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PORTFOLIO BALANCE, GROWTH, AND LONG-RUN BALANCE-OF-PAYMENTS ADJUSTMENT

by

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ABSTRACT

PORTFOLIO BALANCE, GROWTH, AND LONG-RUN

BALANCE-OF-PAYMENTS ADJUSTMENT

by

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This study is an extension of the existing analysis of macroeconomic policy and the balance of payments exemplified by the work of Professor Robert Mundell. It is argued that the Mundell analysis is applicable only in the short-run, since the Keynesian model on which it is based defines asset equilibrium in terms of flows rather than stocks and is thus incompatible with conditions of portfolio balance. It is further argued that existing attempts to correct this deficiency by imposing conditions of zero investment and constant asset stocks are equally untenable as descriptions of long-run equilibrium.

In order to analyze the long-run effects of government policy in a situation of both portfolio balance of asset growth, a variant of the recently developed neoclassical monetary growth model is employed. That model is extended by adding interest bearing government debt and foreign financial assets to wealth portfolios and including international trade in goods. In this setting it is found most convenient to abandon the traditional Keynesian specification of government

policy in terms of monetary and fiscal policy in favor of a specification in terms of debt management policy. In particular, government policy is defined in terms of three fundamental policy variables, the level, rate of growth, and composition of government debt, acting through three intermediate policy variables, the price level, the rate of inflation, and the bond interest rate.

In this framework it is shown that the balance-of-payments effect of changes in the capital account depend importantly on the flow of interest earnings on foreign assets, this flow being roughly equal in size to the flow of new foreign investment. A distinction is drawn between real and nominal magnitudes and it is argued that, contrary to the conclusion of the static portfolio balance models, continuing capital flows at a constant income level are compatible with portfolio equilibrium when price flexibility is introduced.

Composition-neutral increases in the overall level of government debt are shown to raise the level of the steady-state capital outflow and worsen the trade balance. The long-run effects of changes in the composition of a given total debt on international capital and trade flows are shown to be significant but directionally uncertain, the results depending on savings behavior, conditions of production, and relative asset preferences.

The influence of exchange rate flexibility is also investigated, and it is shown that differing rates of inflation between countries can be made compatible with balance-of-payments equilibrium through an appropriate steady crawl of the exchange rate. Central to this process is the direct effect of exchange rate variation on international capital flows acting through capital gains or losses.

Finally, the problem of the optimal composition of the balance of payments is examined. Here it is shown that purely financial capital flows have important effects on the real investment position, and that a policy of adjusting the capital account to accomodate the trade balance is sub-optimal.

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INTRODUCTION

A substantial body of literature has recently developed applying Keynesian analysis to the problem of macroeconomic policy in an open economy. Following the lead of Professor Mundell,[1] the discussion has focused on the possibility of assigning fiscal and monetary policy to the goals of domestic full employment and international payments equilibrium in such a manner that both internal and external balance can be achieved simultaneously. Such an assignment has been found to be possible as long as international capital flows respond to interest rate adjustments. This conclusion provides a solution to the Meadean dilemma faced by policymakers unable to adjust exchange rates.

Several criticisms of the Mundell analysis have arisen.[2] A recurring theme in all is that the suggested policy solutions achieve only temporary adjustments and are inadequate in the long-run. This is because the Keynesian model on which they are based involves only short-run equilibrium. More specifically, the Keynesian model describes asset equilbirium in terms of equality between flow demand and supply, where flow demands are functions of return variables only

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[1]The seminal reference is Mundell [38]. For further references see footnote 1, Chapter II.

[2]For example, Niehans [46] and McKinnon [34].

and are independent of existing asset stocks. In the short-run such stocks can be treated as given parameters defining flow demands, but in the long-run they must be explicitly considered. If not, resulting behavior is inconsistent with the theory of portfolio equilibrium. Thus for example, a government deficit which injects a constant flow of bonds into the economy cannot for long coexist with an unchanged money supply and constant bond interest rate. This long-run inconsistency of the Keynesian model is of key importance for the Mundell-type analysis since it calls into question the theory of international capital movements upon which the solution of the assignment problem crucially depends.

Attempts have been made to recast the internal-external balance analysis in terms of models incorporating full stock equilibrium in the asset sector.[3] Their conclusions have been pessimistic as to the possibility of reaching both targets employing only monetary and fiscal policy instruments, since they suggest that permanent capital flows cannot be induced by discrete interest rate adjustments. The strength of this conclusion is lessened however, due to the fact that portfolio balance is included in these models by assuming a stationary non-growth world, an assumption equally untenable in the long-run.

The aim of this dissertation is to develop a model which incorporates both portfolio equilibrium and asset growth and to employ it in an analysis of the effect of government macro-policy instruments

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[3]Notable here are McKinnon [34] and McKinnon and Oates [35].

on internal and external balance in the long-run. The traditional Keynesian framework will be abandoned in favor of an extension of the recently developed neoclassical monetary growth model. The conclusions reached differ considerably from those of both the Mundell and static portfolio balance analyses. In particular, it will be argued that interest rate policy does have continuing effects on international capital flows but that these effects are more complicated and differently motivated than suggested in the standard internal-external balance literature.

The analysis proceeds as follows. Chapter II reviews critically the Mundell model and several portfolio balance variants. Chapter III develops a theory of international capital movements based on stock equilibrium and asset growth, and analyses the effects of government policy instruments on the steady-state capital account. In Chapter IV the analysis is extended to include the trade account and the assignment of policies needed to achieve internal and external balance is derived. The implications of exchange rate flexibility are considered in Chapter V. Chapter VI consists of an examination of the composition problem, i.e. the optimal mix between the trade and capital accounts in a zero overall payments balance, and Chapter VII contains final conclusions and comments.

CHAPTER II

A CRITIQUE OF EXISTING MACROECONOMIC MODELS OF BALANCE-OF-PAYMENTS ADJUSTMENT

2.1 The Mundell-Keynesian Model and the Internal-External Balance Problem

Much of the recent analysis of the assignment or internal-external balance problem has been based on an amended version of the standard Keynesian model. What may be referred to as the Mundell model can be described by the following equations:[1]

(1) $Y = C(Y) + I(r) + G + T(Y, e)$,

(2) $\dot{R} = T + K(r)$,

(3) $M = R + Q = L(Y, r)$,

(4) $\dot{M} = \dot{R} + \dot{Q} = 0$,

where,

Y = national income,[2]

C = consumption

[1]Variants of this model are used by Mundell [38], [39] and [40], Fleming [9], Krueger [29], Johnson [22], Jones [24], Kemp [26], Niehans [46], Helliwell [20], Sohmen [55], and Sohmen and Schneeweiss [56].

[2]All variables are expressed in real terms since prices are assumed constant. Krueger [29] and Helliwell [20] depart from this assumption and allow prices to vary according to a relation of the form $p = P(Y)$, $dp/dY > 0$. This addition does not qualitatively alter the conclusions which follow.

G = government expenditure,[3]

I = investment,

T = the balance of trade,

r = the bond interest rate,

e = the exchange rate (the price of foreign currency),

R = the stock of foreign exchange reserves,

$\dot{R} = \frac{dR}{dt}$ = the rate of government purchases of foreign exchange reserves, or the balance of payments,

Q = the level of central bank holdings of government bonds

$\dot{Q}$ = the rate of central bank open market purchases,

K = the rate of net capital inflow,

L = the demand for money,

and M = the money supply, or total central bank liabilities.[4]

Equation (1) describes flow equilibrium in the goods market. Equation (2) defines the balance of payments as the sum of the trade balance and net capital inflow, where the latter is assumed to depend only on the interest rate.[5] Equilibrium in the money market is described by

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[3] Taxes are assumed to be zero.

[4] Money is created only by central bank purchases of bonds or foreign exchange and 100 per cent reserve banking is assumed. Note that the exclusion of G from the money supply equations requires that the government deficit be financed entirely by bond sales to the public rather than the central bank.

[5] An exception is Johnson [22] who includes income in the capital flow function on the grounds that direct investment in the home country is more attractive at higher output levels.

equation (3) which states that desired and actual stocks must be equal, and equation (4) which states that in equilibrium the money supply is unchanging.

The total differential of equations (1) - (4) can be written in matrix notation as

$$(5)\quad \begin{bmatrix} (1-c+m) & -I_r & -T_e & 0 \\ L_y & L_r & 0 & 0 \\ m & -K_r & -T_e & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dY \\ dr \\ de \\ d\dot{R} \end{bmatrix} = \begin{bmatrix} dG \\ dM = dQ + dR \\ 0 \\ -d\dot{Q} \end{bmatrix},$$

where

$c = C_y = \frac{dC}{dY}$, $0 < c < 1$, $m = - T_y$, $0 < m < 1$, $I_r < 0$, $T_e > 0$, $L_y > 0$, $L_r < 0$ and $K_r > 0$.

Most writers on the internal-external balance problem have assumed that the domestic money supply is "sterilized" from the effects of foreign exchange movements. This assumption requires that open market operations be separated analytically into two components: offsetting open market operations which are undertaken to neutralize the monetary effect of reserve changes, and independent open market operations which are used to change the money supply. In terms of equation (5), $dQ = dQ_1 + dQ_2$, and under conditions of sterilization, $dQ_1 = - dR$, $d\dot{Q}_1 = - d\dot{R}$ and $dQ_2 = dM$.[6] In this case the balance-of-payments effects of monetary, fiscal, and exchange rate policy are given by

$$(6) \quad \frac{d\dot{R}}{dQ_2} = \frac{d\dot{R}}{dM} = \frac{K_r\ (1-c+m)\ -mI_r}{L_r\ (1-c+m)\ +\ L_yI_r} < 0,$$

$$(7) \quad \frac{d\dot{R}}{dG} = - \frac{L_rm + K_rL_y}{L_r\ (1-c+m)\ +\ L_yI_r} \gtrless 0 \text{ as } \phi = L_rm + K_rL_y \gtrless 0,$$

and

$$(8) \quad \frac{d\dot{R}}{de} = \frac{(1-c)\ L_r + L_y\ (I_r-K_r)}{L_r\ (1-c+m)\ +\ I_rL_y} T_e > 0.$$

Equation (6) indicates that a decrease in the money supply will always improve the balance of payments. As the money supply declines, the interest rate rises. This leads directly to a capital inflow and indirectly to a reduction of imports as investment and income decline. The balance-of-payments effect of a change in government expenditure is uncertain, as the capital and current accounts move in opposite directions. For example, a decrease in government expenditure improves the trade balance by lowering income. With the money supply fixed, however, this fall in income lowers the interest rate causing a capital outflow. If the interest elasticity of international capital movements is large enough ($\phi > 0$) the net effect will be a decrease in reserves. A

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[6] These assumptions are often not made explicitly and occasionally misspecified. For example, Sohmen [55], writes equations (2) and (3) as,

$$(2') \quad R - R_o = T + K(r),$$

and $(3') \quad Q + R = L(Y, r)$,

and uses Q as the monetary policy variable. This requires the variable R to be a flow in equation (2') and a stock in equation (3'). The correct specification of this stock-flow reserve relation can be found in Sohmen and Schneeweiss [56] and Roper [52].

devaluation unambiguously improves the balance-of-payments. The direct effect is an improvement of the trade balance. This improvement is partially offset by an induced rise in imports caused by an increase in income. This income increase, however, has a positive effect on the capital account, as it causes a higher interest rate. Notice that devaluation has no direct effect on international capital flows, i.e., $K_e = 0$.[7]

The two policy goals in this system are desired or full employment income and an equilibrium level of reserve changes, (possibly zero). With complete knowledge of the structure of the economic system, policymakers can set the correct levels of the two policy tools needed to achieve these goals. If, however, policymakers do not have full knowledge, i.e. if they are unsure of the relative magnitudes or possibly even the directional effects of changes in policy parameters, an efficient policy decision rule may be to assign each policy tool exclusively to a particular goal or target.[8] The correct assignment or matching of tools and goals can be discovered from an examination of Figures 1 and 2. In each figure combinations of policy variables giving the desired level of income (XX') and rate of change of reserves (FF') are plotted. The internal balance lines

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[7] Johnson does note that there may be an impact, "depending on the effect of the exchange rate on the domestic value of capital imports, and possibly including speculative effects of exchange rate changes . . ." [22, p. 348] but assumes $K_e = 0$ in his formal analysis.

[8] The reasoning leading to this conclusion is spelled out in Mundell [41, Chapters 14 and 21].

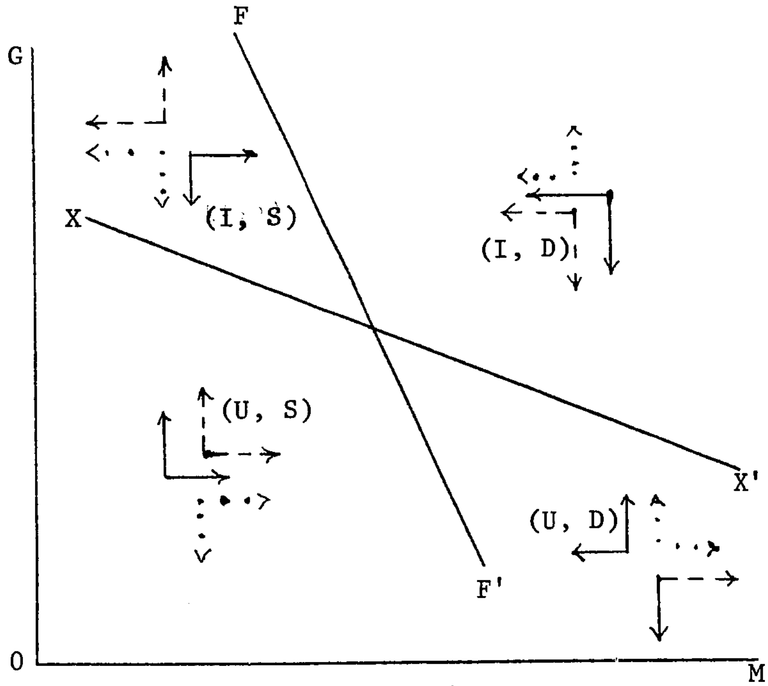

Figure 1.

Internal - External Balance Lines ($\phi < 0$)

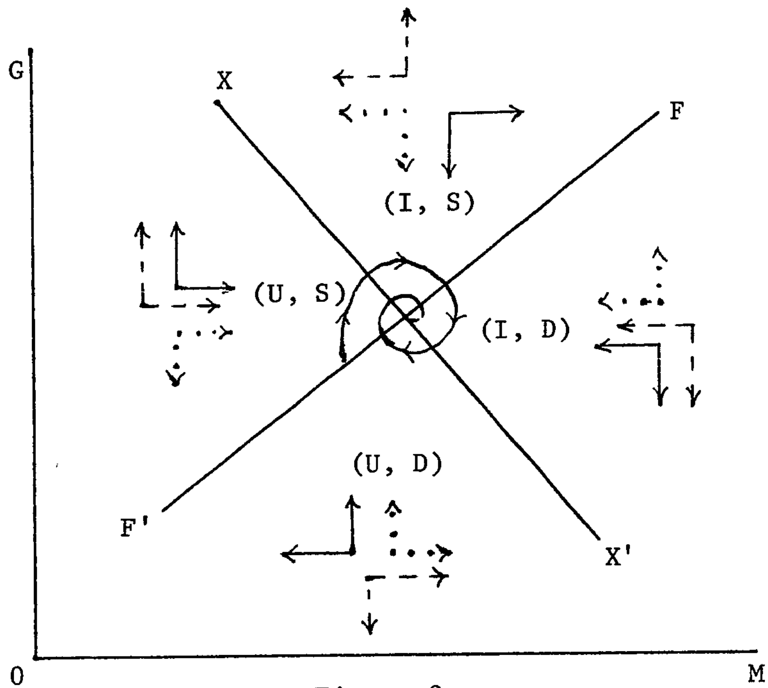

Figure 2.

Internal - External Balance Lines ($\phi > 0$)

separate positions of unemployment (U) and inflation (I), while the external balance lines separate positions of balance-of-payments deficit (D) and surplus (S). The slopes of the XX' and FF' lines can be computed from (5), and are, respectively,

$$(9)\ \left(\frac{dG}{dM}\right)_{dY=0} = -\frac{I_r}{L_r} < 0,$$

and

$$(10)\left(\frac{dG}{dM}\right)_{\dot{d}R=0} = \frac{K_r\ (1-c+m)\ -\ mI_r}{\phi} \gtrless 0 \text{ as } \phi \gtrless 0,$$

where ϕ was defined previously.

The XX' line is always downward sloping since increases in G and M always raise income. The sign of the slope of the FF' line is uncertain. If an increase in government expenditure worsens the balance-of-payments, FF' is downward sloping (Figure 1). If an improvement results, the slope of the FF' curve becomes positive (Figure 2). There are two possible assignments that can be made.

I. Assign monetary policy to external balance and fiscal policy to internal balance.

A. Raise (lower) the money supply if the balance-of-payments is in surplus (deficit) and raise (lower) government expenditure in a situation of unemployment (inflation).

II. Assign fiscal policy to external balance and monetary policy to internal balance.

A. <u>Raise</u> (lower) government expenditure if the balance-of-payments is in surplus (deficit) and increase (decrease) the money supply in a situation of unemployment (inflation).

B. Lower (raise) government expenditure if the balance-of-payments is in surplus (deficit) and increase (decrease) the money supply in a situation of unemployment (inflation).

The division of II into A and B reflects the uncertainty of policy-makers as to the effect of fiscal policy on the balance-of-payments. The results of assignments I, II A and II B are shown in the Figures 1 and 2 by the solid, dashed and dotted lines respectively.

Consider first Figure 1. With $\phi < 0$ the FF' line will be downward sloping and steeper than the XX' line. It can be seen that in this case assignment I is stable, in the sense that from any initial position a movement toward equilibrium occurs. Assignment II is unstable for both cases A and B. For case A, where the effect of fiscal policy on the balance-of-payments is correctly understood, equilibrating movements result in two quadrants, but in the dilemma situations -- unemployment and payments deficit or inflation and surplus -- this assignment causes movements away from target values. In case B, dynamically unstable cyclical movements tend to occur, regardless of the initial position. Figure 2 shows the results when $\phi > 0$ and FF' is upward sloping. In this case assignment I is also stable, but the movement to equilibrium may be cyclical as indicated graphically. If, for example, a situation of unemployment and a balance-of-payments deficit exists, government expenditure is raised and the money supply reduced. These measures both improve the balance-of-payments but have conflicting effects on income, and there is a tendency for the payments deficit to be eliminated before full employment is reached. As the economy moves into the unemployment-surplus quadrant, monetary policy becomes expansionary and both variables act to raise income but have conflicting effects on the balance-of-payments. In this case inflationary conditions develop before the payments surplus is eliminated, moving the economy into the inflationary-surplus quadrant, and so on. Assignment II B causes similar stable movements, with the direction of any cycles reversed. In both cases any cyclical movements could be avoided by adding to assignments I and II B the rule that only fiscal policy is used in dilemma situations and only monetary policy at other times. Assignment II A, where fiscal policy is incorrectly applied, causes unstable movements.

Consideration of Figures 1 and 2 indicates, therefore, that only assignment I assures a stable movement to policy targets in all cases. This conclusion that monetary policy should be assigned to external balance and fiscal policy to internal balance was also reached by Mundell in his original formulation of the assignment problem [38], with the difference that Mundell defined monetary policy in terms of the interest rate rather than the money supply.[9] This distinction is of crucial importance for Mundell's further assertion

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[9]Following Jones [24] r may be called an intermediate policy variable and M and G fundamental policy variables. Notice that under Mundell's definition a change in monetary policy involves a change in only one fundamental policy variable, M, while a change in fiscal policy requires a change in both, adjustments of M being required in the latter case in order to peg r.

that his assignment followed from his principle of effective market classification [41, Chapters 11 and 15], under which policy tools are assigned to targets on which they have the greatest relative effect. This principle suggests that monetary policy should be assigned to external balance and fiscal policy to internal balance if and only if

$$(11) \qquad \left| \frac{d\dot{R}/dX}{dY/dX} \right| > \left| \frac{d\dot{R}/dG}{dY/dG} \right| ,$$

where X is the monetary policy variable, M or r. Defining monetary policy in terms of the interest rate and setting X = r, condition (11) reduces to

$$(12) \qquad K_r \; (1\text{-}c\text{+}m) > 0,$$

and therefore holds unambiguously. However, if monetary policy is defined in terms of the money supply and X = M, inequality (11) will hold only if

$$(13) \qquad \psi = K_r \; [L_r \; (1\text{-}c\text{+}m) - I_r L_y] - 2m \; L_r I_r < 0.$$

If $\psi > 0$, application of the policy comparative advantage criterion leads to the opposite assignment of monetary policy to internal balance and fiscal policy to external balance. If, therefore, ψ is not known with certainty and monetary policy is conducted in terms of the money supply, an incorrect assignment may be made, causing instability.[10] This result casts doubt on the general applicability of Mundell's principle of assignment. An alternative criterion which precludes the possibility of an unstable assignment would be to avoid directing policy variables to targets on which their influence is uncertain. According to this rule, fiscal policy should be assigned the task of domestic stabilization since its directional impact on the balance-of-payments is unknown.[11] Whatever the exact assignment however, the most important conclusion remains: as long as

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[10]Uncertainty in this case arises from the fact that the external balance line may be upward or downward sloping in (M, G) space and the system may be described by Figures 1 or 2. This uncertainty disappears if monetary policy is conducted in terms of the interest rate since the external balance line is always sloped in the same direction as, and steeper than, the internal balance line in (r, G) space. Mathematically,

$$0 < \left(\frac{dG}{dr}\right)_{dY=0} = - I_r < \left(\frac{dG}{dr}\right)_{d\dot{R}=0} = - I_r + \frac{K_r}{m} (1-c+m),$$

and the dichotomy depicted by Figures 1 and 2 cannot arise.

This result in conjunction with the analysis presented above would indicate that in a world of uncertainty monetary policy should be conducted in terms of interest rate rather than money supply levels in order to minimize fluctuations of target variables. Other factors may of course influence the choice of the monetary policy target. (See, for example, Poole [50].)

[11]This alternative criterion is not satisfactory, however, if policy-makers approach goals one at a time. In this case a cobweb cycle is generated with $\phi > 0$, and the stability of assignments will depend on the relative slopes of the XX' and FF' lines. Since these slopes are

$$\left(\frac{dG}{dM}\right)_{dY=0} = - \frac{dY/dM}{dY/dG} \text{ and } \left(\frac{dG}{dM}\right)_{d\dot{R}=0} = - \frac{d\dot{R}/dM}{d\dot{R}/dG},$$

this is determined by inequality (11). If, for example, fiscal policy has a relatively stronger effect on the balance-of-payments, the XX' curve will be steeper than the FF' curve and assignment I will produce an explosive cobweb pattern. To avoid instability an attempt must be made to apply the principle of effective market classification, i.e. determine the direction of the inequality in (11), even if this cannot be done with certainty.

international capital movements respond to interest rate changes, monetary and fiscal policy tools are sufficient to achieve both employment and balance-of-payments goals.

Two further situations frequently discussed in the internal-external balance literature -- changes in monetary and fiscal policy under conditions of no sterilization and flexible exchange rates -- should be mentioned briefly. In these situations an assignment problem as such does not arise, since in both, equilibrium in the balance-of-payments is assured and external balance ceases to be an explicit goal. Interest here has therefore been focused on the difference in income effects of changes in policy variables under alternative exchange rate systems.[12] The results for non-sterilization can be derived by setting $d\dot{Q}_1 = dQ_1$ in equation (5). This gives[13]

$$(14) \quad \frac{dY}{dQ_2} = 0, \; \frac{dR}{dQ_2} = -1,$$

and,

$$(15) \quad \frac{dY}{dG} = \frac{K_r}{(1-c+m)\,K_r - mI_r} > 0, \; \frac{dR}{dG} = \frac{K_r L_y + mL_r}{(1-c+m)\,K_r - mI_r} \gtrless 0 \text{ as } \phi \gtrless 0.$$

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[12]Among those prominent in this discussion have been Fleming [9], Krueger [29], Mundell [39], Sohmen [55], and Sohmen and Schneeweiss [56].

[13]If capital is perfectly mobile, $K_r \longrightarrow \infty$. Equation (14) remains unchanged and equation (15) becomes

$$(15') \quad \frac{dY}{dG} = \frac{1}{1-c+m} > 0, \; \frac{dR}{dG} = \frac{L_y}{(1-c+m)} > 0.$$

Equations (14) and (15') describe mathematically the conclusions reached by Mundell [39] as to the effects of perfect capital mobility under fixed exchange rates. As pointed out by Roper [52], these results depend upon Mundell's implicit assumption of a non-sterilization policy, as well as perfect capital mobility.

Attempts to change the money supply by open market operations have no effect on income, leading only to an equivalent change in the reserve level. This is because any increase in the money supply tends to cause a worsening of the current and capital accounts. Reserves are lost and the money supply declines until it reaches its original level and the balance-of-payments returns to equilibrium. Fiscal policy does have a positive income effect since changes in fiscal policy move the current and capital account in opposite directions.[14]

The situation under flexible exchange rates is described by (5) with $d\dot{R} = dR = 0 = dQ_1 = d\dot{Q}_1$, de being determined endogenously. Here,

$$(16) \qquad \frac{dY}{dQ_2} = \frac{dY}{dM} = \frac{I_r - K_r}{(I_r - K_r) L_y + L_r (1-c)} > 0,$$

and

$$(17) \qquad \frac{dY}{dG} = \frac{L_r}{(I_r - K_r) L_y + L_r (1-c)} > 0.$$

Induced exchange rate adjustments will heighten or reduce the expansionary effects of money supply or government expenditure increases by directly affecting the trade balance.

The relationship between income changes under conditions of sterilization, non-sterilization and flexible exchange rates is summarized in Table 1. It indicates that for changes in the money

[14]With a non-sterilization policy, $\dot{R}$ must be zero in equilibrium. Fiscal expansion therefore causes an increase in the capital inflow and worsening of the trade balance of equal magnitude, i.e.

$$\frac{d\dot{R}}{dG} = 0, \quad \frac{dK}{dG} = - \frac{dT}{dG} > 0.$$

Table 1

Income Effects of Monetary and Fiscal Policy
Under Varying Exchange Rate Systems

	Fixed Exchange Rates: No Sterilization	Fixed Exchange Rates: Sterilization	Flexible Exchange Rates
Monetary Policy $\frac{dY}{dQ_2}$	0	$\frac{I_r}{\theta}^{a} > 0$	$\frac{I_r - K_r}{\theta - \phi}^{a} > 0$
Fiscal Policy $\frac{dY}{dG}$	$\frac{L_r}{\theta - I_r K_r \phi} > 0$	$\frac{L_r}{\theta} > 0$	$\frac{L_r}{\theta - \phi} > 0$

a. also equal to $\frac{dY}{dM}$

$$\theta = I_r L_y + L_r (1-c+m) < 0$$

$$\phi = L_r m + K_r L_y$$

supply, the relative movements of income are,

(18) No sterilization < Sterilization < Flexible exchange rates.

For changes in fiscal policy,

(19) No sterilization $\gtrless$ Sterilization $\gtrless$ Flexible exchange rates as $\phi \gtrless 0$. With no sterilization a deficit in the balance-of-payments results in either an exchange rate depreciation or a decrease in the money supply as foreign exchange reserves decline. A depreciation <u>raises</u> income by increasing the trade balance. A decrease in the money supply <u>lowers</u> income, thereby lowering imports and improving the trade balance. Since under conditions of sterilization, expansionary monetary policy causes a balance-of-payments deficit, the relationship indicated by (18) holds. An increase in government expenditure may worsen or improve the payments position, and the relative income-effectiveness of fiscal policy under the varying exchange rate systems will depend on ϕ as indicated in (19).

2.2 The Static Portfolio Balance Model

The model outlined in the previous section and the policy conclusions based upon it are subject to several criticisms. These may be summarized by saying that the equilibrium positions described by it are only short-run in nature. Central here is the lack of portfolio equilibrium in the underlying Keynesian model. The portfolio theory of asset holdings as outlined by Tobin [60] asserts that the desired stock holdings of a particular asset will depend on the size of the overall wealth portfolio and the risks and rates of return associated with it and competing assets. For real capital and bonds, the Keynesian model specifies asset demands which are functions only of return variables and independent of the underlying stock holdings. This is clear in the case of capital goods, since an investment function of the form $I = I(r)$ is used. This is less obvious in the case of bonds, since Walras' Law is invoked to exclude explicit consideration of the bond market. An examination of the implicit definition of equilibrium in the bond market is facilitated by making the simplifying assumption that all government expenditures and private investment is financed by the issuance of bonds which are perfect substitutes, and that all private saving (S) goes into bond purchases. The flow demand for, and supply of, bonds can be described by

$$(20) \quad \dot{B}_d = S + K, \; \dot{B}_s = I + G - \dot{Q}.$$

Demand comes from domestic savers and foreign investors,[15] and supply arises from government deficits, private entrepreneurs undertaking capital investment and open-market sales. In equilibrium excess flow demand should be zero, i.e.

$$(21) \quad \dot{B}_e = \dot{B}_d - \dot{B}_s = 0.$$

This is assured by equilibrium in the goods and money markets, since from equations (1), (2), (4) and (20),

$$(22) \quad \dot{B}_d - \dot{B}_s = (Y-C) + (R-T) - I - G - \dot{R} = 0.$$

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[15] If domestic residents are net purchasers of foreign bonds, $K < 0$ and $S + K$ still gives net demand for domestic bonds.

Equation (20) implies a flow demand function of the form

(23) $\dot{B}_d = B(Y, r)$.

Portfolio selection theory would suggest that the B function in (23) describes rather the stock bond demand, and that in an equilibrium situation where Y and r are constant, bond market equilibrium requires $\dot{B}_e = \dot{B}_s = \dot{B}_d = 0$. Clearly equations (20) and (21) are inconsistent with equilibrium described in this manner.[16] From this reasoning it follows also that an international bond demand function of the form K(r) is not compatible with portfolio equilibrium theory.

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[16]Thus Krueger [29] is in error when she specifies a stock bond demand function of the form $B_d = B(Y, r)$ in conjunction with equations (1) - (4). She does not specifically include this function in her analysis, explaining that

> . . . the demand for bonds, and the demand for money are interdependent. If the demand for money is satisfied, then the bond market must also be in equilibrium. Hence [the stock bond demand equation] is not required in the system, but is written as a reminder that bonds are taken into account. [29, p. 198.]

While it is true that equilibrium in the Keynesian system implies a type of equilibrium in the bond market, it does not imply stock equilibrium; Walras' Law does not assure that the omitted market is consistent with demand functions however specified. Rather it determines the demand function in this market consistent with general equilibrium.

This confusion may arise from the fact the Krueger defines a change in the government deficit as $dG = dB + dM$ [29, p. 199], and the supply of bonds as $B_s = B_o + dB$. [29, p. 197.] The symbol dB has two different dimensions in these two equations, in the first being a flow per unit time and in the second a stock.

A similar mis-specification of bond market equilibrium is made by Arndt [1] and Helliwell [20].

In contrast with the situation in the bond market, money market equilibrium <u>is</u> described in terms of stocks in the Keynesian model. This can be made clear by rewriting equations (3) and (4) as

(24) $M_d = L(Y, r) = M_s$,

and

(25) $\dot{M}_e = \dot{M}_d = \dot{M}_s = 0$.

A model describing a constant equilibrium income level which is consistent with the theory of portfolio selection requires a symmetric specification for the bond market. This can be done by restricting the Keynesian model to positions of stationary equilibrium.[17] Such a static portfolio balance model may be described by[18]

(26) $Y = C + G + T(Y, e)$,

(27) $M = D + R - Q = L(Y, r)$,

(28) $B = Q = J(Y, r)$,

(29) $F = N(Y, r)$,

(30) $H = V(Y, r)$,

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[17] An alternative approach avoiding the asymmetrical treatment of the bond and money markets is provided by Komiya [28], who assumes flow demand functions of the form

$$\dot{B}_d = H\ (Y, r, M_o, B_o),$$

and $\dot{M}_d = J\ (Y, r, M_o, B_o)$.

Equilibrium must be short-run only, since B_o and M_o will change over time if $\dot{B}_s$ and $\dot{M}_s \neq 0$. For a further analysis of the Keynesian system using symmetrical asset demand functions see Edwards [8].

[18] This formulation is based on, but not identical to, that found in McKinnon [34]. Variants of this model are also used by Levin [32, Ch. VI], McKinnon and Oates [35], and Harkness [19].

(31) $G = \dot{D}$,

(32) $T = \dot{R}$,

and

(33) $\dot{M} = \dot{B} = \dot{F} = \dot{H} = 0$,

where Y, C, r, M, L, G, T, R, $\dot{R}$ and e were defined previously,

B or Q is the stock of government bonds held by the public,[19]

J is the public's stock demand for government bonds,

F(N) is the stock supply of (demand for) foreign bonds,

H(V) is the stock supply of (demand for) real capital,

and D is the total stock of government bonds held by the central bank (D-Q) and the public (Q).

With asset demand functions as specified in equations (27) - (30), portfolio equilibrium at a stationary income level requires that flow investment in all assets be zero as indicated in equation (33). In particular, this means that equilibrium capital flows, $\dot{F}$, must be zero.[20] These requirements constrain government policy actions. The nature of this constraint can be seen by examining equations (27), (28), (31) and (32). As indicated in equation (27), the government is assumed to finance its deficits by selling bonds (D) to the central

[19] This definition of Q differs from that of the previous section, where Q represented the bond holdings of the central bank.

[20] A possible exception is the case of perfect capital mobility where foreigners are assumed willing to purchase unlimited quantities of domestic bonds at a given interest rate. On this point see footnote 27.

bank. Some of these bonds (Q) are then resold by the central bank to the public, but in equilibrium, the rate of these open market sales must be zero, i.e., $\dot{B} = \dot{Q} = 0$. The equilibrium condition in the money market is, therefore,[21]

(34) $\dot{M} = \dot{D} + \dot{R} - \dot{Q} = G + T(Y, e) = 0.$

The equilibrium position can be determined recursively starting with the basic equilibrium equation (34), which alone determines the level of income. Since both a government deficit and a trade balance surplus cause the money supply to increase, portfolio equilibrium requires a level of income and therefore imports which makes their sum zero. Given Y, and the central bank's open market policy, which determines B, equation (28) can be used to determine the equilibrium level of the interest rate. Given Y and r, equation (27) gives the equilibrium level of the demand for money. The money supply adjusts to this level through temporary changes in the balance-of-payments. Similarly, equations (29) and (30) give the equilibrium levels of foreign bond and capital stock holdings, which can be achieved by one-shot purchases from abroad and temporary domestic investment respectively. Given equation (34), equation (26) reduces to,

(35) $Y = C.$

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[21]Equation (34) is based on the implicit assumption that reserve changes are allowed to effect the money supply. Alternatively, it could be assumed that the government both pursues a sterilization policy and finances its deficits by selling bonds directly to the public. In this case $\dot{B} = T + G$ and equation (34) describes equilibrium in terms of the bond rather than money market.

If equations (27) - (30) and (33) are satisfied for given levels of income and the interest rate, wealth holders will be at their desired portfolio positions, and therefore desired investment and savings will be zero and all income will be consumed, as indicated in equation (35).[22]

The effects of monetary, fiscal and exchange rate policy on the target variables can be determined entirely from equation (34). They are,

$$(36) \quad \frac{dY}{dQ} = 0, \frac{d\dot{R}}{dQ} = 0,$$

$$(37) \quad \frac{dY}{dG} = \frac{1}{m}, \frac{d\dot{R}}{dG} = -1,$$

and,

---

[22] McKinnon [34] writes equation (35) as

$$(35') \quad Y = C(Y, r, M, B, R, H),$$

and seems to imply that the C function is an independent behavioral relation. This function cannot, however, be independent, since, as indicated above, the equilibrium values of all variables are determined independently of equation (35); the form of the C function depends on the forms of the L, J, N and V functions specified in equations (27)-(30). In this model, it is the goods market which is omitted in determining the equilibrium position, and, as argued previously in the case of the bond market in the Keynesian model, Walras' Law can be used to show that specification of equilibrium in the omitted market is endogenously determined.

Alternatively, the C function could be made independent and the equilibrium conditions analyzed in terms of equation (35'). In this case the L, J, N and V functions could not be specified independently but would depend on the form of the C function chosen.

(38) $\frac{dY}{de} = \frac{T_e}{m}, \frac{d\dot{R}}{de} = 0.$[23]

These results are most easily understood by depicting equilibrium adjustments graphically. Figure 3 shows the adjustment resulting from contractionary monetary policy. The MM', BB', and FF' lines give equilibrium conditions in the money, domestic bond and foreign bond markets respectively.[24] The PY_o line is based on the basic equilibrium equation (34).

The original equilibrium is at point a. An open market sale of bonds by the central bank ($dQ > 0$), lowers the money supply and caused M_oM_o' and B_oB_o' to shift originally to M_1M_1' and B_1B_1', asset equilibrium being provisionally established at point b. However, asset equilibrium requires income to fall to Y_1, improving the trade balance. This causes the money supply to increase as foreign exchange reserves rise, and MM' continues to shift down along B_1B_1' until income returns to its original level at c.[25]

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[23]A devaluation does have a positive reserve impact if a constant tax rate (h) is in existence. In this case (34) becomes

(34') $G - h\,Y + T\,[(1-h)\,Y, e] = 0,$

and

$$\frac{d\dot{R}}{de} = \frac{hT_e}{h + (1-h)m} > 0.$$

[24]The capital goods line has been omitted for the sake of simplicity. The slopes of the MM', BB' and FF' lines follow from the assumption that L_r and $N_r < 0$, and L_y, J_y, J_r and $N_y > 0$.

[25]Figure 3 is drawn on the assumption that $|J_r| > |L_r|$. If this inequality was reversed, point b would be south-east of point c and income would temporarily rise, a temporary worsening of the balance-of-payments decreasing the money supply and restoring equilibrium at c.

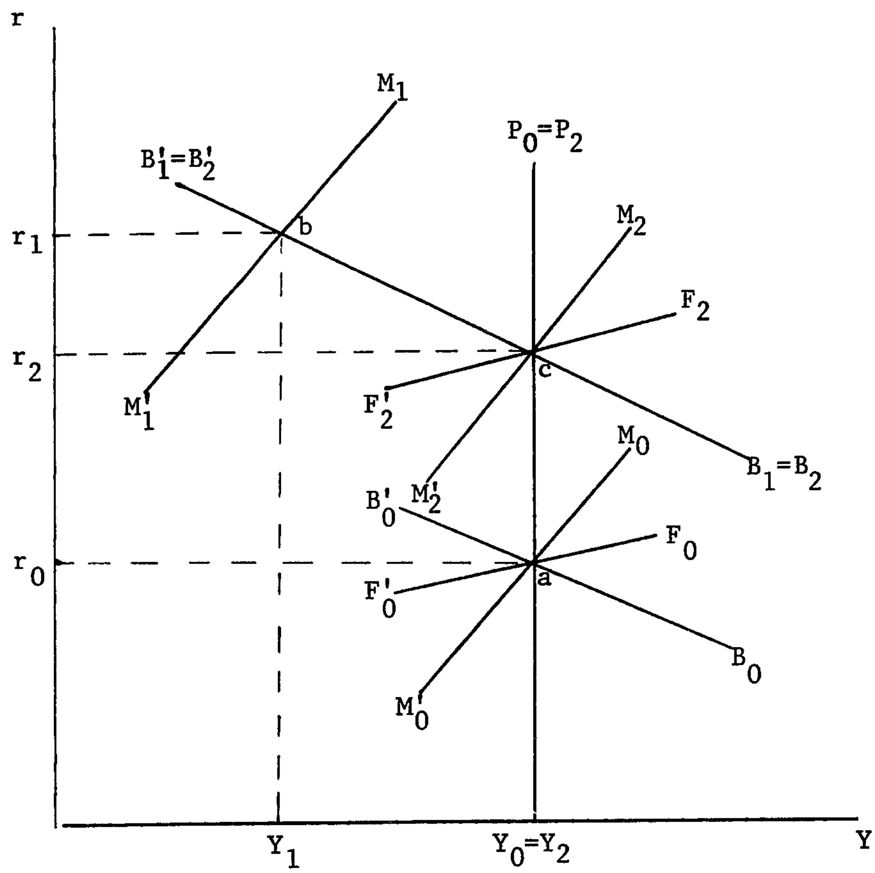

Figure 3.

Contractionary Open Market Operations in the Static Portfolio Balance Model

In the final equilibrium position, therefore, the steady-state level of income and the balance of payments are unchanged. However, the <u>stock</u> of foreign exchange reserves declines during the adjustment to the new equilibrium position, and the stock holdings of foreign bonds decrease (F_oF_o' shifts to F_2F_2') as a result of the rise in the interest rate.

The results of fiscal expansion are depicted in figure 4.

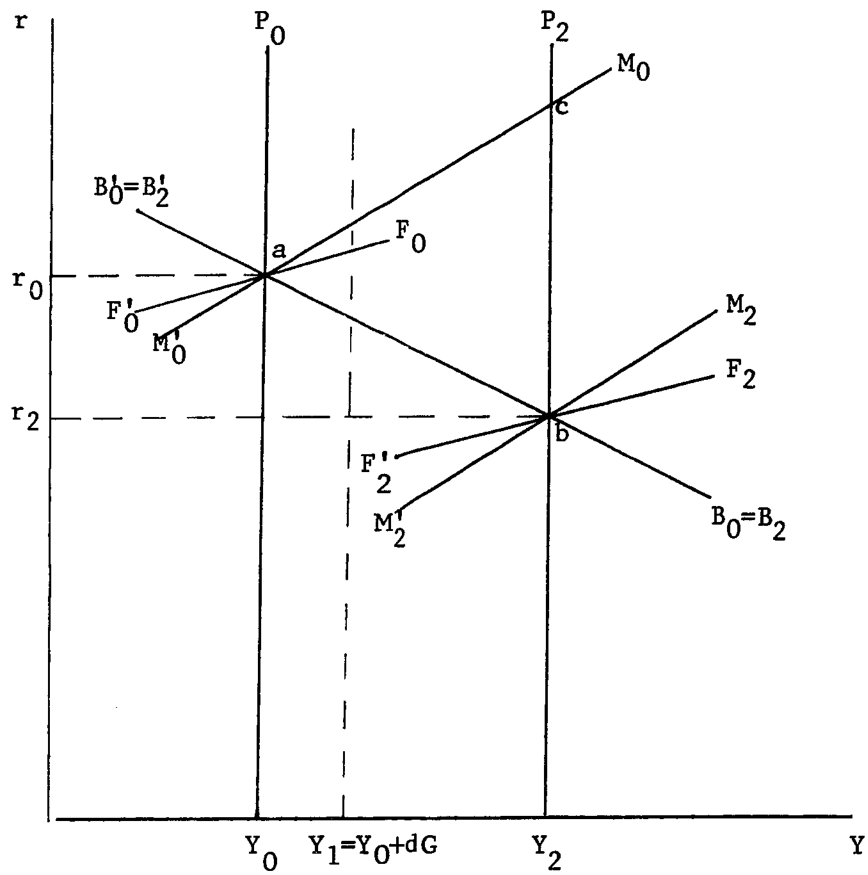

Figure 4.

Expansionary Fiscal Policy in the
Static Portfolio Balance Model

An increase in government expenditure has an impact effect raising income from Y_0 to Y_1. This worsens the trade balance, but by an amount less than the increase in government expenditure, and therefore the money supply is increased. The M_0M_0' curves shifts down the BB' curve until equilibrium is re-established at point b, with increased stock holdings of both money and foreign bonds.[26] At the new equilibrium point, the trade balance has deteriorated by the amount of the increase in government expenditure, and the balance-of-payments worsens by the same amount, the steady-state capital flow remaining zero.[27]

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[26] If reserve changes are sterilized and government deficits financed by direct bond sales to the public as discussed in footnote 21, the M_0M_0' curve remains stationary and the BB' curve rises until equilibrium is reached at c. The changes in income and the balance of payments are identical in this case, but the interest rate increases and foreign bond holdings may decline.

[27] These results differ in the perfect capital mobility case analyzed by McKinnon and Oates [35]. If government deficits are financed by bond sales to the public, reserve changes are not sterilized, and an infinite foreign demand for domestic bonds at a given interest rate exists, portfolio equilibrium in the bond and money markets is described by

$$\dot{M} = \dot{R} = T + \dot{B}_f = 0,$$

and $$\dot{B} = \dot{D} - \dot{B}_f = G - \dot{B}_f = 0,$$

where $\dot{B}_f$ is the foreign rate of purchase of domestic bonds or the capital inflow. The portfolio equilibrium condition is again $\dot{M} + \dot{B} = T + G = 0$. Monetary policy remains ineffectual under these conditions, causing only stock changes in reserves and asset holdings. The income effect of fiscal policy is also unchanged, but the balance-of-payments effect is altered considerably. Under conditions of perfect capital mobility

2.3 The Floyd Model

The static portfolio balance model outlined in Section 2.2 provides a satisfactory solution to the stock-flow inconsistency embedded in the Keynesian model. This consistency is obtained, however, only by imposing stationary equilibrium conditions with all investment flows constrained to be zero, and, as McKinnon admits: "This is obviously a very strong assumption and unlikely to be valid empirically." [34, p.207.] In particular, in a world of growth, portfolio equilibrium requires positive investment flows -- including investment in international capital assets -- and changes in return variables should have continuing steady-state effects as they alter the placement of increases in wealth. For this reason, the ~~static~~ portfolio balance model does not provide an adequate analysis of the long-run effects of government policy variables on the balance-of-payments.[28]

footnote 27 continued

$$\frac{d\dot{R}}{dG} = 0, \quad \frac{d\dot{B}_f}{dG} = -\frac{dT}{dG} = 1.$$

An increase in government expenditure does not affect balance-of-payments equilibrium, a worsening of the trade balance being matched by an increase in the capital inflow.

It should be noted that these are also the conclusions of the Mundell model under conditions of perfect capital mobility. (See Mundell [39] and footnote 13.)

[28] A second shortcoming of the model for long-run analysis is its neglect of the return flow of interest payments resulting from foreign investment. This factor is emphasized by Gray [14], Levin [32], Willett and Forte [66], and Niehans [46].

In a recent article [11], J.E. Floyd attempts to develop a model incorporating both portfolio equilibrium and asset growth and employ it in such a long-run analysis under conditions of perfect capital mobility.[29] Unfortunately, Floyd's assumptions as to investment and savings behavior are inconsistent with conditions of long-run portfolio equilibrium, and his conclusions as to the steady-state effects of changes in policy variables are therefore invalid.

The central difficulty with the investment function assumed by Floyd is that it makes investment in capital goods independent of the productivity of capital. This fact tends to be obscured by Floyd's shifting methods of measuring capital. For some purposes, non-human wealth or capital is " . . . measured by the perpetual income stream that the stock of capital goods generates." [11, p.505.] The supply of capital measured in this manner ($\bar{S}_c$) depends on capital's share of output ($\bar{S}_c = f_c\ (Y_c)$). Portfolio equilibrium is described by [11, p.507.]:

(39) $\quad i = A_c\ (M_c,\ S_c)$,

where M_c is the real[30] money supply, $S_c = Q\bar{S}_c$ is the portion of the total domestic capital stock owned by domestic residents, and the interest rate i is defined as " . . . the reciprocal of the price in terms of money of a unit of expected income in perpetuity." [11, p. 505.] Investment, however, is expressed in terms of capital

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[29]A similar model is used by Floyd in [10] and [12].

[30]Price are assumed constant. No explanation is given as to why, in the long-run, this should be so.

measured in physical units ($\overline{K}$), and the level of investment is assumed to depend only on i. The exact relation is [11, p.509]:

(40) $P_k \; (d\overline{K}/dt) = I_c \; (i)$,

where P_k is " . . . the price of a unit of capital goods in terms of consumer goods." [11, p.509.] The level of i is assumed fixed by the assumption of perfect international capital mobility, and the level of investment is therefore constant.

The relationship between the two measures of capital under such conditions can be seen most clearly by rewriting Floyd's equation (17) as,

(41) $r/P_k = i$,

introducing the variable r, defined as $r = \overline{S}_c/\overline{K}$, or the physical productivity of capital. In order to understand the implication of (40) as a steady-state relationship, it is necessary to consider time explicitly. The assumption that i is fixed over time means that the current valuation of the capital stock (P_{k_t}) must adjust to the current productivity of capital goods (r_t). This assures that <u>at each moment in time</u> the current real rate of return to investment (i.e., r_t/P_{k_t}) is constant. This return can be maintained <u>over time</u>, however, only if the productivity of capital remains unchanged. More specifically, the real rate of return at time t on capital goods purchased at some previous time t_o is $i_o = r_t/P_{k_o}$, and will differ from i if $r_t \neq r_o$, this difference being reflected in a capital gain or loss in the value of capital goods (i.e., $P_{k_t} - P_{k_o}$). Including only i in investment equation (40) requires, therefore, the assumption that investors expect the current rate of return on newly purchased capital goods to remain constant over time, (i.e., that $i_t = i$ for all t). However, there is no reason in Floyd's model why this should be so, since the levels and rates of change of Y_c and $\bar{K}$, upon which r depends, are totally unrelated;[31] dr/dt and dP_k/dt will, in general, be non-zero, and the assumption of (40) as a long-run equilibrium condition implies that investors refuse to alter expectations which are continuously and systematically proven incorrect.[32]

Similar difficulties arise with respect to the long-run implications of the savings behavior assumed by Floyd. The propensity to save (ϕ_c) is assumed constant and independent of thelevels of income or wealth [11, p.504], while asset demands as described by

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[31] The level of ($d\bar{K}/dt$) depends only on i, which is exogenously determined. The levels of Y_c and (dY_c/dt) depend, under fixed exchange rates, only on G_c and (dG_c/dt) respectively, where G_c is the level of "government production of goods and services." [11, p.504.]

[32] It should be noted that Floyd drops the term dP_k/dt from his basic capital flow equation on the grounds, not that it must be zero in equilibrium, but rather because " . . . it represents capital gains or losses which are not purchased and therefore do not enter as foreign exchange transactions." [11, p.510.] This statement would seem consistent with Floyd's interpretation of perfect international capital mobility. The perfect capital mobility assumption should mean that the price of the internationally mobile assets are fixed at the world market level. Fixing i as defined would imply that the assets in question are permanent income assets, i.e., promises to pay a dollar a year forever. The issuance or purchase of such assets, however, is inconsistent with a continuously changing productivity of capital goods. For example, rational wealth holders would refuse to purchase anasset with a fixed future return if the alternative of purchasing capital goods, whose return was and had been continuously increasing, was available.

equation (39) are independent of the level of income.[33] Thus, for example, with no growth in the money supply, a position of growing income, constant wealth, and positive and growing savings is possible under Floyd's assumptions! Similarly, it is difficult to reconcile the assumption that foreigners are willing to purchase a continuous flow of domestic capital assets with the assumption that " . . . the levels of the money stock and output in the rest of the world are constant . . ." [11, p.508.]

A model incorporating such savings and investment behavior is inconsistent with conditions of steady-state equilibrium growth; a long-run equilibrium model cannot be derived simply by taking the time derivative of a short-run model. As a result, Floyd's conclusions as to the steady-state effects of changes in policy variables cannot be accepted. For example, Floyd makes directly contradictory statements regarding the question of whether a change in the level of the money supply causes a temporary or continuing capital flow. He asserts at one point that:

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[33] This fact tends to be obscured by Floyd's introduction of the function $Q = S_c/\bar{S}_c$. For example, equation (39) is rewritten as [11, p.508],

(39') $i = A_c\,[M_c,\ Qf_c(Y_c)]$,

which appears to show the demand for money as a function of income. The Q function, however, is not an independent behavioral relation, but rather is defined by the A_c and f_c functions. Thus any variation in Y_c must cause Q to vary in such a manner that $S_c = Qf_c(Y_c)$ remains constant, as a comparison of equations (39) and (39') makes clear.

> Movements of capital resulting from changes in the level of the . . . money stock . . . lead to once-and-for-all changes in the balance of indebtedness, not to permanent and continuous flows of capital through time. [11, p.509.]

At another point, however, he states that as a result of a

> once-for-all increase in the money stock . . . in the long-run a smaller portion of the flow of new capital investment . . . per unit time will be accruing to foreigners and . . . the steady-state net capital inflow will be smaller. [11, p.513.]

The latter conclusion is of central importance, since, as was shown in the previous section, both the Mundell and static portfolio balance models suggest that changes in the money supply have no continuing effects on capital flows when capital is perfectly mobile. Although never derived explicitly by Floyd, his second conclusion can be shown to follow from his basic capital flow equation (20) and to result from the unsuitable long-run implications of his assumptions as to investment and savings behavior. If government policy is such that $(dM_c/dt) = (dG_c/dt) = 0$, equation (20) [11, p.510] reduces to

(42) $F = (1-Q)\, I_c\, (i),$

where F is the steady-state capital inflow. Using Floyd's equation (I) it follows directly that,

(43) $dF/dM_c = -\,(\partial Q/\partial M_c)\, I_c\, (i) < 0.$

With no growth in government production, and therefore, under fixed exchange rates, no growth in income, the stock of capital measured in permanent income units $(\bar{S}_c)$ is constant over time. However capital measured in physical units $(\bar{K})$ is assumed to continue to grow. It was argued previously that under these conditions the productivity of capital goods would be constantly declining $(dr/dt < 0)$ causing continuous capital losses ($dP_k/dt < 0$). In spite of the fact that their permanent income is not increasing, investors are assumed to continue to invest in capital goods with the expectation that they are acquiring permanent income assets! Under these conditions, a change in the level of M_c can cause a change in the stock demand for capital measured one way (S_c) and a flow demand for capital measured another way ($\bar{K}$).[34]

Similar confusions arise with respect to fiscal policy. For example, by an argument similar to that made above, it can be shown that

(44) $dF/dG_c = - (\partial Q/\partial Y_c) (\partial Y_c/dG_c) I_c (i) > 0.$

Despite this implication of his own model, Floyd concludes that fiscal policy has no effect on the steady-state capital flow. [11, p.514.]

Even if Floyd's model is reinterpreted as applying only to short-run adjustment, it is marred by the neglect of the important relationship between fiscal policy, portfolio balance, and capital movements suggested by the static portfolio balance model outlined previously. That model emphasizes the international movement of purely financial capital and the fact that government deficits inject a continuous flow of financial assets into the economy. Floyd, however, restricts capital movements to claims over capital goods and " . . . assumes a complete absence of . . . government

[34]The difficulty with equation (42), is that Q is defined in terms of S_c, and I in terms of $\bar{K}$.

bonds from the system."[35] [11, p.505.] This omission is justified on the grounds that government bonds will " . . . have no effect on non-monetary wealth . . . as long as the public capitalizes the interest liabilities on the public debt." [11, p.505.] While it is true that under these conditions changes in the level of government debt will have no effect on capital movements as defined by Floyd, the exclusion of the government's financial liabilities from the aggregate wealth portfolio does not prevent government bonds from moving internationally or imply that such movements are unimportant to the balance-of-payments.[36]

Floyd's emphasis on the importance of portfolio balance considerations and the distinction between stock and flow adjustments is laudable. Unfortunately, his attempt to incorporate these factors in his analysis is unsuccessful.

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[35]This presumably is why the fiscal policy variable G_c is defined as government production rather than government expenditure, although the mechanism by which this change avoids the deficit financing problem is never made explicit. Also left unclear is the method of money creation and foreign exchange reserve sterilization, since the absence of a bond market rules out open market operations.

[36]Floyd's neglect of the direct financial effect of fiscal policy makes questionable his criticism of Mundell, since this effect is central to Mundell's analysis. For example, Mundell concludes that, under fixed exchange rates and perfect capital mobility, expansionary fiscal policy will cause a capital inflow if the resulting deficit is financed by government bonds [39, p.479], and a capital outflow if the deficit is financed by the central bank. [39, p.480.]

CHAPTER III

A PORTFOLIO BALANCE GROWTH MODEL OF INTERNATIONAL CAPITAL MOVEMENTS

It was argued in Chapter II that an analysis of the long-run effects of government policy variables on international capital movements requires a model incorporating both full portfolio balance and asset growth. It was also argued that the standard Mundell model is deficient with respect to the first consideration, the static portfolio balance model deficient with respect to the second, and the attempted synthesis of Floyd less than fully successful. An alternative model which attempts to incorporate both factors is developed in this chapter. First, some basic relationships between investment and interest flows are analyzed in Section 3.1. A model incorporating the government sector explicitly is then developed in Sections 3.2, 3.3 and 3.4. Section 3.2 examines a variant of existing neoclassical monetary growth models. This model is extended in Section 3.3 to include government bonds. In Section 3.4 foreign financial assets are added, and the impact of government policy variables on the capital account is examined.

3.1 Investment and Interest Flows Under Conditions of Portfolio Balance and Growing Wealth

In order to isolate some basic relationships which will be encountered frequently in the subsequent analysis, consider the following model.[1] It is assumed that the world is divided into two sectors, the home country (h) and the rest-of-world or foreign sector (f). Financial wealth (W) can be held in two forms, consols issued by the home (B) or foreign (F) governments, with prices equal to one and rates of return r_h and r_f respectively, constant over time.[2] At any moment in time

$$(1) \qquad W_t^h = B_t^h + F_t^h \quad \text{and} \quad W_t^f = B_t^f + F_t^f ,$$

where superscripts refer to the country of ownership. Here F^h equals international investment and B^f international borrowing from the standpoint of the home country. The exchange rate between domestic and foreign currency is fixed at one. The desired composition of total financial wealth in each country is determined by[3]

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[1] This model was developed independently of, but is similar to, that found in Willett and Forte [66]. See also Domar [7], Grubel [15], and Branson and Willett [5].

[2] Consols are used to avoid the complications of debt refunding. The conclusions which follow are not dependent on this assumption, since a discrete time period version of the model, with all debt refunded each period, yields similar results.

[3] This formulation is consistent with the theory of portfolio adjustment under conditions of uncertainty as set forth by e.g., Tobin [60] and Royama and Hamada [51]. Note that the constancy of interest and exchange rates over time -- and thus the absence

$$(2) \qquad \frac{F_t^h}{W_t^h} = \lambda_h(r_f - r_h) \quad \text{and} \quad \frac{B_t^f}{W_t^f} = \lambda_f(r_h - r_f) \; ,$$

where $\lambda_h' \left(= \frac{d\lambda_h}{d(r_f - r_h)} \right)$ and $\lambda_f' > 0.$

Wealth in the two sectors grows at rates g_h and g_f (e.g., $W_t^h = W_o^h e^{g_h t}$). By definition, the rate at which the foreign exchange reserves of the home country are increasing at any moment of time is

$$(3) \qquad \dot{R}_t = \dot{B}_t^f - \dot{F}_t^h + r_f \; F_t^h - r_h \, B_t^f \; , \quad \text{where, for example,}$$

$\dot{R}_t = \frac{dR_t}{dt}$.

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footnote 3 continued

of the forward exchange rate from equation (2) -- calls into question the conditions of uncertainty postulated. To include elements of randomness in the model would, however, greatly complicate the analysis without changing basic conclusions about the nature of the equilibrium growth path.

Note also that equation (2) assumes that the demand for all assets is homogeneous of degree one in wealth. This assumption is traditional and seems reasonable for long-run analysis, since its alternative would imply that the proportion of an asset in the total wealth portfolio would go to zero or infinity over time. For a model using a non-zero wealth elasticity see Munk and Shapiro [44].

A more general specification of the λ functions would be

$$\lambda = \lambda(r_h, r_f), \quad \text{where} \quad \left|\frac{d\lambda}{dr_h}\right| \neq \left|\frac{d\lambda}{dr_f}\right| \; .$$

Again ease of exposition is gained and qualitative conclusions remain unchanged by the choice of the more restrictive formulation.

Using the fact that in equilibrium all components of wealth grow at the same rate, we get

$$(4) \qquad \dot{R}_t = B_t^f(g_f - r_h) + F_t^h(r_f - g_h) .$$

Clearly the sign of equation (4) depends on the relative values of the growth and interest rates. To see why this is so, consider the second term of equation (4). At any moment in time, the flow of interest income on the foreign security holdings of the home country equals the stock of holdings times the foreign interest rate. Similarly, the rate of flow purchase of foreign bonds (i.e. the capital outflow) equals existing stock holdings times the growth rate of domestic wealth. If new purchases exceed interest income, international financial investment draws down home country foreign exchange reserves along the growth path and conversely. An exactly analogous relationship holds for the home country's international borrowing.

Now consider the difference in reserves along paths identical except for values of r_h. Mathematically, differentiating equation (4) with respect to r_h gives[4]

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[4]It should be emphasized that the comparison here is between positions on two alternative growth paths at a moment in time and says nothing about the adjustment occurring over time in moving from one path to another.

$$(5) \qquad \frac{d\dot{R}_t}{dr_h} = W_t^h \lambda_h'(g_h - r_f) + W_t^f[\lambda_f'(g_f - r_h) - \lambda_f] \; .$$

The first term in equation (5) reflects changes in domestic holding of foreign assets. Its sign depends on the relative size of g_h and r_f. An increase in r_h makes foreign assets less attractive relative to domestic assets and decreases the level -- but not the rate of growth -- of foreign bond holdings. If $g_h > r_f$ and continuing international investment is causing the home country to steadily lose reserves, an increase in r_h will,therefore, reduce this loss and conversely.

The sign of the second term in equation (5), reflecting changes in foreign holdings of domestic bonds (i.e. home country borrowing), is less easy to determine. An increase in r_h increases the level of the home country's borrowings from abroad and if interest payments continuously exceed new borrowings (i.e., if $r_h > g_f$) an increase in the level of this activity worsens the reserve position. If, on the other hand, new borrowings exceed interest costs, increasing the scale of this operation tends to increase foreign exchange reserves. However, raising r_h also raises the cost per unit of borrowing, an effect shown by the unambiguously negative term $-\lambda_f$. Which factor will predominate in the case where $g_f > r_h$, depends upon the interest elasticity of borrowing from abroad. More specifically, the second term of equation (5) will be positive (negative) if r_h is less (greater) than r_h*, where r_h* is defined by

$$(6) \qquad \frac{g_f - r_h^*}{r_h^*} = \frac{1}{\epsilon_{\lambda_f, r_h}} ,$$

and $\epsilon_{\lambda_f, r_h}$ is the elasticity of λ_f with respect to r_h. If foreign investors consider domestic and foreign bonds close substitutes (i.e. if $\epsilon_{\lambda_f, r_h}$ is large), substantial increases in foreign borrowing can be made by the home country at a small increase in interest costs, and the optimal level of the domestic interest rate will be only slightly below g_f. Conversely, as $\epsilon_{\lambda_f, r_h} \rightarrow 0$, $r_h^* \rightarrow 0$. In this case, the portfolio holdings of foreigners are insensitive to a change in the domestic interest rate, and lowering r_h reduces interest payments with no cost in terms of disinvestment by foreigners.

These results indicate that, under conditions of portfolio balance and growing wealth, an increase in the domestic interest rate does raise the steady-state capital inflow, but may worsen or improve the long-run reserve position. The direction of the reserve effect depends on the relative size of interest and growth rates, and the substitutability of assets.

The model used to arrive at this conclusion also suggests another variable that may be of interest to policymakers attempting to manipulate the capital account -- the growth rate of financial wealth. From equation (4) we get

$$(7) \qquad \frac{d\dot{R}_t}{dg_h} = \lambda_h W_t^h [t(r_f - g_h) - 1] .$$

In the limit the sign of this expression is determined by the relative sizes of r_f and g_h, and the results here are straightforward compared to the previous case. Since a change in g_h does not alter return variables, its only effect is on the rate of purchase of foreign bonds. If new purchases continuously exceed interest income, resulting in a net loss of reserves (i.e., if $g_h > r_f$), this loss can be reduced by lowering the rate of purchase and conversely.

In interpreting the above results it should be remembered that they refer only to the long-run implications of various policies; the short and intermediate-run results differ considerably and should be mentioned at this point. Consider first changes in interest rates. At the time when r_h is raised there is a once-and-for-all adjustment of existing wealth portfolios in both countries out of foreign into home country securities. This causes an increase in reserves as the flows necessary to make this stock adjustment occur. In future periods the higher interest rate only affects reserves through the flow investment of additions to wealth and interest payments resulting. Since at any moment in time stock holdings are large relative to new investment flows, the short-run stock effect is large relative to the continuing flow effect.[5]

[5] To clarify this point consider the static case, $g_h = g_f = 0$. With a fixed stock of wealth, once-and-for-all adjustments equal to

$$\frac{d\bar{R}}{dr_h} = \lambda_h' W_o^h + \lambda_f' W_o^f > 0$$

occur at the moment when r_h is raised. The continuing effect in this case is, from equation (5), The difference between the short- and long-run implications of changes in g_h is less marked. The time profile of reserve changes in this case can be seen from equation (7), which is based on the assumption that g_h is changed at time t = 0. In this case there is no stock adjustment at the time of the change, since the interest rates which determine portfolio allocations remain unaltered. If g_h is lowered, the capital outflow resulting from new acquisitions of foreign securities will be lower. Since interest payments depend on stock holdings -- the same at t = 0 whatever the new value of g_h -- it will take some time before lower interest earnings "catch up" with the lower capital outflows. If $g_h > r_f$, the reduction in interest earnings remains smaller than the reduction in new capital outflows forever, as was shown previously. If $r_f > g_h$, the reduced interest

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footnote 5 continued

$$\frac{d\dot{R}_t}{dr_h} = -\lambda_h' W_o^h r_f - \lambda_f' W_o^f r_h - \lambda_f W_o^f < 0.$$

This reflects higher interest payments and lower interest income resulting from the original adjustment. That the impact effect is large in magnitude relative to the steady-state effect can be seen from

$$\left(\frac{d\bar{R}}{dr_h} + \frac{d\dot{R}_t}{dr_h}\right) = \lambda_h' W_o^h (1 - r_f) + \lambda_f' W_o^f (1 - r_h) - \lambda_f W_o^f ,$$

which is positive for reasonable parameter values.

If r_h is subsequently lowered to its original level an equal stock adjustment in the opposite direction takes place. The net result over the entire period is a lower level of reserves due to higher interest payments and lower interest income during the time when r_h was at its higher level.

payments effect will dominate, but it may be many periods before this reversal occurs.[6]

The conclusions of this section may be summarized as follows. If the behavior of asset holders is described by the theory of portfolio balance and wealth is growing: (1) Higher domestic interest rates may worsen or improve the net reserve impact of international capital movements in the long-run. The result depends on the value of a number of different parameters (interest rates, growth rates and substitution elasticities). (2) This differs sharply from the short-run situation, where increases in domestic interest rates always increase reserves, and the increase is large in magnitude relative to the continuing effects in any single near-future period. (3) Another variable with important long-run implications for the capital account is the growth rate of domestic wealth. The effects of changes in this variable are also uncertain, but less so than in the case of interest rate changes, the results depending only on the rate of return on foreign assets. (4) The short-run effects of lowering the wealth growth rate are relatively small but positive.

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[6]The number of periods elapsing before the reversal, can be calculated approximately by setting equation (7) equal to zero and solving for t. The result is

$$n = \frac{1}{r_f - g_h} .$$

For reasonable values of both variables this number will be large, especially if r_f only slightly exceeds g_h.

These conclusions, while suggestive, do not give a full indication of the impacts of government policy decisions on the capital account. For the model on which they were based viewed a portion of the financial market in isolation from the rest of the economy. What is needed is a model integrating the real and financial sectors in a situation of equilibrium growth. Such a model is developed in the next sections.

3.2 A Neoclassical Monetary Growth Model

Recently the effects of introducing a financial asset into the traditional neoclassical growth model have been investigated. In this section such a model is presented and its properties analyzed.[7]

Production takes place under the usual neoclassical conditions. Output (Q) is produced by labor (L) and non-depreciating capital (K), where output and capital are identical goods. The production function is constant over time, exhibits constant returns to scale, and marginal products are positive and diminishing. The labor force is assumed to grow at an exogenously determined rate n. Mathematically,

$$(8) \qquad Q_t = G(K_t, L_t) = L_t\, G\left(\frac{K_t}{L_t}, 1\right) = L_t\, g(k_t),\ g' > 0,\ g'' < 0,$$

$$L_t = L_o e^{nt}.$$

[7] On money growth models, see Tobin [62], [63] and [64], Marty [33], Johnson [23], Stein [58] and [59], Hadjimichalakis [16], and Levhari and Patinkin [31]. The model used here is adapted from [31] and [62].

Now we introduce a government whose only activity is the issuance of non-interest bearing debt (M) by means of direct transfer payments.[8] The value of a unit of this asset is $1/p$ in terms of goods and the price level p is assumed to be perfectly flexible. Real disposable income (Y) now differs from output:

$$(9) \qquad Y = Q_t + \frac{\dot{M}_t}{p_t} - \frac{M_t}{p_t}\Pi = Q_t + \left(\frac{\dot{M}}{p}\right)_t, \quad \text{where } \Pi = \frac{\dot{p}_t}{p_t}.$$

The term $\frac{\dot{M}_t}{p_t}$ represents the real value of the increase in the nominal quantity of money (or the real value of government transfer payments) and the term $-\frac{M_t}{p_t}\Pi$ represents the decrease in the real value of existing money holdings due to price increases.[9]

We assume that individuals increase their real wealth (W) by saving a fraction s of disposable income, where real wealth portfolios now consist of two components, capital and real money balances:

$$(10) \qquad S_t = \dot{W}_t = \left(\dot{\frac{M}{p}}\right)_t + \dot{K}_t = sY_t.$$

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[8] All financial assets in this system are of the outside type, i.e., claims of the private sector on the government. The distinction here is crucial since it is the outside nature of the financial assets that causes the public to view them as net wealth. On the role of inside money in these models see, for example, Marty [33, p. 863-869], Johnson [23] , and Stein [59, p. 91-94].

[9] Alternative formulations of the exact role of money in such models have been suggested, e.g. adding an independent utility yield of money to disposable income or the inclusion of money balances in the production function. Since the exact nature of money is not the main concern here, the simpler formulation is used.

The real rate of return on a unit of capital is equal to the marginal product of capital, $g'(k)$. The real rate of return on real money balances is $-\pi$. The desired holding of each asset is an increasing function of its rate of return, but both assets will still be demanded if these rates differ. Specifically,

$$(11) \qquad \frac{(M/p)_t}{K_t} = \alpha(i), \ \alpha' < 0 \qquad \text{where } i = g'(k) + \pi = \text{the nominal rate of interest.}[10]$$

It is assumed that the government fixes the rate of growth of the money supply $(\frac{\dot{M}_t}{M_t} = \mu)$ and that the economy is on a steady-state growth path with all real magnitudes growing at rate n. Using equations (8) - (11) and noting that along such a path per capita real money balances, $m = \frac{(M/p)_t}{L_t}$, are constant, which implies that $\pi = \mu - n$, the equilibrium values along this path are described by

$$(12) \qquad \frac{f(k)}{k} = a(k) = \frac{n}{s}\,[1 + \alpha(1 - s)] \quad .$$

Assuming that s is constant,[11] and using the value of π implied by

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[10]Generally, i should be defined in terms of the expected rate of inflation (π^e) rather than that actually prevailing. However, only steady-state positions where π is constant over time are considered here, and $\pi = \pi^e$ under these conditions for any reasonable assumptions as to expectations behavior, for example the two given in footnote 12.

[11]In general, the savings rate would be an increasing function of the rates of return of the two components of wealth, i.e.

$$s = s[g'(k), -\pi], \qquad s_1 \text{ and } s_2 > 0.$$

Some modification of the conclusions that follow is required if the assumption of a constant s is relaxed. Even with this restrictive assumption, the effects of changes in policy parameters are ambiguous as to sign in most cases.

the government's decision as to μ, equation (12) can be used to determine the equilibrium value of the capital-labor ratio k along the steady-state growth path.[12]

The method by which the introduction of money alters the nature of the equilibrium path can be seen by examining the real savings rate. Combining equations (8) and (9) gives

$$(13) \qquad (\dot{\frac{M}{p}})_t + \dot{K}_t = s[Q_t + (\dot{\frac{M}{p}})_t] \, .$$

In its transfer payment aspect money creation enters the right-hand side of the equation causing disposable income to exceed output. In its asset aspect it enters the left-hand side as a component of savings. Equation (13) can be rewritten, using equation (11), as

$$(14) \qquad s_p = \frac{\dot{K}_t}{Q_t} = s - (1-s)n \frac{(M/p)_t}{Q_t} \quad ,$$

where s_p = the physical savings rate. With a positive $(M/p)_t$, $s_p < s$; by diverting a portion of total savings into the accumulation of real money balances, capital accumulation is reduced and the growth path of output lowered.

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[12]The stability of this equilibrium path has been shown by Hadjimichalakis [16] and Stein [59] to depend on the specification of price expectations behavior. A sufficiently sluggish expectations function, for example, $\pi^e = \mu - n$, or $d\pi^e = \beta(\pi - \pi^e)$ with β small, will assure the dynamic stability of the system.

The main policy variable in this system is μ. A variation in the rate of growth of the nominal stock of money leaves the growth of real money balances unchanged because of compensating variations in the price level. However this change in the rate of inflation, which alters the <u>return</u> on real money balances, changes the amount of savings devoted to their accumulation, the real savings rate and the capital intensity of production. The exact relationship is found by differentiating equation (12) implicitly with respect to μ, giving

$$(15) \qquad \frac{dk}{d\mu} = \frac{n(1-s)\alpha'}{sa'-n(1-s)\alpha' g''} \quad .$$

Since $\alpha' < 0$, $a' < 0$, $g'' < 0$, and $0 < s < 1$, $\frac{dk}{d\mu}$ is clearly positive. This result should not be surprising. A higher rate of monetary expansion means a higher rate of inflation. This reduces the real rate of return on money balances and makes capital, an asset whose nominal return is not fixed, a relatively more attractive asset. As wealth holders attempt to move out of real money balances into capital, the real savings rate is increased and k rises.[13]

Before concluding this section, the relation between the μ variable and monetary and fiscal policy are defined in Chapter II should be mentioned. Since changes in μ involve changes in the rate

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[13]It should be remembered that equation (15) and similar equations which follow, compare positions along alternative growth paths and say nothing about movements from one path to another. The verbal explanation of equation (15) given above is therefore not rigorous, but is consistent with the type of final adjustment indicated.

of growth of the money supply, an identification of μ with monetary policy would seem warranted. However, money is created through government transfer payments, and thus μ can equally be thought of as a fiscal policy variable. This duality can be seen by rewriting the government budget equation as

$$(16) \qquad T_t = \dot{M}_t = \mu M_t ,$$

where T_t is the level of government transfer payments or negative taxes. Specifying T_t as $z(p_t Y_t)$ gives

$$(17) \qquad z = \frac{\alpha}{a(k) + n\alpha} \mu.$$

For a given choice of μ, the equilibrium values of α and k can be used to calculate the corresponding implicit negative tax rate z, and either z or μ -- but not both -- could be thought of as the policy instrument.[14] Since the central mechanism by which government action influences the economy is through portfolio adjustments

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[14] A further fiscal variable that might be considered is the level of government expenditure on goods. If the government purchases a fraction q of output its budget equation becomes:

$$q(p_t Q_t) + T_t = M_t \mu.$$

In this case both q and μ become independent policy instruments. Recalculating the equilibrium equation with this addition, it can be shown that

$$\frac{dk}{dq} = \frac{-as}{(1 - s)n\ \alpha'\ g'' - s(1 - q)a'} < 0 .$$

A movement to goods purchases instead of transfer payments as the method of money creation reduces disposable income and therefore consumption and investment in capital goods, and lowers the level of output.

It must be remembered, however, that efficient policy matches policy instruments with targets. Since the optimal long-run share of full employment output devoted to the government sector becomes a target variable as soon as a non-zero q is introduced, the use of this tool for any other purpose becomes inefficient. For this reason q is assumed zero for purposes of expositional clarity.

and asset substitutions, we choose to emphasize this role by using μ as the policy instrument. The final transfer policy needed to maintain the desired money stock is then determined endogenously.

3.3 A Monetary Growth Model With Interest-Bearing Government Debt

In this section it is assumed that two forms of government debt exist, money (M) and interest-bearing money or consols (B). Consols have a coupon rate c_b and are sold at a price p_b, having a nominal rate of return $r_b = c_b/p_b$. The government budget equation is now

$$(18) \qquad T_t + c_b B_t = \dot{M}_t + p_b \dot{B}_t \; .$$

Untied tranfers (T) and interest payments on the national debt are financed by the issuance of new money and by bond sales. Real disposable income becomes

$$(19) \qquad Y_t = Q_t + \frac{T_t}{p_t} + \frac{c_b B_t}{p_t} - \frac{M_t + p_b B_t}{p_t} \pi \; .$$

The two types of government transfer payments add to disposable income while inflation, which erodes the real value of wealth held in the form of financial assets, reduces it.[15] There are now three forms of

[15]The last term of equation (19) is the rate of purchase of financial assets needed to keep real financial wealth from declining due to inflation. If, for example, $r_b = \pi$, bond interest payments, if spent on new bond purchases, would just be sufficient to keep the real value of bond holdings constant, and B would not appear in the disposable income equation.

wealth that may be held by the public, the two types of government debt and capital. The desired composition of real wealth portfolios is determined by the rates of return on these assets, and is described by[16]

$$\frac{M_t}{p_b B_t} = \beta_1(r_b) \ , \ \beta_1' < 0, \tag{20}$$

and

$$\frac{p_b B_t/P_t}{K_t} = \beta_2(r_b - i) \ , \ \beta_2' > 0 \ . \tag{21}$$

If it is assumed that all real variables grow at rate n and that the government increases the outstanding stock of bonds and money at rate μ, equations (18) - (21) can be solved simultaneously for the steady-state equilibrium equation:

$$a(k) = \frac{n}{s} [1 + (1-s)\beta_2(1+\beta_1)] \ . \tag{22}$$

With the addition of a second debt instrument the government gains an additional policy tool; variation in the composition of government debt as well as variations in the rate of growth of total debt will effect the steady-state value of real variables. The effects of changes in μ remain similar to those discussed in the last section and described by equation (15). An increase in the rate of

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[16] For a discussion of this general formulation, see footnote 3. It does not, as might at first appear, ignore cross-substitutions between money and capital, since

$$\frac{(M/p)_t}{W_t} = \frac{\beta_1\beta_2}{\beta_1\beta_2 + \beta_2 + 1} \ ,$$

a function of i as well as r_b.

debt creation, holding debt composition constant, causes a movement by wealth holders out of real financial assets as a whole into capital, the capital-labor ratio increasing.

The effect on the capital-labor ratio of changes in the composition of government debt can be found by implicitly differentiating equation (22) with respect to r_b[17] holding μ constant. This gives

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[17]A more intuitively appealing definition of the debt composition variable would be B/M. The definition in terms of r_b was found to be more convenient, and is entirely equivalent, in the sense that a particular value of B/M implies a unique value of r_b and conversely. In order to understand this equivalence, consider two alternative methods of implementing debt composition policy. In the first, the coupon rate on bonds is fixed (at e.g. $\bar{c}_b$) and bonds are bought and sold until a target level of B/M is achieved. Asset preferences will then determine the bond interest rate at which the public is willing to hold this particular mix of government financial liabilities, as can be seen by rewriting equation (20) as

$$M/B = (\bar{c}_b/r_b)\beta_1(r_b) = \beta_1^*(r_b) \text{ or } r_b = \beta_1^{*-1}(M/B).$$

In the second method, the government offers to buy or sell unlimited quantities of bonds at a fixed price (e.g. $p_b = 1$) and varies the coupon rate which these bonds carry. The asset preferences of the public then determine the B/M ratio, i.e. $M/B = \beta_1(r_b = c_b)$. The only difference in the two cases is that in the second, the bond price is invariant to changes in debt composition policy; in the first, changes in debt composition cause capital gains or losses on existing consol holdings. Again rewriting equation (20),

$$M/B = p_b\beta_1(\bar{c}_b/p_b) = \beta_1^{**}(p_b) \text{ or } p_b = \beta_1^{**-1}(M/B).$$

For a further discussion of this point see footnote 26.

$$(23) \qquad dk = \frac{(1-s)\Delta_1}{\eta_1} dr_b \quad ,$$

where $\eta_1 = sa' + n(1-s)(1+\beta_1)\beta_2' g'' < 0$,

and $$\Delta_1 = n[(1+\beta_1)\beta_2' + \beta_2\beta_1'] = n \frac{d}{dr_b} \left. \frac{(M_t + p_b B_t)/p_t}{K_t} \right|_{i = \text{constant}}$$

The sign of equation (23) is, in the general case, uncertain. The determining factor is the sign of Δ_1 which depends on the relative size of the asset substitution elasticities β_1' and β_2'. In order to illustrate the nature of this dependency, consider two extreme cases. If bonds and capital are perfect substitutes, the return differential between bonds and capital cannot vary. Open market sales which drive up the bond interest rate will therefore lead to an attempt by asset holders to move out of capital, and the capital-labor ratio will decline until the return to capital is raised to a level equal to that on bonds.[18] Conversely, if bonds and money are perfect substitutes, attempts to raise the bond interest rate by open market sales will be unsuccessful. However, in order for the public to be willing to hold the resulting larger stock of bonds, the bond-capital return differential must move in favor of bonds. Since the bond interest rate cannot

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[18]Mathematically, as bonds and capital become perfect substitutes, $\beta_2' \longrightarrow \infty$ and

$$\frac{dk}{dr_b} \longrightarrow \frac{1}{g''} < 0.$$

rise, the return on capital must fall as attempts are made to increase capital holdings and the capital-labor ratio rises.[19] In general, "contractionary" open market operations and increases in the bond interest rate, will lower the level of output only if bonds are a closer substitute for capital than money.[20] An alternative and complementary interpretation of this condition is provided by the second version of Δ_1 given in equation (23). It indicates that a change in r_b will raise or lower k depending on the direction of the original (i constant) desired shift between capital and real financial assets <u>as a whole</u>.[21]

This second interpretation of Δ_1 provides an explanation of the change in desired holdings of real financial assets <u>relative</u> to capital. The change in the absolute <u>level</u> of desired real financial

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[19]In the extreme case where $\beta_1' \longrightarrow \infty$, debt composition policy should be defined in terms of the bond-money ratio rather than r_b. With this change, equation (23) becomes

$$\frac{dk}{d(M/B)} = \frac{n(1-s)\beta_2/p_b}{sa' + n(1-s)(1+\beta_1)\beta_2' g''} < 0,$$

which shows mathematically that open-market sales always lower k if bonds and money are perfect substitutes.

[20]A similar conclusion is reached in a static framework by Tobin [61, p. 145-149.]. He points out that the Keynesian model employs the implicit assumption that bonds and capital are perfect substitutes and that, therefore, an increase in the bond interest rate always lowers the level of investment and output.

[21]This interpretation suggests that a value of $\Delta_1 > 0$ is most probable, since an increase in the bond interest rate raises the average return on all financial assets, the return on money remaining unchanged. In the subsequent analysis, a value of $\Delta_1 > 0$ will accordingly be viewed as the "normal" case, but the implications of a value of $\Delta_1 < 0$ will also be considered.

wealth depends in addition on the change in the level of total real wealth. This scale effect will, in turn, depend on the change in real disposable income, since the ratio of wealth to disposable income is constant.[22] Since, from equations (19) - (23),

$$(24) \qquad d\left(\frac{Y_t}{L_t}\right) = dy = \frac{\Delta_1 \Delta_2}{\eta_1} dr_b ,$$

where $\Delta_2 = g' - n$ and Δ_1 and η_1 are defined above, the effect of changes in debt composition on total desired real wealth will depend on the signs of Δ_1 and Δ_2. If, for example, $\Delta_1 > 0$, a decrease in r_b will cause a relative substitution in the real wealth portfolio out of real financial assets into capital. This increase in k has conflicting effects on disposable income; it raises the level of output (by g'), but also increases the required level of capital goods investment (by n[23]). If the output effect is greater than the investment effect (i.e. if $\Delta_2 > 0$), real disposable income, and therefore total real wealth, can increase. Thus, with Δ_1 and $\Delta_2 > 0$, a decrease in r_b causes a substitution effect tending to reduce desired holdings of real financial assets, and a scale effect tending to increase desired holdings. The net effect depends on the sign of a third expression, $\Delta_3 = sg' - n$. If the increase in desired savings

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[22] This follows from the fact that in equilibrium

$$\frac{W}{Y} = \frac{W}{\dot{W}} \frac{\dot{W}}{Y} = \frac{1}{\dot{W}/W} \frac{sY}{Y} = \frac{s}{n} \quad .$$

[23] By definition $\dot{K}/L = nk + \dot{k}$. In equilibrium $\dot{k} = 0$. Therefore

$$\frac{d(\dot{K}/L)}{dk} = n.$$

resulting from the increase in k and output (sg') is greater than the increase in required investment in capital goods, this "excess" savings will take the form of an increased demand for real financial assets. The exact mathematical relationship can be derived from equations (20), (21) and (23), and is

$$(25) \qquad d(m + b) = \frac{\Delta_1 \Delta_3}{\eta_1} dr_b ,$$

where $m = M_t/p_t L_t$ and $b = p_b B_t/p_t L_t$ are per capita real money and bond holdings.[24]

This completes the analysis of the effect of government policy variables on the important real variables in the model developed so far. Before expanding the model to include foreign assets in the next section, it is necessary to develop a more complete specification of government policy using Jones' [24] distinction between fundamental and intermediate policy variables. The fundamental policy variables in the model used here are the size and composition of government debt. Policy affecting the size of government debt may in turn be divided into two fundamental policy variables. The size of government debt at any moment in time along a steady-state growth path is

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[24]As can be seen by comparing equations (24) and (25), an increase in disposable income is a necessary but not a sufficient condition for an increase in real financial asset holdings. Mathematically, $\Delta_2 < 0$ implies $\Delta_3 < 0$ but $\Delta_2 > 0$ does not necessarily imply $\Delta_3 > 0$. With $\Delta_2 < 0$, a decrease in r_b lowers disposable income and the scale and substitution effects are reinforcing.

$$(26) \qquad M_t + p_b B_t = (M_o + p_b B_o)e^{\mu t} ,$$

and thus the size of government debt will be affected by changes in the <u>level</u> of government debt $(M_o + p_b B_o)$[25] and the <u>rate of growth</u> of government debt (μ). The corresponding intermediate policy variables are the price level, p_o, and the rate of inflation, π, where

$$(27) \qquad p_t = p_o e^{\pi t} .$$

The relationship between these fundamental and intermediate policy variables arises from the fact that price changes are generated by differences between nominal and desired real financial wealth. The exact relationship is

$$(28) \qquad p_t = \frac{m_t^* + b_t^*}{m + b} = \frac{(m_o^* + b_o^*)e^{(\mu - n)t}}{m + b} ,$$

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[25] It should be emphasized that changes in the level of debt as defined here cause an absolute change in debt size that grows over time, i.e.

$$\left[\frac{d(M_t + p_b B_t)}{d(M_o + p_b B_o)}\right] = \mu e^{\mu t} > 0 \text{ if } \mu > 0 .$$

Changes in the level of the money supply as defined in the models of Chapter II can be thought of as a special case where $\mu = 0$ and therefore $dM_t/dM_o = 1$.

In a growth situation it is most convenient to define changes in the level of government debt in percentage terms since

$$\frac{d(M_o + p_b B_o)}{M_o + p_b B_o} = \frac{d(M_t + p_b B_t)}{M_t + p_b B_t} ,$$

and, for example, a ten percent increase in the level of debt variable causes the stock of outstanding debt to be ten percent higher at each moment in time.

where $m_t^* = M_t/L_t$ and $b_t^* = p_b\, B_t/L_t$ are per capita nominal debt holdings and, as defined previously, m and b are the corresponding real holdings. Since in equilibrium real per capita holdings must be constant, $\Pi = \mu - n$ and $d\mu = d\Pi$; any increase in the rate of debt creation causes an equivalent increase in the inflation rate, leaving the rate of growth of real financial wealth (n) unchanged. For this reason, equation (28) reduces to

$$(29) \qquad p_o = \frac{m_o^* + b_o^*}{m + b} \quad ,$$

or

$$(30) \qquad \frac{dp_o}{p_o} = \frac{d(m_o^* + b_o^*)}{m_o^* + b_o^*} - \frac{d(m + b)}{m + b} \; .$$

In the model used in this section, the equilibrium values of all real variables are independent of the level of debt; $(m_o^* + b_o^*)$ does not appear in equation (22). Under these conditions, equation (30) indicates that increases in the level of nominal government debt cause only equal percentage increases in the price level, leaving real financial wealth holdings unchanged.

This distinction between fundamental and intermediate policy variables requires that changes in the composition of government debt be defined in terms of two variables also. For analysis in terms of fundamental policy variables, changes in debt composition will be defined in terms of a debt level neutral change in r_b, i.e. $d(m_o^* + b_o^*)/dr_b = 0$.[26] For analysis in terms of intermediate policy variables, changes in debt composition will be defined in terms of price level neutral changes in r_b, i.e., $dp_o/dr_b = 0$. As indicated in equation (25), a change in r_b will, in general, change the level of desired real financial wealth holdings, and equation (30) shows that under these conditions a change in r_b will alter the price level if an accomodating policy of debt level variation is not undertaken. For example, if Δ_1 and $\Delta_3 > 0$, an increase in r_b will cause prices to rise unless the accompanying decrease in the level of desired real debt holdings is matched by a fall in the level of nominal debt outstanding.

The distinction made here between fundamental and intermediate policy variables was also encountered in Chapter II. There it was shown that monetary policy could be defined in terms of a fundamental policy variable (the money supply) or an intermediate policy variable (the interest rate). Depending on which definition was used, fiscal policy was implicitly defined in terms of money supply neutral or interest rate neutral changes in government expenditure. Subsequent

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[26] As was shown in footnote 17, a change in r_b may, under some conditions, lead to a change in p_b. In this case, the definition of debt level neutrality given above fixes the value of total debt rather than the number of debt instruments outstanding; if $dp_b/dr_b < 0$ and $d(M_o^* + p_b B_o^*)/dr_b = 0$, $d(M_o^* + B_o^*)/dr_b > 0$. The divergence of these two measures of the debt level will be less, the shorter is the maturity of interest bearing debt. The second method of conducting debt composition policy mentioned in footnote 17, with the government offering to buy or sell unlimited quantities of bonds at a fixed price, reduces the effective maturity period to zero, since all outstanding bonds can potentially be refunded at any moment.

analysis of government policy actions in this study will be carried out in terms of both fundamental and intermediate policy variables. It will be concluded that, as in the model discussed in Chapter II, results differ significantly depending on which approach is taken, and that policy rules stated in terms of intermediate policy variables are usually less ambiguous.

3.4 Foreign Bonds, International Capital Movements, and Government Debt

In this section foreign bonds are added to the domestic wealth portfolio and the effects of government policy variables on the resulting international capital movements are analyzed.[27] It will

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[27]The only other attempt, of which the author is aware, to investigate the relationship between macroeconomic policy and the balance of payments in a monetary growth context is that of Niehans. [46, p.902-907 and 918-919.] While in many respects the basic model used by Niehans is similar to that employed in this study, it differs in important respects which makes a comparison of results difficult. First, Niehans looks only at the trade balance and does not consider international capital movements. Second, the specification of policy variables employed by Niehans differs considerably from that used here.

The latter point may be seen by rewriting the government budget equation (18), replacing T_t by $z(p_tY_t)$ and imposing steady-state conditions. This gives

$$(18') \qquad z = \frac{\mu\beta_2(1+\beta_1) - r_b\beta_2}{a(k) + n\beta_2(1+\beta_1)} \quad ,$$

which defines the endogenously determined implicit transfer policy rate, z, as a function of the independent policy variables r_b and μ. An alternative arrangement of equation (18) is

$$(18'') \qquad p_t z = \sigma_1(r_b - \mu) - \mu\sigma_2,$$

where $\sigma_1 = p_b\ B_t/Y_t$ and $\sigma_2 = M_t/Y_t$. Niehans chooses as independent policy variables z and σ_1 and sets μ=n. This choice makes both the money supply (σ_2) and the price level endogenous variables, and constrains the rate of inflation to zero.

A comparison of this policy variable choice with that used here is complicated by the fact that the σ_1 variable effects both debt level be assumed: (1) that foreign consols (F) bearing a coupon rate c_f can be purchased in unlimited quantity at a constant price p_f;[28] (2) the exchange rate between domestic and foreign currency is fixed at unity;[29] (3) that the government buys and sells foreign exchange to domestic residents on demand; and (4) that interest earnings on foreign assets are fully repatriated. Under these conditions real disposable income becomes

$$(31) \qquad Y_t = Q_t + \frac{T_t}{p_t} + \frac{c_b B_t}{p_t} + \frac{c_f F_t}{p_t} - \frac{M_t + p_b B_t + p_f F_t}{p_t} \pi .$$

Interest income on foreign asset holdings increases disposable income and the depreciation in the real value of the stock of foreign assets due to domestic inflation decreases it. The flow increase in the government's holdings of foreign exchange reserves equals interest income on foreign bond holdings minus the capital outflow resulting

footnote 27 continued

and composition. Furthermore, the relation between changes in independent and endogenous variables is uncertain. More specifically, differentiation of equation (18') shows that both dz/dr_b and $dz/d\mu$ are complicated expressions of uncertain sign. This means that results reported by Niehans as to the effects of changes in "fiscal" policy (i.e. z) cannot be readily restated in terms of debt size and composition policy.

The procedure used in this study of viewing government policy in terms of debt management policy was adopted in order to focus attention on the capital account. Whatever the framework choosen, in a model of equilibrium growth some "important" policy variable (e.g. z or σ_2) must be made endogenous. This policy interdependence is not apparent in the Keynesian model, since the endogenous policy variable is the level of government bonds outstanding, and the bond market has no independent effect on the equilibrium position.

[28]The constancy of p_f can be justified on the grounds that the home country is too small to have a significant impact on the international bond market.

[29]Exchange rate flexibility is discussed in Chapter V.

from new foreign bond purchases, and is given by[30]

$$(32) \qquad \dot{R}_t = c_f F_t - p_f \dot{F}_t \ .$$

The government budget equation now becomes

$$(33) \qquad T_t + c_b B_t + \dot{R}_t = \dot{M}_t + p_b \dot{B}_t \ .$$

Direct transfers, interest payments on internal debt and the purchase of foreign exchange is financed by money creation and domestic bond sales.[31] The division of total real wealth portfolios between the four available assets is determined by

$$(34) \qquad \frac{M_t}{p_b B_t} = \gamma_1(r_b) \ , \ \gamma_1' < 0,$$

$$(35) \qquad \frac{p_f F_t}{p_b B_t} = \gamma_2(r_f - r_b) \ , \ \gamma_2' > 0,$$

---

[30] It is assumed provisionally that the government holds an unlimited stock of foreign exchange reserves and therefore that a continuous non-zero value of $\dot{R}_t$ along the growth path is possible. Reserves are, of course, not in fact infinite, but a full discussion of this problem is postponed until the next chapter when goods trade is introduced.

[31] The choice of policy variables made here implies a policy of complete sterilization, in the sense that the level and rate of change of the money (and bond) supply is determined independently of reserve changes. The specification of policy variables used by Niehans in [46] and discussed in footnote 27 implies what might be called a semi-sterilization policy; changes in the balance of payments are allowed to affect the level of the money supply, but not its rate of growth.

In two recent papers, [42] and [43], Mundell examines a situation where the balance of payments is allowed to redistribute the world money supply under conditions of monetary growth and non-sterilization.

and

$$(36) \qquad \frac{p_b B_t / p_t}{K_t} = \gamma_3(r_b - i) \ , \quad \gamma_3' > 0, \quad i = g'(k) + \Pi \ .$$

Equations (31) - (36) can be combined to give

$$(37) \qquad a(k) = \frac{n}{s}\left[1 + (1-s) \ \gamma_3 \ (1+\gamma_1+\gamma_2)\right],$$

defining the steady-state equilibrium capital-labor ratio. The three fundamental policy variables in this system are the level, rate of change, and composition of government debt. The target variable is $\dot{R}_t$, or the balance of payments.[32]

Consider first the effect on reserves of changes in the level of government debt. This problem can be seperated into two parts. First, the effect of changes in debt level on the level of foreign bond holdings, and second, the effect of changes in the level of foreign bond holdings on the balance of payments. From equation (37) it can be seen that, as in the previous section, the basic equilibrium condition is independent of the level of government debt;[33] increases in the debt level cause only equal increases in the price level.[34]

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[32]The achievement of the second target of the traditional internal-external balance literature, full employment output, is assured in this model by the neoclassical assumptions. In the context of a long-run equilibrium model, the second policy target becomes the optimal level of domestic and foreign investment. This problem is the subject of Chapter VI.

[33]For the moment, international trade in goods is ignored. In Chapter IV when goods trade is introduced, the real sector ceases to be independent of the debt level, since price changes alter the trade balance.

[34]For this reason it is immaterial, in this instance, whether debt level changes are defined in terms of the fundamental policy variable, $m_o^* + b_o^*$, or the intermediate policy variable, p_o.

Since the desired nominal value of per capita foreign bond holdings, $f_t^* = p_f F_t / L_t$, is related to the value of desired real holdings, f, by the relation $f_t^* = p_t f$, and real holdings do not change, it follows directly that

$$(38) \qquad \frac{df_t^*}{f_t^*} = \frac{dp_o}{p_o} = \frac{d(m_o^* + b_o^*)}{m_o^* + b_o^*} \quad .$$

With return variables unchanged by changes in the debt level, the desired relative holdings of all assets in the wealth portfolio remain unchanged. The government, by forcing the public to hold a greater nominal amount of its financial liabilities, causes an increased nominal demand for foreign financial assets, as wealth holders attempt to restore the desired composition of their portfolios. After equilibrium is restored, all real magnitudes remain unchanged, an equal percentage increase in the nominal value of all components of wealth being balanced by an equivalent increase in the price level. If nominal foreign bond holdings were not increased, the real value of wealth held in the form of foreign assets would decline, leaving domestic residents with excess real levels of domestic assets. An increase in the level of government debt causes, therefore, an equal percentage increase in the level of foreign bond holdings.

The effects of this change on the balance of payments can be seen by rewriting equation (32) as[35]

(39) $\dot{R}_t = L_t(r_f - \mu)f^*_t$.

For a given change in f^*_t, the direction of the change in the reserve flow depends on the relative sizes of r_f and μ. This condition is similar to that discussed in Section 3.1. Interest income from foreign investment at any moment in time depends, given the stock of foreign assets held, on the rate of interest on foreign bonds, r_f. The flow purchase of new foreign bonds depends on the desired rate of growth of the existing stock holdings. For the same reasons discussed above, the desired rate of growth of nominal foreign bond holdings along the steady-state growth path will be governed by the rate of growth of domestic financial assets; if B and M are growing at rate μ, portfolio equilibrium requires that F grow at rate μ also. For this reason, foreign exchange reserves will be decreasing if $\mu > r_f$ and capital outflows exceed the return flow of interest

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[35] It should be noted at this point that foreign ownership of domestic bonds is not considered explicitly in the model used here, and that equation (39) does not, therefore, give the reserve effect of the full capital account. To include foreign borrowing explicitly would greatly complicate the analysis, since it would require a complete specification of the foreign sector and its equilibrium interdependencies with the domestic economy, and this added complexity would tend to obscure some of the basic relationships we wish to emphasize. Modification of the results which follow to take account of changes in foreign borrowing may be made on the basis of the model developed in Section 3.1 where this factor was considered. Such modification should be of minor importance in the case of composition neutral changes in overall debt size, since, as indicated in Section 3.1, changes in "scale" variables affect mainly domestic holdings of foreign bonds.

income. The reserve effect of changes in the level of government debt will, therefore, depend on the sign of $(r_f - \mu)$; if foreign investment is draining off foreign exchange reserves, an increase in the debt level which increases the level of this activity, increases the reserve drain.

The effect of a change in the rate of growth of government debt is similar to that just analyzed for a change in the level of debt. Since the equilibrium rate of growth of nominal foreign bond holdings is determined by the rate of growth of nominal domestic debt, an increase in μ will raise the rate of foreign investment by an equal amount. Whether or not this improves the reserve position will depend on the relative sizes of r_b and the existing μ. The difference in this case is that a change in μ changes the desired level of real as well as nominal holdings of foreign assets. The exact relation can be found by differentiating equation (39), using equations (34) - (37):

(40) $$d\dot{R}_t = p_t L_t \left[(r_f - \mu)\left(\frac{df}{d\mu} + tf\right) - f \right] d\mu ,$$

where

(41) $$\frac{df}{d\mu} = \frac{\gamma_2 \gamma_3 \sigma_1}{\tau_1} ,$$

$$\tau_1 = sa' - n(1-s)(1+\gamma_1+\gamma_2)\gamma_3' g'' < 0,$$

and $$\sigma_1 = sg' - n.$$

The sign of $d\dot{R}_t$ will be determined in the long-run (i.e. as $t \longrightarrow \infty$)

by the sign of the term $p_t L_t (r_f - \mu) f d\mu$;[36] and an increase in the rate of debt creation, which raises the rate of foreign investment, will improve or worsen the reserve position as μ is less or greater than r_f. It should be emphasized that this result is independent of the directional effect of changes in μ on <u>real</u> foreign bond holdings, i.e. the sign of $d\dot{R}_t$ in equation (40) is independent of the sign of $df/d\mu$.[37] This is because the balance of payments is a monetary rather than a real phenomenon, and it is nominal rather than real magnitudes which are crucial in its determination.

This distinction between nominal and real magnitudes is not made in the Mundell or static portfolio balance models discussed in Chapter II, and its neglect is of crucial importance to the conclusions derived on the basis of those models. For example,a central conclusion of the static portfolio balance model is that, at a constant income level, steady-state investment in foreign bonds and continuous international capital flows are inconsistent with long-run portfolio equilibrium. When price flexibility is introduced, however,

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[36]Equation (40) is derived on the assumption that $dp_o/d\mu = 0$. If it was assumed instead that $d(m_o^* + b_o^*)/d\mu = 0$, the term $(r_f - \mu)(f/p_o)$ $dp_o/d\mu$ would be added to the bracketed expression in equation (40). Since this term does not involve t, it will not, in the long-run, affect the sign of $d\dot{R}_t$, and results will be the same under either assumption.

[37]Since $d\mu = d\Pi$, a change in the growth rate of nominal debt leaves the growth rate of real debt unchanged at n. The <u>level</u> of real holdings are, however, influenced by the change is the real rate of return on financial assets. As indicated in equation (41), the direction of this effect will depend on the sign of σ_1. This term is identical to the term Δ_3 encountered in Section 3.3, and its interpretation in this instance similar to that given previously. A term analogous to Δ_1 does not appear in equation (41), since the effect of a change in μ on k is unambiguous.

this conclusion no longer holds. In terms of the model developed here, if n = 0 the real value of total foreign bond holdings remains constant over time since $p_f F_t / p_t = L_o f$. However, if $\mu > 0$, nominal holdings continue to grow, and continuous capital flows occur, since $p_f \dot{F}_t = \mu p_f F_t > 0$. With no growth in output or the capital stock, a policy of continuous creation of new financial assets by the government generates an equivalent rate of inflation (i.e. $\mu = \pi$), and portfolio equilibrium requires domestic residents to continuously augment their nominal foreign bond holdings in order to keep the real value of foreign holdings constant.[38] Central in this process is the role of price flexibility, which allows a government policy of continuous debt creation at a constant income level to be compatible with asset equilibrium.

The distinction between nominal and real magnitudes is also important in considering changes in the composition of government debt, and corresponds to the distinction between price level neutral and debt level neutral interest rate changes. Consider first a price level neutral change in debt composition. With prices pegged, nominal and real foreign bond holdings will move together in this case. Mathematically, from equations (34) - (37) and the fact that in equilibrium $f = k\gamma_2\gamma_3$,

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[38]The effect of inflation on financial investment is analogous to that of physical depreciation on capital goods investment; if the stock of capital goods depreciates, continuous new investment is required just to maintain a constant level of capital and output.

$$(42) \qquad \frac{df^*_t}{f^*_t} = \frac{df}{f} = \frac{1}{f}\left[\sigma_3 + \frac{(1-s)\tau_2}{\tau_1}\sigma_2\right] dr_b \quad ,$$

where τ_1 was defined previously, and

$$\tau_2 = \gamma_2(\gamma_3 - k\ \gamma_3' g'') > 0 \ ,$$

$$\sigma_2 = n\left[\gamma_3(\gamma_1' - \gamma_2') + (1+\gamma_1+\gamma_2)\gamma_3'\right]$$

$$= n\frac{d}{dr_b}\ \frac{(M_t + p_b B_t + p_f F_t)/p_t}{K_t}\ \Bigg|\ \ i = \text{constant},$$

and

$$\sigma_3 = k(\gamma_2\gamma_3' - \gamma_3\gamma_2') = k\frac{d}{dr_b}\ \frac{p_f F_t/p_t}{K_t}\ \Bigg|\ \ i = \text{constant}.$$

The direction of the change in desired foreign bond holdings and the sign of equation (42) will depend on the signs of σ_2 and σ_3. If, for example, foreign and domestic bonds are close substitutes but domestic bonds and capital are poor substitutes (i.e., if γ_2' is large relative to γ_3'), $\sigma_3 < 0$ and an increase in r_b will, <u>ceteris paribus</u>, lower the desired foreign bond to capital ratio. If $\sigma_2 > 0$, the absolute level of capital holdings will decline also,[39] and desired per capita foreign bond holdings will decline.

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[39] Differentiating equation (37) with respect to r_b gives

$$(42') \qquad dk = \frac{(1-s)\sigma_2}{\tau_1} dr_b \ .$$

Since $\tau_1 < 0$, if $\sigma_2 > 0$, dr_b and dk will have opposite signs, and an increase in the interest rate will lower per capita capital holdings.

The sign of σ_2 depends on the relative substitutability of assets (i.e. on the relative sizes of γ_1', γ_2' and γ_3') as indicated in equation (42). The nature of this dependency is similar to that discussed in the previous section in relation to Δ_1 and β_1' and β_2'. Since an increase in r_b leaves the return to money and foreign bonds unaffected, it would again be expected normally to cause a movement out of capital into financial assets as a whole, and therefore σ_2 will generally be assumed positive.

If, however, foreign and domestic bonds are poor substitutes relative to domestic bonds and capital, making $\sigma_3 > 0$, holdings of foreign bonds relative to capital will tend to increase and, even if $\sigma_2 > 0$ and capital holdings decline absolutely, the level of foreign bond holdings may increase. In general, therefore, a price neutral increase in r_b may raise or lower the level of foreign investment and the capital outflow, depending on the nature of asset preferences. The effect of this change on foreign exchange reserves will depend in addition on the sign of $(r_f-\mu)$, as can be seen from equation (39).

For a debt level neutral change in r_b, changes in nominal and real foreign bond holdings differ. From the relation $f_t^* = p_t f$ it follows that

$$\text{(43)} \qquad \frac{df_t^*}{f_t^*} = \frac{df}{f} + \frac{dp_o}{p_o} \quad .$$

Since real variables are independent of the price level, df/f is still given by equation (42). However, with a policy of debt level neutrality, it can no longer be assumed that $dp_o = 0$. The direction and magnitude of the change in prices will depend on the change in desired real wealth holdings. Assume for the moment that debt level neutrality is defined as holding constant the total stock of nominal financial assets outstanding. From the relation $p_t = (m_t^* + b_t^* + f_t^*)/$

$(m + b + f)$, it follows that, if $d(m^*_o + b^*_o + f^*_o)/dr_b = 0$,[40]

(44) $$\frac{dp_o}{p_o} = - \frac{d(m + b + f)}{m + b + f} = - \frac{\sigma_1\sigma_2}{nk\gamma_3(1+\gamma_1+\gamma_2)\tau_1} dr_b \quad .$$

where σ_1, σ_2 and τ_1 were defined previously. The direction of price movements will depend on the signs of the terms σ_1 and σ_2. This condition is almost identical to that described by equation (25) and discussed in the last section. Again the change in desired real financial wealth will depend on the directions and relative magnitudes of a substitution effect (the shift between financial assets and capital), and a scale effect (the change in total desired real wealth).

Using the original definition of debt level neutrality as a change in r_b holding the total outstanding nominal financial liabilities of the government constant, i.e. $d(m^*_o + b^*_o)/dr_b = 0$, the change in the price level can be shown to be,

(45) $$\frac{dp_o}{p_o} = \frac{n\tau_1\sigma_3 - \sigma_2\left[\sigma_1 - (1-s)n\tau_2\right]}{nk\gamma_3(1+\gamma_1)\tau_1} dr_b \quad ,$$

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[40] This definition of debt level neutrality has less intuitive appeal than the definition $d(m^*_o + b^*_o)/dr_b = 0$ used subsequently. However, it makes debt composition policy correspond more closely to open market operations under conditions of reserve sterilization as defined in the Keynesian model, where sterilization is defined as a policy which keeps the money supply constant in the face of reserve changes. Suppose, for example, that an original bond sale is made, i.e. $dB_1 = - dM_1 > 0$. As a result of the increase in r_b, however, a capital inflow occurs, increasing reserves and the money supply, i.e. $- dF_2 = dR_2 = dM_2 > 0$. If an open-market sale is made to keep the money supply unchanged, $dB_3 = - dM_3 = dM_2 > 0$. The net result of the last two operations is $dM = 0$, $dB = -dF_2 > 0$, and after all adjustments are made, $d(M + B) > 0$ and $d(M + B + F) = 0$.

where, again, all terms have been defined previously. With the level of only government debt held constant, price changes will be determined by the change in desired real debt holdings, and will depend, therefore, on the change in desired real foreign bond holdings (determined by σ_2 and σ_3 as in equation (42)), as well as the desired change in total real financial wealth (determined by σ_1 and σ_2 as in equation (44)).

Whichever definition of debt level neutrality is used, the change in nominal foreign bond holdings for a given change in r_b will be similar. Specifically, substituting df/f from equation (42), and dp_o/p_o from either equations (44) or (45), into equation (43), gives

$$(46) \qquad \frac{df_t^*}{f_t^*} = - \frac{\gamma_2'(1+\gamma_1) + \gamma_1'\gamma_2}{(1+\gamma_1)\gamma_2} \, dr_b$$

$$= \text{the percentage change in } \frac{p_f F_t}{M_t + p_b B_t} \, ,$$

if $d(m_o^* + b_o^*)/dr_b = 0$, and

$$(47) \qquad \frac{df_t^*}{f_t^*} = - \frac{\gamma_2'\,(\gamma_1' - \gamma_2') + \gamma_2'\,(1+\gamma_1+\gamma_2)}{(1+\gamma_1+\gamma_2)\gamma_2} \, dr_b$$

$$= \text{the percentage change in } \frac{p_f F_t}{M_t + p_b B_t + p_f F_t} \, ,$$

if $d(m_o^* + b_o^* + f_o^*)/dr_b = 0$.

In both cases, the change in the level of nominal foreign bond holdings depends on the relative substitutability of financial assets (i.e. on the relative sizes of γ_1' and γ_2'.) If, for example, money and domestic bonds are close substitutes, large open market sales will be needed to achieve a given increase in the interest rate on domestic bonds. The resulting yield premium on domestic relative to foreign bonds will not, if the two types of bonds are poor substitutes, be sufficient to induce wealthholders to accept such a large relative shift in their bond portfolio toward domestic assets. To restore equilibrium, the existing domestic-foreign bond ratio must fall, and, since the public cannot change its holdings of domestic bonds, this can only be accomplished by an increase in the level of foreign bond holdings. Generally, a shift in the composition of domestic debt toward bonds, holding the overall level of debt constant, will lower the level of foreign investment only if domestic and foreign bonds are close substitutes relative to domestic bonds and money. The exact relationship, for both equations (46) and (47)[41] is

$$(48) \qquad \frac{df_t^*}{dr_b} \lesseqgtr 0 \quad \text{as} \quad \epsilon_{\frac{f_t^*}{b_t^*}, r_b} \gtrless \epsilon_{\frac{m_t^*}{b_t^*}, r_b} \left(\frac{\gamma_1}{1+\gamma_1} \right) ,$$

where the ϵ's are elasticities with respect to the indicated variables.

A comparison of equations (42) and (46) or (47) shows that the effect on foreign investment of a change in the domestic interest rate is less certain for price level neutral changes than for debt level neutral changes. This is because a price level neutral change in r_b involves shifts in both debt level and composition. If the nominal level of total government debt (or total financial wealth)

41 Since results are identical, it makes little difference whether debt level neutrality is defined in terms of the level of total financial wealth or total government debt.

is held constant as r_b is changed, only knowledge of the shift in the desired holdings of foreign bonds relative to total government debt (or total financial wealth) is required, as indicated in the second versions of equations (46) and (47), and only the substitutions among financial assets need be considered. The further substitution between capital and financial assets as a whole is immaterial in this case. However, it is important in the case of a price neutral change in r_b, since the nature of this second substitution determines the change in nominal debt level needed to peg prices.

The results of this section may be summarized as follows. A composition neutral increase in the level or rate of growth of government debt will cause an increase in the level of rate or growth of foreign investment. The effect of a change in the composition of government debt is uncertain. For a purely compositional shift holding the size of total debt constant, the result will depend only on the relative substitutability of financial assets. For a change in composition holding prices constant, the outcome depends on the desired changes in all components of the wealth portfolio.[42] The reserve effect of these changes in foreign investment will depend on the relationship between the rate of interest earned by foreign assets and the rate of government debt creation.

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[42]It should be noted that, whatever the direction of the effect, a change in r_b has a continuing flow effect on the capital account, not just a one-shot stock adjustment effect as suggested by the static portfolio balance model of Chapter II. Mathematically, the steady-state capital outflow is $p_f\dot{F}_t = \mu L_t f_t^*$, and equations (42), (46) or (47) show that df_t^*/dr_b is, in general, non-zero.

CHAPTER IV

GOVERNMENT POLICY AND LONG-RUN BALANCE-OF-PAYMENTS ADJUSTMENT

In Chapter III we developed a portfolio balance model of the economy which included both domestic and foreign financial assets and analyzed the properties of the model under conditions of steady-state growth. In particular, the impact on foreign asset holdings of changes in three fundamental government policy parameters, the level, composition and rate of growth of government debt, acting through three intermediate target variables, the level and rate of change of the price level, and the bond interest rate, was examined. In this chapter international trade in goods as well as financial assets is introduced and we derive the policy mix needed for overall balance-of-payments equilibrium.

Section 4.1 deals with the balance of trade impacts of price and interest rate changes and determines the combinations of these two variables which produce external balance. Section 4.2 then restates the analysis in terms of the related fundamental policy parameters debt level and composition. It will be concluded that while a composition neutral decrease in the level of government debt always improves the trade balance and lowers the level of foreign bond holdings, increasing the proportion of domestic bonds in a given total debt has an ambiguous effect on both the current and capital accounts.

4.1 The Trade Account and External Balance in Terms of Prices and the Interest Rate

In introducing international trade of goods as well as assets, it is assumed that the balance of trade will depend upon real disposable income and prices in the home country (Y and p), and income and prices abroad ($\bar{Y}$ and $\bar{p}$).[1] More specifically, the value of imports of the home country is given by

$$(1) \quad \text{imports} = \delta_t p_t Y_t, \ \delta_t = \delta(q_t), \ q_t = \frac{p_t}{\bar{p}_t}, \ 0 < \delta < 1, \ \delta' > 0.$$

Imports are a fraction of money disposable income, the fraction varying positively with the price of domestic relative to foreign goods. Exports will depend in a similar manner, on relative prices and money income abroad. However, if we ignore the foreign repercussions of changes in domestic variables and assume that $\bar{Y}$ and $\bar{p}$ grow at constant rates n and $\bar{\pi}$ respectively, equation (1) alone, now defined as net imports, can be used to analyze changes in the full trade account, the total balance of trade being divided

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[1] Complete rigor would suggest the use of a two-country two-good model, instead of the one-good model we employ. In a growth context, however, analysis of trade patterns using such a model becomes mathematically complex (see, for example, Oniki and Uzawa [47]), and such added complexity would make the model used here analytically unmanageable. The formulation chosen assumes that the goods produced by each country are differentiated, and therefore that both domestic and foreign goods will be purchased even if their prices differ. This is analogous to the assumption made previously that both domestic and foreign bonds will be held at differing interest rates.

analytically into net imports and autonomous exports, the latter omitted for the sake of expositional clarity.[2]

Using equations (31), (33) and (34)-(36) of Chapter III, defining real disposable income, the government budget constraint and desired asset holdings respectively, and

(2) $\dot{R}_t = -\delta_t p_t Y_t + c_f F_t - p_f \dot{F}_t$,

which defines the rate of government purchase of foreign exchange reserves, the equation giving the equilibrium capital-labor ratio along a balanced growth path can be derived, and is

(3) $a(k) = \frac{n}{s}\,[(1-\delta) + (1-\delta-s)\ \gamma_3\ (1+\gamma_1+\gamma_2)]$.[3]

It is clear from equation (3) that the existence of an equilibrium steady-state growth path depends on the constancy of δ over time. If, for example, the domestic inflation rate is greater than that prevailing abroad ($\Pi > \bar{\Pi}$), the price of domestic relative to foreign goods will increase over time, and δ_t will increase, asymtotically approaching some value $\delta_0 < 1$. Thus, in a world of fixed exchange rates, external equilibrium can only be

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[2]This procedure is again adopted in order to avoid a complete specification of the rest-of-world sector. For a discussion of the implications of explicitly including autonomous exports, see footnote 9.

[3]It is assumed throughout that the δ function is such that $(1-\delta-s) > 0$. This is equivalent to the assumption that the marginal propensity to consume domestic goods is positive (see equation (16)).

attained if the domestic rate of inflation equals the world rate; the rate of government debt creation ceases to be an independent policy parameter and it must equal $n + \overline{\pi}$. Only in the case of flexible exchange rates discussed in Chapter V can the rate of domestic inflation be freed from this constraint.

With this restriction on the domestic inflation rate, two independent policy variables, the level and composition of government debt, remain. What we wish to determine is the method by which government policy can, using these two tools, achieve steady-state equilibrium growth and zero change in foreign exchange reserves. Mathematically, the procedure used can be described as follows. The conditions necessary for internal equilibrium are given by equation (3) and can be written as an implicit function of the form $\phi_1(k, r_b, p_o) = 0$. The conditions necessary for external equilibrium are described by equation (2) with $\dot{R}_t = 0$ and can be written as $\phi_2(k, r_b, p_o) = 0$. The equation relating nominal and real government debt is $m_o^* + b_o^* = (m+b)\, p_o$ and can be written as $\phi_3(k, r_b, p_o, m_o^* + b_o^*) = 0$. There are two possible descriptions of full external equilibrium. In solution I, ϕ_1 and ϕ_2 are solved simultaneously eliminating k to obtain $\phi_4(r_b, p_o) = 0$. ϕ_4 gives the full external balance line in (r_b, p_o) space, and movements along it can be thought of as resulting from changes in the two intermediate policy variables, (I-1) dr_b for $dp_o = 0$,

and (I-2) dp_o for $dr_b = 0$.

In this case ϕ_3 merely defines the level of $m_o^* + b_o^*$ needed to achieve equilibrium; $m_o^* + b_o^*$ is endogenously determined and varies for (I-1) and (I-2).

In solution II, ϕ_1, ϕ_2 and ϕ_3 are solved simultaneously eliminating k and p_o to obtain ϕ_5 $(r_b, m_o^* + b_o^*) = 0$. ϕ_5 defines the external balance line in $(r_b, m_o^* + b_o^*)$ space, and movements along it can be thought of as resulting from changes in the two fundamental policy variables

(II-1) dr_b for $d\,(m_o^* + b_o^*) = 0$,

and (II-2) $d\,(m_o^* + b_o^*)$ for $dr_b = 0$.

In this case prices are endogenously determined and vary for both (II-1) and (II-2). In this section solution I is obtained. First the intermediate policy variables (I-1) and (I-2) are analyzed and then the properties of the ϕ_4 function described in terms of this analysis. In section 4.2 solution II is obtained and analyzed in terms of the corresponding fundamental policy variables (II-1) and (II-2).

Consider first variable (I-1), a change in the domestic price level holding the interest rate constant. The effect of such a change in prices on the equilibrium capital-labor ratio can be determined by differentiating equation (3) with respect to p_o, giving

(4) $$dk = - \frac{nq\delta'\psi_2}{\psi_1} \frac{dp_o}{p_o},$$ [4]

where

$$\psi_1 = sa' + n\,(1-\delta-s)\,(1+\gamma_1+\gamma_2)\,\gamma_3'\,g'' < 0,$$

and $\psi_2 = 1 + \gamma_3\,(1+\gamma_1+\gamma_2) > 0.$

From equation (4) it can be seen that increases in the price level will always increase the equilibrium capital-labor ratio and conversely. Per capita disposable income ($y = \frac{Y_t}{L_t}$) will also move directly with the price level since

(5) $$dy = \frac{q\delta'}{(1-\delta)} [y - \frac{n\psi_2\psi_3}{\psi_1}] \frac{dp_o}{p_o},$$

where ψ_1 and ψ_2 are defined as above and,

$$\psi_3 = g' + n\,(1+\gamma_1+\gamma_2)\,(\gamma_3 - k\gamma_3'g'') > 0.$$

Although output and disposable income both increase with increases in the price level, the expansion in disposable income is greater. This follows from

(6) $$d\left(\frac{Y_t}{Q_t}\right) = \left[\frac{q\delta'y}{(1-\delta)g} + \frac{n^2q\delta'\,(1+\gamma_1+\gamma_2)\,\psi_2}{\psi_1}\,(a\gamma_3'g'' + a'\gamma_3)\right] \frac{dp_o}{p_o},$$

since the expression in equation (6) is always positive.

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[4]The time subscript on q is omitted since its constancy over time is assured by the fact that $\mu = n + \Pi$. Again,

$\frac{dp_o}{p_o} = \frac{dp_t}{p_t}$, $\frac{dm_o^*}{m_o^*} = \frac{dm_t^*}{m_t^*}$, etc. follows from, e.g. $p_t = p_o e^{\Pi t}$.

In order to understand the mechanism by which price changes work through the trade balance to affect output and income, it is instructive to consider the simple case where money is the only financial asset and the rate of inflation is zero. Here, the <u>expenditure</u> of disposable income is defined by

(7) $Y = C + I$,

(8) $C = C_d + C_f$,

and

(9) $I = \dot{K} + \dot{M}$.

Total consumption is divided between domestic goods (C_d) and foreign goods or imports (C_f), and investment takes the form of acquisition of capital goods ($\dot{K}$) and hoarding ($\dot{M}$). The <u>sources</u> of disposable income are given by

(10) $Y = Q + T$,

and

(11) $Q = C_d + \dot{K}$.

Disposable income equals output (domestic production of consumer and capital goods) plus government transfer payments.

The income and expenditure definitions of disposable income are related through the budget equation.

(12) $T + \dot{R} = \dot{M}$, $\dot{R} = - C_f$.

Substitution of (8), (9), (11) and (12) into (10) yields (7). Equations (7) - (12) are accounting identities and must hold for all values; equilibrium is determined by savings and consumption behavior.

(13) $S = sY = I$,

(14) $C = cY$, where $c = (1-s)$,

(15) $C_f = c_f Y$, where $c_f = \delta$,

(16) $C_d = C - C_f = c_d Y$, where $c_d = (1-\delta-s)$,

and

(17) $c = c_f + c_d$.

The total propensity to consume is constant. The propensities to consume domestic and foreign goods move inversely with changes in their relative price.

Given a position of equilibrium where equations (7) - (17) are satisfied, assume a _ceteris_ _paribus_ decrease in domestic prices. The initial impact will be a switching of the given total consumption from foreign to domestic goods. This will lead to a situation where

(18) $C_d + \dot{K} > Q$ and $T - C_f > \dot{M}$,

i.e. an excess demand for domestic output and an excess supply of money. With a policy of sterilization, however, the increase in foreign exchange reserves leads to a decrease in government transfer payments. This causes a decrease in the money supply, a reduction in disposable income -- and therefore consumption of both domestic and foreign goods -- and a decrease in real investment ($\dot{K} = sY - \dot{M}$). In the new equilibrium transfer payments and imports have decreased by an equal amount and disposable income has decreased relative to output (see equation (10)). Thus an _expenditure-switiching_ policy is accompanied by an expenditure-reduction policy.[5] The analysis here is similar to that discussed in footnote 14 of Chapter III for the case of government purchase of goods. It will be remembered that an increase in government purchases led to a decrease in government transfer payments, disposable income and output. Indeed, if we think of the government as selling abroad the goods purchased from domestic residents, that analysis can be stated in terms of an increase in exports.

We are now in a position to determine the effects of changes in the price level on the balance of payments. Consider first the balance of trade. From equations (1) and (5) it can be seen that δ, p_t and Y_t will each increase as a result of an increase in the price level and that therefore the balance of trade is worsened by an increase in the price level and conversely. With higher domestic prices, foreign goods become relatively less expensive and a higher proportion of disposable income is spent on imports. This is reinforced by the increase in the level of money disposable income; real income increases as imports rise and money income increases more than proportionally due to the increase in the price level. The result with respect to foreign bond holdings is also unambiguous since

$$(19) \qquad \frac{df_t^*}{f_t^*} = \frac{(1-\delta)\ k\ \gamma_2\gamma_3\psi_1 - n\ \delta'\psi_2\psi_4}{(1-\delta)\ k\ \gamma_2\gamma_3\psi_1}\,\frac{dp_o}{p_o},$$

[5]This distinction between expenditure-switching and expenditure-reduction and the need for the latter to accompany the former under conditions of full employment is discussed by Johnson [21, especially p. 195-6].

where $\psi_4 = (1-\delta)\ \gamma_2\ (\gamma_3 - k\ \gamma_3'g'') > 0,$

and $f_t* = p_t f,\ f = \frac{p_f F_t}{p_t L_t}$.

For given real bond holdings, nominal holdings are scaled up as the price level rises. A reinforcing factor is the increase in real holdings resulting from the substitution away from lower yielding capital goods to real financial assets. This increase in foreign bond holdings will decrease the level of reserves provided that $r_f < n + \overline{\pi}$. This is the condition, familiar from the last chapter, that new purchases exceed interest income.

The balance-of-payments impact of changes in the price level can be conveniently summarized by rewriting equation (2) as,

(20) $\dot{R}_t = L_t p_t\ [-\delta y + f\ (r_f - n - \overline{\pi})].$

We have shown that increases in the price level holding r_b constant increases p_t, δ, y and f. In the "normal" case this will mean a worsening of the balance of payments and a decrease in the level of reserves. A sufficient condition for this result is $r_f < n + \overline{\pi}$.

The analysis of (I-2), price neutral interest rate changes, is almost identical to that for the no-trade case previously analyzed in Chapter III, since with no change in relative prices, δ is invariant with respect to interest rate movements. The relevant differentials are,[6]

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[6]The equations given here correspond respectively to equations (42'), (24) and (42) of Chapter III, and an interpretation of their meaning can be found in the text following each.

(21) $$dk = \frac{(1-\delta-s)\ \theta_1}{\psi_1}\ dr_b,$$

(22) $$dy = \frac{\theta_1 \theta_3}{\psi_1}\ dr_b,$$

(23) $$df^* = p_o\ df = p_o \left[\theta_2 + \frac{(1-\delta-s)\,\psi_4}{(1-\delta)\ \psi_1}\ \theta_1\right] dr_b,$$

where,

$$\theta_1 = n \left[\gamma_3{}'\ (1+\gamma_1+\gamma_2) + \gamma_3\ (\gamma_1{}'-\gamma_2{}')\right]$$

$$= n \frac{d}{dr_b}\ \frac{(M_t+p_b B_t+p_f F_t)/p_t}{K_t}\ \Bigg|_{i\ =\ \text{constant}}$$

$$\theta_2 = k\ (\gamma_2\gamma_3{}'-\gamma_3\gamma_2{}') = k \frac{d}{dr_b}\ \frac{p_f F t/p_t}{K_t}\ \Bigg|_{i\ =\ \text{constant}}$$

$$\theta_3 = g' - n.$$

With prices constant, the change in the trade balance will depend only on the change in real disposable income. The direction of this change will depend, for the reasons discussed in Chapter III, on the signs of θ_1 and θ_3. The change in foreign bond holdings is determined in addition by the sign and size of θ_2.

This completes the analysis of the intermediate policy variables r_b and p_o. Full external balance stated in terms of these variables is given graphically in Figure 5.

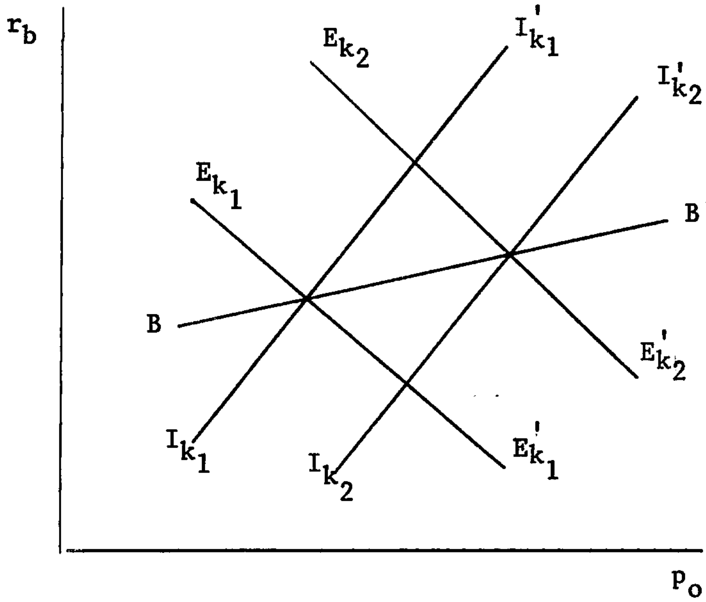

Figure 5.

Internal-External Balance in (r_b, p_o) Space

Internal balance lines (II') are obtained by mapping combinations of r_b and p_o which satisfy equation (3) for different values of k. Combinations of r_b and p_o, given different values of k, satisfying equations (2) or (20) with $\dot{R}_t = 0$ give external balance lines (EE'). Their intersection at the common values of k give positions of full balance (BB').[7] The slope of the BB' line depends on the sign and relative magnitude of the slopes of the II' and EE' lines. It can be found by taking the total differential of equations (3) and equation (2) with $\dot{R}_t = 0$ and combining to eliminate dk. The result is

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[7] In terms of the notation used earlier in this section II', EE' and BB' correspond respectively to ϕ_1, ϕ_2 and ϕ_4. Equation (24) which follows is the differential of $\phi_4 = 0$.

$$(24) \quad dr_b = \frac{q\delta' \left[-\psi_1\psi_5 + n\delta\psi_2\psi_3 - (r_f - n - \overline{\Pi})\, n\psi_2\psi_4 \right]}{\delta(1-\delta)\,\theta_1\theta_3 - (r_f - n - \overline{\Pi})\left[(1-\delta)\,\psi_1\theta_2 + (1-\delta-s)\,\psi_4\theta_1 \right]} \frac{dp_o}{p_o},$$

where $\psi_1 < 0$, $\psi_2 > 0$, $\psi_3 > 0$, $\psi_4 > 0$, θ_1, θ_2 and θ_3 were defined previously and

$$\psi_5 = g + k\,\gamma_3\,[n(1+\gamma_1) + (r_f - \overline{\Pi})\,\gamma_2] > 0.$$ [8]

The sign of equation (24) cannot in general be determined, but depends as would be expected on the signs of θ_1, θ_2, and θ_3 and $(r_f - n - \overline{\Pi})$. Several sufficient conditions can be discovered:[9]

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[8] It is assumed that $r_f - \overline{\Pi}$, the real rate of return on foreign bonds, is positive.

[9] As was noted earlier, autonomous exports were excluded from the balance-of-payments equation on the grounds that they were independent of changes in policy variables. They do however affect the <u>level</u> of the balance of payments and thus should strictly speaking be included in the definition of zero overall balance; setting $\dot{R}_t = 0$ in equation (2) would imply $\dot{R}_t > 0$ including autonomous exports. This adjustment can easily be made and leads to no alteration in the conclusions derived on the basis of equation (24). Following the assumptions made previously, assume that autonomous exports are some constant fraction h of money foreign income $\overline{p}_t\overline{Y}_t$. Assuming further that foreign and domestic real income grow at the same rate we get

$$\text{Autonomous exports} = \overline{p}_t\,(h\overline{Y}_t) = \overline{p}_t X_t = \overline{p}_t L_t x,$$

and the reserve equation becomes

$$\dot{R}_t = \overline{p}_t X_t - \delta p_t Y_t + p_f F_t\,(r_f - n - \overline{\Pi}).$$

For $\dot{R}_t = 0$ this gives

$$(2') \quad x = -\,\delta\,q\,y + q\,f\,(r_f - n - \overline{\Pi}).$$

Using equation (2') instead of (2) gives equation (24) with the single change that ψ_5 becomes $\overline{\psi}_5$, where

$$\overline{\psi}_5 = \psi_5 + \frac{(1-\delta) + q\delta'}{\delta' q^2}\,x > 0.$$

This added term arises because of the scale effect of price changes which raise or lower imports and foreign bond holdings proportionally leaving autonomous exports unchanged. Since this effect only reinforces the other effects of price changes, it does not alter the qualitative conclusions reached as to balance-of-payments policy; mathematically, the dependency of equation (24) on the signs of θ_1, θ_2 and θ_3 remains unchanged when ψ_5 is replaced by $\overline{\psi}_5$.

A. If $\theta_1 > 0$, $\theta_2 < 0$, $\theta_3 > 0$ and $r_f < n + \overline{\pi}$, the expression in equation (24) is positive and the BB' curve is upward sloping.

B. If $\theta_1 < 0$, $\theta_2 > 0$, $\theta_3 > 0$ and $r_f < n + \overline{\pi}$, the expression in equation (24) is negative and the BB' curve is downward sloping. The economic significance of these conditions can be found by referring to the analysis of changes in r_b and p_o just completed.

Consider first the conditions described by A. On the basis of equations (5) and (19) it was concluded that, with $r_f < n + \overline{\pi}$, increases in the price level always worsen the reserve position. From equations (21) - (23) it can be seen that as the bond interest rate increases there is an initial movement into financial assets and out of capital if $\theta_1 > 0$, and the capital-labor ratio and the level of output therefore decline. If in addition $\theta_3 > 0$, the decrease in output is greater than the decrease in required real investment, and disposable income declines also. Under these conditions a price neutral increase in the interest rate will decrease imports and improve the trade balance. If $\theta_2 < 0$, foreign bond holdings will also decline. $\theta_2 < 0$ indicates that as r_b rises the initial substitution between foreign and domestic bonds is greater than that between domestic bonds and capital. With $\theta_1 > 0$, this tendency is reinforced by the movement into all financial assets which decreases the holdings of real capital. This decrease in foreign asset holdings will also improve the balance of payments if $r_f < n + \overline{\pi}$. Under the conditions described in A, therefore, increases in the bond interest rate increase foreign exchange reserve levels and an offsetting increase in the price level is needed to maintain payments equilibrium, and the BB' curve is upward sloping. Under the conditions described in B, increases in the interest rate raise output, disposable income and foreign bond holdings, and worsen the balance-of-payments. An offsetting decrease in prices is needed in this case and the BB' curve slopes down. Intermediate cases are of course possible, where for example $\theta_1 < 0$ and $\theta_2 < 0$, and the trade and capital accounts may move in different directions in response to interest rate changes.

4.2 External Balance and Government Debt Management Policy

In this section external balance policies are restated in terms of the fundamental policy variables debt level and composition. Such a restatement is necessary, since the correspondence between intermediate and fundamental policy variables is not direct. Consider the case where external balance required matching interest rate and price increases. If desired real financial wealth holdings decrease sufficiently, the equilibrium price increase may be accompanied by a decrease in debt levels; in this case the BB' curve would be upward sloping in (r_b, p_o) space and downward sloping in $(r_b, m_o^* + b_o^*)$ space.

This discrepancy arises because changes in the level of government debt affect only the price level, while changes in debt composition have both interest rate and price effects. Consider first the relationship between changes in price and debt levels holding debt composition constant. Using equation (4) and the relation $(b_o^* + m_o^*) = (b + m)\,p_o$ gives

$$\text{(25)}\qquad \frac{dp_o}{p_o} = \left[\frac{(1-\delta)\ \gamma_2\gamma_3 k\psi_1}{(1-\delta)\gamma_2\gamma_3 k\psi_1\ -\ nq\delta'\psi_2\psi_4}\right]\frac{d(m_o^*+b_o^*)}{m_o^*+b_o^*}\ .$$

Since the bracketed expression is positive, it follows that debt and price levels move together. This means that the conclusions in Section 4.1 as to the directional balance-of-payments effects of changes in the price level can be restated without change in terms of changes in the level of government debt. In particular, increases in the level of government debt will always increase imports and foreign bond holdings. The magnitude of changes resulting from equal changes in price and debt levels will differ however. Since the bracketed expression above is a positive fraction, the increase in prices is less than the increase in nominal debt levels. This is because, with the return on capital reduced, the demand for real financial wealth holdings increases, moderating the price rise.

Stating the change in foreign bond holdings in terms of the change in the level of government debt by substituting equation (25) into equation (19) gives

$$\text{(26)}\qquad \frac{df_t^*}{f_t^*} = \frac{d(m_o^*+b_o^*)}{m_o^*+b_o^*}\ .$$

Foreign bond holdings increase or decrease in the same proportion as government debt. This is precisely the same result as in the no-trade situation of Chapter III. With no change in the rate of return on the three financial assets, their desired positions in wealth portfolios relative to each other remain unchanged, and an increase in the nominal quantity of domestic financial assets leads to an equal increase in the desired level of foreign financial assets. Since for balance-of-payments purposes we are only concerned with nominal foreign asset holdings, the substitution in real wealth portfolios between real foreign bond holdings and capital is immaterial.

An analysis of changes in the composition of government debt holding the level of government debt constant is more complicated. The relevant equations can be found by solving simultaneously the total differential of the basic equilibrium equation (3) with respect to r_b and p_o and

$$(27) \qquad \frac{d(m_o{*}+b_o{*}+f_o{*})}{m_o{*}+b_o{*}+f_o{*}} = \frac{d(m+b+f)}{m+b+f} + \frac{dp_o}{p_o},$$

setting $d(m_o{*}+b_o{*}+f_o{*}) = 0.$[10]

In this case the change in prices is determined endogenously, and is

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[10]The analysis for a pure change in composition defined by $d(m_o{*}+b_o{*}) = 0$ is omitted. Conclusions using either definition are similar, the difference being in the conditions under which prices increase or decrease as discussed in Chapter III. The definition in terms of the level of total financial wealth was used in order to minimize mathematical complexity. On this point, see also footnote (15).

$$(28) \quad \frac{dp_o}{p_o} = - \frac{\theta_1\theta_4}{n(1+\gamma_1+\gamma_2)[k\gamma_3\psi_1 - nq\delta'\psi_2(\gamma_3 - k\gamma_3'g'')]} dr_b,$$

where

$$\theta_4 = sg' - (1-\delta)n.$$

The direction of price movements will depend on the signs of θ_1 and θ_4. The condition here is almost identical with that given by equations (25) and (44) of Chapter III, for the case where trade was absent, and the interpretation of θ_4 similar to that given for Δ_3 and σ_1.[11] If, for example, $\theta_1 >$ and 0 and $\theta_4 > 0$, a decrease in the bond interest rate will decrease prices. With $\theta_1 > 0$, the level of output will increase (see equation (29)), and $\theta_4 > 0$ indicates that the desired increase in savings resulting will be greater than the needed increase in capital-goods investment. This "excess" savings leads to an increased demand for real fianacial assets causing a decrease in the price level.

Given the direction of price changes, the size of the movement in prices will depend on the price sensitivity of import demand. In particular, the greater the substitutability of domestic and foreign goods, the less the absolute change in prices resulting from a given interest rate change. This is because price-induced adjustments in the trade balance tend to offset movements in desired real financial

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[11] The expressions Δ_3 and σ_1 of Chapter III and θ_4 differ only by the factor δn. This item appears in θ_4 because with trade, changes in output differ from changes in "goods available" and income by the factor $\frac{1}{(1-\delta)}$ due to changes in imports.

asset holdings which trigger the original price changes. Suppose, for example, that a change in r_b causes a *ceteris paribus* increased demand for real financial asset holdings. This necessitates a decline in domestic prices. This price decrease, however, lowers imports which leads *ceteris paribus* to a decrease in the desired level of real financial assets,[12] offsetting partially the original increase. Thus the price decrease will be smaller than it would have been in the absence of trade. In the limit, when the price elasticity of import demand becomes infinite, no price change can occur and domestic prices are pegged at the world level (i.e., $\frac{dp_o}{p_o} \rightarrow 0$ as $\delta' \rightarrow \infty$ in equation (28)).

Consider next the change in output resulting from a purely compositional change in government debt, given by

$$(29) \qquad dk = \frac{k[(1-\delta-s)(1+\gamma_1+\gamma_2)\gamma_3+q\delta'\psi_2]\theta_1}{(1+\gamma_1+\gamma_2)[k\gamma_3\psi_1 - nq\delta'\psi_2(\gamma_3-k\gamma_3'g'')]} dr_b.$$

The conditions under which output will increase or decrease are identical to those holdings for a price neutral change in debt composition (see equation (21)). The size of the resulting change will differ however. More specifically, it can be shown that the absolute change in output resulting from a purely compositional change in the interest rate will be greater than that resulting

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[12]This follows from the discussion of price changes holding r_b constant in Section 4.1.

from an equal price neutral change if, in the former case, prices and output move in the same direction, and less if they move in the opposite direction.[13] Furthermore, this discrepancy in the size of output adjustments resulting from the two types of interest rate changes will be greater the greater is the price sensitivity of import demand. Consider the case where $\theta_1 > 0$ and $\theta_4 < 0$. Here an increase in r_b will cause output to decline. If the increase in r_b is unaccompanied by a change in the level of debt, prices will decline also. Since it was concluded in Section 4.1 that price decreases, working through the trade balance, lower the level of output, this secondary decline reinforces the first. If prices increase ($\theta_4 > 0$), the movement is offsetting.

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[13]The proof of this statement is as follows. Rewrite equations (29) and (21) as

$$(29') \quad dk = \frac{(1-\delta-s)\theta_1+\delta' \dfrac{q\psi_2\theta_1}{(1+\gamma_1+\gamma_2)\gamma_3}}{\psi_1 - \delta' \dfrac{nq\psi_2(\gamma_3-k\gamma_3'g'')}{k\gamma_3}} dr_b = \frac{a + \delta' b}{c + \delta' d} dr_b$$

and

$$(21') \quad dk = \frac{a}{c} dr_b,$$

where c and d are negative and a and b have the same sign. It is clear that $\left|\frac{a + \delta'b}{c + \delta'd}\right| \gtrless \left|\frac{a}{c}\right|$ as $\left|\frac{b}{d}\right| \gtrless \left|\frac{a}{c}\right|$.

It can be shown that $\frac{b}{d}$ as defined by equation (29') reduces to $\frac{b}{d} = \dfrac{a}{c - \frac{\theta_4}{k}}$, which implies that $\left|\frac{b}{d}\right| \gtrless \left|\frac{a}{c}\right|$ as $\theta_4 \lessgtr 0$.

Therefore, the output change resulting from a purely compositional interest rate adjustment will be greater than that resulting from a price neutral change if $\theta_4 < 0$. But $\theta_4 < 0$ is the condition under which output and prices move together (see equations (28) and (29)), from which the statement in the text follows directly.

The effect of price changes can alter the direction as well as the magnitude of changes in disposable income. The exact relationship is

(30) dy =

$$\frac{\theta_1\{n(1-\delta)k[(1-\delta-s)(1+\gamma_1+\gamma_2)\gamma_3+q\delta'\psi_2]\theta_3-q\delta'[(1-\delta-s)y+nk\psi_2]\theta_4\}}{n(1-\delta)(1-\delta-s)(1+\gamma_1+\gamma_2)[k\gamma_3\psi_1-nq\delta'\psi_2(\gamma_3-k\gamma_3'g'')]}\,dr_b.$$

The change in disposable income depends on the signs of θ_1 and θ_3 in the same manner as in the case of a price neutral interest rate change. It depends in addition, however, on θ_4, i.e. on the direction of price changes. If θ_1 and $\theta_3 > 0$, income declines following a price neutral increase in r_b. Under these conditions an increase in r_b holding the debt level constant will always lower output. Further, there is a tendency for disposable income to decrease due to the fact that $\theta_3 > 0$. If $\theta_4 > 0$, however, prices will increase, and the analysis of Section 4.1 indicated that such an increase would, working through the trade account, cause an increase in disposable income. If the latter effect is greater than the former, disposable income could in this case increase.[14]

[14]Per capita disposable income can be written as

$$y = \frac{1}{(1-\delta)}\,[g+nk\gamma_3(1+\gamma_1+\gamma_2)].$$

The directional change in the bracketed expression with respect to interest rate changes can be shown to depend only on θ_1 and θ_3 using either definition of interest rate policy. The relative magnitude of its movement based on differing definitions will depend upon θ_4, in a manner similar to that discussed in footnote (13). Changes in the first term, $\frac{1}{(1-\delta)}$, however, will depend only on price movements. For example, if prices increase, this term will increase and y may increase even if the bracketed expression declines. Relevant here is equation (6) which shows that price movements affect disposable income more than output.

We are now in a position to determine the conditions under which a change in the bond interest rate holding the level of debt constant will improve or worsen the trade account. For a price neutral interest rate change, the change in imports depended only upon real disposable income. With prices not pegged, it depends in addition on changes in the propensity to import and money disposable income. Since price changes cause directionally inverse movements in the trade balance, for the conditions under which a price neutral interest rate increase improves the trade balance ($\theta_1 > 0$, $\theta_3 > 0$), the improvement will be even larger for an equal increase holding debt level constant, provided that the accompanying price movement is downward (i.e., if $\theta_4 < 0$). If prices increase ($\theta_4 > 0$), any improvement will be smaller, and the trade balance may actually worsen if the price effects are sufficiently large. In particular, an increase in imports may occur even if real disposable income declines.

A similar conclusion holds for the capital account. The conditions sufficient for a decrease in real foreign bond holdings are the same for both definitions of a change in r_b. The flexible price analogue to equation (23) is

$$(31) \qquad df = \left\{\theta_2 + \frac{k\psi_4[(1-\delta-s)(1+\gamma_1+\gamma_2)\gamma_3+q\delta'\psi_2] \qquad \theta_1}{(1-\delta)(1+\gamma_1+\gamma_2)[k\gamma_3\psi_1-nq\delta'\psi_2(\gamma_3-k\gamma_3'g'')]}\right\} dr_b.$$

Since nominal and real holdings are related by $f_t{}^* = p_t f$, conditions sufficient to produce a decrease in nominal foreign bond holding in the price neutral case ($\theta_1 > 0$, $\theta_2 < 0$) will cause an even larger decrease if an accompanying fall in prices occurs (i.e., if the added sufficient condition $\theta_4 < 0$ is satisfied). As opposed to the situation for imports, however, a simple necessary condition can be determined. Using equations (28) and (31) gives equation (47) of Chapter III, or

$$(32) \qquad \frac{df_t^*}{f_t^*} = - \frac{\gamma_2(\gamma_1' - \gamma_2') + \gamma_2'(1+\gamma_1+\gamma_2)}{\gamma_2(1+\gamma_1+\gamma_2)}\, dr_b,$$

$$= \text{the percentage change in } \frac{p_f F_t}{M_t + p_b B_t + p_f F_t}^{15} .$$

If total nominal financial wealth is held constant, changes in the nominal amount held of a particular financial asset resulting from a change in the rate of return on domestic bonds depends only on the substitutions among financial assets. Attempted substitution between nominal financial assets and capital will only lead to price changes which equilibrate holdings in real terms. Thus while changes in desired real holdings differ when trade is introduced, changes in nominal holdings are the same in both cases. An increase in the domestic bond interest rate will therefore lower foreign bond holdings if domestic bonds are a closer substitute for foreign bonds than money.[16]

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[15] If a purely compositional interest rate change was defined by $d(m_o^* + b_o^*) = 0$, equation (32) would become identical with equation (46) of Chapter III.

[16] The exact condition under which dr_b and df_t^* in equation (32) have the opposite sign is given by equation (48) of Chapter III.

Conditions needed to achieve full external balance can now be stated in terms of the fundamental policy variables debt level and debt composition. In Section 4.1 two conditions sufficient to determine the slope of the BB' line in Figure 5 were given. These conditions will still hold if prices are replaced by the level of government debt in Figure 5, provided that the condition $\theta_4 < 0$ is added in both cases. Under condition A as stated, an interest rate increase without an accompanying price change will decrease real disposable income and the level of foreign bond holdings. If in addition prices are allowed to vary and decrease, these movements are reinforced. The same holds for condition B with directions reversed. Under the amended conditions A and B, the BB' curve expressed in terms of $m_o* + b_o*$ instead of p_o will have a greater absolute slope and the same sign. If the condition $\theta_4 > 0$ is added to A and B on the other hand, the BB' curve becomes flatter and its slope may change signs.

Our results are summarized in Table 2, which shows the effect of increases in the indicated policy variables on the levels of output, prices, foreign bond holdings and imports. The most striking feature of Table 2 is the unambiguous effect of changes in the level of government debt and the uncertain effect of changes in debt composition.

This difference hinges on the distinction between scale and substitution effects. An increase in the level of government debt, since it leaves unaltered the return on financial assets, causes a proportionate variation in each component of the financial wealth

Table 2

Effects of Increases in Policy Variables on Indicated Target Variables

	Output (Q_t)	Prices (p_t)	Foreign Lending ($p_f F_t$)	Imports ($\delta_t p_t Y_t$)
Debt Level $M_o + p_b B_o$ (or p_o)	+[a]	+	+	+
Rate of debt Increase μ (or π)	+[b]	+	+	+
Debt Composition r_b (Prices Constant)	- if and only if $\frac{d}{dr_b} \frac{(M+p_b B+p_f F)/p}{K} > 0$[c]	0	- if $\frac{d}{dr_b} \frac{(M+p_b B+p_f F)/p}{K} > 0$[c] and $\frac{d}{dr_b} \frac{p_f F/p}{K} < 0$[c]	- if $\frac{d}{dr_b} \frac{(M+p_b B+p_f F)/p}{K} > 0$[c] and $g' > n$
Debt Composition r_b (Debt Level Constant)	- if and only if above holds	- if output declines and $(1-\delta)n > sg'$	- if above holds and $(1-\delta)n > sg'$ or - if and only if $\frac{\gamma_2'}{\gamma_2} > -\frac{\gamma_1'}{(1+\gamma_1)}$	- if above holds and $(1-\delta)n > sg'$

a. 0 = no change, + = increase, - = decrease
b. unstable
c. derivative taken holding i, return on capital, constant

portfolio -- in particular foreign bonds. An increase in nominal wealth occurs, and since this increase is entirely in financial form, wealth holders attempt to move out of financial assets into real capital. Portfolio equilibrium is re-established partly through an increase in real capital holdings and partly through a decrease in the real value of financial wealth as prices rise. A change in the composition of government debt has no initial impact on the level of nominal wealth or its division between real and financial assets; it affects only the return on one component of the financial wealth portfolio, domestic bonds. With no scale changes, both the substitutions within the financial sector (on which the change in holdings of foreign bonds depends) and between financial assets and capital (on which the changes in prices and output depend) are uncertain, the result depending on relative asset preferences and savings behavior; the stimulus of a change in yield is weaker than that of a change in quantity.

Two further points should be re-emphasized at this point. First, debt <u>level</u> policy as described here has continuing steady-state effects on the balance of payments. In a world of growth, government financial policy equilibrates the steady-state balance of payments through changes in $(\dot{M+B})_t$. Since this variable is determined by the level of debt as defined here as well as the debt growth rate, a classification of these two policy variables based on a distinction between stock and flow effects as traditionally defined is unwarranted.

The second point which should be reiterated is that the distinction between changes in the level and composition of government debt evident in Table 2 is not made in analyses of the external balance problem based on the standard Keynesian model. In that model, monetary policy, consisting of open-market operations, is concerned with the composition of government debt. The level of government debt is determined as a by-product of the chosen fiscal policy, government deficits or surpluses being financed entirely by bond sales or purchases. Debt level is thus not a separate policy tool. Indeed the overall level of debt is immaterial. It is only the level of a component, non-interest bearing government debt or money, which matters in the determination of the interest rate; the level of bonds outstanding has no effect on the bond interest rate!

This failure of the Keynesian model to consider the effect on the bond market of changes in monetary and fiscal policy, means that conclusions derived on the basis of that model differ considerably from those derived here. Suppose, for example, that government debt is equally divided between money and bonds and that an open-market purchase is accompanied by a non-recurring government deficit of twice the magnitude. The net result of this operation is a composition neutral increase in government debt. Under the assumptions of the Keynesian model, however, after the temporary income effects of the one-shot government deficit had ceased, the interest rate would be lower, not unchanged. In fact, the situation would be exactly the same if only the open-market operation had occurred unaccompanied by any fiscal policy and increase in debt levels.

A second striking instance of the different conclusions which emerge from use of the standard model and that developed here concerns the change in international capital movements resulting from a change in fiscal policy. In the Keynesian model an increased government deficit increases income, leading, since the money supply is unchanged, to an increase in the interest rate and a <u>decreased</u> capital outflow. In terms of our model, the change can be thought of as a combination of an increase in government debt and a shift in composition from money to bonds. The result would be a higher bond interest rate but, although the effects of the compositional change are uncertain, a probable <u>increase</u> in the level of foreign bond holdings. The effects on reserve levels differ further since interest payments on foreign asset holdings are neglected in the traditional analysis. The lack of portfolio balance considerations in the Keynesian model means that policy conclusions based upon it cannot be automatically translated into prescriptions for long-run external equilibrium.

CHAPTER V

EXTERNAL BALANCE, INFLATION AND FLEXIBLE EXCHANGE RATES

Our analysis of external equilibrium under a system of fixed exchange rates has shown that government debt creation policy must be assigned the task of keeping domestic prices in line with those prevailing abroad. This restriction posed no problem for the domestic economy, since prices were assumed to be completely flexible. A more reasonable assumption would be that prices were inflexible downward or even that the rate of price increase could not be reduced below some positive minimum π_o. In this case if an attempt were made to reduce μ below n or n + π_o, real per capita money holdings would constantly decline, a situation inconsistent with a positive rate of capital accumulation and portfolio balance. In order to avoid this decline in output the government would be forced to "accomodate" the price increases by the necessary monetary expansion. In this situation government policy based solely on manipulation of government debt might be insufficient to achieve both internal and external equilibrium simultaneously. What is needed in this case is the introduction of an additional policy tool which will allow a divergence of domestic and foreign monetary policy, exchange rate variation.

A distinction must be made at the outset between two possible types of exchange rate variation. This distinction is analogous to that made previously between changes in the level and rate of growth of government debt. Specifically, the exchange rate e_t, defined as the price of a unit of foreign currency, is

$$(1) \qquad e_t = e_o \exp. (\phi t).$$

Changes in e_o will be referred to as changes in the level of the exchange rate and changes in ϕ as changes in its rate of change.

Consider first changes in e_o. It is assumed provisionally that $\phi = 0$ and that $\mu = n + \overline{\Pi}$ where $\overline{\Pi}$ is the foreign inflation rate, so that $\Pi = \overline{\Pi}$. The reserve equation expressed in units of domestic currency now becomes

$$(2) \qquad \dot{R}_t = - \delta_t p_t Y_t + e_t(c_f F_t - p_f \dot{F}_t) ,$$

where

$$\delta_t = \delta(q_t), \; q_t = \frac{p_t}{e_t \overline{p}_t} .$$

p and Y are expressed in terms of domestic currency and $\overline{p}$, c_f and p_f in foreign currency. The relative price of goods produced at home and abroad involves e, since each price must be a measure of the amount of domestic currency needed to purchase one unit of each good. Portfolio equilibrium in the holding of domestic and foreign bonds is now expressed as

$$(3) \qquad \frac{e_t p_f F_t}{p_b B_t} = \gamma_2 \; (r_f - r_b) \; , \; \gamma_2' > 0 .$$

The other behavioral relations and definitions in the model remain unchanged, and the equilibrium equation is identical to equation (3) of chapter IV, with δ and γ_2 defined by equations (2) and (3). Using this equation it can be shown that, for example,

$$(4) \qquad dk = \frac{nq(1-\delta)k\gamma_2\gamma_3\psi_2}{(1-\delta)\gamma_2\gamma_3k\psi_1 - nq\delta'\psi_2\psi_4}\frac{de_o}{e_o} \quad ,$$

where all symbols have been defined previously. Equation (4) shows that a devaluation by the home country ($de > 0$) always decreases the level of output and conversely. The reason for this can be seen by examining the price effects of exchange rate adjustment, given by

$$(5) \qquad \frac{dp_o}{p_o} = \frac{nq\delta'\psi_2\psi_4}{nq\delta'\psi_2\psi_4 - (1-\delta)\gamma_2\gamma_3k\psi_1}\frac{de_o}{e_o} \quad .$$

The expression in equation (5) is a positive fraction. From this it follows that a devaluation causes domestic prices to increase but by an amount less than that of the exchange rate increase. As a result q, the price of domestic goods relative to imports, is decreased by a devaluation, but less than proportionally. The impact of a change in q on the equilibrium growth path has already been analyzed in Chapter IV for the case where q was changed as a result of a change in the level of government debt. Analogously in this case, it can be shown that a devaluation reduces the propensity to import, the level of output (equation (4)) and real disposable income. The "original" effect of the devaluation is to lower q proportionally. But this leads to a decrease in desired financial asset holdings causing, with fixed debt levels, a secondary price increase partially offsetting the effects of the devaluation. It follows that a devaluation always reduces net imports measuerd in terms of foreign currency ($p_t\delta Y_t/e_t$), and conversely. Since domestic prices increase, however, it may or may not reduce imports measured in terms of domestic currency.

The effect of a devaluation on the capital account is also similar to that of a change in debt level. It can be shown that[1]

$$(6) \qquad \frac{df_{t^*}}{f_t^*} = - \frac{de_o}{e_o} \quad , \text{ where } f_t^* = \frac{p_f F_t}{L_t} \quad ,$$

i.e. the number of foreign bonds held decreases in the same proportion as the exchange rate devalues. This result hinges on the fact that a change in the exchange rate causes a capital gain or loss in the domestic currency value of foreign bond holdings. Since the rate of return on financial assets does not change as a result of a devaluation,[2] and the nominal quantity of domestic financial assets is unaltered, the desired domestic currency value of foreign bond holdings also remains constant. Since the domestic currency value of each foreign bond is $e_t p_f$, with an increase in the exchange rate less bonds need be held to achieve this constant domestic currency value of foreign bonds. Put alternatively, if the number of foreign bonds held remained constant, the domestic currency value of these bonds would increase in the same proportion as the devaluation and throw financial wealth portfolios out of equilibrium. There is again a distinction between the change in the capital account measured in terms of the two currencies. In terms of domestic

[1]Equation (6) is derived by using the relation $f_t^* = \left(\frac{p_t}{e_t}\right) f$, where $f = \frac{e_t p_f F_t}{p_t L_t} = \gamma_2 \gamma_3 k$. Differentiating and substituting equations (4) and (5) gives equation (6).

[2]This only holds for changes in e_o not ϕ as is shown subsequently.

currency, the capital account remains unchanged; in terms of foreign currency, it improves or worsens as r_f is less or greater than $n + \overline{\pi}$.

This impact of devaluation on the capital account working through capital gains and losses is ignored in the Mundell analysis discussed in Chapter II. Analysis employing the Keynesian model does imply that devaluation causes a capital inflow, since in this case an exchange rate depreciation raises income by improving the trade balance. With a constant money supply, this leads to an increase in the interest rate causing a capital inflow. Although the result is the same in both cases, the causation is entirely different. It is not surprising that capital gains or losses are ignored in the Mundell model, since stock holdings are never explicitly considered.

The symmetry between changes in exchange rate and government debt levels suggested above can be demonstrated clearly by considering a combined change in both variables. Combining equation (4) of Chapter IV and equations (4) and (5) of this chapter gives

$$(7) \qquad dk = \frac{nq\delta'(1-\delta)\gamma_2\gamma_3 k\psi_2}{(1-\delta)\gamma_2\gamma_3 k\psi_1 - nq\delta'\psi_2\psi_4}\left[\frac{de_o}{e_o} - \frac{d(m_o^* + b_o^*)}{m_o^* + b_o^*}\right].$$

It follows that an increase in the level of government debt accompanied by an equivalent exchange rate depreciation leaves the level of output, and more generally all real variables, unchanged. The price level, however, does not remain constant, for

$$(8) \qquad \frac{dp_o}{p_o} = \frac{de_o}{e_o} - \frac{(1-\delta)\gamma_2\gamma_3 k\psi_1}{(1-\delta)\gamma_2\gamma_3 k\psi_1 - nq\delta'\psi_2\psi_4}\left[\frac{de_o}{e_o} - \frac{d(m_o^* + b_o^*)}{m_o^* + b_o^*}\right].$$

A proportionate increase in the exchange rate and the level of government debt causes a proportionate increase in the price level.

It follows from these results that a matching change in debt level and devaluation leaves the balance of payments expressed in terms of foreign currency unchanged. This can be seen clearly by writing $\dot{R}_t$, measured in foreign currency, as

$$\text{(9)} \qquad \dot{R}_t = L_t \left[-\left(\frac{p_t}{e_t}\right) \delta y + \left(\frac{p_t}{e_t}\right) \; f\,(r_f - n - \bar{\Pi}) \right].$$

If $\dot{R}_t = 0$, this result also holds in terms of domestic currency.[3] We may say, therefore, that under a system of freely fluctuating exchange rates changes in the level of government debt change only the price level. With exchange rates flexible, external balance constraints do not impinge on the ability of domestic authorities to set any desired price level.[4]

[3] If $\dot{R}_t = 0$ the payments balance equation becomes $(p_t/e_t)\ \delta y = (p_t/e_t)\ f(r_f - n - \bar{\Pi})$, and multiplication by e_t does not affect the equality; the domestic currency value of both sides increases equally. This result is still true if autonomous exports are included. The above relation becomes in this case, $(p_t/e_t)\ \delta y = (p_t/e_t)\ f(r_f - n - \bar{\Pi}) + x$, or $p_t \delta y = p_t f(r_f - n - \bar{\Pi}) + e_t x$. Since δ, y, f and x remain constant, equality still holds for proportionate changes in p_t and e_t.

[4] From this symmetry of exchange rate and debt level changes it follows that a decrease in the level of government debt will always improve the balance of payments if the foreign exchange market is stable. Stability in the foreign exchange market requires that $d\dot{R}_t/de_t > 0$. It can be shown that this will be true provided that

$$(r_f - n - \bar{\Pi}) < \frac{(1-\delta)^2 \psi_1^2 \left\{ \delta' q [\delta n \psi_2 \psi_3 - (1-\delta) y \psi_1] - \delta y \psi_1 [\delta' q + (1-\delta)] \right\}}{n q \delta' \psi_2 \psi_4 - (1-\delta) \gamma_2 \gamma_3 k \psi_1}.$$

Since the right-hand expression is positive, questions as to stability only arise if $(r_f - n - \bar{\Pi}) > 0$, i.e. if the foreign exchange impact of a decline in foreign bond holdings is adverse, offsetting the always positive trade balance effects of a devaluation. Assuming exchange market stability obviates the need for the qualifying sufficient condition $(r_f - n - \bar{\Pi}) < 0$ given previously in the analysis of changes in debt level on the balance of payments.

The analogous freeing of the rate of domestic inflation from the world level can be achieved by changes in the rate of exchange depreciation, ϕ. The equilibrium rate of depreciation is determined by the need to keep q, relative world prices, constant, and is

(10) $\phi = \pi - \bar{\pi}$.

From this it follows that in equilibrium

(11) $d\mu = d\pi = d\phi$.

If equations (10) and (11) hold, the domestic inflation rate is freed from the constraint imposed by the need to keep domestic prices competitive in international markets.

A similar result holds for the capital account. From the fact that in equilibrium f, the per capita domestic currency value of real foreign bond holdings, is constant over time, and equation (10), it follows that

(12) $g_f = n + \bar{\pi}$, where $g_f = \frac{\dot{F}_t}{F_t}$.

With increases in domestic monetary expansion being matched by exchange rate depreciation, the rate of growth of foreign bonds held by domestic residents is independent of the rate of domestic inflation. The key to this result is again the capital gains or losses in the domestic currency value of foreign currency denominated assets resulting from a change in the exchange rate. Suppose, for example,

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Footnote 4 continued

It should be pointed out that stability as defined here is long-run i.e. it ignores temporary stock adjustment effects. Assuming that these will swamp interest payments in the short-run, exchange stability in the short-run is assured, since the stock adjustment effects of a devaluation are always positive.

that originally $\mu = n = 5\%$, and $\pi = \bar{\pi} = \phi = 0$. In this case foreign bonds must be purchased at a rate of 5% in order to keep domestic portfolios in balance. If μ increases to 7%, $\pi = \phi = 2\%$. Now the domestic currency value of foreign bond holdings must increase at the rate of 7%. This does not, however, require a 7% rate of purchase. For if <u>no</u> new purchases were made, the domestic currency value of the foreign bond portfolio would increase at a rate of 2% due to appreciation caused by capital gains. The equilibrium rate of foreign bond purchases must, therefore, again be 5%, the additional 2% not involving any transactions through the foreign exchange market. Thus, equation (9) still describes the reserve equation. The difference in this case is in the increased attractiveness of foreign bonds for wealth portfolios, and equilibrium between domestic and foreign bond holdings is now described by[5]

$$(13) \qquad \frac{e_t p_t F_t}{p_b B_t} = \gamma_2 \, (r_f + \phi - r_b) \; , \; \gamma_2' > 0 \; .$$

In the example described above, if $r_f = r_b$, foreign bonds would enjoy a 2% return differential after the increase in monetary expansion. A similar adjustment must also be made in the definition of disposable income, which now becomes

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[5]This premium on foreign assets could reflect either expectations as to future spot exchange rates based on past experience, or a forward discount on domestic currency which would be reflected in the covered interest differential. Equation (13) would hold in either case.

(14) $$Y_t = Q_t + \frac{T_t}{p_t} + \frac{c_b B_t}{p_t} + \frac{e_t c_f F_t}{p_t} - \pi\left(\frac{M_t + p_b B_t + e_t p_f F_t}{p_t}\right) + \phi\,\frac{e_t p_f F_t}{p_t},$$

or

(15) $$Y_t = Q_t + \frac{T_t}{p_t} + \frac{M_t}{p_t}\,(0-\pi) + \frac{p_b B_t}{p_t}\,(r_b-\pi) + \frac{e_t p_f F_t}{p_t}\,(\ r_f + \phi\ -\ \pi)\ .$$

In the second equation the last three terms give the net income from each financial asset written in a uniform manner. Each gives the real value in domestic currency of stock holdings times the nominal domestic currency rate of return minus the domestic currency rate of inflation. Using equations (13) and (14) the basic equilibrium equation is again given by equation (3) of Chapter IV.[6]

It was concluded previously that equal proportionate changes in the level of government debt and the exchange rate did not affect the value of any real variable. The same is not true for matching changes in the rate of change of these variables. The situation is similar to that arising from a change in the domestic bond interest rate and as in the case for a change in debt composition, results will differ depending upon whether the level of government debt (m_o* + b_o*) or the level of prices (p_o) are held constant. Differentiating equation (3) of Chapter IV, using equations (10), (11) and (13) and setting $dp_o = 0$, gives

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[6]For this reason, previously reached conclusions as to the effects of changes in e_o derived on the assumption that $\phi = 0$ also hold in the general case of $\phi \neq 0$.

$$(16) \qquad dk = \frac{(1-\delta-s)\theta_5}{\psi_1} d\phi \Bigg|_{d\phi = d\mu},$$

and

$$(17) \qquad dy = \frac{\theta_5\theta_3}{\psi_1} d\phi \Bigg|_{d\phi = d\mu},$$

where

$$\theta_5 = n \, [\gamma_3\gamma_2' - (1+\gamma_1+\gamma_2)\gamma_3']$$

$$= n \frac{d}{d\phi} \frac{(M_t + p_b B_t + e_t p_f F_t) / p_t}{K_t} \Bigg|_{\substack{d\phi = d\mu \\ i = \text{constant}}}$$

These equations are identical to equations (21) and (22) of Chapter IV giving the corresponding results for price neutral changes in r_b, with the difference that θ_1 is replaced by θ_5, where $\theta_5 = - \theta_1 + \gamma_3\gamma_1'$. This near symmetry results from the fact that increases in μ and decreases in r_b both create a yield differential favoring capital over bonds, and increases in ϕ and decreases in r_b both move the domestic-foreign bond yield differential in favor of foreign assets. The term $\gamma_3\gamma_1'$ is present in θ_1 and absent from θ_5 because a change in r_b affects the desired money-domestic bond ratio, while a change in ϕ and μ leaves this ratio unaffected. As with the case of a change in r_b, if $m_o* + b_o*$ instead of p_o is held constant, the size but not the sign of the output effect given by equation (16) is altered, while the disposable income relation (17) may change direction as well as magnitude. In summation, the balance of trade may worsen or improve as a result of matching increases in the rate of debt creation and exchange rate depreciation.

Although the effect on foreign bond holdings for a price neutral change is ambiguous,[7] the result for a debt level neutral change is not. In the latter case

$$(18) \qquad \frac{df_t^*}{f_t^*} = \frac{\gamma_2'}{\gamma_2}\, d\phi \,\Bigg|_{d\phi = d\mu} .$$

For a $d\mu = d\phi > 0$, the number of foreign bonds held increases. The scale effects of increases in nominal debt and capital gains on foreign bonds exactly cancel out. The substitution effect resulting from an increase in the return on foreign relative to domestic financial assets therefore predominates.

This last conclusion has been used as an argument against the crawling peg exchange rate system. It has been argued that a steady "crawl" of the exchange rate over time will push the covered interest differential against the devaluing country, causing a capital outflow which will partially or completely offset improvements in the current account.[8] The fallacy of this argument in terms of the model presented here is that it ignores the favorable scale effects on the capital account of changes in ϕ. Given the fact that μ has increased, if

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[7]The exact relationship is

$$\frac{df_t^*}{f_t^*} = \frac{1}{f}\left[-\theta_2 + \frac{(1-\delta-s)\psi_4}{(1-\delta)\quad\psi_1}\,\theta_5\right] d\phi \,\Bigg|_{d\phi = d\mu} ,$$

which corresponds to equation (23) of Chapter IV.

[8]For a discussion of the crawling peg and the interest rate "constraint", see Willett [65], Black [4], Meade [36], and Willett, Katz and Branson [67].

the exchange rate did not depreciate, $dg_f = d\mu$, and the rate of foreign bond purchases would increase. A matching increase in ϕ prevents an increased rate of purchase and this scale effect is clearly larger than the substitution effect resulting from increases in ϕ; an upward crawl in the exchange rate prevents rather than causes a capital outflow.

In summation, matching adjustments in the rate of inflation and depreciation have a directionally uncertain non-zero impact on the balance of payments. These effects can, however, easily be neutralized by changes in other policy variables such as the bond interest rate. Such a balancing of changes in the rate of debt creation was impossible in a fixed exchange rate system. The problem posed at the beginning of this section has therefore been satisfactorily resolved. Under a system of flexibility in foreign exchange rates, any desired domestic price structure, both in terms of price level and rates of inflation, can be made compatible with full international payments balance; price inflexibility need not lead to a conflict between long-run internal and external equilibrium.

CHAPTER VI

THE OPTIMAL COMPOSITION OF THE BALANCE OF PAYMENTS

In Chapter IV the combinations of policy variables needed to achieve long-run internal and external balance were derived. The definition of internal balance employed in that analysis differs from that of the assignment problem literature discussed in Chapter II. In the latter case only one income level which achieves full employment is optimal. In the former, neoclassical assumptions assure that all income levels are compatible with full employment. In this sense a unique optimal policy mix for our model has yet to be determined; what is needed is a criterion which can be used to select a point on the BB' line derived in Chapter IV. The problem can be formulated in terms of the choice of the mix between the trade and capital accounts in a zero overall payments balance. This composition problem has received little attention in existing balance-of-payments literature.[1] This is not surprising, since a solution of the composition problem involves a determination of the optimal level of foreign investment,[2] and the short-run Keynesian framework used by

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[1]The lone contribution is that of Miller [37].

[2]This problem is discussed in isolation from financial considerations by Kemp [25] and [27, Chapter 13], Negishi [45], Hamada [17], Bardhan [3], Baldwin [2], and Hanson and Neher [18].

Mundell et al. is ill equipped to deal with capital theoretic considerations involving time as an integral element.[3]

A solution to the composition problem emerges from a consideration of policies which maximize per capita real consumption along the growth path.[4] Real consumption consists of domestic production plus net imports minus real domestic investment. In the notation developed previosly, per capita real consumption (c) is

$$(1) \qquad c = g + \delta y - nk.^{5}$$

Assuming that the balance of payments is in equilibrium, c can be written as, using equation (20) of Chapter IV

$$(2) \qquad c = g - nk + f(r_f - \overline{\pi} - n) = \phi_6(k,f).^{6}$$

Written in this form, the problem becomes the choice of the optimal levels of domestic and foreign real investment. The possible

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[3] Indicative of this is the fact that Johnson, in his otherwise masterly "Towards a General Theory of the Balance of Payments" [21], virtually ignores the capital account, noting that:

> To complete the analysis of 'flow' disequilibria, it would be necessary to ... consider alternative policies for correcting a deficit on current and capital account combined ... This raises ... problems too difficult to pursue here ... When capital account transactions are introduced into the analysis, the choice between policy alternatives requires references to growth considerations not readily susceptible to economic analysis. [21, p. 168.]

[4] This is the "Golden Age" criterion first discussed by Phelps [49].

[5] Per capita real investment is, by definition, $\dot{K}/L = \dot{k} + nk$. Along a steady-state growth path, $\dot{k} = 0$.

[6] Autonomous exports have again been omitted. Their inclusion would change c to $c_1 = c - x$. Since x is exogenously determined, policies which maximize c also maximize c_1.

combinations of these variables can be found by translating the BB' curve derived in Chapter IV into (k,f) space. Mathematically, the internal balance equation $\phi_1(k, r_b, p_o) = 0$, the external balance equation $\phi_2(k, r_b, p_o) = 0$, and the equation of asset equilibrium $\phi_7(k, f, r_b) = 0$,[7] can be combined, eliminating r_b and p_o, to obtain $\phi_8(k, f) = 0$. The optimal balance-of-payments composition can be found by maximizing ϕ_6 with respect to k and f, subject to the constraint $\phi_8 = 0$. Using equations (2) - (5) and (21) - (23) of Chapter IV, it can be shown that the conditions for a maximum are described by

$$(3) \qquad (1 - \delta)(r_f - \overline{\Pi} - n)\,[n\delta\psi_2 - \psi_1\psi_5 + n(1 - \delta)\psi_2\theta_3]\,\theta_2$$

$$= [n\delta k\psi_2 + (1 - \delta - s)\psi_5][\psi_4(r_f - \overline{\Pi} - n) + (1 - \delta)\theta_3]\,\theta_1,$$

where all symbols have been defined previously.

In order to understand the economic meaning of this condition, it is instructive to consider the problem in graphical form. This is done in Figure 6. The II' curves connect points of constant per capita consumption. These isoconsumption curves are derived by plotting $\phi_6(k, f) = c$ for various levels of c. The shape of these curves can be found by taking the differential of equation (2), giving

$$(4) \qquad \frac{df}{dk} = -\frac{(g' - n)}{(r_f - \overline{\Pi} - n)}\ .$$

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[7] ϕ_7 is derived from the equation $f = k\gamma_2\gamma_3$.

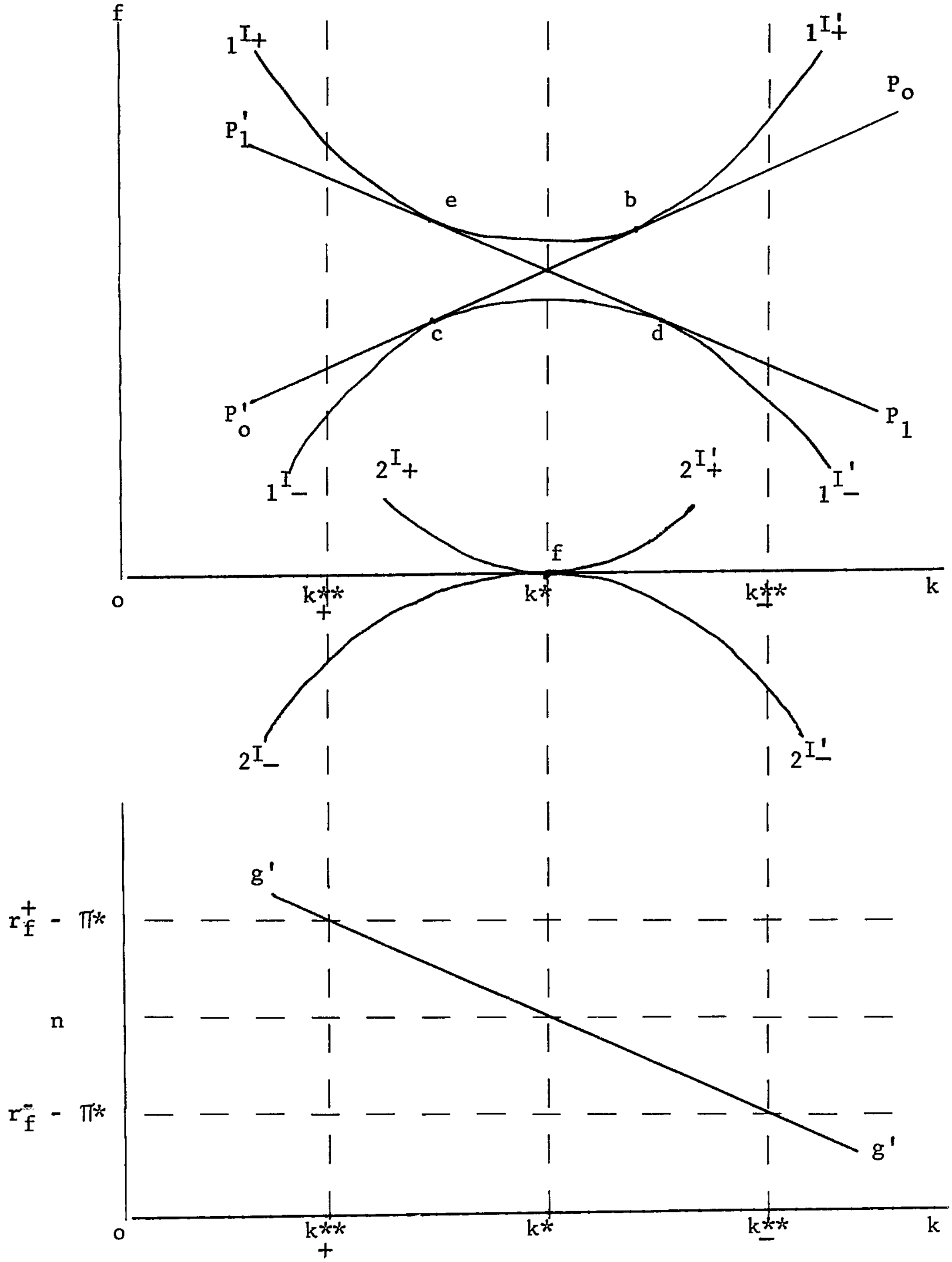

Figure 6.

A Graphical Exposition of the Composition Problem

Both terms in this expression can be interpreted as differences between "marginal revenues" and "marginal costs" of increased investment.[8] In the numerator of (4), g' equals the marginal increase in per capita output resulting from a unit increase in the per capita capital stock. The increase in capital goods investment needed to bring this about in n.[9] If "revenues" exceed "costs", i.e. if $g' > n$ and $(g'-n) = \theta_3 > 0$, domestic goods available for consumption will increase with k.

A similar interpretation can, less obviously, be made for the denominator of (4). The real return on foreign financial investment depends on three factors: (1) the nominal rate of return on foreign currency denominated assets; (2) the rate of exchange between domestic and foreign currency; and (3) the value of foreign and domestic currency in terms of goods. Consider first a situation where domestic and foreign rates of inflation are identical and the exchange rate is constant over time, i.e. where $\Pi = \overline{\Pi}$, $\phi = 0$. The nominal rate of return on foreign investment, in terms of either currency, is r_f. However, since the return on domestic or foreign currency in terms of goods is $-\overline{\Pi}$, the real rate of return is $r_f - \overline{\Pi}$. Put alternatively, if z units of goods were converted into foreign bonds and the interest income used to purchase goods, at home or abroad, the sustainable consumption flow per time period would equal $(r_f - \overline{\Pi})z$. This result

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[8]This interpretation is suggested by Hanson and Neher [18] and Solow [57]. Similar reasoning was used in the discussion of changes in disposable income in Chapter IV.

[9]In footnote (4) it was shown that along the steady-state growth path, $\dot{K}/L = nk$. It follows directly that $d(\dot{K}/L)/dk = n$.

also holds if foreign and domestic inflation rates differ, provided that exchange rates are flexible. In this case the rate of return resulting from using interest income to purchase goods abroad (increase imports) is still $r_f - \bar{\pi}$. If earnings are repatriated and goods purchased domestically (exports reduced), the real rate of return to foreign investment is $r_f + \phi - \pi$. The two become identical, however, as long as $\phi = \pi - \bar{\pi}$. The marginal cost term for foreign investment is n, as it is for domestic investment. In this case, n represents an opportunity cost -- the foreign exchange used (and therefore the imports foregone) in order to raise real foreign bond holdings by df. If marginal revenues exceed marginal costs, i.e. if $(r_f - \bar{\pi} - n) > 0$, an increase in the level of foreign investment increases per capita consumption. This follows directly from the fact that net imports are increased in this case, as a comparison of equations (1) and (2) indicates.[10]

Since we have assumed an infinitely elastic supply of foreign bonds, the marginal return on foreign investment is independent of f. The return on domestic investment is a declining function of k ($g'' < 0$). Domestic consumption will increase (decrease) as k increases, provided that $k < (>) k^*$, where $g'(k^*) = n$.

These considerations explain the shape of the isoconsumption curves of Figure 6. $(I_+I_+')_1$, for example, is drawn on the assumption

[10]Foreign borrowing may be treated in an entirely symmetrical manner by allowing f to be negative. Here marginal "return" equals new borrowing and the marginal cost term equals interest payments.

that $(r_f - \bar{\pi} - n) > 0$. Under these conditions the curve is downward sloping in the range where $k < k^*$, as an increase in k must be accompanied by a compensating decrease in f in order to keep consumption constant. The slope of this curve is less than -1 for $k < k^{**}_{+}$ where $g'(k^{**}_{+}) = r_f^{+} - \bar{\pi}$, and greater than -1 for $k > k^{**}_{+}$. Similarly, $(I_+I_+')_1$ slopes up for $k > k^*$, becoming horizontal at $k = k^*$.[11] Changes in the isoconsumption level cause vertical displacements of the II' curves, lower curves (e.g. $(I_+I_+)_2$) corresponding to lower consumption levels. Similarly, if $(r_f - \bar{\pi} - n) < 0$, isoconsumption curves such as $(I_-I_-')_1$ will be concave to the k axis, and lower curves, such as $(I_-I_-)_2$ will correspond to <u>higher</u> consumption levels. In this case, the slopes of the curves will be positive for $k < k^*$, negative for $k > k^*$, and less than -1 for $k > k^{**}_{-}$, where $g'(k^{**}_{-}) = r_f^{-} - \bar{\pi}$.[12]

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[11]The upward concavity of the curve follows from the fact that $d^2k/df^2 = - g''/(r_f - \bar{\pi} - n) > 0$.

[12]If foreign investment is impossible, admissable points lie along the k axis. Examination of Figure 6 indicates that $k = k^*$ maximizes consumption, i.e. reaches the "highest" II' curve, regardless of the sign of $(r_f - \bar{\pi} - n)$. This solution of $g' = n$ is Phelps' original Golden Rule of Accumulation [49].

If consumers have some positive rate of time preference, λ, optimal consumption is found by maximizing

$$\int_0^\infty c(t)e^{-\lambda t}dt.$$

The solution in this case is of the form $g' = n + \lambda$. (See, for example, Samuelson [54, p. 271-2.]) A positive preference for present consumption raises the marginal "cost" of investment and the optimal investment level is lower. The results that follow may be altered to take this factor into account by replacing n by $n_1 = n + \lambda$. Notice that in this case foreign investment might be undesirable even if it yielded a net import surplus.

The PP' curve gives the locus of consumption possibility points, and its shape is determined by the ϕ_8 function discussed previously. Its slope is

$$(5) \qquad \frac{df}{dk} = \frac{(1-\delta)(n\delta\psi_2 - \psi_1\psi_5)\theta_2 - \psi_4[n\delta k\psi_2 + (1-\delta-s)\psi_5]\theta_1}{n(1-\delta)^2(r_f-\overline{\pi}-n)\ \psi_2\theta_2-(1-\delta)[n\delta k\psi_2 +(1-\delta-s)\psi_5]\theta_1}.$$

This result may be interpreted by reference to the analysis of Chapters III and IV. The equilibrium portfolio holdings of f and k will depend on r_b, the bond interest rate, and i, the return on capital. If k is increased holding r_b constant, i declines, increasing desired f holdings. A change in r_b can either raise or lower the desired f/k ratio, the result depending on the relative substitutability of domestic and foreign bonds vs. domestic bonds and capital. The slope of PP' will depend on the answer to the question: as k increases, what change in r_b is needed to keep the balance of payments in equilibrium? A consideration of several polar cases is instructive. If $\theta_1 = 0$, the slope of PP' reduces to

$$(6) \qquad \frac{df}{dk} = \frac{n\delta\psi_2 - \psi_1\psi_5}{n(1-\delta)(r_f-\overline{\pi}-n)\psi_2} \gtrless 0 \text{ as } (r_f-\overline{\pi}-n) \gtrless 0.$$

Suppose that under these conditions the interest rate was changed in such a manner that f increased.[13] With $\theta_1 = 0$, k, and therefore the balance of trade, will remain unchanged. If $(r_f-\overline{\pi}-n) > 0$, the capital

[13]Whether an increase or decrease in r_b is needed to accomplish this depends on the sign of θ_2. (See equation (23), Chapter IV.)

account will improve, necessitating an offsetting increase in the price level, which raises imports and worsens the current account. This price increase raises k, making df/dk > 0 in this case. If $(r_f - \overline{\pi} - n) < 0$, an offsetting improvement in the trade account is needed, and k falls as p is lowered, making df/dk < 0 as indicated in (6). If $\theta_2 = 0$, (5) becomes,

$$(7) \qquad \frac{df}{dk} = \frac{\psi_4}{(1-\delta)} > 0.$$

With $\theta_2 = 0$, changes in the bond interest rate have no effect on the desired f/k ratio, and changes in both policy variables work only through changes in i. As was shown above, f and k move together in this case.

If θ_1 and θ_2 are both non-zero, (5) will be positive and the PP' curve will be upward sloping if θ_1 and θ_2 have opposite signs and $(r_f-\overline{\pi}-n) > 0$. Under these conditions changes in both p_o and r_b cause f and k to move in the same direction, and, with $(r_f-\overline{\pi}-n) > 0$, cause changes in the trade and capital accounts with offsetting balance-of-payments effects. If $(r_f-\overline{\pi}-n) < 0$ or if θ_1 and θ_2 have the same sign, the result is uncertain and PP' may be downward sloping.[14]

Whatever the exact shape of the curves, points of maximum consumption occur at positions of tangency between the consumption

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[14] In particular, under conditions A and B as described in Chapter IV, the sign of df/dk is indeterminante, although the slope of the BB' curve is unambiguous.

possibility[15] and isoconsumption curves.[16] The several possibilities, indicated by points b-e in Figure 6, are given in Table 3. For example,

Table 3

Conditions for Optimal Foreign Investment

		$r_f - \overline{\pi} > n$	$r_f - \overline{\pi} < n$
Slope of PP' > 0		(point b) $g' < n < r_f - \overline{\pi}$	(point c) $r_f - \overline{\pi} < n < g'$
Slope of PP' < 0	< - 1	$n < r_f - \overline{\pi} < g'$	(point d) $g' < r_f - \overline{\pi} < n$
	> - 1	(point e) $n < g' < r_f - \overline{\pi}$	$r_f - \overline{\pi} < g' < n$

if foreign investment is profitable and domestic and foreign investment must be raised or lowered jointly, a point such as b results. In this case at points such as c where domestic investment is marginally profitable, consumption is raised by moving up the P_oP_o' curve

[15] Introduction of an additional policy tool, such as a tax or subsidy on foreign investment, will widen the range of admissable (k,f) points and make possible a higher level of consumption.

[16] At points of tangency, the slopes of each line as defined by (4) and (5) will be equal. This mathematical equality leads to condition (3). For example, if $(r_f - \overline{\pi} - n) > 0$, $\theta_1 > 0$ and $\theta_2 < 0$, an optimum point such as b in Figure 6 will result, where $k > k^*$ and $\theta_3 < 0$. That a point where $k < k^*$ cannot be optimal in this case, is indicated by the fact that under the above conditions with $\theta_3 > 0$, the left-hand side of equation (3) is negative and the right-hand side positive. Other conditions can be checked similarly by reference to equation (3).

and increasing both forms of investment. Domestic investment will be carried past the point where it is marginally unprofitable (to the right of k*) in this case, the excess capital goods investment being the cost of acquiring additional foreign assets.[17]

With foreign investment unprofitable ($r_f - \overline{\pi} < n$) and domestic and foreign investment substitutes, a point such as d will be optimal. At point e where domestic investment is marginally profitable, consumption can be raised by substituting it for foreign investment and moving down P_1P_1'. Capital goods deepening is carried beyond the point of optimality (k*) in order to reduce the holding of unprofitable foreign assets. The marginal return to domestic investment may remain above or be driven below that of foreign investment depending on substitution possibilities. If increases in domestic investment increase (decrease) total real wealth holdings (i.e., if the slope of $P_1P_1' > (<) -1$), foreign investment will have a marginally lower (higher) return at the optimum point (i.e. $k < (>) \underline{k}^{**}$).[18] An

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[17]Point b corresponds to the optimal foreign investment solution reached by Hamada [17]. Since with $r_f - \overline{\pi} > n$ an unconstrained maximum involves $f \to \infty$, a constraint of the form $f \leq \beta k$, $\beta > 0$ is introduced. Feasible solutions in this case are, in terms of Figure 6, on or below a line such as $\underline{P}_0P_0'$ with slope β, and conditions for optimality are $f = \beta k$, $g' < n < r_f - \overline{\pi}$. [17, p. 61].

[18]In the model used by Negishi [45] marginal returns to both forms of investment are equal at the optimum point, i.e., points e and d lie directly above k_f^{**} and $\underline{k}^{**}$ respectively. Negishi assumes a consumption possibilities function of the form $s[g + (r_f - \overline{\pi})f] = n(k+f)$. This condition assures that savings equal the total investment required for steady-state growth, where investment can only take the form of f or k. Differentiating this function gives the slope of the PP' curve as

$$(5') \qquad \frac{df}{dk} = - \frac{sg' - n}{s(r_f - \overline{\pi}) - n} .$$

Equating (4) and (5') gives, (3') $g' = r_f - \overline{\pi}$. [45, p. 631].

explanation of the optimality of points c and e under the appropriate conditions can be developed in a similar manner.[19]

These results depend on the assumption that foreign assets are available in unlimited quantity at a fixed price, i.e., that the average and marginal returns to foreign investment are equal. In the more general case, the return on foreign assets is a declining function of foreign investment and

(8) $r_f = r_f(f),\ r_f' < 0.$

This case is shown in Figure 7.

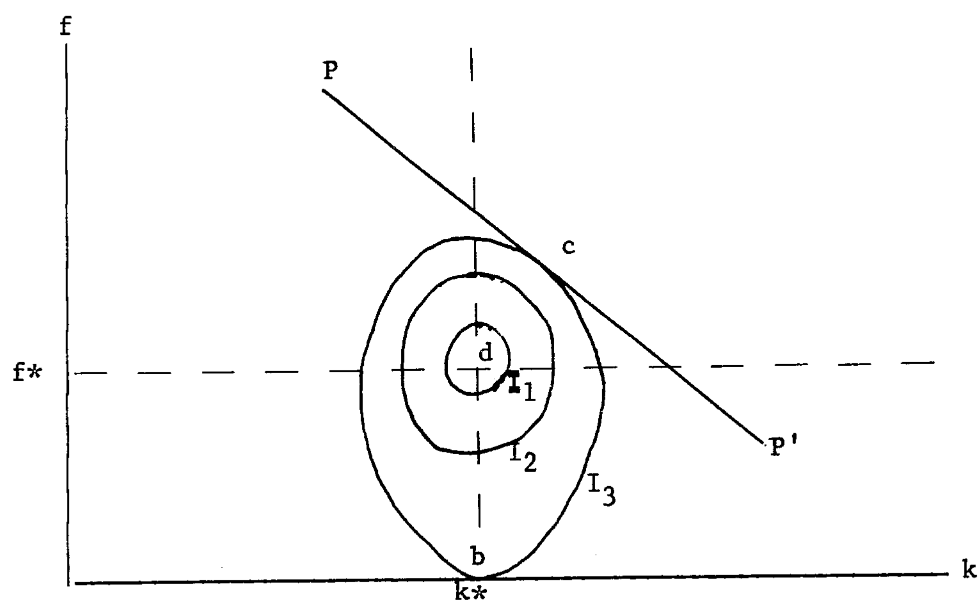

Figure 7.

Isoconsumption Lines With a Decreasing Return on Foreign Investment

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[19] An interesting special case for point e is the optimal allocation of a given wealth portfolio in a static economy. Here $n = \overline{\pi} = 0$ and $(k + f)$ = a constant. With $n = 0$, $k^* \rightarrow \infty$ and the II' curves are downward sloping throughout as long as $r_f > 0$. The PP' curve slopes downward at a 45° angle to the f axis. Tangency occurs where the slope of the II' curve is -1, i.e., where $g' = r_f$. This solution is reached in a different manner by Kemp. [27, Ch. 13.]

At $f^* = [r_f(f^*) - \overline{\pi} - n] / r_f^{\prime}(f^*)$ the marginal return on increased foreign investment just covers marginal costs, and increasing f beyond this point lowers consumption. The maximum consumption point is at d or (k^*, f^*)[20] and isoconsumption curves lying further from d denote lower levels of consumption.[21] Optimal solutions will still be at points of tangency such as c, but the relationships indicated in Table 3 no longer hold.[22] In this case the possibility of foreign investment raises the level of consumption only if the equilibrium point lies within I_3, where c_3 at point b is the maximum consumption level under conditions of autarky. Under the conditions of Figure 6, the opportunity to invest abroad will lead to higher or lower consumption levels as $r_f - \overline{\pi}$ exceeds or falls short of n, as a comparison of points b-e and f indicates.[23]

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[20]At point d, $g' = n = r_f - \overline{\pi} + r_f^{\prime}f$. This is the optimal solution reached by Bardhan [3]. In the static case where $n = \pi^* = 0$ and PP' is a downward sloping 45° line, tangency would occur where $(r_f - g')/g' = r_f^{\prime}f/r_f = \epsilon_{r_f}\, f$. If domestic and foreign assets were perfect substitutes, optimality would, therefore, require a tax on foreign investment equal to the elasticity of the return on foreign assets with respect to additional investment. In the special case where $\epsilon = 0$, foreign investment should be untaxed, making $g' = r_f$. A similar conclusion is reached by Kemp [27, p. 197.].

[21]The slopes of the I curves are given by $\frac{df}{dk} = - \frac{g' - n}{r_f - \overline{\pi} - n + r_f^{\prime}f}$.

[22]The PP' curves of Figure 6, defined by equation (5), will, of course, be altered by assuming that $r_f^{\prime} \neq 0$.

[23]A possible exception is the case where $r_f - \overline{\pi} > n$ and the PP' line lies below the $(I_+ I_+^{\prime})_2$ curve. Here if, e.g. PP' was upward sloping, excessive levels of domestic investment would need to accompany foreign investment.

The above analysis points out the considerations which determine the optimum composition of the equilibrium balance of payments. These same considerations are relevant to the problem of whether to adjust payments imbalances through the current or capital accounts. Suppose, for example, that from an original equilibrium point such as d in Figure 6, exogenous factors caused a shift to a higher desired level of foreign asset holdings. With $r_f - \bar{\pi} < n$, this would cause a deficit in the balance of payments. If the trade account was adjusted to offset this capital account imbalance by lowering k and improving the trade balance, the new equilibrium at a point such as e would be inferior, involving a lower level of consumption. In this case the optimum policy would be to adjust the capital account imbalance and move the equilibrium point back to d. Conversely, if the original equilibrium was at point e, an exogenous drop in desired foreign asset holdings which caused a balance-of-payments surplus should be accomodated by an increase in k and imports in order to move to point d. Similar reasoning holds if the disturbance originates in the trade account. There is, in short, no reason to expect that adjustment should be confined to the trade or capital accounts exclusively in all cases. In particular, a policy which attempted to correct all imbalances through an interest rate manipulation of the capital account would be sub-optimal.[24] Adjustment policies should be geared to the goal of movement toward the position of optimum equilibrium balance-of-payments composition.

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[24]For temporary disturbances, capital account adjustments which minimize adjustments in the real sector may be preferable.

This analysis of the composition problem points out the differential welfare effects of the existence of domestic and foreign debt.[25] The existence of interest bearing financial liabilities of the domestic government affects the real sector only indirectly by altering the incentive to hold real capital assets. The resulting government deficit or surplus, equal to $b(r_b - \pi - n)$, can be financed by issuance of non-interest bearing debt, or money. The deficit or surplus resulting from the holding of foreign bonds, $f(r_f - \bar{\pi} - n)$, cannot be financed in this manner. In a situation of long-run equilibrium where foreign exchange reserves are used only to finance temporary external payments imbalances, the deficit must be financed by a movement of goods through the trade account.[26] Although from the point of view of the individual, purchases of domestic or foreign financial debt are similar transactions, from the perspective of the whole country, they differ significantly. In the case of foreign assets, a purely financial transaction is translated, through the requirements of balance-of-payments equilibrium, into real investment, with the return on this investment depending on nominal financial rates of return, rates of monetary inflation, and exchange rates, in the manner discussed previously.

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[25] This problem is discussed within a slightly different framework by Diamond [6].

[26] For a reserve currency country this distinction is blurred, since non-interest bearing government debt can be used to finance external deficits. Even in this case, however, potential claims over domestic goods are created by balance-of-payments deficits, since holdings of domestic currency can be used by foreigners to finance goods imports. In the case of domestic government deficits this cannot occur; money issued to domestic residents cannot be used to purchase goods from the issuing government, only from other domestic residents.

CHAPTER VII

CONCLUSIONS, QUALIFICATIONS AND COMMENTS

This completes our analysis of long-run balance-of-payments adjustment. We asserted originally that the current Keynesian analysis of macroeconomic policy and external adjustment was applicable only in the short-run, and that this qualification was likely to be especially important for the analysis of international capital movements. In order to examine the long-run implications of adjustment policies it was necessary to construct a model incorporating explicitly both financial growth and an asset sector consistent with the theory of portfolio equilibrium. In this framework it was found to be useful to classify government policy decisions into actions which affected the overall size and composition of government debt, altering the usual Keynesian distinction between monetary and fiscal policy.

Several basic conclusions emerge from this analysis. First, a distinction between real and nominal financial magnitudes must be clearly drawn, and the balance-of-payments effect of international financial capital movements should be measured in nominal terms. Second, the effect of international capital movements on foreign exchange reserves depends importantly on the return flow of interest payments. Third, the long-run effect of changes in interest rates on international capital and trade flows is significant, but directionally uncertain, the results depending on savings behavior, relative asset preferences and substitution possibilities. Fourth, without altering return variables, the government can, by changing the overall level of its financial liabilities, alter international asset holdings and the balance of trade in a predictable manner. Fifth, exchange rate variations have important direct effects on the capital account. Finally, policy should be directed at the composition as well as the overall balance of international payments, and purely financial capital flows have important impacts on a country's real investment position.

The model used to arrive at these conclusions employed a number of restrictive assumptions. Many of these have been mentioned previously. In the domestic financial sector, banks and financial intermediaries were assumed not to exist. Because of this the role of inside money was not considered. The specification of the rest-of-world sector was also incomplete. The foreign and domestic real growth rates were constrained to be equal, and domestic assets were excluded from foreign wealth portfolios. All economic conditions in the foreign country were assumed to be unaffected by actions in the domestic economy. In particular, the prices of foreign goods and bonds were treated exogenously. Relaxation of these assumptions, while complicating the model considerably, should not affect qualitatively the conclusions stated above.

A more serious stricture is placed on the applicability of our basic conclusions by the use of what might be called long-run steady-state comparative statics. Throughout, the analysis has focused on the alternative growth paths resulting from variations in government policies. The disequilibrium adjustment from one path to another has been ignored. Since conditions along this adjustment path may vary considerably from those suggested by the ultimate equilibrium target, and since the period of adjustment may be comparatively long, an important class of problems relating to intermediate adjustment is excluded. This problem was encountered in the discussion of stock and flow capital account adjustments in Section 3.1 of Chapter III. There it was argued that while the long-run effects of an increase in domestic interest rates are uncertain, the short-run stock adjustment effects are always positive and large in magnitude. Conversely a change in what has been called the level of government debt has no comparable differential effect.

A related problem arises from the fact that in disequilibrium speeds of adjustment of variables may differ. Suppose, for example, that an increase in the domestic bond interest rate causes a *ceteris paribus* increased demand for foreign bonds and real capital goods. A stock adjustment of financial wealth portfolios can be made to acquire the former. The latter, however, can only be increased by a temporary rise in the real investment rate. In this case financial markets can adjust more rapidly than goods markets. Similarly, expectations will be in a state of flux during periods of adjustment. Thus an increase in the rate of monetary expansion will have a larger wealth effect in the short-run before inflationary expectations have been raised.[1] Because of these factors, separate policies must be

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[1]These considerations are emphasized by Friedman [13] in distinguishing between the short and long-run effects of monetary policy.

designed for periods of adjustment from one long-run equilibrium path to another, and methods devised to distinguish such adjustments from temporary and reversible balance-of-payments disequilibria.[2]

The importance of institutional arrangements and their development over time should also be emphasized at this point. For example, the model developed here would suggest that the countries inflating most rapidly should experience large and persistent capital outflows. No such pattern is clearly observable in the history of capital movements in the Atlantic Community in the postwar period. It can be argued that the above tendency, while operative, was modified or reversed in specific cases by some or all of the following factors: (1) the development of the dollar as a reserve currency; (2) the freeing of exchange controls in Europe, the integration of European capital markets and the development of the Common Market; (3) the secular rise in United States interest rates; and (4) the development and growth of the Euro-dollar market. In short, the world economy is never actually at a position of long-run steady-state equilibrium, but rather in a continuous process of adjustment toward such a position. In order to take account of these factors, the model developed here could be modified by specifying explicitly the dynamics of disequilibrium behavior. The use of simulation techniques might then be employed to analyze the effects of government policies in the important intermediate-run.

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[2]This distinction between temporary and permanent payments imbalances is implicit in Johnson's [21] analysis of stock and flow adjustment policies.

This study has focused on one aspect of the balance-of-payments adjustment problem -- long-run adjustment. The methodology of economic theory often involves the examination of polar cases. Consideration of such cases allows simplifying assumptions which, though abstracting in significant respects from reality, point to variables and produce theorems that illuminate the complex intermediate cases of the real world.[3] While the polar case of short-run balance-of-payments adjustment has been analyzed extensively, the long-run adjustment problem has been virtually ignored. There has been, as a result, a tendency to apply the conclusions of the short-run analysis to all situations in the absence of alternative hypotheses. In this regard the discussion of capital movements under the crawling peg exchange rate system mentioned in Chapter V could be cited. Although the proposed role of the crawling peg as a method of harmonizing differing inflationary trends makes clear that a considerable time horizon should be considered, arguments have focused almost exclusively on the variable which determines capital flows in the short-run Keynesian analysis--interest rates. The long-run model developed here has been used to suggest alternative and complementary approaches to balance-of-payments adjustment policy.

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[3] The assumptions of perfectly mobile and immobile international capital have been justified on these grounds by Mundell [40].

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