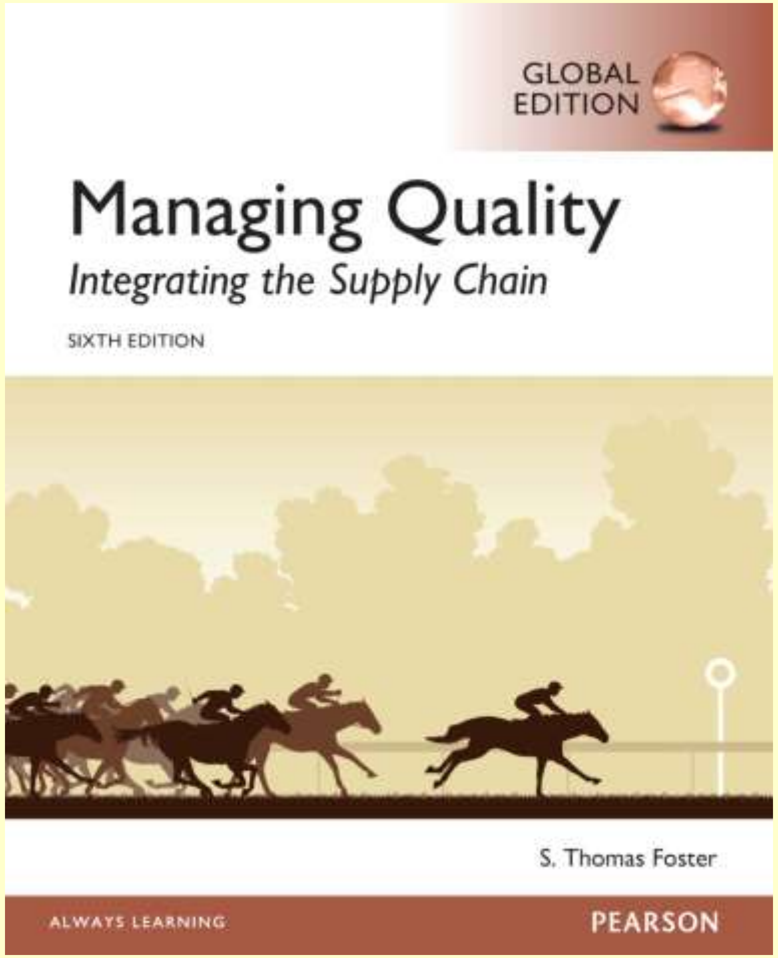


Chapter 12

Statistically Based Quality Improvement for Attributes

Chapter Objectives

1. Discuss the differences between attributes and variables.
2. Implement the process for creating attributes control charts.
3. Interpret attributes control charts.
4. Develop *p*, *np*, *u*, and *c* charts.
5. Develop control charts in Excel.
6. Perform reliability analysis.

Types of Attributes

- Structural attributes
- Sensory attributes
- Performance attributes
- Temporal attributes
- Ethical attributes
- Customer-based attributes
- Production-related attributes

Generic Process for Developing Attributes Charts

1. Identify critical operations in the process where inspection might be needed.
2. Identify critical product characteristics.
3. Determine whether the critical product characteristic is a variable or an attribute.
4. Select the appropriate process chart from the many types of charts.
5. Establish the control limits and use the chart to continually monitor and improve.
6. Update the limits when changes have been made to the process.

Understanding Attributes Charts

- **Attribute charts use binomial and Poisson processes that are not measurements.**

- **Think in terms of defects and defective units.**

Understanding Attributes Charts

- **Defect – an irregularity or problem with a larger unit**
  - Countable; can be several within one unit
  - Monitored using *c* and *u* charts
- **Defective – a unit that, as a whole, is not acceptable or does not meet performance requirements**
  - Monitored using *p* and *np* charts

Process Charts

- **A process chart (or *p* chart) is used to graph the proportion of items in a sample that are defective or nonconforming to specification.**

- **They are also used to determine when there has been a shift in the proportion defective for a particular product or service.**

p Chart Applications

- Late deliveries
- Incomplete orders
- Calls not getting dial tones
- Accounting transaction errors
- Clerical errors on written forms
- Parts that do not mate properly

p Charts Calculations

- **Subgroup sizes**
  - Typically between 50-100 units, and can be of different sizes

- **Formulas for control limits:**

$$\text{Control limits for } p = \overline{p} \pm 3\sqrt{[(\overline{p})(1 - \overline{p})/n]}$$

where:

p = the proportion defective
$\overline{p}$ = the average proportion defective
n = the sample size

Example 12-1

- **Problem:** A city police department was concerned that the number of convictions was decreasing relative to the number of arrests. The suggestion was raised that the district attorney's office was becoming less effective in prosecuting criminals. You are asked to perform an analysis of the situation.

Example 12-1

The data for the previous 27 weeks are provided in the following table.

Sample	Number of Cases Reviewed	Number of Convictions	Proportion
1	100	60	.60
2	95	65	.68
3	110	68	.62
4	142	62	.44
5	100	56	.56
6	98	58	.59
7	76	30	.39
8	125	68	.54
9	100	54	.54
10	125	62	.50
11	111	70	.63
12	116	58	.50
13	92	30	.33
14	98	68	.69
15	162	54	.33
16	87	62	.71
17	105	70	.67
18	110	58	.53
19	98	30	.31
20	96	68	.71
21	100	54	.54
22	100	62	.62
23	97	70	.72
24	122	58	.48
25	125	30	.24
26	110	68	.62
27	100	54	.54
	$\overline{p} = 14.63/27 = .54$		Sum = 14.63

Example 12-1

- **Solution:** Notice that in this problem, the sample size is not constant. When this happens, you have at least two options:
  - Compute the control limits using an average sample size. (This is easier to understand.)
  - Compute the control limits using the different sample sizes. (This is statistically more correct.)

Example 12-1

The results are shown in Figure 12-1.

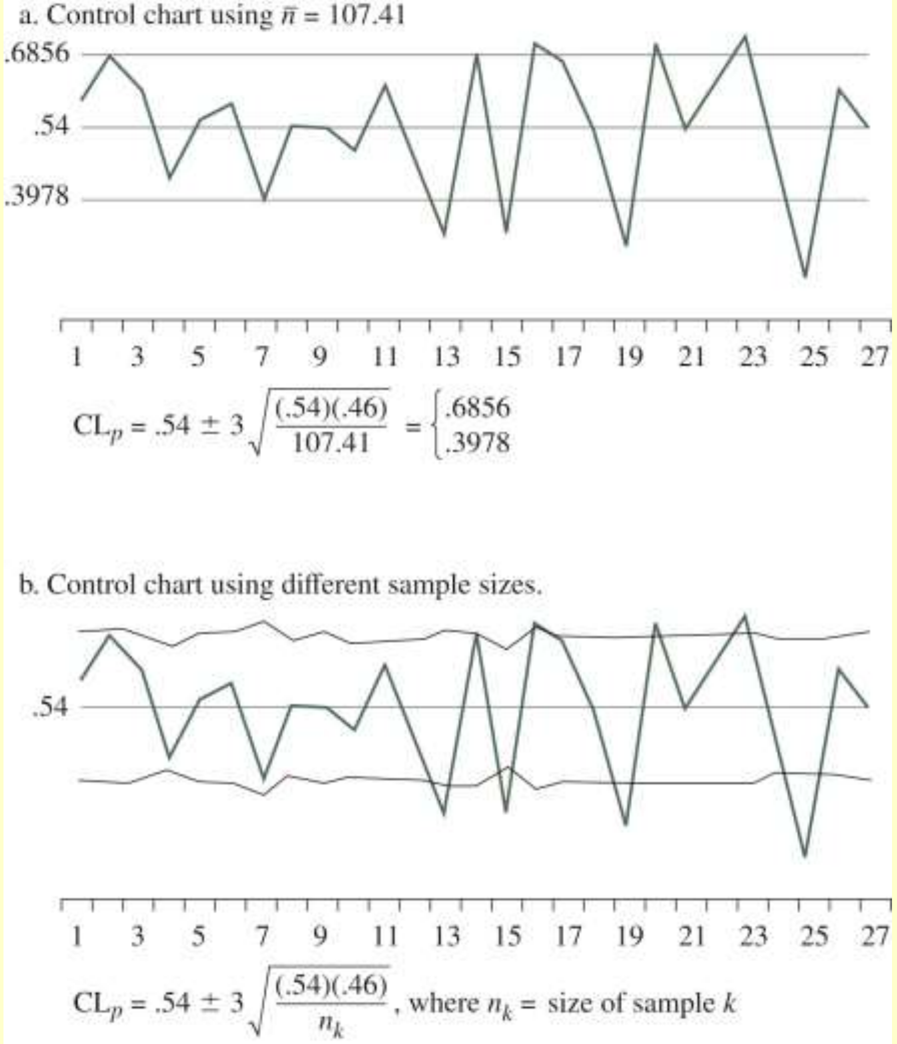


Figure 12-1

Example 12-1

Results using Excel

Sample	Cases	Convictions	Proportion	CL	UCL	LCL
1	100	60	0.6000	0.5417	0.6859	0.3975
2	95	65	0.6842	0.5417	0.6859	0.3975
3	110	68	0.6182	0.5417	0.6859	0.3975
4	142	62	0.4366	0.5417	0.6859	0.3975
5	100	56	0.5600	0.5417	0.6859	0.3975
6	98	58	0.5918	0.5417	0.6859	0.3975
7	[illegible]	[illegible]	0.3947	0.5417	0.6859	0.3975
8	[illegible]	[illegible]	0.5440	0.5417	0.6859	0.3975
9	[illegible]	[illegible]	0.5400	0.5417	0.6859	0.3975
10	[illegible]	[illegible]	0.4960	0.5417	0.6859	0.3975
11	111	70	0.6306	0.5417	0.6859	0.3975
12	116	58	0.5000	0.5417	0.6859	0.3975
13	92	30	0.3261	0.5417	0.6859	0.3975
14	98	68	0.6939	0.5417	0.6859	0.3975
15	162	54	0.3333	0.5417	0.6859	0.3975
16	87	62	0.7126	0.5417	0.6859	0.3975
17	105	70	0.6667	0.5417	0.6859	0.3975
18	110	58	0.5273	0.5417	0.6859	0.3975
19	98	30	0.3061	0.5417	0.6859	0.3975
20	96	68	0.7083	0.5417	0.6859	0.3975
21	100	54	0.5400	[illegible]	0.6859	0.3975
22	[illegible]	[illegible]	0.6200	[illegible]	0.6859	0.3975
23	[illegible]	[illegible]	0.7216	[illegible]	0.6859	0.3975
24	[illegible]	[illegible]	0.4754	[illegible]	0.6859	0.3975
25	125	30	0.2400	0.5417	0.6859	0.3975
26	110	68	0.6182	0.5417	0.6859	0.3975
27	100	54	0.5400	0.5417	0.6859	0.3975
n-bar	107.41	p-bar	0.5417			
		(1-pbar)	0.4583			

Problem Data

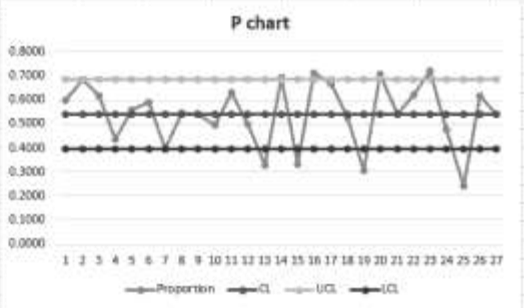
=C23/B23

=$D$29

=($D$29+3*(SQRT(($D$29*$D$30)/$B$29)))

=$D$29-3*(SQRT(($D$29*$D$30)/$B$29))

To develop this chart, follow the procedure in Example 11.1.
Highlight the data in cells D1:G28.



Microsoft Excel, Microsoft Corporation. Used by permission.

Figure 12-2

np Charts

- A graph of the number of defectives (or nonconforming units) in a subgroup
- Requires that the sample size of each subgroup be the same each time a sample is drawn
- If sample sizes are equal, either the *p* or *np* chart can be used.

np Chart Calculations

- **Subgroup sizes**
  - Typically between 50-100 units

- **Formulas for control limits:**

$$\mathrm{CL}_{np} = n(\overline{p}) \pm 3s_{np}$$

where:

n = the sample size

$\overline{p}$ = the average proportion defective

s_{np} = standard error of $\sqrt{n\overline{p}(1 - \overline{p})}$

Example 12-2

- **Problem:** Within the J. Kim Insurance Company of Boston, Massachusetts, management found that too many of its policies were rated incorrectly. Management directed that policy applications be reviewed for the past 24 months on a sampling basis. As an analyst, you are asked to review the policies for correct rating. If any problem is found with the rating of a policy, it is said to be defective.

Example 12-2

One hundred policies from each month were selected for review.

Month	Number of Policies Reviewed	Number of Policies with Rating Errors	*p*
1	100	11	.11
2	100	10	.10
3	100	12	.12
4	100	6	.06
5	100	14	.14
6	100	8	.08
7	100	10	.10
8	100	9	.09
9	100	12	.12
10	100	2	.02
11	100	14	.14
12	100	18	.18
13	100	7	.07
14	100	13	.13
15	100	14	.14
16	100	12	.12
17	100	11	.11
18	100	8	.08
19	100	9	.09
20	100	17	.17
21	100	18	.18
22	100	20	.20
23	100	25	.25
24	100	28	.28
			Mean = .13

Example 12-2

- **Solution:** The results of the control chart are shown in Figure 12-3. The chart shows that rating errors are increasing. Assignable causes should be identified through investigation.

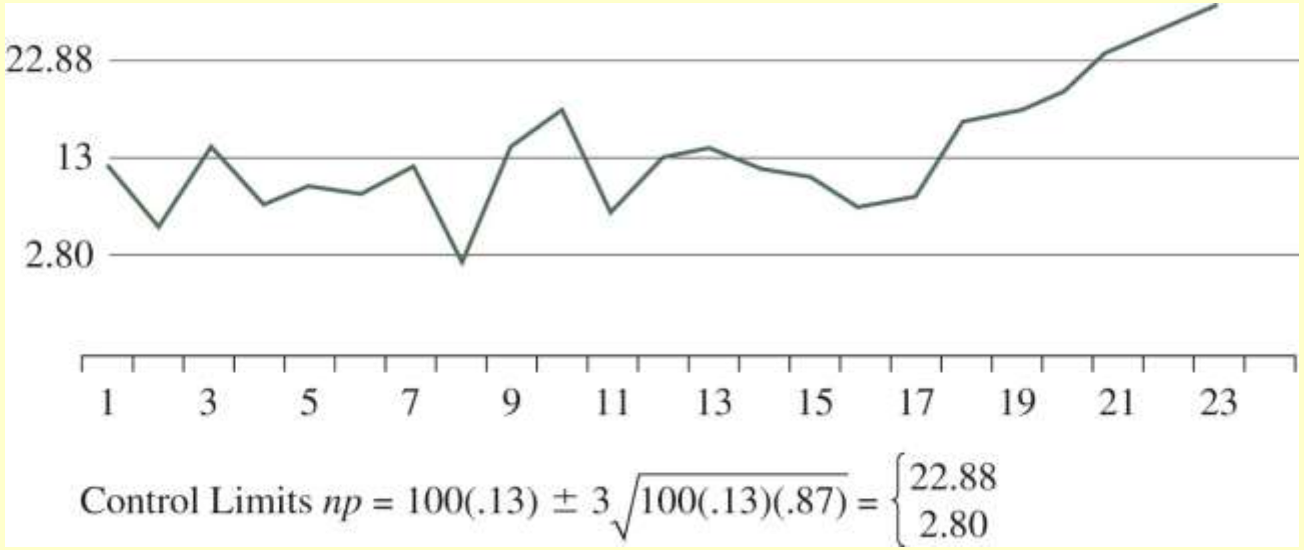


Figure 12-3

Example 12-2

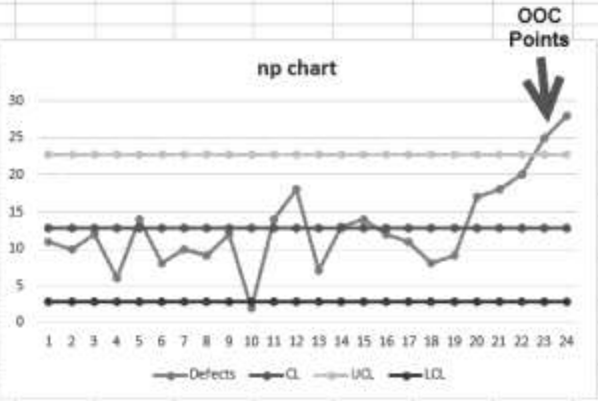
Results using Excel

	A	B	C	D	E	F
1	Month	Policies	Defects	CL	UCL	LCL
2	1	100	11	12.83333	22.86833	2.798333
3	2	100	10	12.83333	22.86833	2.798333
4	3	100	12	12.83333	22.86833	2.798333
5	4	100	6	12.83333	22.86833	2.798333
6	5	100	14	12.83333	22.86833	2.798333
7	6	100	8	12.83333	22.86833	2.79833
8	7	100	10	12.83333	22.86833	2.79833
9	8	100	9	12.83333	22.86833	2.798
10	9	100	12	12.83333	22.86833	2.79833
11	10	100	2	12.83333	22.86833	2.798333
12	11	100	14	12.83333	22.86833	2.798333
13	12	100	18	12.83333	22.8683	2.798333
14	13	100	7	12.83333	22.86	
15	14	100	13	12.83333	22.86	
16	15	100	14	12.83333	22.86833	2.798333
17	16	100	12	12.83333	22.86833	2.798333
18	17	100	11	12.83333	22.86833	2.798333
19	18	100	8	12.83333	22.86833	2.798333
20	19	100	9	12.83333	22.86833	2.798333
21	20	100	17	12.83333	22.86833	2.798333
22	21	100	18	12.83333	22.86833	2.798333
23	22	100	20	12.83333	22.86833	2.798333
24	23	100	25	12.83		2.798333
25	24	100	28	12.83		2.798333
26		Mean	12.83333			
27		Sample Size	100			
28		p-bar	0.128333			
29		1-p-bar	0.871667			
30		Std err of np	3.345			

=$C$26-3*$C$30

=$C$26+3*$C$30

=$C$26



Microsoft Excel, Microsoft Corporation. Used by permission.

Figure 12-4

c Charts

- A graph of the number of defects (nonconformities) per unit

- Units must be of the same metric such as height, length, volume, and so on.

- Used to detect nonrandom events in the life of the production process and when you are inspecting the same size sample space

c Chart Applications

- **Number of flaws in an auto finish**
- **Number of flaws in a standard typed letter**
- **Number of data errors in a standard form**
- **Number of incorrect responses on a standardized test**

u Charts

- **A graph of the average number of defects per unit**
- **Allows for the units sampled to be different sizes, areas, heights, etc.**
- **The uses for the *u* chart are the same as the *c* chart.**

c and *u* Chart Calculations

Formulas for control limits:

$$\mathrm{CL}_c = \bar{c} \pm 3\sqrt{\bar{c}}$$

$$\mathrm{CL}_u = \bar{u} \pm 3\sqrt{\frac{\bar{u}}{n}}$$

where:

n = average sample size
$\bar{c}$ = process average number of nonconformities
$\bar{u}$ = process average number of nonconformities per unit

Example 12-3

- **Problem:** The J. Grout Window Company makes colored-glass objects for home decoration. J. Grout, the owner, has been concerned about scratches in the finish of recently made product.
- The company makes two products: Demi-Glass, which comes in one standard configuration; and Streakless-Glass, which comes in three similar models.
- As an analyst, you are asked to evaluate the process by determining whether the processes are stable. Assume that, on average, the Streakless are 1.5 times the size of the Demis.

Example 12-3

Using high-power magnifying glasses, the company examined 25 each of the Demi (one style only) and the Streakless (randomly selected in all three styles).

Item Number	Demi Defects	Streakless Defects
1	5	6
2	4	4
3	6	7
4	3	9
5	9	5
6	4	8
7	5	7
8	4	4
9	3	5
10	7	4
11	9	5
12	12	4
13	3	5
14	6	6
15	2	4
16	8	8
17	5	5
18	7	7
19	12	10
20	4	5
21	6	4
22	8	7
23	5	5
24	7	6
	Sum c = 144 $\bar{c} = 6$	Sum u = 140 $\bar{u} = 5.83$

Example 12-3

- **Solution:** As shown in Figure 12-5 and Figure 12-6, the process for Demis appears to be in control. However, the process for Streakless shows a run of five points below the mean. An assignable cause should be sought.

c Chart for Demis

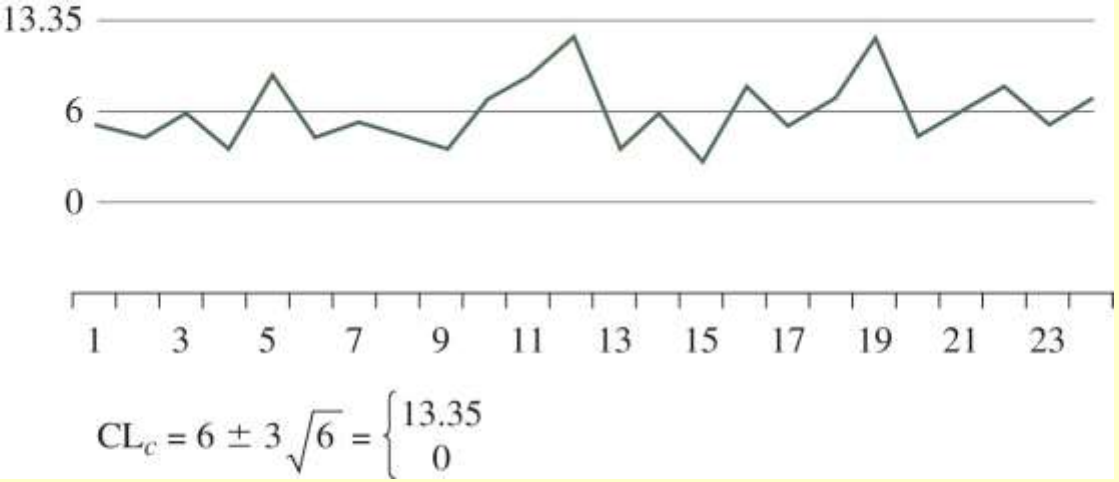
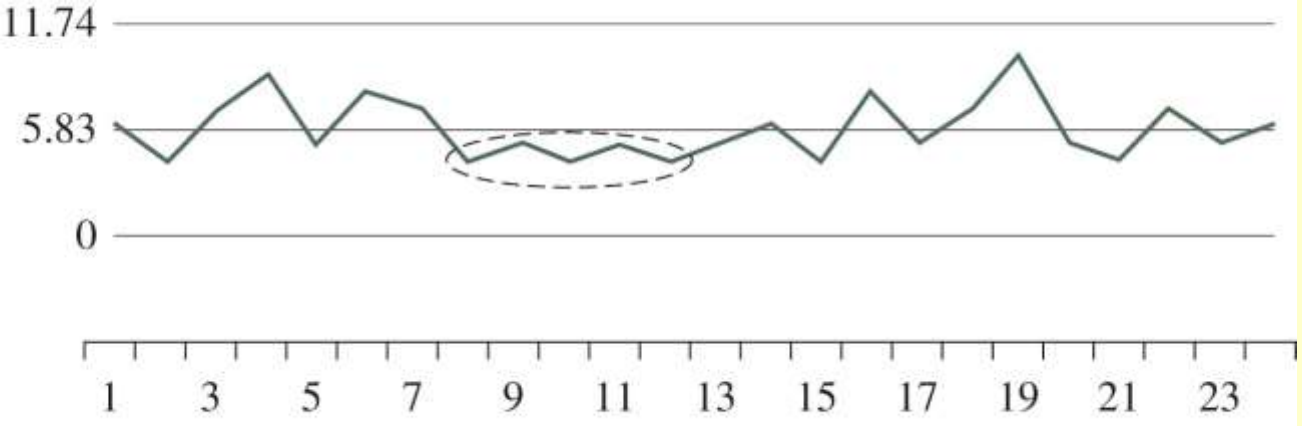


Figure 12-5

Example 12-3

u Chart for Streakless

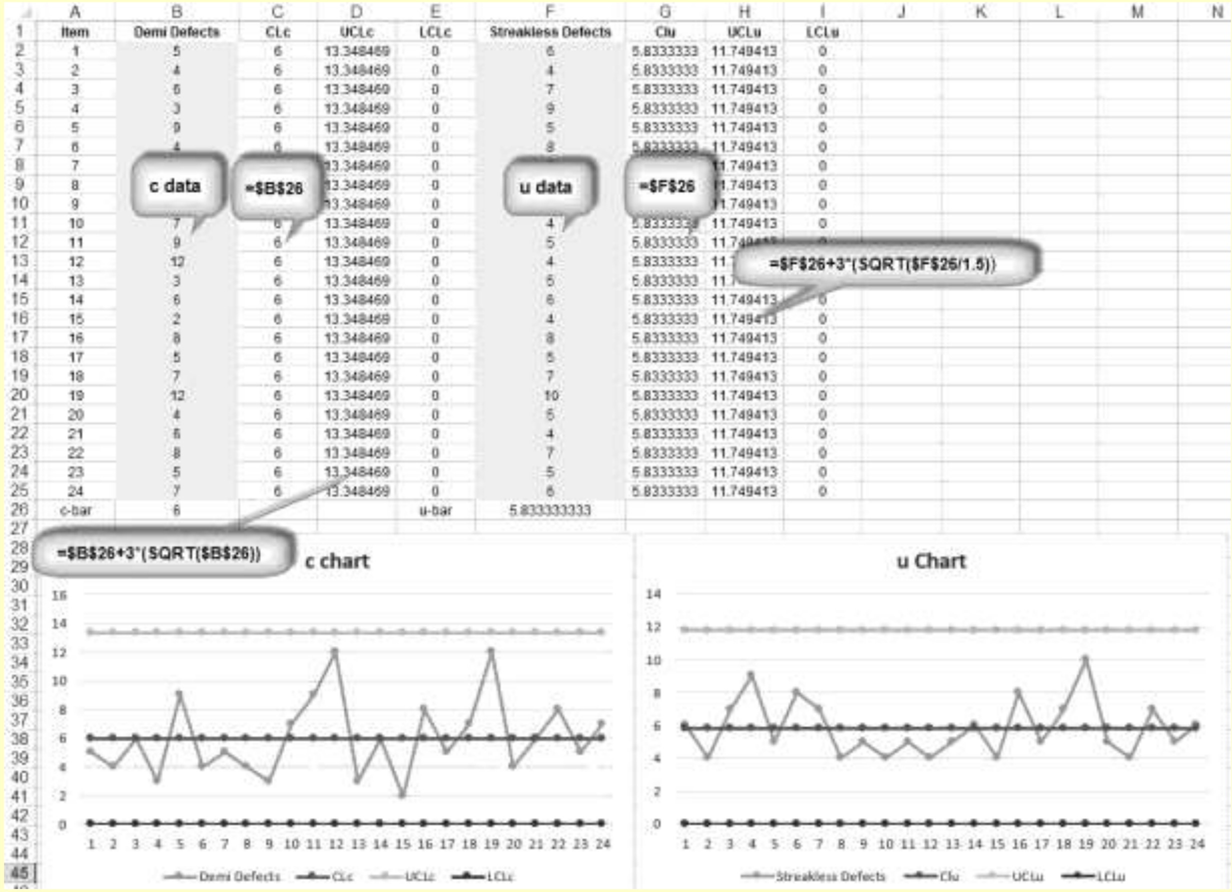


$$CL_u = 5.83 \pm 3\sqrt{\frac{5.83}{1.5}} = \begin{cases} 11.74 \\ 0 \end{cases}$$

Figure 12-6

Example 12-3

Results using Excel



	A	B	C	D	E	F	G	H	I
1	Item	Demi Defects	CLc	UCLc	LCLc	Streakless Defects	Clu	UCLu	LCLu
2	1	5	6	13.348469	0	6	5.8333333	11.749413	0
3	2	4	6	13.348469	0	4	5.8333333	11.749413	0
4	3	6	6	13.348469	0	7	5.8333333	11.749413	0
5	4	3	6	13.348469	0	9	5.8333333	11.749413	0
6	5	9	6	13.348469	0	5	5.8333333	11.749413	0
7	6	4	6	13.348469	0	8	5.8333333	11.749413	0
8	7	[illegible]	[illegible]	13.348469	0	[illegible]	[illegible]	[illegible]	0
9	8	[illegible]	[illegible]	13.348469	0	[illegible]	[illegible]	[illegible]	0
10	9	[illegible]	[illegible]	13.348469	0	[illegible]	[illegible]	[illegible]	0
11	10	7	6	13.348469	0	4	5.8333333	11.749413	0
12	11	9	6	13.348469	0	5	5.8333333	[illegible]	[illegible]
13	12	12	6	13.348469	0	4	5.8333333	[illegible]	[illegible]
14	13	3	6	13.348469	0	5	5.8333333	[illegible]	[illegible]
15	14	6	6	13.348469	0	6	5.8333333	11.749413	0
16	15	2	6	13.348469	0	4	5.8333333	11.749413	0
17	16	8	6	13.348469	0	8	5.8333333	11.749413	0
18	17	5	6	13.348469	0	5	5.8333333	11.749413	0
19	18	7	6	13.348469	0	7	5.8333333	11.749413	0
20	19	12	6	13.348469	0	10	5.8333333	11.749413	0
21	20	4	6	13.348469	0	5	5.8333333	11.749413	0
22	21	6	6	13.348469	0	4	5.8333333	11.749413	0
23	22	8	6	13.348469	0	7	5.8333333	11.749413	0
24	23	5	6	13.348469	0	5	5.8333333	11.749413	0
25	24	7	6	13.348469	0	6	5.8333333	11.749413	0
26	c-bar	6			u-bar	5.833333333			

Microsoft Excel, Microsoft Corporation. Used by permission.

Figure 12-7

Attributes Charts Summary

Chart	LCL	CL	UCL
p	$\bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$	$\bar{p}$	$\bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$
np	$n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$	$n\bar{p}$	$n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$
c	$\bar{c} - 3\sqrt{\bar{c}}$	$\bar{c}$	$\bar{c} + 3\sqrt{\bar{c}}$
u	$\bar{u} - 3\sqrt{\bar{u}/n}$	$\bar{u}$	$\bar{u} + 3\sqrt{\bar{u}/n}$

Table 12-2

Reliability Models

Bathtub-shaped hazard functions:

- The vertical axis is the failure rate.
- The horizontal axis is time.
- Shows that products are more likely to fail either very early or late in their useful lives.

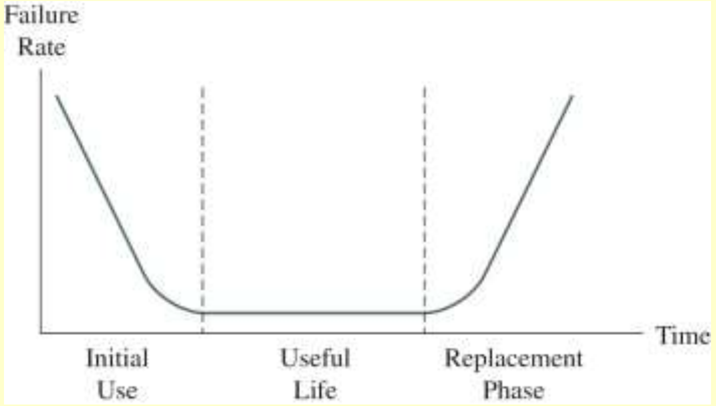


Figure 12-8

Reliability Models

Series reliability:

- Components in a system are in a series if the performance of the entire system depends on all the components functioning properly.
- The components need not be physically wired sequentially for the system to be in series.
- All parts must function for the system to function.

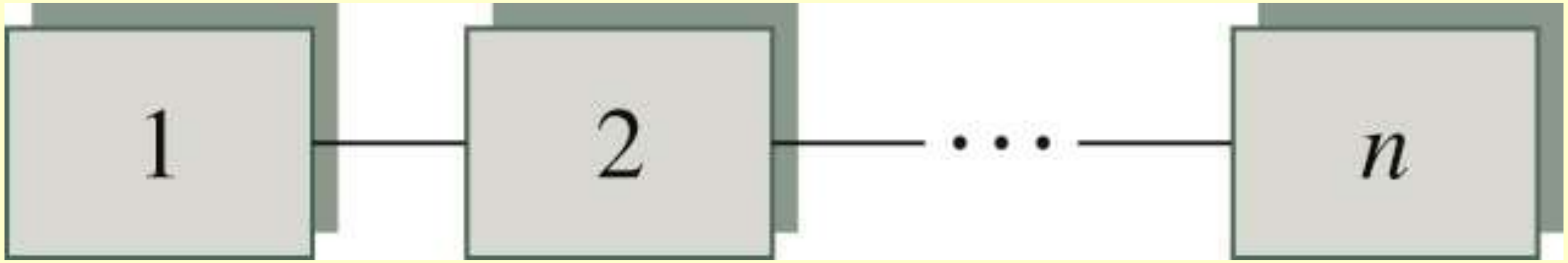


Figure 12-9

Reliability Models

- **System reliability for the series is expressed as:**

$$R_s = P(x_1 x_2 \cdots x_n)$$
$$= P(x_1)P(x_2|x_1)P(x_3|x_1 x_2) \cdots P(x_n|x_1 x_2 \ldots x_{n-1})$$

where:

R_s = system reliability
$P(x)$ = 1 − probability of failure for component x_i

- **System unreliability can be modeled as:**

$$Q_s = 1 - R_s$$

where: Q_s = system unreliability

Reliability Models

Parallel reliability:

- High reliability systems often require extremely high component reliability.
- When such high reliability is an impossibility, an alternative is to use a backup system.
- Another word for backup is *redundant* or *parallel*.

Reliability Models

- **System reliability for the series is expressed as:**

$$R_p = P(x_1 + x_2 + \cdots + x_n) = 1 - P(\bar{x}_1\bar{x}_2\cdots\bar{x}_n)$$

- **Redundant reliability can be modeled as:**

$$R_p = 1 - P(\bar{x}_1)P(\bar{x}_2|\bar{x}_1)P(\bar{x}_3|\bar{x}_1\bar{x}_2)\cdots P(\bar{x}_n|\bar{x}_1\bar{x}_2\cdots\bar{x}_{n-1})$$

- **System unreliability can be modeled as:** $Q_p = \prod_{i=1}^{n} Q_i$

Example 12-4

- At times, systems have some components in series and some components in parallel (or redundancy).
- Figure 12-10 has one such system.
- Overall reliability for this system is R = .98 * .99 * 11 - 1.1 * .122 * .97 = .932

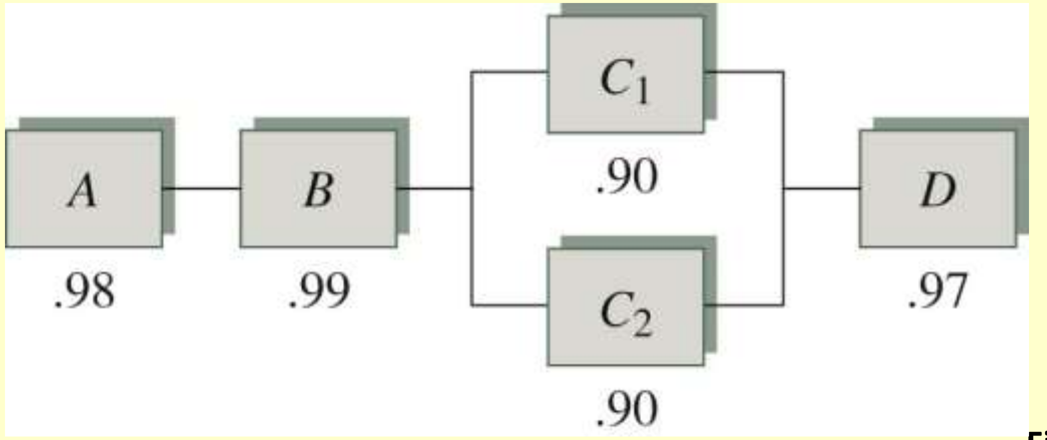


Figure 12-10

Example 12-4

- To continue the example, it is interesting to compare the overall reliability of this system without component C_2.

- It equals $R = .98 * .99 * .90 * .97 = .847$

- Thus the overall improvement in system reliability by adding the additional component is $D = .932 - .847 = .085$

Measuring Reliability

Failure rate:

$$\text{Failure rate} = \lambda = \text{number of failures}/(\text{units tested} \times \text{number of hours tested})$$

Example 12-5

- **Problem:** Suppose that we tested 25 ski exercise machines under strenuous conditions for 100 hours per machine. Of the machines tested, three experienced malfunctions during the test. What is the failure rate for the exercise machines?

- **Solution:**

$$\text{Failure rate} = 3/(25 * 100)$$
$$= .0012 \text{ failures per operating hour}$$

Mean Time to Failure (MTTF)

- **Reliability:** $R(T) = 1 - F(T) = e^{-\lambda T}$

  where:

  $R(T)$ = reliability of the product
  $F(T)$ = unreliability of a product
  λ = failure rate
  T = product's useful life expressed as a function of time

- **Mean time to failure:**
  - Average time before the product will fail $1/\lambda$

Example 12-6

- **Problem:** Suppose that a product is designed to operate for 100 hours continuously with a 1% chance of failure. Find the number of failures per hour incurred by this product and the MTTF.

- **Solution:**

$$
\begin{aligned}
0.99 &= e^{-\lambda(100)} \\
\ln 0.99 &= -100\lambda \\
\lambda &= -(\ln 0.99)/100 \\
&= .01005/100 \\
&= .0001005 \\
\text{MTTF} &= 1/.0001005 = 9950.25 = 1/\lambda
\end{aligned}
$$

Mean Time between Failures (MTBF)

- The average time from one failure to the next when a product can be repaired

- MTBF = total operating hours/number of failures

Example 12-7

- **Problem:** A product has been operated for 10,000 hours and has experienced four failures. What is the MTBF?

- **Solution:**
  - MTBF = 10,000>4 = 2,500 hours between failures
  - The failure rate is then calculated as l = 1>2,500 = .0004 failures per hour.

System Availability

- A useful measure for maintainability of a product that considers both MTBF and a new statistic: mean time to repair (MTTR)
- Gives the "uptime" of a product or system
- SA = MTBF> (MTBF + MTTR)

Example 12-8

- **Problem:** Jami Kovach has to decide between three suppliers for a network server. Other factors being equal, she will base her decision on system availability. Given the following data, which supplier should she choose?

Supplier	MTBF (h)	MTTR (h)
A	67	4
B	45	2
C	36	1

Example 12-8

- **Solution:**
  - SAA = 67> 167 + 42 = .944
  - SAB = 45> 145 + 22 = .957
  - SAC = 36> 136 + 12 = .973
- **Choose supplier C. As you can see, service does matter.**