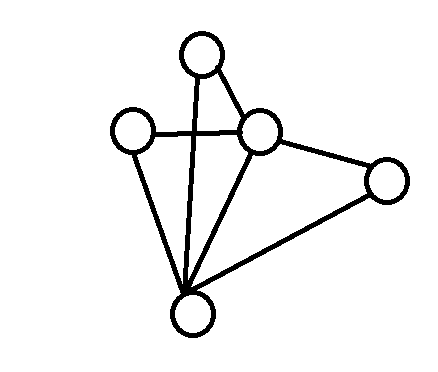
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CS 4800 – Algorithms and Data

Homework 4

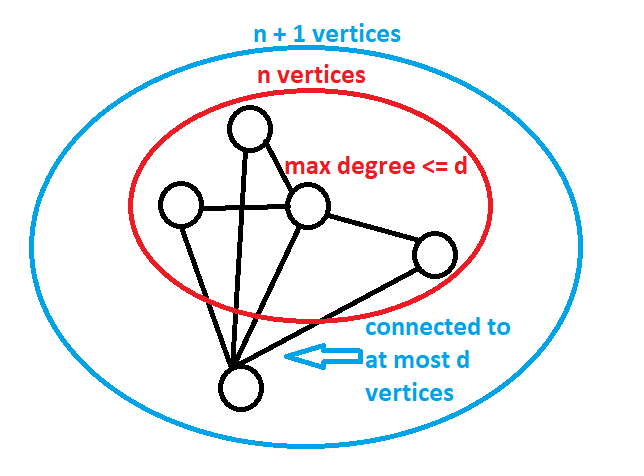
* 1. Let us say we have a graph that looks like this:



This graph contains some vertices and some edges connecting those vertices to

each other.

Now, we must prove that if the degree of every vertex is at most d, then the graph is d+1 colorable.

Let’s make our graph a little easier to visualize so we can prove this:

When we take out the vertex at the bottom and only look at the graph in the red circle, we can say that there are some number of vertices with a maximum degree of less than or equal to . However, when we add the bottom vertex and consider the graph in the blue circle, we know that there is vertices and is connected to at most vertices, so the degree is less than or equal to .

This means that this vertex must have one additional color because it cannot have any of the colors of the vertices it is connected to, which ultimately means that an additional vertex means the graph must require colors.

* 1. Say a graph has vertices. This means that the number of edges that this graph can have to connect all the vertices is , which is , which approximately equals . We can write this as:

Now based on the problem, we know:

Now, we can simplify this:

Which means:

And finally:

To answer the question, there exists a constant which is equal to .

1. Proof by induction on n, the number of vertices in a tree T.

Basis step: If n= 1 or 2 then the center is the entire tree which is a vertex or an edge.

Induction hypothesis. Let n>2. Let T be a tree with n vertices. Assume the center of every tree with less than n vertices is a vertex or an edge. Form T' by deleting the leaves of T. Since the internal vertices on paths between leaves remain, T' has at least one vertex. The eccentricity of a leaf in T is greater than that of its neighbor. Hence the vertices of minimal eccentricity in T are the same as the vertices of minimal eccentricity in T'. This implies that the center of T' is the same as the center of T. By induction hypothesis the center of T' is a vertex or an edge.