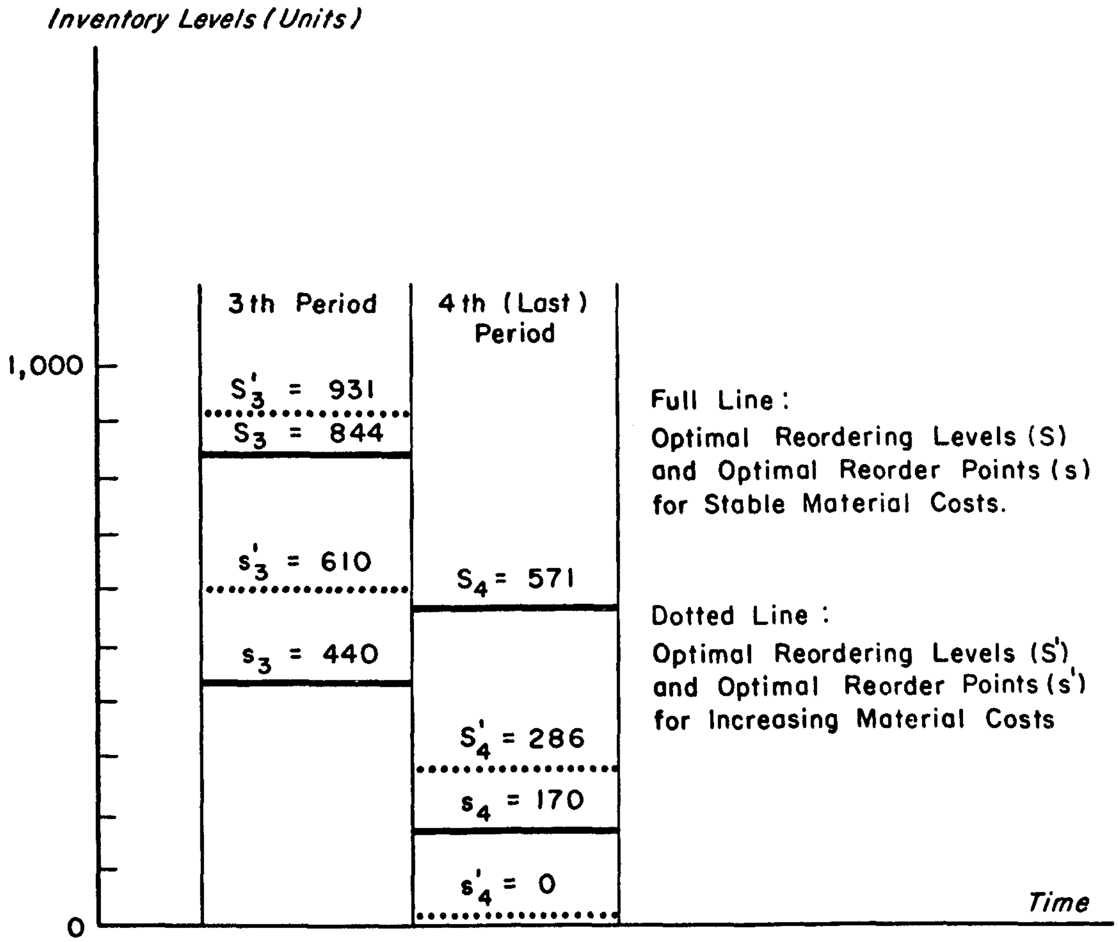

Figure 3-13.

Reordering Levels and Reorder Points for Stationary and Increasing Costs of Material similar cost curve, which would be a vertical translate of the curve shown in the figure.

Figure 3-12 represents inventory costs $G_1(x_1)$ for the last (in time) period, and for the stable cost (full line) and inflationary cost (dashed line) case, respectively; entering inventory in last period is x_1.

Now, if setup cost $K = \$3{,}000$, we see, from inspection of the full line curve of Figure 3-11, that if the initial inventory x_2 had been 500 units, it would not be advantageous to reorder up to the level 844, because, by so doing, one would incur a total cost of \$6,651 + 3,000 = \$9,651, larger than the cost incurred by staying at a level of 500 units, \$8,773. Figure 3-11 shows that the reorder point, that is, the inventory level under which it is adequate to reorder more stock, is approximately 440 units for the stable cost case and 610 units for the inflationary cost case for the third period in time. Figure 3-12 indicates, similarly, for the last period in time, that the reordering points are 170 and 0.

We see that the anticipation of a cost increase changes the distance between reordering level and reordering point. In this example, the distance has decreased in both the third and the last periods.

This result is recorded in Figure 3-13, which shows the s-S levels for the two periods focused, under stable cost and inflationary conditions. The anticipation of a cost increase makes us stock at a higher level in the period preceding the cost increase and makes us initiate a reorder in that period even when the entering inventory is already quite high.

3.3. Potential Savings

In order to appraise the potential savings which can be obtained from the use of the formulas derived previously, some examples will be given:

1. Assume $D = 1{,}000$ units, $C = \$1.00$, $P = \$5.00$, $i + a =$ 16% p.a. When costs are stable, the EOQ is found to be 250 units. If a cost increase of $I_1 = 10\%$ is foreseen for the year, the total inventory costs for an EOQ of 250 units are \$1,077.5 .

Using formula (5), section 3.1.1.4., we find: $Q^o = 408$ units, and the total inventory costs are \$1,076.0 . Computed as a percentage over the sum of administrative, financial, and inflationary costs, which amount to \$77.5, we obtain percentual savings of $\frac{1.5}{77.5} = 1.9\%$.

2. Same data as before, but I_1 is now anticipated to be 20% per year; then $Q^o = 1{,}000$, with costs of \$1,085. If the firm had persisted with a lot size of 250 units, its costs would have been \$1,115. The percentual savings in this case are: $\frac{30}{115} = 26\%$.

3. Same data as before, except that now $I_3 = 20\%$ and $I_1 = 11\%$. The traditional formual (7), section 3.1.1.4., gives: $Q^o = 167$ units; the correct formula, (5), section 3.1.1.4., gives $Q^o = 200$ units; costs are respectively \$1,106 and \$1,105. Savings are $\frac{1}{106} = 1\%$.

4. Assuming the same data as in 3. above, if the firm had shifted to a hand-to-mouth purchasing policy, ordering 20 units at a time, which corresponds to one purchase per week, the cost would be: \$1,159. The savings obtained from the use of the correct formula and a Q^o of 200 units would be: $\frac{54}{159} = 33\%$.

5. If the firm adopts hand-to-mouth purchasing of 20 units per week and decides to purchase in advance of a 10% forthcoming step cost increase, the optimal amount to purchase for $i + a = 16\%$ p.a. is: $n^o = 31$ weeks, computed from formula (2), section 3.2.1. Ignoring the setup costs savings, annual percentual savings in inflationary and holding costs are: $\frac{30}{102} = 29\%$.

6. As a further example, consider the following situation: $D = 100{,}000$ units, $C = \$1.00$, $P = \$70.00$, $i + a = 16\%$ p.a. The traditional EOQ formula gives EOQ = 9,350 units, which corresponds to approximately one purchase a month. If a 12% cost increase in materials is forecasted, and one still purchases once a month, the cost will be \$107,007. If one uses the correct formula, which gives a Q^o of 18,700 units, corresponding to a purchase approximately every two months; and if one buys every two months, the cost becomes \$106,753. The percentual savings over controllable costs are $\frac{254}{7{,}007} = 3.6\%$.

If the forecasted cost increase had been 18% per year, the yearly costs for a monthly purchase would have been \$109,757. The correct formula gives $Q^o = 100{,}000$ units, with costs of \$108,070. Percentual savings on controllable costs are: $\frac{1{,}687}{9{,}757} = 17\%$.

If the manager uses a minimax-cost decision rule, instead of a minimum expected cost decision rule, the methods presented in this study can also be utilized in order to compute potential savings. As an example, assume that demand for some material could be either 750, 1,000, or 1,250 units per year; a step cost increase of 11% is announced by the vendor to be effective next day; management believes that the actual cost will be either 11%, or 5%, but could also be 17%. The firm follows a hand-to-mouth purchasing policy and buys 20 units a week. Unit cost C is \$1.00, and interest and storage costs $i + a$ are 16% per year. Management computes the convenience of doing advance purchasing.

The maximum savings attainable with advance purchasing are obtained if demand were going to be 1,250 units and the step increase were going to be 17%. The optimal amount to purchase in advance would be 1,250 units, and the savings in relationship to hand-to-mouth purchasing would be:

$$1{,}250 \times 17\% - \frac{1{,}250}{2} \times 16\% = \$112.5$$

If demand turned out to be 750 units and the effective cost increase 5% only, the excess cost resulting from an advance purchase of 1,250 units in relationship to hand-to-mouth policy would be:

$$500 \times 16\% + \frac{750}{2} \times 16\% - 1{,}250 \times 5\% = \$77.5$$

According to whether he is more sensitive to a potential gain of \$112.5 or to a potential loss of \$77.5, the manager would decide in favor or against advance purchasing.

A further reason ought to be mentioned to help in explaining why managers are often reluctant to increase their inventories through forward purchasing in advance of increases in costs: the classical EOQ formula does not take into account the discounted value of money. The cost of purchasing a unit is considered to be the same whether it is purchased at the start or at the end of the year. This seems justified for practical purposes so long as the nominal interest rate is small. If nominal interest rate is large, discounting becomes important. By using a technique similar to the one used in section 3.1.1.3., the traditional EOQ formula, equation (2), section 3.1., can be shown to become:

$$EOQ = \sqrt{\frac{2DP}{C(2i + a)} (1 - \frac{i}{2})}$$

and equation (5), section 3.1.1.4., becomes:

$$Q^o = \sqrt{\frac{2DP(1 + \frac{I_2}{2})(1 - \frac{i + I_3}{2})}{C(2i + a + 2I_3 - I_1)}}$$

The conclusions reached earlier on the effect of inflation on the size of inventories remain the same, when discounting is taken into consideration, but the absolute effect of discounting is to decrease inventory sizes considerably.

We see that the savings resulting from the utilization of optimal scheduling constitute a percentage of controllable costs that is a function of the size and nature of the forecasted cost increases. Optimal scheduling of purchases and production, taking future costs increases into consideration, can contribute to cutting down controllable costs at least some percentage points in most cases. Savings are generally larger for jump increases than for continuous increases; they are not a linear function of the size of the increases in costs: a small increment in cost increase at some point can result in a large increment in potential savings.

The use of formulas which take anticipated costs in inflationary periods; an equally important conclusion is the existence of inflation must be no cause for discarding economic lot size formulas; even when traditional EOQ formulas are used, without inflation correction, costs will be smaller than when not using any cost minimization formula at all.

CHAPTER IV

RESULTS OF THE FIELD STUDY

This chapter incorporates the results of a field study undertaken by the author between July and December 1970. Interviews were conducted with business executives in 34 companies--17 in Brazil and 17 in the United States--concerning the influence of price changes on inventory policies.

The sample of firms was based upon convenience. It is not to be regarded as representative. The essential purpose of the study was to obtain knowledge about utilization of economic considerations in inventory management. For obvious reasons, the choice of firms was biased so as to include mostly large and well-staffed companies.

The main characteristics common to the interviewed firms were the following:

- they were large units in their respective sector, regardless of whether the criterion was market share, number of employees, or sales volume;

- all were expanding, profit-making companies, even though the survey was conducted at a time when practically all business sectors--both in Brazil and in the United States--were facing serious difficulties;

- all were essentially manufacturing companies, although many were also engaged--directly or through subsidiaries--in other activities, such as farming, mining, and shipping;

- most were part of multinational organizations and were internationally known; the majority operated in several countries.

According to the main sector in which they operated, the firms can be classified as follows:

- food and beverages: 12 firms;
- chemical, cosmetics, and pharmaceutical products: 10 firms;
- electronics, electrical products, machinery, steel: 7 firms;
- textile and apparel: 2 firms;
- paper: 3 firms;

total: 34 firms.

Only 6 of the companies interviewed had less than 1,000 employees; the smallest had 400 employees. Based on the whole organization to which the division or establishment interviewed belonged, 4 had between 1,000 and 5,000 employees, and the remaining 24 had more than 5,000 employees.

Of the companies interviewed in the United States, most were based on the West Coast. In all, 23 firms were American, most of them with foreign divisions. Of the 17 firms interviewed in Brazil, 11 were subsidiaries of foreign companies, 6 of which were American, and 5 European; 6 enterprises were Brazilian.

Twenty-four companies reported having a department, or a "team," of operations research or of management science. The positions held by the officials interviewed were the following: 6 presidents, or main executive officials; 19 vice-presidents in charge of finance, controllers, or assistant controllers; 3 divisional vice-presidents; 2 purchasing vice-presidents; 1 vice-president of manufacturing; 1 vice-president of corporate planning; and 2 heads of planning and analysis departments. No deliberate effort was made to interview more than one officer in each company, although, in ten firms, it so happened that several executives were interviewed, the most highly placed being the one mentioned in the list just given, and the others, inventory managers. A typical interview lasted about one hour.

4.1. Immediate Economic Conditions in Brazil and the United States

Before presenting the results of the survey, a few words should be said concerning economic conditions in the countries in which interviews were held. In the main characteristics of importance to this study, namely, evolution of price-level changes and of interest rate, Brazil and the United States were strongly similar at the time of the interviews, because both were engaged in a systematic fight against inflation. Attempts to control inflation had been reinforced from year to year in Brazil since 1964 and were initiated in the United States in early 1969, one year and a half before the start of the inquiry. In Brazil, the rate of inflation was about 20% per year in 1970; in the United States, it was about 5% p.a. In both countries, the real interest rate charged to firms was perceived as being unusually high, in comparison with the usual rate of previous periods.

Due to these similarities, and despite considerable differences in the causes, size, duration, and regularity of inflationary movements, it seems justified to make a joint presentation of the results of interviews made in two separate countries. Actually, inasmuch as many companies were international, regional differences were somewhat lessened by the worldwide corporate policy of many firms, which further encouraged aggregation of the results of this survey conducted in both Americas.

In the United States, the years 1968-1970 were marked by an inflationary surge unparalleled since the Second World War, excepting the inflationary upshoot of 1951, which took place during the Korean War. From early 1969 on, the administration tried to exercise indirect control of the inflationary trend by slowing down the economy; the money supply was restricted, the Federal Reserve Board's discount rate was raised; the government committed itself to a balanced budget; many governmental expenses, especially in aerospace and defense sectors, were cut down. Table 4.1 and Figure 4-1 show relevant data for this period.

In the second half of 1970, when the interviews were held, the rate of inflation, measured by changes in GNP deflator, was showing a decrease from its 5.6% peak, reached in mid-1970, toward a value of less than 5% for the second semester of 1970. But the fight against inflation, which had been waged for two years, since its inception in early 1969, had brought a decline of 1% in the real gross national product for 1970 as a whole, as compared with 1969. The unemployment rate reached the 6% level at the end of 1970. A large number of corporations reported losses or profit declines for 1970. The prime interest rate, which had reached a 8.5% peak in early 1970, started to decline in

TABLE 4.1.

SELECTED INTEREST RATES AND MEASURES OF INFLATION RATE

U. S. A. 1960-1970

Year	(1) Prime Commercial Interest Rate at Start of Year	(2) Finance Company Paper, Annual Average	(3) Annual Change in Wholesale Price Index, All Commodities	(4) Annual Change in Gross National Product Deflator
	(All Rates in Percent Per Year)			
1960	5.0	3.54	.1	1.7
1961	4.5	2.68	-.4	1.3
1962	4.5	3.07	.3	1.1
1963	4.5	3.40	-.3	1.4
1964	4.5	3.83	.2	1.6
1965	4.5	4.27	2.0	1.9
1966	5.0	5.42	3.3	2.7
1967	6.0	4.89	.2	3.0
1968	6.0	5.69	2.5	4.5
1969	7.0	7.16	3.9	4.7
1970	8.5	7.68	3.6	5.2

Sources of Data:

1. Standard & Poor's, Trade and Securities Statistics (New York: Standard & Poor's Corporation, 1970).

2. Survey of Current Business, passim.

3. Data were calculated from wholesale price index and gross national product deflator figures published in Survey of Current Business, passim.

4. Ibid.

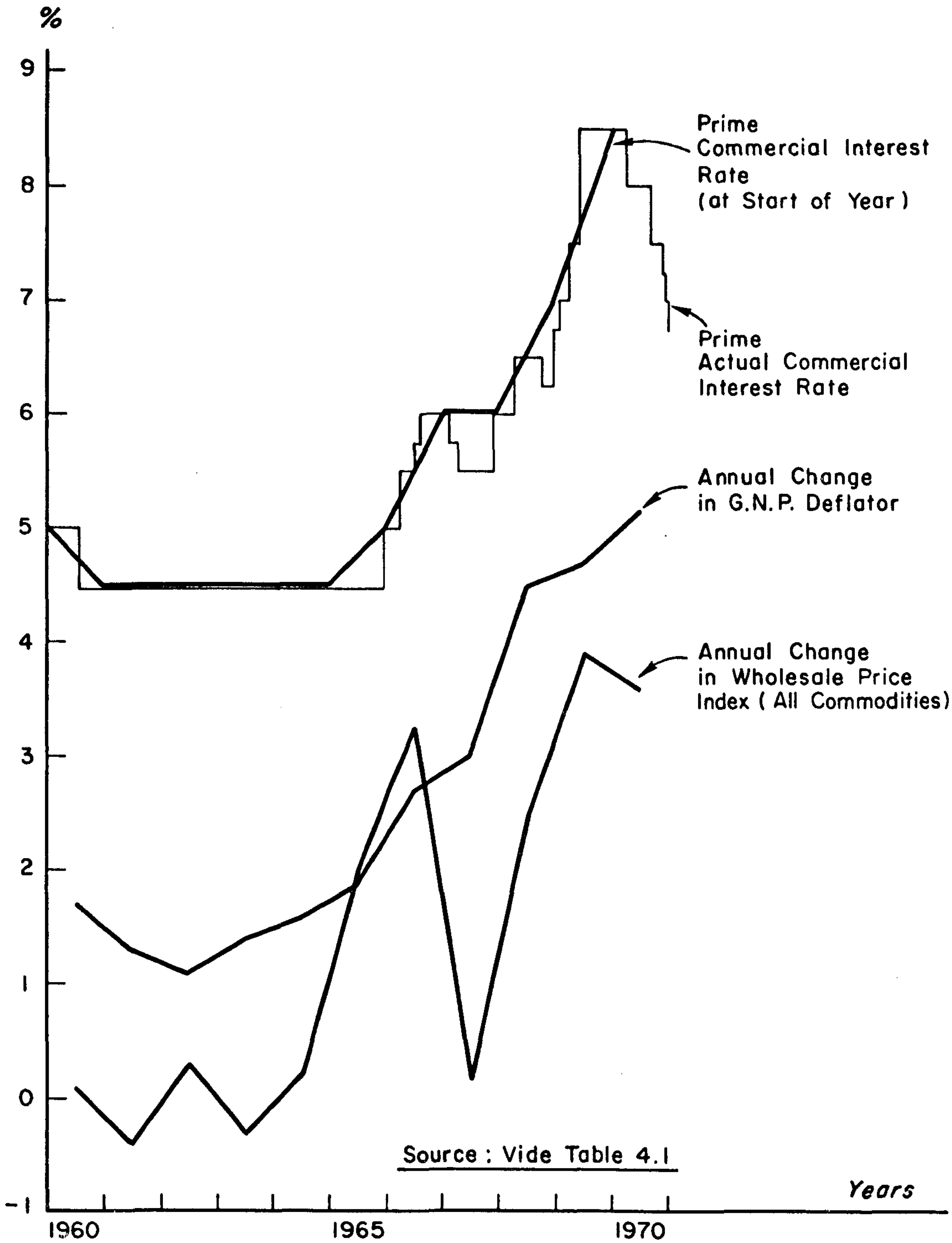

Figure 4-1.

Evolution of Inflation Indexes and Interest Rate,
United States: 1960-1970

mid-1970, due to the recession and faltering demand for money.

It may be observed that the economic situation of the United States in the second half of 1970 typified what was described in the introductory chapters as "controlled inflation," or "disinflation." Prices were rising at an annual rate of 5 to 6%, but, if it is recalled that the effective interest rate was 10 or 12% p.a. (due to compensating balances kept at the banks), it may be seen that the real cost of money was 5 or 6% p.a.--a rather high rate for the United States. Many industries had difficulties in raising their prices, not so much because of the rather mild governmental exhortations, called "inflation alerts," but because of severely reduced demand for their products.

In a major policy reversal, in December 1970, the administration opted for easing up inflationary controls, planned a deficit ("full employment") budget for fiscal year 1971-72, created some business tax incentives, and tried to bring about a revival of the economy. At the end of December 1970, when the interviews were concluded, the economic situation had still not changed appreciably and could continue to be characterized as an environment of controlled inflation.

In Brazil, the period 1968-1970 had seen an unprecedented growth in real gross national product, which climbed at a rate of 9% per year, following eight years, 1960-1967, of stagnation or of slow growth per capita. Inflation, which had been fluctuating in the 10 to 20% per year range during the 1940's and 1950's, accompanied by appreciable prosperity and real growth, had accelerated in the early 60's. In the first quarter of 1964, it reached the annual rate of 130%, accompanied by paralysis of foreign investments and levelling of the real GNP. From 1964 on, the government established as one of its fundamental economic goals reduction of the rate of inflation to 25% a year, to be followed later on by a further reduction to 10%.

The 25% target was reached in 1966, mainly through use of fiscal policy--giving tax cuts to firms which increased their prices only moderately--and through a severe monetary policy--reducing and even stopping completely for months at a stretch the issue of new currency and of other means of payment. Foreign investments were encouraged again. Extensive tax reform took place. Index-price accounting became legal. During the disinflationary period, from 1964 to 1967, a considerable economic crisis occurred. Most marginal firms, unprepared for reduction in their activities and unable to adapt to a situation in which almost monthly price increases could not take place, any longer, faced bankruptcy, sold out to foreign capital, or were absorbed by the government. Underemployment of resources became considerable, and average capacity utilization of industry fell to 60%.

During the interviews, in July-August 1970, the Brazilian economic situation, like that of the United States in the same period, was typically one of "controlled inflation." The rate of inflation was about 20% a year, materials increases being about 18% a year and wage increases averaging 25%. The interest rate for short-term loans and the discount rate for commercial paper were in excess of 30% for most companies and transactions. The cost of money was, therefore, higher than the price-level change rate would justify.

Any intended price increase had to be communicated beforehand to an Interministerial Commission of Prices, accompanied by detailed cost justifications. The Commission normally delayed price increase concessions by 3 to 6 months and conceded only part of the price increase asked for. On the average, no more than two price increases per year were conceded for each product, in contrast to the almost monthly increases common in the early 60's.

4.2. Questions and Answers

In presenting results of the interviews, the general trend of answers to each question asked will be indicated. Some typical comments made by interviewees will be included along with a few comments of the author.

1. The first question asked was: "Which methods do you use in order to determine the levels of your materials inventories?"

The justification for asking this question derives from the following: a firm is more likely to take price changes and interest rate into consideration in its inventory policy if it uses quantitative methods, especially quantitative methods of an economic nature, that is, those in which cost considerations, as opposed to purely physical considerations, play an explicit role.

The majority of respondents, 25 out of 34, answered that they did not use any economic method, and, instead, relied basically upon physical considerations, such as size of outstanding orders or production and storage constraints, which completely determined inventory levels.

Of the eight positive answers, four were given in Brazil and four in the United States. One executive said: "We use economic order quantity formulas for all raw materials and parts; we have a rather elaborate computer program which our engineers started two years ago." (Cosmetic products) Another reply was: "We have just started using EOQ, now that we have a centralized materials department." (Heavy machinery) Two other answers were that the lack of funds prevented using economic order quantities, except for a few items. (Pharmaceutical products, paper) In the United States, one company declared that, to some extent, it used EOQ in purchasing. Another stated: "We use EOQ sparingly, because, for most expensive materials, we get yearly contracts." A third American company declared that it used EOQ for many parts. These three companies were all manufacturers of electronic products. A fourth company, a manufacturer of machinery, said that it utilized the EOQ formula for minor items but not for expensive parts. Still another firm (oil) set up an economic model in order to minimize operational costs, but inventory costs in this model seemed to be secondary to production and transportation costs, and inventory levels were determined as "by-products" of the refineries' and tankers' operations.

The following quotations illustrate the kinds of reasons invoked by executives for not using an economic approach to materials inventories policy. "We have to buy all our cotton at crop time, otherwise we will never find the required quality. All is dependent of our production needs. We cannot find anybody who wants to carry our cotton inventory for one year." "We do not have to carry inventory; our supplier does it for us, and we can get delivery in a couple of days." "We do not have to use the formula, since we have yearly purchasing contracts at a fixed price or approximately at a fixed price." "Batch capacity is our real determinant," was the tenor of more than one answer. "We tried to use scientific rules, but there is too much demand fluctuation," sighed an official. "Our product is just too seasonal," explained an ice-cream manufacturer, "and inventory accumulation in more freezers is out of the question. Our raw materials cannot be stored long either." Various administrators pointed to the tendency of EOQ formulation to lead to a higher level of inventories than a periodic review policy does and declared that financial constraints obliged them to follow an inventory minimization policy rather than a strategy of cost minimization. Physical storage constraints, transportation constraints, and obsolescence were also negative factors mentioned.

Whenever the inventory level is not dictated by physical imperatives and constraints, it seems that the system of purchasing used by most companies for important items and, moreover, advocated by executives as the "best," is a periodic review method. It works as follows: the review time interval is established empirically, based upon the company's experience, and can be as low as a few days for expensive materials. The review time is flexible and accompanies usage fluctuations. The amount to reorder is usually chosen as "the minimum compatible with safety." The reserve level--if any--is not established on the basis of statistical demand, probability of stockout, nor balancing of the opposite costs of having and not having inventory. Some crude consideration is given to physical distance from the supplier. Reserve levels, when they exist, show fixity, while amounts to reorder are variable.

Advocates of the EOQ formula pointed out that the popular empirical system of purchasing just described is costly in clerical expense, because reviews of inventory position are carried out so frequently. But clerical time is a cost element overlooked by executives, especially in comparison with the cost of carrying stock.

A few managers who seemed to be willing to use the EOQ method stressed the difficulties in estimating proper values for the setup cost of purchasing, as well as for the cost of holding inventories. These uncertainties delayed or prevented introduction of the EOQ method.

Except for one U. S. firm (electronics), which used a 50% p.a. money cost rate for holding purchased inventories in its EOQ formula, all firms of the sample used prime interest rate. When other costs, like storage, are added, the total cost of holding inventories ranged from 15% to 25% p.a. in the U. S. and were about 45% p.a. in Brazil.

2. The second question was: "Which methods do you use in order to determine the inventory levels of your work-in-process?"

Only 5 our of the 34 respondents said that they used an economic formula in order to determine the optimal size of manufacturing lots.[+] They referred to their method as the "textbook" economic lot size of fabrication. All five firms were located in the United States. Two were manufacturers of electronic goods, one of which made extensive use of the ELS formula and the other of which made some usage but with admittedly "not satisfying" results. The third was a wine and liquor producer, who

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[+]An oil company and a food manufacturer had computerized linear programming models, which optimized some elements of their manufacturing operations but did not deal with the size of the lots and, therefore, did not relate to the inventory problem.

resorted to the ELS formula in order to determine the size of lots which should be bottled as a batch. The fourth was a drug company, in which the ELS method was applied to compute "pharmaceutical runs." The fifth was a steel mill, which utilized a computer program to schedule sequence and size of runs.

The majority of enterprises which did not utilize an economic formula to justify the nonutilization of an economic attempt to optimize their batch sizes presented the same arguments as they did in reply to the first question. Either the fabrication was based upon outstanding orders, or "economic lot sizes would be too big for us," or physical constraints, such as storage and production capacities, governed.

3. The third question read: "Which methods do you use in order to determine the level of your finished-goods inventories?"

Two situations obtained here, depending upon whether or not the firm had its own system of distribution. Those who possessed their own distribution systems maintained inventory at their plants and shipped products at periodic intervals to regional warehouses, which in turn shipped to wholesalers and retailers. The problem was how much to stock ideally at plants and at warehouses?

Some companies established maximum inventory levels to be maintained at their regional distribution centers and replenishment levels, which, whenever reached, triggered a new shipment to the depleted distribution center. All firms in the sample resorted exclusively to physical considerations--such as average demand, demand dispersion, distances from plant, and storage constraints--in order to fix the maximum and replenishment levels of their distribution system. Economic optimization, taking into account cost of a stockout and carrying cost, was not mentioned by any of those interviewed.

An oil company and a canned food company established a 98% level of protection at their terminals and warehouses against depletion of their products. But this figure was based strictly upon physical or judgmental considerations and was not the result of an explicit calculation involving stockout and holding costs. An American drug company declared: "Our objective is to carry a 90-day supply at each distribution center, depending also on the distance to our plant."

For many food processors, the amount of finished goods inventory was stated as being a given datum, depending solely on acreage contracted months earlier and on crop yield. No economic considerations needed to be taken into account nor could they be.

The large majority of those companies which did not operate their own distribution system said--especially in Brazil--that they tried to keep their finished goods inventory at the minimum possible level. In other words, whatever existed as finished goods inventories for these firms was unintended accumulation of stock.

Only one company (pharmaceutical) reported that it was contractually obliged to carry a large amount of finished goods inventory for their main clients. A steel company also mentioned the need to carry a permanent inventory of tin plate for can manufacturers. Whatever stock of finished goods these companies carried was not determined by economic computation.

Many an executive pointed out that the written policies of their firm, or of the purchasing department, stated that inventory should "be kept at the minimum level compatible with safe operations" of the company. No written policy mentioning cost minimization of inventories was exhibited.

For the vast majority of managers interviewed, whatever their function, the concept of stock minimization seemed to be a clear and sound one and (except for some hedging against price increases of materials) an acceptable goal, while the concept of cost minimization of inventory was not identified nor acknowledged. Besides, in many cases, even if cost minimization had been accepted as a feasible objective for inventory policy, practical difficulties in implementing it would have constituted a decisive obstacle to its adoption; most frequently mentioned was the difficulty in determining the cost of placing an order, the cost of holding inventory, and the stockout cost. Fluctuations of demand were also constantly remembered as a serious difficulty for any quantitative approach to the inventory problem.

Still, it can be asked whether difficulties were not somewhat exaggerated by interviewees. Defending economic lot size formulation, a financial manager observed that any error in determination of economic parameters would represent a much less serious mistake than altogether rejecting an economic approach.

A further reason for lack of economic thinking about inventories lay in the failure of companies interviewed to charge interest cost on carried inventories to operating divisions. Storage cost, whenever charged, was fixed. An operations man observed: "We are not really under pressure from the finance division in terms of bringing interest costs down. Maybe in five years from now, they will think of that. For the time being, we put pressure on ourselves to make a good performance with small inventories. I am sure that we could do as well with half our present level of inventories. But they [the top management] are extremely anxious over the possibility of any stockout. Stockout cost, as estimated, is much too high."

Much performance reporting was traditionally done in terms of inventory turnover rather than in terms of inventory costing. This probably contributed to thinking about inventory minimization rather than about cost minimization.

4. The next question was: "Did the inflationary situation originate any modification in your inventory policy?"

The predominant answer, for both Brazilian and American firms, was that inflation, if anything, had strengthened efforts to keep inventories at a minimum. It must be stressed again that the interviews were conducted during a period of controlled inflation in both countries. Interviewees, therefore, had reference to an economic environment marked by a high interest rate and repressed, though substantial, cost increases.

In Brazil, the situation of controlled inflation had existed for three years in an extreme form and, before that, for three other years in a less intense form. As a result, inventories were reported to have reached extremely low levels. Brazilian executives were talking about bare minimal, literally hand-to-mouth inventories. It should be observed that this situation of disinflation was accompanied, for the three years of 1968-70, by an economic boom. Nonetheless, inventories were aimed at absolute minima.

Typical answers heard in Brazil follow: "We always keep a minimum inventory; now, more than ever." (Food processor) "Ten days of finished product. Zero days of raw materials." (Chemicals) "Our stock of finished goods is now 5-10 days; 7 years ago, it was 30-50 days." (Food processor) "We eliminated all local warehouses. Rush transportation is now cheaper than to have local inventories." Indeed, all pharmaceutical companies had abolished their elaborated local warehousing system from 1967 on. Aside from the cost of carrying inventories, two other reasons for the move were improvement in the freeway system, with resulting decrease in transportation costs, and a change in the taxation system, making it 2% more expensive to transship goods through regional warehouses. "Fifteen days of stock versus 3 months 2 years ago. It seemed impossible, but we did it." (Cosmetic products) "Stocks are minimum now, as in the U.S.A. The situation has become quite different from before [This is a thoughtful reference to the easier pre-1964 period.] We have days of sugar inventory, a few weeks of cacao." (Ice-cream and candy manufacturer)

In the United States, the following comments were received: "Our stock of scrap [this was a steel factory] is down to less than a week. Constraints are always stronger. It is a very interesting challenge, almost a game, to manage, year after year, with less and less inventories. We used to have excess inventory of refractory bricks. Now all surplus has been cut."

"High interest rate sure hurts us; now, as far as raw materials are concerned, nothing can be done; but, in work-in-process and in finished goods, that is, in our distribution system, we must become scientific." (Beverages) "Higher interest rate in the EOQ formula makes stocks decrease automatically." (Food processor) "We stock parts instead of finished products. We have also eliminated all price breaks for quantity discounts, except for a few items." (Electronics) "Inventories were not a major concern. They are becoming so now." (Pharmaceuticals) "We are going to start to squeeze inventories. Our production people have suggested it themselves in order to help us in the present financial strain. Still, inventories are not the name of the game in our business." (Pharmaceuticals) "The first thing we have cut is fixed capital investment. The second thing is inventories. The last thing we want to cut is research." (Electronics)

5. The next question was: "Do you use advance purchasing when your vendor announces an imminent increase in his prices?"

Three positive answers came from Brazil and six from the United States. But all of these answers were qualified by restrictions, such as "rarely," "it all depends," "not as a policy," "not to a great extent," etc.

Brazilian answers were the following: "Previously, we might have speculated some, but now it is extremely dangerous.+ We use break-even computation to decide on the convenience of advance purchasing." (Textiles) "We never purchase in advance. Always the minimum. Speculation is terribly dangerous. We have to buy for one year anyway. What we do in those cases is to play one raw material against

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+It is assumed that the interviewee meant "economically dangerous."

another, like, say, soya against corn." (Food processor) "No speculation any more, but, formerly, inventories contributed to profit. We used high stocks when inflation was high (before 1964). Since January 1970, by decision of the Board of Directors, no advance purchasing whatsoever is made." (Food processor) "Formerly yes, not any more." (Food processor) "We make break-even computations, we compare alternatives." (Paper products) "There was speculation before. Not any more since 1967, because the interest rate is larger than the inflation." (Paper) "Cost variations have become small. No advantage in purchasing in advance. The cost of cacao has actually decreased, due to overproduction. Too much money is required anyway." (Food processor) "If there is a 'rumor' of increase in prices, I increase my raw materials." (Food processor)

In conclusion, much advance purchasing was carried on during the uncontrolled inflationary period prior to 1964. In the subsequent period of controlled inflation, much less advance purchasing occurred, partly because the cost of carrying inventory becomes prohibitive when the interest rate is high.

The most significant answers given by United States firms follow: "We purchase some copper in advance of needs, very little." (Electronics) "As a rule, we do not hedge against price increases; in some cases, we use break-even computation. When there is an increase in an important part, I (the divisional vice-president) go to the phone and talk to the supplier. I holler till they roll back the increase." (Electronics) "Not as a policy, only on specific instances. It has happened but is not very significant." (Electronics) "No, we never speculated. We try yearly contracts." (Electronics)

"We do not speculate, we hedge; well, we do some speculation, a little stockpiling, only when price increases will be considerable. We use that break-even formula. Our company went broke in the 1920's with inventory speculation." (Food processor) "Only a little." (Food processor) "Yes, at times." (Pharmaceuticals) "We do not speculate as a rule, but we cannot be stupid either. You have to buy ahead when an increase is expected." (Paper) "Yes, as a rule for big raw materials, such as aluminum. For other products, sometimes, yes." (Machinery)

It should be noticed that, although the expression "advance purchasing" was used in the question, many used the word "speculation" in their answers. Few used the word "hedge." It should also be observed that, even when a general increase takes place in all prices, some commodities might still fluctuate up and down, like cacao and copper in 1970. Therefore, one is never assured that all prices will go upwards, and this might be a reason why executives referred to advance purchasing as being a "speculative" practice rather than a defensive measure.[+]

The next two questions were destined to help in throwing some light on the degree of economic computation used by firms in an area close to inventory management, namely, fixed assets investment, as well as upon the crucial influence of the inflation rate on the desired rate of return.

[+]Many U. S. executives referred to advance purchasing in anticipation of a physical shortage as a standard practice in their firm or industry.

6. The sixth question was: "Which methods do you use in the selection of fixed assets investments?"

The large majority of interviewees, 29 out of 34, answered that they used one or more economic methods to appraise investment projects. Four answered negatively, and one answer was not clear.

Examining negative answers first, three came from Brazil and the fourth from the United States. The arguments presented were the following: "We always purchase the more expensive, because the cheapest turns out regularly to be the worst. As a rule, we buy, therefore, the newest, the most modern." "Up to now, we have used no method; the decision was a matter of felt convenience, of opportunity and power politics within the firm; now, for the first time, we will use some scientific method, either payback or discounted cash flow, for our next project, a large pulp plant." "No new investments are planned now." The fourth firm said that a verbal justification by the proponent, who would submit to a quiz by the management committee, was all that was required for a project to be approved.

Nineteen firms said that they use discounted cash flow, either alone or in conjunction with other methods; ten said that they do not use a method based upon discounting but, instead, one method or a mix of methods such as payback time, straight cash flow, or nondiscounted rate of return over investment. Some of the 19 which use discounted cash flow stressed that they do not use the payback approach as being too crude a method; a few of the 10 which use the payback time procedure shrugged at the discounted cash flow system for being too sophisticated.

Nine companies had standard procedure manuals and forms for project evaluations. Five of these nine used computer programs to handle capital budgeting studies. "Our computer program spits out the whole thing--discounted cash flow, rate of return over investment, sensitivity analysis, payback time, everything; the problem is to choose the output."

The payback time method or the cumulative payback time method still constitute the main analytical tools of 4 companies, but they seem to be yielding to other methods, even in these 4 companies, especially for large projects. The range of desired rate of return mentioned was: 10 to 15% per year after taxes; 15 to 25% per year before taxes.

As will be elaborated upon later, it was clear that Brazilian managers were talking about real rates of return; the distinction between real and apparent rates of return was not made by any U. S. manager.

Most respondents, both in Brazil and in the United States, commented spontaneously that, in reality, projects habitually yielded much less than the rate of return shown on the drawing board and less, even, than the indicated minimal desired rates of return. A large majority also indicated a need in their company to institute some control over real rate of return of the project as opposed to anticipated rate. Only three companies disclosed that they actually follow up the rate of return of their projects.

The consensus was that economic considerations were decisive only for ventures in diversification. But equipment replacement, cost cutting, quality improvement, and expansion proposals were usually accepted by financial management if operations or marketing people felt very strongly about them.

7. The seventh question was: "Has inflation changed the method used to select investments or the desired rate of return of the company?"

In the American interviews, the answer was invariably that methods had not been modified and that the desired rate of return had not been changed. The impression was given that most American managers felt that a level of 5 or 6% of inflation per year was not large enough to need any correction for distortions created by different rates of increases of the several cost components.

Many pointed out that uncertainties of all kinds surrounding the projects were such that even a 6% correction in the rate of return would not affect the decision. The majority seemed to believe that the rate of inflation would fall very soon to a 4% level and that they would be able to come up with an annual improvement in productivity of 3%, which would mean a drop of only 1% every year--assuming that they could not readjust their selling prices. But in some sectors--heavy industry, especially--it was felt that improvements in productivity were levelling off and that diversification would ultimately have to be resorted to as the way out of the cost-profit squeeze. But the possibility of being able to slow this erosion of the rate of return in the economic computations themselves through introduction of corrected price-level changes was not being utilized by any of the 17 companies visited.

In the Brazilian interviews, on the other hand, it was noted that six companies were going to considerable effort in order to forecast the effect of future costs and price changes on their real cash flow. The method they used to this effect consisted in making separate forecasts of cost changes of the main materials, of labor, and of administrative expenses for each year of the project's duration and, after discount of the total yearly expenses incurred, obtaining the present value of the project in constant currency; yearly sectoral costs changes are thus corrected.

One reason why Brazilian firms found it necessary to go into these details was the high annual rate of inflation--in the order of magnitude of 20%. Another reason was the considerable difference in rates of increase of materials, labor, and imported equipment, respectively. A third was that, during the seven years of controlled inflation which had taken place from 1964 to 1970, most industrial firms were not able to readjust their prices at the same rate than their inputs were climbing. A fourth reason was that many firms as subsidiaries of multinational concerns were obliged to present estimates in strong currency, usually dollars, which made a careful projection of real cash flow almost indispensable. A fifth was that many firms were competing for funds with sister companies located in other countries, where different inflationary distortions were taking place, and therefore had to justify their projects in a common, inflation-free monetary unit.

Most Brazilian managers acknowledged that the method just described depends totally on the precision with which price-level changes, sectoral and aggregate, can be forecasted. This necessarily involves a substantial proportion of guessing.[+] A manager explained: "The government says that it intends to reduce the rate of inflation to 10% next year. It is about 25% now. Down from 30% three years ago. So we figure that it will be hard to get much less than a 20% rate for the next couple

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[+]The forecast of deflators in most U. S. econometric models has been one of their weakest features.

of years. We also assume that increases of each component will follow past trends."

8. The last question was: "Can you increase your prices in the same proportion as the cost increases you suffer yourselves?"

In Brazil, interviewees unanimously answered that price increases were small and tardy in relationship to increases in inputs. The cause was the need to obtain governmental authorization for all price increases. Competition was also considered a factor.

In the United States, the effects of the 1970 depression must be separated from those of inflation. Heavy industries considered inflation their "vital problem," because they felt that increases in their products were more strictly controlled by the government, more noticed by the public, and more difficult to be matched by increases in productivity than in the instance of most other industries. The solution could be to establish still larger units of production so as to obtain economies of scale. But each such unit added would represent a huge increment to the sector's existing capacity and would therefore make competition increasingly tougher.

Other industries were suffering more from recession brought about by control of inflation than by inflation itself, in other words, more by the drop in physical demand for their products than by rising costs or high interest rate, though these were additional factors, not negligible when inventories pile up. Electronic industries, typically, said that they could offset cost increases by very large gains in productivity in normal times and that the product is so frequently modified that it becomes obsolete before requiring any price readjustment.

Food processors intimated that the main deterrent for price increases was stiff competition; overproduction of the sector was also an element preventing price readjustments. Pharmaceutical and chemical companies interviewed indicated that they felt they had some freedom in increasing prices but that they knew little about price elasticity of their products. They believed that their productivity potential was still enough to absorb further cost increase, inclusive in the inventory management area, which only lately had caught their attention.

In summary, the following trends emerge from this survey and seem to be valid for both the Brazilian and American firms interviewed:

1. A minority of industrial firms used cost minimization as the main objective of inventory policy. Most used predominantly physical considerations--such as sales-inventory ratios, turnover indices, and demand requirements--to determine and control the size of stocks. Inventory policy of many firms was largely dictated by physical and financial constraints.

2. Some manufacturing firms adopted the policy of stockpiling in face of imminent cost increases in their most important materials and supplies but only if the cost increase was a certainty and of great consequence. When advance purchasing was done, precise economic calculations were rarely used in order to determine the optimal amount to buy. The firms interviewed tried to avoid any uncertainty in regard to cost changes by shifting speculation risks to suppliers. All of the firms

interviewed denied speculating with final inventories.

3. The majority of firms seemed sensitive to interest rate, especially in Brazil, where most companies used borrowed funds to finance at least part of their inventories. The firms seemed to be very aware that any financing beyond a "normal" credit line entails a considerably higher interest rate. The cash shortage and high interest rate which characterized the economies of Brazil and the United States during 1968-70 contributed decisively to making firms try to minimize their inventories, even below the minimum level suggested by inventory theory. By 1970, minimization of physical inventories, rather than minimization of costs of the total inventory system, had become the main objective of firms in Brazil and was becoming the main concern of firms in the United States. This appeared to be likely to persist so long as a situation of controlled inflation, with credit restrictions or price regulations or both, continued.

CHAPTER V

REGRESSION STUDIES OF AGGREGATE INVENTORIES ON PRICE-LEVEL CHANGES AND ON INTEREST RATE

Chapter II reviewed the main currents of ideas concerning the influence of price-level changes and changes in interest rate upon inventory accumulation. In this chapter, attention will be turned to an examination of the empirical evidence regarding these effects. First, consideration will be given to previous work on the subject; then the results of the author's regression studies will be presented.

5.1. Review of Previous Regression Studies

Aggregate studies performed in order to uncover some relationship between price changes and inventory investment have so far been contradictory and inconclusive. The reasons for such difficulties are many.

First of all, advance purchasing is based upon anticipations of changes in costs. But how could one measure estimates that businessmen make of future changes in costs? Some surrogate, usually the actual subsequent price change, has been used instead. But businessmen's estimates might have been very much off.

A second reason is the difficulty of ascertaining the lag with which the actual cost increase follows the forward purchase. As has been seen in Chapter III, advance purchases should be made on the eve of cost increases. If this is the practice followed by managers, then inventory fluctuations should be correlated with price index fluctuations which occur in the <u>same</u> quarter. If, on the contrary, advance purchasing is carried out one quarter ahead of the cost increase, inventory fluctuations should be correlated with price index fluctuations occurring one quarter ahead.

A further reason is that most studies have lumped together inventories at all stages of fabrication; inasmuch as the reasons impelling managers to accumulate materials and supplies, namely, future costs rises, would decrease their finished goods' stocks through larger purchases by their own customers, who would also want to increase their inventories, the effects could cancel each other in an aggregate inventory study.

While investments in fixed assets are always purposeful, inventory investments are many times undesired and result from an unexpected slow demand. No way exists for distinguishing intended from unintended inventory investment in a regression study. This factor also contributes to obscuring any causal relationship between inventory accumulation and the variables under study, such as interest rate and price changes.

A reason somewhat related to an anticipation of cost increase, and which could cause an accumulation of materials and supplies inventories, is an anticipation of a shortage in some product. The increase in stocks brought about by the forecast of a major strike in a basic industry introduces a perturbation in aggregate inventory levels. The introduction of dummy variables in regression analysis is designed to single out the periods in which this kind of distortion is thought to have occurred.

Attention will now be turned to previous quantitative studies intended to inquire into the relationship between inventories, price changes, and interest rate. First mention will be given to the findings of some authors, who, although they did not conduct actual regression studies of aggregate inventories with price level and interest rate as independent variables, have nevertheless made empirical studies of aggregate inventories levels and postulated or concluded that these two variables could be dismissed.

Moses Abramovitz reminds us that:

> The negligible influence of interest rates upon inventory policy was substantiated by the well-known inquiry by J. E. Meade and P. W. S. Andrews (Summary of Replies to Questions on Effects of Interest Rates, Oxford Economic Papers, October, 1938). None of the managers interviewed said that the cost of borrowed funds affected his calculations about the volume of stocks he should hold.[1]

Abramovitz performs some calculations, based upon the assumption that the range of variation in the interest cost of customers' loans is not larger than 20%, and concludes that the influence of interest rate cannot be detected in inventory records.

For Abramovitz, the influence of price speculation upon stocks stands on "a quite different footing" than the influence of the interest rate. He agrees that:

> Obviously prices often fluctuate so violently as to stimulate speculative investment in commodities. And whatever the common opinion about good practice, there can be little doubt that some manufacturers do on occasion attempt to anticipate

[1]Moses Abramovitz, Inventories and Business Cycles with Special Reference to Manufacturers' Inventories (New York: National Bureau of Economic Research, Inc., 1950), p. 126.

price rises or declines by modifying their inventory policy.[2] He points to a number of difficulties in measuring the specific influence of price speculation on stock accumulation. Through observation of movements in business cycles, he suggests that "price speculation is not a regular influence of great importance" but agrees that the issue is not settled, because the statistical materials at hand are neither rich enough nor adequate for this purpose.

Thomas M. Stanback, Jr. reached conclusions for the postwar period analogous to the ones established by Abramovitz for the period between the two World Wars. He stresses the reciprocal relationship of inventory changes and price changes:

> During periods of rising demand, tightening supply, and rapid inventory accumulation, price increases are likely to occur. Under certain conditions such price rises may, in turn, set off inventory buying which will give rise to still further pressure upon prices. . . .[3]

Nestor E. Terleckyj draws charts which show changes in inventory book values (unadjusted and adjusted for valuation adjustment) for 1948-59 and concludes that:

> It is quite clear that rapid changes in prices have been accompanied by parallel movements in inventory values. . . . . When these valuation consequences of price changes are removed, a certain parallel to price changes still exists in the inventory data. Moreover, the chart indicates that there has been some lead of price changes over the rate of inventory accumulation. . . . This lead might be considered as evidence

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[2] Ibid., p. 127.

[3] Thomas M. Stanback, Jr., Postwar Cycles in Manufacturers' Inventories (New York: National Bureau of Economic Research, Inc., 1962), p. 51.

of the fact that a "speculative" element exerts an independent influence on inventory plans.[4]

In the same technical paper, Terleckyj reports a multiple regression analysis conducted for the period of 1948-59. The objective was to obtain an estimating equation to explain percentage changes in business inventory book values over successive six-month periods, staggered at quarterly intervals. His results are quoted here at length, with small explanatory additions.

The estimating equation which was obtained for changes in inventories for successive six-month periods, staggered at quarterly intervals, appears as follows:

$$Y = -121.3 - \underset{(3.84)}{18.48\,X_1} + \underset{(9.73)}{72.51\,X_2} + \underset{(.111)}{.788\,X_3} - \underset{(.461)}{.028\,X_4} - \underset{(.310)}{.026\,X_5}$$

where:

Y is the percentage change in inventory book values for trade and manufacturing over a six-month period,

X_1 = inventory-sales ratio at the beginning of the period,

X_2 = average ratio of new orders to sales for the quarter preceding the period,

X_3 = percentage change in the index of industrial wholesale prices, during the period,

X_4 = percentage change in inventories attributable to changing mix, during the period,

[4]Nestor E. Terleckyj, assisted by Alfred Tella, Measures of Inventory Conditions (New York: National Industrial Conference Board, Inc., 1960), p. 22.

X_5 = interest rate on four-to-six months' prime commercial paper: average for the first quarter of the period.

Terleckyj continues as follows:

> The numbers written in parentheses under the regression coefficients are the standard errors of these coefficients. The ratio of a regression coefficient to its own standard error provides a test of significance of the coefficient. This amounts to testing whether there is a real correlation between the variable to which the coefficient applies and the dependent variable to be explained. When a probability standard of one chance in twenty is applied, the ratio should equal at least 2.0 if the coefficient is to be accepted as significant. . . . . . The coefficients for the first three variables--inventory-sales ratio, new orders-sales ratio and price change--thus appear to be highly significant; those for the other two variables--change indicated by shifting mix and the interest rate--are not significant. The statistical fit of the equation is excellent. The independent variables explain 91% (R^2 = .91) of the total variation. . . .[5]

Terleckyj believes that the significant relationship between inventory changes and changes in price index is ". . . mainly built upon the induced effect of price changes on inventory valuations . . . but a mild indication of a lead of price movements over inventory changes also appears."[6] As for the influence of the interest rate, the fact that the variable has the correct sign means little, inasmuch as the regression coefficient is less than one-tenth the magnitude of its estimated standard error.

Lawrence R. Klein reports a significant positive association between aggregate inventory investment and change in Gross National

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[5] *Ibid.*, p. 25.

[6] *Ibid.*, p. 24.

Product deflator.[7] The total inventory investment has a 269.3 $(p - p_{-1})$ term, where p is the implicit GNP deflator. T. M. Brown also obtains a positive relationship in his study of Canadian inventory behavior: aggregate business inventories are related to price-level change.[8]

In a study of manufacturers' holdings of stocks of purchased materials and goods in process, Michael C. Lovell found that the relationship with price changes was insignificant for both durable and nondurable goods and had the wrong sign for total manufacturing stocks.[9] Lovell also reports that Paul G. Darling

> . . . found that price change, while of correct sign, was insignificant at the 5 per cent level in the equation explaining manufacturers' holdings of purchased materials and goods in process and in regressions for wholesale and retail trade; in other regressions, which constituted the majority, the sign was incorrect. This evidence is compatible with the null hypothesis that firms do not speculate in stocks. Of course, the test is not conclusive; for one thing, firms may simply change the composition rather than the magnitude of their holdings; in addition, they may seriously misjudge price movements.[10]

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[7] Lawrence R. Klein, "A Postwar Quarterly Model: Description and Applications," in Conference on Research in Income and Wealth, Models of Income Determination (Princeton, N. J.: Princeton University Press, for the National Bureau of Economic Research, 1964), vol. 28, pp. 11-57.

[8] T. M. Brown, "A Forecast Determination of National Product, Employment, and Price Level in Canada, from an Econometric Model," Ibid., pp. 59-96.

[9] Michael C. Lovell, "Manufacturers' Inventories, Sales Expectations, and the Acceleration Principle," Econometrica, XXIX (July, 1961), 293-314.

[10] Michael C. Lovell, "Determinants of Inventory Investment," in Models of Income Determination, op. cit., p. 212.

Albert Ando, E. Cary Brown, Robert M. Solow, and John Kareken performed a study concerning the nature of the lags involved in the functioning of monetary and fiscal policy. These authors report that:

> As it happens, in the course of some statistical exploration of inventory investment we did come upon one model in which a measurable interest rate effect on inventories does appear. The estimated interest-elasticity, as will appear, is not large, but it has the right sign and is statistically significant.[11]

They found that the actual stock of undeflated, aggregate manufacturers' inventories is related to the average interest rate charged on short-term bank loans to business toward the end of period $t - 1$, with a regression coefficient of -1.1547, the standard error being .4054. The period studied was 1947-3rd quarter to 1960-1st quarter. This suggests that a 1 percent rise in the interest rate reduces inventory investment by $1.15 billion in the following quarter and, ultimately, after the chain of disinvestments is completed, by $4.86 billion.

But these authors warn us that all of the results in this connection should be viewed cautiously. In a second regression, involving a more complicated lag structure, the regression coefficient becomes approximately equal to its standard error, and one can rightly be skeptical about the relationship between inventory and interest rate.

Paul F. McGouldrick of the Board of Governors of the Federal Reserve System reports disappointing results in attempts to determine the influence of the bank rate on short-term business loans upon inventory

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[11] Albert Ando, E. Cary Brown, Robert M. Solow, and John Kareken, "Lags in Fiscal and Monetary Policy," in E. Cary Brown and others, _Stabilization Policies: A Series of Research Studies Prepared for the Commission on Money and Credit_ (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1963), pp. 39-40.

holdings of durable goods manufacturers for the 1952-61 period:

> . . . Interest rates were found to have an uncertain influence on inventory accumulation. In three regressions . . . the partial regression coefficient for the interest rate was negative each time, but it was never statistically significant at the 5 percent level. However, all three coefficients were close to their standard errors in size. [A footnote states: "If a net regression coefficient equals its standard error, and if the distribution of such coefficient estimates is the normal one, there are 83 chances out of 100 that the true coefficient in the universe is of the same sign . . . ."].[12]

Michael C. Lovell also found "quite disappointing" results in attempts to correlate inventories with interest rates:

> . . . The business loan rate . . . invariably had a significantly positive coefficient. This was true for both durable and nondurable manufacturing.
>
> . . . .
>
> The rate of prime commercial paper, the bank rate on short-term business loans, the 3-month bill rate, and the rate on prime bankers acceptances were correlated with the residuals from the various industry regressions. None of the correlation coefficients was significant at the 5-percent level. This second test offered no support to the conjecture that interest rate costs have a direct effect on inventory investment.[13]

The preceding results help in explaining why most current econometric models do not incorporate changes in price levels nor interest rate terms in the inventory investment equations.

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[12] U. S. Congress, Joint Economic Committee, "The Impact of Credit Cost and Availability on Inventory Investments," in Inventory Fluctuations and Economic Stabilization, by Paul F. McGouldrick, Joint Committee Print, Part II, Causative Factors in Movements of Business Inventories (Washington, D. C.: Government Printing Office, 1961), p. 105.

[13] U. S. Congress, Joint Economic Committee, "Factors Determining Manufacturing Inventory Investment," by Michael C. Lovell, ibid., pp. 133-134.

The Wharton Econometric and Forecasting Unit Model uses two inventory equations, one for investment in aggregate manufacturing inventories, and the other for inventory investment in the nonmanufacturing sector.[14] The first considers inventory investment as a function of lagged sales, lagged inventory level, unfilled orders, and a dummy variable for steel strike. The second relates inventory investment to lagged sales, lagged inventory levels, lagged purchases of consumer durables, gross output increment, and a factor: $40.43\ (p_m - p_{m-2})$, where p_m is wholesale price level excluding food and farm products in period m . This factor means that merchants increase their stocks in advance of price increases.

The "revised Klein-Goldberger" model uses an aggregate inventory equation, in which the inventory in a period is a function of gross national product, inventory level in last period, and changes in inventory level from last period.[15] Likewise, the model used by Daniel B. Suits,[16] as well as the F.R.B.-M.I.T.[17] and Brookings[18] models, in their present formulation, do not include price-level nor interest rate factors in their inventories investment equations.

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[14] Evans, op. cit., p. 434.

[15] Ibid., p. 498.

[16] Daniel B. Suits, "Forecasting and Analysis with an Econometric Model," American Economic Review, LII (March, 1962), 104-132.

[17] Robert H. Rasche and Harold T. Shapiro, "The F.R.B.-M.I.T. Econometric Model: Its Special Features," Ibid., XLVIII (May, 1968), 123-154.

[18] Evans, op. cit., p. 507.

5.2. Inventory Accumulation in a Sustained Inflation

The normative models developed in Chapter III were built for small planning horizons: The EOQ model, equation (5), section 3.1.1.4, was based upon a one-year horizon; the other models concerned a once-and-for-all cost increase, or, at most, a few consecutive increases in some inputs. It is now necessary to discover the implications of these models for sustained changes in costs over prolonged periods in order to observe the evolution of inventories over years in a situation of continued inflation.

Consider the EOQ model, formula (5), section 3.1.1.4. It suggests that a rational manager who foresees a linear increase I_1 in materials during the next year and an apparent interest rate $i + I_3$ would purchase economic lots of larger size than he would if he had foreseen stable costs conditions, if $I_1 > I_3$, and of smaller size, if $I_1 < I_3$. In the case of $I_1 > I_3$, his average inventory level will increase; in the case of $I_1 < I_3$, it will decrease, in comparison with periods of anticipated stable costs.

Let it be assumed that, after the first year, during which conditions I_1^1 and I_3^1 prevailed, the value of the anticipated increase rate in materials becomes I_1^2 for the second year, and the anticipated apparent interest rate becomes $i + I_3^2$. We neglect the effects of changes in I_2.

The inventory change which occurs during the first year, compared to year zero, where costs were stable, is:

$$\text{Change in Inventory} = \sqrt{\frac{2PD}{C(i + a + I_3^1 - I_1^1)}} - \sqrt{\frac{2PD}{C(i + a)}} \qquad (1)$$

To keep matters simple, it is asssumed that, at the start of the second year, readjustments in the values of P and C cancel each other. The inventory change which occurs during the second year, in comparison with the first year, is:

$$\text{Change in Inventory} = \sqrt{\frac{2PD}{C(i + a + I_3^2 - I_1^2)}} - \sqrt{\frac{2PD}{C(i + a + I_3^1 - I_1^1)}} \qquad (2)$$

In general, the change in inventory, from year $n - 1$ to year n , will be given by formula (3):

Change in Inventory for year n =

$$= \sqrt{\frac{2PD}{C(i + a + I_3^n - I_1^n)}} - \sqrt{\frac{2PD}{C(i + a + I_3^{n-1} - I_1^{n-1})}} \qquad (3)$$

Equation (3) reveals that a change must take place in either I_1 or I_3 , or in both, in order to justify a change in inventory level. Stated otherwise, it is a change in the anticipated inflation rates I_1 and I_3 that will cause changes in inventory levels. Changes in inventories will stop as soon as inflation stops accelerating. A continuation of the same anticipated rates I_1 and I_3 year after year would not justify any further changes in inventory levels.

When we want to study the influence of inflation on inventories for a number of years, the model suggests that <u>changes</u> in inventories would be related with the difference

$$(i + a + I_3^n - I_1^n) - (i + a + I_3^{n-1} - I_1^{n-1}) , \qquad (4)$$

that is, with <u>changes</u> in the <u>real</u> interest rate $i + I_3 - I_1$.

We see from (3) that, if:

$i + I_3^n - I_1^n > i + I_3^{n-1} - I_1^{n-1}$, the inventory change is negative,

and, if:

$i + I_3^n - I_1^n < i + I_3^{n-1} - I_1^{n-1}$, the inventory change is positive.

We would therefore expect, in a regression study of inventories of materials and work in process, that inventory investments (that is, inventory changes) would be correlated negatively with changes in the real interest rate.

The preceding conditions can also be rewritten in the following way: if

$I_3^n - I_3^{n-1} > I_1^n - I_1^{n-1}$, the inventory change is negative,

and if:

$I_3^n - I_3^{n-1} < I_1^n - I_1^{n-1}$, the inventory change is positive.

Inventory changes are therefore correlated negatively with the difference between the change in nominal interest rate and the acceleration of inflation in costs of materials.

We can, alternatively, perform regression studies with inventory levels instead of dealing with inventory changes. Equation (5), section 3.1.1.4, reveals that inventory levels should be correlated positively with the forecasted inflation rate I_1 in materials and negatively with the apparent interest rate $i + I_3$, or, equivalently, that inventory levels correlate negatively with the real interest rate $i + I_3 - I_1$.

The inflation rate in materials, I_1, would normally be the annual change in wholesale price index. For work in process, and for aggregate inventories over all stages of fabrication, the annual percentual change in Gross National Product implicit deflator, which we call:

$$\frac{p - p_{-1}}{p_{-1}} ,$$

could be equally utilized. The preceding conditions can therefore be rewritten by substituting $\frac{p - p_{-1}}{p_{-1}}$ for I_1.

These considerations were based upon the EOQ model and anticipated linear cost increase in materials. The step-increase model, the hand-to-mouth purchasing model, and the s, S model indicate that the level of materials inventories is a negative function of the anticipated nominal interest rate and a positive function of the anticipated rate of changes in costs of materials. It should therefore be expected in general that the level of inventories be correlated negatively with nominal interest rate and positively with changes in costs. Alternately, it would be expected that changes in materials inventories be correlated negatively with changes in nominal interest rate and positively with changes in rates of changes in materials costs (that is, with accelerations in changes in materials costs).

5.3. The Brazilian Case, 1948-1967

Initially, some results obtained with Brazilian data are presented. Statistical data for manufacturers' inventories could not be found. The only figures that we came across were national account aggregate inventories. We were also unable to obtain commercial actual interest rates, which, anyway, are very variable from one borrower to the other in Brazil and quite different from official rates quoted by the banks or financial institutions, due to high commissions and compensating balances charged to borrowers. We were therefore unable to study the relation between inventory accumulation and interest rate and had to restrict ourselves to regression studies involving inventory investment and price-level changes.

The data used are reproduced in Table 5.1.[19] They represent, in million of cruzeiros, at constant 1953 prices, the National Account Changes in Inventories and the Real Gross National Income, and, in percentages, the annual changes in implicit Gross National Product Deflator. The period is 1948-1967.

Except for the years 1965-67, when inflation control started, the period under consideration was one of uncontrolled inflation. It is postulated therefore that nominal interest rates were always lagging behind changes in price indexes. The condition:

Inventory changes are positive if $I_1^n - I_1^{n-1} > I_3^n - I_3^{n-1}$

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[19] "Brazil's National Accounts - New Estimates," Conjuntura econômica (Rio de Janeiro) (English ed.), XVI (October, 1969), 35-73.

TABLE 5.1

NATIONAL ACCOUNT INVENTORY INVESTMENT
BRAZIL: 1948-1967

(1) Year	(2) Change in Inventory (Inventory Investment) at Constant 1953 Prices (Million of cruzeiros)	(3) Gross National Income at Constant 1953 Prices (Million of cruzeiros)	(4) Annual Change in GNP Implicit Deflator (%) $\frac{P - P_{-1}}{P_{-1}}$
1948	-.4	331.8	9.9
1949	-2.9	356.3	10.7
1950	-2.0	397.5	11.2
1951	-2.7	422.2	12.0
1952	1.7	455.0	13.2
1953	-2.3	465.9	15.3
1954	4.7	521.7	21.4
1955	9.6	550.0	16.8
1956	-.2	567.9	23.2
1957	14.7	614.2	13.2
1958	8.3	661.0	11.1
1959	19.5	695.9	29.2
1960	10.3	763.2	26.3
1961	16.5	834.3	33.3
1962	19.2	881.9	54.8
1963	9.1	893.8	78.0
1964	17.4	929.3	87.8
1965	32.0	955.3	55.4
1966	-.2	1000.5	38.8
1967	5.8	1047.0	32.4

Source of (2), (3), and (4):

"Brazil's National Accounts - New Estimates," Conjuntura econômica (Rio de Janeiro) (English ed.), XVI (October, 1969), 35-73.

can be substituted by the condition:

Inventory changes are positive if $I_1^n - I_1^{n-1} > 0$

since changes in I_3 are postulated to be negligible.

Our model says, then, that under the prevailing conditions of Brazil at that time, inventory changes would correlate positively with changes in inflation rate of materials, i.e., with acceleration of inflation in materials costs or with its surrogate, the acceleration of inflation as measured with successive changes in percentual changes of the GNP deflator.

We first regressed annual changes in inventory against annual changes in inflation rates, that is, against inflation acceleration, taken with a one-year lag. We considered that a lag of this order of magnitude would reflect the length of time necessary to obtain information concerning inflation rate, to transmit it through the firm and to accumulate or diminish physical inventories. In other words, we correlated the changes in inventory which occurred between 1949 and 1950 with the acceleration in inflation which occurred between 1949 and 1948, and so on from year to year, up to 1967. The result of this regression was equation (1):

$$\text{Inventory Investment} = 8.5 + \underset{(.164)}{.2949}\left[\left(\frac{p - p_{-1}}{p - 1}\right) - \left(\frac{p - p_{-1}}{p - 1}\right)_{-1}\right] \qquad (1)$$

Beta Coefficient = .410

Coefficient of
Determination = .1162
(Corrected for Degrees of Freedom)

We see that the coefficient of correlation is .34, so that there is some relationship, though not strong, between the variables examined. If we eliminate the data from years 1965 and 1966, the coefficient of correlation becomes negative, which indicates that we cannot claim with confidence that managers did follow a rational policy of advance purchasing. We can at most find some possibility that this was the case.

We now regressed annual inventory investments against annual <u>changes</u> of GNP deflator $\left(\frac{p - p_{-1}}{p_{-1}}\right)$, initially without any lag, and in the second trial, with a one-year lag; the results are represented in the two following equations:

Without lag:

$$\text{Inventory Investment} = .770 + \underset{(.081)}{.240} \left(\frac{p - p_{-1}}{p_{-1}}\right) \tag{2}$$

Beta Coefficient = .572

Coefficient of Determination: $R^2 = .289$

With lag:

$$\text{Inventory Investment} = 1.11 + \underset{(.080)}{.245} \left(\frac{p - p_{-1}}{p_{-1}}\right)_{-1} \quad . \tag{3}$$

Beta Coefficient = .594

Coefficient of Determination: $R^2 = .315$

We next regressed annual inventory investments against annual changes in GNP deflator and annual percentual changes in Gross National Income, initially without lag, and, in the second trial, with a one-year lag. Equations (3) and (4) show the results obtained.

Without lag:

$$\text{Inventory Investment} = -\ 2.909 + \underset{(.101)}{.266}\left(\frac{p - p_{-1}}{p_{-1}}\right) + \underset{(.775)}{.490}(GNI - GNI_{-1}) \quad (4)$$

Beta Coefficients = .633 .153

Coefficient of Multiple Determination: $R^2 = .238$

With lag:

$$\text{Inventory Investment} = -\ 3.123 + \underset{(.094)}{.278}\left(\frac{p - p_{-1}}{p_{-1}}\right)_{-1} + \underset{(.735)}{.518}(GNI - GNI_{-1}) \quad (5)$$

Beta Coefficients = .676 .161

Coefficient of Multiple Determination: $R^2 = .294$

Equations (2) and (3) suggest that a highly significant relationship exists between aggregate inventory investment and inflationary price changes. It is possible that this relationship is entirely spurious and due to a third variable. It is plausible to believe, for instance, that inventory investments are correlated with changes in Gross National Income. Equations (4) and (5) indicate, indeed, that such a relationship exists but that it is not significant and that changes in GNI are about four times less important than the rate of inflation in determining inventory investment.

Performing a two-tail t-test on the regression coefficients in equations (4) and (5), one finds that a significant positive relationship between inventory investment and the rate of inflation exists at the 98% pro' ility level; the same test, performed in equations (2) and (3), in es that such a positive relationship is significant at the 99%

probability level. A one-year lag mechanism gives, therefore, better results than no lag.

The coefficient of correlation of equation (3) is .56. This means, for our sample of 19 observations, that the true coefficient of correlation is at least .23, with a 95% chance of being correct.[20]

In summary, we have found that inventory investments in Brazil, for the period studied, are very significantly correlated with inflationary rate and weakly correlated with inflationary acceleration. The assumption of rational advance purchasing, which requires that the inventory investment would be performed only if inflation were accelerating, is only mildly supported by the data. Some other reason must have caused inventory accumulation. Among the possibilities, one is that speculative motives, flight from money, hoarding, or agricultural stockpiling by the government might have taken place. Errors in the forecast of inflation and nonoptimal advance purchasing might also have occurred.

5.4. Regressions Based on U. S. Data, 1955-70

Initially, we regressed manufacturers' annual raw materials and supplies inventories—shipments ratios against real annual interest rate for the 1955-1970 period.

Inasmuch as anticipated inflationary and interest rates are unobserved variables, and, as the recorded values of these variables fluctuate considerably during the period analyzed, it seemed preferable

[20] See William A. Spurr and Charles P. Bonini, Statistical Analysis for Business Decisions (Homewood, Illinois: Richard D. Irwin, Inc., 1967), p. 576.

to work with annual, rather than quarterly, values, all the more because the planning period adopted in the theoretical models was usually one year.

We worked with inventories-shipments ratios. Inventories are strongly correlated to shipments; taking these ratios, we eliminate the major source of fluctuations in inventories, which is the anticipated demand, assuming that shipments are a reasonable surrogate for forecasted demand in the same period. In the models, the inventory level was either a linear or a square-root function of demand. In either case, the use of the ratio does not cause a considerable distortion in the models.

We are testing the influence of changes in costs and of interest rate on the physical size of inventories. But available data on inventories are in dollars. Ideally, we should deflate the values of both dollars inventories and dollars shipments to a constant dollar value; then, take the ratios of these values and get physical values. The deflators of inventories are different from the shipments deflators, inasmuch as shipments contain a larger proportion of labor, and changes in labor costs have been different from changes in materials costs. If labor costs have risen more than material costs, the ratio of dollars inventories to dollars shipments will show a decrease over the years, while the physical ratio could actually have not changed or even have increased. In other words, we should like to study the annual ratio:

$$\frac{\text{Physical Inventories of Materials and Supplies}}{\text{Physical Shipments}} ,$$

but we are actually working with the following annual ratios:

$$\frac{\text{Physical Inventories of Materials and Supplies} \times \text{Unit Costs}}{\text{Physical Shipments} \times \text{Unit Prices}} .$$

If the ratio Costs-Prices changes over time, a possibly considerable distortion will have been introduced into our computations. Correction to this effect was taken in a second series of runs, as will be seen later.

The interest rate selected was the prime rate for commercial loans made by banks. We used start-of-year values, based upon data published by Standard & Poor's.[21] They are shown in Table 5.2.

The inflation measure utilized was the annual changes in Wholesale Price Index, for all commodities, computed from data published in Survey of Current Business.[22] They correspond to the symbol I_1 of the normative models. The values used in the regressions are shown in Table 5.2.

The values of manufacturers' inventories and shipments are taken from publications of the U. S. Department of Commerce.[23] We computed annual averages of inventories and divided them by average annual shipments. The values used in the regressions are reproduced in Table 5.2.

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[21] Prime Commercial Loan Rates are published by Standard & Poor's in Trade and Securities Statistics (New York: Standard & Poor's Corporation).

[22] Survey of Current Business (Office of Business Economics, U. S. Department of Commerce).

[23] U. S. Bureau of the Census, Manufacturers' Shipments, Inventories, and Orders: 1947-1963 (Revised)(U. S. Government Printing Office, Washington, D. C., 1963). Also: U. S. Bureau of the Census, Manufacturers' Shipments, Inventories, and Orders: 1961-1970 (Washington, D. C.: 1970).

TABLE 5.2

MANUFACTURERS' RAW MATERIALS AND SUPPLIES
INVENTORIES-SHIPMENTS RATIOS
UNITED STATES: 1955-1970

(1) Year	(2) Manufacturers' Raw Materials and Supplies Inventories-Shipments Ratios	(3) Prime Commercial Interest Rate at Year's Start (%)	(4) Annual Change in Wholesale Price Index (%)	(5) Annual Real Interest Rate (%) = (3) - (4)
1954	--	3.2	.2	3.0
1955	.6322	3.0	.3	2.7
1956	.6698	3.5	3.2	.3
1957	.6744	4.0	2.9	1.1
1958	.6849	4.5	1.4	3.1
1959	.6428	4.0	.2	3.8
1960	.6482	5.0	.1	4.9
1961	.6152	4.5	-.4	4.9
1962	.6178	4.5	.3	4.2
1963	.5959	4.5	-.3	4.8
1964	.5672	4.5	.2	4.3
1965	.5614	4.5	2.0	2.5
1966	.5644	5.0	3.3	1.7
1967	.5880	6.0	.2	5.8
1968	.5504	6.0	2.5	3.5
1969	.5256	7.0	3.9	3.1
1970	.5313	8.5	3.6	4.9

The sources of these data are indicated in references 21, 22, and 23 of this section.

We first regressed ratios without using any time lag. That means, we correlated inventories-shipments ratios for 1955 with the real interest rate prevailing in 1955; this rate is obtained by subtracting the annual change in Wholesale Price Index from 1954 to 1955 (column 4 of Table 5.2) from the Prime Commercial Interest Rate prevailing at the start of 1955 (column 3 of Table 5.2) and shows on column 5 of Table 5.2.

We next used a one-year lag, correlating inventory-shipment ratios with the real rate of interest prevailing at the start of the previous year. That is, the 1955 ratio was correlated with the real interest rate prevailing at the start of 1954.

The results of these two regressions are represented, respectively, by equations (1) and (2).

Without lag:

$$\left.\begin{array}{l}\text{Ratio of Manufacturers'}\\ \text{Raw Materials and Supplies}\\ \text{to Manufacturers' Shipments}\end{array}\right\} = .6407 - \underset{(.0085)}{.0104}\ (\text{Real Interest Rate}) \qquad (1)$$

Beta Coefficient = - .3114

Coefficient of Determination: $R^2 = .0325$

With a one-year lag:

$$\left.\begin{array}{l}\text{Ratio of Manufacturers'}\\ \text{Raw Materials and Supplies}\\ \text{to Manufacturers' Shipments}\end{array}\right\} = .6615 - \underset{(.0080)}{.0170}\ (\text{Real Interest Rate})_{-1} \qquad (2)$$

Beta Coefficient = -.4914

Coefficient of Determination: $R^2 = .187$

The coefficient of determination of equation (1) has no significance; the coefficient of determination of equation (2) is not significant either at the 95% level, because the correlation coefficent is .43, and a sample of size 16 would require a correlation coefficient of .49 in order to enable us to state that the correlation coefficient is larger than zero with a 95% chance of being correct.[24]

We next regressed these same inventory-shipments ratios against nominal interest rates and annual inflation rates. The interest rate was the prime rate, column 3 of Table 5.2. The inflationary rate was the annual change in Wholesale Price Index, column 4 of Table 5.2.

As usual, we made two trials, one without, and the other with a one-year lag. The results are shown in equations (3) and (4), where symbol r stands for annual nominal prime interest rate, and symbols I_1 and $(I_1)_{-1}$ represent percent annual changes in the Wholesale Price Index, without and with lag, respectively.

Without lag:

$$\left.\begin{array}{l}\text{Ratio of Manufacturers'}\\ \text{Raw Materials and Supplies}\\ \text{to Manufacturers' Shipments}\end{array}\right\} = .738 - \underset{(.0082)}{.027}\, r + \underset{(.0072)}{.00018}\, I_1 \qquad (3)$$

$$\text{Beta Coefficients} = \qquad -.717 \qquad .005$$

Coefficient of Multiple Determination: $R^2 = .437$
(corrected for degrees of freedom)

[24]Spurr and Bonini, op. cit., p. 576.

With a one-year lag:

$$\left.\begin{array}{l}\text{Ratio of Manufacturers'}\\ \text{Raw Materials and Supplies}\\ \text{to Manufacturers' Shipments}\end{array}\right\} = .795 - \underset{(.0080)}{.0435(r)_{-1}} + \underset{(.0057)}{.0079(I_1)_{-1}} \qquad (4)$$

Beta Coefficients = -.882 .225

Coefficient of Multiple Determination: $R^2 = .64$
(corrected for degrees of freedom)

The usual t-tests of significance, performed on the regression coefficients of equation (4), indicate that the regression coefficient of the interest rate is highly significant, while the regression coefficient of the inflation rate is not significantly different from zero at the 95% level of confidence. The beta coefficients indicate that interest rate is about four times as important in determining the inventory-shipment ratio than anticipated inflation rate. An F-test, performed on the coefficient of determination of equation (4), indicates that this coefficient is not significantly different from zero at the 95% level of confidence.[25]

In conclusion, the hypothesis that inventory levels are negatively correlated with real interest rate cannot be accepted with confidence; neither can the hypothesis that they are negatively correlated with nominal interest rate and positively correlated with inflation rate be accepted. Though statistical tests of significance oblige rejection of these hypotheses, the regression coefficients have the right signs and indicate that interest rate and inflation rate have some effect on

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[25] William L. Hays, Statistics for Psychologists (New York: Holt, Rinehart and Winston, 1963), p. 573.

raw materials-shipments ratios.

We further performed regression studies on ratios of manufacturers' raw materials and supplies inventories to manufacturers' shipments at a more disaggregate level. Table 5.3 shows the results of the regressions performed on real interest rate with a one-year lag. The five last lines of that table refer to ratios in which correction for differences in price indices has been incorporated. For these five runs, we divided materails and supplies by a yearly index obtained averaging the Wholesale Price Index of Crude Materials for Further Processing and the Wholesale Price Index for Intermediate Materials, Supplies and Components, both obtained from U.S. Bureau of the Census and the Department of Commerce publications. And we divided shipments by Wholesale Price Indexes for Manufactures, either Total, Durable, or Nondurable, according to the case.

Table 5.4 shows the results of regressions performed on apparent interest rate and change in annual GNP deflator, with a one-year lag. Again, the first five runs were made without correction of price indexes, and the last five, with correction. Although the signs of the regression coefficients are all correct, except only two, lack of statistical significance is evident, especially when data are corrected for differences in price indexes.

The last lines of Tables 5.3 and 5.4 show that the inventory-shipments ratio of chemical and allied products, even with price-indexes correction, is significantly correlated with the real interest rate as well as with price-level changes and apparent interest rate.

Moreover, Table 5.4 indicates that inventory-shipments ratios

TABLE 5.3

RESULTS OF REGRESSIONS PERFORMED ON MANUFACTURERS' RATIOS OF RAW MATERIALS AND SUPPLIES INVENTORIES TO MANUFACTURERS' SHIPMENTS, UNITED STATES, 1955-1970. REGRESSIONS PERFORMED ON REAL INTEREST RATE, WITH ONE-YEAR LAG

Industry Group	Regression Coefficient	Standard Error of Regression Coefficient	Beta Coefficient	Coefficient of Determination
	Without Price-Indexes Correction			
Total, All Manufacturing Industries	-.0170	.0080	-.491	.187
Total, Durable Goods Industries	-.0183	.0078	-.530	.230
Total, Nondurable Goods Industries	-.0129	.0108	-.304	.027
Stone, Clay, and Glass Products	-.0102	.0050	-.479	.174
Chemicals and Allied Products	-.0182	.0052	-.685	.432
	With Price-Indexes Correction			
Total, All Manufacturing Industries	-.0125	.0073	-.418	.116
Total, Durable Goods Industries	-.0113	.0077	-.364	.071
Total, Nondurable Goods Industries	-.0110	.0109	-.259	.001
Stone, Clay, and Glass Products	-.0044	.0062	-.188	-.034
Chemicals and Allied Products	-.0165	.0052	-.647	.378

TABLE 5.4

RESULTS OF REGRESSIONS PERFORMED ON MANUFACTURERS' RATIOS OF RAW MATERIALS AND SUPPLIES INVENTORIES TO MANUFACTURERS' SHIPMENTS, UNITED STATES, 1955-1970. REGRESSIONS PERFORMED ON APPARENT INTEREST RATE AND CHANGE OF GNP DEFLATOR

Industry Group	Regression Coefficient	Standard Error of Regression Coefficient	Beta Coefficient	Regression Coefficient	Standard Error of Regression Coefficient	Beta Coefficient	Coefficient of Multiple Determination
	of Apparent Interest Rate			of Change of GNP Deflator			
	Without Price-Indexes Correction						
Total, All Manufacturing Industries	-.0435	.0080	-.882	.0079	.0057	.226	.640
Total, Durable Goods Industries	-.0328	.0110	-.666	.0133	.0079	.382	.327
Total, Nondurable Goods Industries	-.0474	.0113	-.783	.0010	.0080	.024	.539
Stone, Clay and Glass Products	-.0054	.0077	-.176	.0119	.0054	.552	.158
Chemicals and Allied Products	-.0257	.0076	-.676	.0157	.0055	.584	.457
	With Price-Indexes Correction						
Total, All Manufacturing Industries	-.0339	.0082	-.794	.0052	.0059	.170	.501
Total, Durable Goods Industries	-.0142	.0121	-.321	.0103	.0086	.329	.007
Total, Nondurable Goods Industries	-.0463	.0112	-.766	-.0012	.0080	-.027	.541
Stone, Clay, and Glass Products	.0111	.0078	.331	.0098	.0056	.410	.273
Chemicals and Allied Products	-.0246	.0077	-.672	.0139	.0055	.534	.414

of all industry groups correlate well with apparent interest rate, when no price-indexes corrections are made, and that inventory-shipments ratios of aggregate manufacturing industries and of nondurable goods industries are significantly correlated with apparent interest rate, even when price-indexes corrections are made.

Confirmation of these results is obtained when a regression is performed on first differences of aggregate inventory-shipments ratios against first differences of real interest rate, with a one-year lag. It was stated in section 5.2. that theoretically one should obtain same results in this way as when inventory-shipments ratios are correlated against real interest rate values. The regression coefficient is found to be -.0086 with a standard error of .0039 and a coefficient of determination, corrected for degrees of freedom, of .202. A multiple regression of first differences of aggregate inventory-shipments ratios against acceleration in price-level movements and against changes of nominal interest rate resulted in the following regression coefficients and standard errors: .00289 (.0049) and -.01365 (.0168), therefore, with right signs in the regression coefficients, but low significance.

Further confirmation of previous regression studies is obtained from the following tests of hypotheses concerning inventory-shipments ratios as a function of real interest rate. First, aggregate inventory-shipments ratios are segregated in two categories, according to whether they correspond to periods of low ($\leq$ 2.5% p.a.) real interest rate and to periods of rather high real interest rate (> 2.5% p.a.). Table 5.2 indicates that there are 4 years of low real interest rate: 1956, 1957, 1965, and 1966. The twelve remaining years of the period studied have a real interest rate larger than 2.5% p.a. We compute the average value of inventory-shipments ratios of years 1957, 1958, 1966, and 1967, and the average value of this ratio for the 12 remaining years. The difference between these average ratios, .6281 - .5965 = .0316, is subjected to a two-tail t-test of differences; the standard deviation of the ratios is .0515; the value of t is found to be 1.1, which indicates that the difference between groups is significant at a little better than a 70% level of confidence. Repeating the procedure for aggregate ratios with price-indexes correction, one obtains significance at a slightly lower than 70% level, for a difference of .0253 and a standard deviation of the ratios of .0445.

Next, the six years (with a one-year lag) corresponding to high real interest rate (>4% p.a.), namely, 1960, 1961, 1962, 1963, 1964 and 1967, are segregated from the ten remaining years. The average ratios of both groups are computed; the difference is .5846 - .6162 = -.0316, yielding, therefore, also, a 70% level of confidence in the test of the difference between groups; a 53% level of confidence is obtained when the test is repeated with price-indexes correction.

Similar t-tests of differences lead to results of the same order of magnitude for disaggregated inventory-shipment ratios in the four industry groups analyzed. It must be recalled that the 15-years period studied is marked by ups and downs in the movements of price-level changes and nominal interest rate, as Figure 1-5 and 4-1 indicate, resulting in periods of inflation, deflation, stability, uncontrolled inflation, and controlled inflations succeeding each other, at rather short intervals of time; short-lived movements in price-level changes and interest rate are likely not to be perceived nor acted upon by inventory managers. Besides, price-level changes measured in terms of wholesale price index have been far from parallel with price-level changes measured in terms of GNP. As a result, a year which would be characterized as one of uncontrolled inflation on the basis of one price-level index could be characterized as a year of controlled inflation on the basis of another price-level index.

For year 1967, for instance, real interest rate based on change in WPI was 6% p.a., and, based upon change in GNP deflator, it was 3% p.a., making it probably difficult for the inventory manager to follow a consistent policy of increase or decrease of his inventories. In contrast, inflation in Brazil in the period shown in Table 5-1 had been marked by uncontrolled inflation clearly followed by controlled inflation, easily identifiable by the inventory manager.

These results may be interpreted to mean that some manufacturers correctly forecast future interest rates and changes in costs of materials, basing their forecasts upon past data, and modify their inventories-shipments ratio in accordance with anticipated trends in those variables, giving more attention to interest rates than to changes in costs.

The correlation results are not significant, either because actually no relationship exists between cur variables, or because the proportion of managers who take these factors into account is not considerable, as our field survey had indicated. The aggregate model postulates that all managers would shape up their inventory policy at the same period of the year, which is certainly not a fact and contributes to decreasing the correlation significance.

More regression studies of inventories on interest rate and cost changes, conducted at a lesser degree of aggregation, should be undertaken in order to reach definitive conclusions on this problem.

SUMMARY AND CONCLUSIONS

This paper is written from the point of view of an operations manager who is working in an economic environment characterized by rising costs of materials, labor, and interest rate. A study was made of how he should take into account anticipated increases in costs in the inputs, so as to minimize inventory costs.

In an inflationary situation, three factors are of major importance in shaping inventory policy: the rise in costs of material and labor, the rise in interest rate, and the ability of the firm to readjust its selling prices. Inasmuch as this last factor is not subject to his control, the inventory manager must take in consideration the distortions in the size and the lags in timing of the increases in the two first factors. Among the several possible inflationary patterns, two seemed of particular importance for this study: they were called, for convenience, uncontrolled and controlled inflation. Uncontrolled inflation is characterized by continuous small increases and a lag of increase in interest rate in relation to increases in other inputs--that is, by a low real interest rate. Controlled inflation is characterized by intermittent step increases and a real interest rate larger than the increase in material and labor would justify.

An examination was made of how to incorporate anticipated costs changes into the structure of traditional quantitative inventory models. Elaborating on models published by diverse authors, the constant review inventory system was analyzed, and the optimal amount to purchase in advance of an anticipated step cost increase was determined. Rules valid for the case in which the size of the increase or its timing are of a probabilistic nature were formulated; these rules could only be made operational for a uniform density distribution of size and time of increases in costs.

Formulas were obtained, or extended, for the case of successive, linear increases in costs; they take into account different rates of increases in material, labor, and interest rate and make it possible to compute a revised economic lot size. The case of successive irregular step increases in costs was also studied. Several problems involving joint probability distributions of cost increases in material and interest rate and discounting for time value of money were solved.

Periodic review inventory models were examined next. The appropriate amount which should be purchased in advance when the firm follows a hand-to-mouth purchasing policy was determined. Rules for the proper amount and timing to purchase in advance when increases in costs are probabilistic in size, in timing, or in both were derived for a case in which simplifying assumptions concerning the independence of the variables and the nature of the probability distributions are made. The effect of anticipated changes in costs upon the reorder point and the reordering level of probabilistic demand models was investigated: an increase in costs of material tends to increase the s-S levels in anterior periods and to decrease them in later periods in comparison with levels prevailing under stable cost conditions.

Potential savings to be secured by adopting formulas which take into account anticipated changes in costs were calculated. It was found that traditional models can be used without appreciably impairing cost optimality, so long as anticipated increases in costs are small; when they are large, savings generated by the revised formulas constitute a sizable percentage of controllable costs.

The previous analysis was extended to the case of a sustained inflation, covering a series of periods. It was shown that, theoretically, inventory levels are a function of annual changes in costs and of the value of the nominal interest rate prevailing each year; alternatively, inventory changes--that is, annual investments and disinvestments in inventories--are a function of the acceleration in inflation and of the annual changes in nominal interest rate. The effect of changes in costs can be combined with the effect of changes in nominal interest rate through the use of the concept of real interest rate.

A number of managers were asked how they took anticipated changes in costs into consideration in their inventory strategy. Seventeen executives in the United States and seventeen in Brazil were interviewed. The economic situation of both countries was dominated by a state of rather high, controlled inflation--after years of uncontrolled inflation. The survey showed that a minority of firms accepted cost minimization as the main objective of their inventory policy. Nonetheless, most firms in the sample were sensitive to both interest rate and anticipated increases in costs, and many used to take action in consequence. The previous periods of uncontrolled inflation, especially in Brazil, were remembered as periods in which large accumulation of inventories took place, while the prevailing period of controlled inflation forced a contraction in inventories, to the point that minimization of inventories was the main objective of inventory management, to the complete exclusion of cost minimization. Demand considerations and physical or financial contraints were more important than economic computation in modeling the inventory policy of the firms analyzed.

Regression studies performed on aggregate inventories in Brazil for the period 1948-1967 showed high positive correlation between inventory accumulation and inflation rate, and mild positive correlation between inventory accumulation and inflation acceleration, suggesting that, aside from rational cost minimization, other factors, possibly flight from money, speculation or hoarding, played a role in inventory buildup in the Brazilian case.

Regression studies performed in the United States for the period 1955-1970 on the ratios of manufacturers' inventories of raw materials and supplies to manufacturers' shipments indicated the existence of negative correlation between inventory levels and real interest rate, as theory suggests should happen for rational inventory management. Regressions were performed at the aggregate level and for four industry groups: namely, durable goods; nondurable goods; stone, clay, and glass products; chemical and allied products.

In relationship to the same period and the same kind of inventories, tests of hypotheses were conducted upon the difference of inventory levels in periods of low and non-low real interest rate and upon the difference of inventory levels in periods of high and non-high real interest rate. Regression studies were also conducted on inventory accumulation as a function of real interest rate change.

Although the results of the regression studies and of the tests of hypotheses were not always statistically significant at a high confidence level, the bulk of the evidence gathered indicates that the variables analyzed--price-level changes and interest rate--had an influence on actual inventory decisions.

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