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Transportation and the Urban Economy

by

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ABSTRACT

In the dissertation, the relationship between transportation cost and various economic activities of cities is studied. The first activity studied is governmental services such as schools and libraries. Three models are developed to determine the impact of changes in transport cost, demand, and production cost on the size and spacing of services. In an illustrative calculation surprisingly high imputed transportation cost for children traveling to school in Boston is uncovered.

The second activity considered is retailing services. A model developed earlier by Mills and Lav is used to study the effect of transport cost, demand, production cost and population density on retail prices within a city. In the context of this model high population densities are found to be a source of high prices.

The third activity considered is the location of residents around a central business district. The relationship between the type of transportation system and the locational pattern is studied in the context of a general equilibrium model. In particular, the impact of a subway system on a city and the benefits from such an improvement in the transportation system are considered. Finally, a numerical analysis is performed on the model to compare the benefits of a subway improvement under a variety of assumptions.

PREFACE

This dissertation is the result of research conducted between 1970 and 1972. Within this period, I benefited from the fellowship support of Resources for the Future during the 1970-71 academic year.

I am indebted to many people for assistance in a variety of ways. My greatest debt is to Professor Edwin S. Mills who directed the research. Much of the research here is based upon his own pioneering works in the area. In addition, earlier, his encouragement and later, his comments and suggestions have been invaluable to the final product.

Valuable assistance was given by Professors William Oakland and Peter Newman and my fellow graduate students in the dissertation seminar at Johns Hopkins University during 1970-71. Particularly helpful were numerous conversations with Eli Burokov who was interested in similar problems.

Special appreciation goes to Kathleen Sayers for providing the stability necessary for the completion of a work of this scale.

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CHAPTER 1

THE CITY IN THE UNITED STATES

Cities in the United States have a long tradition of being held in low esteem. Numerous prominent Americans have waxed acerbic when queried about urban life, and while Americans have never been unanimously hostile to the city, there has been a virulent undercurrent of anti-urbanism.

Thomas Jefferson's agrarianism, for example, was tinted with anti-urbanism, as he indicated in a comment to Benjamin Rush in 1800 concerning the yellow fever epidemics in Philadelphia.

> When great evils happen, I am in the habit of looking for what good may arise from them as consolations to us, and Providence has in fact so established the order of things, most evils are the means of producing some good. The yellow fever will discourage the growth of great cities, and I view great cities as pestential to the morals, the health, and the liberty of man. True, they nourish some of the elegant arts, but the useful ones can thrive elsewhere, and less perfection in the others, with more health, virtue, and freedom, would be my choice. [10]

This kind of anti-urbanism has found its most vocal spokesmen among agrarian movements. In 1850, the editor of the <u>Prairie Farmer</u> said, "City life crushes, enslaves, and ruins so many thousands of our young men, who are insensibly made the victims of dissipation, of reckless speculation, and of ultimate crime." [10]

Agricultural interests have not been the only source of anti-urbanism. In philosophy the Transendentalists Emerson and Thoreau both preached the need for man to return to the woods. In recent years a strong "back to nature" movement has existed, fed by both youthful romanticism and environmentalism. In literature, Melville, Hawthorne, and Poe all had "bad dreams of the city." [10]

While moralistic diatribes are easily uncovered in the history of the city in American thought, they certainly are not the only attitude represented. The amoral position is also common and of long standing. For example, the Jacksonian leader, Theodore Sedgwick II, argued that the reputation of cities did not involve a moral question at all but simply an economic one. If polled, most economists (this author among them) would probably agree with this position.

The modern American city has become an enormously complicated institution. However, the historical growth pattern has led many writers to some reasonable simplifications.

The growth of a city in United States history has depended on its accessibility to cheap forms of transportation. In colonial times, cheap transportation was synonymous with water transportation. The important cities of that time - New York, Philadelphia, Baltimore, and Boston - all were endowed with good harbor facilities.

The midwestern cities grew rapidly in the second half of the 19th century. Chicago, St. Louis, Cleveland, and Detroit all were located on the great lakes or the Mississippi River.

During the 19th century, rail transportation became increasingly important. Many cities owed either their existence or their rapid growth to the location of a rail terminal in the city. Chicago outpaced St. Louis because of its excellent rail links. Los Angeles boomed in the 1880's as a result of a rate war between the Southern Pacific and the Santa Fe railroads. Some cities like Cheyenne and Duluth did not even exist before a rail terminal was built.

Therefore, it can be seen that rail terminals and port facilities have been extremely important to the growth of American cities. Due to the importance of railroads and ports, the land around the terminals became valuable for business uses, so that central business districts generally grew up around them.

Economists have long been aware of the importance of centrality and have based much locational theory upon it. The Von Thünen rent model is an early example.

In the 20th century, the rapid growth of cities has continued as has a concommitant growth in their complexity. Two factors can be singled out here. First, the advent of modern modes of transportation, the automobile and, to a lesser extent, the airplane, has reduced the importance of locations close to the old rail and harbor terminals.

Secondly, as cities grow, an increasing number of activities are able to exist at more than one location. An example has been the location of large department stores outside the central business district (CBD) in the suburbs. As a result, the location of activities outside the CBD can not be disregarded.

In the chapters that follow the importance of transportation cost within the city is studied. Its effect on the location of both governmental and business activities is analyzed as well as its effect on the location of residences.

A simplified conception of the city underlies the entire work. In this conception, there is a central business district around which the employees locate their residences. The desire to be close to work so that transportation costs are minimized causes employees to bid up the price of land near the CBD in the equilibrium configuration. The result is a rent-distance function as in Figure 1-1.

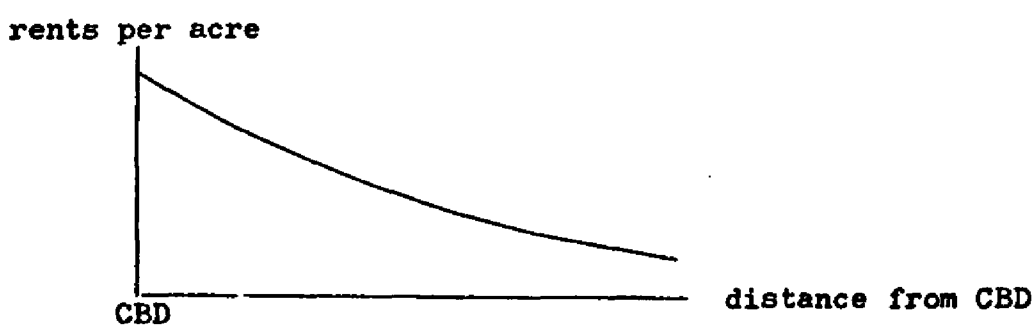

Figure 1-1. A rent-distance function

In addition, governmental and retail services must be supplied to the CBD employees and their families. These activities, then, disperse themselves throughout the residential area locating in the center of hexagonal market areas as in Figure 1-2.

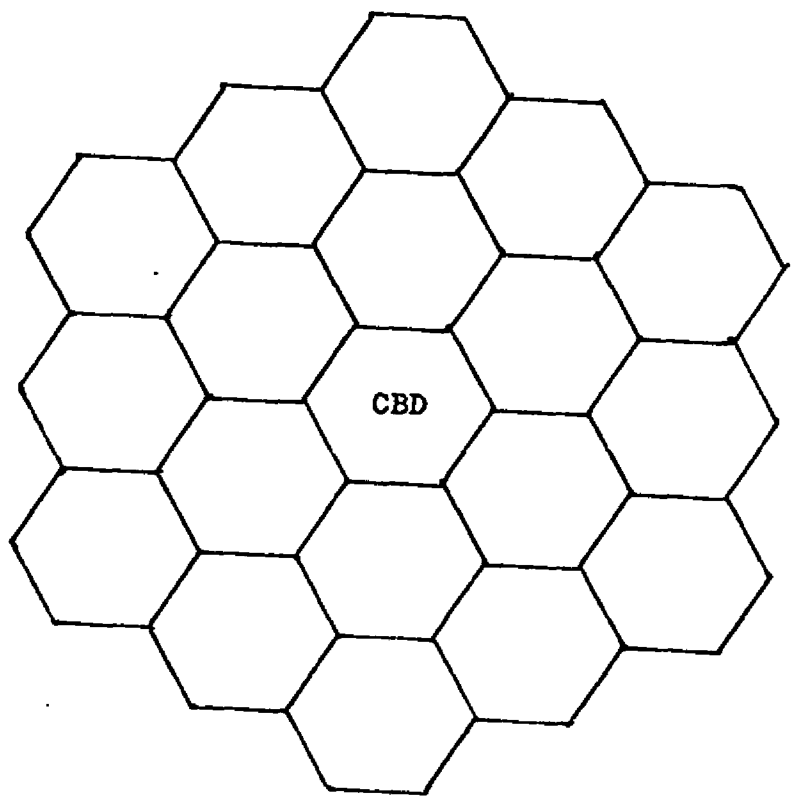

Figure 1-2. Hexagonal market areas

The governmental services locate (or should locate) in this manner to minimize costs while the retail services locate to maximize profits.

The abstraction, then, is one where there is an exogenously determined CBD. The desire to minimize commuting costs results in land rents near the CBD being bid up by the CBD workers. Governmental and retail services disperse throughout the residential area with hexagonal market areas. The land rents determine population densities, and in turn, the population density at any given location (among other things) determines the equilibrium size market area. Higher densities imply smaller market areas as in Figure 1-3.

Chapter 2 entitled "Transportation Cost and the Provision of Urban Governmental Services" analyzes the optimal spacing of governmental services such as schools, libraries, fire protection, etc. Several assumptions about the nature of demand are compared. The important variables which determine the optimal size in every case are the density of population, transportation cost, and the extent of scale economics. In the last section of the chapter the actual spacing of schools in Boston is compared with the theoretically optimal one. The results indicate a great over-investment in school construction in Boston.

Chapter 3 entitled "Transportation Cost and Urban Retail Trade" analyzes the equilibrium spacing of firms in

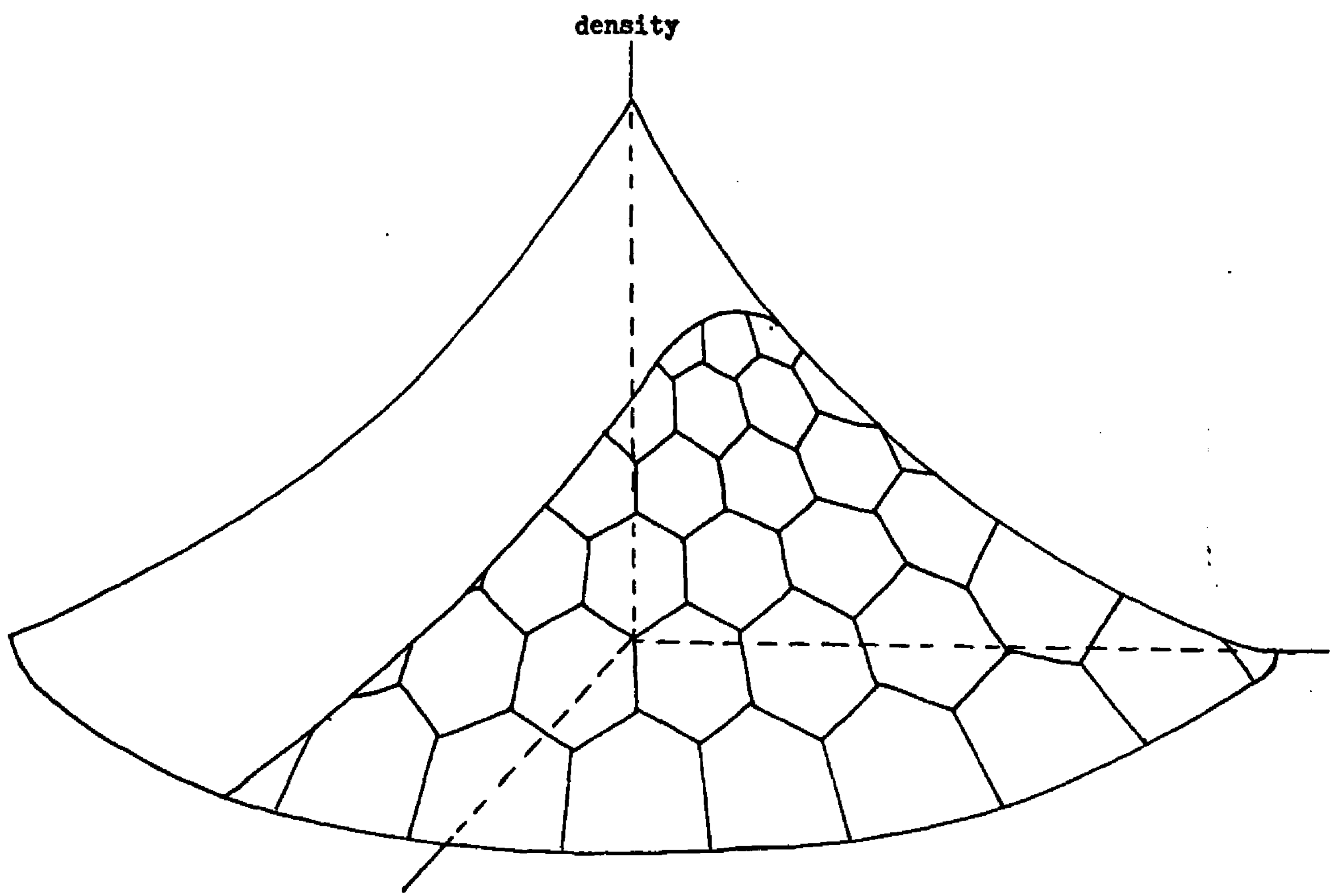

Figure 1-3. Market areas and population density

the retail trade industry. The equilibrium spacing of firms is found to depend on the demand and cost parameters, transportation cost, and density of population. The effect of changes in these parameters on prices is studied and a comparison between high and low income areas is made. In the comparison it is found that high population densities in low income areas are the principle cause of higher mill prices (prices at the store) in low income areas. The other parameters have a negative or ambiguous effect on mill prices. However, if delivered prices are studied it is uncertain that prices are higher in low income areas. Relative prices, then, depend on the nature of the good in question.

Chapter 4 entitled "The Benefits of An Urban Transportation System Improvement" analyzes the potential benefits of a fall in transportation cost both to users and non-users. In particular, the reorganization benefits are studied for governmental services, retail trade, and residential locations. A new method of calculating the benefits of a subway improvement is proposed; and the long run and short run benefits are discussed.

In Chapter 5 entitled "An Aggregative Urban Commuter Model" an urban model with roads and subways is developed. The structure of a road-based city is compared to that of a subway-based city. In Appendix A the assumption of an exogenously determined central business district is discussed.

In this appendix it is demonstrated that central locations are important even if a firm does not ship output through a rail terminal or port.

In the final chapter, numerical analysis is applied to the model of Chapter 5 to determine the benefits of building a subway in a city of three million people. The policy implications are discussed.

CHAPTER 2

TRANSPORTATION COST AND THE PROVISION OF URBAN GOVERNMENTAL SERVICES

Market area models have become an important tool in location theory both in the study of the location of firms within an industry and in the study of the spatial patterns of cities and their functional heirarchy. Underlying all these models are the assumptions that (1) agglomeration economies or economies of scale encourage larger size plants or cities while (2) the diseconomies of transporting inputs and outputs larger distances encourage smaller sizes.

Although these market area models have been widely applied to the theoretical and empirical problems of the free market economy, a comparable theory for decisions in the public sector is not well developed. A notable exception is an article by Mycielski and Trzeciakowski [28] which analyzes the problem of the location and size of service stations in a centrally planned economy, a situation similar to that found in the public sector of a decentralized economy.

The purpose here is to look at the provision of governmental services and provide models that will indicate what the optimal size and spacing of facilities are under a variety of assumptions. For example, how many schools, parks, libraries, fire stations, etc. should a city have, and how large should each facility be in the presence of economies of scale and diseconomies in transportation? Various conditions of demand are considered. The effects of changes in population densities, transport costs, scale parameters, and demand elasticities on the optimal size and spacing are investigated.

Optimization Criteria

First, some discussion of the possible optimization criteria is necessary. The issue can be looked at in the framework of separate "allocation" and "distribution" functions of government, as Musgrave [26] suggests.

On the allocation side, the local government should provide a facility whose size minimizes the sum of production cost and transportation cost. Where the population is insufficient or barely adequate to demand this cost minimizing output, only one facility would be provided. Where demand is more than adequate, multiple facilities would be provided and located in such a way that transport costs are minimized. (Indivisibility of plants implies that a second facility will not be built until the first is beyond the minimum cost size).

There are three regular polygons that will cover a plane without leaving gaps. These are the triangle, square, and hexagon. Of these, the hexagon can serve a given population with the lowest total transport cost. A circular area has lower transport cost than an hexagonal one but has the disadvantage of leaving some population unserved in the interstices.[1] In what follows, service areas are assumed to be either circular or hexagonal.

The distributional criteria seem to be unsettled. In facilities where benefits fall off with distance, a taxation scheme where tax rates fall with distance can be optimal under certain restrictive assumptions.[2] However, this problem is neglected. The allocation aspect which involves minimizing costs is of primary interest here.

The Provision of Services

In this section we develop some simple models to study the trade-off between facility size and transportation costs. In the cases studied, production of the services is assumed to exhibit falling average cost over the relevant range of values and constant marginal cost. Three types of demand equations are considered. First is the perfectly inelastic demand characteristic of public education. Each school age child is required by law to attend school, and therefore the demand for education and the quality of education will be unaffected by the distance to the school. In this situation, the economies of scale must be balanced against the increasing costs of

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[1]See Mills and Lav [19] for a discussion of this problem.

[2]For a more complete discussion of taxation schemes associated with local public goods see Borukov [6].

transporting children to the school from larger distances. Even if the children walk to the school and the valuation of their time is zero there will still be some costs. There are pedestrian risks involved in having small children walk to school. In addition, there are costs to the parents who may have to drive or accompany the children.

The second case considered is the one in which the demand for the service is perfectly inelastic, but the quality of the service, for example fire protection, falls off with distance from the facility. The third case considers a situation where demand is influenced by transport costs and by income.

Perfectly Inelastic Demand - Public Schools

Urban planners commonly use rules of thumb about the size and spacing of schools when making construction decisions. Some common rules include building elementary schools so that all children are within one-half mile or building to a capacity of 850 pupils. Underlying any rule should be explicit assumptions about the density of population and the imputed transport costs of pupils. With the following model we can calculate the effect of the various assumptions.

Suppose there are a children per family and D families per square mile. Transport cost is t dollars a year per pupil-mile. The total yearly transport costs for pupils at a school serving a circular (2-1C) or an hexagonal

(2-1H) area of radius R will be[3]

(2-1C) $$T = \int_0^R aDtr\, 2\pi r\, dr = \frac{2}{3}\, aDt\pi R^3$$

(2-1H) $$T = 12aD \int_0^{\pi/6} \left\{ \int_0^{R/\cos\theta} tr\, rdr \right\} d\theta = 2.430\ aDtR^3$$

The production of education, it is assumed, has falling average cost and constant marginal cost as in

(2-2) $$C_p = A + kx$$

where C_p is production cost, A is fixed costs, k is marginal cost, and x is the number of pupils.

Given the density of school age population, aD, the level of output at a school serving an area of radius R will be

(2-3C) $$X = \int_0^R aD\, 2\pi r\, dr = aD\pi R^2$$

(2-3H) $$X = 12 \int_0^{\pi/6} \left\{ \int_0^{R/\cos\theta} aDrdr \right\} d\theta = 3.462\ aDR^2$$

The total costs to the community, C, of providing education is

(2-4C) $$C = C_p + T = A + kaD\pi R^2 + \frac{2}{3}\, aDt\pi R^3$$

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[3]For the hexagon, R is the minimum distance to a point on the border.

(2-4H) $$C = C_p + T = A + 3.462\ aDkR^2 + 2.430\ aDtR^3$$

Average cost per pupil will be

(2-5C) $$AC = \frac{C_p + T}{X} = \frac{A}{aD\pi R^2} + k + \frac{2}{3}tR$$

(2-5H) $$AC = \frac{C_p + T}{X} = \frac{A}{3.462\ aDR^2} + k + \frac{2.430}{3.462}tR$$

Optimal allocation of resources would minimize the cost per pupil. Therefore, differentiating with respect to R and setting equal to zero we have

(2-6C) $$\frac{dAC}{dR} = \frac{-2A}{aD\pi R^3} + \frac{2}{3}t = 0$$

(2-6H) $$\frac{dAC}{dR} = \frac{-2A}{3.462\ aDR^3} + \frac{2.430}{3.462}t = 0$$

and solving

(2-7C) $$R = \sqrt[3]{\frac{3A}{\pi aDt}}$$

(2-7H) $$R = \sqrt[3]{\frac{.823A}{aDt}}$$

(2-7) implies that, rather than following a rigid rule of thumb which fixes the area served by a school to a radius of a half mile, a school should serve an area that is smaller, the greater the density of school age population.

Further, increasing valuations of transport costs over time imply that there should be smaller service areas.

Perfectly Inelastic Demand with Decreasing Quality - Fire Protection

The demand for fire protection is similar to that for schools. Both are inelastic; one by law, the other by the emergency created in a fire. The difference between them arises in the quality of service. The quality of education is the same no matter how far away from the school a pupil lives, but the effectiveness of the local firemen is a function of how quickly they can respond to a call, and the speed of response is a function of the distance the fireman must travel. Police protection and broadcasting stations also have a quality component associated with distance from the facility.

This quality difference can be treated in the same way transport costs were earlier. Suppose a person living close to a fire station is 90% protected against fire damage to his property, while a person 10 miles away is only 80% protected. Then fire protection, we might say, falls by 10 percentage points over 10 miles or the expected loss rises by 10 percentage points.

If l is the increase in expected fire damage as one moves one mile farther from the fire station, then the total expected loss, L, from fire in an areas of radius R served by a station will be

(2-8C) $$L = \int_0^R aDlr\ 2\pi r\ dt = \frac{2}{3} aDl\pi R^3$$

(2-8H) $$12aD \int_0^{\pi/6} \left\{ \int_0^{R/\cos\theta} lrdr \right\} d\theta = 2.430\ aDlR^3$$

a, in this case, is the amount of property per person which requires protection. The other variables are as before.

Production and transport costs will bear the same relationships as for education. Total costs will be

(2-9C) $$C = C_p + T + L = A + kaD\pi R^2 + (t+1)\ aD\pi R^3$$

(2-9H) $$C = C_p + T + L = A + 3.462\ aDkR^2 + (t+1)\ 2.430\ aDR^3$$

It follows from cost minimization that the radius of the optimally spaced stations will be

(2-10C) $$R = \sqrt[3]{\frac{3A}{aD\pi(t+1)}}$$

(2-10H) $$R = \sqrt[3]{\frac{.823A}{aD(t+1)}}$$

This result is similar to the earlier one. The only addition is that any increase in the effectiveness of firemen reflected by a fall in l will permit fewer stations with larger service areas.

Elastic Demand

The addition of an elastic demand adds another dimension to the problem. Since the level of service is not exogenously determined by legal or physical factors, it must be incorporated into the size-location decision. In the absence of a pricing mechanism, the political process must determine the most desirable level of service either by the edicts of elected officials or by the vote of the people on bond issues or referenda.

Neither of these methods guarantees that there will be an optimal level of service provided. In fact, except in the case of a pure public good, no matter what level of service is provided the facilities are likely to be overused unless a system of fees is charged. Most local public services are not pure public goods, for example, libraries, parks, playgrounds, or courts. Nevertheless, we can only assume that a decision which is as nearly optimal as possible is made concerning the level of service. What remains then is to decide the size and location of facilities.

Let us consider the decision of locating libraries. Suppose it has been decided to provide an average of $\bar{a}$ units of library service per person. If demand is elastic with respect to the transport cost involved in getting to the library, a service area radius can then be found such that transport costs are sufficient to bring the average demand to the desired level.

Consider the following demand equation which is elastic with respect to transport costs and income

(2-11) $$\text{Demand per person} = B(tr)^{\theta}\, Y^{\varphi}$$

where B is a constant, Y is income per capita, and θ and φ are the "price" and income elasticities.

The total demand over a circular area of radius R will be[4]

(2-12) $$X = \int_0^R DB(tr)^{\theta} Y^{\varphi}\, 2\pi r dr = \frac{2\pi DB\, Y^{\varphi} t^{\theta} R^{(2+\theta)}}{2+\theta}$$

The average demand per person in this area is

(2-13) $$\frac{X}{\pi R^2 D} = \frac{2B\, Y^{\varphi} t^{\theta} R^{\theta}}{2+\theta}$$

The radius of the service area can be chosen so that transport costs choke off demand to the desired level $\bar{a}$. The radius which does this is

(2-14) $$R = \left[\frac{\bar{a}(2+\theta)}{2B\, Y^{\varphi} t^{\theta}}\right]^{1/\theta}$$

This value of R is not necessarily the cost minimizing one. In fact, it will be cost minimizing only if, by some fortuitous event, the level $\bar{a}$ chosen is the same as would result under an optimizing procedure. This

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[4]Only circular areas are considered in this section.

is easily demonstrated.

Suppose production costs are again given by $C_p = A + kx$, then

$$(2\text{-}15) \qquad C_p = A + \frac{k2\pi DBY^{\varphi}t^{\theta}R^{2+\theta}}{2+\theta}$$

Transportation costs will be

$$(2\text{-}16) \quad T = \int_0^R (DB(tr)^{\theta}Y^{\varphi})(tr)2\pi r dr = \frac{2\pi DBt^{1+\theta}Y^{\varphi}R^{3+\theta}}{3+\theta}$$

Substituting to obtain average costs, and differentiating and setting equal to zero gives the cost minimizing R ·

$$(2\text{-}17) \qquad R = \left[\frac{A(2+\theta)\ (3+\theta)}{2\pi DBt^{1+\theta}Y^{\varphi}}\right]^{\frac{1}{(3+\theta)}}$$

(2-14) and (2-17) will not be equivalent unless

$$(2\text{-}18) \qquad \bar{a} = \frac{2BY^{\varphi}t^{\theta}}{2+\theta} \left[\frac{A(2+\theta)\ (3+\theta)}{2\pi DBt^{1+\theta}Y^{\varphi}}\right]^{\frac{\theta}{(3+\theta)}}$$

Since all the terms on the right are either parameters or exogenous variables, (2-18) will not hold except by coincidence. This is an important result. It implies that <u>without a fee structure</u> (i.e., if the service is provided free of charge) <u>the local government cannot chose both the level of consumption of a service and the cost minimizing locations for the facilities</u>.

Thus, if the government choses the level of consumption and locates facilities to bring that level into effect, there will exist a different spacing of facilities that will reduce the per capita costs to the community (though the costs to the government may be higher). Similarly, it can chose to minimize the costs to the community only if the level of consumption is allowed to seek its own level. To select both the level of consumption and the minimum cost locations, the government must institute a fee or subsidy structure.

The cost minimizing R given in (2-17) has the same qualitative relationship to density of population, transport costs, and fixed costs that was found for an inelastic demand. In addition, (2-17) indicates that increases in income imply smaller optimal radii.

School Location and Transport Costs

In this section we calculate the radius-density trade-off for elementary schools for an American city. This calculation is meant to be illustrative of a typical situation. Excellent cost data is available for the Boston Public Schools and is used in the calculation.

To make the calculations, reasonable values for production costs, transport costs, and population densities are needed. Regression of total cost on average daily attendance using 56 schools in the Boston public schools system for the year 1969 as data gave the following equation.

$$(2\text{-}19) \qquad C_p = 257{,}330 + 368x \qquad (R^2 = .686)$$

The average daily attendance of 52,127 elementary school pupils divided by the 47.8 square mile area of Boston gives an average pupil density (aD) of 1090.52 per square mile. Since the average school in the system has a daily attendance of 921.58 pupils, the service area (hexagonal) of an average school in an average density section of the city would have a radius ($R = \sqrt{\frac{921.58}{\pi aD}}$) of .494 miles. These data,

$$R = .494$$

$$A = 257{,}330$$

$$aD = 1090.52$$

when substituted into (2-7H) imply a transport cost per pupil/mile-year of \$1610.93. Assuming there are 2 round trips per day for 175 days in a school year, the cost per mile per pupil is \$2.30. The average pupil in the average school travels .299 miles to school or 1.20 miles a day. The transport cost per day for the average pupil then is \$2.75 a day.

Using the value of \$1610.93 per pupil/year-mile, one can calculate what the radius of the service area and school size should be for schools in areas of lower or higher than average pupil density by substituting in (2-7H). The radius and size for various densities are shown in Table 2-1.

TABLE 2-1

OPTIMAL SERVICE AREA RADIUS
AND SCHOOL SIZE

Pupil Density (pupils sq. mile)	Radius (miles)	School Size (pupils)
100	1.100	415.47
500	.641	711.24
600	.603	755.29
700	.573	795.67
800	.548	831.72
900	.527	865.35
1000	.508	893.42
1100	.493	925.58
1200	.478	949.21
1300	.466	977.33
1400	.455	1003.41
1500	.444	1023.73
2000	.404	1127.75
4000	.320	1420.88

Increases in pupil density by a factor α imply that the radius of the service area of a school should be decreased by a factor $\frac{1}{\sqrt[3]{\alpha}}$ and that the school size should be increased by a factor $\sqrt[3]{\alpha}$.

Within the Boston School System elementary schools vary in size from 404 pupils to 1,924 pupils ($\mu = 921.58$ $\sigma = 336.33$). This is considerable variation and probably more than can be accounted for by the variance in pupil densities in Boston. Another source of variance would arise if different administrations valued travel cost differently and built schools accordingly. The effects of different travel costs on the optimal radius and school size for a given density are shown in Table 2-2.

If transport costs differ by a factor β then the optimal radius will differ by a factor $1/\sqrt[3]{\beta}$ and the optimal school size by a factor $1/\sqrt[3]{\beta^2}$.

There are, of course, many other considerations in addition to pupil density and transport cost when locating schools. Some of these might include availability of land, proximity to supporting facilities such as playgrounds, a desire for racial or economic balance, avoidance of traffic barriers, and topography.

The value of \$2.30 per mile for elementary school pupil transport cost in Boston is strikingly high. Normally one would expect a very low valuation of travel time for small children but this figure suggests an

TABLE 2-2

TRANSPORT COSTS, THE OPTIMAL RADIUS AND SCHOOL SIZE

(aD = 1090.52)

Transport Cost ($ per mile)	Radius (miles)	School Size (pupils)
1.00	.652	1604.93
1.50	.570	1226.62
2.00	.518	1013.03
2.50	.480	869.85
3.00	.452	778.17
3.50	.429	694.82
4.00	.411	637.74

opportunity cost of more than $5/hr. The high value could be a reflection of the pedestrian risks or the salaries of crossing guards.

Even so, the figure still appears high and suggests looking for possible sources of bias in the model. A non-proportional linear transport cost (T_0 + tr instead of tr) would not effect the figure; but a non-linearity in the production cost function would cause the economies of scale to be over or under estimated. It seems more likely that costs would be increasing at a decreasing rate than at an increasing rate. If so, the economies of scale would be underestimated in a linear regression and, similarly, the transport cost would be underestimated in the calculations rather than overestimated.

Therefore, we are left with a high implied cost of transporting elementary school pupils and a possible over-investment in the number of school buildings.

Conclusions

Transport costs are an important factor in the provision of local governmental services. These costs should be given explicit consideration in any size and location decisions connected with the facilities providing the service. If transport costs are not given explicit consideration an inefficient configuration is likely to arise in which a higher level of service is possible at a lower per capita social cost.

Definition of Symbols

T	transportation cost in an area of radius R
a	demand per family
D	density of families
t	transport cost per unit per mile
R	radius of the area
C_p	production cost
C	total production costs and transport costs in an area of radius R
x	quantity demanded in area of radius R
A	fixed costs of production
k	marginal costs of production
l	expected loss per mile
B	scale parameter in the demand equation
θ	"price" elasticity in the demand equation
ω	income elasticity in the demand equation

CHAPTER 3

TRANSPORTATION COST AND URBAN RETAIL TRADE

Within any city there will be many activities that can support more than one firm. For example, in a small town only one grocery store may be able to operate profitably. If the town is small enough the grocery store may be combined with other retailing activities, such as hardware and drugs, into a general store. In a large city, on the other hand, there will be numerous supermarkets retailing groceries, as well as many hardware stores, drugstores, barber shops, etc.

Questions arise concerning the number and spacing of firms engaged in a particular activity. How will the number and spacing be affected by changes in relevant parameters? Does the free market result in a social optimum?

These and other questions have been investigated in a series of recent articles including those of Mills and Lav [19], Hoover [15] and Beckman [4]. Mills and Lav, and Beckman consider the problem of whether the equilibrium spacing is also the optimum one. Hoover, using numerical analysis on a model which defies explicit solution, looks at how the equilibrium spacing is affected by changes in parameters. The basic model of all three papers first appears in that of Mills and Lav. This model is then modified in one manner or another to suit the purposes of the authors. This tradition is followed here.

As in the earlier discussion of the location of government activities, there are two forces acting in opposite directions to determine the spacing of firms. On the one hand, economies of scale[1] imply that production costs per unit output can be reduced as output expands and encourage larger market areas. On the other hand, transportation costs increase at an increasing rate and encourage smaller areas.

The Mills and Lav model will be summarized briefly here. The model assumes that there are fixed costs, f, and marginal costs, c, in producing output. c is less than f so that average costs are falling. Total production costs, then, are

(3-1) $$f + cx$$

Demand per unit area is a linear function of the mill price, p, plus transport cost, tr, where t is transport cost per mile and r is the distance to the store or factory.

(3-2) $$\text{demand per household} = a-b(p+tr)$$

where a and b are the price intercept and slope of the demand curve.

Profits for a firm with a circular market area of radius, R, and density of households, D, will be

[1] Actually, falling average cost is all that is needed.

$$(3\text{-}3) \qquad Z = D\int_0^R 2\pi r(p-c)(a-bp-btr)dr - f$$

$$= \pi R^2 D(p-c)(a-bp - 2/3\ btR) - f$$

which must be equal to zero in equilibrium if there is free entry.

The profit maximizing price for a given R can be found by maximizing (3-3) with respect to p and is given in (3-4).

$$(3\text{-}4) \qquad p = \frac{a}{2b} + \frac{c}{2} - \frac{tR}{3}$$

Retail Prices and Transportation Costs

One of the most controversial issues in recent years has been the question of whether or not prices are higher in urban low income areas and, if so, why. The existence of higher prices in the ghetto has been documented by several authors, for example, Caplovitz [7]. A subsidiary issue concerns whether chainstores raise their prices in low income sections of the city. Evidence of this type of price discrimination also exists, for example, U. S. Department of Agriculture [33], but is much less conclusive.

Explanations of why prices vary within a city range from racial discrimination and moral turpitude on the part of store owners to the more traditional economic causes of higher fixed or marginal costs. Locational factors have generally been ignored.

Explanations which suggest that excess profits are being earned must rely on the existence of a short-run disequilibrium because in the long run, if there is free entry into the industry, profits must be no greater than the normal profits earned in other areas. Therefore, it is unlikely that high prices are due to excess profits.

In the long run, underlying differences in supply and demand could be a cause of higher prices. In the context of the earlier assumptions, if costs are given by (3-1), and the firms face downward sloping demand curves of the form $a-bp$ where a and b are the intercept and slope, then the profit maximizing price will be given by

$$p = \frac{a}{2b} + \frac{c}{2} \tag{3-5}$$

From (3-5) we can see that higher marginal costs mean higher prices but that fixed costs do not affect the price. A shift in demand will increase price and a shallower sloped demand curve also will increase price. These results are summarized for future reference in Table 3-1.

There is reason to expect that both marginal (c) and fixed costs (f) might be higher in low income areas: marginal costs because of greater theft and vandalism, greater temporal concentration of demand, and greater risks; fixed costs because of higher insurance premiums and land rents. We might expect demand to be lower and more elastic in a low

TABLE 3-1

EFFECT ON PRICE OF A CHANGE IN PARAMETERS FOR THE MONOPOLISTIC FIRM, IGNORING LOCATION

		Value	Sign
demand intercept	$\frac{\partial p}{\partial a}$	$\frac{1}{2b}$	Pos.
demand slope	$\frac{\partial p}{\partial b}$	$-\frac{2}{2b^2}$	Neg.
marginal costs	$\frac{\partial p}{\partial c}$	$\frac{1}{2}$	Pos.
fixed costs	$\frac{\partial p}{\partial f}$	0	Zero

income area (smaller a and b).[2] Thus, for prices to rise in this non-locational model the positive effect of higher costs and more elastic demand must be greater than the negative effect of a lower demand curve. Fixed costs make no difference.

Next, the Mills and Lav model is used to investigate what effect changes in the relevant parameters have on the mill price when locational factors are considered. Some quite startling results appear when the equilibrium spacing of firms is taken into account. For example, high costs are more likely to decrease than increase prices. The principal source of high prices in the ghetto becomes high population densities.

To determine the effect of the various parameters on prices one need only take the partial derivatives of (3-4).

First, it is noted that the first two terms of (3-4) are identical to those of (3-5). The changes in these two terms are called the monopoly effect.[3] While changes in the last term are called the free entry effect

---

[2]For a linear demand curve the smaller the intercept the more elastic demand is even if the slope remains constant. Therefore, it is not necessary for b to fall for demand to be more elastic.

[3]Of course a firm need not be a monopoly to experience this monopoly effect. A downward sloping linear demand curve is sufficient.

here.

The derivatives are summarized in Table 3-2. R_0 is the equilibrium radius.

To determine the sign of these partials, it is necessary to know what effect the parameter changes will have on the market area radius. Unfortunately, the explicit solutions for R_0 and its partial derivations are very complicated and difficult to interpret a priori. Hoover [15], however, has done a numerical analysis of this free entry model for a wide range of parameter values. His results suggest the signs given in Table 3-3.

The signs of these derivatives can now be used to determine the signs of the free entry effect and the sign of the price change. These are given in Table 3-4.

Earlier it was argued that one would expect a, and possibly b, to be smaller in a low income area; c and t higher. Population density would characteristically be much higher in a ghetto than it would in the suburbs. Transportation costs are usually said to be higher for low income areas because of greater congestion and less availability of cars. On the other hand the valuation of time spent traveling will be less for low income groups and may offset the other components of transport cost. Therefore, it is difficult to say whether transport costs are higher or lower in ghetto areas.

TABLE 3-2

EFFECT OF PARAMETER CHANGES ON PRICE
IN THE FREE ENTRY LOCATION MODEL

		Monopoly Effect	Free Entry Effect
demand intercept	$\frac{\partial p}{\partial a}$	$\frac{1}{b}$	$-1/3t\ \frac{\partial R_0}{\partial a}$
demand slope	$\frac{\partial p}{\partial b}$	$-\frac{a}{2b^2}$	$-1/3t\ \frac{\partial R_0}{\partial b}$
marginal costs	$\frac{\partial p}{\partial c}$	$\frac{1}{2}$	$-1/3t\ \frac{\partial R_0}{\partial c}$
fixed costs	$\frac{\partial p}{\partial f}$	0	$-1/3t\ \frac{\partial R_0}{\partial f}$
population density	$\frac{\partial p}{\partial D}$	0	$-1/3t\ \frac{\partial R_0}{\partial D}$
transportation costs	$\frac{\partial p}{\partial t}$	0	$-1/3R_0 - 1/3t\ \frac{\partial R_0}{\partial t}$

TABLE 3-3

SIGNS OF THE PARTIAL DERIVATIVES

Partial	Sign
$\frac{\partial R_O}{\partial a}$	Neg.
$\frac{\partial R_O}{\partial b}$	Pos.
$\frac{\partial R_O}{\partial c}$	Pos.
$\frac{\partial R_O}{\partial f}$	Pos.
$\frac{\partial R_O}{\partial D}$	Neg.
$\frac{\partial R_O}{\partial t}$	Pos.

TABLE 3-4

EFFECT OF PARAMETER CHANGES ON PRICE

	Monopoly Effect	Free Entry Effect	Net Effect
$\frac{\partial p}{\partial a}$	Pos.	Pos.	Pos.
$\frac{\partial p}{\partial b}$	Neg.	Neg.	Neg.
$\frac{\partial p}{\partial c}$	Pos.	Neg.	?
$\frac{\partial p}{\partial f}$	Zero	Neg.	Neg.
$\frac{\partial p}{\partial D}$	Zero	Pos.	Pos.
$\frac{\partial p}{\partial t}$	Zero	Neg.	Neg.

These statements are summarized in Table 3-5 along with the implied effect on prices.

Only one parameter change unambiguously produces higher prices in the ghetto - higher population densities. Two others may increase prices. If the demand slope is lower, prices will be higher; but this source of increase is likely to be more than offset by the unambiguous reduction in prices resulting from a lower demand intercept. Higher marginal costs will raise or lower prices depending upon the relative magnitudes of the monopoly and free entry effects. The monopoly effect will tend to increase prices while the free entry effect will decrease prices.

As marginal costs rise, the monopoly effect will tend to raise prices. The higher prices reduce the demand per capita making a larger market area necessary for zero profit. The larger market area then reduces the profit maximizing price. In the results of the numerical analysis given in the appendix to this chapter, the mill price first increases as marginal cost rises and then begins to fall as the free entry effect overcomes the monopoly effect.

The other likely parameter differences cause price reductions. Of particular note is the negative effect on prices of higher fixed costs such as insurance premiums or rent. This, of course, is a result of the fact that higher fixed costs make a larger market area necessary for break even; the larger market area causes a lower profit maximizing

TABLE 3-5

PARAMETER VALUES IN LOW VERSUS HIGH INCOME AREAS

Parameter	Value in Low Income Area	Effect on Prices
a	lower	neg.
b	lower (?)	pos. (?)
c	higher	?
f	higher	neg.
D	much higher	pos.
t	higher (?)	neg. (?)

price.

On balance, the most important parameter difference between low income and other areas is probably the population density difference. Population density in the inner city can be one or two orders of magnitude greater than in the suburbs. There will be, however, considerable variation in the importance of the several parameter differences depending on the nature of the product or industry in question.

Delivered Prices

If one is trying to determine whether or not the poor suffer economically by living in the typical low income area, a much better measure than the mill price would be the delivered price (p+tr). It is terribly convenient to be within walking distance of the stores one patronizes; and a person could be just as well off paying higher prices if he did not have to travel as far to purchase the goods.

The average delivered price paid in an area of radius R_0 served by a store at the center will be given by

$$(3\text{-}6) \qquad Pd = \frac{D \int_0^{R_0} (p+tr)(a-bp-btr)2\pi r dr}{D \int_0^{R_0} (a-bp-btr)2\pi r dr}$$

This reduces to

$$(3\text{-}7) \qquad Pd = \frac{-3bt^2R_0^{\ 2} + 4(at-2btp)R_0 + 6(ap-bp^2)}{-4btR_0 + 6(a-bp)}$$

Once again the partial derivatives are too complicated to yield information about the expected signs, making numerical analysis necessary. The results are given in the appendix to this chapter and summarized in Table 3-6.

Two differences between the average delivered price and the mill price are apparent. First, the impact of a given change in a parameter is much smaller for delivered price than it is for the mill price. Second, the impact of a parameter change is ambiguous for four of the six parameters as opposed to only one for the mill price. Therefore, it is much less certain that the economic system discriminates against low income groups by causing higher prices when one looks at the average delivered price rather than the mill price. Whether or not the average delivered price is higher or lower depends for the most part on the nature of the particular product involved.

Conclusion

The results indicate that free market forces will tend to make the mill price (price at the store) higher in low income areas. The characteristic which plays the largest role in raising prices in low income areas is the high population densities rather than higher costs.

TABLE 3-6

SIGNS OF THE PARTIAL DERIVATIVES
FOR DELIVERED PRICES

Partial	Sign	Partial	Sign
$\frac{\partial Pd}{\partial a}$	+	$\frac{\partial p}{\partial a}$	+
$\frac{\partial Pd}{\partial b}$	-	$\frac{\partial p}{\partial b}$	-
$\frac{\partial Pd}{\partial D}$	±	$\frac{\partial p}{\partial D}$	+
$\frac{\partial Pd}{\partial c}$	±	$\frac{\partial p}{\partial c}$	±
$\frac{\partial Pd}{\partial f}$	±	$\frac{\partial p}{\partial f}$	-
$\frac{\partial Pd}{\partial t}$	±	$\frac{\partial p}{\partial t}$	-

On the other hand, residents of low income areas generally benefit from having stores located closer to their homes. A measure of the total economic cost (product price and travel cost) is the average delivered price. Free market forces may cause the price to be either higher or lower in a low income area. Which forces prevail depends upon the characteristics of the product or industry involved.

Definition of Symbols

a	price intercept of the demand curve per person
b	slope of the demand curve per person
c	marginal costs
f	fixed costs
t	transportation costs
D	density of population
R_0	radius of smallest zero profit market area
p	mill price
Pd	average delivered price

The assumed parameter values and values of R_0 are taken from Hoover [15].

APPENDIX TO CHAPTER 3

MILL PRICES AND DELIVERED PRICES

TABLE 3-7

EFFECT OF LOWER DEMAND INTERCEPT (a)

Values Assumed						
a	100	93	86	79	72	71.13
b	1	1	1	1	1	1
c	25	25	25	25	25	25
f (in millions)	2	2	2	2	2	2
t	.5	.5	.5	.5	.5	.5
D	1	1	1	1	1	1
Results						
R_0	23.79	63.17	31.63	38.88	56.97	69.19
p	58.54	54.49	50.24	45.52	39.01	36.57
Pd	66.24	63.17	60.23	57.46	54.72	53.85

TABLE 3-7 (Continued)

EFFECT OF STEEPER DEMAND CURVE (b)

Parameter Values Assumed						
a	100	100	100	100	100	100
b	1.00	1.12	1.24	1.36	1.48	1.50
c (in millions)	25	25	25	25	25	25
f	2.0	2.0	2.0	2.0	2.0	2.0
t	.5	.5	.5	.5	.5	.5
D	1	1	1	1	1	1
R_0	23.79	27.33	31.82	38.24	51.94	62.50
p	58.54	52.59	47.52	42.89	37.63	35.42
Pd	66.24	61.29	57.50	54,50	51.98	51.04

TABLE 3-7 (Continued)

EFFECT OF LARGER MARGINAL COST (c)

Parameter Values Assumed						
a	100	100	100	100	100	100
b	1.00	1.00	1.00	1.00	1.00	1.00
c	25	31	37	43	49	53.87
f (in millions)	2.0	2.0	2.0	2.0	2.0	2.0
t	.5	.5	.5	.5	.5	.5
D	1	1	1	1	1	1
R_0	23.79	26.53	30.13	35.27	56.97	69.19
p	58.54	61.08	63.48	65.63	67.01	65.41
Pd	66.24	69.60	73.05	76.62	82.78	82.70

TABLE 3-7 (Continued)

EFFECT OF LARGER FIXED COSTS (f)

Parameter Values Assumed						
a	100	100	100	100	100	100
b	1.00	1.00	1.00	1.00	1.00	1.00
c (in millions)	25	25	25	25	25	25
f	2.0	4.9	7.8	10.7	13.6	13.98
t	.5	.5	.5	.5	.5	.5
D	1	1	1	1	1	1
R_O	23.79	40.65	55.91	72.71	99.37	112.50
p	58.54	55.73	53.19	50.39	45.94	43.75
Pd	66.24	68.53	70.29	71.73	73.51	71.88

TABLE 3-7 (Concluded)

EFFECT OF LARGER TRANSPORT COSTS (t)

Parameter Values Assumed						
a	100	100	100	100	100	100
b	1.00	1.00	1.00	1.00	1.00	1.00
c (in millions)	25	25	25	25	25	25
f	2.0	2.0	2.0	2.0	2.0	2.0
t	.5	.7	.9	1.1	1.3	1.322
D	1	1	1	1	1	1
R_0	23.79	25.24	27.19	30.19	37.71	42.55
p	58.54	56.61	54.35	51.43	46.16	43.75
Pd	66.24	67.84	69.53	71.25	72.53	71.88

CHAPTER 4

BENEFITS OF AN URBAN TRANSPORTATION SYSTEM IMPROVEMENT

Several controversies have arisen recently concerning the measurement of the benefits of government investment projects in general and of the benefits of transportation investments specifically. One controversy centers around the use of consumer surplus as a measure of benefits. Consumer surplus seems to have fallen into considerable disrepute as a result of attacks by Samuelson [30] and others. More recently consumer surplus has been reasserting itself as a result of defenses by Harberger [11], Mohring and Harwitz [24], and Friedlaender [9].

Harberger refutes several common objections to the use of consumer surplus, including:

> (i) Consumer-surplus analysis is valid only when the marginal utility of real income is constant.
>
> (ii) Consumer-surplus analysis does not take account of changes in income distribution caused by the action(s) being analyzed.
>
> (iii) Consumer-surplus analysis is partial-equilibrium in nature, and does not take account of the general-equilibrium consequences of the actions whose effects are being studied.
>
> (iv) Consumer-surplus analysis, though valid for small changes, is not so for large changes.
>
> (v) The concept of consumer surplus has been rendered obsolete by revealed-preference analysis. [11]

The objections contain varying amounts of truth but do not invalidate the use of consumer surplus. (i), (ii), and iv), however, do imply that a consumer surplus measure will be an approximation of the benefits.

A second difficulty arises concerning whether or not the benefits which accrue to users of a transportation investment accurately measure the total benefits to society. Mohring and Williamson [25], Tinbergen [32], Bos and Koyck [5], and Friedlaender [9] analyze situations where user benefits underestimate the social benefits.

There seem to be at least two situations where user benefits underestimate the social benefits of a transportation investment. One situation occurs when there are economies of scale in production which are being left unutilized because of the increasing transport costs as output is shipped farther from the production facility. A reduction in transport cost will reduce the delivered price of the good in question but in addition it may (depending on the organization of the industry) make possible an expansion of output at each production site thereby further reducing the cost by allowing more of the economies of scale to be implemented. This type of secondary benefit carries many labels in discussions of transport cost and is perhaps most aptly called the reorganization benefit. This reorganization benefit has been estimated by Mohring and Williamson [25] to be between 7% and 12% of the total benefits of a transportation improvement, depending on the assumption made concerning the extent of scale economies.

A second situation where user benefits may underestimate the social benefits appears when the improvement is important enough to affect absolute prices and therefore real incomes.

Tinbergen [32] and Bos and Koyck [5] discuss this kind of improvement with special reference to a small, developing country. Friedlaender [9] confronts the same problems in calculating the benefits of the interstate highway system in the United States.

When the improvement is large in relation to the economy it can affect both relative and absolute prices. A decrease in absolute prices increases real incomes and makes possible higher levels of consumption and welfare which may not be fully reflected in user benefits. In the context of two models, Friedlaender [9] is able to show that social benefits will diverge from user benefits. The sign and extent of the divergence depends on the conditions of consumer demand and factor supply. Reasonable values of the relevant elasticities suggest that the situation in which user benefits underestimate the social benefits is the more likely.

The Urban Transportation Improvement

A major improvement in the transportation system of an urban area will result in both benefits to users of the improvement and secondary benefits of the kinds discussed above. Since economies of scale are inherent in all cities, there will be reorganization benefits; and for a major improvement, there will also be changes in prices that will affect the benefits. For expository purposes, the benefits of a reduction in transportation costs will be discussed for the three sectors of government services, retail trade, and residential housing in turn.

A. Government Services

A reduction in transport cost as a result of an improvement will reduce the costs of providing governmental services in two ways. First, there will be the direct benefit from the reduction in the cost of getting to and from the facility and second, there will be the reorganization benefit arising from the implementation of additional scale economies as the optimal service area increases.

Using the models of Chapter 2, it is straightforward to calculate both these benefits. In the case of perfectly inelastic demand, the total production and transport costs for an hexagonal area of radius R from Equation (2-4H) were

$$C = (A + 3.462\ aDkR^2) + (2.430\ aDtR^3) \tag{4-1}$$

If the total area of the city is S square miles, it will take $S/3.462R^2$ hexagons to serve the entire city. Total costs will be $SC/3.462R^2$.

By substituting Equation (2-7H) from Chapter 2 in the above, we can express total costs as a function of the parameters and transport cost per mile.

$$(4\text{-}2) \quad \text{Social Costs} = \frac{SC}{3.462R^2}$$

$$= (.329\ SA^{1/3}(aDt)^{2/3} + .289\ SaDk) + (.657\ S(aDt)^{2/3}A^{1/3})$$

The first parentheses in (4-2) gives production costs and the second transport costs. The change in social costs as transport costs fall is

$$(4\text{-}3) \quad \frac{d(\text{Social Costs})}{dt} = \frac{.657\ SA^{1/3}(aD)^{2/3}}{t^{1/3}}$$

$$(4\text{-}4) \quad \text{Benefit} = \int_{t_0}^{t_1} \frac{.657SA^{1/3}(aD)^{2/3}}{t^{1/3}}\,dt$$

where t_0 is the initial transport cost and t_1 the terminal.

In this case the reduction in production costs, i.e., the reorganization benefits, amounts to 1/3 of the total benefits.

The results are analogous for the other cases discussed in Chapter 2. There will be direct and reorganization benefits in each case, with the size of the benefits depending on the relevant parameters. However, there is one exception in the case of an elastic demand for the service. If the government choses the level of service rather than the cost minimizing locations, then the service areas of the facilities must be enlarged as marginal transport costs fall. To maintain the same level of service, the new service areas must result in the same average transport cost. Therefore, in such a situation, all of the benefit will be in the form of reduced production costs, i.e., reorganization benefit only, since total transport costs remain constant.

B. Retail Trade

In the consumption of retailing services a transportation cost reduction will result in both a user benefit and a reorganization benefit as it did in the case of governmental services. However, in this case the reorganization benefit may be negative.

To see why this is so the free entry model of Chapter 3 is referred to. The benefit can be found by measuring the change in consumer surplus as the delivered price changes. This difference is given for a city of S square miles by the following

$$(4\text{-}5) \qquad B = \frac{SD}{\pi R_0'^2} \int_0^{R_0'} 1/2(a-p'-t'r)(a-bp'-bt'r)2\pi r dr$$

$$- \frac{SD}{\pi R_0^2} \int_0^{R_0} 1/2(a-p-tr)(a-bp-btr)2\pi r dr$$

where the primed values are the post-improvement values. The two terms in this expression correspond to the shaded area in Figure 4-1 summed over the entire city.

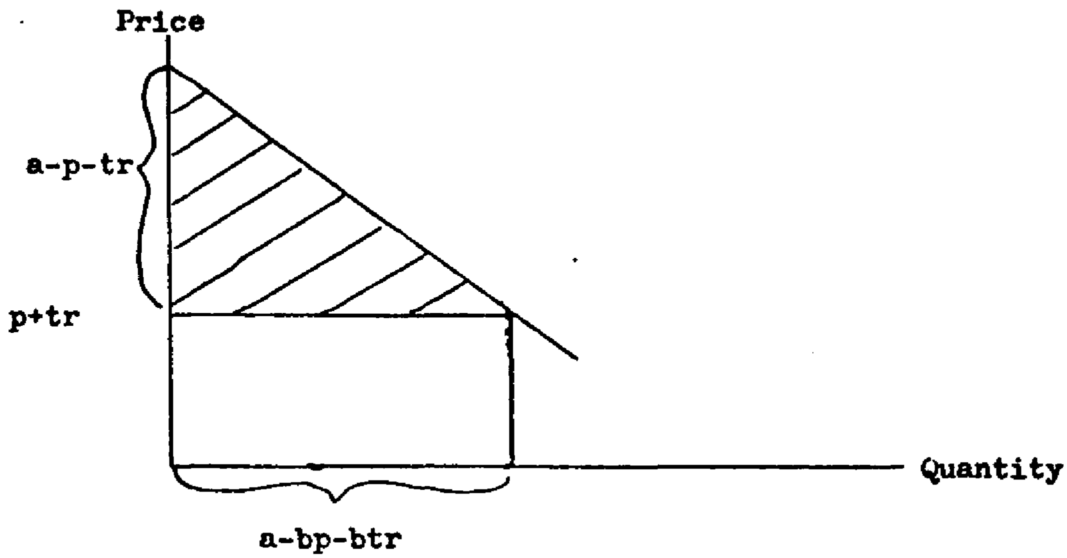

Figure 4-1. Consumer Surplus

To see what happens to (4-5) when transportation costs fall refer to the last table in the appendix to Chapter 3. There it can be seen that as transport costs fall the mill price rises, indicating that the reorganization benefit is negative. The fall in transport costs makes the equilibrium size of the market areas smaller by increasing demand. From Equation (3-2) in Chapter 3 it may be seen that the profit maximizing price falls if either transport costs or the radius of the market area falls.

To determine whether the transport cost reduction is sufficient to offset the negative reorganization benefit it is necessary to look at the average delivered price. From the table it may be seen that the average delivered price may rise or fall, and therefore, the net benefit may be either positive or negative, depending on the parameter values.

Thus, for industries where free entry prevails, the advent of reduced transport costs does not necessarily benefit consumers. The negative reorganization benefit may be greater than the positive user benefit and, therefore, the net benefits may be negative.

C. Housing

There is considerable confusion over how the benefits of an urban transportation improvement can be valued. Models of residential location (e.g., Alonso [1]) postulate a relationship between housing prices and transport costs. Since both housing and transportation enter into the utility functions of consumers, it would appear that changes in the quantities or prices of either should enter into the measurement of the benefits.

However, under a set of strict assumptions (fixed coefficients in housing production, perfectly inelastic demand for housing) all of the reduction in the price of housing will be a result of a fall in land values. The fall in land values, while a benefit to housing consumers, will be a loss to land owners. Thus, the two will cancel each other (if the marginal utility of income is the same for both groups) leaving only the fall in transport cost as the benefit.

If we relax the assumption of fixed coefficients in the production of housing, the fall in land values as a result of the transportation improvement may make it possible to substitute land for other factors, thereby reducing housing costs an additional amount. In this situation housing costs may fall more than land rents. There will be benefits over and above the reduced transport cost. This is another example of a reorganization benefit.

Relaxing the fixed coefficients assumption introduces another complication. If more land is used in housing production, the city will cover a larger area and more miles of transportation will be necessary to bring commuters to the CBD. The fall in the marginal cost of travel can then be valued either at the old quantity or at the new. Valuing at the old quantity will give a lower bound to the benefits while valuing at the new will give an upper bound. Taking an average of the two will give a good approximation of the increase in consumer surplus or true benefits (particularly if the demand for transportation is linear in the relevant range).

To summarize this case: if factor substitution is possible in housing production but demand for housing is perfectly inelastic, then the benefits of a transportation system improvement can be measured by the excess of the fall in housing costs over the fall in land values, plus the increase in consumer surplus resulting from the fall in commuting costs. The two are, respectively, a reorganization benefit and a direct benefit. Symbolically this can be represented by

$$(4\text{-}6) \quad \int_0^k P(u)X(u)du - \int_0^{k'} p'(u)X'(u)du - \int_0^k R(u)L(u)du + \int_0^{k'} R'(u)L(u)du + \int_0^k [T(u)-T'(u)] \left[\frac{X(u)+X'(u)}{2}\right] du$$

The assumption of a perfectly inelastic demand can be relaxed in two ways. Either the quantity demanded per household can increase as the price falls or the number of households demanding the original quantity can increase. The latter would occur if labor were mobile between cities and the lower transport and housing costs attracted new residents from other areas.

If demand is elastic the above result need only be modified to take into account the problem of whether to measure the cost reduction at the old quantity or the new. Again an average of the two will give a good approximation of the consumer surplus.

The earlier representation becomes

$$(4\text{-}7) \quad \int_0^{k'} [p(u)-p'(u)] \left[\frac{X(u)+X'(u)}{2}\right] du - \int_0^{k} R(u)L(u)du$$

$$+ \int_0^{k'} R'(u)L(u)du + \int_0^{k'} [t(u)-T'(u)] \left[\frac{X(u)+X'(u)}{2}\right] du$$

One interesting extreme case arises. If demand is perfectly elastic the first and last terms of (4-7) drop out leaving only the increase in land values as the measure of the benefits. A perfectly elastic demand could occur in the long run. A condition of equilibrium between cities is that the standard of living in all cities be nearly identical. A transportation improvement which reduced both housing and travel cost would attract new residents to the city until the standards of living are again equalized. If the city in question were small relative to the entire country the new equilibrium would occur when housing and travel costs were driven back to the higher level.

Thus, there is likely to be a big difference between the short-run beneficiaries and the long run beneficiaries. In the short run, demand will be very inelastic and commuters will benefit from lower housing and travel costs while land owners will lose some rents. In the long run demand will be very elastic. Housing and travel costs will return to the early level. Commuters will no longer benefit from lower prices while land owners will benefit from higher rents.

How long it takes to go from the short run to the long run becomes an important consideration. Mills [22] has estimated that adjustment to changes in the equilibrium structure of a city proceed at about 44% per decade. However this estimate was for internal adjustments, not for migration into the city, and may be inaccurate if applied in this situation.

In order to estimate the benefits of an improvement, Equations (4-6) and (4-7) indicate that it is necessary to be able to predict the response of housing prices, land values, transport costs, quantities of housing and land, and the passenger miles traveled. In the next chapter a model with this ability is developed.

Definition of Symbols

C	total production and transport costs for an area of radius R
A	fixed costs in production of governmental services
a	demand intercept
D	density of population
k	marginal costs in production of governmental services
R	radius of service area
t	transport cost per mile
S	area of the city
p	price of output
b	demand slope
R_0	smallest zero profit radius of the market area of a firm in an industry with free entry
u	distance to the CBD
$p(u)$	price of housing at u
$X(u)$	quantity of housing at u
$R(u)$	land rent at u
$T(u)$	cost of traveling to the CBD from u
$L(u)$	quantity of land at u

CHAPTER 5

AN AGGREGATIVE URBAN COMMUTER MODEL

This chapter describes an aggregative urban commuter model. Its progenitor is a model by Mills [21] and, like Mills' model, it is continuous and highly aggregative in its treatment of output. Unlike Mills' model, however, it disaggregates the transportation sector into two sectors - a land-intensive one and a land-economizing one (presumably roads and subways). The intent is to focus attention on the difference in urban structure that can be anticipated between cities with road transportation and those with subway transportation.

This chapter is organized as follows: the first section discusses the assumptions underlying the Von Thünen-Ellet rent model which is the basis for the model described in the second section. The section section gives a description of the model and solves for the rent-distance function. The third section indicates how some of the initial assumptions can be relaxed and analyzes their effect on the earlier results.

The Von Thünen-Ellet (VTE) Rent Model

Underlying the VTE model (and others derived from it) is the abstraction that cities consist of an exogenously determined employment center which hires a certain work force. The journey to work from residences surrounding the employment center or central business district (CBD) and the desire to minimize commuting costs make locations near the CBD more desirable than the ones further away. As a result, land rents arise in such a way as to equalize the desirability of all locations to workers. Since workers prefer to live close to work in order to bear as little transport costs as possible, rents will have to be high close to the CBD and fall off with distance if workers are to be indifferent among locations, as is necessary for equilibrium. In other words, if a worker is to be in locational equilibrium, the rate of change of housing cost with respect to distance from the CBD must equal the negative of marginal transportation costs.

$$\frac{dp}{du} = -t$$

where p is housing prices.

If all workers have the same perfectly inelastic demand for housing, and there is no factor substitution in the production of housing, this becomes

$$\frac{dR}{du} = -t$$

where R is land rents.

The above model has come to be known as the Von Thünen-Ellet model, which, despite its extreme simplification, is the starting point for many studies of urban areas including this one. The assumptions that there is an exogenously determined employment center and that rents will arise to make workers indifferent among locations are commonly accepted. The assumptions of inelastic demand, homogeneous consumers,[1] and fixed coefficients in housing production, however, are clearly unrealistic and accepted only for convenience or simplicity's sake. The latter three assumptions are often modified, as they will be here later. Wide acceptance of the former two, however, does not mean that they do not merit some comment.

There is very little discussion of why a central business district arises within a metropolitan area. One possible explanation would follow the hierarchy of cities idea.[2] The reasoning is that cities arise to service an agricultural population with retail stores and certain manufactured goods. Larger cities exist to serve not only a rural population but also systems of smaller cities. Larger cities are further along in the hierarchy, and the largest city serves the entire country. The usual purpose of this type of discussion is to derive a size distribution for cities. However, it is only a small step to note that, since cities are furnishing manufactured goods to rural populations and other cities, transportation of these goods

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[1]John Kain [16] gives an illuminating discussion of the effect of family size, income and other variables on residential location.

[2]See, for example, M. J. Beckman [2].

will be an important consideration. Thus, the employment center might arise as firms cluster around a transport terminal (e.g., a rail head or port) to minimize transportation costs.

There are two problems with this explanation. First, a large proportion of employers in CBD's have never been tied to transport terminals (e.g., law offices); and second, more and more manufacturer's are using truck transportation which does not necessitate location near a terminal. If a CBD is going to exist, something more than just a transport terminal is needed.

Another line of thought considers external economies to be important. A supplier (e.g., a machine shop) may be able to offer its output at a lower price if it is serving several firms, whereas spatial separation may precipitate multiple suppliers and higher costs. The need for face to face negotiation in some business affairs (e.g., banks) may also contribute to the clustering of firms in a CBD. These explanations are difficult to evaluate, since measurement of the relevant variables is either difficult or impossible.

A third explanation centers around transportation costs within the city to distribute output or collect inputs. If a firm is distributing output throughout an area, it will want to locate where the distribution costs are minimized. The location which does this will be more desirable than other locations and will command high rents. If the cost-minimizing location is the same for a number of firms, a CBD will arise. A detailed discussion is given in the appendix to this chapter.

Certainly none of these explanations is sufficient in itself to explain the existence of a modern CBD. All the factors mentioned here plus, in all likelihood, several others are operating together to make a CBD. In the model that follows, the CBD is taken as an empirical fact, and attention is focused on the surrounding land area. This non-CBD land usually comprises over 99% of the land in a typical city.

The second important assumption of a VTE rent model, that land rents are determined by accessibility to the CBD, is, of course, an oversimplification of a modern city. A wide variety of factors influence the value of a parcel of land.

Hoover [13] states that the three determinants of the relative desirability of locations in a city are (1) access, (2) environmental characteristics, and (3) cost. The VTE model studies (1) and (3) but ignores (2). Hoover [13] continues

> A site has value according to its access but also according to its physical features and to the character of its immediate surroundings. Neighborhood character in terms of cleanliness, interest, and general appearance is important in attracting some kinds of use and repelling others.

The VTE model "assumes away all differentiation of sites with respect to topography, amenity, and environmental advantage." [13]

Nevertheless, several studies indicate that accessibility to the CBD is one of the most important, if not the most important, variable in explaining land values. Three such studies - those of Pendleton [29], Muth [27], and Mills [22] - will be surveyed briefly here.

Pendleton, in attempting to measure the value of accessibility as expressed in property values in the Washington, D. C. area, regresses sales prices of FHA homes on an accessibility variable and on variables describing housing style and characteristics. In so doing, he is able to explain between 82% and 92% of the variance in sales price. In each sample, accessibility is important and highly significant.

Richard Muth [27] studies the population densities on the south side of Chicago. Population densities are a good proxy for land values for which good data is much more difficult to obtain. Production theory indicates that increasing the price of a factor of production where substitution is possible will result in "more intensive use of the factor." Increased population densities are one reflection of more intensive land use.

In the study (p. 216) Muth regresses net population density (population/square mile of land used for residential purposes in natural logs) on three accessibility measures.

Accessibility explains between 80% and 82% of the variance.

Mills [22] approaches the importance of accessibility from two additional angles. Using historical data of Chicago land values at 5 selected years from 1836 to 1928, he regresses land values on distance to the CBD and finds in log-log regressions that distance explains 46% to 83% of the variance in land values. Then, using data for Chicago in 1959 he regresses floor space per acre on distance. Again, floor space can be considered a proxy for land values, since the increased capital intensity manifested in higher floor space per acre would be a natural economic reaction to higher land values. In this case distance explains between 81% and 88% of the variance in the log-log regressions for various land uses.

Clearly the available empirical studies reaffirm the preeminent importance of accessibility to the CBD. This assumption is also important for other reasons. Accessibility is one of the few variables affecting land values that is both easily quantifiable and present in all cities. These characteristics are necessary in order for an urban model to be useful in a practical sense and to be widely applicable.

While accessibility is commonly measured by airline distance to the CBD for simplicity, there are many complications if accuracy is desired. Generally, traffic moves more slowly in denser areas; circuitous routes, often necessitated by irregular topography, impose more costs than direct routes; and proximity to high-speed arterial roads or transit lines reduces costs.

All of these factors will affect accessibility and, therefore, land values. Evidence of this is the usual star-shaped rather than circular lines of equal land value around the CBD due to high-speed arterial roads as in Figure 5-1.

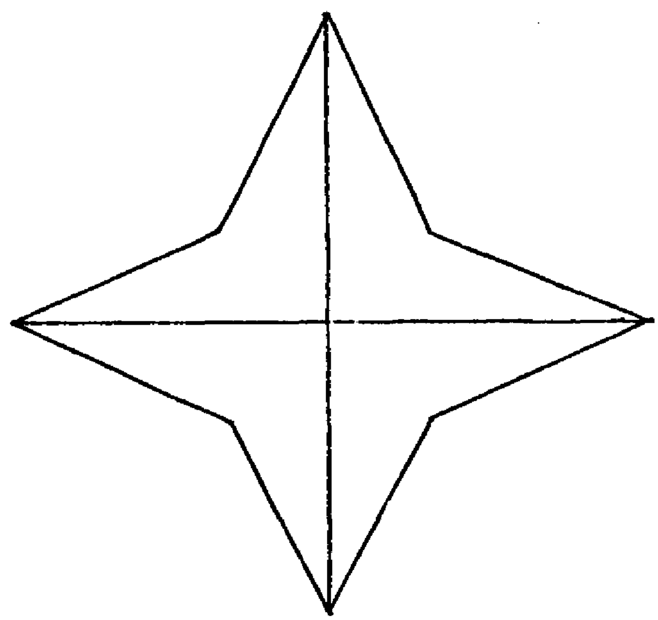

Figure 5-1. The star-shaped city

Hartwick and Hartwick [12] analyze residential development in this star-shaped city and show how accessibility is effected by arterial roads or subways. It is clear that, in theory, accessibility means economic distance, which includes the time, vehicle, and external costs of travel to the CBD.

An Aggregative Commuter Model

The model here is a sophisticated VTE model. As in a VTE model, the city's characteristics are determined primarily on the supply side, while the demand side is somewhat underdeveloped.

The model assumes that there is a CBD which employs a fraction ρ of the workers living in the city. This fraction employed in the CBD is exogenously determined. The workers are part of CBD worker units (WU). Each CBD worker unit requires on the average $\frac{1-\rho}{\rho}$ other workers for support - for example, employees in retail stores, barber shops, etc.

Each WU demands a unit of housing, which uses land, and each CBD worker commutes to work on a transportation mode which may or may not use land depending on factor prices. The interaction of the demand for land from the housing and transportation sectors with the available supply at each location determines land rents.

The model is divided, for expository purposes, as follows: the housing sector, supply and demand for CBD workers, and the transportation sectors.

The Housing Sector

In the housing sector it is assumed that factor and output markets are competitive. The prices of capital and labor are exogenously determined while land rents are endogenous. A Cobb Douglas production function for housing is assumed.

$$X_1(u) = \overline{A}_1 L_1(u)^{\alpha_1} N_1(u)^{\beta_1} K_1(u)^{\gamma_1} \tag{5-1}$$

$$\alpha_1 + \beta_1 + \gamma_1 = 1$$

$X_1(u)$ - housing output in an annulus of width du at a distance of u miles from the CBD (equal to CBD workers in the annulus)

$L_1(u)$ - land used for housing production in this annulus

$N_1(u)$ - labor used in housing production in this annulus

$K_1(u)$ - capital used in housing production in this annulus.

Housing, here, is defined to include not only the actual living area of the CBD worker but also a share of the local roads and retail trade establishments which he uses and the housing of the supporting workers.

The effective wage a CBD worker receives is the money wage (w) minus the cost of commuting from a location u miles from the CBD (T(u)), that is [w-T(u)] is the effective wage. Since factor markets are competitive we have the following marginal productivity conditions:

$$R(u) = p_1(u)\alpha_1 \frac{X_1(u)}{L_1(u)} \tag{5-2}$$

$$(5\text{-}3) \qquad w - T(u) = p_1(u)\beta_1 \frac{X_1(u)}{N_1(u)}$$

$$(5\text{-}4) \qquad r = p_1(u)\gamma_1 \frac{X_1(u)}{K_1(u)}$$

where $R(u)$ is the land rent at u

$p_1(u)$ the price of housing at u

r is the rental rate of capital.

These equations say that factor prices must equal the marginal physical product of the factor times the price of output. Equation (5-3) uses $(w-T(u))$ as the price of labor at u since it is not necessary to pay the CBD wage at u. A worker who does not have to commute should be willing to work for less than a commuter.

Substituting (5-2), (5-3), (5-4) into (5-1) solving for $p_1(u)$ and adding the assumption that workers spend all their wages gives

$$(5\text{-}5) \qquad p_1(u) = A_1 R(u)^{\left(\frac{\alpha_1}{\alpha_1+\gamma_1}\right)}$$

where $A_1 = \left[\left[\bar{A}_1 \alpha_1^{\alpha_1} \beta_1^{\beta_1} \gamma_1^{\gamma_1}\right]^{-1} r^{\gamma_1}\right]^{\frac{1}{\alpha_1+\gamma_1}}$

Equation (5-5) says that the effective wage rate equals the cost of producing housing for a CBD worker. If we differentiate (5-5) with respect to u, assuming w constant:

$$p_1'(u) = \alpha A_1 R(u)^{\alpha-1} R'(u) \tag{5-6}$$

where $\alpha = \dfrac{\alpha_1}{\alpha_1+\gamma_1}$.

For locational equilibrium to prevail among workers, reductions in the price of housing must offset the increases in transportation cost as one moves away from the CBD. That is

$$p_1'(u) = -T'(u) \tag{5-7}$$

or substituting

$$-T'(u) = \alpha A_1 R(u)^{\alpha-1} R'(u) \tag{5-8}$$

This is similar to the condition in the VTE model that $R'(u) = -t$. Condition (5-7) is much more general since it allows transportation costs to vary, rather than be constant, as one moves away from the CBD. It permits the offsetting rents to occur not only in residential land prices, as in the VTE model, but also in any aspect of 'housing' production. For example, food prices are often lower in the suburbs and these lower prices are an incentive for people to live there. (5-8) considers the possible variance in these other production costs.

Demand for CBD Workers

The employer of CBD workers will be interested primarily in the money wage, w, he must pay. If he is competing with national firms he will be able to offer more than the prevailing national wage only if there are offsetting cost reducing advantages to location in the particular city. For example, the need for a deep water harbor may tie many firms to a few locations with such a harbor. The greater labor's share of output is and the fewer attractions there are to the particular city, then the greater the elasticity of demand will be.

This demand is given in (5-9).

(5-9) $$D_{CBDW} = F(w)$$

and it can be expected to be relatively elastic. In the long run labor demand will be affected not only by the expansion and contraction of output by existing firms, but also by the migration of firms into or out of the city.

Supply of CBD Workers

In the short run, before migration can occur, the supply of CBD workers is limited by the size of the work force in the city. CBD employment can be increased only by out-bidding the other sectors for workers.

In the long run, any deviation in the local standard of living (real income) from the national will attract or repulse workers to or from the area. The level of money wages and local prices will determine real incomes.

In the model, the two important prices are the prices of housing and of transportation. For workers to be in locational equilibrium within the city, the sum of housing and transportation cost must be constant. That is

$$(5\text{-}10) \qquad p_1(u) + T(u) = K$$

$$\text{for all } u \leq k$$

where K is a constant and k is the radius of the city. Therefore, K can be taken as a measure of the housing plus transport cost in the city. The difference between money wages and K is a measure of the surplus income available to be spent on items other than 'housing' and transportation.[3] We call this surplus, w-K, disposable earnings (DE).

Workers will be attracted to the area if the local disposable earnings, DE, are greater than those prevailing in the rest of the country, DE*, which will equal zero in equilibrium.[4] That is,

$$(5\text{-}10) \qquad S_W = g(DE - DE^*) = g(DE)$$

where S_W is the supply of workers. If DE = DE* = 0, then S_W reduces to the number of workers already in the city. This supply, of course, will be more elastic, the longer

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[3]Data presented by Hoover [14] indicate that of the four most important items in the family budget of an urban worker, housing and transportation exhibit the greatest variance from city to city. Therefore, netting these two items out of the budget gives a better measure of the standard of living than money wages.

[4]This is true because 'housing' is defined to be the average consumption bundle, excepting transportation.

the time period under consideration.

If the number of CBD workers are a constant proportion, ρ, of the total number of workers we have

$$S_{CBDW} = \rho g(DE)$$

If the city is, initially, in equilibrium, the local DE must equal those prevailing nationally. Any reduction in housing or transportation costs will increase (decrease) disposable earnings and attract (repel) workers to (from) the city. In the very long run workers will continue to migrate until disposable earnings are again equalized between the city and the rest of the country.

Equilibrium occurs when supply and demand are equal, that is when

$$f(w) = \rho g(DE) \tag{5-12}$$

which can be written as

$$f(w) = \rho g(w-K) \tag{5-13}$$

Transportation Sector

Equation (5-7) specifies the rent structure of the city once the nature of transport costs are given. To complete this, we need to describe the production of transportation.

At any point in the city u miles from the CBD, two types of transportation service can be produced, a land intensive one, roads, $(X_2(u))$, and a land economizing one, subways, $(X_3(u))$. In both modes, factor substitution is possible according to the following production functions:

$$X_2(u) = \overline{A}_2 L_2(u)^{\alpha_2} N_2(u)^{\beta_2} K_2(u)^{\gamma_2} \tag{5-14a}$$

$$\alpha_2 + \beta_2 + \gamma_2 = 1$$

$$X_3(u) = \overline{A}_3 N_3(u)^{\beta_3} K_3(u)^{\gamma_3} \tag{5-14b}$$

$$\beta_3 + \gamma_3 = 1$$

As with CBD worker production, there are marginal conditions corresponding to each.

$$p_2(u)\alpha_2 \frac{X_2(u)}{L_2(u)} = R(u) \tag{5-15a}$$

$$p_2(u)\beta_2 \frac{X_2(u)}{N_2(u)} = \overline{W} \tag{5-15b}$$

$$p_3(u)\beta_3 \frac{X_3(u)}{N_3(u)} = \overline{W} \tag{5-16b}$$

$$p_2(u)\gamma_2 \frac{X_2(u)}{K_2(u)} = r \tag{5-15c}$$

$$p_3(u)\gamma_3 \frac{X_3(u)}{K_3(u)} = r \tag{5-16c}$$

where $\overline{W}$ is the valuation of time spent commuting.

It is assumed that the only labor input into transportation is the labor of commuter himself.

$\overline{W}$ is likely to differ from the wage rate but be dependent on it. The following simple relationship is assumed:

(5-17) $$\overline{W} = \delta W$$

where δ is a constant.

Equations (5-15) and (5-16) and their marginal conditions yield marginal cost prices for transportation of

(5-18) $$p_2(u) = A_2 R(u)^{\alpha_2} \qquad A_2 = \frac{\overline{W}^{\beta_2} r^{\gamma_2}}{\overline{A}_2 \alpha_2^{\alpha_2} \beta_2^{\beta_2} \gamma_2^{\gamma_2}}$$

(5-19) $$p_3(u) = A_3 \qquad A_3 = \frac{\overline{W}^{\beta_3} r^{\gamma_3}}{\overline{A}_3 \beta_3^{\beta_3} \gamma_3^{\gamma_3}}$$

The total cost of commuting from a point u miles from the CBD, then, is the integral of marginal transport costs ($p_2(u)$ or $p_3(u)$) from the CBD to u. If the commuter uses roads to travel to the CBD his cost will be

(5-20) $$T(u) = \int_0^u p_2(u)du = \int_0^u A_2 R(u)^{\alpha_2} du$$

If he uses the subway his cost is

(5-21) $$T(u) = \int_0^u A_3 du = A_3 u$$

In the case where both modes are used on the way to the CBD, subways will be used close to the CBD and roads further out because if subways are less costly, they will be so primarily because they economize on land. The land economizing nature of subway makes them most useful where land prices are very high, close to the CBD. If switching of modes is done at some point u* total cost will be

$$T(u) = \int_{0}^{u^*} A_3 du + \int_{u^*}^{u} A_2 R(u)^{\alpha_2} du$$

$$T(u) = A_3 u^* + \int_{u^*}^{u} A_2 R(u)^{\alpha_2} du \tag{5-22}$$

Equations (5-20) and (5-21) are the special cases of (5-22) where u* is zero and where u is less than u*, respectively.

To assure that no more land than that which is available is used:

$$L_1(u) + L_2(u) = \theta u \tag{5-23}$$

where θ is a constant between θ and 2π.

A θ of less than 2π indicates that there is a pie slice missing from a circle of land. For example, a waterfront city would have θ equal π since 180°, or half the pie, would be missing (see the appendix for a discussion of CBD location in non-circular cities).

One additional condition that

$$R(k) = R_a \tag{5-24}$$

says that the price of land at the edge of the city, (k), equals the price of land for agriculture, from which it must be bid away.

Demand for Transportation

The total demand for transportation services in an annulus u miles from the CBD depends on how many commuters must pass through the annulus in their journey to work.

Since any CBD worker living beyond u must pass through u we have

$$\text{(5-25)} \qquad \text{Demand at } u = \int_u^k X_1(u')du'$$

The transportation authority then supplies as much as is needed, at the marginal cost price. That is,

$$\text{(5-26)} \qquad X_2(u) + X_3(u) = \int_u^k X_1(u')du'$$

The Structure of the City

Equation (5-8), the condition for locational equilibrium of CBD workers, determines the rent structure of the city once transport costs are known. By differentiating (5-20), (5-21) or (5-22) with respect to u and substituting into (5-8), differential equations in R(u) are obtained which can be solved for a land rent-distance function (R(u) as a function of u).

Combining (5-20) and (5-8) gives

$$(5\text{-}27) \qquad -A_2R(u)^{\alpha_2} = \alpha A_1R(u)^{\alpha-1}R'(u)$$

which can be rewritten as

$$(5\text{-}28) \qquad R'(u) + \frac{A_2}{A_1\alpha} R(u)^{1+\alpha_2-\alpha} = 0$$

and solved to obtain

$$(5\text{-}29) \qquad R(u) = \left[C' - \frac{A_2}{\alpha A_1} (\alpha-\alpha_2)u\right]^{1/(\alpha-\alpha_2)}$$

where C' is a constant which depends on initial conditions.

If (5-24) is used as the initial condition, (5-29) becomes

$$(5\text{-}30) \quad R(u) = \left[Ra^{\alpha-\alpha_2} + \frac{A_2}{\alpha A_1} (\alpha-\alpha_2)k - \frac{A_2}{\alpha A_1} (\alpha-\alpha_2)u\right]^{1/(\alpha-\alpha_2)}$$

This is of the form

$$(5\text{-}31) \qquad R(u) = \left[c_1 - c_2u\right]^{c_3}$$

Equation (5-31) is analogous to Mills [21] (p. 244, Equation (14)) and results when there is land substitution in both production and transportation.

Combining Equations (5-21) and (5-8) gives

$$(5\text{-}32) \qquad -A_3 = \alpha A_1R(u)^{\alpha-1}R'(u)$$

which can be rewritten and solved to obtain

$$(5\text{-}33) \qquad R(u) = \left[C'' - \frac{A_3}{A_1} u\right]^{1/\alpha}$$

Again using (5-24) as an initial condition we have

$$(5\text{-}34) \qquad R(u) = \left[Ra^{\alpha} + \frac{A_3}{A_1} k - \frac{A_3}{A_1} u\right]^{1/\alpha}$$

This is the case of a city in which there is land substitution possible in production, but none in transportation because no land is needed in its production.[5]

The two cases, one of a city with a land-intensive transportation system and one of a city with a land-economizing one, offer comparison to the expressway and subway transit issues. Suburbanization and the related problem of the fiscal solvency of cities are part of this issue.

Suburbanization

Proponents of subway mass transit are at times given to lauding mass transit for its ability to slow or reverse the trend towards suburbanization. The urban model given here of course says nothing about the relative merits of centralized and decentralized cities. However, from the

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[5]Equation (5-34) reduces to a linear function if $\alpha=1$, which would be the case if housing used only land so that no substitution among factors would be possible. The VTE model also arrives at a linear rent-distance function because of a similar assumption of fixed coefficients in housing and transportation.

viewpoint of city governments suburbanization generally means that there is a migration of population into other jurisdictions. This migration tends to reduce land values in the city relative to outlying areas, thus decreasing the tax base of the city and exacerbating fiscal problems.

In terms of the ability of the city to raise money, suburbanization is a potential evil. Whether or not subway construction is the best answer is, again, outside the scope of this urban model.

Concerning suburbanization it is observed that a land intensive transportation system can influence the structure of the city four ways, in the context of the above model.

First, for a given population, a city with a land intensive road transportation system will require more land than a city with a subway system, other things being equal. This is so because the land used for roads could have been used to house some of the population. If the suburbs are in other political units, then the city will in fact have a smaller taxable population. Of course, it will also have fewer demands on its revenues. The road-based city in this example is suburbanized more than the subway-based city. If we define degree of suburbanization as the proportion of people living beyond a given distance from the CBD, then the road city is more suburbanized. Also, if we define suburbanization as the slope of the rent distance or density (net) distance function, the road city is more suburbanized.

Equation (5-7) says that the rate of change of housing prices must equal marginal transportation costs. If we compare a situation where the roads and subways are equally desirable, i.e., where the sum of housing and transport costs, K, are equal, the two price gradients will start from the same level, K, at u = 0, but the subway city's gradient will decline faster because the land area of the subway city is smaller. Rents and net population densities will show a similar relationship, i.e., the road city more suburbanized.

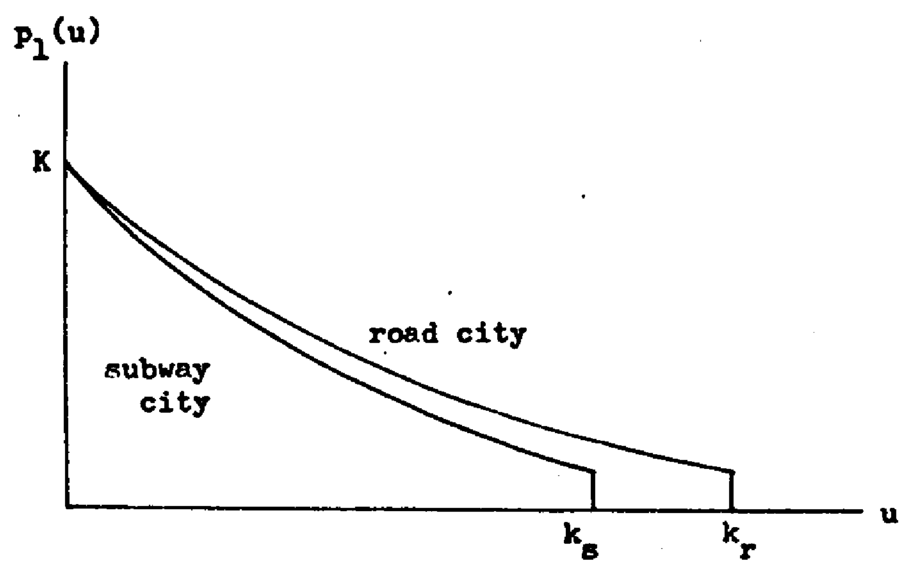

Figure 5-2. Housing prices in the road and subway cities

The second way city structure is influenced is through the effect on intercity migration. In Equation (5-9), it was postulated that workers decide which city to work in on the basis of the wage rate minus the anticipated transport plus housing expense they can anticipate in a particular city. It is common, for example, to hear government employees in Washington, D. C. talk about how much more they would have to earn in New York City to be comparably well-off in terms of housing and commuting time. In the model, w-(K) is taken as a measure of the surplus over housing plus transport cost one could expect to gain in the city. If w-K is equal in the two cities, workers, if given a choice between two such cities, would not migrate. Similarly, differences in K's between cities must be offset by differences in the money wage rate.

One of the things assumed constant here is marginal cost pricing policies for both modes. It is sometimes felt that if there is congestion, or if land use taxes are not charged for the roads, urban road users are charged less than the marginal cost price. If so, the effective wage would be higher in the road city and migration would result in the road city gaining population relative to the subway city.

A third way that road transportation influences city structure is through its effect on marginal transport costs. The subway city has constant marginal transport costs while the road city has marginal transport costs which increase as one near the CBD. The result is that the rent gradient in a road city will diverge from a straight line more than that of the subway city as in Figure 5-3.

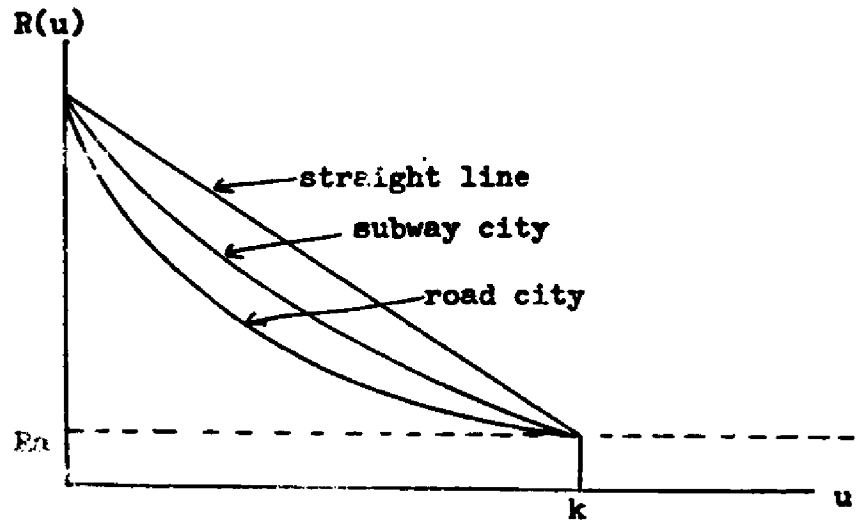

Figure 5-3. Curvature of the rent gradient

Both Equations (5-30) and (5-34), the rent gradients for the two types of cities, are of the form of Equation (5-31). In such a function the exponent, C_3, determines the extent to which the rent gradient diverges from the linear. A value of 1 for C_3 makes the function linear, while values greater than 1 make the function increasingly convex to the origin. Now, then, the road-city will be more convex to the origin if $1/\alpha-\alpha_2$ is greater than $1/\alpha$ and it will be as long as $\alpha_2 > 0$. α_2 is the share of land in road production which is assumed to be greater than zero; therefore, we can expect the rent gradient of the road-city to be more convex to the origin than that of the subway-city.

The fourth way the transportation system may effect the city's structure is through its effect on the proportion of the population working in the CBD. All variables other than factor prices effecting this proportion are assumed constant in the above model. The transportation system seems to be an important influence on this proportion. Muth [27] has found in empirical estimates that the type of transportation system is an important determinant of the density of population gradient. Mills [22] presents estimates which indicate a correlation between the density of population gradient and the gradient of employment for various sectors. Thus, we would expect cities with extensive freeway systems to have a different proportion of total employment in the CBD when compared to cities with rail or subway transport (compare Los Angeles and Chicago, for example).

A possible explanation of the phenomenon may be the fact that, while roads are able to transport both goods and people equally well, subways are much better suited to moving people than they are to moving goods. This difficulty of goods transport in a subway city forces firms that exchange goods to locate more closely together than they would have to in a road city.

The proportion working in the CBD can also be influenced by city size. Studies of the economic base of a region indicate that employment in geographically oriented industries as a percentage of total employment falls as city size increases. Czamanski [8] presents estimates that imply that geographically oriented employment as a percent of total employment is 36% in cities of 50-100 thousand and 5% in cities of over 800 thousand. Of course, CBD employment and geographically oriented employment are not identical, but we might expect a similar trend.

The Hybrid City

The two earlier cases, of a subway-city and road-city, are limiting cases of the city with both roads and subways in its transport system - the hybrid city. The subway city is an unrealistic characterization of a modern city since in any city with a subway system there are also roads that carry commuters to the CBD. This section begins to correct the earlier shortcoming.

Returning to the model and combining (5-22) and (5-8) we have

$$-A_3 - A_2 R(u)^{\alpha_2} = \alpha A_1 R(u)^{\alpha-1} R'(u) \qquad (5\text{-}35)$$

and

$$R'(u) = \frac{-A_3}{\alpha A_1} R(u)^{1-\alpha} - \frac{A_2}{\alpha A_1} R(u)^{1-\alpha+\alpha_2} \qquad (5\text{-}36)$$

Solution of (5-36) is easiest if we first apply economic intuition to divide the city into two rings. Since all CBD workers are assumed homogeneous, once a transport mode is preferred (cheaper) by one worker it is preferred by all. Thus all workers will switch modes at the same distance from the CBD dividing the city into an inner and outer ring. This distance is called u* here.

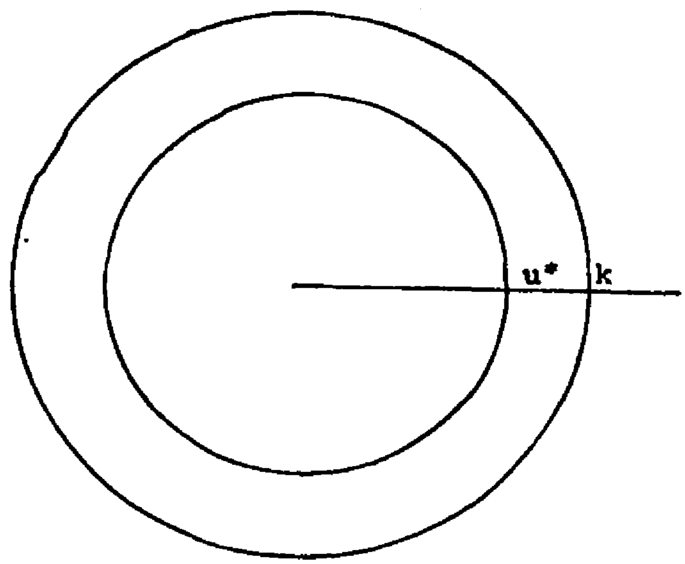

Figure 5-4. The hybrid city

The question remains, which mode is the inner and which the outer? If the subway has a cost advantage anywhere in the city it will have an advantage close to the

CBD.[6] It is there that rents will be highest, making the land economizing nature of subways important.

For the time being, an assumption will be made that there are only roads in the outer ring and only subways in the inner. We can then drop the first term of (5-35), reducing it to Equation (5-28). The solution of (5-28), after application of the appropriate initial condition, was (5-30) which now also describes the rent structure of the hybrid city from u* to k.

$$(5\text{-}30)\quad R(u) = \left[Ra^{\alpha-\alpha_2} + \frac{A_2}{\alpha A_1}(\alpha-\alpha_2)k - \frac{A_2}{\alpha A_1}(\alpha-\alpha_2)u\right]^{1/(\alpha-\alpha_2)}$$

where $u^* < u < k$.

In the inner ring there are no roads therefore we can drop the second term of (5-35), reducing it to (5-32). The solution of (5-32) was (5-33); but now we must apply a new initial condition because rents at u* where the subway ends will be greater than the agricultural rents Ra assumed for the subway city. The rents at u* are determined by (5-30). Substituting u* into (5-30),

[6]The only price that varies over the city is land rents, since capital and labor prices to the transport sector are constant. In the model the marginal cost of subway transport is constant while the marginal cost of roads increases with land rents. Therefore relative prices, $p_3(u)/p_2(u)$, move in favor of subways as u declines.

$$(5\text{-}37)\quad R(u^*) = \left[Ra^{\alpha-\alpha_2} + \frac{A_2}{\alpha A_1}(\alpha-\alpha_2)k - \frac{A_2}{\alpha A_1}(\alpha-\alpha_2)u^* \right]^{1/\alpha-\alpha_2}$$

$$= R_{u^*}$$

Now using (5-37) as the initial condition in (5-33) we have

$$(5\text{-}38)\qquad R(u) = \left[R_{u^*} + \frac{A_3}{A_1}u^* - \frac{A_3}{A_1}u \right]^{1/\alpha}$$

where $0 < u < u^*$. Equations (5-30) and (5-38) now define the rent structure of the hybrid city.

It is interesting to compare the hybrid city with a road-city. The addition of a subway to a road-city results in rents being lower than they otherwise would in the ring served by the subway.

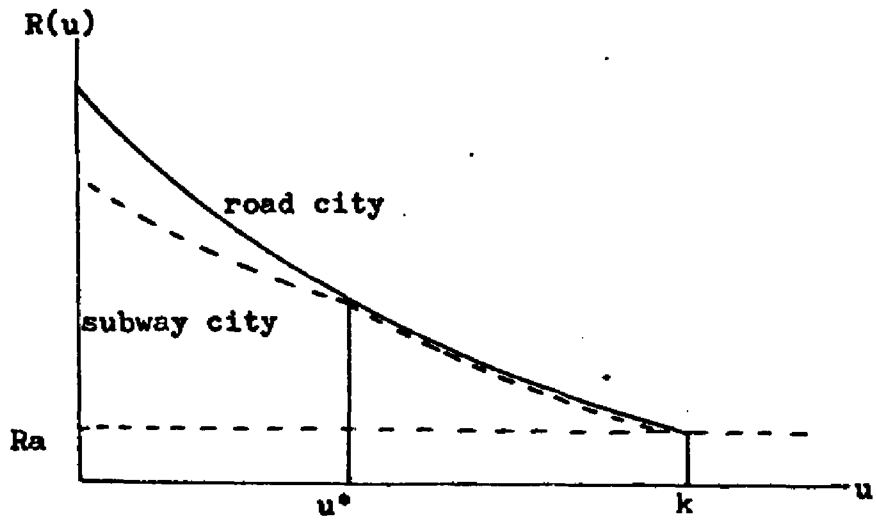

Figure 5-5. The effect of a subway on land rents

Whether or not the hybrid city is larger or smaller than the comparable road city depends on the relative magnitudes of a variety of effects. First the subway will make it possible for some land to be transferred from road use to housing in the subway ring. This will tend to make the city smaller as in Figure 5-6.

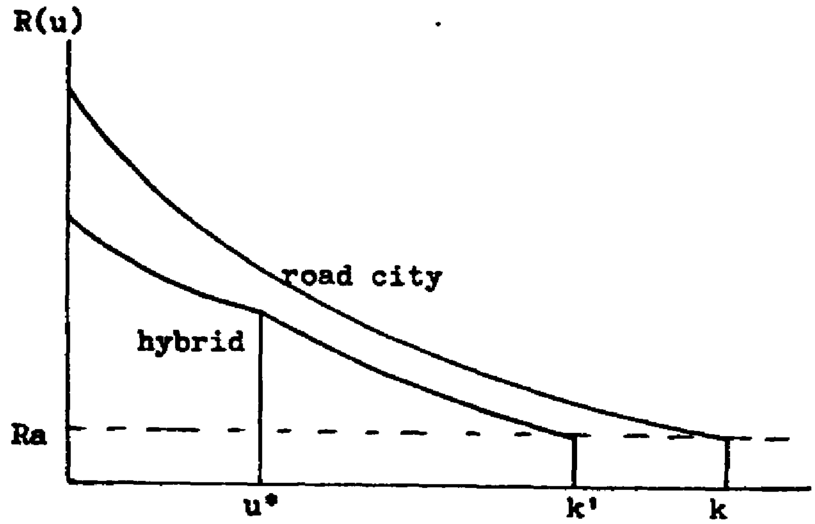

Figure 5-6. The effect of transferring land from transportation to housing production

Second, the reduction in land rents will make housing production use more land per unit. This will tend to increase the size of the city as in Figure 5-7.

And third, the reduction in transportation costs, T(u), will increase the effective wage, w-T(u), and attract new workers to the city. Again, this will tend to increase the size of the city.

In actual practice land is rarely transferred from

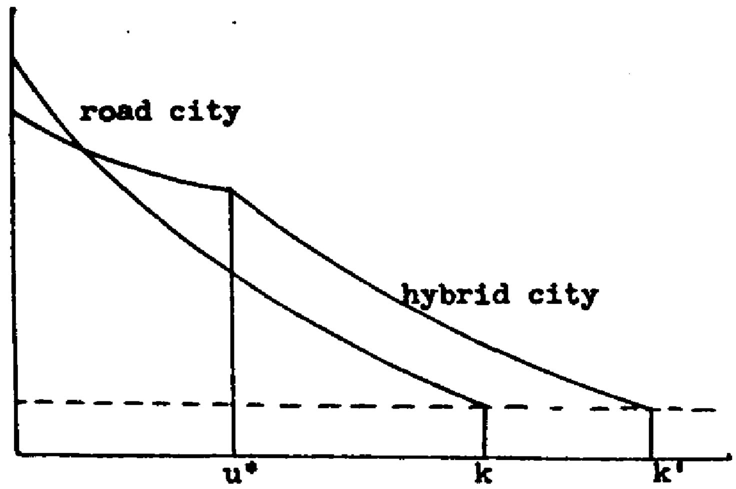

Figure 5-7. The effect of factor substitution

road use to housing. Therefore, an increase in city size is the most likely outcome. The city, then, will be more suburbanized in terms of both the steepness of the rent and density gradients and the number of people living beyond any given distance.

This is an important result since it implies that a city planner who attempts to slow the rate of suburbanization by constructing a subway is likely to be frustrated in the effort, and, in fact, achieve the opposite result. The above result holds, however, only for the comparison of the existing city and the city with an added subway. If the choice is between constructing a subway and improving the road system, perhaps by construction of a freeway, the road improvement is likely to be more suburbanizing than a new subway because all three effects cited above, rather than two, will tend to increase city size. In addition, the freeway system may reduce the proportion of employment found in the CBD more than the subway would, and thereby increase suburbanization even more (see the earlier discussion).

The Switching Point

CBD workers will switch modes whenever the relative marginal cost prices turn against the mode they are traveling on.[7] In the model the marginal cost of subway transport is constant while that of road transport rises monotonically as one moves towards the CBD. There will, therefore, be, at most, one switching point, u*.

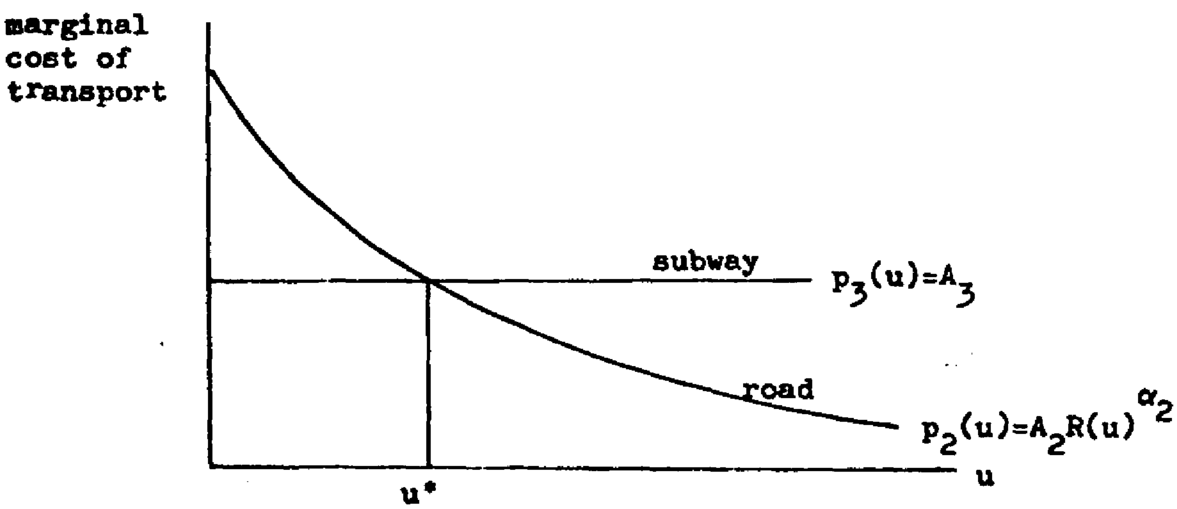

Figure 5-8. Marginal cost prices of transportation

[7]Switching costs are assumed to be zero here.

Equating the marginal cost prices of the two modes at u* gives (from (5-18) and (5-19))

$$A_3 = A_2 R(u^*)^{\alpha_2} \tag{5-39}$$

Substituting from (5-30) into (5-39) yields

$$u^* = k - \frac{\left(\frac{A_3}{A_2}\right)^{\left(\frac{\alpha-\alpha_2}{\alpha_2}\right)} - Ra^{(\alpha-\alpha_2)}}{\frac{A_2}{\alpha A_1}(\alpha-\alpha_2)} \tag{5-40}$$

and for (5-40) to have economic meaning u* must be within the city limits. It will be if the second term on the right hand side is negative and smaller in absolute value than k. Otherwise there is only one mode in the city, roads if (5-40) is less than or equal to zero, and subways if (5-40) is greater than or equal to k.

As might be expected, the factors which determine where the switching point is include: first, the radius of the city k. The larger the city is, the larger will be the area of the subway ring in the optimal configuration. Second, the production parameters for the two modes are important. Both the scale parameters and the factor elasticities enter into the relation. If technical progress increases the scale parameter for subway, $\overline{A}_3$, faster than for roads, $\overline{A}_2$, the ratio A_3/A_2 will fall, and u* will increase (recall the definition (A_3 and A_2)).

Third, an increase in the price of land outside the city, Ra, will increase the size of the subway ring (as long as $\alpha > \alpha_2$).

The above assumes that there are no switching costs. The effect of switching costs on the hybrid city is discussed next.

Switching Costs

It would be unusual if commuters could switch modes without bearing some cost. Normally, there is considerable cost in terms of time and inconvenience involved in switching modes. These costs are well known to commuters and weighed heavily in their transportation decisions. What effect will switching costs have on the desirability of a subway system?

In the presence of switching costs rational commuters will require not only that marginal transport costs be less, but also that the cumulative savings be greater than the switching cost. If the switching cost is denoted by Q, then these two conditions can be expressed as

$$p_3(u^*) \le p_2(u^*) \tag{5-41}$$

and

$$\int_0^{u^*} p_2(u)du - \int_0^{u^*} p_3(u)du > Q \tag{5-42}$$

As long as (5-42) holds, commuters will continue to switch at the same u* that existed without the switching cost, and in the absence of economies of scale in subway production, the subway should be built as long as savings can be realized for some commuters.

What happens when switching costs are so high that they make it uneconomic for anyone to switch once they have started on a given mode? In such a situation, commuters living beyond u* (which is no longer a switching point but only the radius of the subway ring) will use road transport the entire distance to the CBD. Those living inside u* will continue to take the subway.

There will be only roads beyond u*, while inside u* there will be both roads and subways. An amount of land sufficient to transport all those living beyond u* will be withdrawn from housing production in the inner ring. Therefore, the city will be larger than it would be if the switching costs were low.

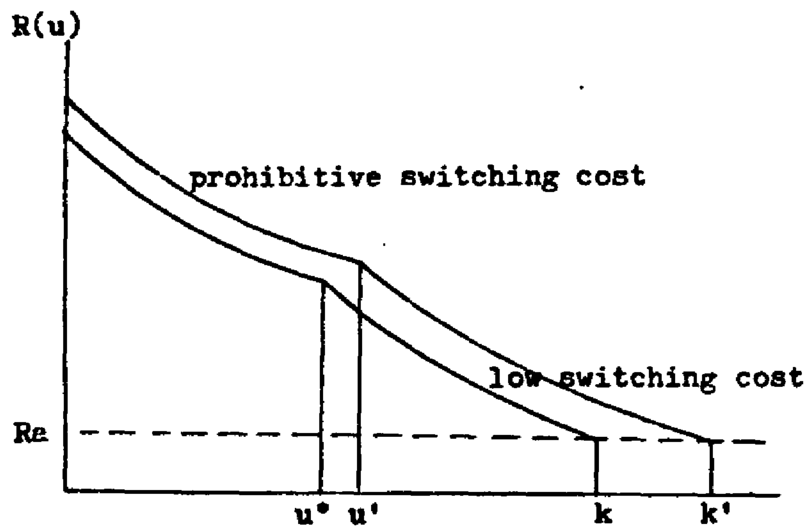

Figure 5-9. The effect of high switching costs

Parking Costs

In any large city, parking becomes increasingly expensive as one travels towards the CBD. In most cities the increased costs are reflected in higher prices for off-street parking; but even in cities that still permit on-street parking in downtown areas, there are higher parking costs reflected in the increased amount of time it may take to find an open space. In the latter case the pricing system is very imperfect. In either case the realization that parking costs will be higher closer to the CBD is an important consideration for commuters. High parking costs will encourage subway use.

Consider the manner that parking costs enter into the commuting decision. Suppose parking is produced, using land only, according to the following simple production function.[8]

(5-43) $$X_4(u) = \overline{A}_4 L_4(u)$$

The marginal cost price of parking spaces will be

(5-44) $$p_4(u) = A_4 R(u)$$

where $A_4 = 1 / \overline{A}_4$.

[8] Parking space production generally involves more than land. Meyer, Kain and Wohl [18] list construction, and operating and maintenance in addition to land costs. Construction costs in particular are important for multi-story garages. A more general function which takes account of these other costs would be $X_4(u) = \overline{A}_4 L_4(u)^{\alpha_4} N_4(u)^{\beta_4} K_4(u)^{\gamma_4}$. (5-43) is chosen for simplicity.

The parking cost incurred in traveling through an annulus of width du on the way to the CBD is the difference between parking cost at u and at u-du, or

$$A_4R'(u)du$$

and the total increase in parking cost incurred in traveling from u to the CBD will be

$$\int_u^0 A_4R'(u)du = A_4(R(0) - R(u)) \tag{5-45}$$

The condition for taking the subway now becomes that the savings in marginal costs on the subway be greater than the switching costs minus the increased parking costs, i.e.,

$$\int_0^{u^*} (p_3(u) - p_2(u))du > Q - A_4(R(0) - R(u^*)) \tag{5-46}$$

The parking costs offset some of the switching costs and therefore encourage use of the subway by commuters living beyond u^*. A policy which increases parking costs, either by taxing parking or by eliminating the subsidy inherent in allowing low cost or free on-street parking, will increase subway use. The latter, eliminating the subsidy, promotes a more efficient use of the transportation system and should increase the economic well-being of a city.

Increasing Returns in Subway Production

One weakness of the subway transportation production function is the assumption of constant returns. There seem to be substantial economies of scale in rail transport. Meyer, Kain and Wohl [18], for example, estimate a linear total cost function for rail transport which exhibits falling average cost in response to both track miles and passenger volume.

We will consider one type of scale economy here - that which occurs in response to an increase in the number of track miles, i.e., the size of the system. Suppose that instead of the production function specified for subways in (5-16) we allow output to vary with changes in the size of the system which we measure by u*, the radius of the subway ring.

Borrowing from the technical progress functions used by Solow [31] and others, we can make the scale parameter of (5-16) a function of system size.

$$(5\text{-}47) \qquad X_3(u) = \bar{A}_3(u^*)N_3(u)^{\beta_3}K_3(u)^{\gamma_3} \qquad \beta_3 + \gamma_3 = 1$$

where $\bar{A}_3(u^*)$ is an increasing function of u*.

The marginal cost price is then

$$(5\text{-}48) \qquad p_3(u) = A_3(u^*)$$

where $A_3(u^*)$ is a decreasing function of u*.

Economies of scale will affect both the decision to build the subway and the switching point or radius of the subway ring, as well as making small systems less beneficial and large ones more so.

Determination of the city stfucture becomes a two step recursive procedure. First the switching point is fixed by comparing the relative marginal costs of the two modes. (Compare Figures 5-8 and 5-10).

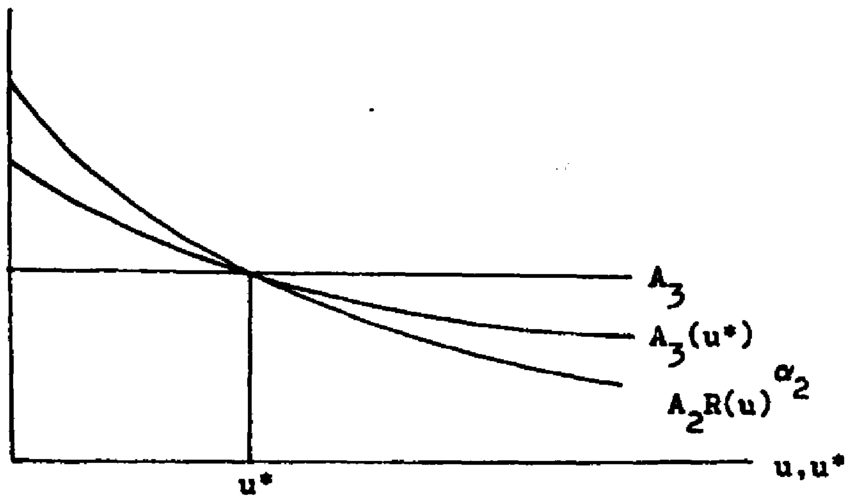

Figure 5-10. Marginal transport cost and increasing returns in subway production

Once the point u* is determined, the value of $A_3(u^*)$ becomes a constant again, A_3 in Figure 5-10, and can be used to determine the rental structure of the city.

For small u*, A_3 will be larger and, as determined earlier from (5-40), a larger A_3 implies a smaller efficient u* for the city. Similarly, in larger cities where u* would be larger, the efficient u* will be larger than it would in the constant returns case.

Definition of Symbols

t	transport cost per passenger mile
$R(u)$	land rent at u
w	distance to the CBD
CBD	central business district
ρ	fraction of workers employed in the CBD
$X_1(u)$	housing output at u
$L_1(u)$	land used in housing production at u
$N_1(u)$	labor used in housing production at u
$K_1(u)$	capital used in housing production at u
$\bar{A}_1$	scale parameter in housing production function
α_1	elasticity of housing output with respect to land input
β_1	elasticity of housing output with respect to labor input
γ_1	elasticity of housing output with respect to capital input
W	daily wage rate
$T(u)$	cost of commuting to the CBD from u
$p_1(u)$	price of housing at u
K	sum of housing plus transport cost at k
k	radius of the city
DE	disposable earnings
$X_2(u)$	quantity of road transport at u
$L_2(u)$	land used in road output at u

$N_2(u)$	labor used in road output at u
$K_2(u)$	capital used in road output at u
$\bar{A}_2$	scale parameter in road transport production function
α_2	elasticity of road transport output with respect to land
β_2	elasticity of road transport output with respect to labor
γ_2	elasticity of road transport output with respect to capital
$X_3(u)$	quantity of subway transport at u
$N_3(u)$	labor used in subway transport output at u
$K_3(u)$	capital used in subway transport output at u
$\bar{A}_3$	scale parameter
β_3	elasticity of subway output with respect to labor
γ_3	elasticity of subway output with respect to capital
$p_2(u)$	marginal cost price of road travel at u
$p_3(u)$	marginal cost price of subway travel at u
$\bar{w}$	valuation of time spent traveling
C'	constant
C''	constant
c_1	constant
c_2	constant
c_3	constant
u^*	switching point between modes
Q	switching cost

$p_4(u)$ marginal parking cost at u

$X_4(u)$ parking output at u

APPENDIX TO CHAPTER 5

CENTRALITY IN AN URBAN MODEL

Models of urban structure often rely on either (1) the assumption that goods are produced anywhere but must be shipped to the city center, e.g., to a rail terminal or port, or (2) the assumption that all goods are produced in the central business district (CBD); see for example [17] or [20]. Either assumption is sufficient to derive the centrality of rents, densities, etc. observed in a city. Models employing assumption (1) are open to the criticism that they are unrealistic representations of modern cities because of the increased use of trucks to transport goods out of the city. Those employing assumption (2) do not explain why employers locate in the CBD. The purpose, here, is to show that central location is also important for firms that do not ship output through a port or rail head.

In general, firms must transport inputs, mainly raw materials and labor, to the factory; and transport outputs, final products, away from the factory to the ultimate consumers. Firms may be classified according to the origin of these inputs and the destination of the outputs as in Table 5-1. The nature of the economies of scale for the firm determine the market area for its inputs and outputs and, thus, its position in the table. There

TABLE 5-1

TYPES OF FIRMS

	Origin of Inputs	Destination of Outputs	Possible Examples
1)	outside city	outside city	steel mill
2)	whole city	outside city	offices
3)	locally in the city	outside city	small manufacturing
4)	outside	whole	wholesaler
5)	whole	whole	diaper service
6)	locally	whole	bread manufacturing
7)	outside	locally	newsstand
8)	whole	locally	lawyer
9)	locally	locally	barber shop

may, of course, be some overlap of classification.

A Simple Von Thünen Model

In a city where all communication with the outside world is through a port or rail head located in the CBD, firms of the type 1, 2, 3, 4 and 7 will be drawn to the CBD. For example, a type 1 firm will offer land rents which are highest at the CBD and fall off going toward the edge of the city. This is true because a firm should be indifferent between paying an additional dollar of rent or an additional dollar of transport cost. If there are fixed coefficients in production and zero land rents outside the city, the highest rents a firm would be willing to pay for a location u (this may not be the same as what the firm actually pays unless all markets are competitive) would be:

$$R(u) = Q_i(K_i - T_i(u)) + Q_o(K_o - T_o(u)) \tag{5-49}$$

where

$R(u)$ is rent offer

u is distance to CBD

K_i is transport cost per unit of input from the edge of the city to the CBD

K_o is transport cost per unit of output from the edge of the city to the CBD

$T_i(u)$ is transport cost per unit input from u to the CBD

Q_i is the quantity of input per square mile

Q_o is the quantity of output per square mile.

It is easy to see that truck transport unties this type 1 industry from the CBD. If inputs and outputs can be transported by truck, the firm can locate with impunity wherever rents are lowest. Similarly, firms of type 2, 3, 4, and 7 have their bindings loosened by the advent of truck transport.

At this point, it may be asked whether or not a city would be of uniform density if no firms were tied to a port or rail terminal and all transports were by truck. The answer to this question is negative. Even if central location no longer minimizes transport costs for type 1 firms, it will continue to do so for type 2, 4, 5, 6 and 8, i.e., for firms which must transport inputs or outputs over the whole city. The following model demonstrates the transport cost minimizing aspect of central location for these firms.

A Von Thünen-Weber Model

The model combines elements of both Von Thünen rent models and Weber market area models. It looks at firms whose market area is the whole city and then investigates what transportation cost advantages there are for the firm to locate in or near the CBD. These transport cost savings can then be translated into rent offers on the part of the firms, using the reasoning of the Von Thünen model.

The model starts with the purest case of a type 5 firm which transports both inputs and outputs over an entire city which is circular and uniformly populated. An example might be a diaper service which collects its inputs, soiled diapers, from customers spread uniformly throughout the city, processes the diapers, and returns clean diapers to the customers.

What rents will such a firm offer for a plant site? As before, this type firm will be willing to pay an amount for rent equal to the transport cost savings the firm accrues, if any, when it moves away from the edge of the city. That is, with fixed coefficients in production

$$R(u) = q(K - T(u))$$

where

$R(u)$ is rent offer at u

q is level of output per acre of plant site

K is total cost of transport (round trip) per unit at the edge of the city

$T(u)$ is total transport cost at a point u

$$u = \sqrt{x_o^2 + y_o^2}$$

where (x_o, y_o) are the cartesian coordinates of a location.

If transport cost is proportional to distance and equal to one, then K, the transport cost at a point $(\sqrt{r^2-y^2}, y)$ on the fringe of the city, will be

$$(5\text{-}50) \qquad K = \iint_{\text{disk}} \sqrt{(r-x)^2 + y^2}\, dydx$$

From symmetry all points on the circumference of the city will have equal K.

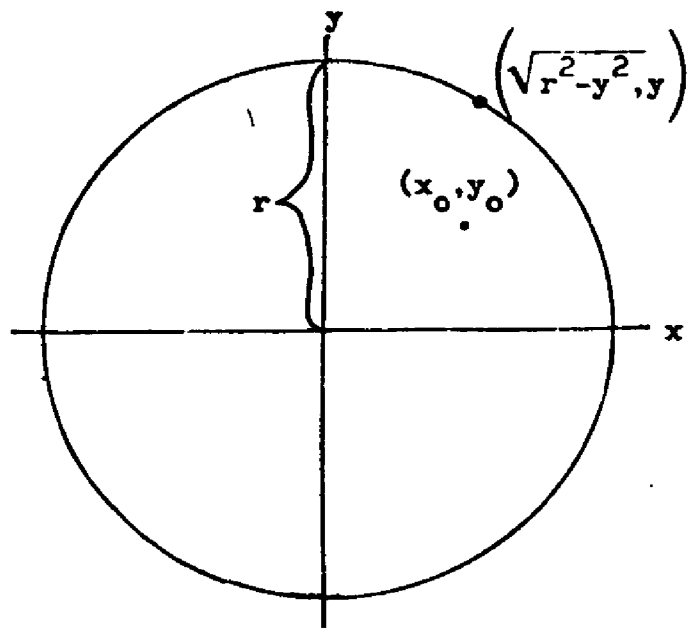

Figure 5-11. The coordinates of a location

The transport cost at a point (x_o,y_o) inside the city will be

$$(5\text{-}51) \quad T(u) = T\sqrt{x_o^2 + y_o^2} = \iint_{\text{disk}} \sqrt{(x-x_o)^2 + (y-y_o)^2}\, dydx$$

The minimum transport cost location and thus the maximum rent location would be the minimum of $T\left(\sqrt{x_o^2 + y_o^2}\right)$.

This extremum problem is analogous to the center of mass problem in physics. It is well known that the center of mass of a disk of uniform density is at the center of the disk (see the last section), and therefore the most desirable location in terms of transport cost will be at the center of the city.

It is possible to drop the uniform density assumption. Let $\rho(x,y)$ be a density of customers function for the firm and city in question, then the problem becomes the minimum of

$$(5\text{-}52) \quad T(x_o,y_o) = \int\int_{\text{disk}} \sqrt{(x-x_o)^2 + (y-y_o)^2}\, \rho(x,y)\,dy\,dx$$

For the commonly used density functions such as the ones that fall off linearly or exponentially with distance from the center, the above result remains.

Unfortunately, an explicit solution of (5-51) in terms of x_o and y_o is unavailable so that we cannot make quantitative judgments about the nature of the rent offer curve. However, qualitatively it seems that the attraction of the center of the city is less for this type 5 firm than it would be for a firm shipping to a rail head or port in the CBD. This is so because the type 5 firm, even though its transport costs are minimized, still bears some transport cost at the center, while the type 1 firm would bear no transport cost if located next to the terminal.

Ultimately, which type firm is able to bid highest for the center depends on the nature of the inputs and outputs and the cost of transporting them. However, the above indicates that rent offer functions are less steep in the type 5 case. The result is that as more industries are untied from the terminal by truck transport and as the attraction of central location is mainly for firms shipping or receiving from all points in the city, the rent and density gradients of the city, although still centrally focused, will be less steep. Thus, the empirically observed fall in density gradients which is often weakly explained by a presumed fall in transport cost, may also be a result of a change in the nature of the attraction to the CBD.

The center of mass approach gives an indication of where the highest rent-highest density district of a city should be. For a circular city it will, of course, always be the center of the circle no matter what the size of the city, but for a semicircular waterfront city, the transport cost minimizing location will not be at the center of the implied circle but rather inland somewhat. As a semicircular city grows, this point should move further inland as in Figure 5-12.

There is evidence of this phenomenon in many cities. For example, the business center of Philadelphia, which in earlier times was located around 4th or 5th Streets, has crawled one or two miles to the west as the city grew and is currently located in the 13th to 19th Street area.

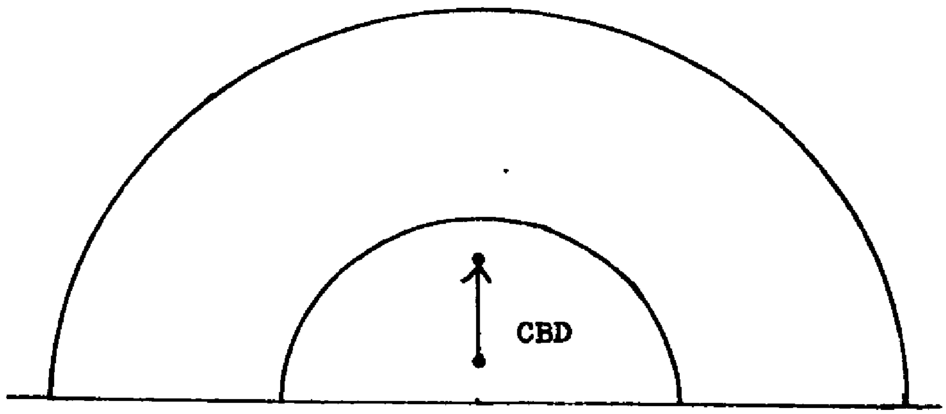

Figure 5-12. The central business district in a semicircular city

What type of firm would be most likely to locate at the center of the city? It will be the firms willing to offer the highest rent, that is, the firms that have the most to gain in transport cost savings from central location. The firms that meet this condition will be those with the most expensive inputs and outputs to transport. Generally we observe offices which employ professionals and white-collar workers intensively at the city center. People, and especially highly paid people, are one of the most expensive commodities to transport. Therefore, it is reasonable that those firms, businesses, and government offices, which employ these factors most are located centrally. This transportation cost explanation is an addition to other explanations - one of the more common of which is that of the "meeting" phenomenon.

In general, firms will not fit neatly into one of the nine classifications of Table 5-1, but will be a combination of more than one class. The more expensive the inputs and outputs are to transport, and the greater their importance in the firm's activity, the greater is the advantage of central location.

The Center of Mass

If we set the cost of transporting a unit of output one mile equal to one, then the total transport cost at the edge of the city of shipping the output uniformly throughout the city will be the sum of the cost of shipping $1/\pi r^2$ units to each square mile of the city. This is the double integral of the distance function over all points. In a city of radius r

$$(5\text{-}53) \qquad K = \int_{-r}^{+r} \int_{-\sqrt{r^2-x^2}}^{+\sqrt{r^2-x^2}} \sqrt{(r-x)^2 + y^2}\, dydx$$

At a point (x_o,y_o) inside the city the total transport cost is

$$(5\text{-}54) \qquad T(x_o,y_o) = \iint_{\text{disk}} \sqrt{(x-x_o)^2 + (y-y_o)^2}\, dydx$$

The rent-offer function is

$$R(x_o,y_o) = q\left[K - T(x_o,y_o)\right]$$

$$(5\text{-}55) \qquad R(x_o,y_o) = q\left[K - \iint_{\text{disk}} \sqrt{(x-x_o)^2 + (y-y_o)^2}\, dydx\right]$$

where K is the constant defined above

q is the level of output per square mile.

If $\rho(x,y)$ is a density function of customers, not necessarily uniform, then the expression for the transport cost at the edge of the city becomes

$$(5\text{-}56) \qquad K^* = \iint_{\text{disk}} \sqrt{(x-x_o)^2 + (y-y_o)^2}\, \rho(x,y)\, dydx$$

and total transport cost is

$$(5\text{-}57) \qquad T^*(x_o,y_o) = \iint_{\text{disk}} \sqrt{(x-x_o)^2 + (y-y_o)^2}\, \rho(x,y)\, dydx$$

The rent-offer function then is

$$(5\text{-}58) \qquad R^*(x_o,y_o) = q\left[K^* - T^*(x_o,y_o)\right]$$

A density function which declines with distance from the center, exponentially or otherwise, will tend to make central location more desirable.

Of course, it has not yet been shown that the center of the city is the most desirable location for this type industry, i.e., the minimum transport cost location. Mathematically, the question is whether or not the minimum of $T(x,y)$ is at the point $(0,0)$. The solution to this extrema problem is

$$\bar{x} = \iint_{\text{disk}} x \, dydx \tag{5-59}$$

$$\bar{y} = \iint_{\text{disk}} y \, dydx$$

i.e.,

$$\bar{x} = 0 \quad \bar{y} = 0$$

the center of mass.

If a density function $\rho(x,y)$ is included the solution is

$$\bar{x}^* = \iint x \, \rho(x,y) dydx \tag{5-60}$$

$$\bar{y}^* = \iint y \, \rho(x,y) dydx$$

with the exact value of the minimum depending on the specification of the density function. In the case where $\rho(x,y) = e^{-\sqrt{x^2+y^2}}$, an exponentially declining function, then $\bar{x}^* = 0$ and $\bar{y}^* = 0$ so that the center city is still the point of minimum transport cost.

CHAPTER 6

MEASURING THE BENEFITS OF AN URBAN TRANSPORTATION IMPROVEMENT: A NUMERICAL ANALYSIS

In the last two chapters, the argument was developed that if one is interested in determining the benefits of a city-wide transportation improvement, it is necessary to consider not only the benefits to the users, but also the indirect effects on housing prices and land values that will occur, inevitably. This approach is in opposition to the more traditional approach, which assumes that the location of households is exogenous. In the traditional approach, the benefits can be measured in terms of transportation cost savings. In the case of a subway improvement, these savings will accrue both to those who switch to subway use and to those who may continue to travel by automobile or bus, since road congestion, if present before the subway, will be reduced after subway construction.

In Chapter 4, it was demonstrated that if land values and housing prices are assumed to be endogenously determined, then the benefits are more accurately approximated by the change in the consumer surplus accruing to consumers from transportation, housing and land.[1] Further, it was shown that if the population of the city is also endogenous and the demand for CBD workers perfectly elastic, then, in the long run, the beneficiaries of the improvement will be the land owners and not the commuters.

One difficulty with assuming that housing and land prices are endogenous is that, until recently, little was known about the relationship between the transportation system and the prices of housing and land. The work of Alonso [1], Mills [20], and others has greatly increased our understanding of this relationship. As a result, it was possible in Chapter 5 to develop a model in which the interaction of the transportation system and the housing technology determined, among other things, the housing and land prices, and transportation costs. Several types of transportation systems were compared including several assumptions relating to subway transportation.

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[1] In Chapter 4, it was noted that a change in transportation costs will also produce reorganization benefits in the governmental services and retail trade sectors. In this chapter, the impact of a subway improvement is the primary interest. As a result, it is radial travel, i.e., the commute to the CBD, that will be most affected. Therefore, since most transportation to governmental services or retail stores will be local rather than radial, the benefits in these sectors are ignored. To the extent that there may be positive benefits associated with these sectors, the benefit calculations in this chapter are underestimates.

In this chapter, numerical analysis is applied to the model of Chapter 5. The impact of the various assumptions about the transportation system on the structure of the city is considered, and the benefits of a subway are investigated numerically. There are several steps in the analysis. First, it is necessary to solve the model of Chapter 5 for the desired variables. In this case, from Chapter 4, it is clear that the prices of land, housing, and transportation, and the total expenditure on these items are of interest in the measurement of benefits. Second, realistic values of the parameters must be chosen, based on the characteristics of the type of city to be analyzed. Third, a numerical solution and evaluation of the results must be made.

Solving for the Desired Variables

Since both housing prices and road transportation costs depend upon land rents, these rents are the logical starting point. Land rents at each distance, u, from the CBD are given by Equations (5-30) and (5-34) of Chapter 5 for the cases of road transportation and subway transport, respectively. They are repeated, here, for convenience.

$$(5\text{-}30) \qquad R(u) = \left[Ra^{(\alpha-\alpha_2)} + \frac{A_2(\alpha-\alpha_2)}{\alpha A_1}(k-u) \right]^{1/\alpha-\alpha_2}$$

$$(5\text{-}34) \qquad R(u) = \left[Ra^{\alpha} + \frac{A_3}{A_1}(k-u) \right]^{1/\alpha}$$

Housing prices, then, are a function of these rents in Equation (5-5).

$$p_1(u) = A_2R(u)^{\alpha} \tag{5-5}$$

Transportation prices for roads and subways were, respectively

$$p_2(u) = A_2R(u)^{\alpha_2} \tag{5-18}$$

and

$$p_3(u) = A_3 \tag{5-19}$$

In Chapter 5, it was assumed that the CBD did not use any land and was located at the center of the city. It is possible to be somewhat more realistic by assuming, instead, that the CBD requires some land and has a radius, $\hat{u}$, that is exogenously determined. The cost of commuting to the CBD on roads or on subways, then, is:

$$T(u) = \int_{\hat{u}}^{u} A_2R(u')^{\alpha_2}du' \tag{6-1}$$

if on roads and

$$T(u) = A_3(u-\hat{u}) \tag{6-2}$$

if on a subway. If all land in the CBD rents for $R(\hat{u})$ and the city is a complete circle, total expenditure on land is:

$$TLE = \int_{\hat{u}}^{k} R(u)2\pi u\ du + R(\hat{u})\pi\hat{u}^2 \tag{6-3}$$

Total housing expenditure is

$$\text{(6-4)} \qquad THE = \int_{\hat{u}}^{k} x_1(u)p_1(u)du$$

where $x_1(u)$ = total number of CBD workers at u.

Total transportation expenditure is

$$\text{(6-5)} \qquad TTE = \int_{\hat{u}}^{u} \left[p_i(u'') \left(\int_{u}^{k} x_1(u')du' \right) \right] du''$$

where subscript i is 2 if travel is by road and 3 if by subway.

From Equation (5-2),

$$\text{(6-6)} \qquad x_1(u) = \frac{L_1(u)R(u)}{\alpha_1 p_1(u)}$$

and from (5-23),

$$\text{(6-7)} \qquad L_1(u) = 2\pi u - L_2(u)$$

Since no workers live beyond the distance, k, we can see from Equation (5-25) that no transportation need be provided at k. Therefore, $L_2(k)$ will be zero. As a result, $L_1(k)$ is equal to $2\pi k$. This result, along with Equation (5-24) that $R(k) = Ra$ gives the initial conditions at k and the city can be generated from k on towards the CBD.

One simplification of the model was made in the calculations. The same wage rate, w, was assumed to prevail everywhere in the city. This simplification implies that $\alpha = \alpha_1$.

Parameters

In the model of Chapter 5, there are 19 parameters and exogenous variables. However, a subset of only 12 of these is needed for the calculations. These include ρ, X_1, k, α_1, α_2, A_1, A_2, A_3, w, Ra, ρ_1, and $\hat{u}$. The other parameters enter implicitly into the calculations through the composite parameters A_1, A_2, A_3. In selecting the parameter values, while no empirical studies were undertaken, an effort was made to choose realistic values based on the available information. This task was made considerably easier as a result of an earlier effort by Mills [23] to select similar parameters.

Before selecting the parameter values, it was decided that a city of about three million people would be the most interesting to study with respect to subway benefits. The primary reason for this is that the two metropolitan areas in the United States currently constructing subways, Washington, D. C. and San Francisco, are about this size. Larger cities, with the exception of Los Angeles and Detroit, have some form of rail mass transit in operation. Some smaller cities, e.g., Baltimore, have rail mass transit systems proposed, but not adopted. Thus, three million appears to be about the necessary size for a city to consider subway construction seriously.

Once city size is chosen, several parameter values are chosen to be consistent with a city of that size. These parameters include ρ, X_1, k, α_1, α_2, A_1, A_2, A_3, and W. Values for these parameters were chosen as follows:

ρ (the proportion of total employment in the CBD). As suggested in Chapter 5, this proportion usually varies with city size. A realistic range is between 10% and 20%, with the larger cities at the lower end. Since three million is a fairly large city, a value of 10% (.1) was chosen. Washington, D. C. has a larger proportion than this, but is greatly influenced by the concentration of federal government employees there.

X_1 (CBD employment). In the United States there is about a three to one ratio of population to employment. This implies a total employment of one million in a city of three million. Since a value of 10% was chosen for ρ, a CBD work force of 100,000 was selected.

R (radius of the city in miles). Both the Washington, D. C. and the San Francisco SMSA's have more than 2,400 square miles of land area. However, the land area of an SMSA typically includes vacant land or farm land on the fringes that should not be considered part of the metropolitan area.[2] This occurs because if part of the county is in the metropolitan area, the entire county is included in the SMSA. Therefore, 2,400 square miles

[2]The metropolitan area is meant to describe the economic boundaries of the city, as opposed to the political or statistical boundaries.

probably exaggerates the actual size of the metropolitan area. For these reasons, a radius of 26.4 miles was selected and the resulting land area is 2,190 square miles.

α_1 (the elasticity of housing output with respect to land input). α_1 is an important determinant of population density in this model. To assure that the entire population was housed within the 26.4 mile radius, α_1, was chosen experimentally. A value of .15637 was selected and the constraint was satisfied. Mills [23] suggests that reasonable values lie between .1 and .25. It should be noted that the selected value is well within this range.

α_2 (elasticity of road transport output with respect to land input). This parameter was also chosen experimentally to yield a city with 15% of the land going to transportation if only roads are used. Mills [23] suggests that 20% is more realistic, but roads are defined to include only those used for transporting workers to the CBD. Therefore, the lower figure was chosen. The resulting value of α_2 was .153.

A_1 (a composite parameter equal to $w^{\rho_1} r^{\gamma_1} / \bar{A}_1 \alpha_1^{\alpha_1} \rho_1^{\rho_1} \gamma_1^{\gamma_1}$). The choice of A_1 is important in determining the ratio of CBD to non-CBD employment.[3] In the calculations, A_1 was chosen experimentally to yield a CBD employment equal to 10% of the total city employment. A value of 127.686 for A_1 gave this

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[3] The ratio $\dfrac{x_1(u)}{N_1(u)}$ at any u is $\dfrac{w}{A_1 \rho_1 R(u)^{\alpha_1}}$.

percentage.

A_2 (composite parameter equal to $\dfrac{\bar{w}^{\beta_2} r^{\gamma_2}}{\bar{A}_2 \alpha_2^{\alpha_2} \beta_2^{\beta_2} \gamma_2^{\gamma_2}}$).

This parameter determines road transportation prices and total transportation expenditure. In the United States in 1970, about 10% of income [34, p. 341], was spent on transportation. This implies a total daily expenditure of 3-6 million dollars in this model. This figure, of course, does not include the value of the time spent traveling. The cost of travel time can be more than half of the total travel costs. Therefore, a much higher figure would be more reasonable. In the calculations, A_2 was equal to 3.0 and a daily total transportation expenditure of 16 million dollars resulted. This value appears to be within a reasonable range.

A_3 (composite parameter equal to $\dfrac{\bar{w}^{\beta_3} r^{\gamma_3}}{\bar{A}_3 \beta_3^{\beta_3} \gamma_3^{\gamma_3}}$). This parameter determines the relative desirability of subways compared to roads and the radius of the optimal subway ring. The plans for the subway system in Washington, D. C., show lines extending 16-20 miles from the CBD. To obtain results comparable to this system, A_3 was chosen to yield a subway ring of that size. An A_3 equal to 9.9 was selected and the resulting radius of the subway ring was 17 miles. To be more specific, it is assumed in the calculations that subway technology is such that, in a city of 3 million people, the optimal distance to extend the subway lines is about 17 miles.

W (CBD daily wage rate). Average gross weekly earnings in non-agricultural private employment in 1971 were \$125 a week or \$25 a day. This figure would be higher in a large city and, therefore, a value of \$30 was chosen.

The remaining parameters were chosen independently of city size.

Ra (daily rent on a square mile of agricultural land). Mills [23] suggests that \$800 a day is reasonable, and this is the value used here.

β_1 (elasticity of housing output with respect to labor input). With a linear homogeneous production function, this parameter is also the factor share of labor. While there are no estimates of the "housing" production function available as defined here, the factor share of labor in the housing output should be about the same as that prevailing in the economy as a whole. In 1970, wages and salaries were more than half of total national income [34, p. 309]. The value chosen here for β_1 was .545.

$\hat{u}$ (radius of the CBD). This parameter is assumed to be exogenously determined. However, one technical consideration suggests a lower limit to its value. The amount of land needed to transport all the CBD workers living beyond u must be less than or equal to the total amount of land available at u, i.e., $L_2(u) \leq 2\pi u \, du$.

In the calculations this value was 1.05 miles for the road city and $\hat{u}$ was given this value.

The parameter values are summarized in Table 6-1.

The Numerical Solution

To generate the numerical solution and the parameter values that yield a realistic city, a computer program was written.[4] Figure 6-1 summarizes the logic of the program. Since the quantity of land used for transportation is endogenous, it is necessary to start on the edge of the city, k, where land rents are known and the value of $X_1(k+du)$ is known to be zero since, by assumption, no CBD workers live beyond the city limit, k. The city can then be approximated in intervals of du from the edge of the city to the CBD as follows:

i. In the first annulus from k-du to k, no land is needed for road transport since no workers live beyond k, i.e., since in discrete terms the demand for transportation at w is:

$$X_2(u) = \sum_{u+du}^{k} X_1(u) \text{ (from Eq. 5-25)}$$

This implies that $L_2(k-du) = 0$ and, therefore, all the land in this annulus can be used for

[4]The computer program was written in Fortran IV and run on an IBM 370/155 computer. The entire undertaking required about 5 hours of computing time. One run of the program in its final version required about 15 minutes.

TABLE 6-1

PARAMETER VALUES

ρ	.1
x_1	100,000 workers
k	26.4 miles
α_1	.15637
α_2	.153
A_1	127.686
A_2	3.0
A_3	9.9
w	$30/day
Ra	$800 a day per square mile
$\hat{\rho}_1$	.545
$\hat{u}$	1.05 miles

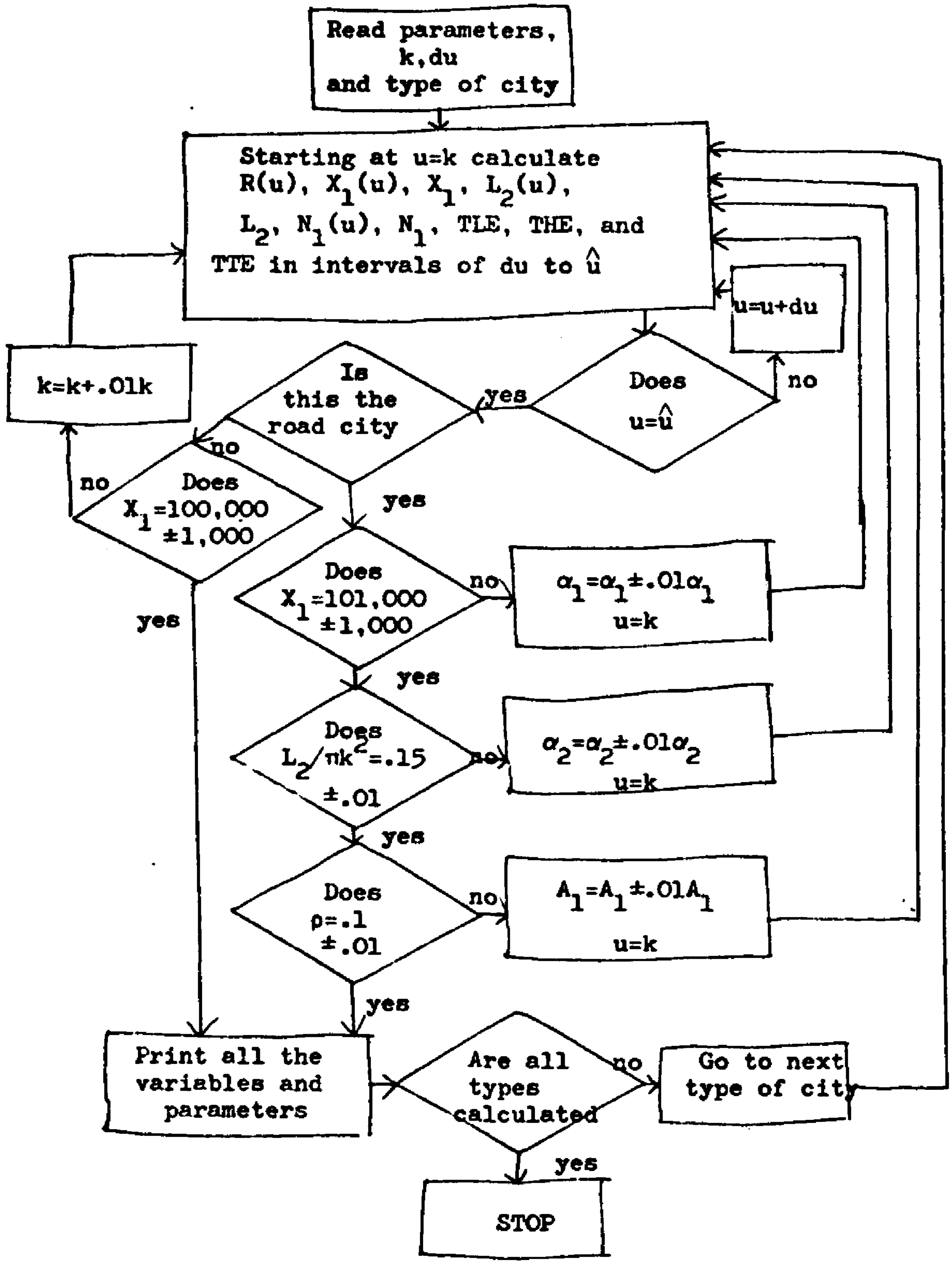

Figure 6-1. Computer program logic

housing, i.e., $L_1(k\text{-}du) = 2\pi(k\text{-}du)du$.

Next, from Equation (5-30), rents in this annulus are known and the number of CBD workers can be found from Equation (6-6).

ii. Once the number of CBD workers in the first annulus is known, the amount of transportation, $X_2(u)$, needed to transport these workers through the next annulus is known (Equation 5-25). The quantity of land, $L_2(k\text{-}2du)$ necessary to provide this amount of transportation can be found from Equation (5-15a) and Equation (5-18), i.e.

$$L_2(u) = \alpha_2 A_2 R(u)^{\alpha_2 - 1} X_2(u) \tag{6-8}$$

Equation (6-8) yields the amount of land left for housing and the number of the CBD workers in this annulus (k-2du to k-du) can again be found from Equation (6-6).

iii. These steps are then repeated until $u=\hat{u}$.

iv. If one of the tests of the city characteristics is violated (see Figure 6-1), a parameter value is adjusted in the appropriate direction and the process begun anew until all the tests are satisfied for the road city.

v. The parameter values, with the exception of city radius, k, are held constant when the calculations are made for the other types of cities. The radius is adjusted for each of the other types to assure that only 100,000 CBD workers reside within the city limits.

The degree of accuracy in these calculations is dependent upon the width of the annulus, du, used. In the final version on which the results that follow are based, there are 250 annuli in the cities being approximated, i.e., du was equal to k/250. The results appear to be quite accurate, since there was very little change in the characteristics of cities when du was decreased from k/100 to k/250.

In all, four types of cities were studied. The first is the road city in which there are only roads available to transport workers to and from the CBD. The second type is the subway city in which there is only subway transportation available for travel to the CBD. In this second type, no land is required for transportation. All the land is used in housing.

Types three and four are hybrid cities having both roads and subways. In the type three city, switching costs between modes are assumed to be zero. As a result, everyone changes modes at the switching point u* and no land is used for transportation within the subway ring. In the type four city, switching costs are assumed to be infinite, so that no one changes modes. Here, some land is required within the subway ring to transport commuters who do not switch to the subway at u^*.

Characteristics of the Cities

The first set of calculations compares cities with an equal number of CBD workers - in this case 100,000 - but different transportation systems. Table 6-2 and Figure 6-2 summarize these calculations. Several items are worth noting.

First, it is noted, as might be expected, that the most efficient transportation system gives the lowest living expenses, K, (housing plus transportation). The type 3 city has the lowest living expenses with types 2, 4, and 1 following in that order. By assumption, the subway is less expensive than roads in only a portion of the city. Therefore, the type 3 city, which implements this assumption, is the least cost city. The type 4 city is more expensive because the switching costs reduce the benefit of having a subway.

Land rents follow the same relationship that living costs do, with the lowest rents at the edge of the CBD in type 3, and types 2, 4 and 1 following. Land rents and transport cost in the model are directly related as can be seen in Equations (5-30) and (5-34).

The ordering of city size differs from the ordering of land rents. Type 2 is the smallest, followed by types 3, 4, and 1. Here two opposing forces act against each other.

TABLE 6-2

CITY CHARACTERISTICS

	1 Road City	2 Subway City	3 Hybrid City No Switching Costs	4 Hybrid City Infinite Switching Costs
city radius (k) (miles)	26.4	24.73	25.32	25.96
land area (square miles)	2189.6	1921.3	2014.1	2117.2
land used for roads (square miles)	295.1	0	64.7	172.3
land for housing (square miles)	1891.5	1928.8	1945.9	1941.4
$p_1(u)+T(u)=K$ (dollars per day)	663	608	607	637
switching point (u^*) (miles)	-	-	17.7	18.3
land rent at k ($ per day per square mile)	800	800	800	800

TABLE 6-2 (Continued)

CITY CHARACTERISTICS

	1 Road City	2 Subway City	3 Hybrid City No Switching Costs	4 Hybrid City Infinite Switching Costs
land rent at .8k	1735	1796	1561	1713
land rent at .6k	3758	3682	3256	3620
land rent at .4k	8122	7020	6377	7129
land rent at .2k	17,516	12,615	11,716	13,156
land rent at $\hat{u}$	32,352	19,452	19,354	20,655

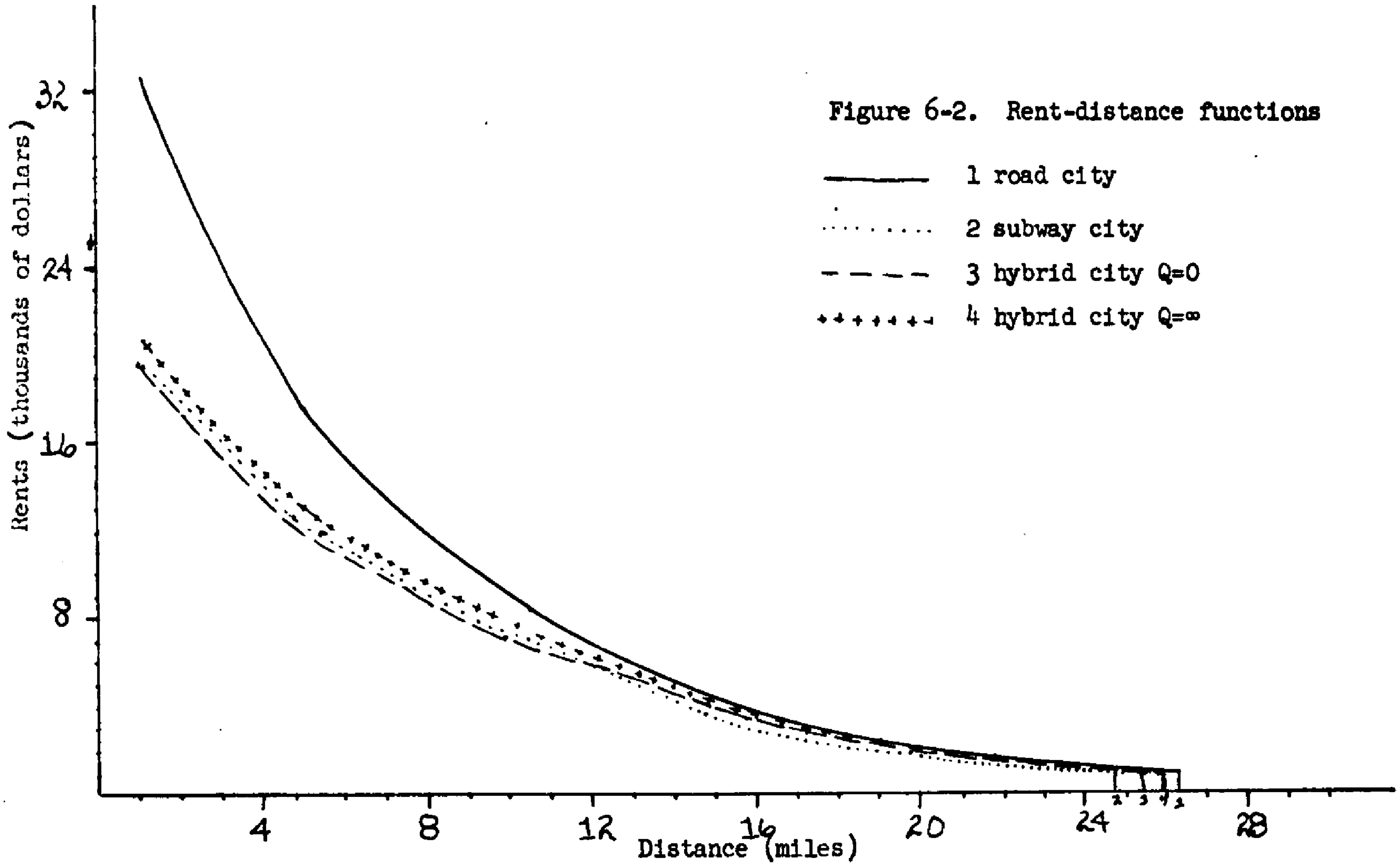

Figure 6-2. Rent-distance functions

On the one hand, the construction of a subway permits land to be transferred from road use to housing, thereby reducing city size. On the other hand, the resulting decrease in land prices causes land to be substituted for other factors in housing production, thereby increasing city size. Cities with subways are smaller than the road city. In this model, the first effect outweighs the second. (Note that the amount of land used for housing is inversely related to land rents.)

Short-run Benefits

The short-run benefits are the benefits of an improvement after the city reorganizes to an efficient pattern, but remains with the same number of workers employed in the CBD. That is, the short-run is defined, here, to mean a period of time long enough for residents to move to the new equilibrium locations, but too short for new residents to move to the city from other areas. From Chapter 4, it can be seen that the necessary data for such a comparison are land rents, housing costs, and transportation costs. These are summarized in Table 6-3.

In the table, the benefits are measured relative to the road city. User benefits are the difference between the transportation costs in the given city and the transportation costs in the road city. Reorganization or non-user benefits are the sum of the decreased housing costs and the increase in total land rents. Total benefits

TABLE 6-3

SHORT-RUN BENEFITS*

	1 Road City	2 Subway City	3 Hybrid City No Switching Costs	4 Hybrid City Infinite Switcing Costs
Total rents on land used for housing	7.86	7.68	7.66	7.69
Total rents on all city land	10.50	7.76	7.85	8.61
Total housing expenditure	50.49	49.11	48.97	49.20
Total transportation costs	16.79	12.05	12.07	13.07
User benefits (fall in transportation costs)	-	4.74	4.72	3.72
Non-user or reorganization benefits (fall in housing costs + increase in total land rents)	-	-1.36	-1.13	- .60
Total net benefits (sum of user & non-user benefits)	-	3.38	3.59	3.12

*figures in millions of dollars per day.

are the sum of user and non-user benefits. The benefits in each case are net benefits, i.e., net of the debt service on construction costs. The debt service on construction costs is included in the pricing of transportation, and, therefore, need not be deducted again.

What is of interest, here, is not so much the actual figures but the relationship between the user benefits and total benefits. Recall that the traditional approach looks at user benefits primarily and ignores the effects on housing prices and land values. The immediately striking result here is that, in each case, the reorganization benefit is negative. The relative magnitude is between 16% and 29% of the user benefit. In other words, if only the user benefit is considered, the total benefits are over-estimated by 19% to 40%.

Also it can be seen in the table that the benefits of the subway are reduced by the presence of high switching costs in the type 4 city. The benefits are reduced by about a seventh.

Long-run Benefits

One effect of introducing a more efficient transportation system was a reduction in the sum of housing and transport costs per CBD worker (K). As indicated in Chapter 5, these two items are a large fraction of the total purchases of the average consumer. Any large reduction in these will improve the standard of living in the city relative to other cities, thus attracting new residents.

In the long run, a new equilibrium will be established when a sufficient number of these new residents have arrived and driven the standard of living back to its original level. Several factors will influence the number of new residents, including the elasticity of demand for CBD workers, the extent of the discrepancy in the standard of living between cities, and the ease of intercity migration. The longer the period under consideration, the more elastic the demand for CBD workers will be and the less important migration costs will be. Since the very long run is of interest, here, it is assumed in the following calculations that the demand for CBD workers is perfectly elastic at the given wage, that migration costs are negligible, and that other cities do not make transportation improvements. As a result, it was only necessary to increase the size of the city (k) until the sum of housing plus transport costs was equalized.

Some characteristics of the resulting cities are summarized in Table 6-4. As can be seen, the long-run effect for type 2 and 3 cities is a very dramatic increase in CBD employment and city population. However, the most realistic comparison is probably between type 1 and type 4 cities. Here there is a 24% increase in CBD employment and a similar increase in city population.

TABLE 6-4

THE CITIES IN THE LONG-RUN

	1 Road City	2 Subway City	3 Hybrid City No Switching Costs	4 Hybrid City Infinite Switching Costs
CBD employment (thousands)	100	208	211	124
City population (millions)	3.05	6.59	6.69	3.76
City radius (miles)	26.4	30.5	31.2	27.5
Land area (square miles)	2190	2922	3058	2375
Switching point (miles)	-	-	23.6	19.9
$R(\hat{u})$* ($ per square mile per day)	32,351	34,131	34,415	34,150

* $\hat{u}$ equals .04k here.

As indicated in Chapter 4, in this situation the benefits accrue primarily to the land owners in the form of higher land rents. The original residents, after an adjustment period, face the same sum of housing and transportation costs. The long-run increase in land rents is presented in Table 6-5 and compared to the short-run benefits.

The long-run benefits differ from the short-run in each case and are larger by from 50% to over 100%. In addition the long-run benefits are larger than the short-run user benefits and therefore benefit calculations based on the user benefits will underestimate the long-run benefits. This contrasts with the overestimate of the short-run benefits.

Some Policy Implications

Several items are worthy of note. First, the result of Friedlaender [9], that, in general, the total benefits may be greater or less than the user benefits depending upon the particular situation and the relevant elasticities, is confirmed here. In the case of short-run benefits, for all three types of improvements considered, the total benefits were less than the user benefits. In the long-run case, the benefits are the changes in land rents. Whether or not there are any user benefits at all depends on whether the original residents remain in the same location or not. Even if they remain in the same location the user benefits are offset entirely by an increase in housing costs. Thus, in the long-run the user benefits underestimate total benefits.

TABLE 6-5

LONG-RUN BENEFITS*

	1 Road City	2 Subway City	3 Hybrid City No Switching Costs	4 Hybrid City Infinite Switching Costs
Total rents on all city land	10.50	17.41	17.82	15.06
Long-run benefits	-	6.91	7.32	4.56
Short-run benefits	-	3.38	3.59	3.12
Difference (long-run minus short-run)	-	3.53	3.73	1.44

*Figures in millions of dollars per day.

A second item of note is the effect of large switching costs on the benefits. In both the short-run and the long-run, the benefits are greatly reduced by the presence of large switching costs. This suggests that if the maximum benefit is to be derived from a subway, every effort should be made to minimize the switching costs. This could be done by providing parking at the stations, a good connecting bus system, short headways for trains, etc.

A third item is the question of who benefits from the transportation improvement. Substantial redistribution of income is likely to occur. In the short-run the users receive more than a full share of the benefits, but in the long-run only the land owners benefit. Therefore, the distribution of income will move away from commuters towards land owners. If this redistribution is considered to be undesirable, then corrective taxes may be warranted.

A fourth item is the common complaint of the anti-freeway lobby that it is useless to build freeways. Their reason is that although freeways initially reduce congestion and transportation costs, it is only a few years before the freeways have encouraged so much more use that congestion reoccurs and the situation is back to where it started. On this basis, the anti-freeway lobby recommends subways. Reducing transportation costs by building freeways will, of course, encourage more use. But the same is true of subways. The reduction in transport costs not only encourages additional use by the existing population, it also attracts new residents. Now, it may be that subway capacity can be expanded without congestion more easily by adding trains than freeways can increase use without congestion by adding more cars. Nevertheless, even if the added use is not reflected in congestion, it will be reflected in higher housing prices as new residents bid up the locational values of housing. Therefore, the construction of subways is also futile, although in a more subtle way.

All of this suggests that if congestion and transportation costs in the cities are to be reduced, a national effort must be made to improve the transportation systems of cities. By so doing, intercity migration will not be encouraged as one city's transportation network improves, since transportation will be improved in other cities where improvements are justified. An increase in per capita use may still counteract some of the reduction in cost initiated by the improvement, but a much larger share of the total benefits will accrue to users instead of land owners.

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