Mark Mynsted, @mmynsted Dave Gurnell, @davegurnell



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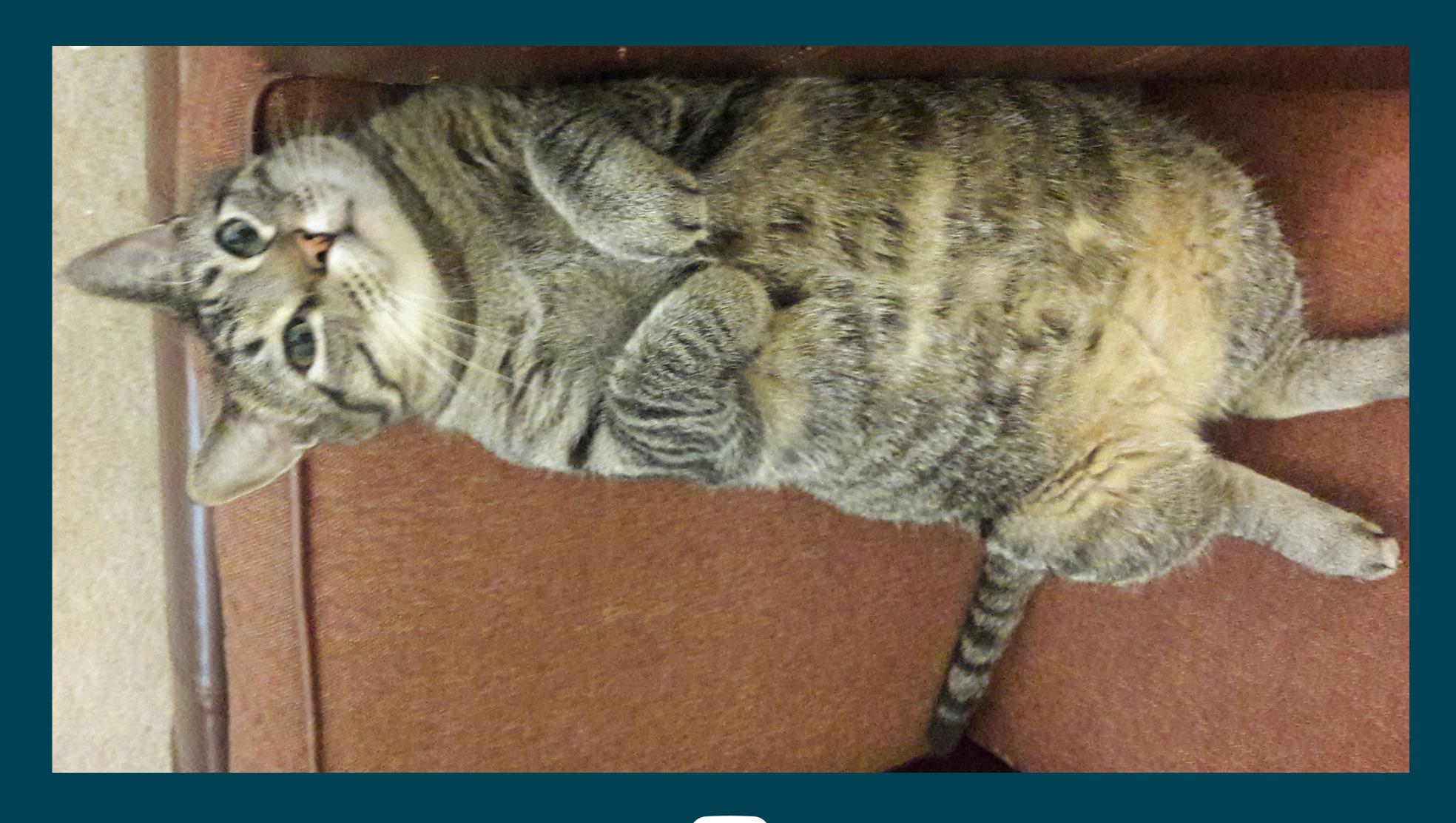


Dave Gurnell, @davegurnell Mark Mynsted, @mmynsted

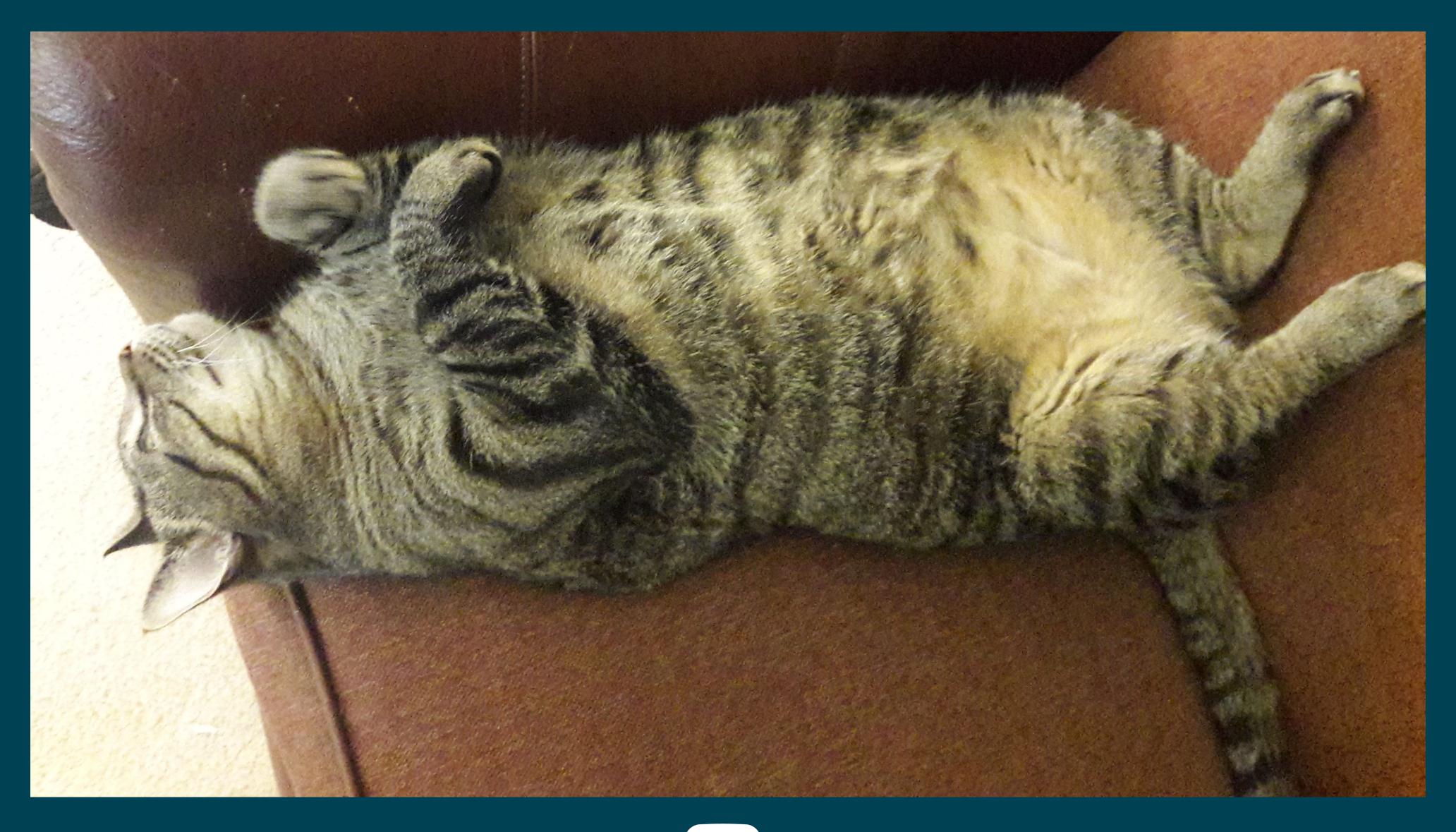


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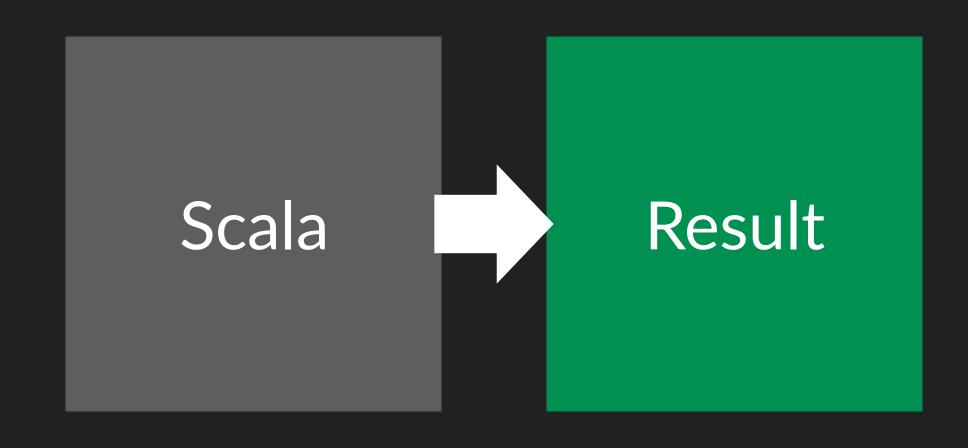


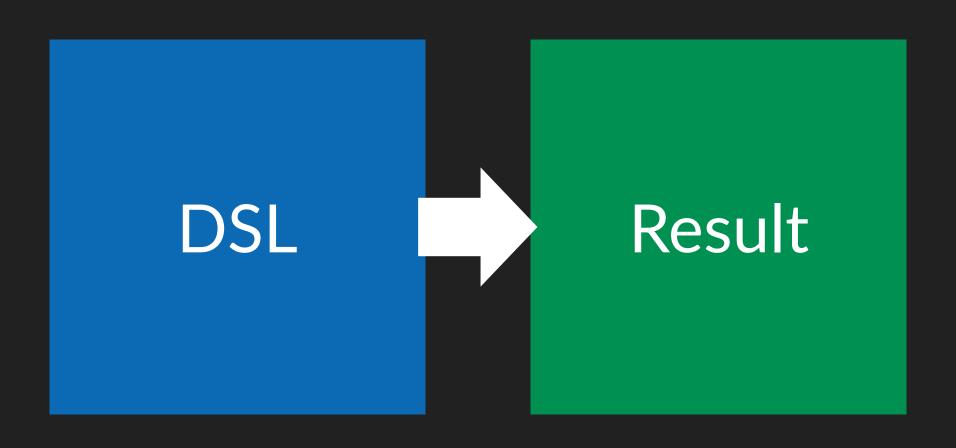


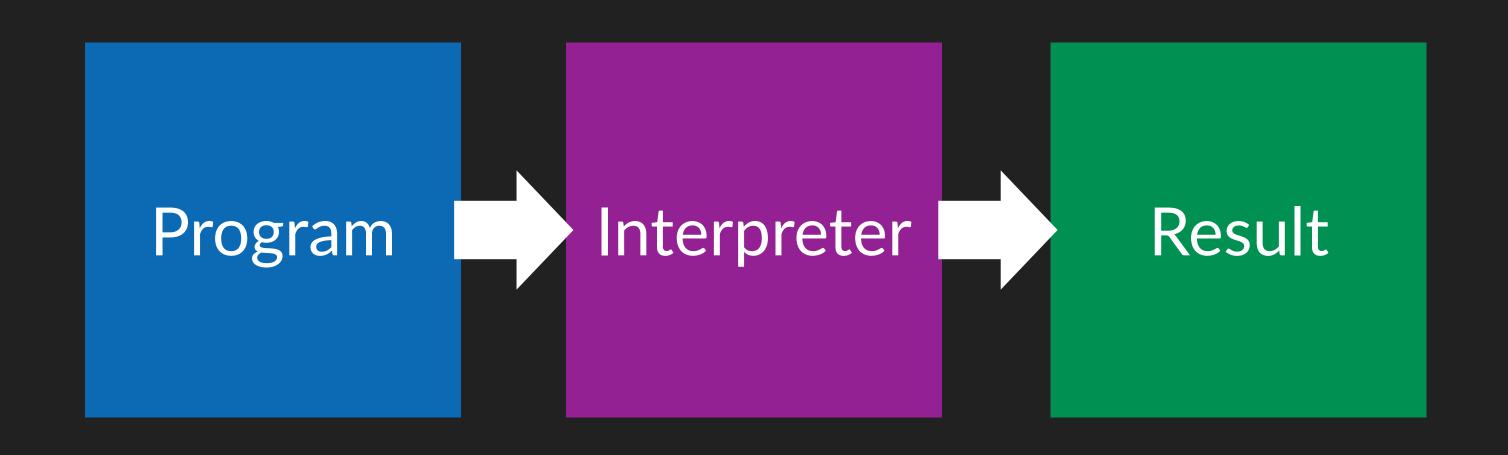


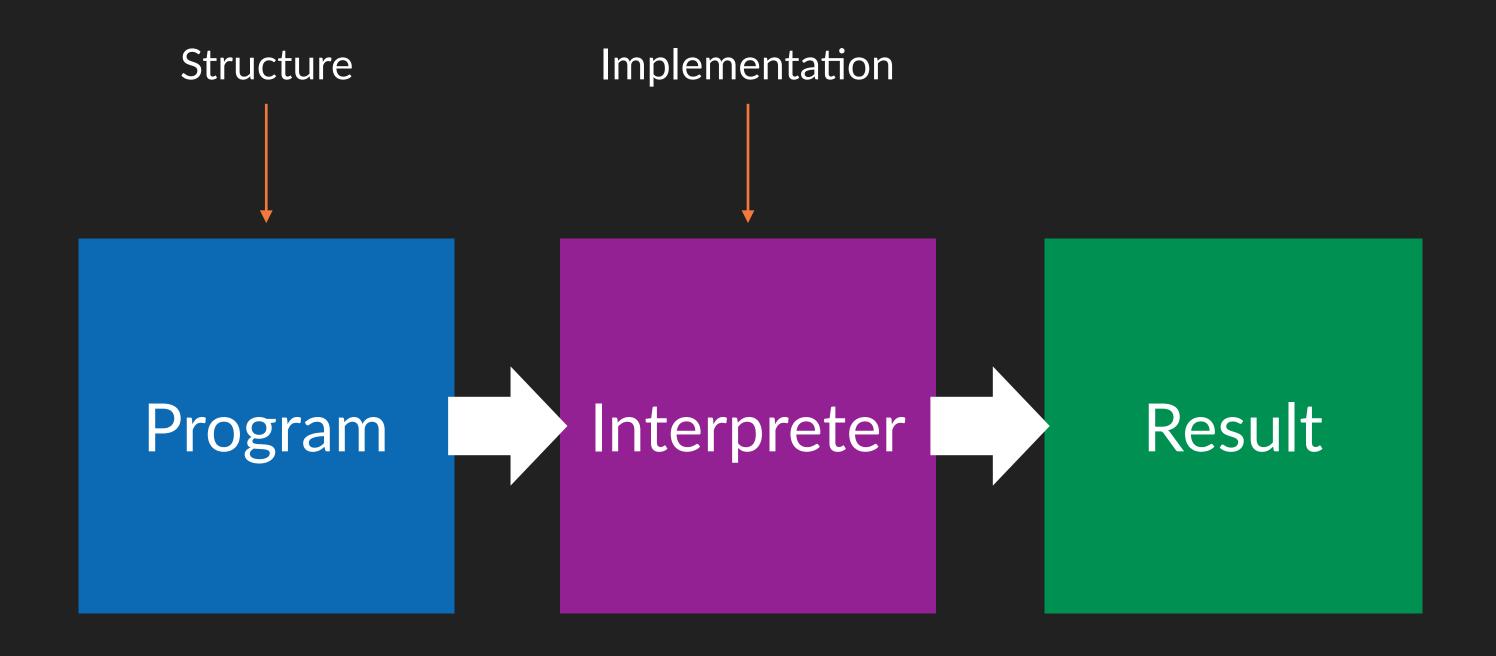
underscore

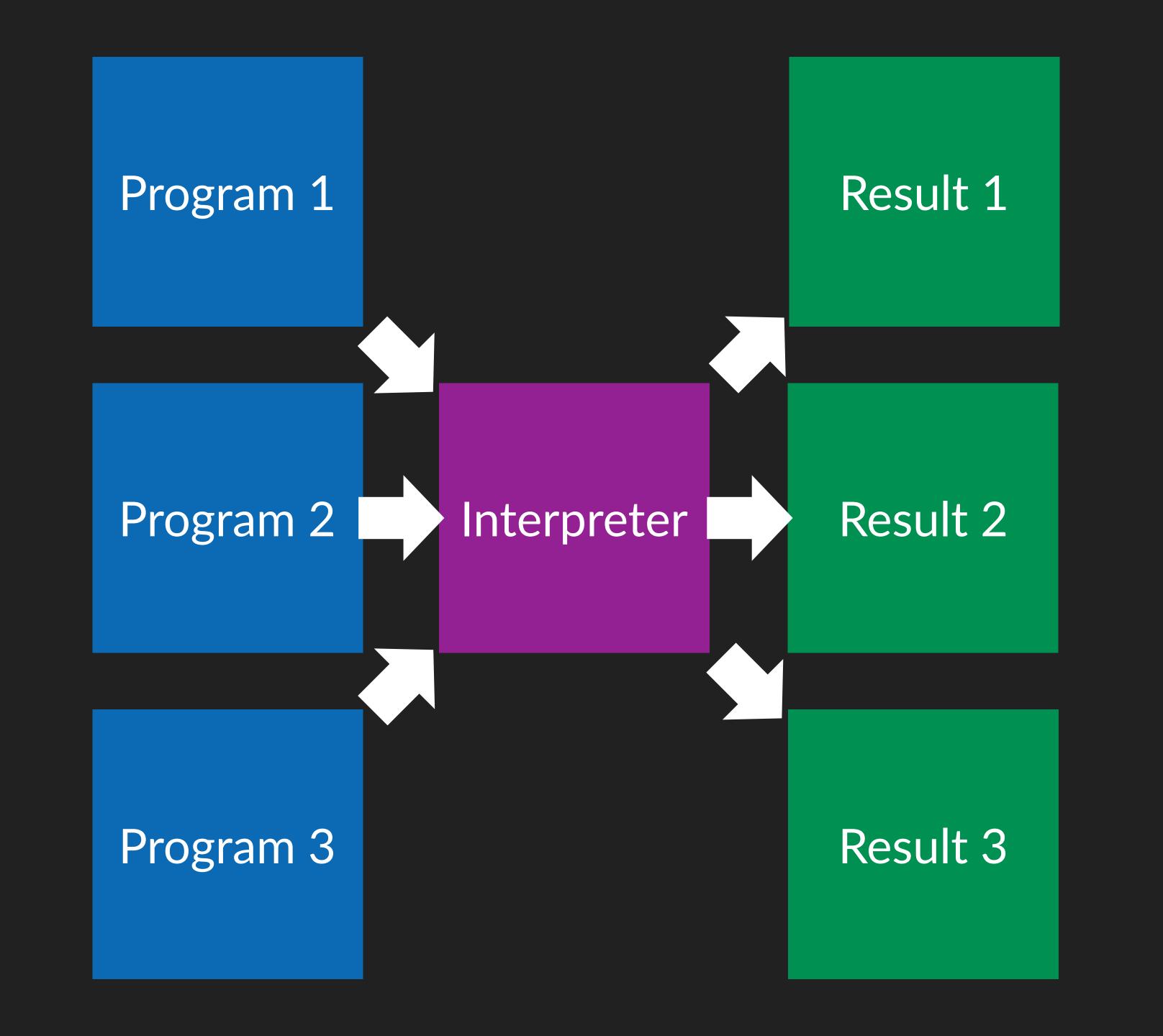
Scala

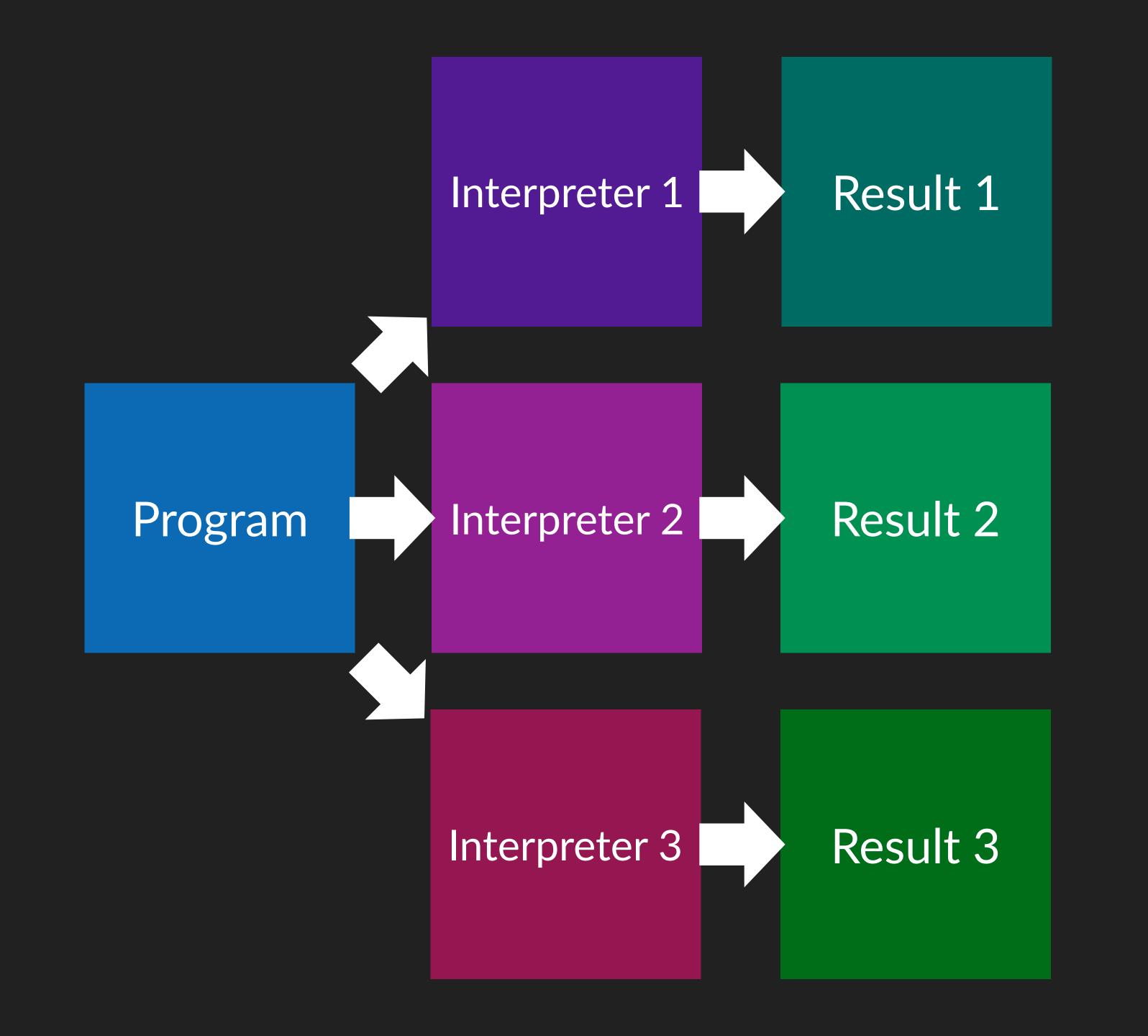


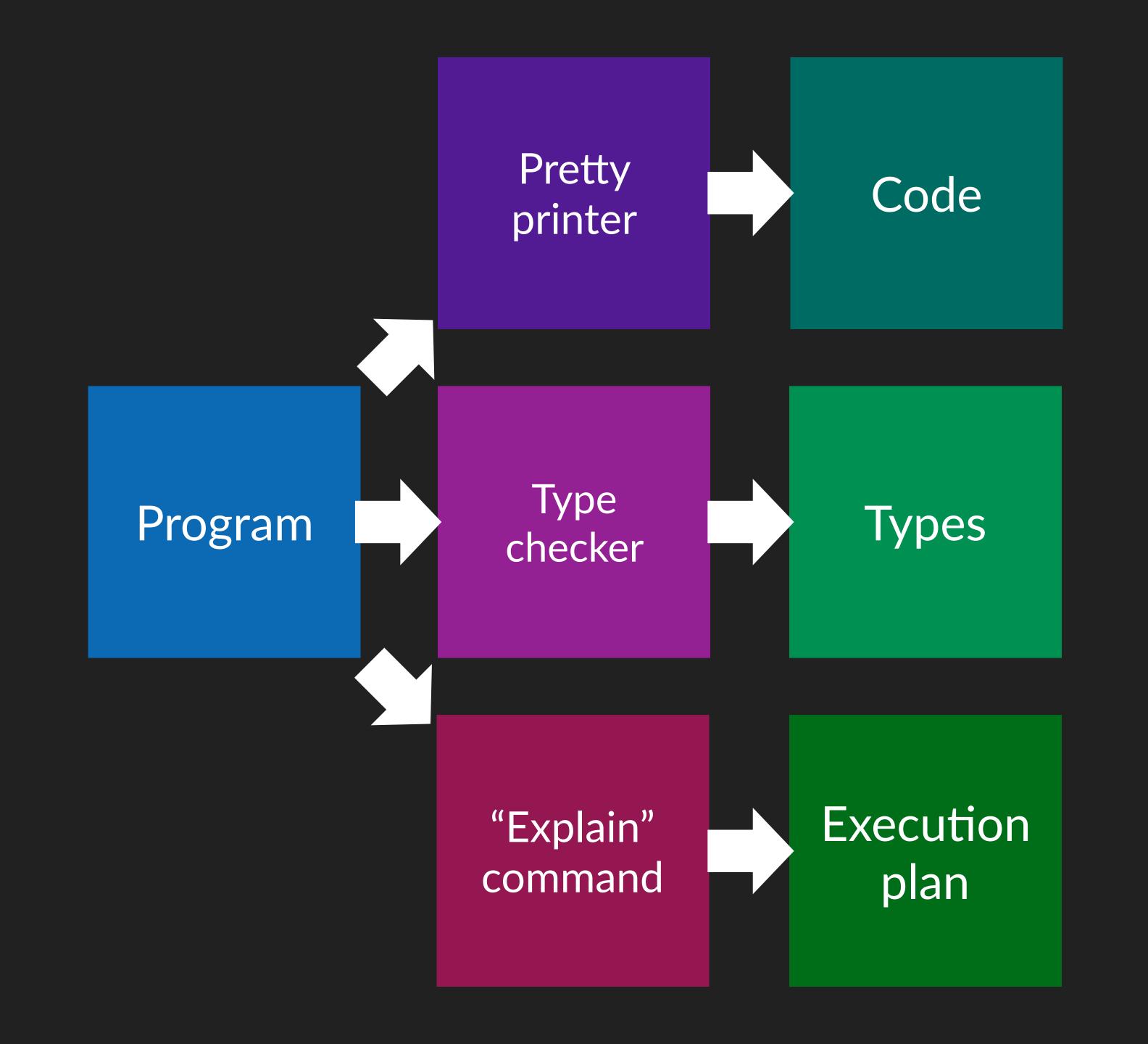


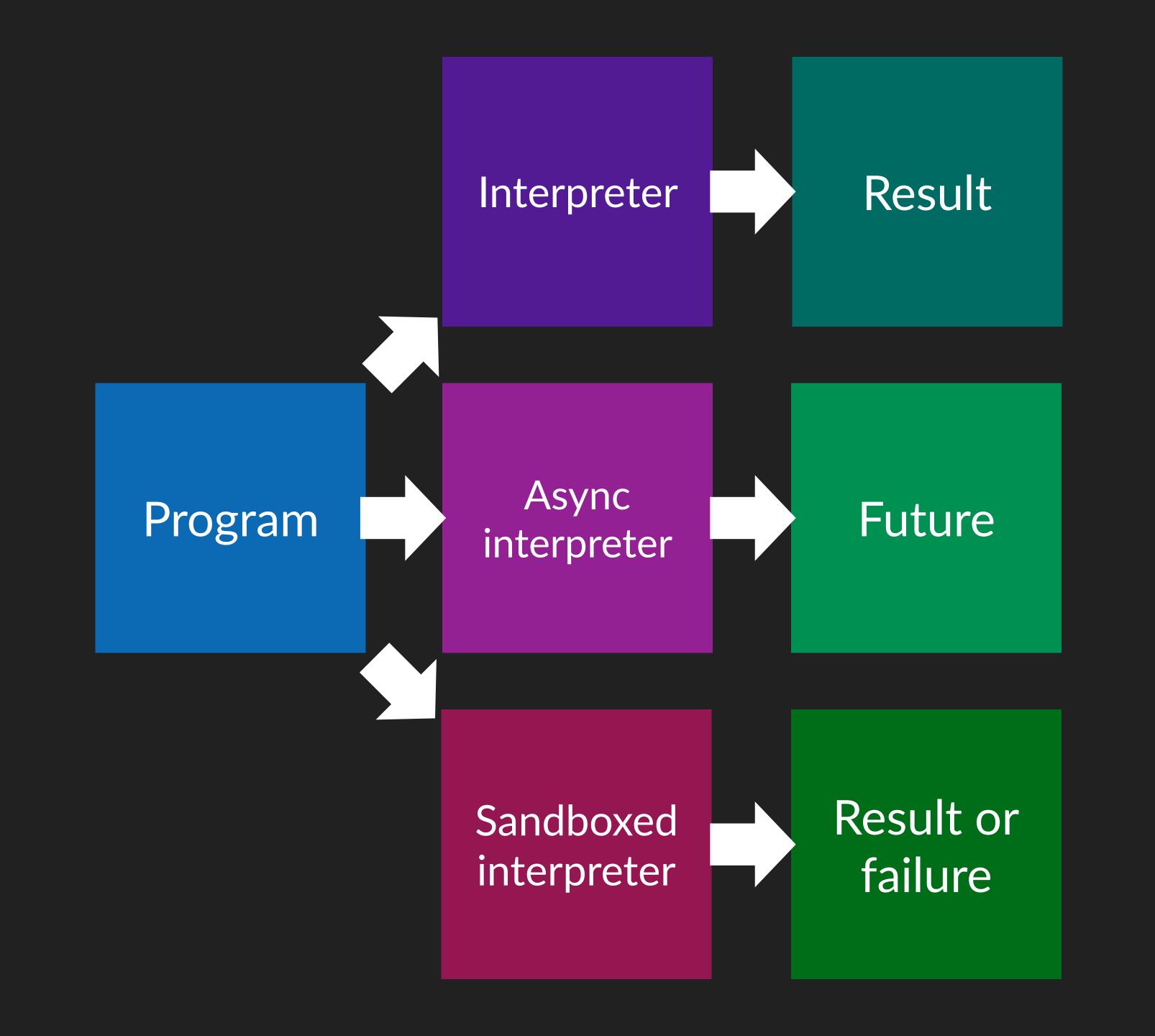


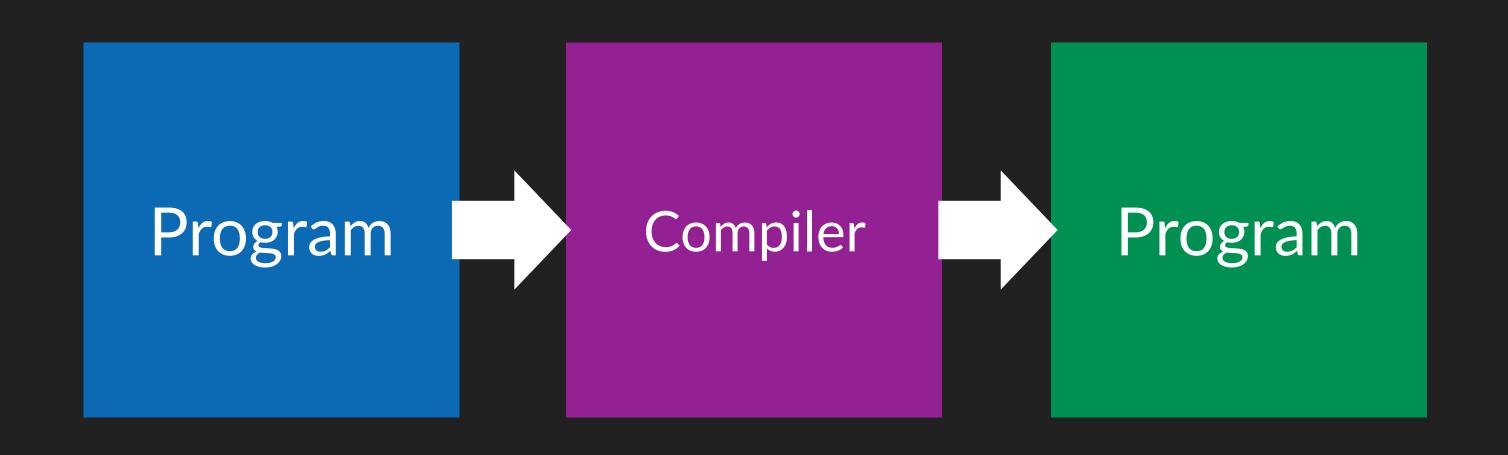












1. Re-use interpreters

2. Multiple interpreters

3. Abstract over effects

(4. Inspect / rewrite / compile programs)

Big approaches

Reification

Model programs as data

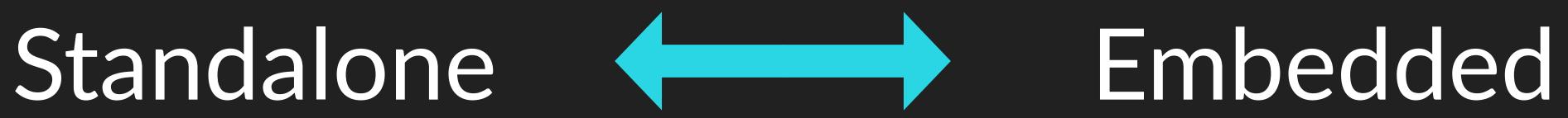
Church encoding

Model programs as sets of method calls

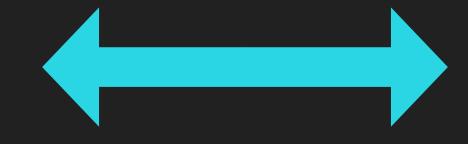
Reification Free Model programs as data

Church encoding Tagless final

Model programs as sets of method calls



Standalone



Embedded

Completely custom Independent of host language

Align with host language Reuse host language features

Reified encodings

A calculated example

$$(1 + 2) * (3 + 4)$$

```
val program = () \Rightarrow (1 + 2) * (3 + 4)
```

```
val program = () ⇒
  (1 + 2) * (3 + 4)

program()
// res0: Int = 21
```

```
val program = () ⇒
  (1 + 2) * (3 + 4)

def eval(prog: () ⇒ Int): Int =
  ???
```

```
val program = () ⇒
  (1 + 2) * (3 + 4)

def eval(prog: () ⇒ Int): Int =
  prog()
```

```
sealed trait Expr

case class Lit(value: Int) extends Expr

case class Add(a: Expr, b: Expr) extends Expr

case class Mul(a: Expr, b: Expr) extends Expr
```

```
sealed trait Expr
case class Lit(value: Int) extends Expr
case class Add(a: Expr, b: Expr) extends Expr
case class Mul(a: Expr, b: Expr) extends Expr

val program: Expr =
  Mul(Add(Lit(1), Lit(2)), Add(Lit(3), Lit(4)))
```

```
sealed trait Expr
case class Lit(value: Int) extends Expr
case class Add(a: Expr, b: Expr) extends Expr
case class Mul(a: Expr, b: Expr) extends Expr
val program: Expr =
  Mul(Add(Lit(1), Lit(2)), Add(Lit(3), Lit(4)))
def eval(expr: Expr): Int =
  expr match {
    case Lit(value) ⇒ value
    case Add(a, b) \Rightarrow eval(a) + eval(b)
    case Mul(a, b) \Rightarrow eval(a) * eval(b)
eval(program)
// res0: Int = 21
```

```
def eval(expr: Expr): Int =
  expr match {
    case Lit(value) ⇒ value
    case Add(a, b) ⇒ eval(a) + eval(b)
    case Mul(a, b) ⇒ eval(a) * eval(b)
}
```

```
def evalAsync(expr: Expr): Future[Int] =
  expr match {
     case Lit(value) ⇒
       Future.successful(value)
     case Add(a, b) \Rightarrow
       val x = evalAsync(a)
       val y = evalAsync(b)
       x.flatMap(a \Rightarrow y.map(b \Rightarrow a + b))
     case Mul(a, b) \Rightarrow
       val x = evalAsync(a)
       val y = evalAsync(b)
       x.flatMap(a \Rightarrow y.map(b \Rightarrow a * b))
```

```
def prettyPrint(expr: Expr): String =
  expr match {
    case Lit(value) ⇒
      value.toString

  case Add(a, b) ⇒
    s"${prettyPrint(a)} + ${prettyPrint(b)}"

  case Mul(a, b) ⇒
    s"${prettyPrint(a)} * ${prettyPrint(b)}"
}
```

```
def simplify(expr: Expr): Expr =
  expr match {
    case Mul(a, Add(b, c)) \Rightarrow
       simplify(Add(Mul(a, b), Mul(a, c)))
    case Mul(Add(a, b), c) \Rightarrow
       simplify(Add(Mul(a, c), Mul(b, c)))
    case Mul(a, b) \Rightarrow
       Mul(simplify(a), simplify(b))
    case Add(a, b) \Rightarrow
       Add(simplify(a), simplify(b))
    case Lit(v) \Rightarrow
       Lit(v)
```

Simplify the language



Empower the interpreter



What about types?

Untyped DSLs

1 < 2 & 3 < 4

```
sealed trait Expr

case class Lit(value: Int) extends Expr

case class Lt(a: Expr, b: Expr) extends Expr

case class And(a: Expr, b: Expr) extends Expr
```

```
val validProgram: Expr =
And(Lt(Lit(1), Lit(2)), Lt(Lit(3), Lit(4)))
```

```
val invalidProgram: Expr =
Lt(And(Lit(1), Lit(2)), And(Lit(3), Lit(4)))
```

```
def eval(program: Expr): ??? =
   ???
```

```
Boolean or integer expression?

def eval(program: Expr): ??? = ???
```

```
Boolean or integer expression ?

def eval(program: Expr): ??? =
???

Model errors here ?
```

```
Boolean or integer expression?
def eval(program: Expr): ??? =
  program match {
     case Lit(n) \Rightarrow
        ???
                             Model errors here?
     case Lt(a, b) \Rightarrow
        ???
     case And(a, b) \Rightarrow
        ???
        Are sub-expressions the right type?
```

```
Boolean or integer expression?
```

```
def eval(program: Expr): Either[Error, ???] =
  program match {
     case Lit(n) \Rightarrow
       ???
     case Lt(a, b) \Rightarrow
       ???
    case And(a, b) \Rightarrow
```

Are sub-expressions the right type?

```
sealed trait Value
case class IntValue(n: Int) extends Expr
case class BooleanValue(b: Boolean) extends Expr
def eval(program: Expr): Either[Error, Value] =
  program match {
    case Lit(n) \Rightarrow
       ???
    case Lt(a, b) \Rightarrow
       ???
    case And(a, b) \Rightarrow
       ???
       Are sub-expressions the right type?
```

```
sealed trait Value
case class IntValue(n: Int) extends Expr
case class BooleanValue(b: Boolean) extends Expr
def eval(program: Expr): Either[Error, Value] =
  program match {
    case Lit(n) \Rightarrow
       Right(IntValue(n))
    case Lt(a, b) \Rightarrow
      val x = evalAsInt(a)
       val y = evalAsInt(b)
       x.flatMap(a \Rightarrow y.map(b \Rightarrow a < b))
    case And(a, b) \Rightarrow
       val x = evalAsBool(a)
       val y = evalAsBool(b)
       x.flatMap(a \Rightarrow y.map(b \Rightarrow a & b))
```

Language mismatch



Complex interpreter

Can this be simplified?

Typed DSLs

```
sealed trait Expr

case class Lit(value: Int) extends Expr

case class Lt(a: Expr, b: Expr) extends Expr

case class And(a: Expr, b: Expr) extends Expr
```

```
sealed trait Expr[A]

case class Lit[A](value: A) extends Expr[A]

case class Lt(a: Expr[Int], b: Expr[Int])
  extends Expr[Boolean]

case class And(a: Expr[Boolean], b: Expr[Boolean])
  extends Expr[Boolean]
```

```
val validProgram: Expr[Boolean] =
And(Lt(Lit(1), Lit(2)), Lt(Lit(3), Lit(4)))
```

```
val invalidProgram: Expr[Boolean] =
  Lt(And(Lit(1), Lit(2)), And(Lit(3), Lit(4)))
// expected Expr[Boolean], found Expr[Int]
```

```
def eval[A](program: Expr[A]): A =
  program match {
    case Lit(v) ⇒
      v

    case Lt(a, b) ⇒
      eval(a) < eval(b)

    case And(a, b) ⇒
      eval(a) & eval(b)
}</pre>
```

```
def evalAsync[A](program: Expr[A]): Future[A] =
  program match {
     case Lit(v) \Rightarrow
     case Lt(a, b) \Rightarrow
       val x = evalAsync(a)
       val y = evalAsync(b)
       x.flatMap(a \Rightarrow y.map(b \Rightarrow a < b))
     case And(a, b) \Rightarrow
       val x = evalAsync(a)
       val y = evalAsync(b)
       x.flatMap(a \Rightarrow y.map(b \Rightarrow a & b))
```

```
def prettyPrint[A](program: Expr[A]): String =
  program match {
    case Lit(v) ⇒
     v.toString

    case Lt(a, b) ⇒
     s"${prettyPrint(a)} < ${prettyPrint(b)}"

    case And(a, b) ⇒
     s"${prettyPrint(a)} & ${prettyPrint(b)}"
}</pre>
```

```
def simplify[A](program: Expr[A]): Expr[A] =
  program match {
     case Lit(n) \Rightarrow
       Lit(n)
    case Lt(Lit(a), Lit(b)) \Rightarrow
       Lit(a < b)
     case Lt(a, b) \Rightarrow
       Lt(simplify(a), simplify(b))
    case And(a, b) \Rightarrow
       And(simplify(a), simplify(b))
```

Deeper embedding Simpler interpreter

Ordering

```
def eval[A](program: Expr[A]): A =
  program match {
    case Lit(v) ⇒
      v

    case Lt(a, b) ⇒
      eval(a) < eval(b)

    case And(a, b) ⇒
      eval(a) & eval(b)
}</pre>
```

Can we have explicit ordering?

Yes, with monads!

Monadic DSLs

```
sealed trait Expr[A]

case class Lit[A](value: A) extends Expr[A]

case class Lt(a: Expr[Int], b: Expr[Int])
  extends Expr[Boolean]

case class And(a: Expr[Boolean], b: Expr[Boolean])
  extends Expr[Boolean]
```

```
sealed trait Expr[A]
case class Lit[A](value: A) extends Expr[A]
case class Lt(a: Expr[Int], b: Expr[Int])
  extends Expr[Boolean]
case class And(a: Expr[Boolean], b: Expr[Boolean])
  extends Expr[Boolean]
case class FlatMap[A, B](
  a: Expr[A],
  f: A \Rightarrow Expr[B]) extends Expr[B]
```

```
sealed trait Expr[A]
case class Lit[A](value: A) extends Expr[A]
case class Lt(a: Int, b: Int)
  extends Expr[Boolean]
case class And(a: Boolean, b: Boolean)
  extends Expr[Boolean]
case class FlatMap[A, B](
  a: Expr[A],
  f: A \Rightarrow Expr[B]) extends Expr[B]
```

```
sealed trait Expr[A]
case class Pure[A](value: A) extends Expr[A]
case class Lt(a: Int, b: Int)
  extends Expr[Boolean]
case class And(a: Boolean, b: Boolean)
  extends Expr[Boolean]
case class FlatMap[A, B](
 a: Expr[A],
 f: A \Rightarrow Expr[B]) extends Expr[B]
```

```
val program: Expr[Boolean] =
And(Lt(Pure(1), Pure(2)), Lt(Pure(3), Pure(4)))
```

```
val program: Expr[Boolean] =
  FlatMap(Pure(1), (a: Int) ⇒
    FlatMap(Pure(2), (b: Int) ⇒
        FlatMap(Lt(a, b), (x: Boolean) ⇒
        FlatMap(Pure(3), (c: Int) ⇒
        FlatMap(Pure(4), (d: Int) ⇒
        FlatMap(Lt(c, d), (y: Boolean) ⇒
        FlatMap(And(x, y), (z: Boolean) ⇒
        Pure(z))))))))
```

```
val program: Expr[Boolean] =
  for {
    a ← pure(1)
    b ← pure(2)
    x ← lt(a, b)
    c ← pure(3)
    d ← pure(4)
    y ← lt(c, d)
    z ← and(a, b)
} yield z
```

```
def eval[A](program: Expr[A]): A =
  program match {
     case Pure(v) \Rightarrow
     case Lt(a, b) \Rightarrow
       a < b
     case And(a, b) \Rightarrow
       a & b
     case FlatMap(a, fn) \Rightarrow
       eval(fn(eval(a))
```

```
def evalAsync[A](program: Expr[A]): Future[A] =
  program match {
    case Pure(v) \Rightarrow
       Future.successful(v)
    case Lt(a, b) \Rightarrow
       Future.successful(a < b)</pre>
    case And(a, b) \Rightarrow
       Future.successful(a & b)
    case FlatMap(a, fn) \Rightarrow
       evalAsync(a).flatMap(a \Rightarrow evalAsync(fn(a))
```

```
def prettyPrint[A](program: Expr[A]): String =
  program match {
    case Pure(v) \Rightarrow
       v.toString
    case Lt(a, b) \Rightarrow
       s"${prettyPrint(a)} < ${prettyPrint(b)}"</pre>
    case And(a, b) \Rightarrow
       s"${prettyPrint(a)} & ${prettyPrint(b)}"
    case FlatMap(a, fn) \Rightarrow
       ???
```

```
def simplify[A](program: Expr[A]): Expr[A] =
  program match {
     case Pure(v) \Rightarrow
       ???
     case Lt(a, b) \Rightarrow
       ???
     case And(a, b) \Rightarrow
       ???
     case FlatMap(a, fn) \Rightarrow
       ???
```

Generality Inspectability

Do we have to write FlatMap ourselves?

Free monadic DSLs

Abstract Pure/FlatMap

```
sealed trait Expr[A]
case class Pure[A](value: A) extends Expr[A]
case class Lt(a: Int, b: Int)
  extends Expr[Boolean]
case class And(a: Boolean, b: Boolean)
  extends Expr[Boolean]
case class FlatMap[A, B](
 a: Expr[A],
 f: A \Rightarrow Expr[B]) extends Expr[B]
```

```
sealed trait Expr[A]
case class Lt(a: Int, b: Int)
  extends Expr[Boolean]
case class And(a: Boolean, b: Boolean)
  extends Expr[Boolean]
case class Pure[A](value: A) extends Expr[A]
case class FlatMap[A, B](
 a: Expr[A],
 f: A \Rightarrow Expr[B]) extends Expr[B]
```

```
sealed trait ExprAlg[A]
case class Lt(a: Int, b: Int)
  extends ExprAlg[Boolean]
case class And(a: Boolean, b: Boolean)
  extends ExprAlg[Boolean]
case class Pure[A](value: A) extends Expr[A]
case class FlatMap[A, B](
  a: Expr[A],
 f: A \Rightarrow Expr[B]) extends Expr[B]
```

```
sealed trait ExprAlg[A]
case class Lt(a: Int, b: Int)
  extends ExprAlg[Boolean]
case class And(a: Boolean, b: Boolean)
  extends ExprAlg[Boolean]
sealed trait ExprMonad[A]
case class Pure[A](value: A) extends ExprMonad[A]
case class FlatMap[A, B](
  a: ExprMonad[A],
  f: A ⇒ ExprMonad[B]) extends ExprMonad[B]
```

```
sealed trait ExprAlg[A]
case class Lt(a: Int, b: Int)
 extends ExprAlg[Boolean]
case class And(a: Boolean, b: Boolean)
  extends ExprAlg[Boolean]
sealed trait ExprMonad[A]
case class Pure[A](value: A) extends ExprMonad[A]
case class Suspend[A](value: ExprAlg[A])
  extends ExprMonad[A]
case class FlatMap[A, B](
 a: ExprMonad[A],
  f: A ⇒ ExprMonad[B]) extends ExprMonad[B]
```

```
sealed trait ExprMonad[A]

case class Pure[A](value: A) extends ExprMonad[A]

case class Suspend[A](value: ExprAlg[A])
  extends ExprMonad[A]

case class FlatMap[A, B](
  a: ExprMonad[A],
  f: A ⇒ ExprMonad[B]) extends ExprMonad[B]
```

```
sealed trait Free[F[_], A]

case class Pure[F[_], A](value: A) extends Free[F, A]

case class Suspend[F[_], A](value: F[A])
  extends Free[F, A]

case class FlatMap[F[_], A, B](
  a: Free[F, A],
  f: A ⇒ Free[F, B]) extends Free[F, B]
```

```
sealed trait ExprAlg[A]

case class Lt(a: Int, b: Int)
  extends ExprAlg[Boolean]

case class And(a: Boolean, b: Boolean)
  extends ExprAlg[Boolean]
```

```
val program: Expr[Boolean] =
  for {
    a ← pure(1)
    b ← pure(2)
    x ← suspend(lt(a, b))
    c ← pure(3)
    d ← pure(4)
    y ← suspend(lt(c, d))
    z ← suspend(and(x, y))
  } yield z
```

import cats.free.Free

type Expr[A] = Free[ExprAlg, A]

```
import cats.free.Free
type Expr[A] = Free[ExprAlg, A]
def lit[A](value: A): Expr[A] =
  Free.pure[ExprAlg, A](value)
def lt(a: Int, b: Int): Expr[Boolean] =
  Free.liftF[ExprAlg, Boolean](Lt(a, b))
def and(a: Boolean, b: Boolean): Expr[Boolean] =
  Free.liftF[ExprAlg, Boolean](And(a, b))
def fail[A](msg, String): Expr[A] =
  Free.liftF[ExprAlg, A](Fail(msg))
```

```
val program: Expr[Boolean] =
  for {
    a ← lit(1)
    b ← lit(2)
    x ← lt(a, b)
    c ← lit(3)
    d ← lit(4)
    y ← lt(c, d)
    z ← and(x, y)
  } yield z
```

```
import cats.arrow.FunctionK
object evalAsync extends FunctionK[ExprAlg, Future] {
  def apply[A](expr: ExprAlg[A]): Future[A] =
    expr match {
     case Lt(a, b) \Rightarrow
         Future.successful(a < b)</pre>
      case And(a, b) \Rightarrow
         Future.successful(a & b)
```

```
val program: Expr[Boolean] =
  for {
     a \leftarrow lit(1)
     b \leftarrow lit(2)
     x \leftarrow lt(a, b)
     c \leftarrow lit(3)
     d \leftarrow lit(4)
     y \leftarrow lt(c, d)
     z \leftarrow and(x, y)
  } yield z
program.foldMap(evalAsync)
// res0: Future[Boolean] = Future(...)
```

```
import cats.arrow.FunctionK
import cats.Id
object eval extends FunctionK[ExprAlg, Id] {
  def apply[A](expr: ExprAlg[A]): A =
    expr match {
     case Lt(a, b) \Rightarrow
        a < b
      case And(a, b) \Rightarrow
        a & b
```

Free provides sequencing

Algebra provides steps

Combine DSLs

```
import cats.data.EitherK
import cats.free.Free

type Alg[A] = EitherK[Alg1, Alg2, A]

type Expr[A] = Free[Alg, A]
```

Lots of boilerplate

(see Freestyle, http://frees.io)

Church encodings

Encode programs as Scala expressions

Simple Church encoding

```
sealed trait Expr[A]

case class Lit[A](value: A) extends Expr[A]

case class Lt(a: Int, b: Int)
   extends Expr[Boolean]

case class And(a: Boolean, b: Boolean)
   extends Expr[Boolean]
```

```
trait ExprDsl {
  def lit[A](n: A): A
  def lt(a: Int, b: Int): Boolean
  def and(a: Boolean, b: Boolean): Boolean
}
```

```
def program(dsl: ExprDsl): Boolean = {
  import dsl._
  and(lt(lit(1), lit(2)), lt(lit(3), lit(4)))
}
```

```
object Interpreter extends ExprDsl {
  def lit[A](n: A): A =
        n

  def lt(a: Int, b: Int): Boolean =
        a < b

  def and(a: Boolean, b: Boolean): Boolean =
        a && b
}</pre>
```

```
def program(dsl: ExprDsl): Boolean = {
  import dsl._
  and(lt(lit(1), lit(2)), lt(lit(3), lit(4)))
}
program(Interpreter)
// res0: Boolean = true
```

Can't abstract over effects

Tagless final encoding

Tagless final encoding (Finally tagless encoding?)

```
trait ExprDsl {
  def lit[A](n: A): A
  def lt(a: Int, b: Int): Boolean
  def and(a: Boolean, b: Boolean): Boolean
}
```

```
trait ExprDsl[F[_]] {
  def lit[A](n: A): F[A]
  def lt(a: Int, b: Int): F[Boolean]
  def and(a: Boolean, b: Boolean): F[Boolean]
}
```

```
object AsyncInterpreter extends ExprDsl[Future] {
   def lit[A](n: A): Future[A] =
      Future.successful(n)

def lt(a: Int, b: Int): Future[Boolean] =
      Future.successful(a < b)

def and(a: Boolean, b: Boolean): Future[Boolean] =
      Future.successful(a & b)
}</pre>
```

```
import cats.Id
object Interpreter extends ExprDsl[Id] {
 def lit[A](n: A): Id[A] =
 def lt(a: Int, b: Int): Id[Boolean] =
   a < b
 def and(a: Boolean, b: Boolean): Id[Boolean] =
   a & b
```

```
import cats.Monad
import cats.syntax.functor._
import cats.syntax.flatMap._
def program[F[_]](dsl: ExprDsl[F])
       (implicit monad: Monad): F[Boolean] = {
  import dsl._
  for {
    a \leftarrow lit(1)
    b \leftarrow lit(2)
    x \leftarrow lt(a, b)
    c \leftarrow lit(3)
    d \leftarrow lit(4)
     y \leftarrow lt(c, d)
     z \leftarrow and(x, y)
  } yield z
```

```
import cats. Monad
import cats.instances.future._
import cats.syntax.functor._
import cats.syntax.flatMap._
def program[F[_]](dsl: ExprDsl[F])
       (implicit monad: Monad): F[Boolean] = {
  import dsl._
  for {
    a \leftarrow lit(1)
    b \leftarrow lit(2)
    x \leftarrow lt(a, b)
    c \leftarrow lit(3)
    d \leftarrow lit(4)
    y \leftarrow lt(c, d)
    z \leftarrow and(x, y)
  } yield z
program(AsyncInterpreter)
//res0: Future[Boolean] = Future(Success(true))
```

```
import cats. Monad
import cats.syntax.functor._
import cats.syntax.flatMap._
def program[F[_]](dsl: ExprDsl[F])
       (implicit monad: Monad): F[Boolean] = {
  import dsl._
  for {
    a \leftarrow lit(1)
    b \leftarrow lit(2)
    x \leftarrow lt(a, b)
    c \leftarrow lit(3)
    d \leftarrow lit(4)
     y \leftarrow lt(c, d)
     z \leftarrow and(x, y)
  } yield z
program(Interpreter)
res0: Id[Boolean] = true
```

Combine DSLs

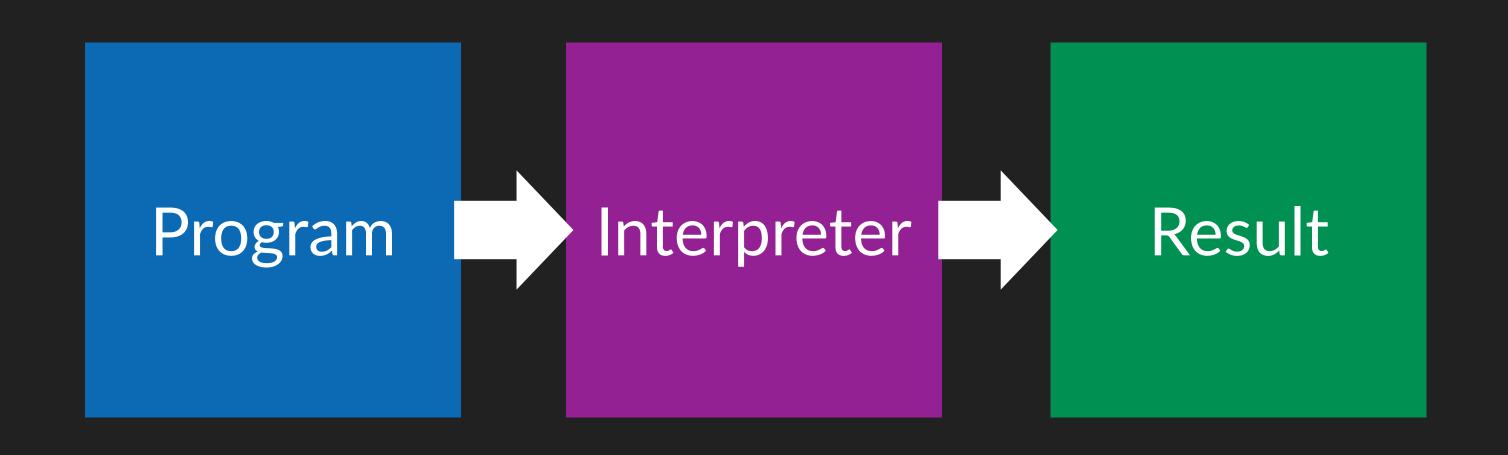
```
def program[F[_], A](dsl1: Dsl1[F], dsl2: Dsl2[F])
    (implicit monad: Monad[F]): A = {
    import dsl1._
    import dsl2._

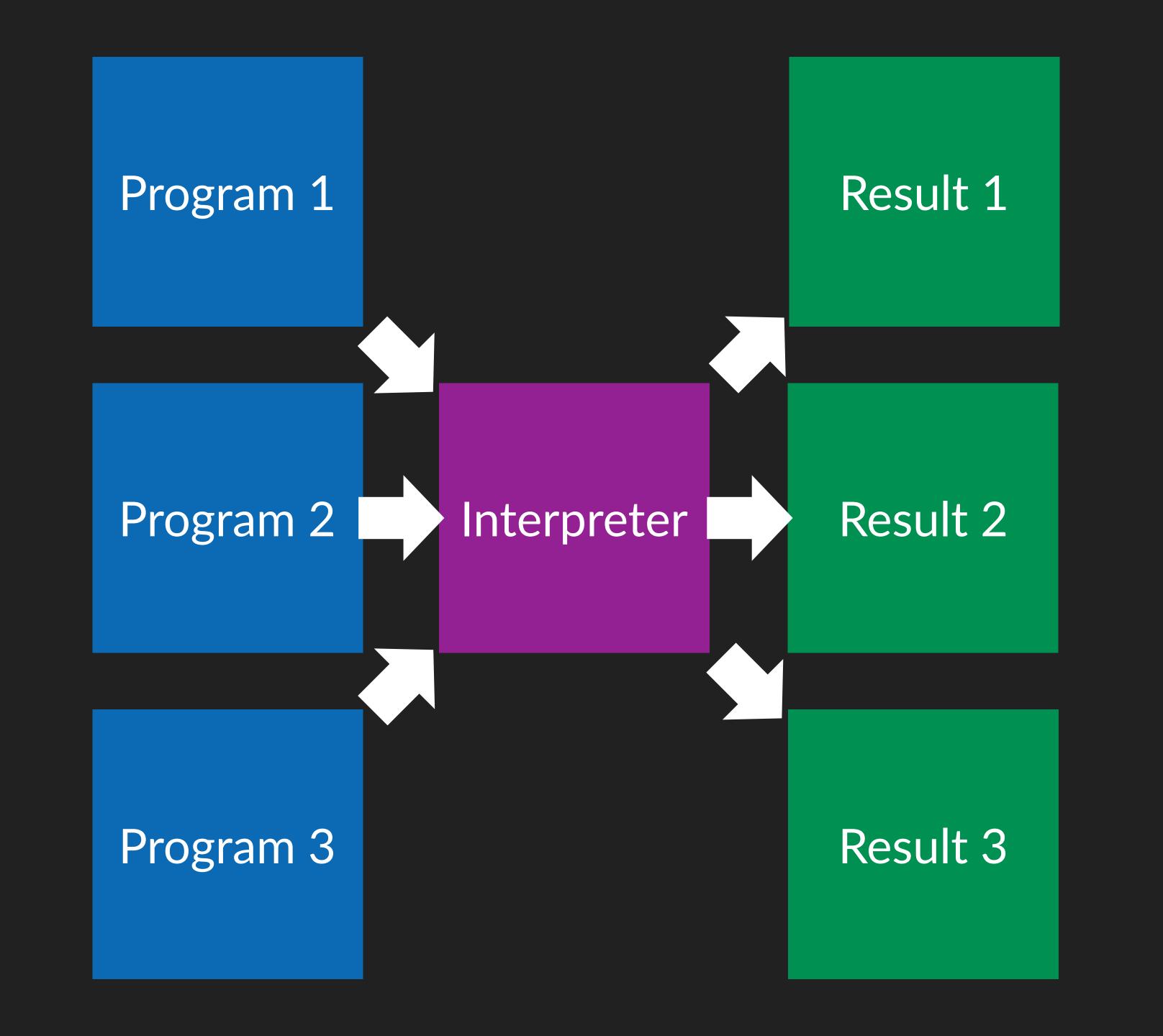
// ...
}
```

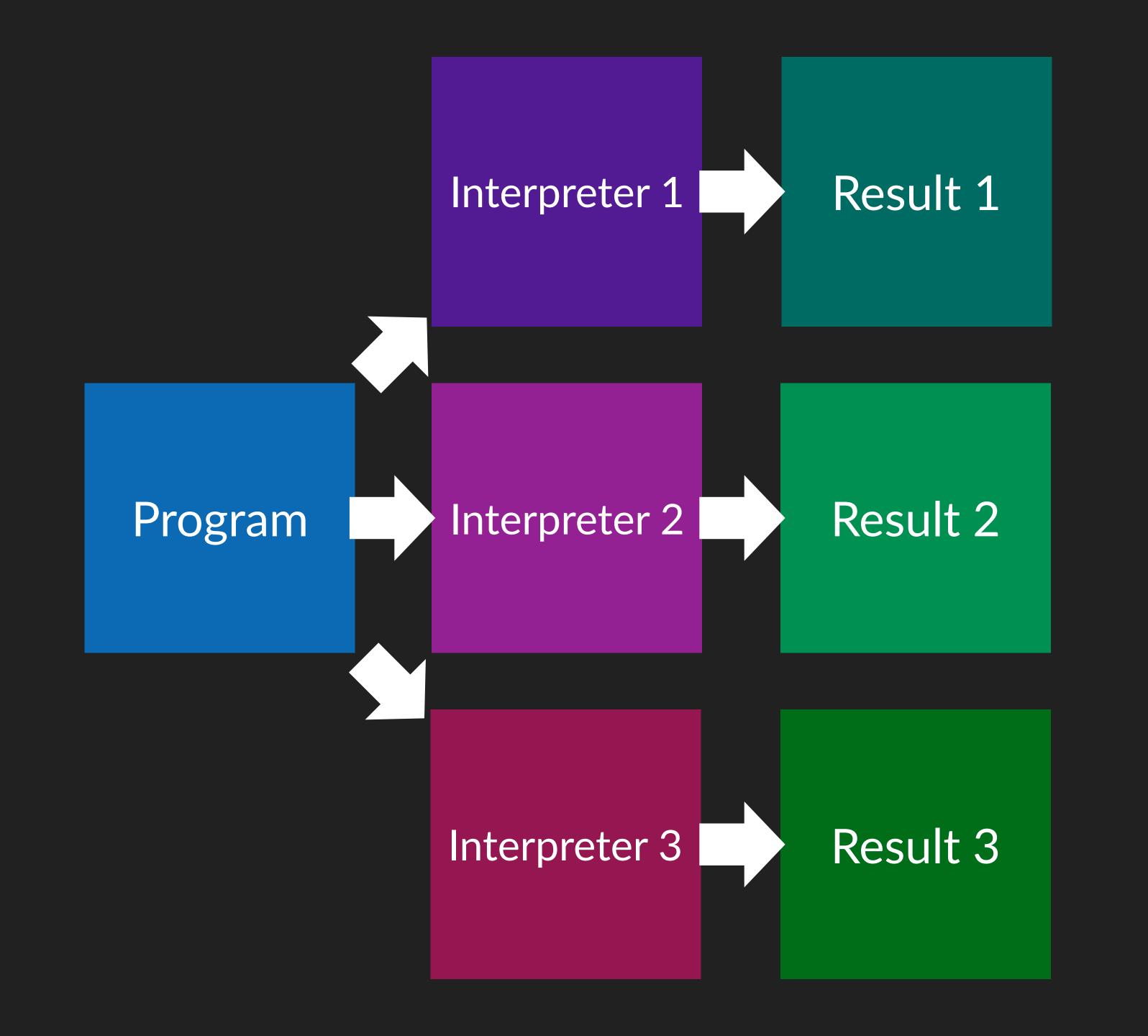
Difficult to inspect

Less boilerplate than Free

Summary







Reification Free Model code as data

Church encoding Tagless final

Model code as ... code

Standalone

Completely custom

Embedded

Reuse host language features

Constraints liberate, liberty constrains

https://www.youtube.com/watch?v=GqmsQeSzMdw

Free

https://underscore.io/blog/posts/2015/04/14/free-monads-are-simple.html

Free with multiple algebras

https://underscore.io/blog/posts/2017/03/29/free-inject.html

Tagless final

https://skillsmatter.com/skillscasts/ 10007-free-vs-tagless-final-with-chris-birchall

Slides, code samples, and notes (WIP)

https://github.com/underscoreio/interpreters-and-you

Thankyou

https://github.com/underscoreio/interpreters-and-you

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