

Managing Inventory

12

**PowerPoint presentation to accompany
Heizer and Render
Operations Management, Global Edition, Eleventh Edition
Principles of Operations Management, Global Edition, Ninth Edition**

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Outline

- ▶ **Global Company Profile:**
Amazon.com
- ▶ The Importance of Inventory
- ▶ Managing Inventory
- ▶ Inventory Models
- ▶ Inventory Models for Independent Demand

Outline - Continued

- ▶ Probabilistic Models and Safety Stock
- ▶ Single-Period Model
- ▶ Fixed-Period (P) Systems

Learning Objectives

When you complete this chapter you should be able to:

- 1. Conduct** an ABC analysis
- 2. Explain** and use cycle counting
- 3. Explain** and use the EOQ model for independent inventory demand
- 4. Compute** a reorder point and safety stock

Learning Objectives

When you complete this chapter you should be able to:

- 5. Apply** the production order quantity model
- 6. Explain** and use the quantity discount model
- 7. Understand** service levels and probabilistic inventory models

Inventory Management at Amazon.com

- ▶ Amazon.com started as a “virtual” retailer – no inventory, no warehouses, no overhead; just computers taking orders to be filled by others
- ▶ Growth has forced Amazon.com to become a world leader in warehousing and inventory management

Inventory Management at Amazon.com

1. Each order is assigned by computer to the closest distribution center that has the product(s)
2. A “flow meister” at each distribution center assigns work crews
3. Lights indicate products that are to be picked and the light is reset
4. Items are placed in crates on a conveyor, bar code scanners scan each item 15 times to virtually eliminate errors

Inventory Management at Amazon.com

5. Crates arrive at central point where items are boxed and labeled with new bar code
6. Gift wrapping is done by hand at 30 packages per hour
7. Completed boxes are packed, taped, weighed and labeled before leaving warehouse in a truck
8. Order arrives at customer within 1 - 2 days

Inventory Management

The objective of inventory management is to strike a balance between inventory investment and customer service

Importance of Inventory

- ▶ One of the most expensive assets of many companies representing as much as 50% of total invested capital
- ▶ Operations managers must balance inventory investment and customer service

Functions of Inventory

1. To provide a selection of goods for anticipated demand and to separate the firm from fluctuations in demand
2. To decouple or separate various parts of the production process
3. To take advantage of quantity discounts
4. To hedge against inflation

Types of Inventory

- ▶ Raw material
 - ▶ Purchased but not processed
- ▶ Work-in-process (WIP)
 - ▶ Undergone some change but not completed
 - ▶ A function of cycle time for a product
- ▶ Maintenance/repair/operating (MRO)
 - ▶ Necessary to keep machinery and processes productive
- ▶ Finished goods
 - ▶ Completed product awaiting shipment

The Material Flow Cycle

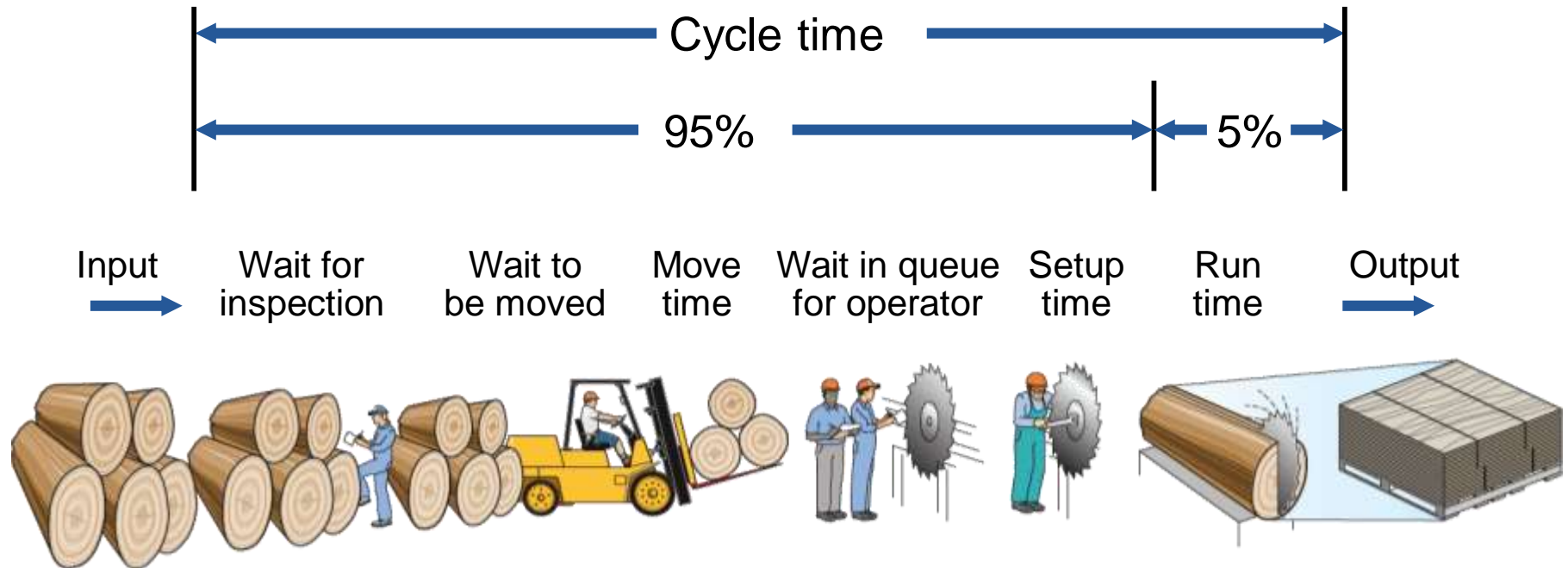


Figure 12.1

Managing Inventory

1. How inventory items can be classified (*ABC analysis*)
2. How accurate inventory records can be maintained

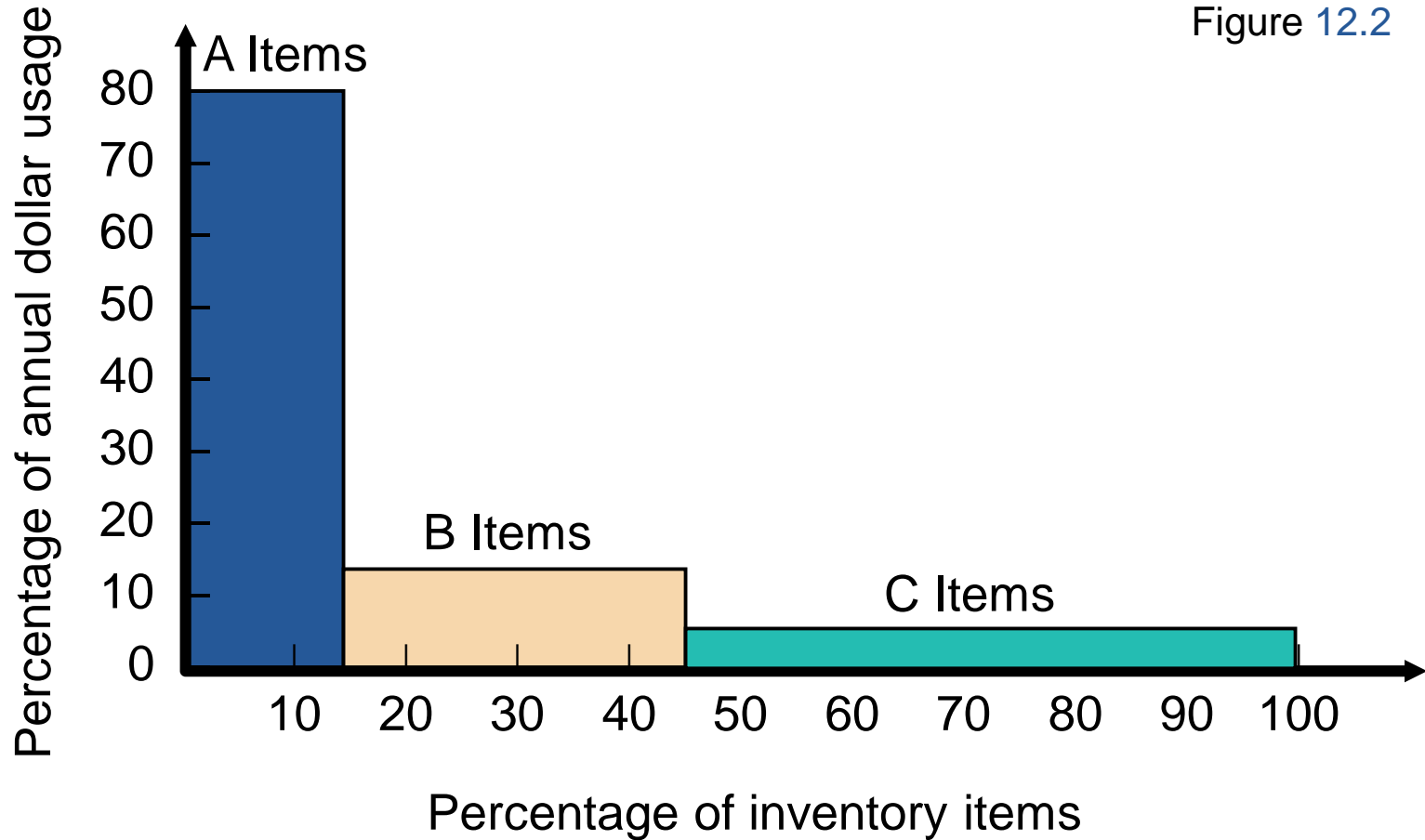
ABC Analysis

- ▶ Divides inventory into three classes based on annual dollar volume
 - ▶ Class A - high annual dollar volume
 - ▶ Class B - medium annual dollar volume
 - ▶ Class C - low annual dollar volume
- ▶ Used to establish policies that focus on the few critical parts and not the many trivial ones

ABC Analysis

ABC Calculation							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	
ITEM STOCK NUMBER	PERCENT OF NUMBER OF ITEMS STOCKED	ANNUAL VOLUME (UNITS)	x UNIT COST	= ANNUAL DOLLAR VOLUME	PERCENT OF ANNUAL DOLLAR VOLUME	CLASS	
#10286	20%	1,000	\$ 90.00	\$ 90,000	38.8%	72%	A
#11526		500	154.00	77,000	33.2%		A
#12760		1,550	17.00	26,350	11.3%	23%	B
#10867	30%	350	42.86	15,001	6.4%		B
#10500		1,000	12.50	12,500	5.4%		B
#12572		600	\$ 14.17	\$ 8,502	3.7%	5%	C
#14075		2,000	.60	1,200	.5%		C
#01036	50%	100	8.50	850	.4%		C
#01307		1,200	.42	504	.2%		C
#10572		250	.60	150	.1%		C
		<u>8,550</u>		<u>\$232,057</u>	<u>100.0%</u>		

ABC Analysis



ABC Analysis

- ▶ Other criteria than annual dollar volume may be used
 - ▶ High shortage or holding cost
 - ▶ Anticipated engineering changes
 - ▶ Delivery problems
 - ▶ Quality problems

ABC Analysis

- ▶ Policies employed may include
 1. More emphasis on supplier development for A items
 2. Tighter physical inventory control for A items
 3. More care in forecasting A items

Record Accuracy

- ▶ Accurate records are a critical ingredient in production and inventory systems
 - ▶ Periodic systems require regular checks of inventory
 - ▶ Two-bin system
 - ▶ Perpetual inventory tracks receipts and subtractions on a continuing basis
 - ▶ May be semi-automated

Record Accuracy

- ▶ Incoming and outgoing record keeping must be accurate
- ▶ Stockrooms should be secure
- ▶ Necessary to make precise decisions about ordering, scheduling, and shipping



Cycle Counting

- ▶ Items are counted and records updated on a periodic basis
- ▶ Often used with ABC analysis
- ▶ Has several advantages
 1. Eliminates shutdowns and interruptions
 2. Eliminates annual inventory adjustment
 3. Trained personnel audit inventory accuracy
 4. Allows causes of errors to be identified and corrected
 5. Maintains accurate inventory records

Cycle Counting Example

5,000 items in inventory, 500 A items, 1,750 B items, 2,750 C items

Policy is to count A items every month (20 working days), B items every quarter (60 days), and C items every six months (120 days)

ITEM CLASS	QUANTITY	CYCLE COUNTING POLICY	NUMBER OF ITEMS COUNTED PER DAY
A	500	Each month	$500/20 = 25/\text{day}$
B	1,750	Each quarter	$1,750/60 = 29/\text{day}$
C	2,750	Every 6 months	$2,750/120 = 23/\text{day}$
			<hr/> 77/day

Control of Service Inventories

- ▶ Can be a critical component of profitability
- ▶ Losses may come from shrinkage or pilferage
- ▶ Applicable techniques include
 1. Good personnel selection, training, and discipline
 2. Tight control of incoming shipments
 3. Effective control of all goods leaving facility



Inventory Models

- ▶ **Independent demand** - the demand for item is independent of the demand for any other item in inventory
- ▶ **Dependent demand** - the demand for item is dependent upon the demand for some other item in the inventory

Inventory Models

- ▶ **Holding costs** - the costs of holding or “carrying” inventory over time
- ▶ **Ordering costs** - the costs of placing an order and receiving goods
- ▶ **Setup costs** - cost to prepare a machine or process for manufacturing an order
 - ▶ May be highly correlated with **setup time**

Holding Costs

TABLE 12.1

Determining Inventory Holding Costs

CATEGORY	COST (AND RANGE) AS A PERCENT OF INVENTORY VALUE
Housing costs (building rent or depreciation, operating costs, taxes, insurance)	6% (3 - 10%)
Material handling costs (equipment lease or depreciation, power, operating cost)	3% (1 - 3.5%)
Labor cost (receiving, warehousing, security)	3% (3 - 5%)
Investment costs (borrowing costs, taxes, and insurance on inventory)	11% (6 - 24%)
Pilferage, space, and obsolescence (much higher in industries undergoing rapid change like PCs and cell phones)	3% (2 - 5%)
Overall carrying cost	26%

Holding Costs

TABLE 12.1

Determining Inventory Holding Costs

Holding costs vary considerably depending on the business, location, and interest rates. Generally greater than 15%, some high tech and fashion items have holding costs greater than 40%.

Interest on inventory (including costs, taxes, and insurance on inventory)	11% (6 - 24%)
Pilferage, space, and obsolescence (much higher in industries undergoing rapid change like PCs and cell phones)	3% (2 - 5%)
Overall carrying cost	26%

Inventory Models for Independent Demand

Need to determine when and how much to order

1. Basic economic order quantity (EOQ) model
2. Production order quantity model
3. Quantity discount model

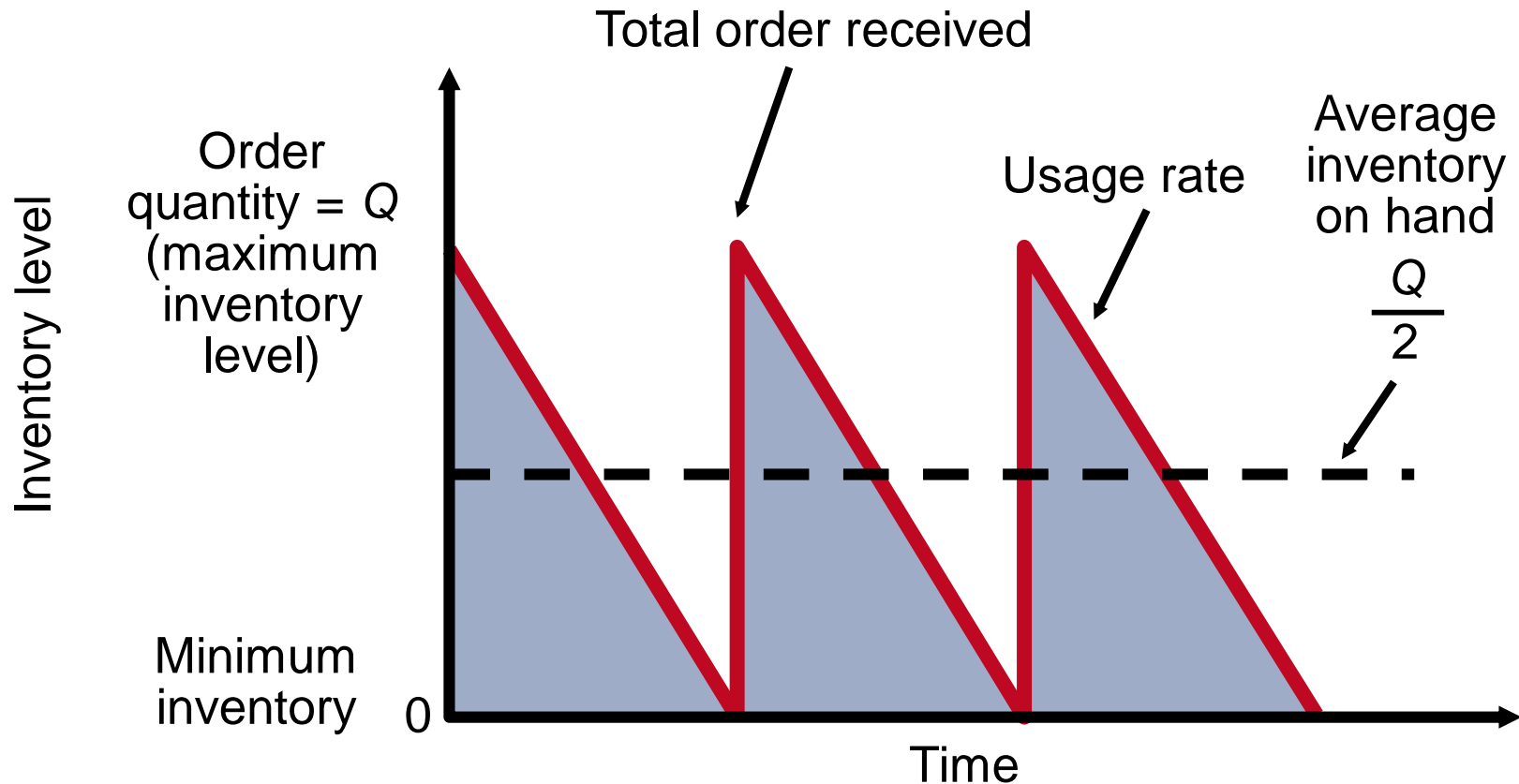
Basic EOQ Model

Important assumptions

1. Demand is known, constant, and independent
2. Lead time is known and constant
3. Receipt of inventory is instantaneous and complete
4. Quantity discounts are not possible
5. Only variable costs are setup (or ordering) and holding
6. Stockouts can be completely avoided

Inventory Usage Over Time

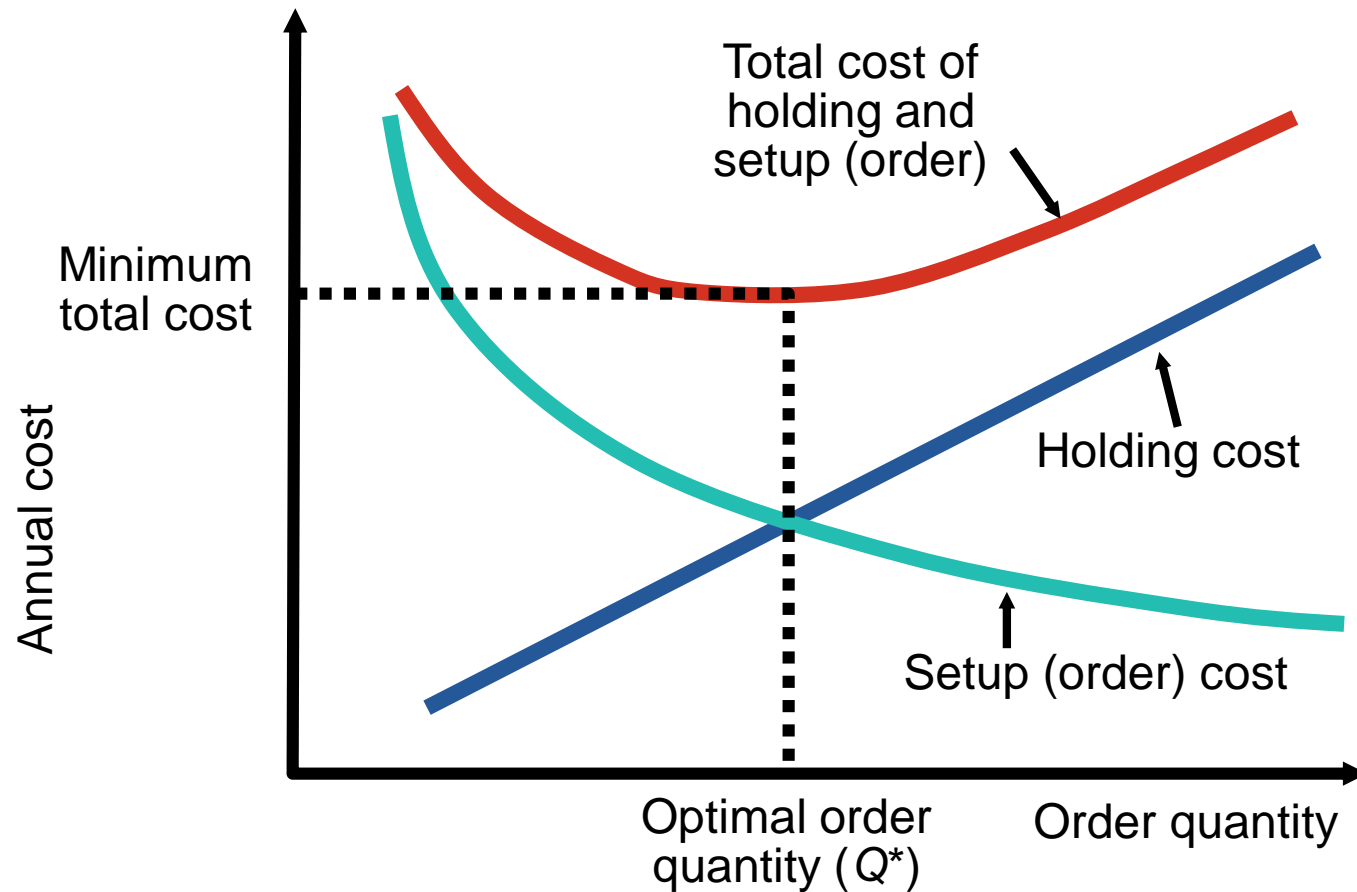
Figure 12.3



Minimizing Costs

Objective is to minimize total costs

Table 12.4(c)



Minimizing Costs

- ▶ By minimizing the sum of setup (or ordering) and holding costs, total costs are minimized
- ▶ Optimal order size Q^* will minimize total cost
- ▶ A reduction in either cost reduces the total cost
- ▶ Optimal order quantity occurs when holding cost and setup cost are equal

Minimizing Cost

$$\text{Annual setup cost} = \frac{D}{Q} S$$

Q = Number of pieces per order

Q^* = Optimal number of pieces per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

Annual setup cost = (Number of orders placed per year)
x (Setup or order cost per order)

$$= \left(\frac{\text{Annual demand}}{\text{Number of units in each order}} \right) \left(\text{Setup or order cost per order} \right)$$

$$= \frac{D}{Q} S$$

Minimizing Cost

$$\text{Annual setup cost} = \frac{D}{Q} S$$

$$\text{Annual holding cost} = \frac{Q}{2} H$$

Q = Number of pieces per order

Q^* = Optimal number of pieces per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

Annual holding cost = (Average inventory level)
x (Holding cost per unit per year)

$$= \left(\frac{\text{Order quantity}}{2} \right) (\text{Holding cost per unit per year})$$

$$= \frac{Q}{2} H$$

Minimizing Cost

$$\text{Annual setup cost} = \frac{D}{Q}S$$

$$\text{Annual holding cost} = \frac{Q}{2}H$$

Q = Number of pieces per order

Q^* = Optimal number of pieces per order (EOQ)

D = Annual demand in units for the inventory item

S = Setup or ordering cost for each order

H = Holding or carrying cost per unit per year

Optimal order quantity is found when annual setup cost equals annual holding cost

$$\frac{D}{Q}S = \frac{Q}{2}H$$

Solving for Q^*

$$2DS = Q^2H$$

$$Q^2 = \frac{2DS}{H}$$

$$Q^* = \sqrt{\frac{2DS}{H}}$$

An EOQ Example

Determine **optimal number of needles to order**

$D = 1,000$ units

$S = \$10$ per order

$H = \$0.50$ per unit per year

$$Q^* = \sqrt{\frac{2DS}{H}}$$

$$Q^* = \sqrt{\frac{2(1,000)(10)}{0.50}} = \sqrt{40,000} = 200 \text{ units}$$

An EOQ Example

Determine **expected number of orders**

$$D = 1,000 \text{ units}$$

$$Q^* = 200 \text{ units}$$

$$S = \$10 \text{ per order}$$

$$H = \$0.50 \text{ per unit per year}$$

$$\begin{array}{l} \text{Expected} \\ \text{number of} \\ \text{orders} \end{array} = N = \frac{\text{Demand}}{\text{Order quantity}} = \frac{D}{Q^*}$$

$$N = \frac{1,000}{200} = 5 \text{ orders per year}$$

An EOQ Example

Determine **optimal time between orders**

$$D = 1,000 \text{ units}$$

$$Q^* = 200 \text{ units}$$

$$S = \$10 \text{ per order}$$

$$N = 5 \text{ orders/year}$$

$$H = \$0.50 \text{ per unit per year}$$

$$\begin{array}{l} \text{Expected} \\ \text{time between} \\ \text{orders} \end{array} = T = \frac{\text{Number of working days per year}}{\text{Expected number of orders}}$$

$$T = \frac{250}{5} = 50 \text{ days between orders}$$

An EOQ Example

Determine the **total annual cost**

$$D = 1,000 \text{ units}$$

$$Q^* = 200 \text{ units}$$

$$S = \$10 \text{ per order}$$

$$N = 5 \text{ orders/year}$$

$$H = \$0.50 \text{ per unit per year}$$

$$T = 50 \text{ days}$$

Total annual cost = Setup cost + Holding cost

$$\begin{aligned} TC &= \frac{D}{Q} S + \frac{Q}{2} H \\ &= \frac{1,000}{200} (\$10) + \frac{200}{2} (\$0.50) \\ &= (5)(\$10) + (100)(\$0.50) \\ &= \$50 + \$50 = \$100 \end{aligned}$$

The EOQ Model

When including actual cost of material P

Total annual cost = Setup cost + Holding cost + Product cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}H + PD$$

Robust Model

- ▶ The EOQ model is robust
- ▶ It works even if all parameters and assumptions are not met
- ▶ The total cost curve is relatively flat in the area of the EOQ

An EOQ Example

Determine optimal number of
 ~~$D = 1,000$ units~~ 1,500 units
 $S = \$10$ per order
 $H = \$0.50$ per unit per year

Only 2% less than
the total cost of
\$125 when the
order quantity was
200

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$= \frac{1,500}{200}(\$10) + \frac{200}{2}(\$0.50)$$

$$= \$75 + \$50 = \$125$$

$$= \frac{1,500}{244.9}(\$10) + \frac{244.9}{2}(\$0.50)$$

$$= 6.125(\$10) + 122.45(\$0.50)$$

$$= \$61.25 + \$61.22 = \$122.47$$

Reorder Points

- ▶ EOQ answers the “how much” question
- ▶ The reorder point (ROP) tells “when” to order
- ▶ Lead time (L) is the time between placing and receiving an order

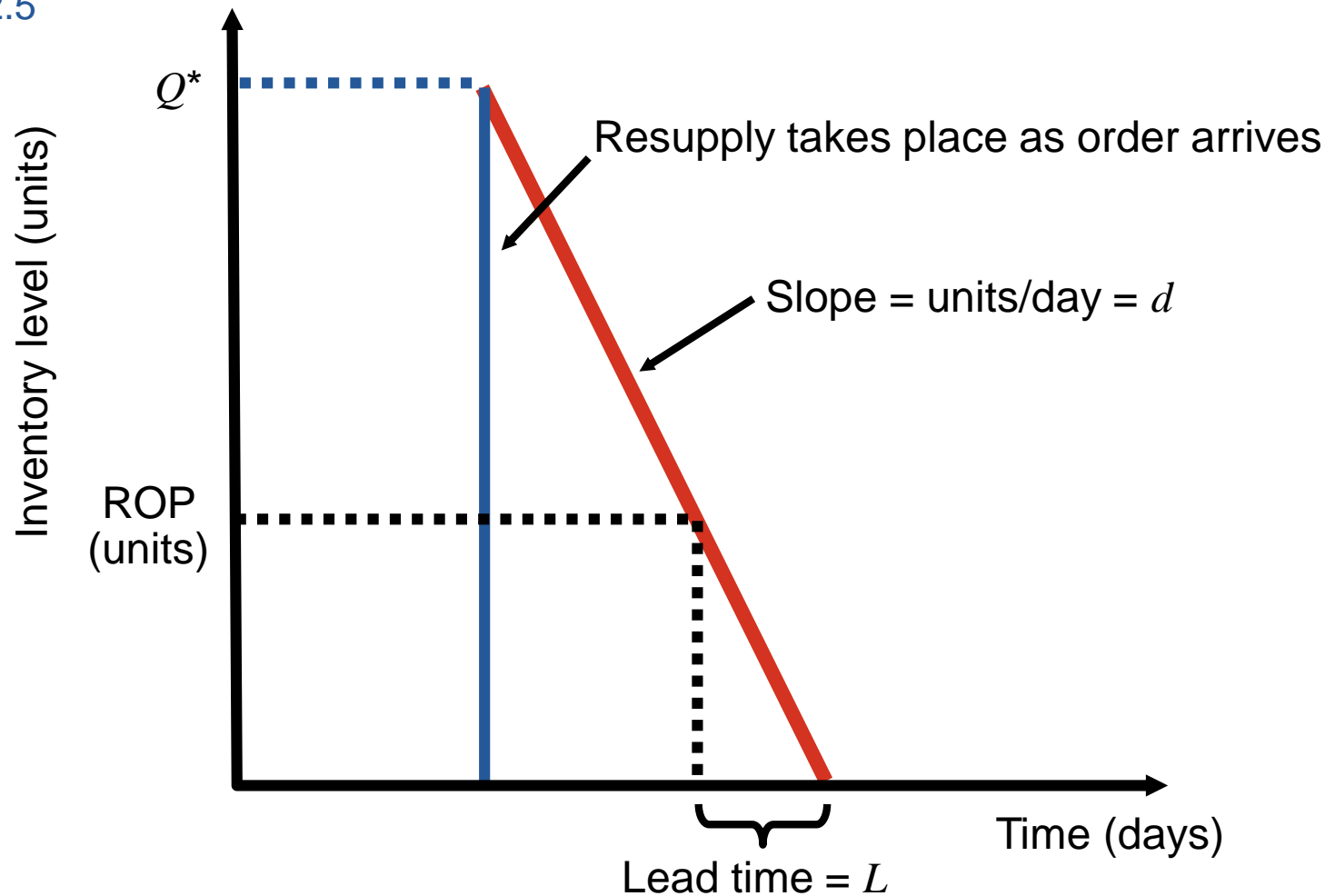
$$\text{ROP} = \left(\begin{array}{c} \text{Demand} \\ \text{per day} \end{array} \right) \left(\begin{array}{c} \text{Lead time for a new} \\ \text{order in days} \end{array} \right)$$

$$= d \times L$$

$$d = \frac{D}{\text{Number of working days in a year}}$$

Reorder Point Curve

Figure 12.5



Reorder Point Example

Demand = 8,000 iPods per year

250 working day year

Lead time for orders is 3 working days, may take 4

$$d = \frac{D}{\text{Number of working days in a year}}$$
$$= 8,000/250 = 32 \text{ units}$$

$$\text{ROP} = d \times L$$

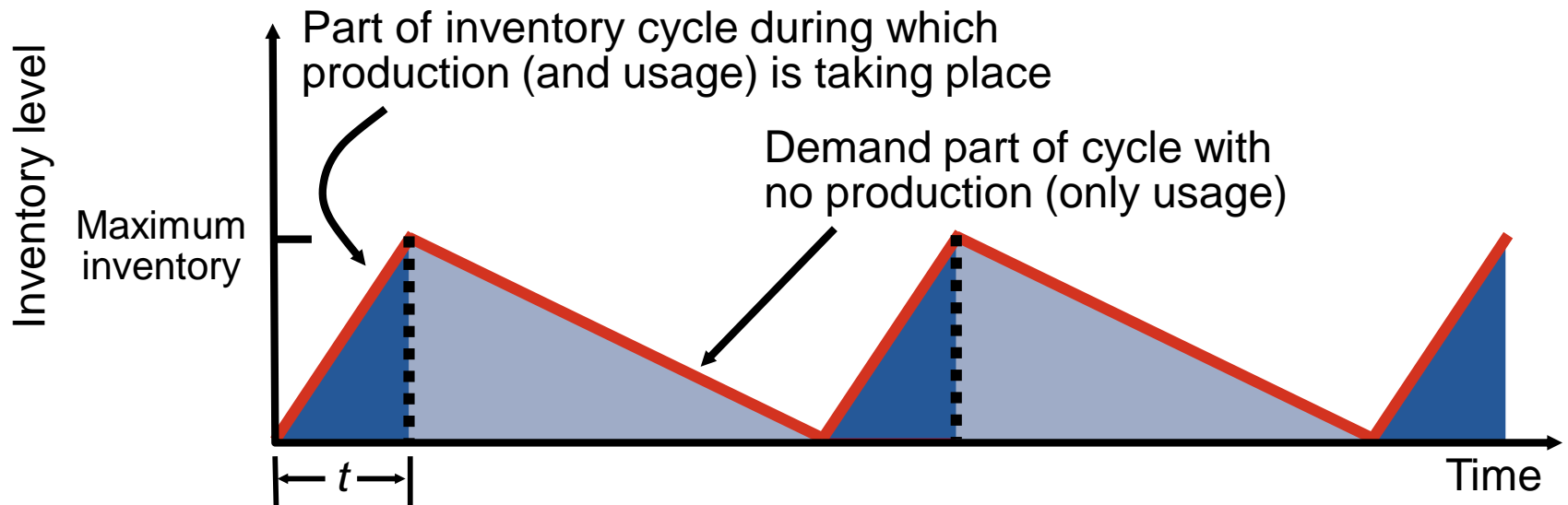
$$= 32 \text{ units per day} \times 3 \text{ days} = 96 \text{ units}$$

$$= 32 \text{ units per day} \times 4 \text{ days} = 128 \text{ units}$$

Production Order Quantity Model

1. Used when inventory builds up over a period of time after an order is placed
2. Used when units are produced and sold simultaneously

Figure 12.6



Production Order Quantity Model

Q = Number of pieces per order p = Daily production rate
 H = Holding cost per unit per year d = Daily demand/usage rate
 t = Length of the production run in days

$$\left(\begin{array}{c} \text{Annual inventory} \\ \text{holding cost} \end{array} \right) = (\text{Average inventory level}) \times \left(\begin{array}{c} \text{Holding cost} \\ \text{per unit per year} \end{array} \right)$$

$$\left(\begin{array}{c} \text{Annual inventory} \\ \text{level} \end{array} \right) = (\text{Maximum inventory level})/2$$

$$\begin{aligned} \left(\begin{array}{c} \text{Maximum} \\ \text{inventory level} \end{array} \right) &= \left(\begin{array}{c} \text{Total produced during} \\ \text{the production run} \end{array} \right) - \left(\begin{array}{c} \text{Total used during} \\ \text{the production run} \end{array} \right) \\ &= pt - dt \end{aligned}$$

Production Order Quantity Model

Q = Number of pieces per order p = Daily production rate
 H = Holding cost per unit per year d = Daily demand/usage rate
 t = Length of the production run in days

$$\left(\begin{array}{c} \text{Maximum} \\ \text{inventory level} \end{array} \right) = \left(\begin{array}{c} \text{Total produced during} \\ \text{the production run} \end{array} \right) - \left(\begin{array}{c} \text{Total used during} \\ \text{the production run} \end{array} \right)$$
$$= pt - dt$$

However, Q = total produced = pt ; thus $t = Q/p$

$$\left(\begin{array}{c} \text{Maximum} \\ \text{inventory level} \end{array} \right) = p \left(\frac{Q}{p} \right) - d \left(\frac{Q}{p} \right) = Q \left(1 - \frac{d}{p} \right)$$

$$\text{Holding cost} = \frac{\text{Maximum inventory level}}{2} (H) = \frac{Q}{2} \left[1 - \left(\frac{d}{p} \right) H \right]$$

Production Order Quantity Model

Q = Number of pieces per order p = Daily production rate
 H = Holding cost per unit per year d = Daily demand/usage rate
 t = Length of the production run in days

$$\text{Setup cost} = (D / Q)S$$

$$\text{Holding cost} = \frac{1}{2}HQ\left(1 - \left(\frac{d}{p}\right)\right)$$

$$\frac{D}{Q}S = \frac{1}{2}HQ\left(1 - \left(\frac{d}{p}\right)\right)$$

$$Q^2 = \frac{2DS}{H\left(1 - \left(\frac{d}{p}\right)\right)}$$

$$Q_p^* = \sqrt{\frac{2DS}{H\left(1 - \left(\frac{d}{p}\right)\right)}}$$

Production Order Quantity Example

$D = 1,000$ units

$S = \$10$

$H = \$0.50$ per unit per year

$p = 8$ units per day

$d = 4$ units per day

$$Q_p^* = \sqrt{\frac{2DS}{H(1 - (d/p))}}$$

$$Q_p^* = \sqrt{\frac{2(1,000)(10)}{0.50(1 - (4/8))}}$$

$$= \sqrt{\frac{20,000}{0.50(1/2)}} = \sqrt{80,000}$$

= 282.8 hubcaps, or 283 hubcaps

Production Order Quantity Model

Note:

$$d = 4 = \frac{D}{\text{Number of days the plant is in operation}} = \frac{1,000}{250}$$

When annual data are used the equation becomes

$$Q_p^* = \sqrt{\frac{2DS}{H \left(1 - \frac{\text{Annual demand rate}}{\text{Annual production rate}} \right)}}$$

Quantity Discount Models

- ▶ Reduced prices are often available when larger quantities are purchased
- ▶ Trade-off is between reduced product cost and increased holding cost

TABLE 12.2		A Quantity Discount Schedule	
DISCOUNT NUMBER	DISCOUNT QUANTITY	DISCOUNT (%)	DISCOUNT PRICE (P)
1	0 to 999	no discount	\$5.00
2	1,000 to 1,999	4	\$4.80
3	2,000 and over	5	\$4.75

Quantity Discount Models

Total annual cost = Setup cost + Holding cost + Product cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}H + PD$$

where Q = Quantity ordered P = Price per unit
 D = Annual demand in units H = Holding cost per unit per year
 S = Ordering or setup cost per order

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

Because unit price varies, holding cost (H) is expressed as a percent (I) of unit price (P)

Quantity Discount Models

Steps in analyzing a quantity discount

1. For each discount, calculate Q^*
2. If Q^* for a discount doesn't qualify, choose the lowest possible quantity to get the discount
3. Compute the total cost for each Q^* or adjusted value from Step 2
4. Select the Q^* that gives the lowest total cost

Quantity Discount Models

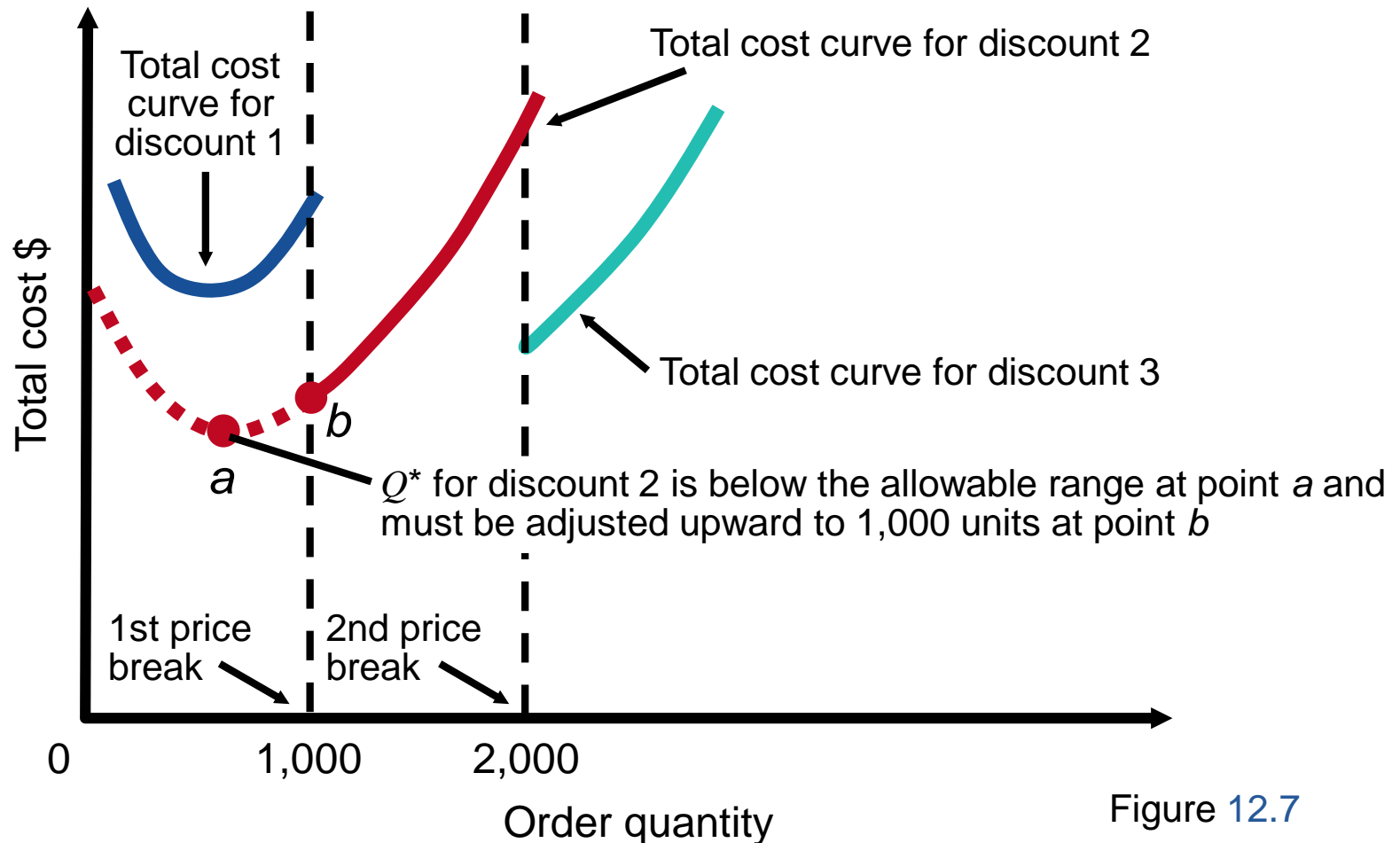


Figure 12.7

Quantity Discount Example

Calculate Q^* for every discount

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_1^* = \sqrt{\frac{2(5,000)(49)}{(.2)(5.00)}} = 700 \text{ cars/order}$$

$$Q_2^* = \sqrt{\frac{2(5,000)(49)}{(.2)(4.80)}} = 714 \text{ cars/order}$$

$$Q_3^* = \sqrt{\frac{2(5,000)(49)}{(.2)(4.75)}} = 718 \text{ cars/order}$$

Quantity Discount Example

Calculate Q^* for every discount

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_1^* = \sqrt{\frac{2(5,000)(49)}{(.2)(5.00)}} = 700 \text{ cars/order}$$

$$Q_2^* = \sqrt{\frac{2(5,000)(49)}{(.2)(4.80)}} = ~~714~~ \text{ cars/order}$$

1,000 — adjusted

$$Q_3^* = \sqrt{\frac{2(5,000)(49)}{(.2)(4.75)}} = ~~718~~ \text{ cars/order}$$

2,000 — adjusted

Quantity Discount Example

TABLE 12.3		Total Cost Computations for Wohl's Discount Store				
DISCOUNT NUMBER	UNIT PRICE	ORDER QUANTITY	ANNUAL PRODUCT COST	ANNUAL ORDERING COST	ANNUAL HOLDING COST	TOTAL
1	\$5.00	700	\$25,000	\$350	\$350	\$25,700
2	\$4.80	1,000	\$24,000	\$245	\$480	\$24,725
3	\$4.75	2,000	\$23,750	\$122.50	\$950	\$24,822.50

Choose the price and quantity that gives the lowest total cost

Buy 1,000 units at \$4.80 per unit

Probabilistic Models and Safety Stock

- ▶ Used when demand is not constant or certain
- ▶ Use safety stock to achieve a desired service level and avoid stockouts

$$ROP = d \times L + ss$$

Annual stockout costs = the sum of the units short x
the probability x the stockout cost/unit
x the number of orders per year

Safety Stock Example

ROP = 50 units

Stockout cost = \$40 per frame

Orders per year = 6

Carrying cost = \$5 per frame per year

NUMBER OF UNITS	PROBABILITY
30	.2
40	.2
ROP → 50	.3
60	.2
70	.1
	<hr/> 1.0

Safety Stock Example

ROP = 50 units

Orders per year = 6

Stockout cost = \$40 per frame

Carrying cost = \$5 per frame per year

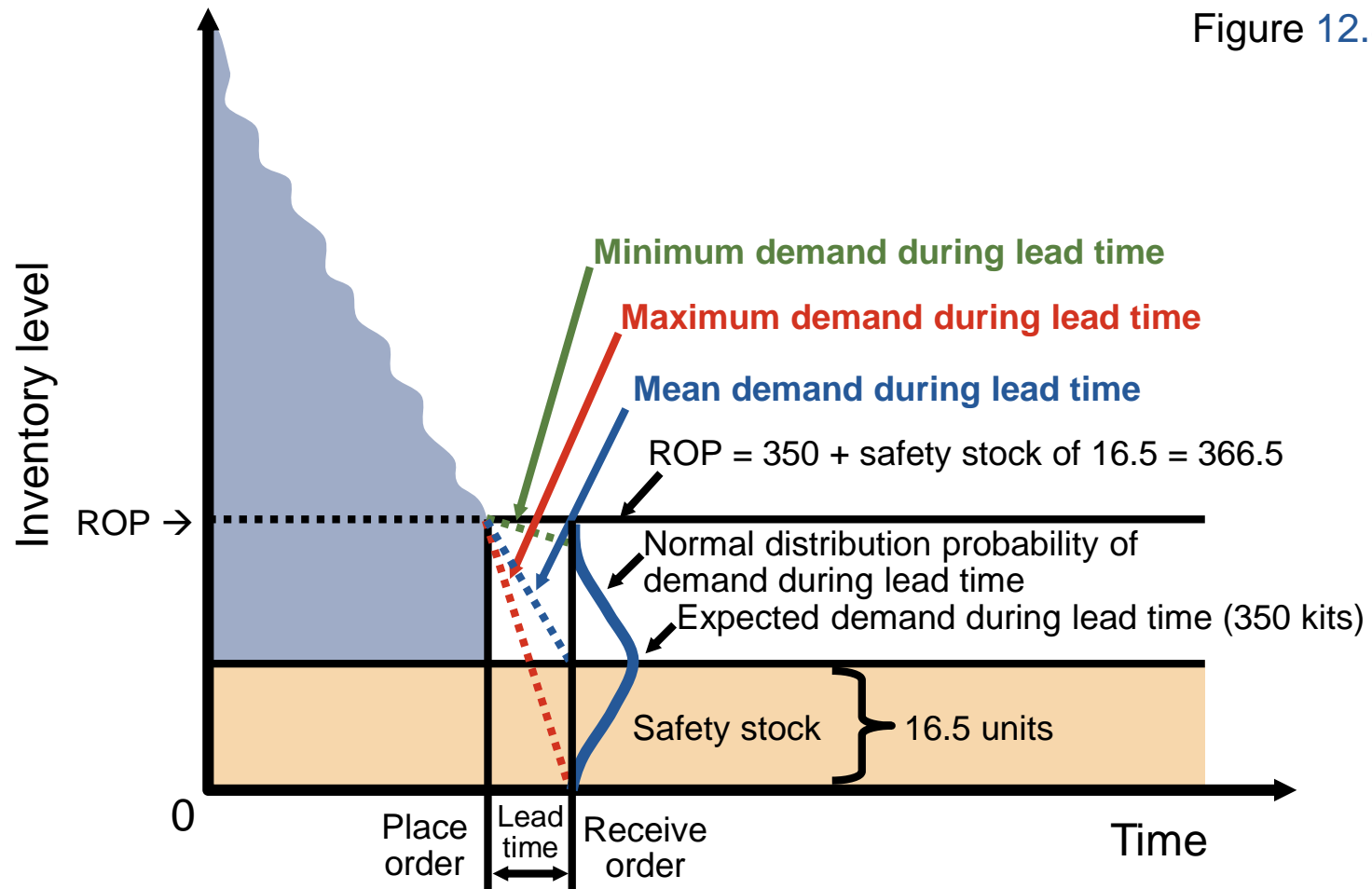
SAFETY STOCK	ADDITIONAL HOLDING COST	STOCKOUT COST	TOTAL COST
20	$(20)(\$5) = \100	\$0	\$100
10	$(10)(\$5) = \$ 50$	$(10)(.1)(\$40)(6) = \240	\$290
0	\$ 0	$(10)(.2)(\$40)(6) + (20)(.1)(\$40)(6) = \$960$	\$960

A safety stock of 20 frames gives the lowest total cost

$$\text{ROP} = 50 + 20 = 70 \text{ frames}$$

Probabilistic Demand

Figure 12.8



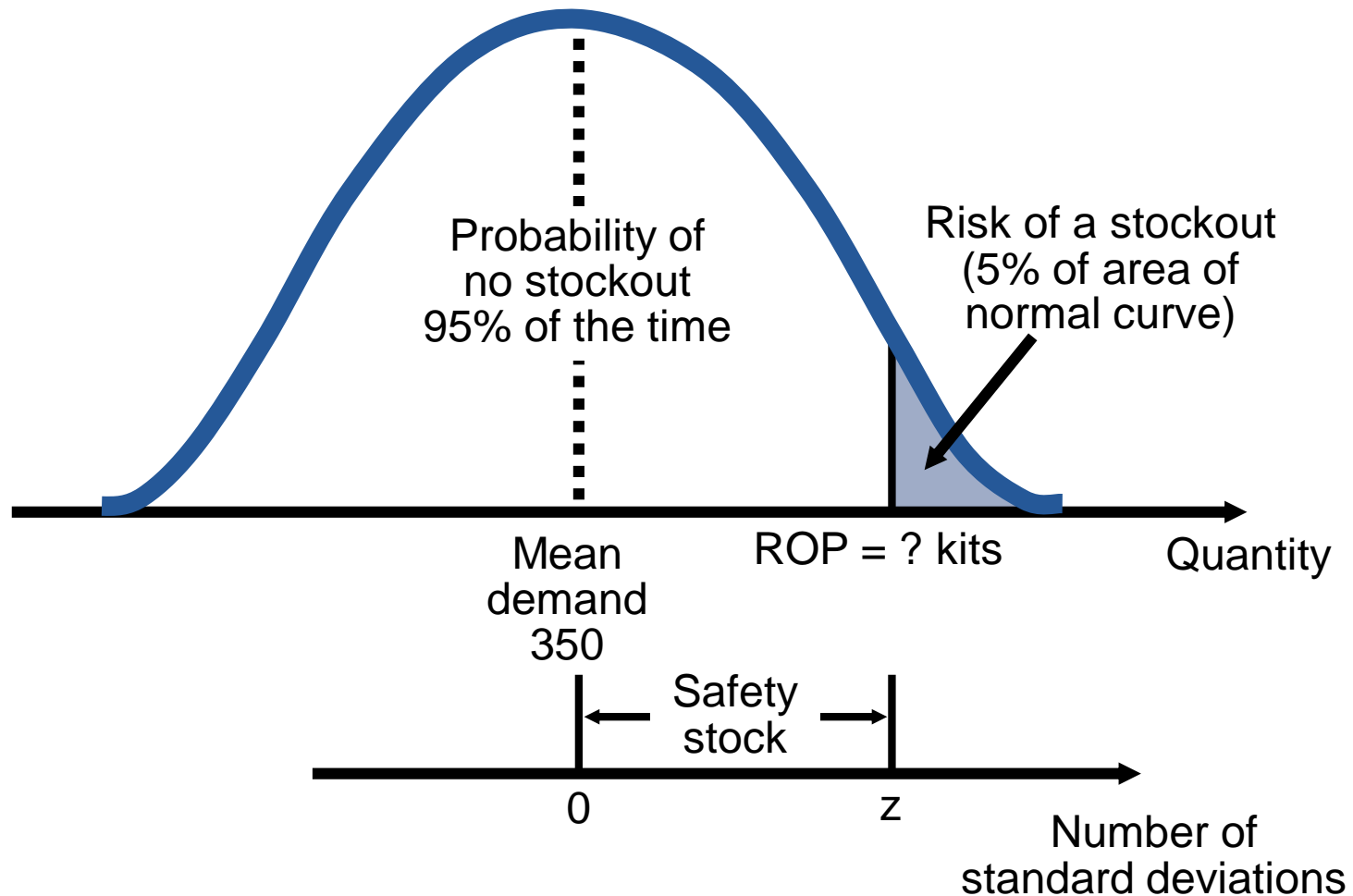
Probabilistic Demand

Use prescribed service levels to set safety stock when the cost of stockouts cannot be determined

$$\text{ROP} = \text{demand during lead time} + Z\sigma_{dLT}$$

where Z = Number of standard deviations
 σ_{dLT} = Standard deviation of demand during lead time

Probabilistic Demand



Probabilistic Example

μ = Average demand = 350 kits

σ_{dLT} = Standard deviation of
demand during lead time = 10 kits

Z = 5% stockout policy (service level = 95%)

Using Appendix I, for an area under the curve of
95%, the $Z = 1.65$

Safety stock = $Z\sigma_{dLT} = 1.65(10) = 16.5$ kits

Reorder point = Expected demand during lead time +
Safety stock
= 350 kits + 16.5 kits of safety stock
= 366.5 or 367 kits

Other Probabilistic Models

- ▶ When data on demand during lead time is not available, there are other models available
 1. When demand is variable and lead time is constant
 2. When lead time is variable and demand is constant
 3. When both demand and lead time are variable

Other Probabilistic Models

Demand is variable and lead time is constant

$$\text{ROP} = (\text{Average daily demand} \times \text{Lead time in days}) + Z\sigma_{dLT}$$

$$\text{where } \sigma_{dLT} = \sigma_d \sqrt{\text{Lead time}}$$

σ_d = standard deviation of demand per day

Probabilistic Example

Average daily demand (normally distributed) = 15

Lead time in days (constant) = 2

Standard deviation of daily demand = 5

Service level = 90%

Z for 90% = 1.28

From Appendix I

$$\begin{aligned}\text{ROP} &= (15 \text{ units} \times 2 \text{ days}) + Z\sigma_{dLT} \\ &= 30 + 1.28(5)(\sqrt{2}) \\ &= 30 + 9.02 = 39.02 \approx 39\end{aligned}$$

Safety stock is about 9 computers

Other Probabilistic Models

Lead time is variable and demand is constant

$$\text{ROP} = (\text{Daily demand} \times \text{Average lead time in days}) + Z \times (\text{Daily demand}) \times \sigma_{LT}$$

where σ_{LT} = Standard deviation of lead time in days

Probabilistic Example

Daily demand (constant) = 10

Average lead time = 6 days

Standard deviation of lead time = $\sigma_{LT} = 1$

Service level = 98%, so Z (from Appendix I) = 2.055

$$\begin{aligned}\text{ROP} &= (10 \text{ units} \times 6 \text{ days}) + 2.055(10 \text{ units})(1) \\ &= 60 + 20.55 = 80.55\end{aligned}$$

Reorder point is about 81 cameras

Other Probabilistic Models

Both demand and lead time are variable

$$\text{ROP} = (\text{Average daily demand} \times \text{Average lead time}) + Z\sigma_{dLT}$$

where σ_d = Standard deviation of demand per day

σ_{LT} = Standard deviation of lead time in days

$$\sigma_{dLT} = \sqrt{(\text{Average lead time} \times \sigma_d^2) + (\text{Average daily demand})^2 \sigma_{LT}^2}$$

Probabilistic Example

Average daily demand (normally distributed) = 150

Standard deviation = $\sigma_d = 16$

Average lead time 5 days (normally distributed)

Standard deviation = $\sigma_{LT} = 1$ day

Service level = 95%, so $Z = 1.65$ (from Appendix I)

$$ROP = (150 \text{ packs} \times 5 \text{ days}) + 1.65 S_{dLT}$$

$$\begin{aligned} S_{dLT} &= \sqrt{(5 \text{ days} \times 16^2) + (150^2 \times 1^2)} = \sqrt{(5 \times 256) + (22,500 \times 1)} \\ &= \sqrt{(1,280) + (22,500)} = \sqrt{23,780} @ 154 \end{aligned}$$

$$ROP = (150 \times 5) + 1.65(154) @ 750 + 254 = 1,004 \text{ packs}$$

Single-Period Model

- ▶ Only one order is placed for a product
- ▶ Units have little or no value at the end of the sales period

C_s = Cost of shortage = Sales price/unit – Cost/unit

C_o = Cost of overage = Cost/unit – Salvage value

$$\text{Service level} = \frac{C_s}{C_s + C_o}$$

Single-Period Example

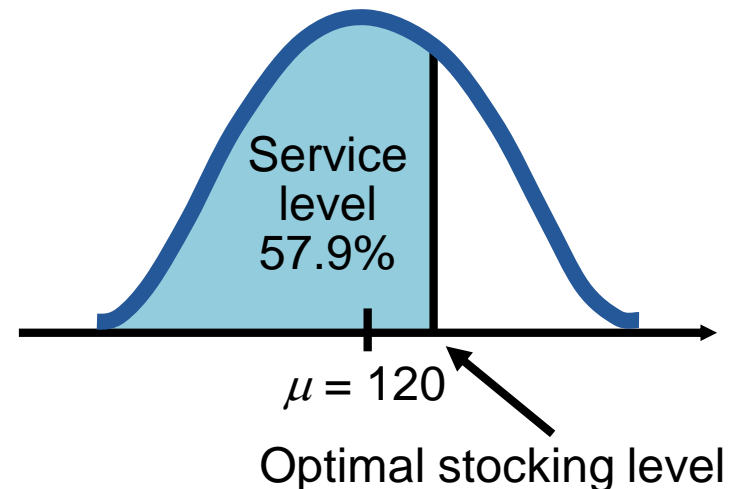
Average demand = $\mu = 120$ papers/day

Standard deviation = $\sigma = 15$ papers

C_s = cost of shortage = $\$1.25 - \$0.70 = \$0.55$

C_o = cost of overage = $\$0.70 - \$0.30 = \$0.40$

$$\begin{aligned}\text{Service level} &= \frac{C_s}{C_s + C_o} \\ &= \frac{.55}{.55 + .40} \\ &= \frac{.55}{.95} = .579\end{aligned}$$



Single-Period Example

From Appendix I, for the area .579, $Z \cong .20$

The optimal stocking level

$$= 120 \text{ copies} + (.20)(\sigma)$$

$$= 120 + (.20)(15) = 120 + 3 = 123 \text{ papers}$$

The stockout risk = $1 - \text{Service level}$

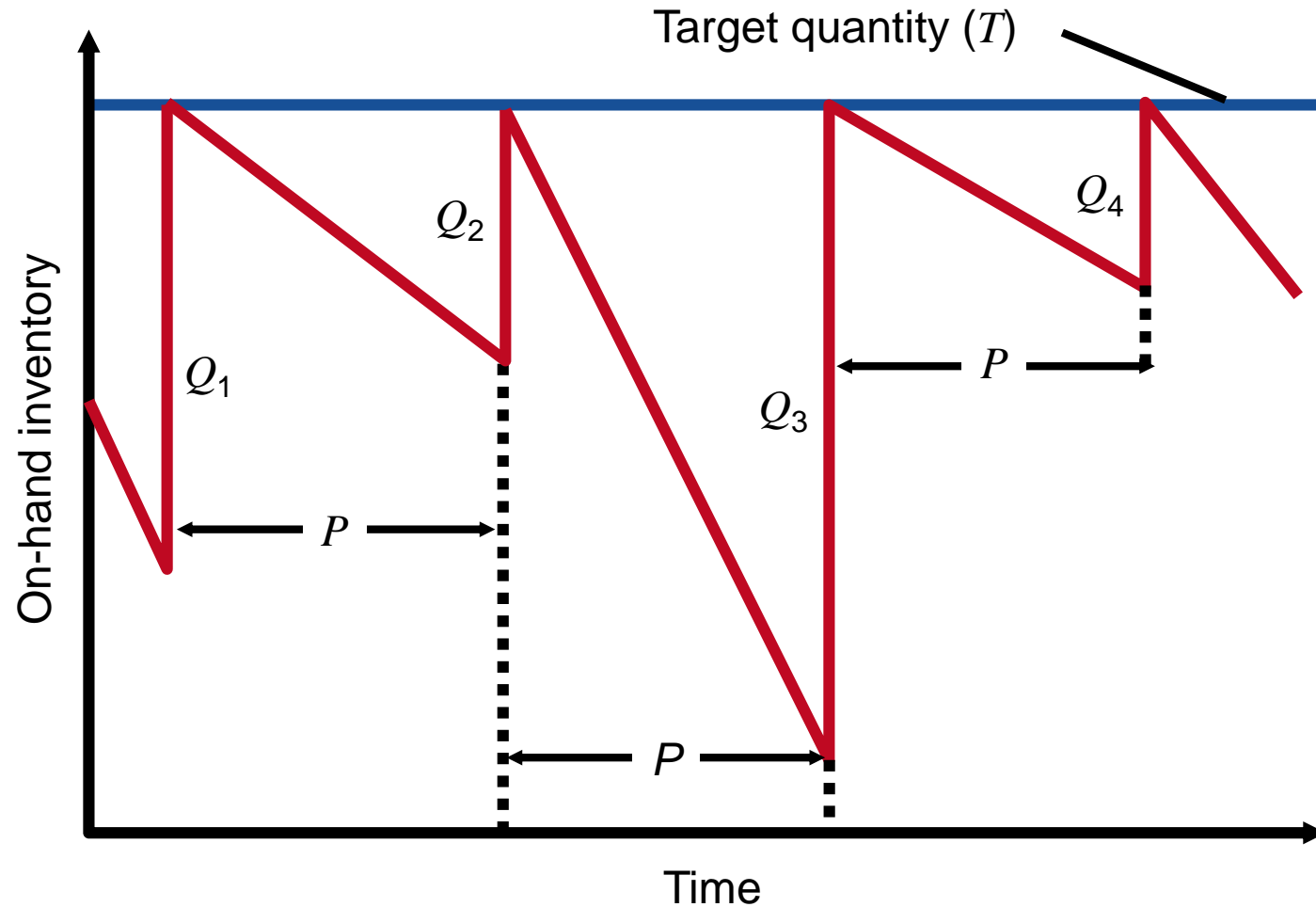
$$= 1 - .579 = .422 = 42.2\%$$

Fixed-Period (P) Systems

- ▶ Orders placed at the end of a fixed period
- ▶ Inventory counted only at end of period
- ▶ Order brings inventory up to target level
 - ▶ Only relevant costs are ordering and holding
 - ▶ Lead times are known and constant
 - ▶ Items are independent of one another

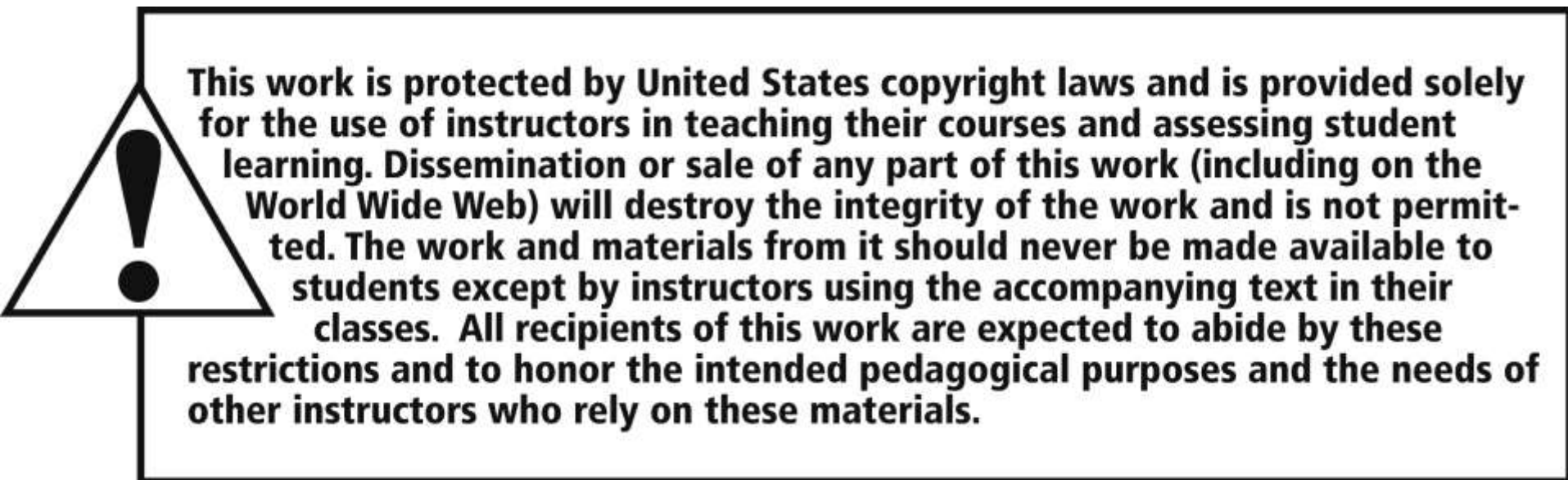
Fixed-Period (P) Systems

Figure 12.9



Fixed-Period Systems

- ▶ Inventory is only counted at each review period
- ▶ May be scheduled at convenient times
- ▶ Appropriate in routine situations
- ▶ May result in stockouts between periods
- ▶ May require increased safety stock



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