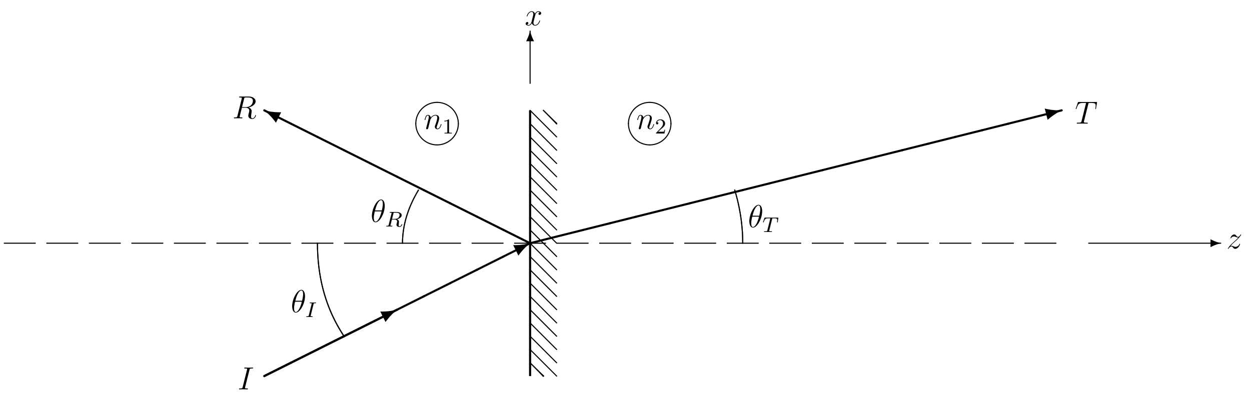
Physics 110B (Electrodynamics)
Problem Set #6
Due by March 17, 2015

1 Fresnel equations for s-polarized light



Derive the reflection coefficient $R = I_R/I_I$ (the ratio of reflected to incident intensities) for the case of an incident plane wave with electric field perpendicular to the plane of incidence. Take the magnetic permeabilities of the two media to be equal ($\mu_1 = \mu_2$).

Express your results in terms of the ratio of indices of refraction and the ratio of cosine of angle of refraction to cosine of angle of incidence:

$$\beta \equiv \frac{n_2}{n_1}, \qquad \alpha \equiv \frac{\cos\theta_T}{\cos\theta_I}.$$

Please follow steps (a)-(d) below, and then solve part (e).

(a) Define the complex amplitudes of the electric fields of the 3 waves to be: $\widetilde{E}_{0I}$, $\widetilde{E}_{0R}$, $\widetilde{E}_{0T}$. They are in the y-direction. Then, find expressions for the x and z components of the (complex) amplitudes of the magnetic fields:

$$(\widetilde{B}_{0I})_x, \quad (\widetilde{B}_{0I})_z, \quad (\widetilde{B}_{0R})_x, \quad (\widetilde{B}_{0R})_z, \quad (\widetilde{B}_{0T})_x, \quad (\widetilde{B}_{0T})_z.$$

The expressions should be in terms of $\widetilde{E}_{0I}$, $\widetilde{E}_{0R}$, $\widetilde{E}_{0T}$.

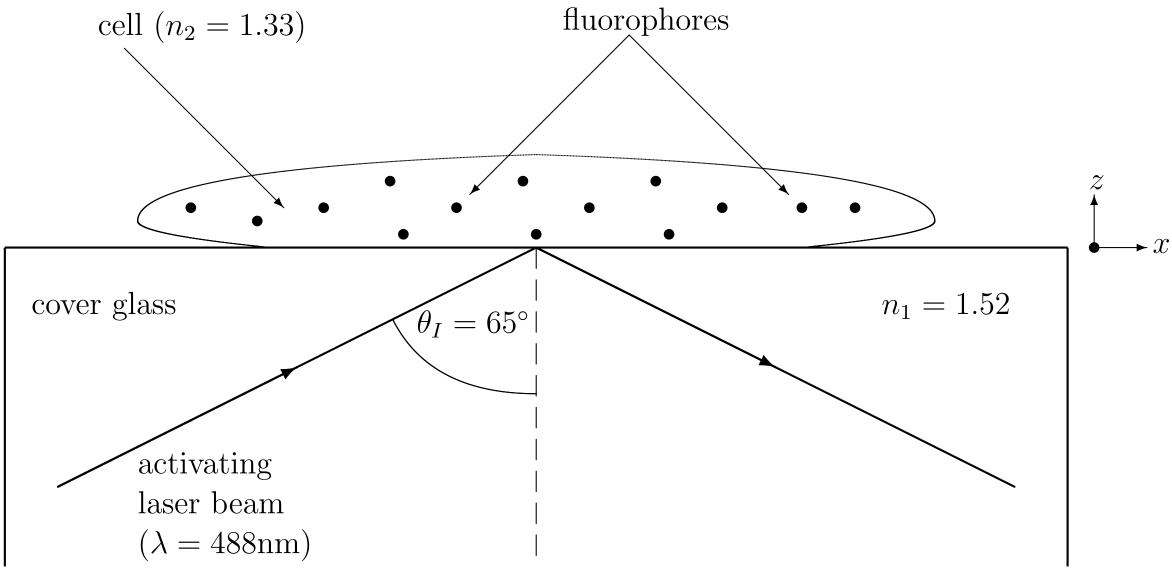
(b) Use the boundary conditions on the field components B_x, B_z and E_y at $z = 0$ to write down three linear equations in $\widetilde{E}_{0I}$, $\widetilde{E}_{0R}$, $\widetilde{E}_{0T}$. Check that only two equations are independent.

(c) Solve the linear equations to express $\widetilde{E}_{0R}$ in terms of $\widetilde{E}_{0I}$.

(d) Calculate the reflection coefficient $R \equiv I_R/I_I$.

(e) Is there an angle θ_I at which $R = 0$?

2 Total Internal Reflection Fluorescence Microscopy



The *evanescent wave* effect is put to good use in life-science research in an important imaging technique known as *Total Internal Reflection Fluorescence Microscopy* (TIRFM). This technique makes it possible to investigate processes near the cell membrane.

A biological cell (made mostly of water with index of refraction $n_2 = 1.33$) is placed on a glass slide with index of refraction $n_1 = 1.52$. The cell is engineered to include fluorescent molecules (fluorophores) that when activated with (blue) light of frequency that corresponds to a wavelength (**in air**) $\lambda = 488$nm, they (later) emit light of longer wavelength (green) that can be detected with a microscope.

The problem, however, is that fluorophores at different depths in the cell would make the picture out of focus if we used a usual microscope to image it.[1] With TIRFM, the activating laser beam is sent at an incidence angle θ_I larger than the critical angle θ_c, so that only the evanescent wave exists within the sample, and only a thin layer of fluorophores very close to the cell membrane is actually activated. Please solve the following problems:

(a) What is the critical angle θ_c?

(b) With $\theta_I = 65°$, what are the sine and cosine, $\sin\theta_T$ and $\cos\theta_T$, of the angle of transmission (also called the angle of refraction)? (**Note:** $\cos\theta_T$ is imaginary, of course.)

(c) Taking the z-direction as "up", and the x-direction as horizontal, what are the z- and x-components of the wave-vector (also called the propagation vector) $\vec{\tilde{k}}_T$?

Notes: (i) some components may be imaginary; (ii) $\lambda = 488$nm is **not** the wavelength in water.

(d) At what height h above the surface of the cover-glass does the electromagnetic field drop to 10^{-3} of its value (compared to the value just above the cover glass, which is touching the

[1] See [`http://lightmicroscopy.ucdenver.edu/img/seminars/June17.png`] for a comparison of images with and without TIRF.

lower cell membrane)?

(e) If the intensity of the laser beam is $I = 10^5 \mathrm{W/m}^2$, and the magnetic field of the incident beam is taken to be

$$\tilde{\vec{B}}_I = B_{0I} e^{i(\vec{k}_I \cdot \vec{r} - \omega t)} \hat{y}$$

what is the amplitude B_{0I} of the magnetic field of the incident light? (You may assume that it is real and that the magnetic constant of glass is the same as of vacuum.)

(f) Assuming P-polarized light, use Fresnel's equations

$$\tilde{\vec{E}}_{0T} = (\frac{2}{\alpha + \beta})\tilde{\vec{E}}_{0I}\,, \qquad \beta \equiv \frac{\mu_1 n_2}{\mu_2 n_1} = \frac{n_2}{n_1}\,, \qquad \alpha \equiv \frac{\cos\theta_T}{\cos\theta_I}$$

to determine the (complex) amplitude $\tilde{\vec{B}}_{0T}$ of the magnetic field of the evanescent wave. Express it in polar form $B_{0T}e^{i\phi}$ for part (g). (First, find the relation between $\tilde{\vec{B}}_{0T}$ and $\tilde{\vec{B}}_{0I}$.)

(g) Write down the *real* expression for the magnetic field of the evanescent wave:

$$\vec{B}(x, y, z, t) = (\cdots)?$$