Forecasting Demand

4

PowerPoint presentation to accompany
Heizer and Render
Operations Management, Global Edition, Eleventh Edition
Principles of Operations Management, Global Edition, Ninth Edition

PowerPoint slides by Jeff Heyl

Outline

- Global Company Profile:
  Walt Disney Parks & Resorts
- What Is Forecasting?
- The Strategic Importance of Forecasting
- Seven Steps in the Forecasting System
- Forecasting Approaches

Outline - Continued

- Time-Series Forecasting
- Associative Forecasting Methods: Regression and Correlation Analysis
- Monitoring and Controlling Forecasts
- Forecasting in the Service Sector

Learning Objectives

When you complete this chapter you should be able to :

1. **Understand** the three time horizons and which models apply for each use
2. **Explain** when to use each of the four qualitative models
3. **Apply** the naive, moving average, exponential smoothing, and trend methods

Learning Objectives

When you complete this chapter you should be able to :

4. **Compute** three measures of forecast accuracy
5. **Develop** seasonal indices
6. **Conduct** a regression and correlation analysis
7. **Use** a tracking signal

Forecasting Provides a Competitive Advantage for Disney

- Global portfolio includes parks in Hong Kong, Paris, Tokyo, Orlando, and Anaheim
- Revenues are derived from people – how many visitors and how they spend their money
- Daily management report contains only the forecast and actual attendance at each park

Forecasting Provides a Competitive Advantage for Disney

- Disney generates daily, weekly, monthly, annual, and 5-year forecasts
- Forecast used by labor management, maintenance, operations, finance, and park scheduling
- Forecast used to adjust opening times, rides, shows, staffing levels, and guests admitted

Forecasting Provides a Competitive Advantage for Disney

- 20% of customers come from outside the USA
- Economic model includes gross domestic product, cross-exchange rates, arrivals into the USA
- A staff of 35 analysts and 70 field people survey 1 million park guests, employees, and travel professionals each year

Forecasting Provides a Competitive Advantage for Disney

- Inputs to the forecasting model include airline specials, Federal Reserve policies, Wall Street trends, vacation/holiday schedules for 3,000 school districts around the world
- Average forecast error for the 5-year forecast is 5%
- Average forecast error for annual forecasts is between 0% and 3%

What is Forecasting?

- Process of predicting a future event
- Underlying basis of all business decisions
  - Production
  - Inventory
  - Personnel
  - Facilities

Forecasting Time Horizons

1. *Short-range forecast*
    - Up to 1 year, generally less than 3 months
    - Purchasing, job scheduling, workforce levels, job assignments, production levels
2. *Medium-range forecast*
    - 3 months to 3 years
    - Sales and production planning, budgeting
3. *Long-range forecast*
    - 3^+ years
    - New product planning, facility location, research and development

Distinguishing Differences

1. Medium/long range forecasts deal with more comprehensive issues and support management decisions regarding planning and products, plants and processes
2. Short-term forecasting usually employs different methodologies than longer-term forecasting
3. Short-term forecasts tend to be more accurate than longer-term forecasts

Influence of Product Life Cycle

Introduction – Growth – Maturity – Decline

- Introduction and growth require longer forecasts than maturity and decline
- As product passes through life cycle, forecasts are useful in projecting
  - Staffing levels
  - Inventory levels
  - Factory capacity

Product Life Cycle

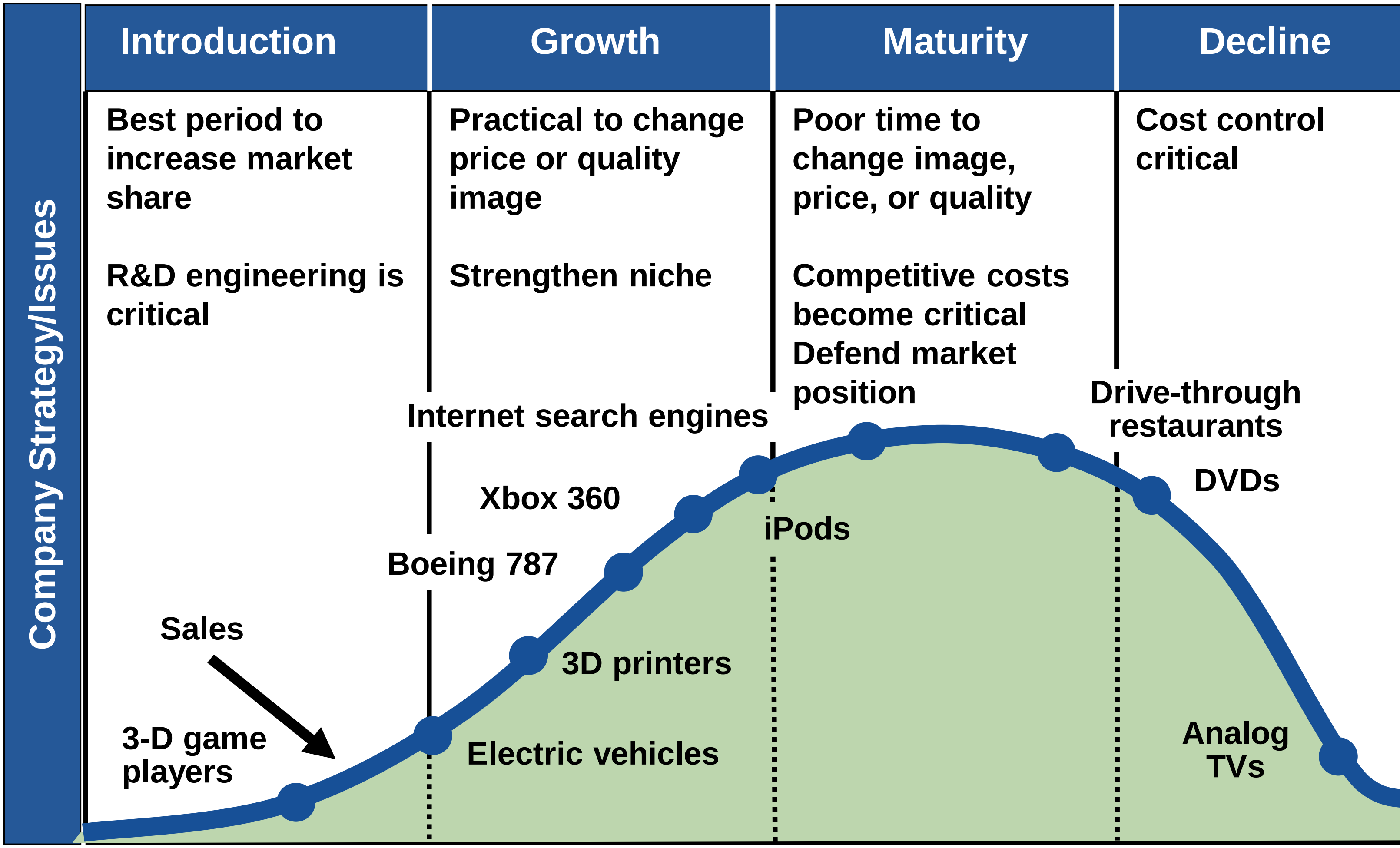


Figure 2.5

Product Life Cycle

	Introduction	Growth	Maturity	Decline
OM Strategy/Issues	Product design and development critical Frequent product and process design changes Short production runs High production costs Limited models Attention to quality	Forecasting critical Product and process reliability Competitive product improvements and options Increase capacity Shift toward product focus Enhance distribution	Standardization Fewer product changes, more minor changes Optimum capacity Increasing stability of process Long production runs Product improvement and cost cutting	Little product differentiation Cost minimization Overcapacity in the industry Prune line to eliminate items not returning good margin Reduce capacity

Figure 2.5

Types of Forecasts

1. Economic forecasts
   - Address business cycle – inflation rate, money supply, housing starts, etc.
2. Technological forecasts
   - Predict rate of technological progress
   - Impacts development of new products
3. Demand forecasts
   - Predict sales of existing products and services

Strategic Importance of Forecasting

- Supply-Chain Management – Good supplier relations, advantages in product innovation, cost and speed to market
- Human Resources – Hiring, training, laying off workers
- Capacity – Capacity shortages can result in undependable delivery, loss of customers, loss of market share

Seven Steps in Forecasting

1. Determine the use of the forecast
2. Select the items to be forecasted
3. Determine the time horizon of the forecast
4. Select the forecasting model(s)
5. Gather the data needed to make the forecast
6. Make the forecast
7. Validate and implement results

The Realities!

- Forecasts are seldom perfect, unpredictable outside factors may impact the forecast
- Most techniques assume an underlying stability in the system
- Product family and aggregated forecasts are more accurate than individual product forecasts

Forecasting Approaches

Qualitative Methods

- Used when situation is vague and little data exist
  - New products
  - New technology
- Involves intuition, experience
  - e.g., forecasting sales on Internet

Forecasting Approaches

Quantitative Methods

- Used when situation is ‘stable’ and historical data exist
  - Existing products
  - Current technology
- Involves mathematical techniques
  - e.g., forecasting sales of color televisions

Overview of Qualitative Methods

1. Jury of executive opinion
   - Pool opinions of high-level experts, sometimes augment by statistical models
2. Delphi method
   - Panel of experts, queried iteratively

Overview of Qualitative Methods

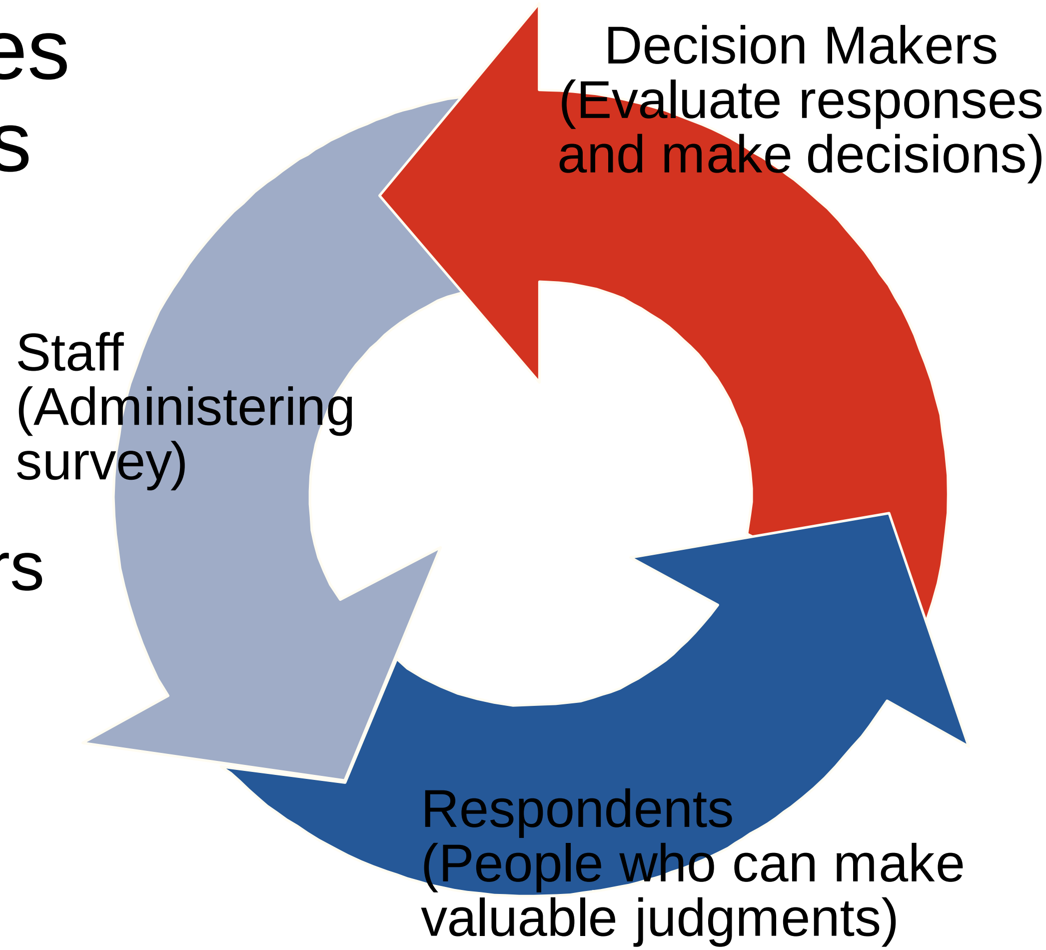
3. Sales force composite
   - Estimates from individual salespersons are reviewed for reasonableness, then aggregated
4. Market Survey
   - Ask the customer

Jury of Executive Opinion

- Involves small group of high-level experts and managers
- Group estimates demand by working together
- Combines managerial experience with statistical models
- Relatively quick
- ‘Group-think’ disadvantage

Delphi Method

- Iterative group process, continues until consensus is reached
- 3 types of participants
  - Decision makers
  - Staff
  - Respondents



Sales Force Composite

- Each salesperson projects his or her sales
- Combined at district and national levels
- Sales reps know customers' wants
- May be overly optimistic

Market Survey

- Ask customers about purchasing plans
- Useful for demand and product design and planning
- What consumers say, and what they actually do may be different
- May be overly optimistic

Overview of Quantitative Approaches

1. Naive approach
2. Moving averages
3. Exponential smoothing
4. Trend projection
5. Linear regression

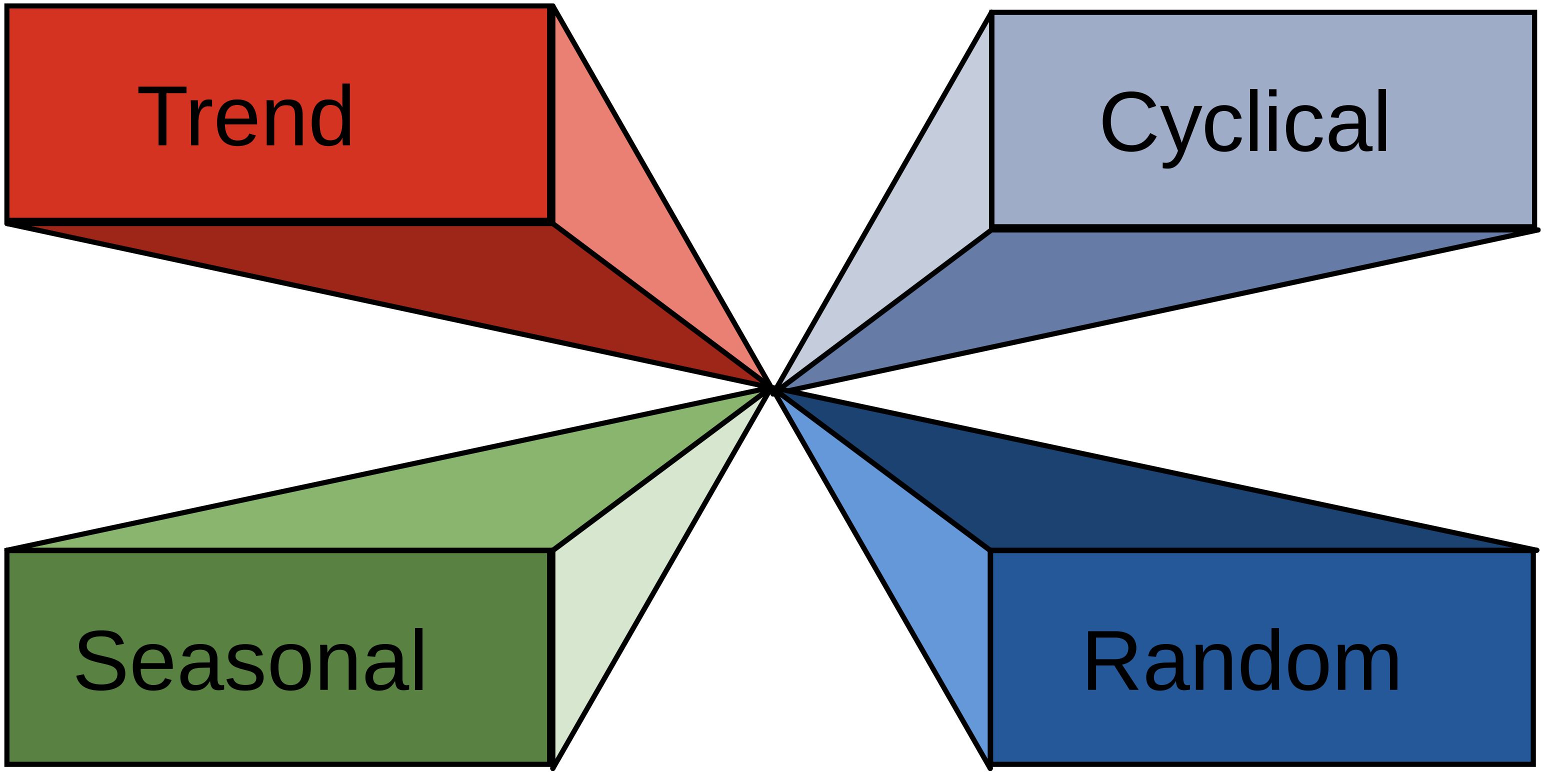
Time-series models

Associative model

Time-Series Forecasting

- Set of evenly spaced numerical data
  - Obtained by observing response variable at regular time periods
- Forecast based only on past values, no other variables important
  - Assumes that factors influencing past and present will continue influence in future

Time-Series Components



Components of Demand

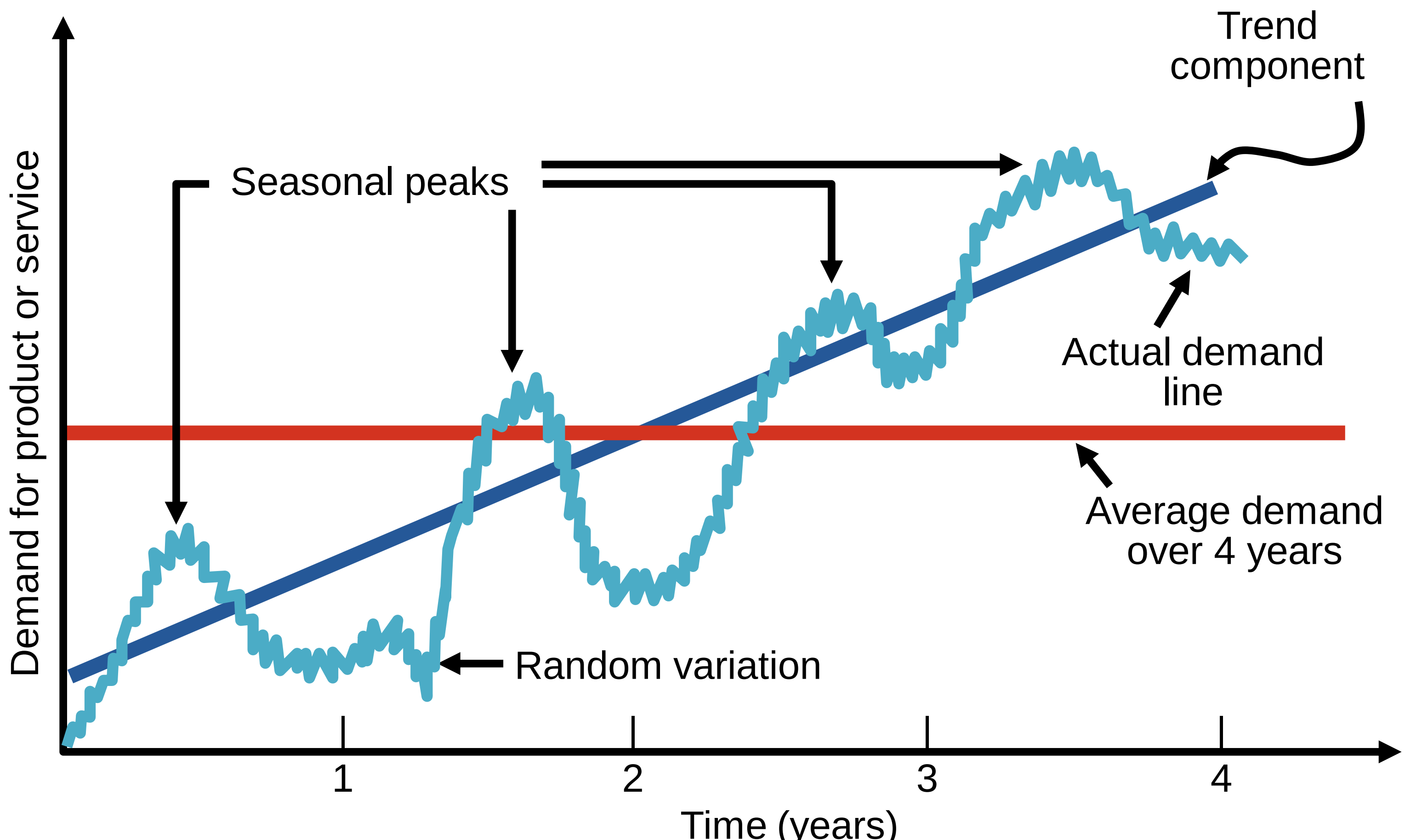
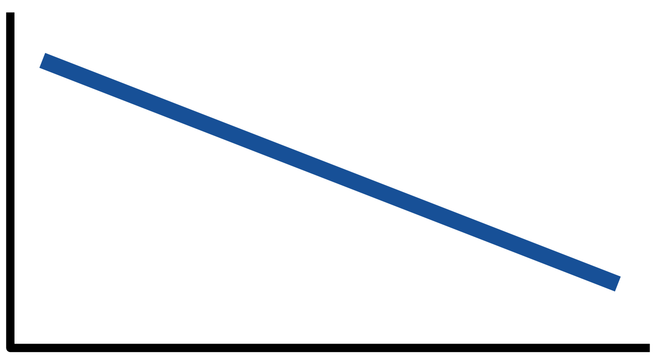


Figure 4.1

Trend Component

- Persistent, overall upward or downward pattern
- Changes due to population, technology, age, culture, etc.
- Typically several years duration



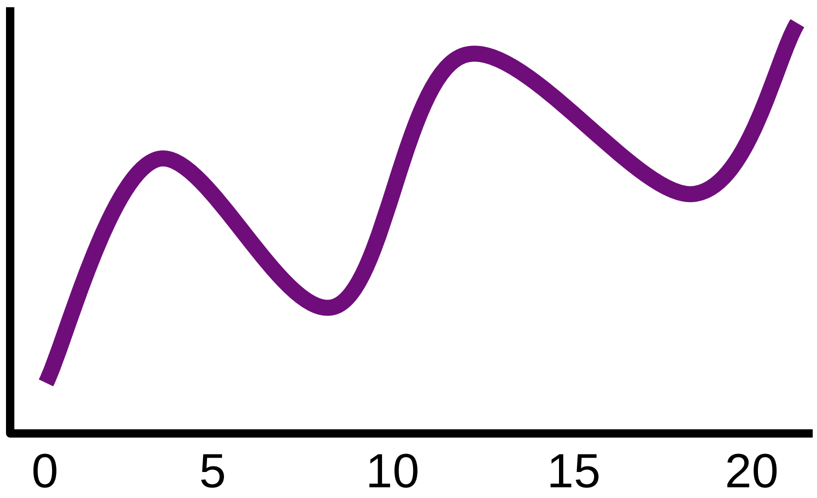
Seasonal Component

- Regular pattern of up and down fluctuations
- Due to weather, customs, etc.
- Occurs within a single year

PERIOD LENGTH	"SEASON" LENGTH	NUMBER OF "SEASONS" IN PATTERN
Week	Day	7
Month	Week	4 – 4.5
Month	Day	28 – 31
Year	Quarter	4
Year	Month	12
Year	Week	52

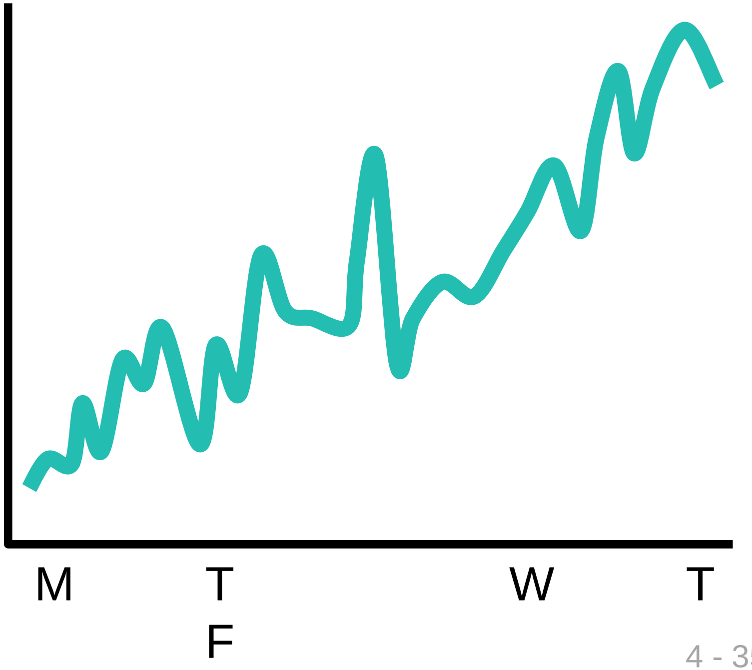
Cyclical Component

- Repeating up and down movements
- Affected by business cycle, political, and economic factors
- Multiple years duration
- Often causal or associative relationships



Random Component

- Erratic, unsystematic, ‘residual’ fluctuations
- Due to random variation or unforeseen events
- Short duration and nonrepeating



Naive Approach

- Assumes demand in next period is the same as demand in most recent period
  - e.g., If January sales were 68, then February sales will be 68
- Sometimes cost effective and efficient
- Can be good starting point

Moving Average Method

- MA is a series of arithmetic means
- Used if little or no trend
- Used often for smoothing
  - Provides overall impression of data over time

$$\text{Moving average} = \frac{\sum \text{demand in previous } n \text{ periods}}{n}$$

Moving Average Example

MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11\ ^2/_3$
May	19	$(12 + 13 + 16)/3 = 13\ ^2/_3$
June	23	$(13 + 16 + 19)/3 = 16$
July	26	$(16 + 19 + 23)/3 = 19\ ^1/_3$
August	30	$(19 + 23 + 26)/3 = 22\ ^2/_3$
September	28	$(23 + 26 + 30)/3 = 26\ ^1/_3$
October	18	$(29 + 30 + 28)/3 = 28$
November	16	$(30 + 28 + 18)/3 = 25\ ^1/_3$
December	14	$(28 + 18 + 16)/3 = 20\ ^2/_3$

Weighted Moving Average

- Used when some trend might be present
  - Older data usually less important
- Weights based on experience and intuition

$$\text{Weighted moving average} = \frac{\sum \left((\text{Weight for period } n)(\text{Demand in period } n) \right)}{\sum \text{Weights}}$$

Weighted Moving Average

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$[(3 \times 13) + (2 \times 12) + (10)]/6 = 12\ {}^{1}/_{6}$
May	19	
June		
July		
August		
September		
October		
November		
December		

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago
6	Sum of the weights

Forecast for this month =

$$\frac{3 \times \text{Sales last mo.} + 2 \times \text{Sales 2 mos. ago} + 1 \times \text{Sales 3 mos. ago}}{\text{Sum of the weights}}$$

Weighted Moving Average

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$[(3 \times 13) + (2 \times 12) + (10)]/6 = 12\ ^1/_6$
May	19	$[(3 \times 16) + (2 \times 13) + (12)]/6 = 14\ ^1/_3$
June	23	$[(3 \times 19) + (2 \times 16) + (13)]/6 = 17$
July	26	$[(3 \times 23) + (2 \times 19) + (16)]/6 = 20\ ^1/_2$
August	30	$[(3 \times 26) + (2 \times 23) + (19)]/6 = 23\ ^5/_6$
September	28	$[(3 \times 30) + (2 \times 26) + (23)]/6 = 27\ ^1/_2$
October	18	$[(3 \times 28) + (2 \times 30) + (26)]/6 = 28\ ^1/_3$
November	16	$[(3 \times 18) + (2 \times 28) + (30)]/6 = 23\ ^1/_3$
December	14	$[(3 \times 16) + (2 \times 18) + (28)]/6 = 18\ ^2/_3$

Potential Problems With Moving Average

- Increasing n smooths the forecast but makes it less sensitive to changes
- Does not forecast trends well
- Requires extensive historical data

Graph of Moving Averages

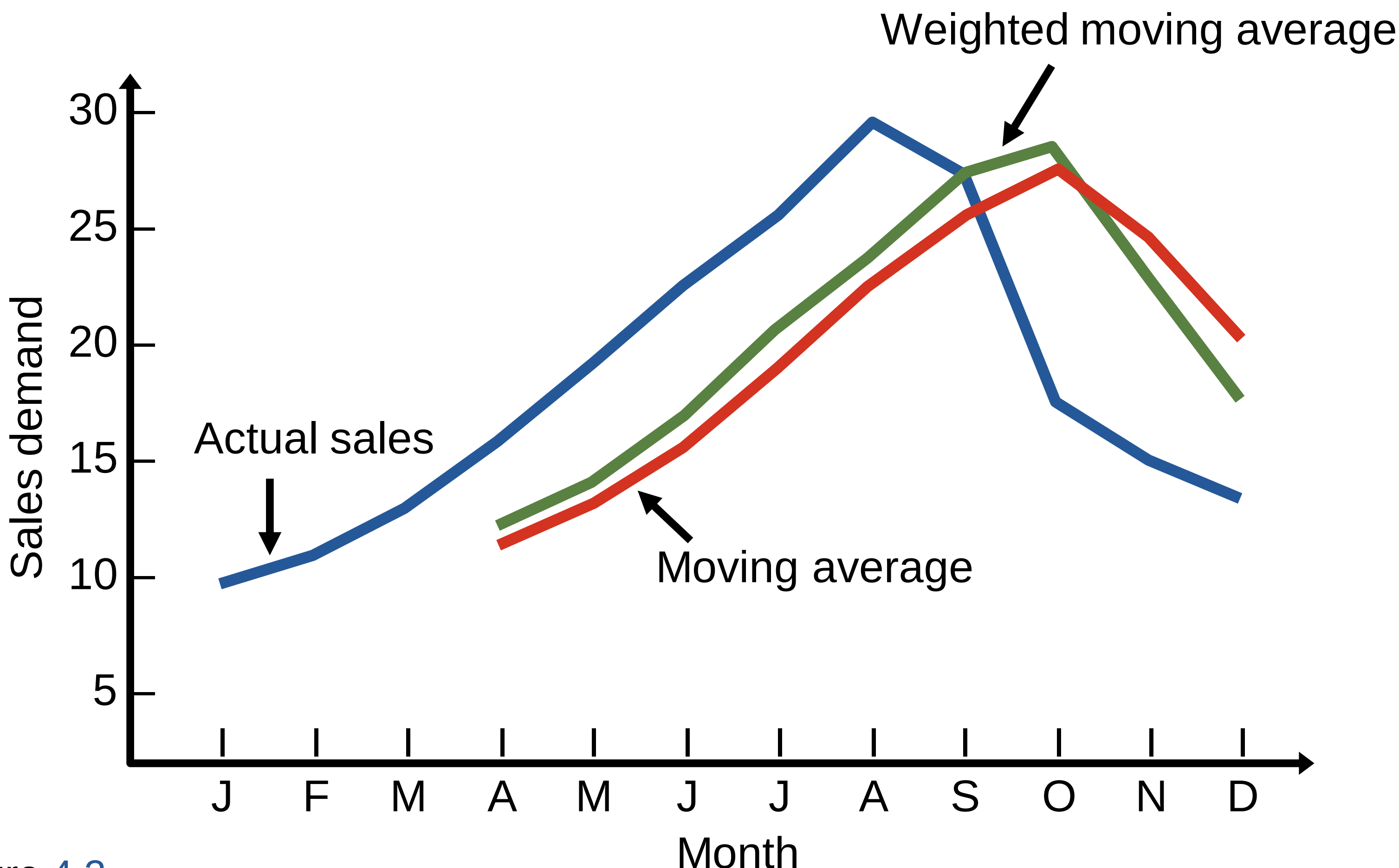


Figure 4.2

Exponential Smoothing

- Form of weighted moving average
  - Weights decline exponentially
  - Most recent data weighted most
- Requires smoothing constant (α)
  - Ranges from 0 to 1
  - Subjectively chosen
- Involves little record keeping of past data

Exponential Smoothing

New forecast = Last period's forecast
+ α (Last period's actual demand
– Last period's forecast)

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

where
F_t = new forecast
F_{t-1} = previous period's forecast
α = smoothing (or weighting) constant ($0 \le \alpha \le 1$)
A_{t-1} = previous period's actual demand

Exponential Smoothing Example

Predicted demand = 142 Ford Mustangs
Actual demand = 153
Smoothing constant α = .20

Exponential Smoothing Example

Predicted demand = 142 Ford Mustangs
Actual demand = 153
Smoothing constant α = .20

New forecast = 142 + .2(153 – 142)

Exponential Smoothing Example

Predicted demand = 142 Ford Mustangs
Actual demand = 153
Smoothing constant α = .20

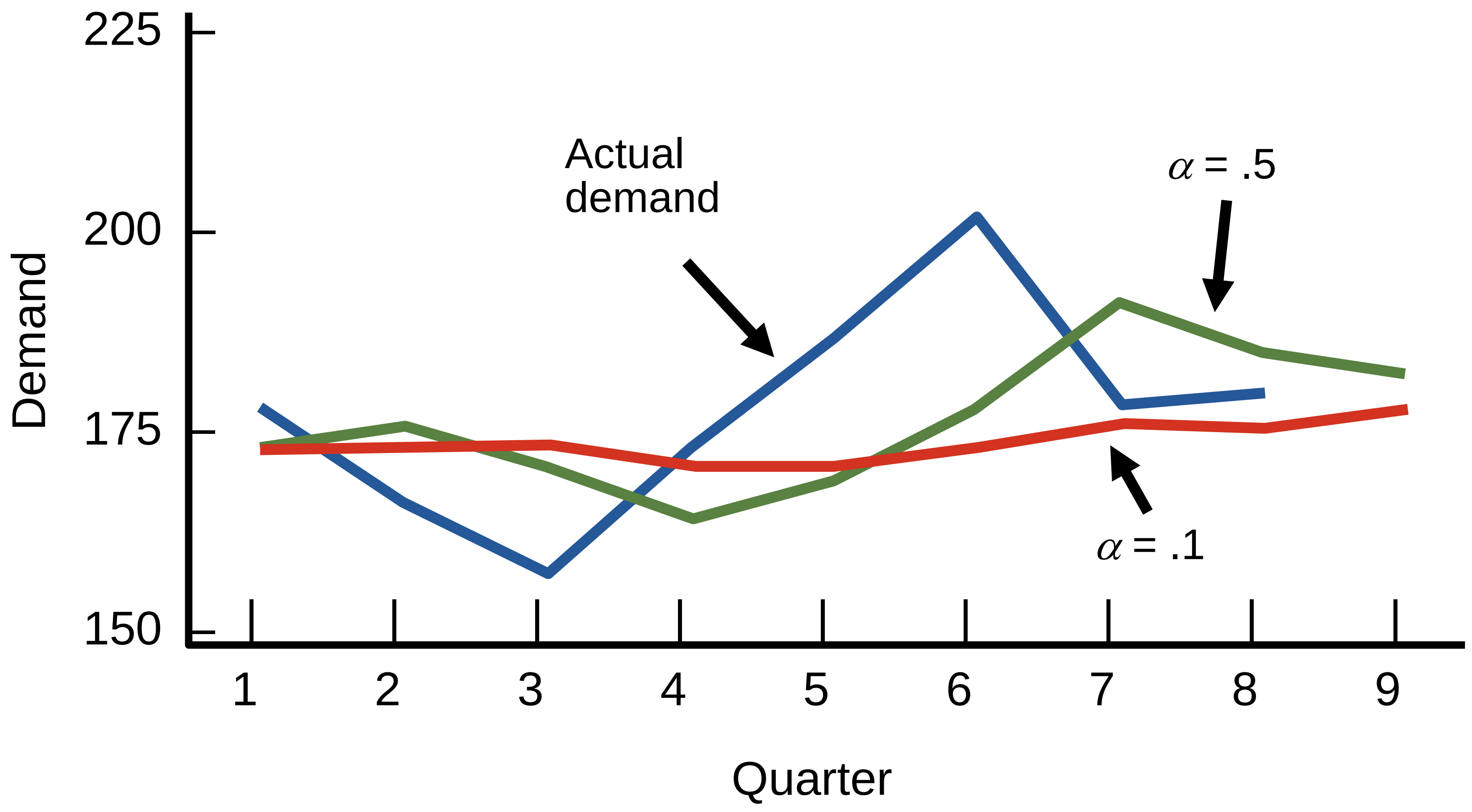
$$
\begin{aligned}
\text{New forecast} &= 142 + .2(153 - 142) \\
&= 142 + 2.2 \\
&= 144.2 \approx 144 \text{ cars}
\end{aligned}
$$

Effect of Smoothing Constants

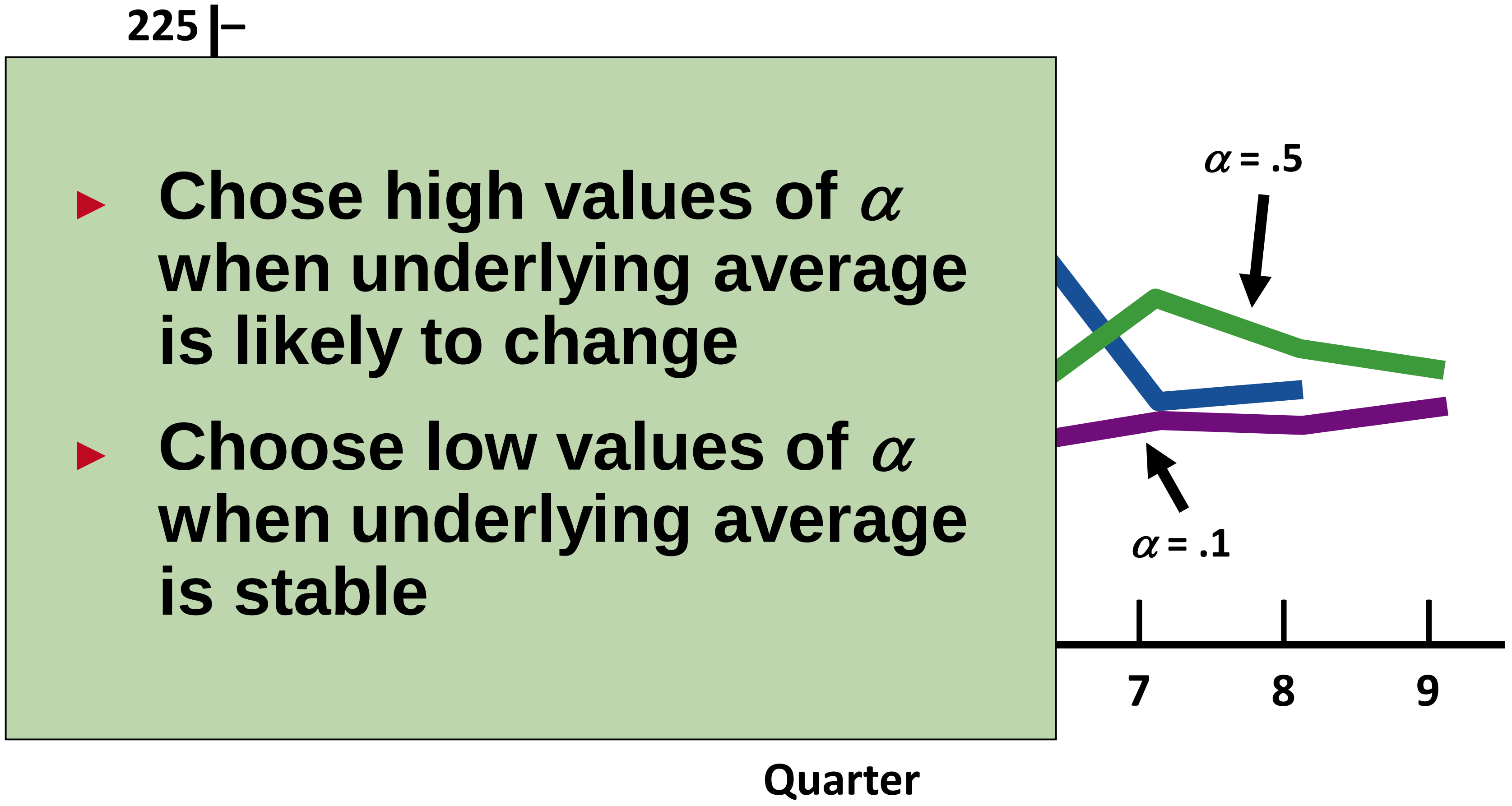
- Smoothing constant generally $.05 \leq \alpha \leq .50$
- As α increases, older values become less significant

	WEIGHT ASSIGNED TO				
SMOOTHING CONSTANT	MOST RECENT PERIOD (α)	2ND MOST RECENT PERIOD $\alpha(1-\alpha)$	3RD MOST RECENT PERIOD $\alpha(1-\alpha)^2$	4th MOST RECENT PERIOD $\alpha(1-\alpha)^3$	5th MOST RECENT PERIOD $\alpha(1-\alpha)^4$
$\alpha = .1$	.1	.09	.081	.073	.066
$\alpha = .5$	.5	.25	.125	.063	.031

Impact of Different α



Impact of Different α



- Chose high values of α when underlying average is likely to change
- Choose low values of α when underlying average is stable

Choosing α

The objective is to obtain the most accurate forecast no matter the technique

We generally do this by selecting the model that gives us the lowest forecast error

$$\text{Forecast error} = \text{Actual demand} - \text{Forecast value}$$
$$= A_t - F_t$$

Common Measures of Error

Mean Absolute Deviation (MAD)

$$\text{MAD} = \frac{\sum \left|\text{Actual} - \text{Forecast}\right|}{n}$$

Determining the MAD

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = .10$	FORECAST WITH $\alpha = .50$
1	180	175	175
2	168	175.50 = 175.00 + .10(180 – 175)	177.50
3	159	174.75 = 175.50 + .10(168 – 175.50)	172.75
4	175	173.18 = 174.75 + .10(159 – 174.75)	165.88
5	190	173.36 = 173.18 + .10(175 – 173.18)	170.44
6	205	175.02 = 173.36 + .10(190 – 173.36)	180.22
7	180	178.02 = 175.02 + .10(205 – 175.02)	192.61
8	182	178.22 = 178.02 + .10(180 – 178.02)	186.30
9	?	178.59 = 178.22 + .10(182 – 178.22)	184.15

Determining the MAD

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH α = .10	ABSOLUTE DEVIATION FOR a = .10	FORECAST WITH α = .50	ABSOLUTE DEVIATION FOR a = .50
1	180	175	5.00	175	5.00
2	168	175.50	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
Sum of absolute deviations:			82.45		98.62
		$\text{MAD} = \dfrac{\Sigma\lvert\text{Deviations}\rvert}{n}$	10.31		12.33

Common Measures of Error

Mean Squared Error (MSE)

$$\text{MSE} = \frac{\sum(\text{Forecast errors})^2}{n}$$

Determining the MSE

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	$(\text{ERROR})^2$
1	180	175	$5^2 = 25$
2	168	175.50	$(-7.5)^2 = 56.25$
3	159	174.75	$(-15.75)^2 = 248.06$
4	175	173.18	$(1.82)^2 = 3.31$
5	190	173.36	$(16.64)^2 = 276.89$
6	205	175.02	$(29.98)^2 = 898.80$
7	180	178.02	$(1.98)^2 = 3.92$
8	182	178.22	$(3.78)^2 = 14.29$
			Sum of errors squared = 1,526.52

$$\text{MSE} = \frac{\sum (\text{Forecast errors})^2}{n} = 1{,}526.52 / 8 = 190.8$$

Common Measures of Error

Mean Absolute Percent Error (MAPE)

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100 \left| \text{Actual}_i - \text{Forecast}_i \right| / \text{Actual}_i}{n}$$

Determining the MAPE

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	ABSOLUTE PERCENT ERROR 100(ERROR/ACTUAL)
1	180	175.00	100(5/180) = 2.78%
2	168	175.50	100(7.5/168) = 4.46%
3	159	174.75	100(15.75/159) = 9.90%
4	175	173.18	100(1.82/175) = 1.05%
5	190	173.36	100(16.64/190) = 8.76%
6	205	175.02	100(29.98/205) = 14.62%
7	180	178.02	100(1.98/180) = 1.10%
8	182	178.22	100(3.78/182) = 2.08%
			Sum of % errors = 44.75%

$$\text{MAPE} = \frac{\sum \text{absolute percent error}}{n} = \frac{44.75\%}{8} = 5.59\%$$

Comparison of Forecast Error

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
1	180	175	5.00	175	5.00
2	168	175.5	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
			82.45		98.62

Comparison of Forecast Error

$$\text{MAD} = \frac{\sum |\text{deviations}|}{n}$$

For $\alpha = .10$

$= 82.45/8 = 10.31$

For $\alpha = .50$

$= 98.62/8 = 12.33$

				Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
				175	5.00
				177.50	9.50
				172.75	13.75
				165.88	9.12
				170.44	19.56
				180.22	24.78
				192.61	12.61
8	182	178.22	3.78	186.30	4.30
			82.45		98.62

Comparison of Forecast Error

$$\text{MSE} = \frac{\sum (\text{forecast errors})^2}{n}$$

For $\alpha = .10$

$= 1{,}526.54/8 = 190.82$

For $\alpha = .50$

$= 1{,}561.91/8 = 195.24$

				Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
				175	5.00
				177.50	9.50
				172.75	13.75
				165.88	9.12
				170.44	19.56
				180.22	24.78
				192.61	12.61
8	182	178.22	3.78	186.30	4.30
			82.45		98.62
		MAD	10.31		12.33

Comparison of Forecast Error

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100|\text{deviation}_i|/\text{actual}_i}{n}$$

For $\alpha = .10$

$= 44.75/8 = 5.59\%$

For $\alpha = .50$

$= 54.05/8 = 6.76\%$

		Absolute Deviation for $\alpha = .50$
		5.00
		9.50
		13.75
		9.12
		19.56
		24.78
		12.61
		4.30
	82.45	98.62
MAD	10.31	12.33
MSE	190.82	195.24

Comparison of Forecast Error

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
1	180	175	5.00	175	5.00
2	168	175.5	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
			82.45		98.62
		MAD	10.31		12.33
		MSE	190.82		195.24
		MAPE	5.59%		6.76%

Exponential Smoothing with Trend Adjustment

When a trend is present, exponential smoothing must be modified

MONTH	ACTUAL DEMAND	FORECAST (F_t) FOR MONTHS 1 – 5
1	100	$F_t = 100$ (given)
2	200	$F_t = F_1 + \alpha(A_1 - F_1) = 100 + .4(100 - 100) = 100$
3	300	$F_t = F_2 + \alpha(A_2 - F_2) = 100 + .4(200 - 100) = 140$
4	400	$F_t = F_3 + \alpha(A_3 - F_3) = 140 + .4(300 - 140) = 204$
5	500	$F_t = F_4 + \alpha(A_4 - F_4) = 204 + .4(400 - 204) = 282$

Exponential Smoothing with Trend Adjustment

Forecast including trend (FIT_t) = Exponentially smoothed forecast (F_t) + Exponentially smoothed trend (T_t)

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1})$$

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1}$$

where F_t = exponentially smoothed forecast average
T_t = exponentially smoothed trend
A_t = actual demand
α = smoothing constant for average ($0 \le \alpha \le 1$)
β = smoothing constant for trend ($0 \le \beta \le 1$)

Exponential Smoothing with Trend Adjustment

Step 1: Compute F_t

Step 2: Compute T_t

Step 3: Calculate the forecast $FIT_t = F_t + T_t$

Exponential Smoothing with Trend Adjustment Example

MONTH (t)	ACTUAL DEMAND (A_t)	MONTH (t)	ACTUAL DEMAND (A_t)
1	12	6	21
2	17	7	31
3	20	8	28
4	19	9	36
5	24	10	?

$\alpha = .2$ $\beta = .4$

Exponential Smoothing with Trend Adjustment Example

TABLE 4.1 Forecast with α - .2 and β = .4

MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80		
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 1: Average for Month 2

$$F_2 = \alpha A_1 + (1 - \alpha)(F_1 + T_1)$$

$$F_2 = (.2)(12) + (1 - .2)(11 + 2)$$

$$= 2.4 + (.8)(13) = 2.4 + 10.4$$

$$= 12.8 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example

TABLE 4.1	Forecast with α - .2 and β = .4			
MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 2: Trend for Month 2

$$T_2 = \beta(F_2 - F_1) + (1 - b)T_1$$

$$T_2 = (.4)(12.8 - 11) + (1 - .4)(2)$$

$$= .72 + 1.2 = 1.92 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example

TABLE 4.1 **Forecast with α - .2 and β = .4**

MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 3: Calculate *FIT* for Month 2

$$FIT_2 = F_2 + T_2$$

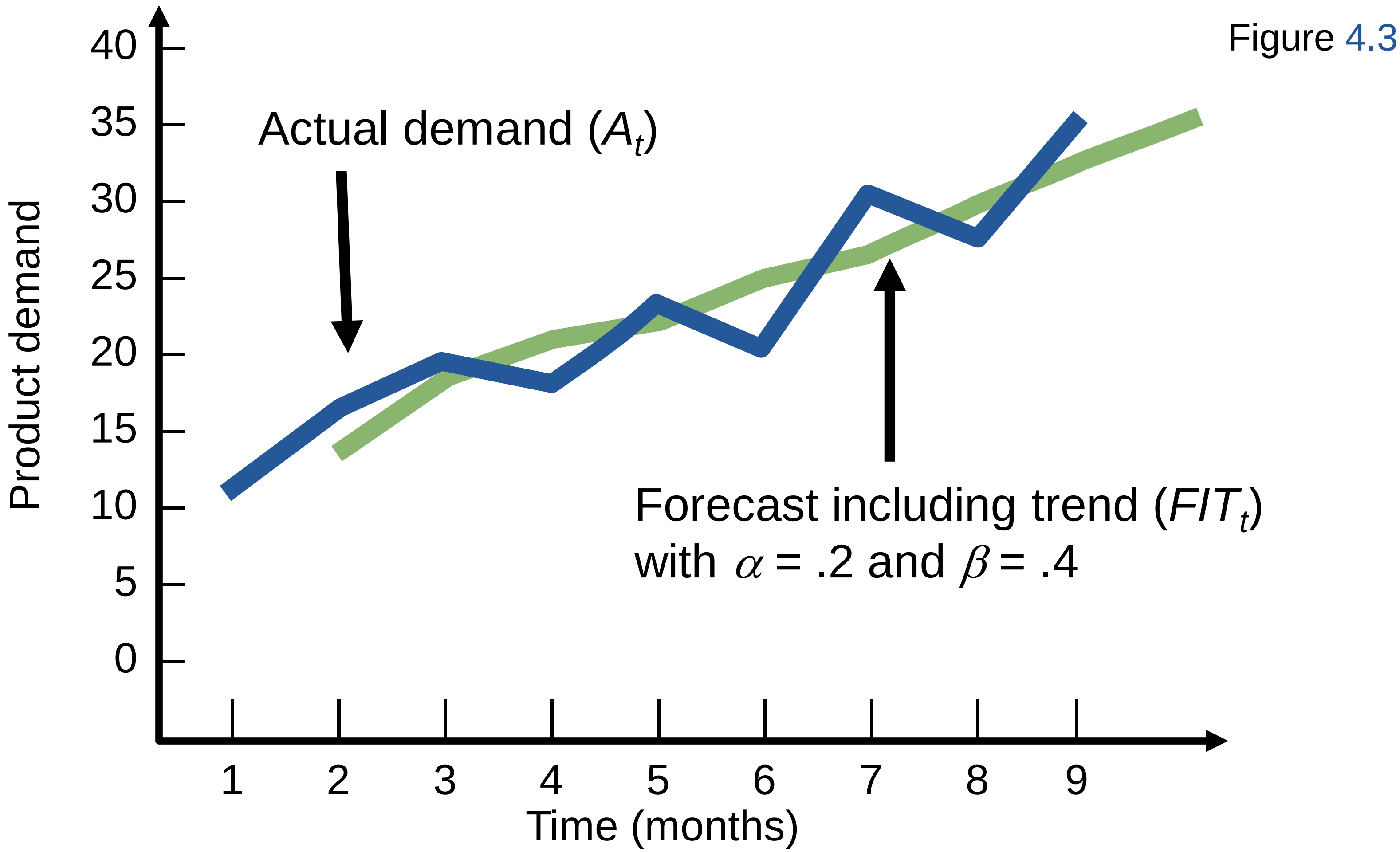
$$FIT_2 = 12.8 + 1.92$$

$$= 14.72 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example

TABLE 4.1	Forecast with α - .2 and β = .4			
MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20	15.18	2.10	17.28
4	19	17.82	2.32	20.14
5	24	19.91	2.23	22.14
6	21	22.51	2.38	24.89
7	31	24.11	2.07	26.18
8	28	27.14	2.45	29.59
9	36	29.28	2.32	31.60
10	—	32.48	2.68	35.16

Exponential Smoothing with Trend Adjustment Example



Trend Projections

Fitting a trend line to historical data points to project into the medium to long-range

Linear trends can be found using the least squares technique

$$\hat{y} = a + bx$$

where $\hat{y}$ = computed value of the variable to be predicted (dependent variable)

a = y-axis intercept

b = slope of the regression line

x = the independent variable

Least Squares Method

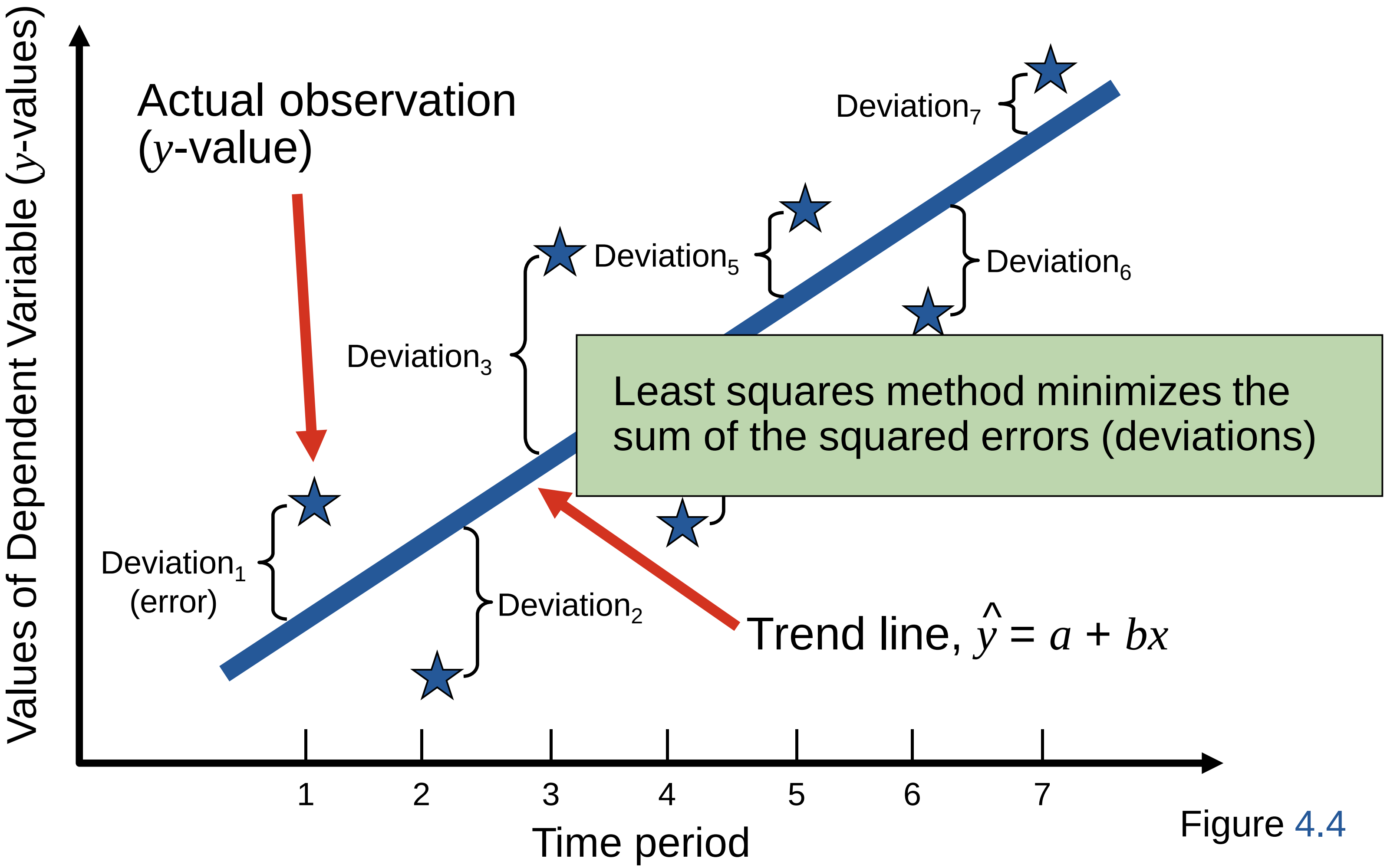


Figure 4.4

Least Squares Method

Equations to calculate the regression variables

$$\hat{y} = a + bx$$

$$b = \frac{\sum xy - n\overline{xy}}{\sum x^2 - n\overline{x}^2}$$

$$a = \overline{y} - b\overline{x}$$

Least Squares Example

YEAR	ELECTRICAL POWER DEMAND	YEAR	ELECTRICAL POWER DEMAND
1	74	5	105
2	79	6	142
3	80	7	122
4	90		

Least Squares Example

YEAR (x)	ELECTRICAL POWER DEMAND (y)	x^2	xy
1	74	1	74
2	79	4	158
3	80	9	240
4	90	16	360
5	105	25	525
6	142	36	852
7	122	49	854
$\Sigma x = 28$	$\Sigma y = 692$	$\Sigma x^2 = 140$	$\Sigma xy = 3{,}063$

$$\bar{x} = \frac{\sum x}{n} = \frac{28}{7} = 4 \qquad \bar{y} = \frac{\sum y}{n} = \frac{692}{7} = 98.86$$

Least Squares Example

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{3{,}063 - (7)(4)(98.86)}{140 - (7)(4^2)} = \frac{295}{28} = 10.54$$

$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$

Thus, $\hat{y} = 56.70 + 10.54x$

Y			
7	122	49	854
$\Sigma x = 28$	$\Sigma y = 692$	$\Sigma x^2 = 140$	$\Sigma xy = 3{,}063$

Demand in year 8 = 56.70 + 10.54(8)
= 141.02, or 141 megawatts

Least Squares Example

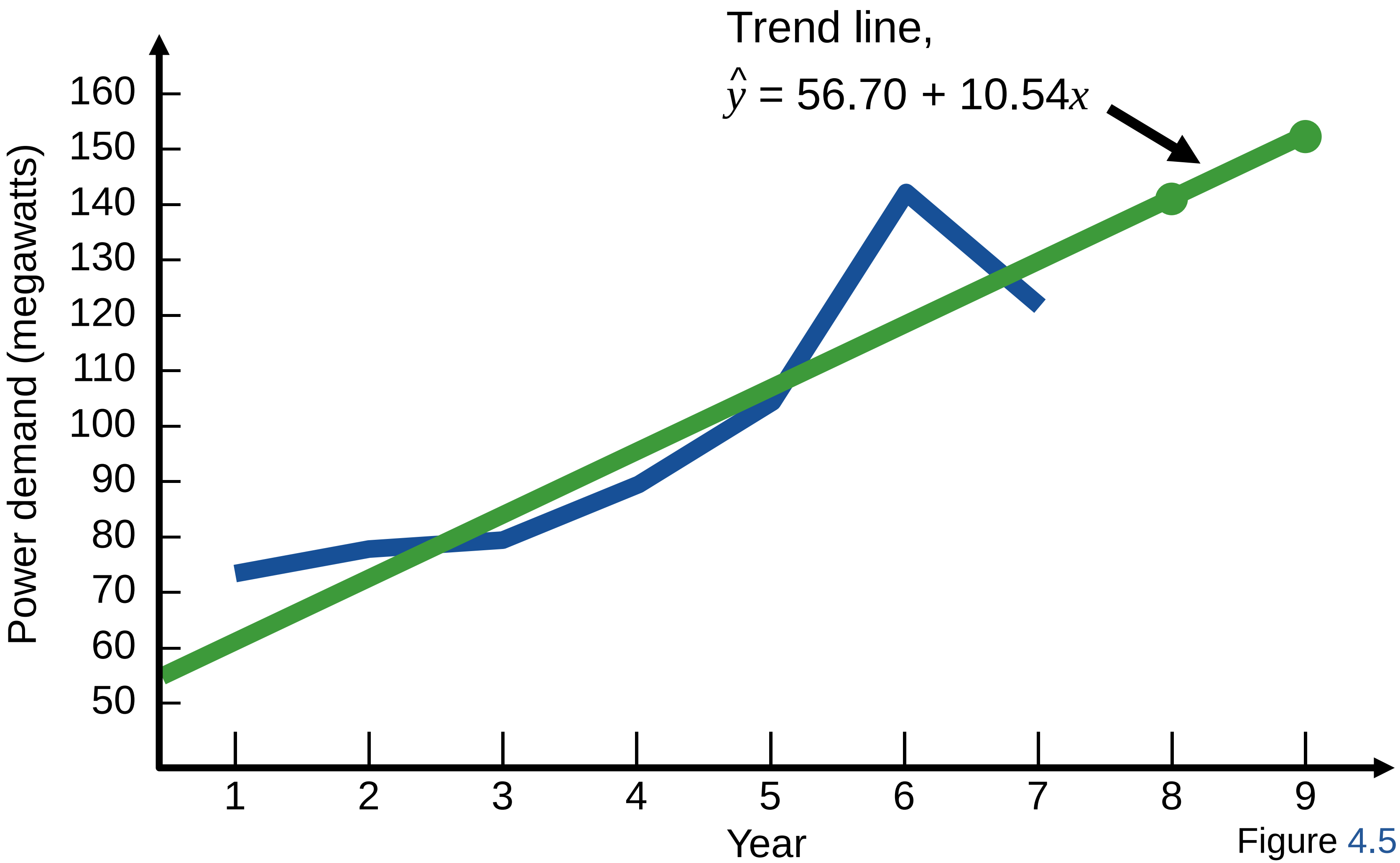


Figure 4.5

Least Squares Requirements

1. We always plot the data to insure a linear relationship
2. We do not predict time periods far beyond the database
3. Deviations around the least squares line are assumed to be random

Seasonal Variations In Data

The multiplicative seasonal model can adjust trend data for seasonal variations in demand

Seasonal Variations In Data

Steps in the process for monthly seasons:

1. Find average historical demand for each month
2. Compute the average demand over all months
3. Compute a seasonal index for each month
4. Estimate next year’s total demand
5. Divide this estimate of total demand by the number of months, then multiply it by the seasonal index for that month

Seasonal Index Example

DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE YEARLY DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90		
Feb	70	85	85	80		
Mar	80	93	82	85		
Apr	90	95	115	100		
May	113	125	131	123		
June	110	115	120	115		
July	100	102	113	105		
Aug	88	102	110	100		
Sept	85	90	95	90		
Oct	77	78	85	80		
Nov	75	82	83	80		
Dec	82	78	80	80		
Total average annual demand =				1,128		

Seasonal Index Example

	DEMAND					
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE YEARLY DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	
Feb	70	85	85	80	94	
Mar	[illegible]	[illegible]	[illegible]	[illegible]	94	
Apr	[illegible]	[illegible]	[illegible]	[illegible]	94	
May	[illegible]	[illegible]	[illegible]	[illegible]	94	
June	[illegible]	[illegible]	[illegible]	[illegible]	94	
July	100	102	113	105	94	
Aug	88	102	110	100	94	
Sept	85	90	95	90	94	
Oct	77	78	85	80	94	
Nov	75	82	83	80	94	
Dec	82	78	80	80	94	
			Total average annual demand =	1,128		

$$\text{Average monthly demand} = \frac{1{,}128}{12 \text{ months}} = 94$$

Seasonal Index Example

	DEMAND					
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE YEARLY DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	.957(= 90/94)
Feb	70	85	85	80	94	
Mar	80	93	82	85	94	
Apr	90	95	115	100	94	
Sept	85	90	95	90	94	
Oct	77	78	85	80	94	
Nov	75	82	83	80	94	
Dec	82	78	80	80	94	
Total average annual demand =				1,128		

$$\text{Seasonal index} = \frac{\text{Average monthly demand for past 3 years}}{\text{Average monthly demand}}$$

Seasonal Index Example

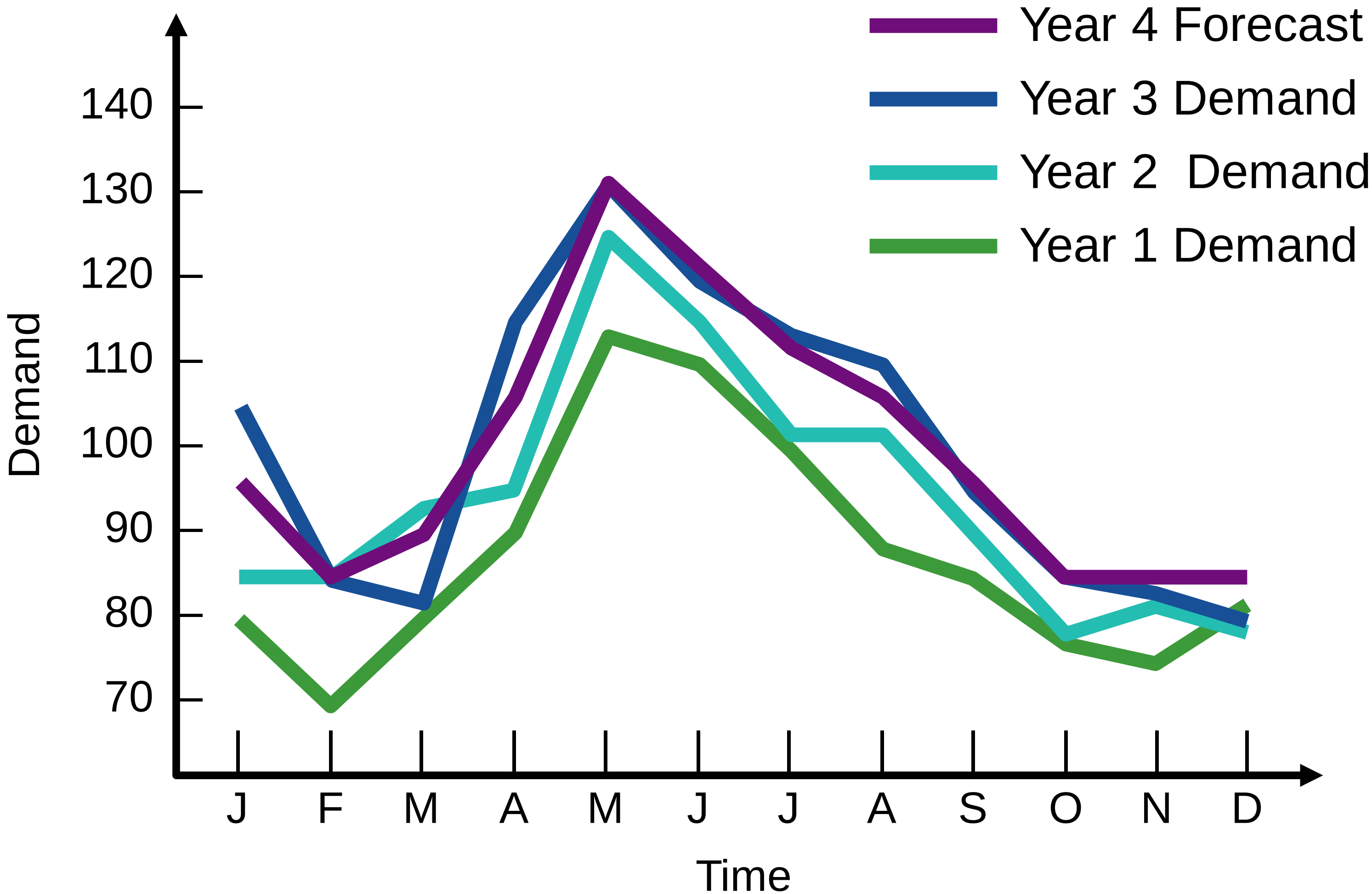
DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE YEARLY DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	.957(= 90/94)
Feb	70	85	85	80	94	.851(= 80/94)
Mar	80	93	82	85	94	.904(= 85/94)
Apr	90	95	115	100	94	1.064(= 100/94)
May	113	125	131	123	94	1.309(= 123/94)
June	110	115	120	115	94	1.223(= 115/94)
July	100	102	113	105	94	1.117(= 105/94)
Aug	88	102	110	100	94	1.064(= 100/94)
Sept	85	90	95	90	94	.957(= 90/94)
Oct	77	78	85	80	94	.851(= 80/94)
Nov	75	82	83	80	94	.851(= 80/94)
Dec	82	78	80	80	94	.851(= 80/94)
Total average annual demand =				1,128		

Seasonal Index Example

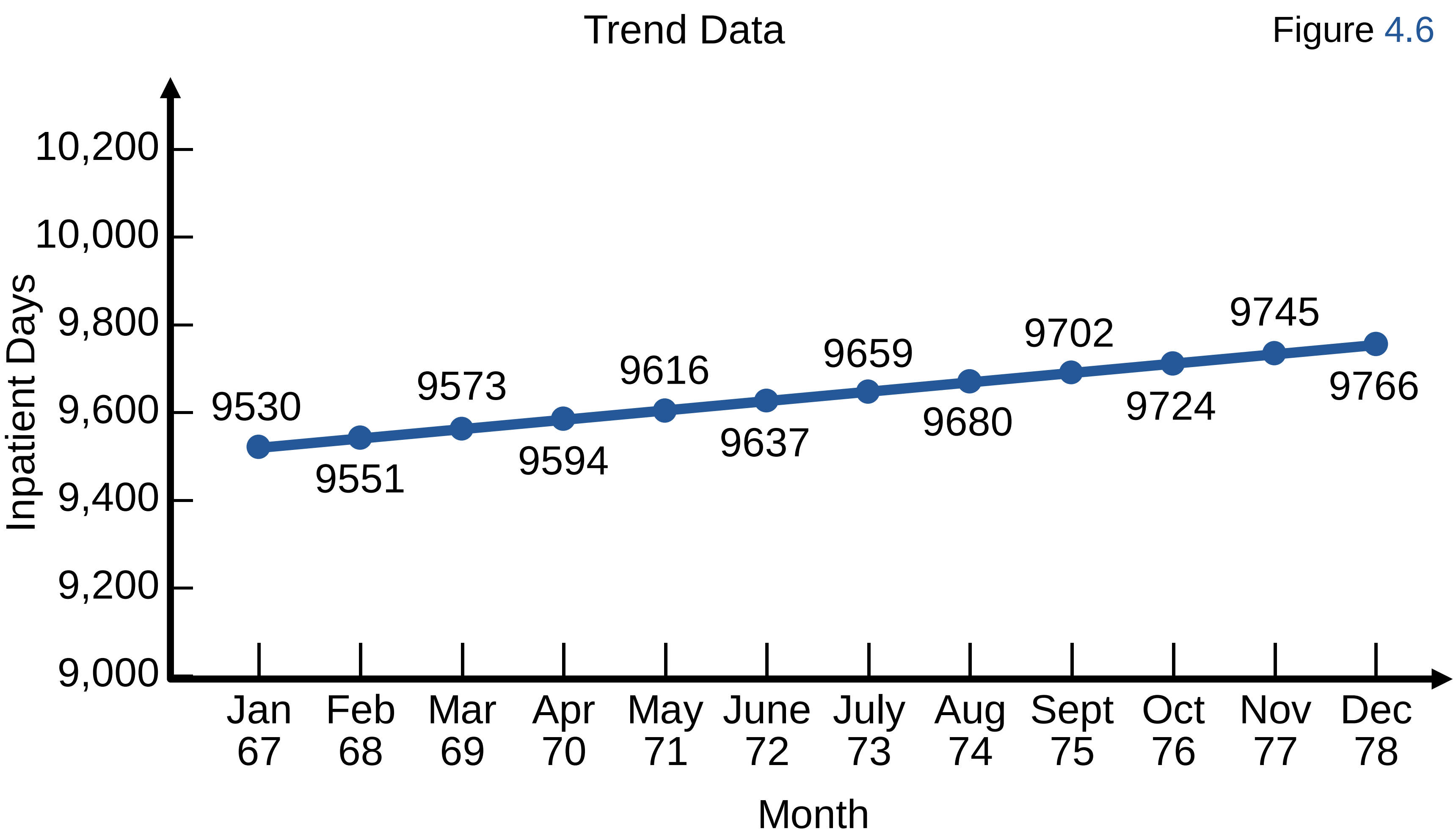
Seasonal forecast for Year 4

MONTH	DEMAND	MONTH	DEMAND
Jan	$\frac{1,200}{12}$ x .957 = 96	July	$\frac{1,200}{12}$ x 1.117 = 112
Feb	$\frac{1,200}{12}$ x .851 = 85	Aug	$\frac{1,200}{12}$ x 1.064 = 106
Mar	$\frac{1,200}{12}$ x .904 = 90	Sept	$\frac{1,200}{12}$ x .957 = 96
Apr	$\frac{1,200}{12}$ x 1.064 = 106	Oct	$\frac{1,200}{12}$ x .851 = 85
May	$\frac{1,200}{12}$ x 1.309 = 131	Nov	$\frac{1,200}{12}$ x .851 = 85
June	$\frac{1,200}{12}$ x 1.223 = 122	Dec	$\frac{1,200}{12}$ x .851 = 85

Seasonal Index Example



San Diego Hospital

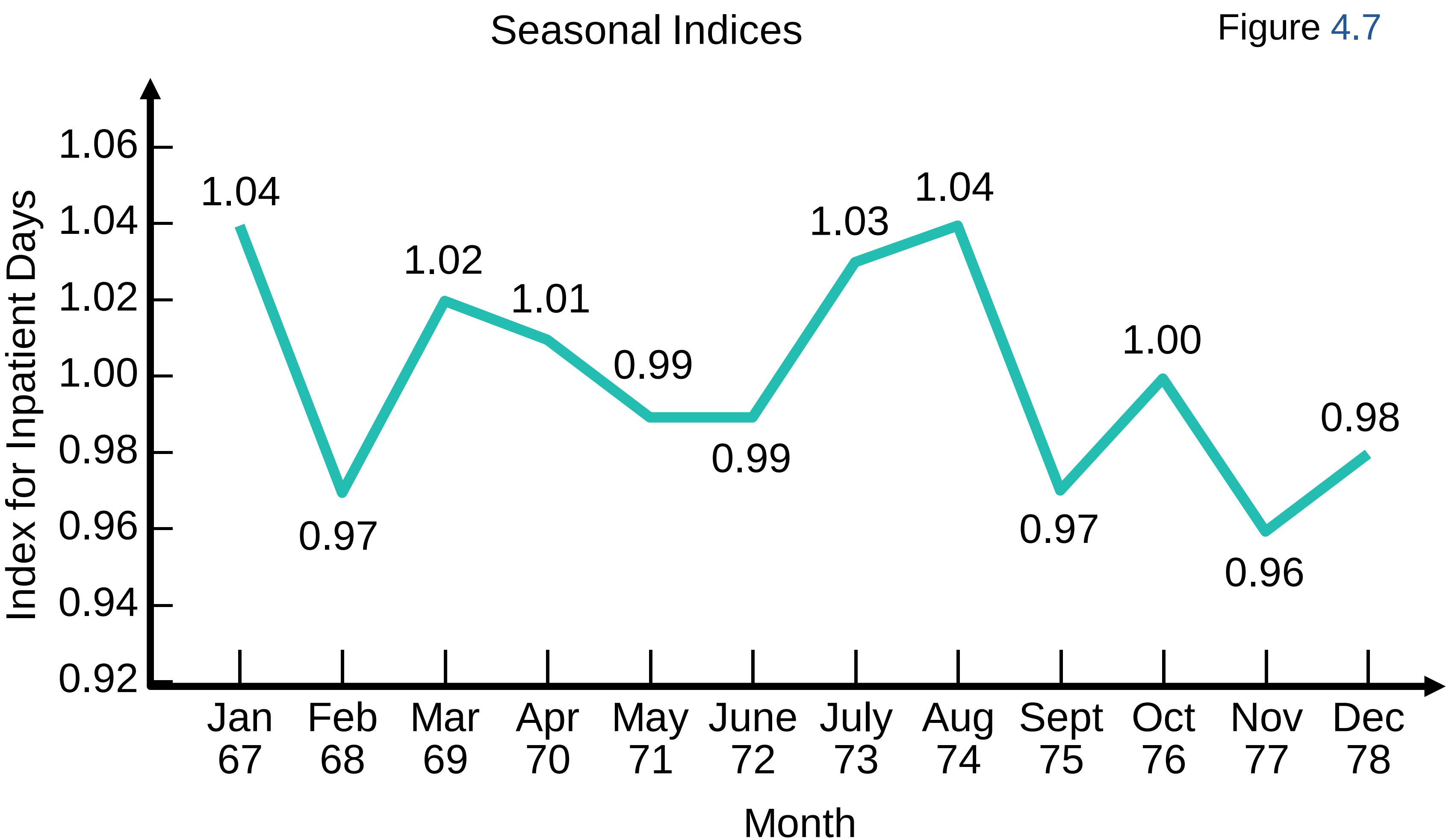


San Diego Hospital

	Seasonality Indices for Adult Inpatient Days at San Diego Hospital		
MONTH	**SEASONALITY INDEX**	**MONTH**	**SEASONALITY INDEX**
January	1.04	July	1.03
February	0.97	August	1.04
March	1.02	September	0.97
April	1.01	October	1.00
May	0.99	November	0.96
June	0.99	December	0.98

San Diego Hospital

Figure 4.7



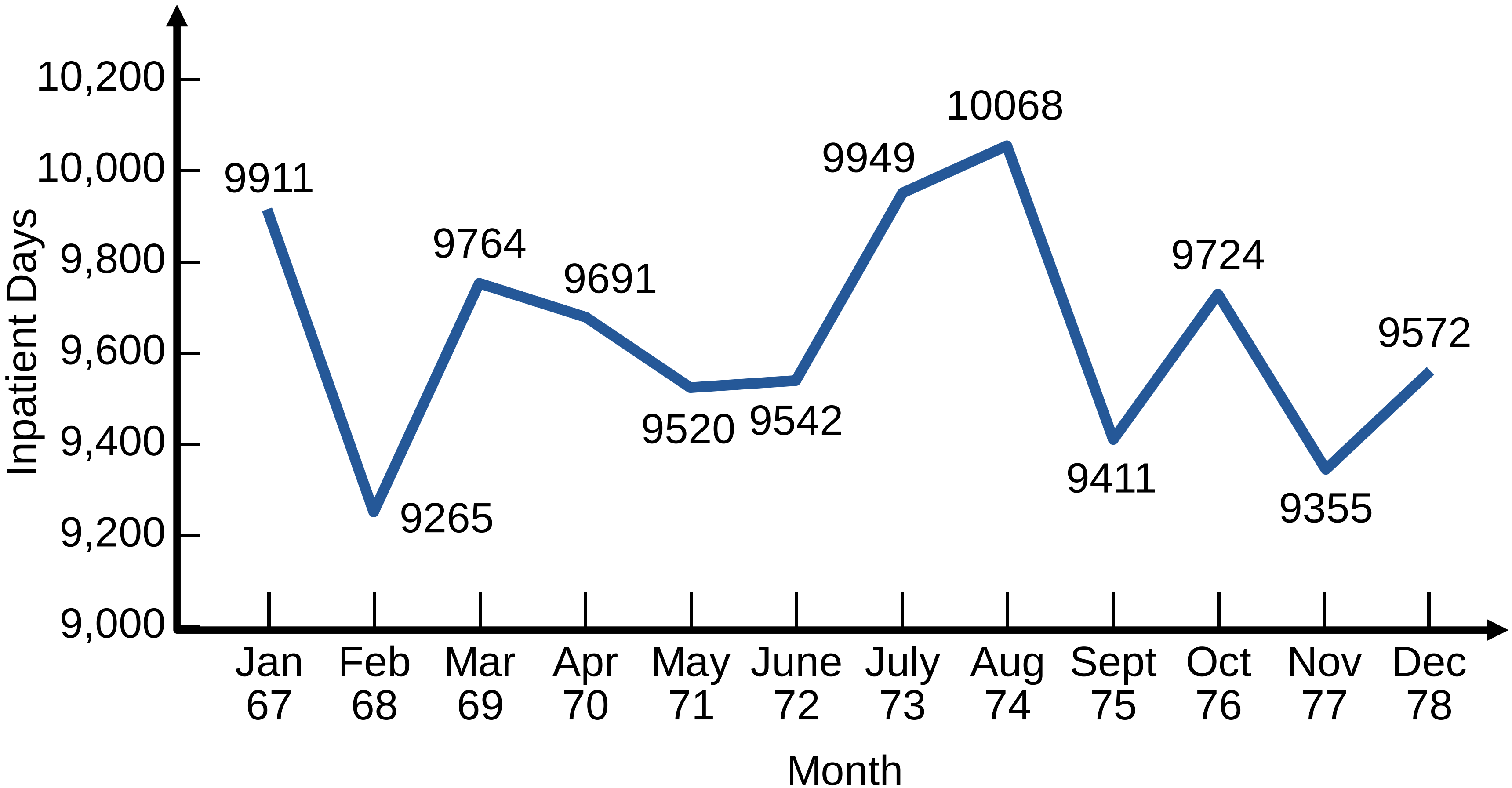
San Diego Hospital

Period	67	68	69	70	71	72
Month	Jan	Feb	Mar	Apr	May	June
Forecast with Trend & Seasonality	9,911	9,265	9,164	9,691	9,520	9,542
Period	73	74	75	76	77	78
Month	July	Aug	Sept	Oct	Nov	Dec
Forecast with Trend & Seasonality	9,949	10,068	9,411	9,724	9,355	9,572

San Diego Hospital

Combined Trend and Seasonal Forecast

Figure 4.8



Adjusting Trend Data

$$\hat{y}_{\text{seasonal}} = \text{Index} \times \hat{y}_{\text{trend forecast}}$$

Quarter I: $\hat{y}_{\text{I}} = (1.30)(\$100{,}000) = \$130{,}000$

Quarter II: $\hat{y}_{\text{II}} = (.90)(\$120{,}000) = \$108{,}000$

Quarter III: $\hat{y}_{\text{III}} = (.70)(\$140{,}000) = \$98{,}000$

Quarter IV: $\hat{y}_{\text{IV}} = (1.10)(\$160{,}000) = \$176{,}000$

Associative Forecasting

Used when changes in one or more independent variables can be used to predict the changes in the dependent variable

Most common technique is linear regression analysis

We apply this technique just as we did in the time-series example

Associative Forecasting

Forecasting an outcome based on predictor variables using the least squares technique

$$\hat{y} = a + bx$$

where $\hat{y}$ = value of the dependent variable (in our example, sales)

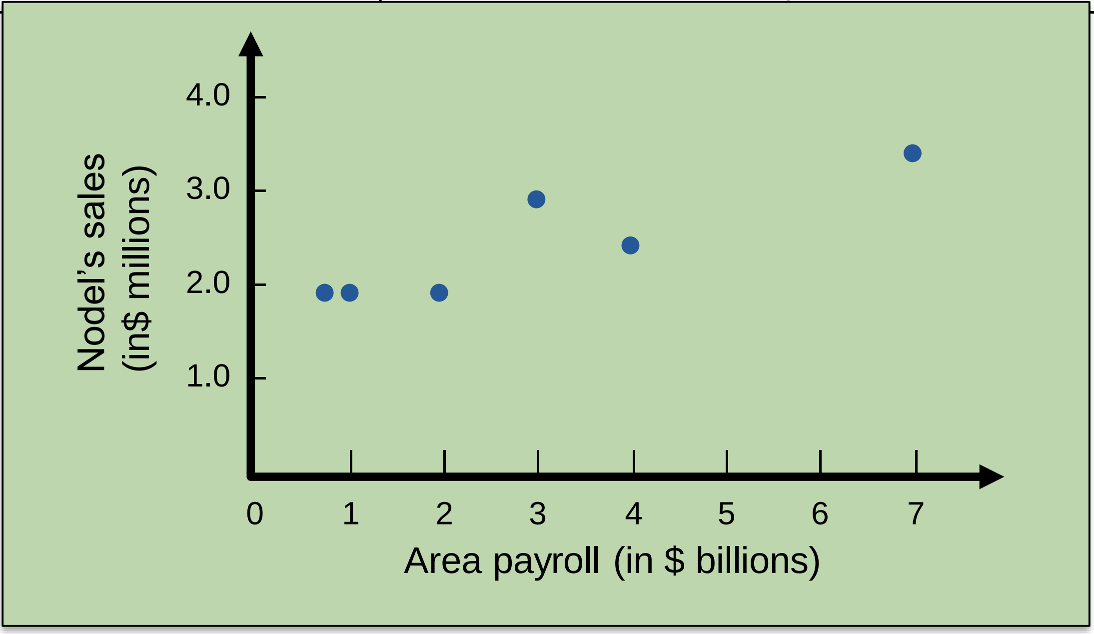
a = y-axis intercept

b = slope of the regression line

x = the independent variable

Associative Forecasting Example

NODEL'S SALES (IN \$ MILLIONS), *y*	AREA PAYROLL (IN \$ BILLIONS), *x*	NODEL'S SALES (IN \$ MILLIONS), *y*	AREA PAYROLL (IN \$ BILLIONS), *x*
2.0	1	2.0	2
3.0	3	2.0	1
2.5	4	3.5	7



Associative Forecasting Example

SALES, y	PAYROLL, x	x^2	xy
2.0	1	1	2.0
3.0	3	9	9.0
2.5	4	16	10.0
2.0	2	4	4.0
2.0	1	1	2.0
3.5	7	49	24.5
$\Sigma y =$ 15.0	$\Sigma x =$ 18	$\Sigma x^2 =$ 80	$\Sigma xy =$ 51.5

$$\bar{x} = \frac{\sum x}{6} = \frac{18}{6} = 3 \qquad \bar{y} = \frac{\sum y}{6} = \frac{15}{6} = 2.5$$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{51.5 - (6)(3)(2.5)}{80 - (6)(3^2)} = .25 \qquad a = \bar{y} - b\bar{x} = 2.5 - (.25)(3) = 1.75$$

Associative Forecasting Example

SALES, y	PAYROLL, x	x^2	xy
2.0			
3.0			
2.5			
2.0			
2.0			
3.5	7	49	24.5
$\Sigma y =$ 15.0	$\Sigma x =$ 18	$\Sigma x^2 =$ 80	$\Sigma xy =$ 51.5

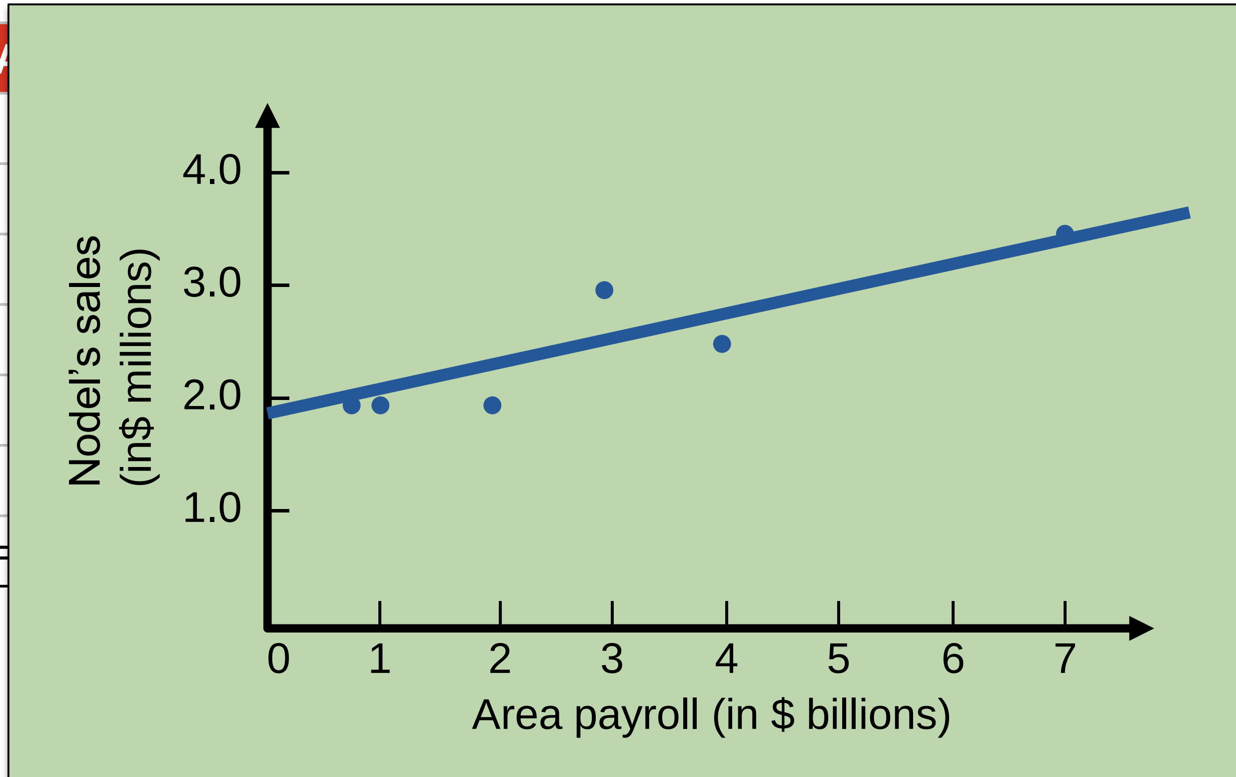
$$\hat{y} = 1.75 + .25x$$

$$\text{Sales} = 1.75 + .25(\text{payroll})$$

$$\bar{x} = \frac{\sum x}{6} = \frac{18}{6} = 3 \qquad \bar{y} = \frac{\sum y}{6} = \frac{15}{6} = 2.5$$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{51.5 - (6)(3)(2.5)}{80 - (6)(3^2)} = .25 \qquad a = \bar{y} - b\bar{x} = 2.5 - (.25)(3) = 1.75$$

Associative Forecasting Example



$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{51.5 - (6)(3)(2.5)}{80 - (6)(3^2)} = .25 \qquad a = \bar{y} - b\bar{x} = 2.5 - (.25)(3) = 1.75$$

Associative Forecasting Example

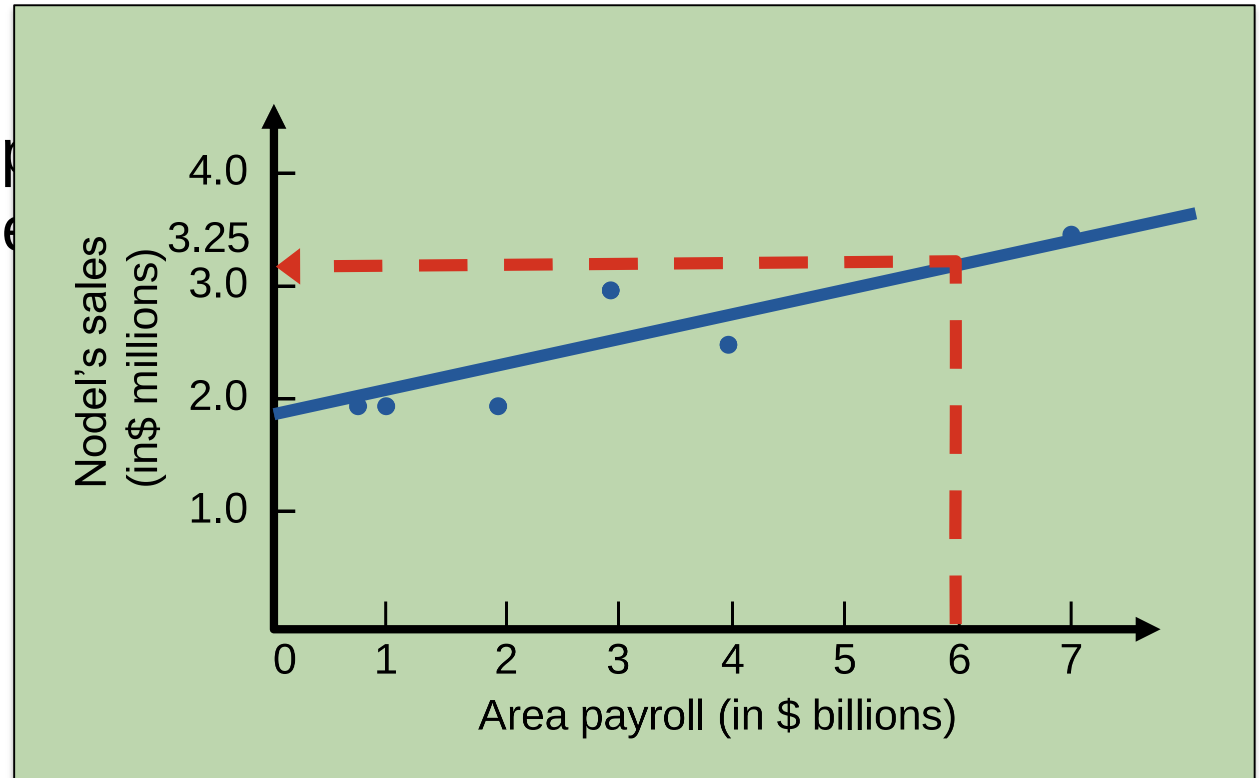
If payroll next year is estimated to be \$6 billion, then:

$$\text{Sales (in \$ millions)} = 1.75 + .25(6)$$
$$= 1.75 + 1.5 = 3.25$$

$$\text{Sales} = \$3{,}250{,}000$$

Associative Forecasting Example

If p ,
the



Standard Error of the Estimate

- A forecast is just a point estimate of a future value
- This point is actually the mean of a probability distribution

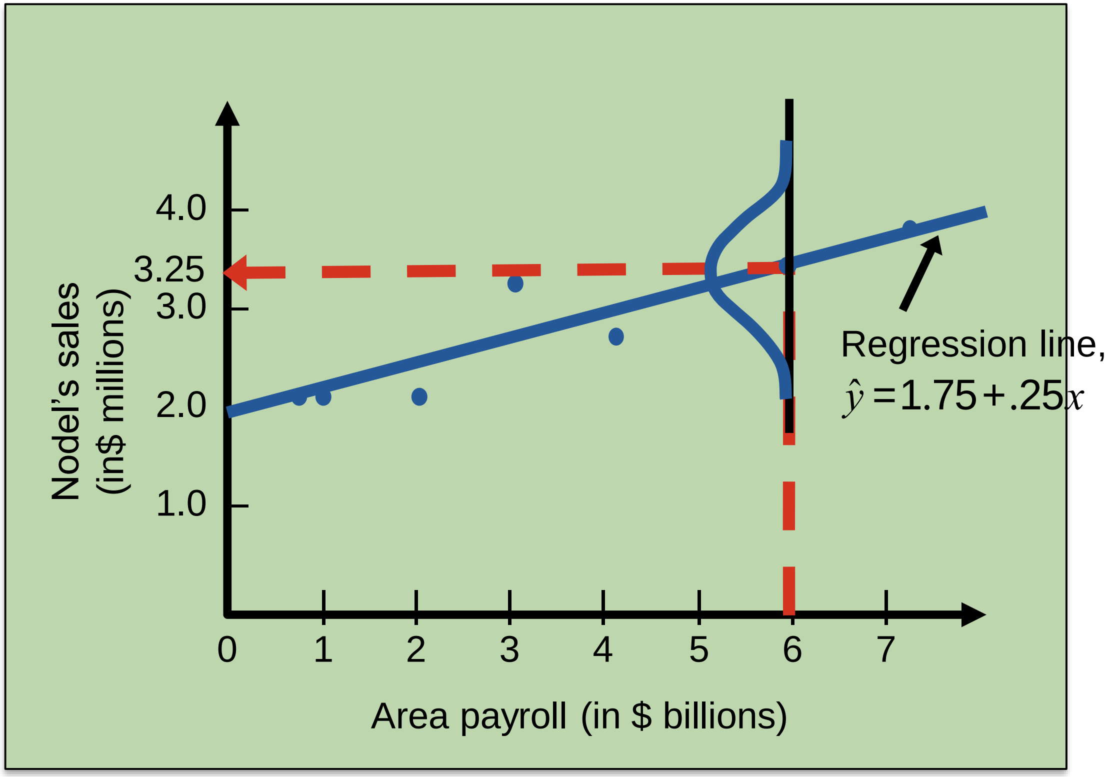


Figure 4.9

Standard Error of the Estimate

$$S_{y,x} = \sqrt{\frac{\sum (y - y_c)^2}{n - 2}}$$

where y = y-value of each data point

y_c = computed value of the dependent variable, from the regression equation

n = number of data points

Standard Error of the Estimate

Computationally, this equation is considerably easier to use

$$S_{y,x} = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n - 2}}$$

We use the standard error to set up prediction intervals around the point estimate

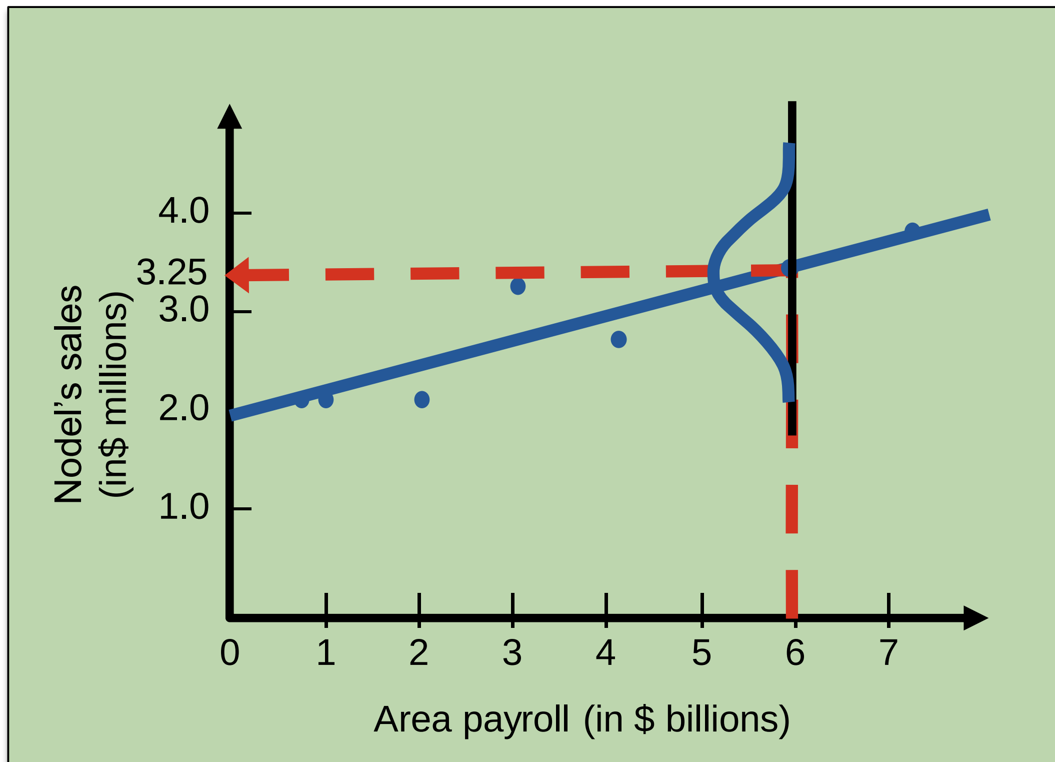
Standard Error of the Estimate

$$S_{y,x} = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n - 2}} = \sqrt{\frac{39.5 - 1.75(15.0) - .25(51.5)}{6 - 2}}$$

$$= \sqrt{.09375}$$

$$= .306 \text{ (in \$ millions)}$$

The standard error of the estimate is $306,000 in sales



Correlation

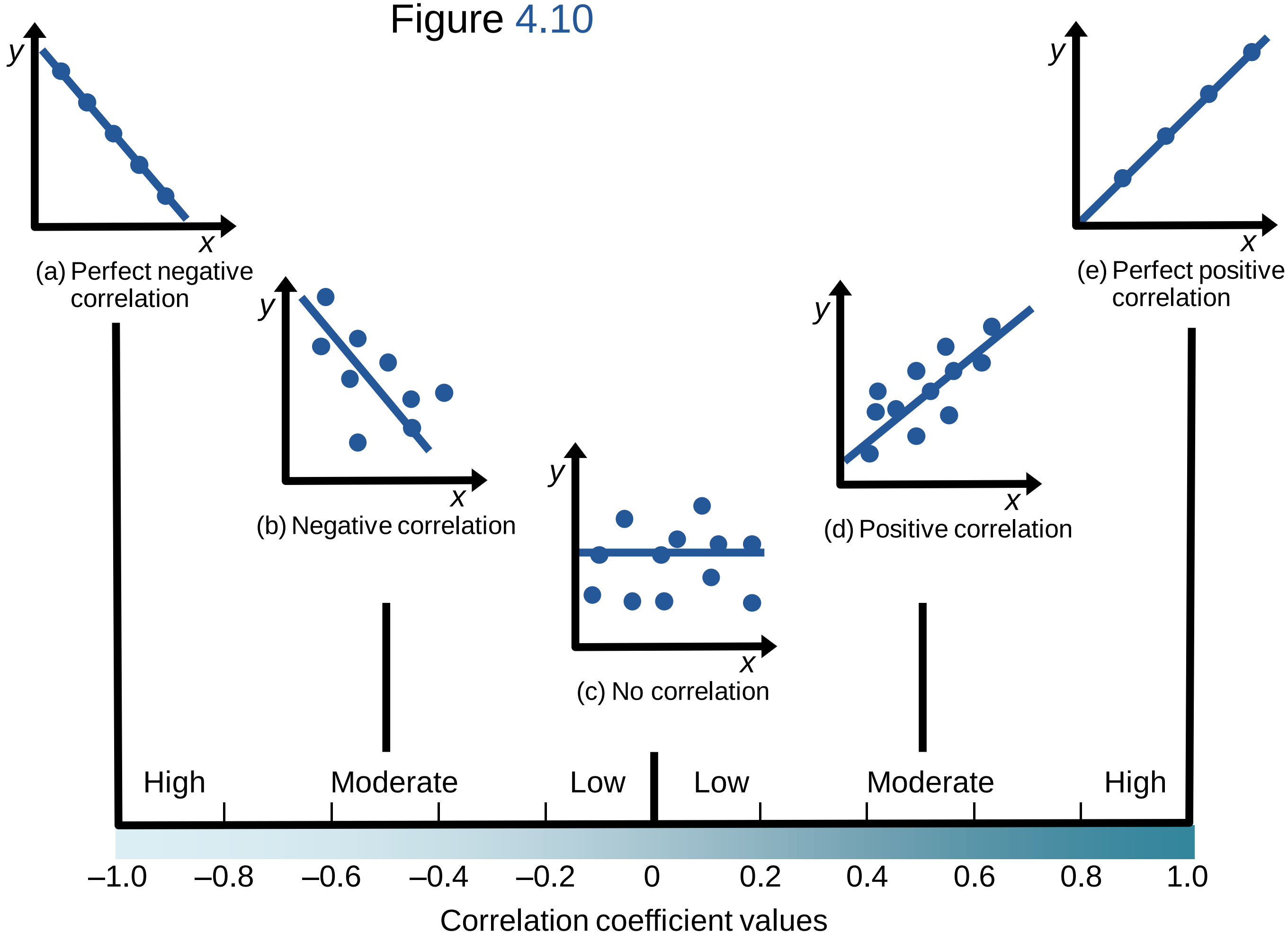
- How strong is the linear relationship between the variables?
- Correlation does not necessarily imply causality!
- Coefficient of correlation, r, measures degree of association
  - Values range from -1 to +1

Correlation Coefficient

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\left[n\sum x^2 - \left(\sum x\right)^2\right]\left[n\sum y^2 - \left(\sum y\right)^2\right]}}$$

Correlation Coefficient

Figure 4.10



Correlation Coefficient

y	x	x^2	xy	y^2
2.0	1	1	2.0	4.0
3.0	3	9	9.0	9.0
2.5	4	16	10.0	6.25
2.0	2	4	4.0	4.0
2.0	1	1	2.0	4.0
3.5	7	49	24.5	12.25
$\Sigma y =$ 15.0	$\Sigma x =$ 18	$\Sigma x^2 =$ 80	$\Sigma xy =$ 51.5	$\Sigma y^2 =$ 39.5

$$r = \frac{(6)(51.5) - (18)(15.0)}{\sqrt{\left[(6)(80) - (18)^2\right]\left[(16)(39.5) - (15.0)^2\right]}}$$

$$= \frac{309 - 270}{\sqrt{(156)(12)}} = \frac{39}{\sqrt{1{,}872}} = \frac{39}{43.3} = .901$$

Correlation

- Coefficient of Determination, r^2, measures the percent of change in y predicted by the change in x
  - Values range from 0 to 1
  - Easy to interpret

For the Nodel Construction example:

$r = .901$

$r^2 = .81$

Multiple-Regression Analysis

If more than one independent variable is to be used in the model, linear regression can be extended to multiple regression to accommodate several independent variables

$$\hat{y} = a + b_1 x_1 + b_2 x_2$$

Computationally, this is quite complex and generally done on the computer

Multiple-Regression Analysis

In the Nodel example, including interest rates in the model gives the new equation:

$$\hat{y} = 1.80 + .30x_1 - 5.0x_2$$

An improved correlation coefficient of r = .96 suggests this model does a better job of predicting the change in construction sales

Sales = 1.80 + .30(6) - 5.0(.12) = 3.00

Sales = $3,000,000

Monitoring and Controlling Forecasts

Tracking Signal

- Measures how well the forecast is predicting actual values
- Ratio of cumulative forecast errors to mean absolute deviation (MAD)
  - Good tracking signal has low values
  - If forecasts are continually high or low, the forecast has a bias error

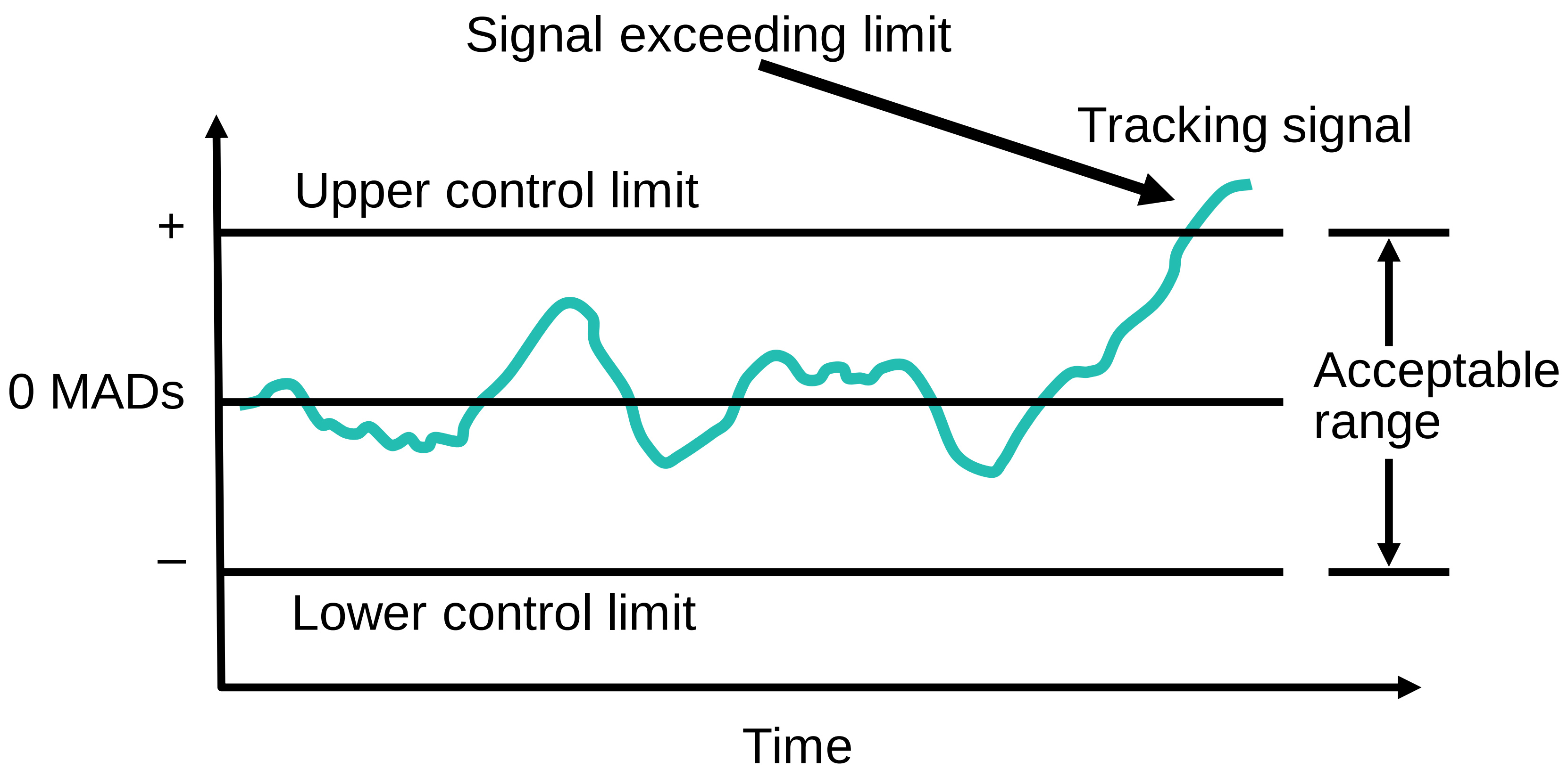
Monitoring and Controlling Forecasts

$$\text{Tracking signal} = \frac{\text{Cumulative error}}{\text{MAD}}$$

$$= \frac{\sum(\text{Actual demand in period } i - \text{Forecast demand in period } i)}{\dfrac{\sum\left|\text{Actual} - \text{Forecast}\right|}{n}}$$

Tracking Signal

Figure 4.11



Tracking Signal Example

QTR	ACTUAL DEMAND	FORECAST DEMAND	ERROR	CUM ERROR	ABSOLUTE FORECAST ERROR	CUM ABS FORECAST ERROR	MAD	TRACKING SIGNAL (CUM ERROR/MAD)
1	90	100	–10	–10	10	10	10.0	–10/10 = –1
2	95	100	–5	–15	5	15	7.5	–15/7.5 = –2
3	115	100	+15	0	15	30	10.	0/10 = 0
4	100	110	–10	–10	10	40	10.	10/10 = –1
5	125	110	+15	+5	15	55	11.0	+5/11 = +0.5
6	140	110	+30	+35	30	85	14.2	+35/14.2 = +2.5

$$\text{At the end of quarter 6, MAD} = \frac{\sum |\text{Forecast errors}|}{n} = \frac{85}{6} = 14.2$$

$$\text{Tracking signal} = \frac{\text{Cumulative error}}{\text{MAD}} = \frac{35}{14.2} = 2.5 \text{ MADs}$$

Adaptive Smoothing

- It's possible to use the computer to continually monitor forecast error and adjust the values of the α and β coefficients used in exponential smoothing to continually minimize forecast error
- This technique is called adaptive smoothing

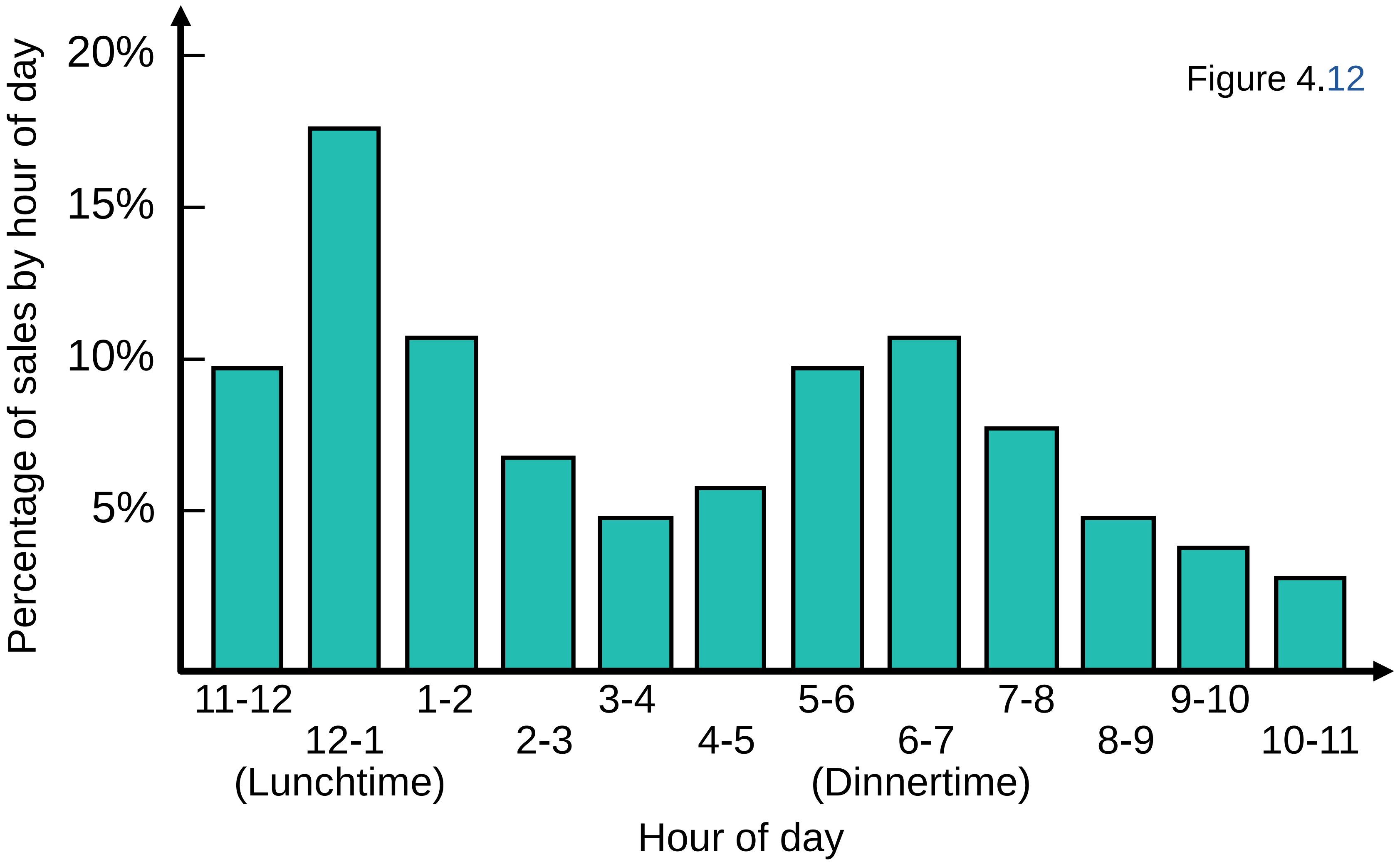
Focus Forecasting

- Developed at American Hardware Supply, based on two principles:
  1. Sophisticated forecasting models are not always better than simple ones
  2. There is no single technique that should be used for all products or services
- Uses historical data to test multiple forecasting models for individual items
- Forecasting model with the lowest error used to forecast the next demand

Forecasting in the Service Sector

- Presents unusual challenges
  - Special need for short term records
  - Needs differ greatly as function of industry and product
  - Holidays and other calendar events
  - Unusual events

Fast Food Restaurant Forecast



FedEx Call Center Forecast