Statistical Process Control

SUPPLEMENT 6

SUPPLEMENT OUTLINE

Alaska Airlines

Alaska Airlines

LEARNING OBJECTIVES

Georgia Tech

As part of its statistical process control system, Flowers Bakery, in Georgia, uses a digital camera to inspect just-baked sandwich buns as they move along the production line. Items that don't measure up in terms of color, shape, seed distribution, or size are identified and removed automatically from the conveyor.

Statistical Process Control (SPC)

Statistical process control (SPC)
A process used to monitor standards by taking measurements and corrective action as a product or service is being produced.

Control chart
A graphical presentation of process data over time.

In this supplement, we address statistical process control—the same techniques used at BetzDearborn, at Arnold Palmer Hospital, at GE, and at Southwest Airlines to achieve quality standards. **Statistical process control (SPC)** is the application of statistical techniques to ensure that processes meet standards. All processes are subject to a certain degree of variability. While studying process data in the 1920s, Walter Shewhart of Bell Laboratories made the distinction between the common (natural) and special (assignable) causes of variation. He developed a simple but powerful tool to separate the two—the **control chart**.

A process is said to be operating *in statistical control* when the only source of variation is common (natural) causes. The process must first be brought into statistical control by detecting and eliminating special (assignable) causes of variation.[1] Then its performance is predictable, and its ability to meet customer expectations can be assessed. The *objective* of a process control system is to *provide a statistical signal when assignable causes of variation are present*. Such a signal can quicken appropriate action to eliminate assignable causes.

Natural variations
Variability that affects every production process to some degree and is to be expected; also known as common cause.

Natural Variations Natural variations affect almost every process and are to be expected. **Natural variations** are the many sources of variation that occur within a process, even one that is in statistical control. Natural variations form a pattern that can be described as a *distribution*.

As long as the distribution (output measurements) remains within specified limits, the process is said to be "in control," and natural variations are tolerated.

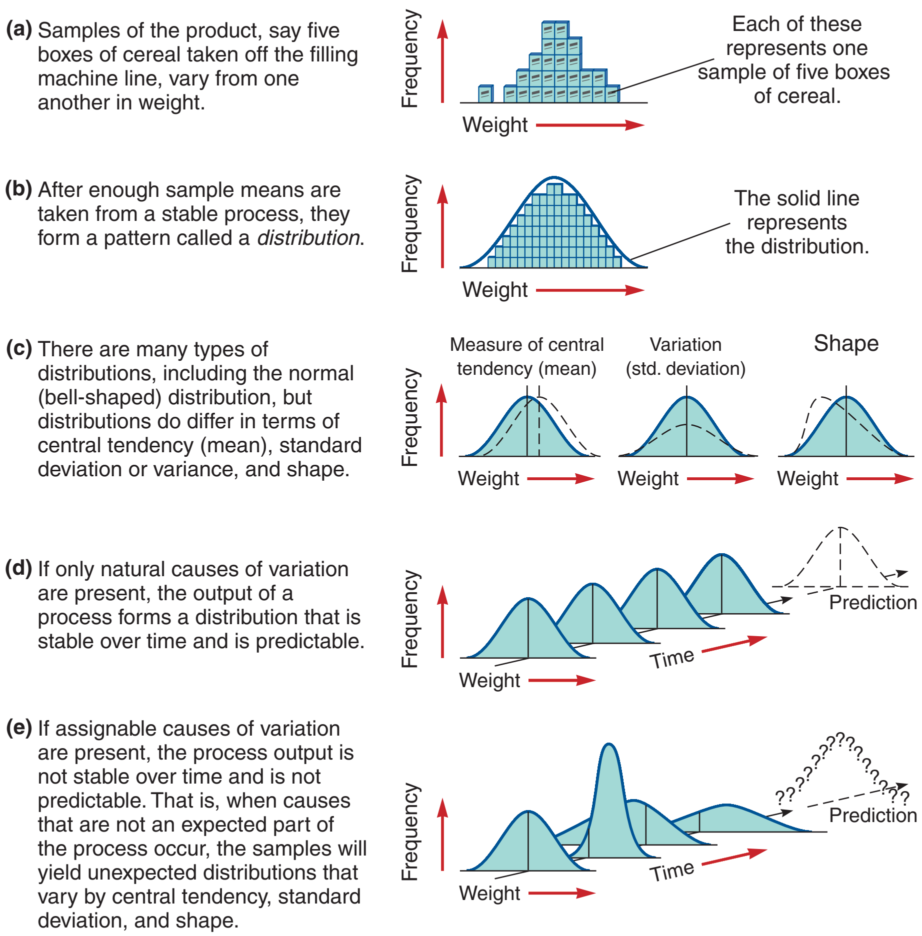


Figure **S6.1**

Natural and Assignable Variation

Assignable Variations **Assignable variation** in a process can be traced to a specific reason. Factors such as machine wear, misadjusted equipment, fatigued or untrained workers, or new batches of raw material are all potential sources of assignable variations.

Assignable variation
Variation in a production process that can be traced to specific causes.

Natural and assignable variations distinguish two tasks for the operations manager. The first is to *ensure that the process is capable* of operating under control with only natural variation. The second is, of course, to *identify and eliminate assignable variations* so that the processes will remain under control.

Samples Because of natural and assignable variation, statistical process control uses averages of small samples (often of four to eight items) as opposed to data on individual parts. Individual pieces tend to be too erratic to make trends quickly visible.

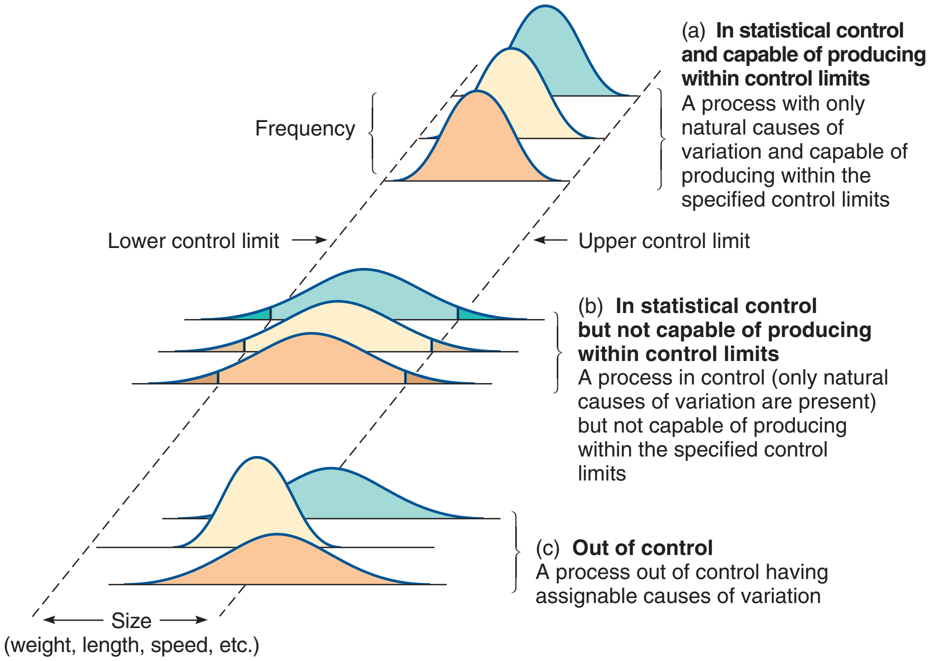
Figure S6.1 provides a detailed look at the important steps in determining process variation. The horizontal scale can be weight (as in the number of ounces in boxes of cereal) or length (as in fence posts) or any physical measure. The vertical scale is frequency. The samples of five boxes of cereal in Figure S6.1 **(a)** are weighed, **(b)** form a distribution, and **(c)** can vary. The distributions formed in **(b)** and **(c)** will fall in a predictable pattern **(d)** if only natural variation is present. If assignable causes of variation are present, then we can expect either the mean to vary or the dispersion to vary, as is the case in **(e)**.

Control Charts The process of building control charts is based on the concepts presented in Figure S6.2. This figure shows three distributions that are the result of outputs from three types of processes. We plot small samples and then examine characteristics of the resulting data to see if the process is within "control limits." The purpose of control charts is to help distinguish between natural variations and variations due to assignable causes. As seen in Figure S6.2, a process is **(a)** in control *and the process is capable of producing within established control limits*, **(b)** in control *but the process is not capable of producing within established*

LO S6.1 *Explain* the purpose of a control chart

Figure **S6.2**

Process Control: Three Types of Process Outputs



limits, or **(c)** out of control. We now look at ways to build control charts that help the operations manager keep a process under control.

Control Charts for Variables

$\bar{x}$-chart
A quality control chart for variables that indicates when changes occur in the central tendency of a production process.

***R*-chart**
A control chart that tracks the "range" within a sample; it indicates that a gain or loss in uniformity has occurred in dispersion of a production process.

Central limit theorem
The theoretical foundation for $\bar{x}$-charts, which states that regardless of the distribution of the population of all parts or services, the distribution of $\bar{x}$s tends to follow a normal curve as the number of samples increases.

The variables of interest here are those that have continuous dimensions. They have an infinite number of possibilities. Examples are weight, speed, length, or strength. Control charts for the mean, $\bar{x}$ or x-bar, and the range, R, are used to monitor processes that have continuous dimensions. The **$\bar{x}$-chart** tells us whether changes have occurred in the central tendency (the mean, in this case) of a process. These changes might be due to such factors as tool wear, a gradual increase in temperature, a different method used on the second shift, or new and stronger materials. The ***R*-chart** values indicate that a gain or loss in dispersion has occurred. Such a change may be due to worn bearings, a loose tool, an erratic flow of lubricants to a machine, or to sloppiness on the part of a machine operator. The two types of charts go hand in hand when monitoring variables because they measure the two critical parameters: central tendency and dispersion.

The Central Limit Theorem

The theoretical foundation for $\bar{x}$-charts is the **central limit theorem**. This theorem states that regardless of the distribution of the population, the distribution of $\bar{x}$s (each of which is a mean of a sample drawn from the population) will tend to follow a normal curve as the number of samples increases. Fortunately, even if each sample (n) is fairly small (say, 4 or 5), the distributions of the averages will still roughly follow a normal curve. The theorem also states that: (1) the mean of the distribution of the $\bar{x}$s (called $\bar{\bar{x}}$) will equal the mean of the overall population (called μ); and (2) the standard deviation of the *sampling distribution*, $\sigma_{\bar{x}}$, will be the *population (process) standard deviation*, divided by the square root of the sample size, n. In other words:[2]

$$\bar{\bar{x}} = \mu \tag{S6-1}$$

LO S6.2 *Explain* the role of the central limit theorem in SPC

and

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \tag{S6-2}$$

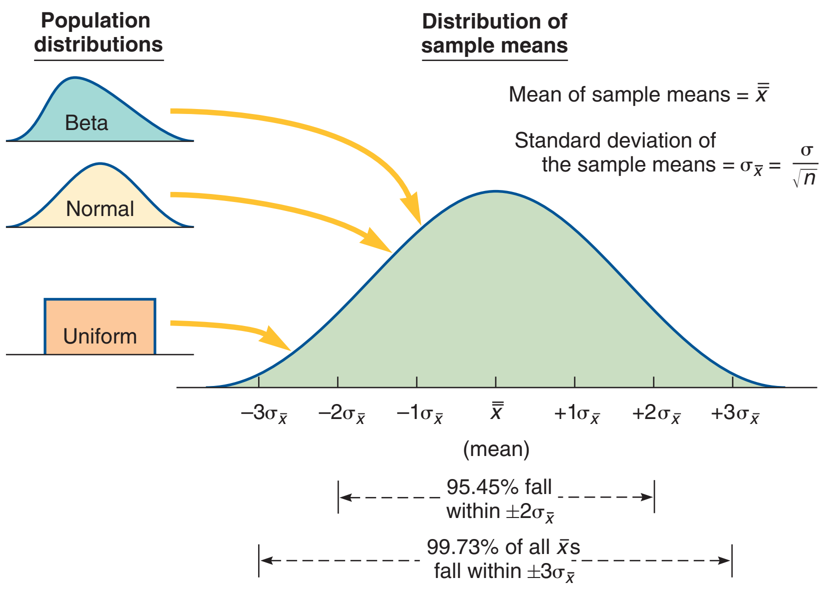


Figure **S6.3**

The Relationship Between Population and Sampling Distributions

Even though the population distributions will differ (e.g., normal, beta, uniform), each with its own mean (μ) and standard deviation (σ), the distribution of sample means always approaches a normal distribution.

Figure S6.3 shows three possible population distributions, each with its own mean, μ, and standard deviation, σ. If a series of random samples ($\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$, $\bar{x}_4$, and so on), each of size n, is drawn from any population distribution (which could be normal, beta, uniform, and so on), the resulting distribution of $\bar{x}_i$s will approximate a normal distribution (see Figure S6.3).

Moreover, the sampling distribution, as is shown in Figure S6.4(a), will have less variability than the process distribution. Because the sampling distribution is normal, we can state that:

- 95.45% of the time, the sample averages will fall within $\pm 2\sigma_{\bar{x}}$ if the process has only natural variations.
- 99.73% of the time, the sample averages will fall within $\pm 3\sigma_{\bar{x}}$ if the process has only natural variations.

If a point on the control chart falls outside of the $\pm 3\sigma_{\bar{x}}$ control limits, then we are 99.73% sure the process has changed. Figure S6.4(b) shows that as the sample size increases, the sampling distribution becomes narrower. So the sample statistic is closer to the true value of the population for larger sample sizes. This is the theory behind control charts.

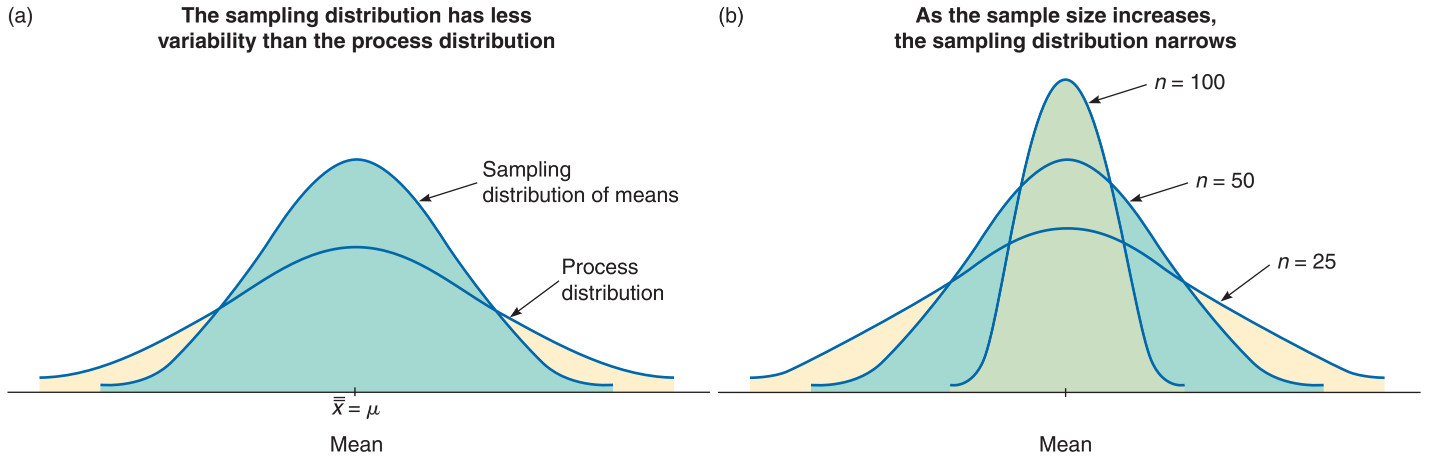


Figure **S6.4**

The Sampling Distribution of Means Is Normal

The process distribution from which the sample was drawn was also normal, but it could have been any distribution.

Setting Mean Chart Limits ($\bar{x}$-Charts)

If we know, through past data, the standard deviation of the population (process), σ, we can set upper and lower control limits[3] by using these formulas:

$$\text{Upper control limit (UCL)} = \bar{\bar{x}} + z\sigma_{\bar{x}} \tag{S6-3}$$

$$\text{Lower control limit (LCL)} = \bar{\bar{x}} - z\sigma_{\bar{x}} \tag{S6-4}$$

LO S6.3 *Build* $\bar{x}$-*charts and* *R*-*charts*

where

$\bar{\bar{x}}$ = mean of the sample means or a target value set for the process
z = number of normal standard deviations (2 for 95.45% confidence, 3 for 99.73%)
$\sigma_{\bar{x}}$ = standard deviation of the sample means $= \sigma/\sqrt{n}$
σ = population (process) standard deviation
n = sample size

Example S1 shows how to set control limits for sample means using standard deviations.

Example S1

SETTING CONTROL LIMITS USING SAMPLES

The weights of boxes of Oat Flakes within a large production lot are sampled each hour. Managers want to set control limits that include 99.73% of the sample means.

APPROACH ▶ Randomly select and weigh nine ($n = 9$) boxes each hour. Then find the overall mean and use Equations (S6-3) and (S6-4) to compute the control limits. Here are the nine boxes chosen for Hour 1:

STUDENT TIP If you want to see an example of such variability in your supermarket, go to the soft drink section and line up a few 2-liter bottles of Coke or Pepsi.

SOLUTION ▶

$$\text{The average weight in the first hourly sample} = \frac{17 + 13 + 16 + 18 + 17 + 16 + 15 + 17 + 16}{9} = 16.1 \text{ ounces.}$$

Also, the *population (process)* standard deviation (σ) is known to be 1 ounce. We do not show each of the boxes randomly selected in hours 2 through 12, but here are all 12 hourly samples:

WEIGHT OF SAMPLE		WEIGHT OF SAMPLE		WEIGHT OF SAMPLE	
HOUR	(AVG. OF 9 BOXES)	HOUR	(AVG. OF 9 BOXES)	HOUR	(AVG. OF 9 BOXES)
1	16.1	5	16.5	9	16.3
2	16.8	6	16.4	10	14.8
3	15.5	7	15.2	11	14.2
4	16.5	8	16.4	12	17.3

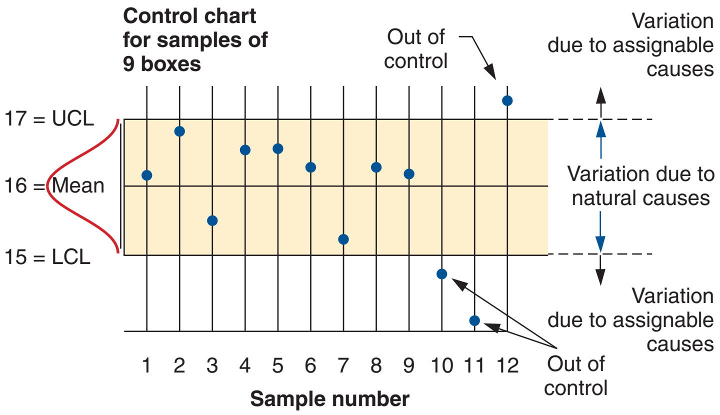
The average mean $\bar{\bar{x}}$ of the 12 samples is calculated to be exactly 16 ounces $\left[\bar{\bar{x}} = \dfrac{\sum_{i=1}^{12} (\text{Avg. of 9 Boxes})}{12}\right]$.

We therefore have $\overline{\overline{x}} = 16$ ounces, $\sigma = 1$ ounce, $n = 9$, and $z = 3$. The control limits are:

$$\text{UCL}_{\overline{x}} = \overline{\overline{x}} + z\sigma_{\overline{x}} = 16 + 3\left(\frac{1}{\sqrt{9}}\right) = 16 + 3\left(\frac{1}{3}\right) = 17 \text{ ounces}$$

$$\text{LCL}_{\overline{x}} = \overline{\overline{x}} - z\sigma_{\overline{x}} = 16 - 3\left(\frac{1}{\sqrt{9}}\right) = 16 - 3\left(\frac{1}{3}\right) = 15 \text{ ounces}$$

The 12 samples are then plotted on the following control chart:



INSIGHT ▶ Because the means of recent sample averages fall outside the upper and lower control limits of 17 and 15, we can conclude that the process is becoming erratic and is *not* in control.

LEARNING EXERCISE ▶ If Oat Flakes's population standard deviation (σ) is 2 (instead of 1), what is your conclusion? [Answer: LCL = 14, UCL = 18; the process would be in control.]

RELATED PROBLEMS ▶ S6.1, S6.2, S6.4, S6.8, S6.10a,b (S6.28 is available in MyOMLab)

ACTIVE MODEL S6.1 This example is further illustrated in Active Model S6.1 in MyOMLab.

EXCEL OM Data File **CH06ExS1.XLS** can be found in MyOMLab.

Because process standard deviations are often not available, we usually calculate control limits based on the average *range* values rather than on standard deviations. Table S6.1 provides the necessary conversion for us to do so. The *range* (R_i) is defined as the difference between the largest and smallest items in one sample. For example, the heaviest box of Oat Flakes in Hour 1 of Example S1 was 18 ounces and the lightest was 13 ounces, so the range for that hour is 5 ounces. We use Table S6.1 and the equations:

$$\text{UCL}_{\overline{x}} = \overline{\overline{x}} + A_2\overline{R} \qquad \text{(S6-5)}$$

and:

$$\text{LCL}_{\overline{x}} = \overline{\overline{x}} - A_2\overline{R} \qquad \text{(S6-6)}$$

where $\overline{R} = \dfrac{\sum_{i=1}^{k} R_i}{k}$ = average range of all the samples; R_i = range for sample i

A_2 = value found in Table S6.1 $\qquad k$ = total number of samples

$\overline{\overline{x}}$ = mean of the sample means

Example S2 shows how to set control limits for sample means by using Table S6.1 and the average range.

TABLE S6.1 Factors for Computing Control Chart Limits (3 sigma)

SAMPLE SIZE, n	MEAN FACTOR, A_2	UPPER RANGE, D_4	LOWER RANGE, D_3
2	1.880	3.268	0
3	1.023	2.574	0
4	.729	2.282	0
5	.577	2.115	0
6	.483	2.004	0
7	.419	1.924	0.076
8	.373	1.864	0.136
9	.337	1.816	0.184
10	.308	1.777	0.223
12	.266	1.716	0.284

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Example S2

SETTING MEAN LIMITS USING TABLE VALUES

Super Cola bottles soft drinks labeled "net weight 12 ounces." Indeed, an overall process average of 12 ounces has been found by taking 10 samples, in which each sample contained 5 bottles. The OM team wants to determine the upper and lower control limits for averages in this process.

APPROACH ▶ Super Cola first examines the 10 samples to compute the average range of the process. Here are the data and calculations:

SAMPLE	WEIGHT OF LIGHTEST BOTTLE IN SAMPLE OF $n = 5$	WEIGHT OF HEAVIEST BOTTLE IN SAMPLE OF $n = 5$	RANGE (R_i) = DIFFERENCE BETWEEN THESE TWO
1	11.50	11.72	.22
2	11.97	12.00	.03
3	11.55	12.05	.50
4	12.00	12.20	.20
5	11.95	12.00	.05
6	10.55	10.75	.20
7	12.50	12.75	.25
8	11.00	11.25	.25
9	10.60	11.00	.40
10	11.70	12.10	.40
			$\sum R_i = 2.50$

$$\text{Average Range} = \frac{2.50}{10 \text{ samples}} = .25 \text{ ounces}$$

Now Super Cola applies Equations (S6-5) and (S6-6) and uses the A_2 column of Table S6.1.

SOLUTION ▶ Looking in Table S6.1 for a sample size of 5 in the mean factor A_2 column, we find the value .577. Thus, the upper and lower control chart limits are:

$$\begin{aligned} \text{UCL}_{\bar{x}} &= \bar{\bar{x}} + A_2\bar{R} \\ &= 12 + (.577)(.25) \\ &= 12 + .144 \\ &= 12.144 \text{ ounces} \end{aligned}$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$$
$$= 12 - .144$$
$$= 11.856 \text{ ounces}$$

INSIGHT ▶ The advantage of using this range approach, instead of the standard deviation, is that it is easy to apply and may be less confusing.

LEARNING EXERCISE ▶ If the sample size was $n = 4$ and the average range $= .20$ ounces, what are the revised $\text{UCL}_{\bar{x}}$ and $\text{LCL}_{\bar{x}}$? [Answer: 12.146, 11.854.]

RELATED PROBLEMS ▶ S6.3a, S6.5, S6.6, S6.7, S6.9, S6.10b,c,d, S6.11, S6.26 (S6.29a, S6.30a, S6.31a, S6.32a, S6.33a are available in MyOMLab)

EXCEL OM Data File **CH06ExS2.xls** can be found in MyOMLab.

Setting Range Chart Limits (*R*-Charts)

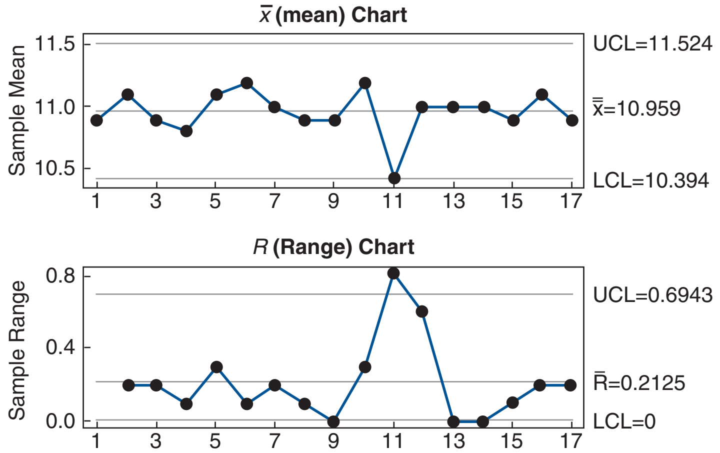
In Examples S1 and S2, we determined the upper and lower control limits for the process *average*. In addition to being concerned with the process average, operations managers are interested in the process *dispersion*, or *range*. Even though the process average is under control, the dispersion of the process may not be. For example, something may have worked itself loose in a piece of equipment that fills boxes of Oat Flakes. As a result, the average of the samples may remain the same, but the variation within the samples could be entirely too large. For this reason, operations managers use control charts for ranges to monitor the process variability, as well as control charts for averages, which monitor the process central tendency. The theory behind the control charts for ranges is the same as that for process average control charts. Limits are established that contain ± 3 standard deviations of the distribution for the average range $\bar{R}$. We can use the following equations to set the upper and lower control limits for ranges:

$$\text{UCL}_R = D_4\bar{R} \qquad \text{(S6-7)}$$

$$\text{LCL}_R = D_3\bar{R} \qquad \text{(S6-8)}$$

where

UCL_R = upper control chart limit for the range
LCL_R = lower control chart limit for the range
D_4 and D_3 = values from Table S6.1



Salmon filets are monitored by Darden Restaurant's SPC software, which includes $\bar{x}$-(mean) charts and R-(range) charts. Darden uses average weight as a measure of central tendency for salmon filets. The range is the difference between the heaviest and the lightest filets in each sample. The video case study "Farm to Fork," at the end of this supplement, asks you to interpret these figures.

VIDEO S6.1
Farm to Fork: Quality of Darden Restaurants

Example S3 shows how to set control limits for sample ranges using Table S6.1 and the average range.

Example S3

SETTING RANGE LIMITS USING TABLE VALUES

Roy Clinton's mail-ordering business wants to measure the response time of its operators in taking customer orders over the phone. Clinton lists below the time recorded (in minutes) from five different samples of the ordering process with four customer orders per sample. He wants to determine the upper and lower range control chart limits.

APPROACH ▶ Looking in Table S6.1 for a sample size of 4, he finds that $D_4 = 2.282$ and $D_3 = 0$.

SOLUTION ▶

SAMPLE	OBSERVATIONS (MINUTES)	SAMPLE RANGE (R_i)
1	5, 3, 6, 10	10 − 3 = 7
2	7, 5, 3, 5	7 − 3 = 4
3	1, 8, 3, 12	12 − 1 = 11
4	7, 6, 2, 1	7 − 1 = 6
5	3, 15, 6, 12	15 − 3 = 12
		$\Sigma R_i = 40$

$$\overline{R} = \frac{40}{5} = 8$$

$$\text{UCL}_R = 2.282(8) = 18.256 \text{ minutes}$$

$$\text{LCL}_R = 0(8) = 0 \text{ minutes}$$

INSIGHT ▶ Computing ranges with Table S6.1 is straightforward and an easy way to evaluate dispersion. No sample ranges are out of control.

LEARNING EXERCISE ▶ Clinton decides to increase the sample size to $n = 6$ (with no change in average range, $\overline{R}$). What are the new UCL_R and LCL_R values? [Answer: 16.032, 0.]

RELATED PROBLEMS ▶ S6.3b, S6.5, S6.6, S6.7, S6.9, S6.10c, S6.11, S6.12, S6.26 (S6.29b, S6.30b, S6.31b, S6.32b, S6.33b are available in MyOMLab)

Using Mean and Range Charts

The normal distribution is defined by two parameters, the *mean* and *standard deviation*. The $\overline{x}$ (mean)-chart and the *R*-chart mimic these two parameters. The $\overline{x}$-chart is sensitive to shifts in the process mean, whereas the *R*-chart is sensitive to shifts in the process standard deviation. Consequently, by using both charts we can track changes in the process distribution.

For instance, the samples and the resulting $\overline{x}$-chart in Figure S6.5(a) show the shift in the process mean, but because the dispersion is constant, no change is detected by the *R*-chart. Conversely, the samples and the $\overline{x}$-chart in Figure S6.5(b) detect no shift (because none is present), but the *R*-chart does detect the shift in the dispersion. Both charts are required to track the process accurately.

LO S6.4 *List* the five steps involved in building control charts

Steps to Follow When Building Control Charts There are five steps that are generally followed in building $\overline{x}$- and *R*-charts:

1. Collect 20 to 25 samples, often of $n = 4$ or $n = 5$ observations each, from a stable process, and compute the mean and range of each.
2. Compute the overall means ($\overline{\overline{x}}$ and $\overline{R}$), set appropriate control limits, usually at the 99.73% level, and calculate the preliminary upper and lower control limits. Refer to Table S6.2 for other control limits. *If the process is not currently stable and in control*, use the desired mean, μ, instead of $\overline{\overline{x}}$ to calculate limits.

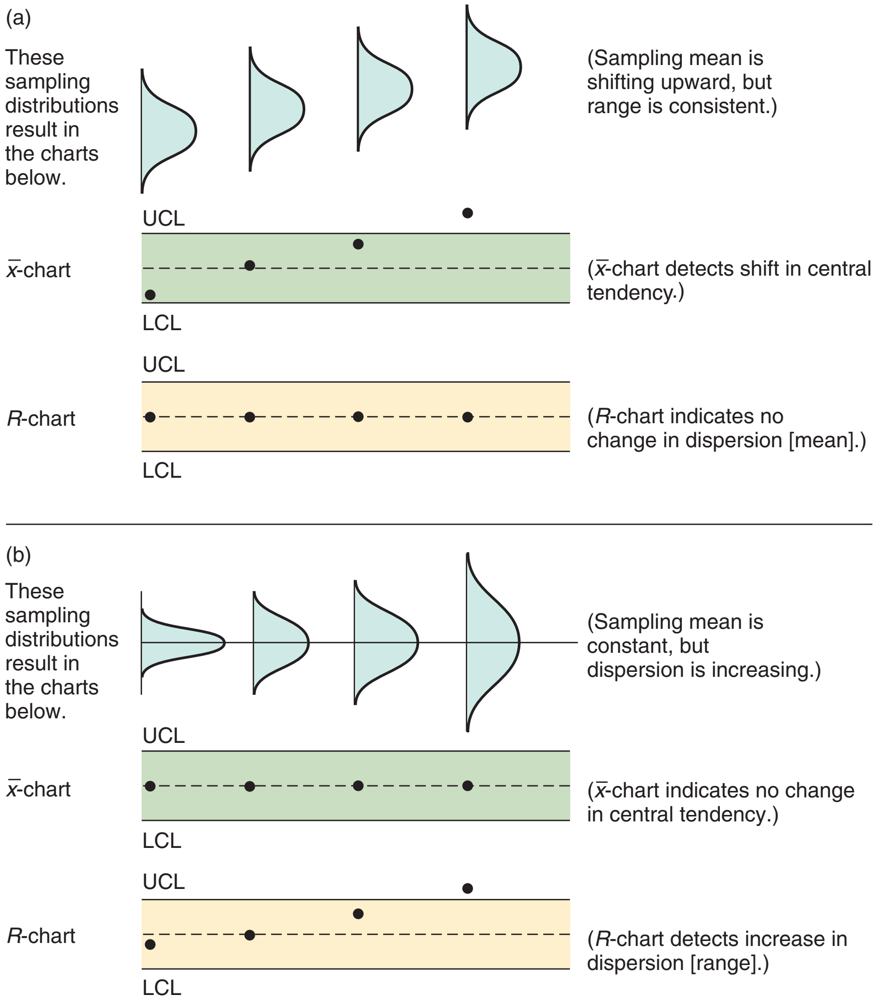


Figure **S6.5**

Mean and Range Charts Complement Each Other by Showing the Mean and Dispersion of the Normal Distribution

STUDENT TIP

Mean ($\bar{x}$) charts are a measure of *central tendency*, while range (R) charts are a measure of *dispersion*. SPC requires both charts for a complete assessment because a sample mean could be out of control while the range is in control and vice versa.

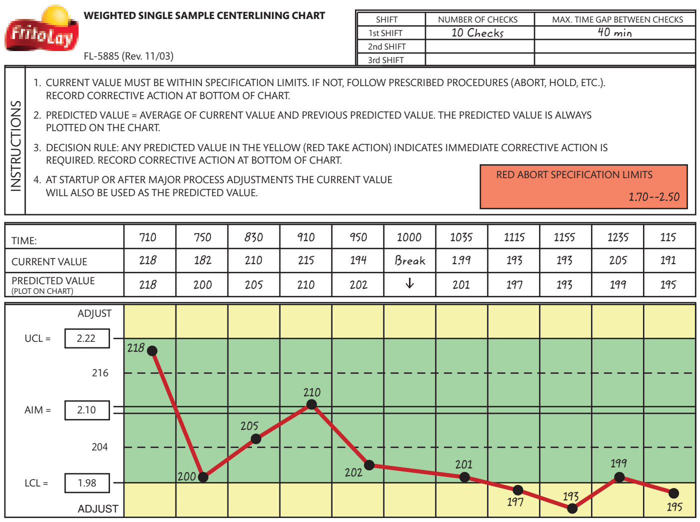
3. Graph the sample means and ranges on their respective control charts, and determine whether they fall outside the acceptable limits.
4. Investigate points or patterns that indicate the process is out of control. Try to assign causes for the variation, address the causes, and then resume the process.
5. Collect additional samples and, if necessary, revalidate the control limits using the new data.

TABLE S6.2

Common *z* Values

DESIRED CONTROL LIMIT (%)	*Z*-VALUE (STANDARD DEVIATION REQUIRED FOR DESIRED LEVEL OF CONFIDENCE)
90.0	1.65
95.0	1.96
95.45	2.00
99.0	2.58
99.73	3.00


Donna McWilliam/AP Images



Frito-Lay uses $\bar{x}$ charts to control production quality at critical points in the process. About every 40 minutes, three batches of chips are taken from the conveyor (on the left) and analyzed electronically to get an average salt content, which is plotted on an $\bar{x}$-chart (on the right). Points plotted in the green zone are "in control," while those in the yellow zone are "out of control." The SPC chart is displayed where all production employees can monitor process stability.

Control Charts for Attributes

LO S6.5 *Build p-charts and c-charts*

Control charts for $\bar{x}$ and R do not apply when we are sampling *attributes*, which are typically classified as *defective* or *nondefective*. Measuring defectives involves counting them (for example, number of bad lightbulbs in a given lot, or number of letters or data entry records typed with errors), whereas *variables* are usually measured for length or weight. There are two kinds of attribute control charts: (1) those that measure the *percent* defective in a sample—called p-charts—and (2) those that count the *number* of defects—called c-charts.

***p*-chart**
A quality control chart that is used to control attributes.

VIDEO S6.2
Frito-Lay's Quality-Controlled Potato Chips

***p*-Charts** Using ***p*-charts** is the chief way to control attributes. Although attributes that are either good or bad follow the binomial distribution, the normal distribution can be used to calculate p-chart limits when sample sizes are large. The procedure resembles the $\bar{x}$-chart approach, which is also based on the central limit theorem.

The formulas for p-chart upper and lower control limits follow:

$$\text{UCL}_p = \bar{p} + z\sigma_p \quad \text{(S6-9)}$$

$$\text{LCL}_p = \bar{p} - z\sigma_p \quad \text{(S6-10)}$$

where $\bar{p}$ = mean fraction (percent) defective in the samples $= \dfrac{\text{total number of defects}}{\text{sample size} \times \text{number of samples}}$

z = number of standard deviations ($z = 2$ for 95.45% limits; $z = 3$ for 99.73% limits)

σ_p = standard deviation of the sampling distribution

σ_p is estimated by the formula:

$$\hat{\sigma}_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \quad \text{(S6-11)}$$

where n = number of observations in *each* sample[4]

Example S4 shows how to set control limits for p-charts for these standard deviations.

Example S4

SETTING CONTROL LIMITS FOR PERCENT DEFECTIVE

Clerks at Mosier Data Systems key in thousands of insurance records each day for a variety of client firms. CEO Donna Mosier wants to set control limits to include 99.73% of the random variation in the data entry process when it is in control.

APPROACH ▶ Samples of the work of 20 clerks are gathered (and shown in the table). Mosier carefully examines 100 records entered by each clerk and counts the number of errors. She also computes the fraction defective in each sample. Equations (S6-9), (S6-10), and (S6-11) are then used to set the control limits.

SAMPLE NUMBER	NUMBER OF ERRORS	FRACTION DEFECTIVE	SAMPLE NUMBER	NUMBER OF ERRORS	FRACTION DEFECTIVE
1	6	.06	11	6	.06
2	5	.05	12	1	.01
3	0	.00	13	8	.08
4	1	.01	14	7	.07
5	4	.04	15	5	.05
6	2	.02	16	4	.04
7	5	.05	17	11	.11
8	3	.03	18	3	.03
9	3	.03	19	0	.00
10	2	.02	20	4	.04
				80	

SOLUTION ▶

$$\bar{p} = \frac{\text{Total number of errors}}{\text{Total number of records examined}} = \frac{80}{(100)(20)} = .04$$

$$\hat{\sigma}_p = \sqrt{\frac{(.04)(1 - .04)}{100}} = .02 \text{ (rounded up from .0196)}$$

(*Note:* 100 is the size of *each* sample $= n$.)

$$\text{UCL}_p = \bar{p} + z\hat{\sigma}_p = .04 + 3(.02) = .10$$

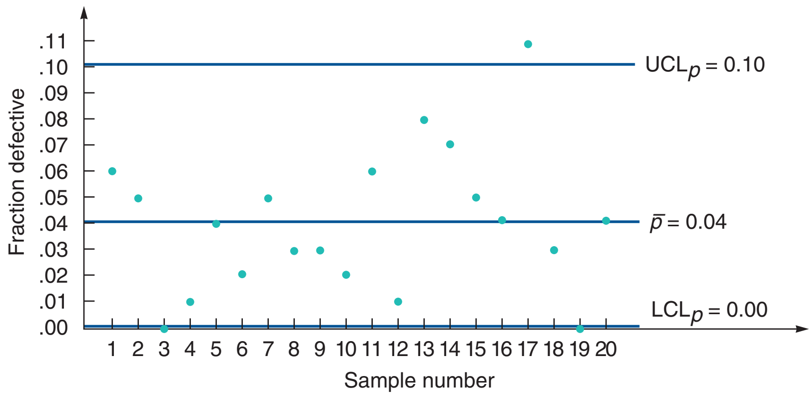
$$\text{LCL}_p = \bar{p} - z\hat{\sigma}_p = .04 - 3(.02) = 0$$

(because we cannot have a negative percentage defective)

INSIGHT ▶ When we plot the control limits and the sample fraction defectives, we find that only one data-entry clerk (number 17) is out of control. The firm may wish to examine that individual's work a bit more closely to see if a serious problem exists (see Figure S6.6).

Figure **S6.6**

***p*-Chart for Data Entry for Example S4**



STUDENT TIP

We are always pleased to be at zero or below the center line in a *p*-chart.

LEARNING EXERCISE ▶ Mosier decides to set control limits at 95.45% instead. What are the new UCL_p and LCL_p? [Answer: 0.08, 0]

RELATED PROBLEMS ▶ S6.13–S6.20, S6.25, S6.27 (S6.35–S6.39 are available in MyOMLab)

ACTIVE MODEL S6.2 This example is further illustrated in Active Model S6.2 in MyOMLab.

EXCEL OM Data File **Ch06ExS4.xls** can be found in MyOMLab.

The *OM in Action* box "Trying to Land a Seat with Frequent Flyer Miles" provides a real-world follow-up to Example S4.

***c*-Charts** In Example S4, we counted the number of defective records entered. A defective record was one that was not exactly correct because it contained at least one defect. However, a bad record may contain more than one defect. We use ***c*-charts** to control the *number* of defects per unit of output (or per insurance record, in the preceding case).

***c*-chart**

A quality control chart used to control the number of defects per unit of output.

OM in Action Trying to Land a Seat with Frequent Flyer Miles

How hard is it to redeem your 25,000 frequent flyer points for airline tickets? That depends on the airline. (It also depends on the city. Don't try to get into or out of San Francisco!) When the consulting firm Idea Works made 280 requests for a standard mileage award to each of 24 airlines' Web sites (a total of 6,720 requests), the success rates ranged from a low of 25.7% and 27.1% (at US Airways and Delta, respectively) to a high of 100% at GOL-Brazil and 99.3% at Southwest.

The overall average of 68.6% for the two dozen carriers provides the center line in a *p*-chart. With 3-sigma upper and lower control limits of 82.5% and 54.7%, the other top and bottom performers are easily spotted. "Out of control" (but in a positive *outperforming* way) are GOL and Southwest, Lufthansa (85.0%), Singapore (90.7%), Virgin Australia (91.4%), and Air Berlin (96.4%).

Out of control *on the negative side* are US Airways and Delta, plus Emirates (35.7%), AirTran (47.1%), Turkish (49.3%), and SAS (52.9%).

Control charts can help airlines see where they stand relative to competitors in such customer service activities as lost bags, on-time rates, and ease of redeeming mileage points. "I think airlines are getting the message that availability is important. Are airlines where they need to be? I don't think so," says the president of Idea Works.

Sources: Wall Street Journal (May 26, 2011); and *Consumer Reports* (November 2014).

Sampling wine from these wooden barrels, to make sure it is aging properly, uses both SPC (for alcohol content and acidity) and subjective measures (for taste).

Charles O'Rear/Corbis

Control charts for defects are helpful for monitoring processes in which a large number of potential errors can occur, but the actual number that do occur is relatively small. Defects may be errors in newspaper words, bad circuits in a microchip, blemishes on a table, or missing pickles on a fast-food hamburger.

The Poisson probability distribution,[5] which has a variance equal to its mean, is the basis for *c*-charts. Because $\bar{c}$ is the mean number of defects per unit, the standard deviation is equal to $\sqrt{\bar{c}}$. To compute 99.73% control limits for $\bar{c}$, we use the formula:

$$\text{Control limits} = \bar{c} \pm 3\sqrt{\bar{c}} \qquad \text{(S6-12)}$$

Example S5 shows how to set control limits for a $\bar{c}$-chart.

Example S5

SETTING CONTROL LIMITS FOR NUMBER OF DEFECTS

Red Top Cab Company receives several complaints per day about the behavior of its drivers. Over a 9-day period (where days are the units of measure), the owner, Gordon Hoft, received the following numbers of calls from irate passengers: 3, 0, 8, 9, 6, 7, 4, 9, 8, for a total of 54 complaints. Hoft wants to compute 99.73% control limits.

APPROACH ▶ He applies Equation (S6–12).

SOLUTION ▶ $\bar{c} = \frac{54}{9} = 6$ complaints per day

Thus:

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 6 + 3\sqrt{6} = 6 + 3(2.45) = 13.35, \text{ or } 13$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}} = 6 - 3\sqrt{6} = 6 - 3(2.45) = 0 \leftarrow \text{(since it cannot be negative)}$$

INSIGHT ▶ After Hoft plotted a control chart summarizing these data and posted it prominently in the drivers' locker room, the number of calls received dropped to an average of three per day. Can you explain why this occurred?

LEARNING EXERCISE ▶ Hoft collects 3 more days' worth of complaints (10, 12, and 8 complaints) and wants to combine them with the original 9 days to compute updated control limits. What are the revised UCL_c and LCL_c? [Answer: 14.94, 0.]

RELATED PROBLEMS ▶ S6.21, S6.22, S6.23, S6.24

EXCEL OM Data File **Ch06SExS5.xls** can be found in MyOMLab.

TABLE S6.3 Helping You Decide Which Control Chart to Use

VARIABLE DATA USING AN $\bar{x}$-CHART AND AN *R*-CHART
1. Observations are *variables*, which are usually products measured for size or weight. Examples are the width or length of a wire and the weight of a can of Campbell's soup. **2.** Collect 20 to 25 samples, usually of $n = 4$, $n = 5$, or more, each from a stable process, and compute the means for an $\bar{x}$-chart and the ranges for an *R*-chart. **3.** We track samples of *n* observations each, as in Example S1.
ATTRIBUTE DATA USING A *p*-CHART
1. Observations are *attributes* that can be categorized as good or bad (or pass–fail, or functional–broken); that is, in two states. **2.** We deal with fraction, proportion, or percent defectives. **3.** There are several samples, with many observations in each. For example, 20 samples of $n = 100$ observations in each, as in Example S4.
ATTRIBUTE DATA USING A *c*-CHART
1. Observations are *attributes* whose defects per unit of output can be counted. **2.** We deal with the number counted, which is a small part of the possible occurrences. **3.** Defects may be: number of blemishes on a desk; flaws in a bolt of cloth; crimes in a year; broken seats in a stadium; typos in a chapter of this text; or complaints in a day, as is shown in Example S5.

STUDENT TIP
This is a really useful table. When you are not sure which control chart to use, turn here for clarification.

Managerial Issues and Control Charts

In an ideal world, there is no need for control charts. Quality is uniform and so high that employees need not waste time and money sampling and monitoring variables and attributes. But because most processes have not reached perfection, managers must make three major decisions regarding control charts.

First, managers must select the points in their process that need SPC. They may ask "Which parts of the job are critical to success?" or "Which parts of the job have a tendency to become out of control?"

Second, managers need to decide if variable charts (i.e., $\bar{x}$ and R) or attribute charts (i.e., p and c) are appropriate. Variable charts monitor weights or dimensions. Attribute charts are more of a "yes–no" or "go–no go" gauge and tend to be less costly to implement. Table S6.3 can help you understand when to use each of these types of control charts.

Third, the company must set clear and specific SPC policies for employees to follow. For example, should the data-entry process be halted if a trend is appearing in percent defective records being keyed? Should an assembly line be stopped if the average length of five successive samples is above the centerline? Figure S6.7 illustrates some of the patterns to look for over time in a process.

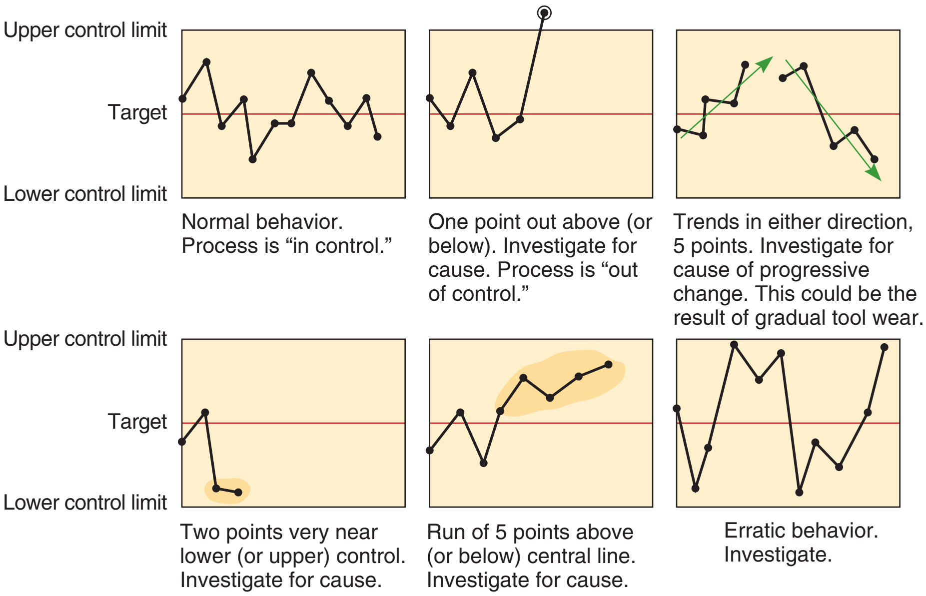


Figure **S6.7**

Patterns to Look for on Control Charts

Source: Adapted from Bertrand L. Hansen, *Quality Control: Theory and Applications* (1991): 65. Reprinted by permission of Prentice Hall, Upper Saddle River, NJ.

STUDENT TIP
Workers in companies such as Frito-Lay are trained to follow rules like these.

Run test
A test used to examine the points in a control chart to see if nonrandom variation is present.

A tool called a **run test** is available to help identify the kind of abnormalities in a process that we see in Figure S6.7. In general, a run of 5 points above or below the target or centerline may suggest that an assignable, or nonrandom, variation is present. When this occurs, even though all the points may fall inside the control limits, a flag has been raised. This means the process may not be statistically in control. A variety of run tests are described in books on the subject of quality methods.

STUDENT TIP Here we deal with whether a process meets the specification it was *designed* to yield.

Process Capability

Statistical process control means keeping a process in control. This means that the natural variation of the process must be stable. However, a process that is in statistical control may not yield goods or services that meet their *design specifications* (tolerances). In other words, the variation should be small enough to produce consistent output within specifications. The ability of a process to meet design specifications, which are set by engineering design or customer requirements, is called **process capability**. Even though that process may be statistically in control (stable), the output of that process may not conform to specifications.

Process capability
The ability to meet design specifications.

For example, let's say the time a customer expects to wait for the completion of a lube job at Quik Lube is 12 minutes, with an acceptable tolerance of ± 2 minutes. This tolerance gives an upper specification of 14 minutes and a lower specification of 10 minutes. The lube process has to be capable of operating within these design specifications—if not, some customers will not have their requirements met. As a manufacturing example, the tolerances for Harley-Davidson cam gears are extremely low, only 0.0005 inch—and a process must be designed that is capable of achieving this tolerance.

LO S6.6 *Explain* process capability and compute C_p and C_{pk}

There are two popular measures for quantitatively determining if a process is capable: process capability ratio (C_p) and process capability index (C_{pk}).

Process Capability Ratio (C_p)

For a process to be capable, its values must fall within upper and lower specifications. This typically means the process capability is within ± 3 standard deviations from the process mean. Because this range of values is 6 standard deviations, a capable process tolerance, which is the difference between the upper and lower specifications, must be greater than or equal to 6.

C_p
A ratio for determining whether a process meets design specifications; a ratio of the specification to the process variation.

The process capability ratio, **C_p**, is computed as:

$$C_p = \frac{\text{Upper specification} - \text{Lower specification}}{6\sigma} \tag{S6-13}$$

Example S6 shows the computation of C_p.

Example S6

PROCESS CAPABILITY RATIO (C_p)

In a GE insurance claims process, $\bar{x} = 210.0$ minutes, and $\sigma = .516$ minutes.

The design specification to meet customer expectations is 210 ± 3 minutes. So the Upper Specification is 213 minutes and the lower specification is 207 minutes. The OM manager wants to compute the process capability ratio.

APPROACH ▶ GE applies Equation (S6-13).

SOLUTION ▶ $C_p = \dfrac{\text{Upper specification} - \text{Lower specification}}{6\sigma} = \dfrac{213 - 207}{6(.516)} = 1.938$

INSIGHT ▶ Because a ratio of 1.00 means that 99.73% of a process's outputs are within specifications, this ratio suggests a very capable process, with nonconformance of less than 4 claims per million.

LEARNING EXERCISE ▶ If $\sigma = .60$ (instead of .516), what is the new C_p? [Answer: 1.667, a very capable process still.]

RELATED PROBLEMS ▶ S6.40, S6.41 (S6.50 is available in MyOMLab)

ACTIVE MODEL S6.3 This example is further illustrated in Active Model S6.3 in MyOMLab.

EXCEL OM Data File **Ch06SExS6.xls** can be found in MyOMLab.

A capable process has a C_p of at least 1.0. If the C_p is less than 1.0, the process yields products or services that are outside their allowable tolerance. With a C_p of 1.0, 2.7 parts in 1,000 can be expected to be "out of spec."[6] The higher the process capability ratio, the greater the likelihood the process will be within design specifications. Many firms have chosen a C_p of 1.33 (a 4-sigma standard) as a target for reducing process variability. This means that only 64 parts per million can be expected to be out of specification.

Recall that in Chapter 6 we mentioned the concept of *Six Sigma* quality, championed by GE and Motorola. This standard equates to a C_p of 2.0, with only 3.4 defective parts per million (very close to zero defects) instead of the 2.7 parts per 1,000 with 3-sigma limits.

Although C_p relates to the spread (dispersion) of the process output relative to its tolerance, it does not look at how well the process average is centered on the target value.

Process Capability Index (C_{pk})

The process capability index, **C_{pk}**, measures the difference between the desired and actual dimensions of goods or services produced.

C_{pk}
A proportion of variation (3σ) between the center of the process and the nearest specification limit.

The formula for C_{pk} is:

$$C_{pk} = \text{Minimum of}\left[\frac{\text{Upper specification limit} - \overline{X}}{3\sigma}, \frac{\overline{X} - \text{Lower specification limit}}{3\sigma}\right] \tag{S6-14}$$

where $\overline{X}$ = process mean

σ = standard deviation of the process population

When the C_{pk} index for both the upper and lower specification limits equals 1.0, the process variation is centered and the process is capable of producing within ± 3 standard deviations (fewer than 2,700 defects per million). A C_{pk} of 2.0 means the process is capable of producing fewer than 3.4 defects per million. For C_{pk} to exceed 1, σ must be less than $\frac{1}{3}$ of the difference between the specification and the process mean ($\overline{X}$). Figure S6.8 shows the meaning of various measures of C_{pk}, and Example S7 shows an application of C_{pk}.

Example S7

PROCESS CAPABILITY INDEX (C_{pk})

You are the process improvement manager and have developed a new machine to cut insoles for the company's top-of-the-line running shoes. You are excited because the company's goal is no more than 3.4 defects per million, and this machine may be the innovation you need. The insoles cannot be more than $\pm$.001 of an inch from the required thickness of .250″. You want to know if you should replace the existing machine, which has a C_{pk} of 1.0.

Mean of the new process $\overline{X}$ = .250 inches.
Standard deviation of the new process = σ = .0005 inches.

APPROACH ▶ You decide to determine the C_{pk}, using Equation (S6-14), for the new machine and make a decision on that basis.

SOLUTION ▶

Upper specification limit = .251 inches

Lower specification limit = .249 inches

$$C_{pk} = \text{Minimum of}\left[\frac{\text{Upper specification limit} - \overline{X}}{3\sigma}, \frac{\overline{X} - \text{Lower specification limit}}{3\sigma}\right]$$

$$C_{pk} = \text{Minimum of}\left[\frac{.251 - .250}{(3).0005}, \frac{.250 - .249}{(3).0005}\right]$$

Both calculations result in: $\frac{.001}{.0015} = .67.$

INSIGHT ▶ Because the new machine has a C_{pk} of only 0.67, the new machine should *not* replace the existing machine.

LEARNING EXERCISE ▶ If the insoles can be $\pm .002''$ (instead of .001") from the required .250", what is the new C_{pk}? [Answer: 1.33 and the new machine *should* replace the existing one.]

RELATED PROBLEMS ▶ S6.41–S6.45 (S6.46–S6.49 are available in MyOMLab)

ACTIVE MODEL S6.2 This example is further illustrated in Active Model S6.2 in MyOMLab.

EXCEL OM Data File **Ch06SExS7.xls** can be found in MyOMLab.

Note that C_p and C_{pk} will be the same when the process is centered. However, if the mean of the process is not centered on the desired (specified) mean, then the smaller numerator in Equation (S6-14) is used (the minimum of the difference between the upper specification limit and the mean or the lower specification limit and the mean). This application of C_{pk} is shown in Solved Problem S6.4. C_{pk} is the standard criterion used to express process performance.

Acceptance Sampling[7]

Acceptance sampling
A method of measuring random samples of lots or batches of products against predetermined standards.

LO S6.7 *Explain* acceptance sampling

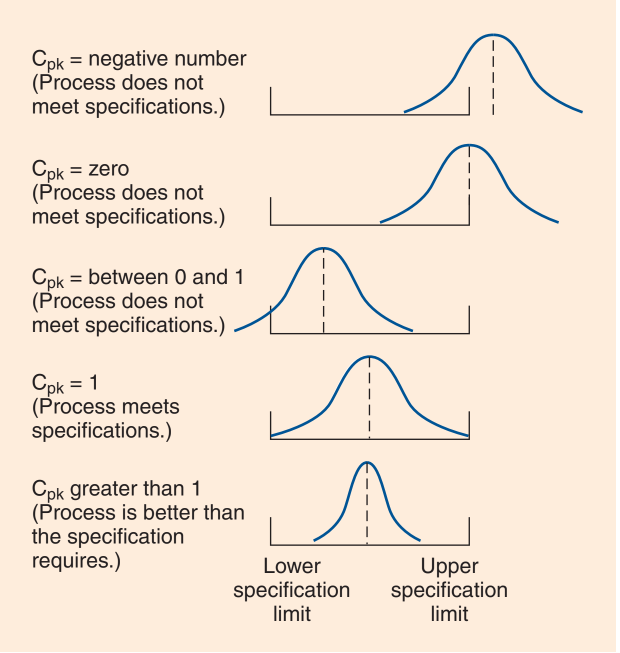
Acceptance sampling is a form of testing that involves taking random samples of "lots," or batches, of finished products and measuring them against predetermined standards. Sampling is more economical than 100% inspection. The quality of the sample is used to judge the quality of all items in the lot. Although both attributes and variables can be inspected by acceptance sampling, attribute inspection is more commonly used, as illustrated in this section.

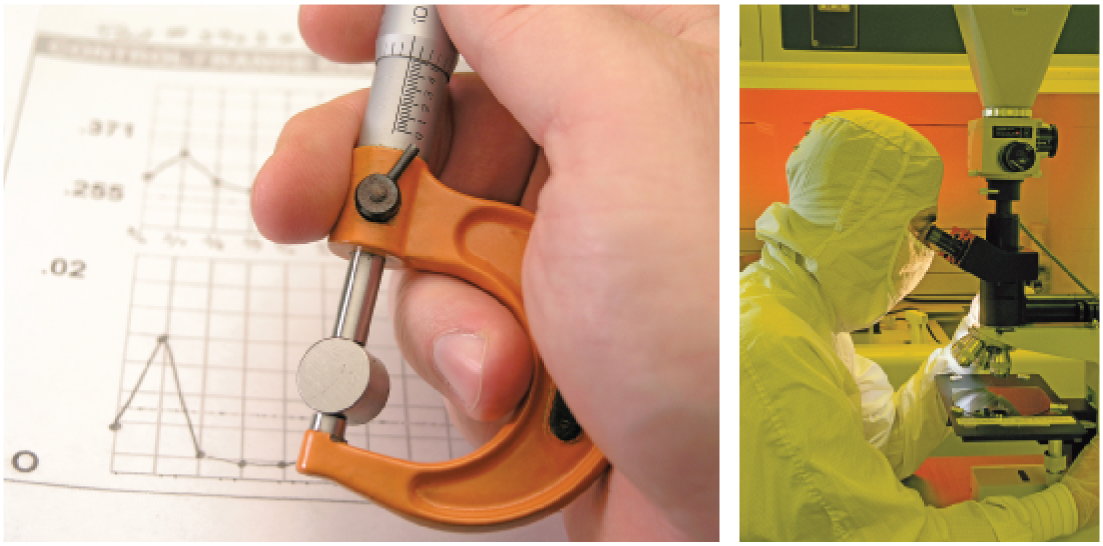
Acceptance sampling can be applied either when materials arrive at a plant or at final inspection, but it is usually used to control incoming lots of purchased products. A lot of items rejected, based on an unacceptable level of defects found in the sample, can (1) be returned to the supplier or (2) be 100% inspected to cull out all defects, with the cost of this screening usually billed to the supplier. However, acceptance sampling is not a substitute for adequate process controls. In fact, the current approach is to build statistical quality controls at suppliers so that acceptance sampling can be eliminated.

Figure **S6.8**

Meanings of C_{pk} Measures

A C_{pk} index of 1.0 for both the upper and lower specification limits indicates that the process variation is within the upper and lower specification limits. As the C_{pk} index goes above 1.0, the process becomes increasingly target oriented, with fewer defects. If the C_{pk} is less than 1.0, the process will not produce within the specified tolerance. Because a process may not be centered, or may "drift," a C_{pk} above 1 is desired.





Lyroky/Alamy

Richard T. Nowitz/Flirt/Alamy

Raw data for Statistical Process Control is collected in a wide variety of ways. Here physical measures using a micrometer (on the left) and a microscope (on the right) are being made.

Operating Characteristic Curve

The **operating characteristic (OC) curve** describes how well an acceptance plan discriminates between good and bad lots. A curve pertains to a specific plan—that is, to a combination of n (sample size) and c (acceptance level). It is intended to show the probability that the plan will accept lots of various quality levels.

Operating characteristic (OC) curve
A graph that describes how well an acceptance plan discriminates between good and bad lots.

With acceptance sampling, two parties are usually involved: the producer of the product and the consumer of the product. In specifying a sampling plan, each party wants to avoid costly mistakes in accepting or rejecting a lot. The producer usually has the responsibility of replacing all defects in the rejected lot or of paying for a new lot to be shipped to the customer. The producer, therefore, wants to avoid the mistake of having a good lot rejected (**producer's risk**). On the other hand, the customer or consumer wants to avoid the mistake of accepting a bad lot because defects found in a lot that has already been accepted are usually the responsibility of the customer (**consumer's risk**). The OC curve shows the features of a particular sampling plan, including the risks of making a wrong decision. The steeper the curve, the better the plan distinguishes between good and bad lots.[8]

Producer's risk
The mistake of having a producer's good lot rejected through sampling.

Consumer's risk
The mistake of a customer's acceptance of a bad lot overlooked through sampling.

Figure S6.9 can be used to illustrate one sampling plan in more detail. Four concepts are illustrated in this figure.

The **acceptable quality level (AQL)** is the poorest level of quality that we are willing to accept. In other words, we wish to accept lots that have this or a better level of quality, but no worse. If an acceptable quality level is 20 defects in a lot of 1,000 items or parts, then AQL is $20/1{,}000 = 2\%$ defectives.

Acceptable quality level (AQL)
The quality level of a lot considered good.

The **lot tolerance percentage defective (LTPD)** is the quality level of a lot that we consider bad. We wish to reject lots that have this or a poorer level of quality. If it is agreed that an unacceptable quality level is 70 defects in a lot of 1,000, then the LTPD is $70/1{,}000 = 7\%$ defective.

Lot tolerance percentage defective (LTPD)
The quality level of a lot considered bad.

To derive a sampling plan, producer and consumer must define not only "good lots" and "bad lots" through the AQL and LTPD, but they must also specify risk levels.

Producer's risk (α) is the probability that a "good" lot will be rejected. This is the risk that a random sample might result in a much higher proportion of defects than the population of all items. A lot with an acceptable quality level of AQL still has an α chance of being rejected. Sampling plans are often designed to have the producer's risk set at $\alpha = .05$, or 5%.

Consumer's risk (β) is the probability that a "bad" lot will be accepted. This is the risk that a random sample may result in a lower proportion of defects than the overall population of items. A common value for consumer's risk in sampling plans is $\beta = .10$, or 10%.

The probability of rejecting a good lot is called a **type I error**. The probability of accepting a bad lot is a **type II error**.

Type I error
Statistically, the probability of rejecting a good lot.

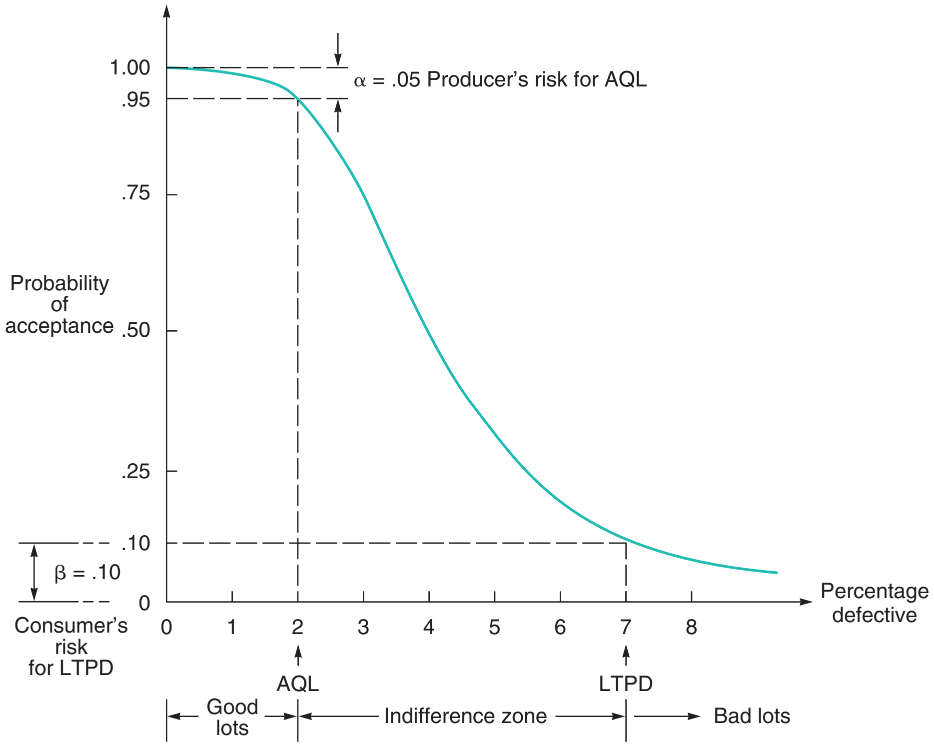
Type II error
Statistically, the probability of accepting a bad lot.

Sampling plans and OC curves may be developed by computer (as seen in the software available with this text), by published tables, or by calculation, using binomial or Poisson distributions.

Figure **S6.9**

An Operating Characteristic (OC) Curve Showing Producer's and Consumer's Risks

A good lot for this particular acceptance plan has less than or equal to 2% defectives. A bad lot has 7% or more defectives.



Average Outgoing Quality

In most sampling plans, when a lot is rejected, the entire lot is inspected and all defective items replaced. Use of this replacement technique improves the average outgoing quality in terms of percent defective. In fact, given (1) any sampling plan that replaces all defective items encountered and (2) the true incoming percent defective for the lot, it is possible to determine the **average outgoing quality (AOQ)** in percentage defective. The equation for AOQ is:

Average outgoing quality (AOQ)
The percentage defective in an average lot of goods inspected through acceptance sampling.

$$\text{AOQ} = \frac{(P_d)(P_a)(N - n)}{N} \qquad \text{(S6-15)}$$

where

P_d = true percentage defective of the lot
P_a = probability of accepting the lot for a given sample size and quantity defective
N = number of items in the lot
n = number of items in the sample

The maximum value of AOQ corresponds to the highest average percentage defective or the lowest average quality for the sampling plan. It is called the *average outgoing quality limit (AOQL)*.

This laser tracking device, by Faro Technologies, enables quality control personnel to measure and inspect parts and tools during production. The portable tracker can measure objects from 262 feet away and takes to up 1,000 accurate readings per second.

Faro Technologies

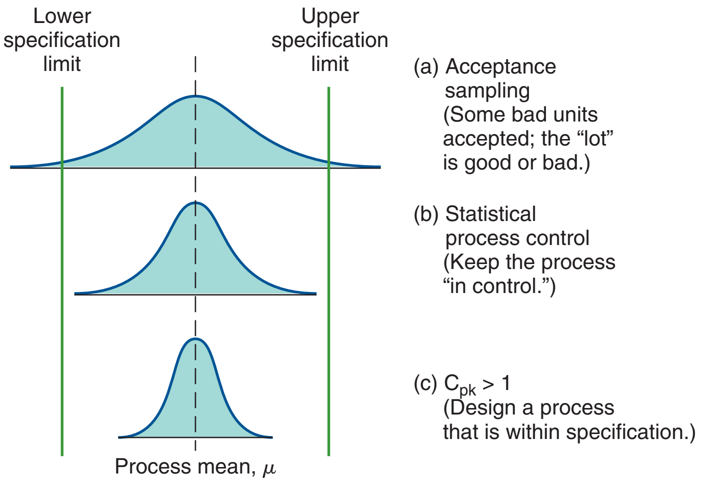


Figure **S6.10**

The Application of Statistical Process Control Techniques Contributes to the Identification and Systematic Reduction of Process Variability

Acceptance sampling is useful for screening incoming lots. When the defective parts are replaced with good parts, acceptance sampling helps to increase the quality of the lots by reducing the outgoing percent defective.

Figure S6.10 compares acceptance sampling, SPC, and C_{pk}. As the figure shows, (a) acceptance sampling by definition accepts some bad units, (b) control charts try to keep the process in control, but (c) the C_{pk} index places the focus on improving the process. As operations managers, that is what we want to do—improve the process.

Summary

Statistical process control is a major statistical tool of quality control. Control charts for SPC help operations managers distinguish between natural and assignable variations. The $\bar{x}$-chart and the R-chart are used for variable sampling, and the p-chart and the c-chart for attribute sampling. The C_{pk} index is a way to express process capability. Operating characteristic (OC) curves facilitate acceptance sampling and provide the manager with tools to evaluate the quality of a production run or shipment.

Key Terms

Statistical process control (SPC) (p. 284)
Control chart (p. 284)
Natural variations (p. 284)
Assignable variation (p. 285)
$\bar{x}$-chart (p. 286)
R-chart (p. 286)
Central limit theorem (p. 286)
p-chart (p. 293)
c-chart (p. 295)
Run test (p. 298)
Process capability (p. 298)
C_p (p. 298)
C_{pk} (p. 299)
Acceptance sampling (p. 300)
Operating characteristic (OC) curve (p. 301)
Producer's risk (p. 301)
Consumer's risk (p. 301)
Acceptable quality level (AQL) (p. 301)
Lot tolerance percentage defective (LTPD) (p. 301)
Type I error (p. 301)
Type II error (p. 301)
Average outgoing quality (AOQ) (p. 302)

Discussion Questions

1. List Shewhart's two types of variation. What are they also called?
2. Define "in statistical control."
3. Can the X-bar chart be used in order to see if the number of defectives in a production timeslot is randomly distributed? Explain.
4. What might cause a process to be out of control?
5. List five steps in developing and using $\bar{x}$-charts and R-charts.
6. List some possible causes of assignable variation.
7. Explain how a person using 2-sigma control charts will more easily find samples "out of bounds" than 3-sigma control charts. What are some possible consequences of this fact?
8. When is the desired mean, μ, used in establishing the centerline of a control chart instead of $\bar{\bar{x}}$?
9. Why is a process termed "out of control" if it temporarily produces less defectives?
10. In a control chart, what would be the effect on the control limits if the sample size varied from one sample to the next?
11. Define C_{pk} and explain what a C_{pk} of 1.0 means. What is C_p?
12. Even if the population distribution isn't normal (eg. Beta or Uniform), can the distribution of sample means be normal?
13. What are the acceptable quality level (AQL) and the lot tolerance percentage defective (LTPD)? How are they used?
14. What is a run test, and when is it used?
15. Discuss the managerial issues regarding the use of control charts.
16. What is a P-chart used for?

17. What is the purpose of acceptance sampling?

18. In the ideal world, we would like our processes to have a high process capability. Discuss.

19. Is a *capable* process a *perfect* process? That is, does a capable process generate only output that meets specifications? Explain.

Using Software for SPC

Excel, Excel OM, and POM for Windows may be used to develop control charts for most of the problems in this chapter.

CREATING YOUR OWN EXCEL SPREADSHEETS TO DETERMINE CONTROL LIMITS FOR A C-CHART

Excel and other spreadsheets are extensively used in industry to maintain control charts. Program S6.1 is an example of how to use Excel to determine the control limits for a *c*-chart. A *c*-chart is used when the number of defects per unit of output is known. The data from Example S5 are used here. In this example, 54 complaints occurred over 9 days. Excel also contains a built-in graphing ability with Chart Wizard.

	A	B	C	D
1	Red Top Cab Company			
2	Setting Control Limits for Number of Defects: *c*-chart			
3				
4	Number of Standard Deviations (z)		3	
5				
6	Number of Samples	Complaints		
7	Day 1	3		
8	Day 2	0		
9	Day 3	8		
10	Day 4	9		
11	Day 5	6		
12	Day 6	7		
13	Day 7	4		
14	Day 8	9		
15	Day 9	8		
16	Sum	54		
17				
18	Center Line (Avg. Defects) c-bar		6.00	
19	Standard Deviation of Defects		2.45	
20				
21	Upper Control Limit UCL_c		13.35	
22	Lower Control Limit LCL_c		0	
23				

Use z = 3 for 99.73% limits.

=SUM(B7:B15)

=AVERAGE(B7:B15)

=SQRT(C18)

=C18+C4*C19

=MAX(0,C18-C4*C19)

Program **S6.1**

An Excel Spreadsheet for Creating a *c*-Chart for Example S5

USING EXCEL OM

Excel OM's Quality Control module has the ability to develop $\bar{x}$-charts, *p*-charts, and *c*-charts. It also handles OC curves, acceptance sampling, and process capability. Program S6.2 illustrates Excel OM's spreadsheet approach to computing the $\bar{x}$ control limits for the Oat Flakes company in Example S1.

	A	B	C	D	E	F	G	H
1	Oat Flakes							
2								
3	Quality Control		x bar chart					
4								
5	Number of samples	12						
6	Sample size	9						
7	Population standard deviation	1						
8	Data				Results			
9		Mean						
10	Sample 1	16.1			x-bar value	16		
11	Sample 2	16.8			z value	3		
12	Sample 3	15.5			Sigma x bar	0.333333		
13	Sample 4	16.5						
14	Sample 5	16.5			Upper control limit	17		
15	Sample 6	16.4			Center line	16		
16	Sample 7	15.2			Lower control limit	15		
17	Sample 8	16.4						
18	Sample 9	16.3						
19	Sample 10	14.8	Below LCL					
20	Sample 11	14.2	Below LCL					
21	Sample 12	17.3	Above UCL					
22	Average	16						

Enter the population standard deviation then enter the data from each sample. Finally, you

Do not change this cell without changing the number of rows in the data table.

Enter the size for each of the hourly samples taken.

Enter the mean weight for each of the 12 samples.

= B22

Enter the desired number of standard deviations.

= B7/SQRT(B6)

Use the overall average as the center line; add and subtract the product of the desired number of standard deviations and sigma *x*-bar in order to create upper and lower control limits (e.g., LCL = F10 – F11*F12).

Calculate $\bar{\bar{x}}$—the overall average weight of all the samples = AVERAGE (B10:B21).

Program **S6.2**

Excel OM Input and Selected Formulas for the Oat Flakes Company in Example S1

USING POM FOR WINDOWS

The POM for Windows Quality Control module has the ability to compute all the SPC control charts we introduced in this supplement, as well as OC curves, acceptance sampling, and process capability. See Appendix IV for further details.

Solved Problems

Virtual Office Hours help is available in MyOMLab.

SOLVED PROBLEM S6.1

A manufacturer of precision machine parts produces round shafts for use in the construction of drill presses. The average diameter of a shaft is .56 inch. Inspection samples contain 6 shafts each. The average range of these samples is .006 inch. Determine the upper and lower $\bar{x}$ control chart limits.

SOLUTION

The mean factor A_2 from Table S6.1, where the sample size is 6, is seen to be .483. With this factor, you can obtain the upper and lower control limits:

$$
\begin{aligned}
UCL_{\bar{x}} &= .56 + (.483)(.006) \\
&= .56 + .0029 \\
&= .5629 \text{ inch} \\
LCL_{\bar{x}} &= .56 - .0029 \\
&= .5571 \text{ inch}
\end{aligned}
$$

SOLVED PROBLEM S6.2

Nocaf Drinks, Inc., a producer of decaffeinated coffee, bottles Nocaf. Each bottle should have a net weight of 4 ounces. The machine that fills the bottles with coffee is new, and the operations manager wants to make sure that it is properly adjusted. Bonnie Crutcher, the operations manager, randomly selects and weighs $n = 8$ bottles and records the average and range in ounces for each sample. The data for several samples is given in the following table. Note that every sample consists of 8 bottles.

SAMPLE	SAMPLE RANGE	SAMPLE AVERAGE	SAMPLE	SAMPLE RANGE	SAMPLE AVERAGE
A	.41	4.00	E	.56	4.17
B	.55	4.16	F	.62	3.93
C	.44	3.99	G	.54	3.98
D	.48	4.00	H	.44	4.01

Is the machine properly adjusted and in control?

SOLUTION

We first find that $\bar{\bar{x}} = 4.03$ and $\bar{R} = .505$. Then, using Table S6.1, we find:

$$
\begin{aligned}
UCL_{\bar{x}} &= \bar{\bar{x}} + A_2\bar{R} = 4.03 + (.373)(.505) = 4.22 \\
LCL_{\bar{x}} &= \bar{\bar{x}} - A_2\bar{R} = 4.03 - (.373)(.505) = 3.84 \\
UCL_R &= D_4\bar{R} = (1.864)(.505) = .94 \\
LCL_R &= D_3\bar{R} = (.136)(.505) = .07
\end{aligned}
$$

It appears that the process average and range are both in statistical control.

The operations manager needs to determine if a process with a mean (4.03) slightly above the desired mean of 4.00 is satisfactory; if it is not, the process will need to be changed.

SOLVED PROBLEM S6.3

Altman Distributors, Inc., fills catalog orders. Samples of size $n = 100$ orders have been taken each day over the past 6 weeks. The average defect rate was .05. Determine the upper and lower limits for this process for 99.73% confidence.

SOLUTION

$z = 3$, $\bar{p} = .05$. Using Equations (S6-9), (S6-10), and (S6-11):

$$
\begin{aligned}
UCL_p &= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = .05 + 3\sqrt{\frac{(.05)(1-.05)}{100}} \\
&= .05 + 3(0.0218) = .1154 \\
LCL_p &= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = .05 - 3(0.0218) \\
&= .05 - .0654 = 0 \quad \text{(because percentage defective cannot be negative)}
\end{aligned}
$$

SOLVED PROBLEM S6.4

Ettlie Engineering has a new catalyst injection system for your countertop production line. Your process engineering department has conducted experiments and determined that the mean is 8.01 grams with a standard deviation of .03. Your specifications are: $\mu = 8.0$ and $\sigma = .04$, which means an upper specification limit of 8.12 [$= 8.0 + 3(.04)$] and a lower specification limit of 7.88 [$= 8.0 - 3(.04)$].

What is the C_{pk} performance of the injection system?

SOLUTION

Using Equation (S6-14):

$$C_{pk} = \text{Minimum of}\left[\frac{\text{Upper specification limit} - \overline{X}}{3\sigma}, \frac{\overline{X} - \text{Lower specification limit}}{3\sigma}\right]$$

where

$\overline{X} =$ process mean

$\sigma =$ standard deviation of the process population

$$C_{pk} = \text{Minimum of}\left[\frac{8.12 - 8.01}{(3)(.03)}, \frac{8.01 - 7.88}{(3)(.03)}\right]$$

$$\left[\frac{.11}{.09} = 1.22, \frac{.13}{.09} = 1.44\right]$$

The minimum is 1.22, so the C_{pk} is within specifications and has an implied error rate of less than 2,700 defects per million.

SOLVED PROBLEM S6.5

Airlines lose thousands of checked bags every day, and America South Airlines is no exception to the industry rule. Over the past 6 weeks, the number of bags "misplaced" on America South flights has been 18, 10, 4, 6, 12, and 10. The head of customer service wants to develop a *c*-chart at 99.73% levels.

SOLUTION

She first computes $\bar{c} = \dfrac{18 + 10 + 4 + 6 + 12 + 10}{6} = \dfrac{60}{6} = 10 \text{ bags/week}$

Then, using Equation (S6-12):

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 10 + 3\sqrt{10} = 10 + 3(3.16) = 19.48 \text{ bags}$$

$$LCL_C = \bar{c} - 3\sqrt{\bar{c}} = 10 - 3\sqrt{10} = 10 - 3(3.16) = .52 \text{ bag}$$

Problems

Note: means the problem may be solved with POM for Windows and/or Excel OM/Excel.

Problems S6.1–S6.39 relate to Statistical Process Control (SPC)

• **S6.1** Boxes of Honey-Nut Oatmeal are produced to contain 14 ounces, with a standard deviation of .1 ounce. Set up the 3-sigma $\bar{x}$-chart for a sample size of 36 boxes.

• **S6.2** The overall average on a process you are attempting to monitor is 50 units. The process population standard deviation is 1.72. Determine the upper and lower control limits for a mean chart, if you choose to use a sample size of 5.
a) Set $z = 3$.
b) Now set $z = 2$. How do the control limits change?

• **S6.3** Thirty-five samples of size 7 each were taken from a fertilizer-bag-filling machine. The results were overall mean = 57.75 lb; average range = 1.78 lb.
a) Determine the upper and lower control limits of the $\bar{x}$-chart, where $\sigma = 3$.
b) Determine the upper and lower control limits of the R-chart, where $\sigma = 3$.

• **S6.4** Rosters Chicken advertises "lite" chicken with 30% fewer calories than standard chicken. When the process for "lite" chicken breast production is in control, the average chicken breast contains 420 calories, and the standard deviation in caloric content of the chicken breast population is 25 calories.

Rosters wants to design an $\bar{x}$-chart to monitor the caloric content of chicken breasts, where 25 chicken breasts would be chosen at random to form each sample.
a) What are the lower and upper control limits for this chart if these limits are chosen to be *four* standard deviations from the target?
b) What are the limits with three standard deviations from the target?

• **S6.5** Ross Hopkins is attempting to monitor a filling process that has an overall average of 705 cc. The average range is 6 cc. If you use a sample size of 10, what are the upper and lower control limits for the mean and range?

• • **S6.6** Sampling four pieces of precision-cut wire (to be used in computer assembly) every hour for the past 24 hours has produced the following results:

HOUR	$\bar{X}$	R	HOUR	$\bar{X}$	R
1	3.25″	.71″	13	3.11″	.85″
2	3.10	1.18	14	2.83	1.31
3	3.22	1.43	15	3.12	1.06
4	3.39	1.26	16	2.84	.50
5	3.07	1.17	17	2.86	1.43
6	2.86	.32	18	2.74	1.29
7	3.05	.53	19	3.41	1.61
8	2.65	1.13	20	2.89	1.09
9	3.02	.71	21	2.65	1.08
10	2.85	1.33	22	3.28	.46
11	2.83	1.17	23	2.94	1.58
12	2.97	.40	24	2.64	.97

Develop appropriate control charts and determine whether there is any cause for concern in the cutting process. Plot the information and look for patterns.

• • **S6.7** Auto pistons at Wemming Chung's plant in Shanghai are produced in a forging process, and the diameter is a critical factor that must be controlled. From sample sizes of 10 pistons produced each day, the mean and the range of this diameter have been as follows:

DAY	MEAN (MM)	RANGE (MM)
1	156.9	4.2
2	153.2	4.6
3	153.6	4.1
4	155.5	5.0
5	156.6	4.5

a) What is the value of $\bar{\bar{x}}$?
b) What is the value of $\bar{R}$?
c) What are the $UCL_{\bar{x}}$ and $LCL_{\bar{x}}$, using 3σ? Plot the data.
d) What are the UCL_R and LCL_R, using 3σ? Plot the data.
e) If the true diameter mean should be 155 mm and you want this as your center (nominal) line, what are the new $UCL_{\bar{x}}$ and $LCL_{\bar{x}}$?

• • **S6.8** A. Choudhury's bowling ball factory in Illinois makes bowling balls of adult size and weight only. The standard deviation in the weight of a bowling ball produced at the factory is known to be 0.12 pounds. Each day for 24 days, the average weight, in pounds, of nine of the bowling balls produced that day has been assessed as follows:

DAY	AVERAGE (lb)	DAY	AVERAGE (lb)
1	16.3	13	16.3
2	15.9	14	15.9
3	15.8	15	16.3
4	15.5	16	16.2
5	16.3	17	16.1
6	16.2	18	15.9
7	16.0	19	16.2
8	16.1	20	15.9
9	15.9	21	15.9
10	16.2	22	16.0
11	15.9	23	15.5
12	15.9	24	15.8

a) Establish a control chart for monitoring the average weights of the bowling balls in which the upper and lower control limits are each two standard deviations from the mean. What are the values of the control limits?
b) If three standard deviations are used in the chart, how do these values change? Why?

•• **S6.9** Organic Grains LLC uses statistical process control to ensure that its health-conscious, low-fat, multigrain sandwich loaves have the proper weight. Based on a previously stable and in-control process, the control limits of the $\bar{x}$- and R-charts are $UCL_{\bar{x}} = 6.56$. $LCL_{\bar{x}} = 5.84$, $UCL_R = 1.141$, $LCL_R = 0$. Over the past few days, they have taken five random samples of four loaves each and have found the following:

SAMPLE	NET WEIGHT			
	LOAF #1	LOAF #2	LOAF #3	LOAF #4
1	6.3	6.0	5.9	5.9
2	6.0	6.0	6.3	5.9
3	6.3	4.8	5.6	5.2
4	6.2	6.0	6.2	5.9
5	6.5	6.6	6.5	6.9

Is the process still in control? Explain why or why not. PX

••• **S6.10** A process that is considered to be in control measures an ingredient in ounces. Below are the last 10 samples (each of size $n = 5$) taken. The population process standard deviation, σ, is 1.36.

SAMPLES									
1	2	3	4	5	6	7	8	9	10
10	9	13	10	12	10	10	13	8	10
9	9	9	10	10	10	11	10	8	12
10	11	10	11	9	8	10	8	12	9
9	11	10	10	11	12	8	10	12	8
12	10	9	10	10	9	9	8	9	12

a) What is $\sigma_{\bar{x}}$?
b) If $z = 3$, what are the control limits for the mean chart?
c) What are the control limits for the range chart?
d) Is the process in control? PX

••• **S6.11** Twelve samples, each containing five parts, were taken from a process that produces steel rods at Emmanuel Kodzi's factory. The length of each rod in the samples was determined. The results were tabulated and sample means and ranges were computed. The results were:

SAMPLE	SAMPLE MEAN (in.)	RANGE (in.)
1	10.002	0.011
2	10.002	0.014
3	9.991	0.007
4	10.006	0.022
5	9.997	0.013
6	9.999	0.012
7	10.001	0.008
8	10.005	0.013
9	9.995	0.004
10	10.001	0.011
11	10.001	0.014
12	10.006	0.009

a) Determine the upper and lower control limits and the overall means for $\bar{x}$-charts and R-charts.
b) Draw the charts and plot the values of the sample means and ranges.
c) Do the data indicate a process that is in control?
d) Why or why not? PX

•• **S6.12** Eagletrons are all-electric automobiles produced by Mogul Motors, Inc. One of the concerns of Mogul Motors is that the Eagletrons be capable of achieving appropriate maximum speeds. To monitor this, Mogul executives take samples of eight Eagletrons at a time. For each sample, they determine the average maximum speed and the range of the maximum speeds within the sample. They repeat this with 35 samples to obtain 35 sample means and 35 ranges. They find that the average sample mean is 88.50 miles per hour, and the average range is 3.25 miles per hour. Using these results, the executives decide to establish an R chart. They would like this chart to be established so that when it shows that the range of a sample is not within the control limits, there is only approximately a 0.0027 probability that this is due to natural variation. What will be the upper control limit (UCL) and the lower control limit (LCL) in this chart? PX

•• **S6.13** The defect rate for data entry of insurance claims has historically been about 1.5%.
a) What are the upper and lower control chart limits if you wish to use a sample size of 100 and 3-sigma limits?
b) What if the sample size used were 50, with 3σ?
c) What if the sample size used were 100, with 2σ?
d) What if the sample size used were 50, with 2σ?
e) What happens to $\hat{\sigma}_p$ when the sample size is larger?
f) Explain why the lower control limit cannot be less than 0. PX

•• **S6.14** You are attempting to develop a quality monitoring system for some parts purchased from Charles Sox Manufacturing Co. These parts are either good or defective. You have decided to take a sample of 100 units. Develop a table of the appropriate upper and lower control chart limits for various values of the average fraction defective in the samples taken. The values for $\bar{p}$ in this table should range from 0.02 to 0.10 in increments of 0.02. Develop the upper and lower control limits for a 99.73% confidence level.

$N = 100$		
$\bar{P}$	UCL	LCL
0.02		
0.04		
0.06		
0.08		
0.10		

PX

•• **S6.15** The results of an inspection of DNA samples taken over the past 10 days are given below. Sample size is 100.

DAY	1	2	3	4	5	6	7	8	9	10
DEFECTIVES	7	6	6	9	5	6	0	8	9	1

a) Construct a 3-sigma p-chart using this information.
b) Using the control chart in part (a), and finding that the number of defectives on the next three days are 12, 5, and 13, is the process in control? PX

• **S6.16** In the past, the defective rate for your product has been 1.5%. What are the upper and lower control chart limits if you wish to use a sample size of 500 and $z = 3$? PX

• **S6.17** Refer to Problem S6.16. If the defective rate was 3.5% instead of 1.5%, what would be the control limits ($z = 3$)? PX

• • **S6.18** Five data entry operators work at the data processing department of the Birmingham Bank. Each day for 30 days, the number of defective records in a sample of 250 records typed by these operators has been noted, as follows:

SAMPLE NO.	NO. DEFECTIVE	SAMPLE NO.	NO. DEFECTIVE	SAMPLE NO.	NO. DEFECTIVE
1	7	11	18	21	17
2	5	12	5	22	12
3	19	13	16	23	6
4	10	14	4	24	7
5	11	15	11	25	13
6	8	16	8	26	10
7	12	17	12	27	14
8	9	18	4	28	6
9	6	19	6	29	12
10	13	20	16	30	3

a) Establish 3σ upper and lower control limits.
b) Why can the lower control limit not be a negative number?
c) The industry standards for the upper and lower control limits are 0.10 and 0.01, respectively. What does this imply about Birmingham Bank's own standards?

• • **S6.19** Houston North Hospital is trying to improve its image by providing a positive experience for its patients and their relatives. Part of the "image" program involves providing tasty, inviting patient meals that are also healthful. A questionnaire accompanies each meal served, asking the patient, among other things, whether he or she is satisfied or unsatisfied with the meal. A 100-patient sample of the survey results over the past 7 days yielded the following data:

DAY	NO. OF UNSATISFIED PATIENTS	SAMPLE SIZE
1	24	100
2	22	100
3	8	100
4	15	100
5	10	100
6	26	100
7	17	100

Construct a *p*-chart that plots the percentage of patients unsatisfied with their meals. Set the control limits to include 99.73% of the random variation in meal satisfaction. Comment on your results.

Corbis Super RF/Alamy

• • **S6.20** Jamison Kovach Supply Company manufactures paper clips and other office products. Although inexpensive, paper clips have provided the firm with a high margin of profitability. Sample size is 200. Results are given for the last 10 samples:

SAMPLE	1	2	3	4	5	6	7	8	9	10
DEFECTIVES	5	7	4	4	6	3	5	6	2	8

a) Establish upper and lower control limits for the control chart and graph the data.
b) Has the process been in control?
c) If the sample size were 100 instead, how would your limits and conclusions change?

• **S6.21** Peter Ittig's department store, Ittig Brothers, is Amherst's largest independent clothier. The store receives an average of six returns per day. Using $z = 3$, would nine returns in a day warrant action?

• • **S6.22** An ad agency tracks the complaints, by week received, about the billboards in its city:

WEEK	NO. OF COMPLAINTS
1	4
2	5
3	4
4	11
5	3
6	9

a) What type of control chart would you use to monitor this process? Why?
b) What are the 3-sigma control limits for this process? Assume that the historical complaint rate is unknown.
c) Is the process in control, according to the control limits? Why or why not?
d) Assume now that the historical complaint rate has been four calls a week. What would the 3-sigma control limits for this process be now? Has the process been in control according to the control limits?

• • **S6.23** The school board is trying to evaluate a new math program introduced to second-graders in five elementary schools across the county this year. A sample of the student scores on standardized math tests in each elementary school yielded the following data:

SCHOOL	NO. OF TEST ERRORS
A	52
B	27
C	35
D	44
E	55

Construct a *c*-chart for test errors, and set the control limits to contain 99.73% of the random variation in test scores. What does the chart tell you? Has the new math program been effective?

•• **S6.24** Telephone inquiries of 100 IRS "customers" are monitored daily at random. Incidents of incorrect information or other nonconformities (such as impoliteness to customers) are recorded. The data for last week follow:

DAY	NO. OF NONCONFORMITIES
1	5
2	10
3	23
4	20
5	15

a) Construct a 3-standard deviation *c*-chart of nonconformities.
b) What does the control chart tell you about the IRS telephone operators?

••• **S6.25** The accounts receivable department at Rick Wing Manufacturing has been having difficulty getting customers to pay the full amount of their bills. Many customers complain that the bills are not correct and do not reflect the materials that arrived at their receiving docks. The department has decided to implement SPC in its billing process. To set up control charts, 10 samples of 50 bills each were taken over a month's time and the items on the bills checked against the bill of lading sent by the company's shipping department to determine the number of bills that were not correct. The results were:

SAMPLE NO.	NO. OF INCORRECT BILLS	SAMPLE NO.	NO. OF INCORRECT BILLS
1	6	6	5
2	5	7	3
3	11	8	4
4	4	9	7
5	0	10	2

a) Determine the value of *p*-bar, the mean fraction defective. Then determine the control limits for the *p*-chart using a 99.73% confidence level (3 standard deviations). Has this process been in control? If not, which samples were out of control?
b) How might you use the quality tools discussed in Chapter 6 to determine the source of the billing defects and where you might start your improvement efforts to eliminate the causes?

••• **S6.26** West Battery Corp. has recently been receiving complaints from retailers that its 9-volt batteries are not lasting as long as other name brands. James West, head of the TQM program at West's Austin plant, believes there is no problem because his batteries have had an average life of 50 hours, about 10% longer than competitors' models. To raise the lifetime above this level would require a new level of technology not available to West. Nevertheless, he is concerned enough to set up hourly assembly line checks. Previously, after ensuring that the process was running properly, West took size $n = 5$ samples of 9-volt batteries for each of 25 hours to establish the standards for control chart limits. Those samples are shown in the following table:

West Battery Data—Battery Lifetimes (in hours)

HOUR SAMPLE TAKEN	SAMPLE						
	1	2	3	4	5	$\overline{X}$	R
1	51	50	49	50	50	50.0	2
2	45	47	70	46	36	48.8	34
3	50	35	48	39	47	43.8	15
4	55	70	50	30	51	51.2	40
5	49	38	64	36	47	46.8	28
6	59	62	40	54	64	55.8	24
7	36	33	49	48	56	44.4	23
8	50	67	53	43	40	50.6	27
9	44	52	46	47	44	46.6	8
10	70	45	50	47	41	50.6	29
11	57	54	62	45	36	50.8	26
12	56	54	47	42	62	52.2	20
13	40	70	58	45	44	51.4	30
14	52	58	40	52	46	49.6	18
15	57	42	52	58	59	53.6	17
16	62	49	42	33	55	48.2	29
17	40	39	49	59	48	47.0	20
18	64	50	42	57	50	52.6	22
19	58	53	52	48	50	52.2	10
20	60	50	41	41	50	48.4	19
21	52	47	48	58	40	49.0	18
22	55	40	56	49	45	49.0	16
23	47	48	50	50	48	48.6	3
24	50	50	49	51	51	50.2	2
25	51	50	51	51	62	53.0	12

With these limits established, West now takes 5 more hours of data, which are shown in the following table:

	SAMPLE				
HOUR	1	2	3	4	5
26	48	52	39	57	61
27	45	53	48	46	66
28	63	49	50	45	53
29	57	70	45	52	61
30	45	38	46	54	52

a) Determine means and the upper and lower control limits for $\overline{x}$ and R (using the first 25 hours only).
b) Has the manufacturing process been in control?
c) Comment on the lifetimes observed.

•••• **S6.27** One of New England Air's top competitive priorities is on-time arrivals. Quality VP Clair Bond decided to personally monitor New England Air's performance. Each week for the past 30 weeks, Bond checked a random sample of 100 flight arrivals for on-time performance. The table that follows contains the number of flights that did not meet New England Air's definition of "on time":

SAMPLE (WEEK)	LATE FLIGHTS	SAMPLE (WEEK)	LATE FLIGHTS
1	2	16	2
2	4	17	3
3	10	18	7
4	4	19	3
5	1	20	2
6	1	21	3
7	13	22	7
8	9	23	4
9	11	24	3
10	0	25	2
11	3	26	2
12	4	27	0
13	2	28	1
14	2	29	3
15	8	30	4

a) Using a 95% confidence level, plot the overall percentage of late flights ($\bar{p}$) and the upper and lower control limits on a control chart.
b) Assume that the airline industry's upper and lower control limits for flights that are not on time are .1000 and .0400, respectively. Draw them on your control chart.
c) Plot the percentage of late flights in each sample. Do all samples fall within New England Air's control limits? When one falls outside the control limits, what should be done?
d) What can Clair Bond report about the quality of service? **PX**

Additional problems **S6.28–S6.39** *are available in* MyOMLab.

Problems S6.40–S6.50 relate to Process Capability

• **S6.40** The difference between the upper specification and the lower specification for a process is 0.6". The standard deviation is 0.1". What is the process capability ratio, C_p? Interpret this number. **PX**

•• **S6.41** Meena Chavan Corp.'s computer chip production process yields DRAM chips with an average life of 1,800 hours and $\sigma = 100$ hours. The tolerance upper and lower specification limits are 2,400 hours and 1,600 hours, respectively. Is this process capable of producing DRAM chips to specification? **PX**

•• **S6.42** Linda Boardman, Inc., an equipment manufacturer in Boston, has submitted a sample cutoff valve to improve your manufacturing process. Your process engineering department has conducted experiments and found that the valve has a mean (μ) of 8.00 and a standard deviation (σ) of .04. Your desired performance is $\mu = 8.0 \pm 3\sigma$, where $\sigma = .045$. What is the C_{pk} of the Boardman valve? **PX**

•• **S6.43** The specifications for a plastic liner for concrete highway projects calls for a thickness of 3.0 mm $\pm .1$ mm. The standard deviation of the process is estimated to be .02 mm. What are the upper and lower specification limits for this product? The process is known to operate at a mean thickness of 3.0 mm. What is the C_{pk} for this process? About what percentage of all units of this liner will meet specifications? **PX**

•• **S6.44** Frank Pianki, the manager of an organic yogurt processing plant, desires a quality specification with a mean of 16 ounces, an upper specification limit of 16.5, and a lower specification limit of 15.5. The process has a mean of 16 ounces and a standard deviation of 1 ounce. Determine the C_{pk} of the process. **PX**

•• **S6.45** A process filling small bottles with baby formula has a target of 3 ounces ± 0.150 ounce. Two hundred bottles from the process were sampled. The results showed the average amount of formula placed in the bottles to be 3.042 ounces. The standard deviation of the amounts was 0.034 ounce. Determine the value of C_{pk}. Roughly what proportion of bottles meet the specifications? **PX**

Additional problems **S6.46–S6.50** *are available in* MyOMLab.

Problems S6.51–S6.55 relate to Acceptance Sampling

•• **S6.51** As the supervisor in charge of shipping and receiving, you need to determine *the average outgoing quality* in a plant where the known incoming lots from your assembly line have an average defective rate of 3%. Your plan is to sample 80 units of every 1,000 in a lot. The number of defects in the sample is not to exceed 3. Such a plan provides you with a probability of acceptance of each lot of .79 (79%). What is your average outgoing quality? **PX**

•• **S6.52** An acceptance sampling plan has lots of 500 pieces and a sample size of 60. The number of defects in the sample may not exceed 2. This plan, based on an OC curve, has a probability of .57 of accepting lots when the incoming lots have a defective rate of 4%, which is the historical average for this process. What do you tell your customer the average outgoing quality is? **PX**

•• **S6.53** The percent defective from an incoming lot is 3%. An OC curve showed the probability of acceptance to be 0.55. Given a lot size of 2,000 and a sample of 100, determine the average outgoing quality in percent defective.

•• **S6.54** In an acceptance sampling plan developed for lots containing 1,000 units, the sample size n is 85. The percent defective of the incoming lots is 2%, and the probability of acceptance is 0.64. What is the average outgoing quality?

•• **S6.55** We want to determine the AOQ for an acceptance sampling plan when the quality of the incoming lots in percent defective is 1.5%, and then again when the incoming percent defective is 5%. The sample size is 80 units for a lot size of 550 units. Furthermore, P_a at 1.5% defective levels is 0.95. At 5% incoming defective levels, the P_a is found to be 0.5. Determine the average outgoing quality for both incoming percent defective levels.

CASE STUDIES

Cecil Rice Export, Alexandria, Egypt

Cecil Rice Export operates a facility in Alexandria, Egypt, where bulk white rice is bagged for export in 70-pound grain bags. The bagged rice is then palletized and brought by truck to a quay in the harbor and loaded onto a break-bulk ship for transport to foreign customers.

Cecil Rice Export has experienced good growth in the past decade due to a concentration on continuous improvement and Lean techniques. It has gained an enviable reputation for being a lowcost, high-value supplier. This growth has led to the need to increase capacity by expanding the number of shifts at Cecil Rice Export's facility. In 2005, it operated only one shift, from 08:00 to 16:00. In 2007, a second shift was added, and the facility operated from 04:00 to 20:00, switching shifts at 12:00. Since the start of 2009, the company has increased to three shifts, operating from 00:00 to 08:00, 08:00 to 16:00, and 16:00 to 00:00. As these shifts were added, most of the more experienced employees, due to their seniority, chose to remain on the day shift. Those originally on the second shift added in 2007 (at that time the 12:00 to 20:00 shift) mostly moved onto the 16:00 to 00:00 shift. Those on the shift added in 2009, the greenest shift, were therefore left with the 00:00 to 08:00 shift. As the new shifts were added, some senior personnel were assigned to the new shifts to break in the new workers. Eventually, these experienced hands returned to their preferred day shift. Traditionally, working bonds between shift members have resulted in a good amount of cohesion in the workforce, and shift members are usually quite averse to changing shifts.

Rice is delivered to the Cecil Rice Export facility and stored in a silo. From the silo, an air-powered transport system moves the grain to the hopper of the bagging machine. The bagging machine fills bags from the hopper to a specific weight, 70 pounds, and past records indicate that the mean is in fact 70.0 pounds, with a standard deviation of 1.0 pounds. This machine, however, is very sensitive and must be calibrated by trained personnel. In the past, this was not a problem due to the experience of the personnel in the original production shift. As shifts were added, there was no effort to examine whether the weight of the bags remained within acceptable limits. Recently, during the loading of the pallets on the merchant ship, the ships' officers have detected abnormalities in the pallets. The pallets are loaded with 30 bags each, for a presumed weight of 2,100 pounds. When confronted with abnormalities, plant management initially suspected the bagging operation, specifically over- or underweight bags. Both under- and overweight bags are undesirable from the firm's point of view. Overweight bags decrease Cecil Rice Export's revenue as product is given away in the overage. Underweight bags, on the other hand, present a customer service issue. Cecil Rice Export's customers have come to expect bags to have a proper weight and are disappointed when the bags are underweight. For these reasons, over- and underweight bags are equally undesirable. The table shows the minimum, maximum, and average from hourly samples of five bags per hour, taken over 3 days.

		RANGE				*RANGE*	
TIME	AVERAGE WEIGHT (POUNDS)	SMALLEST	LARGEST	TIME	AVERAGE WEIGHT (POUNDS)	SMALLEST	LARGEST
6:00 A.M.	49.6	48.7	50.7	6:00 P.M.	46.8	41.0	51.2
7:00	50.2	49.1	51.2	7:00	50.0	46.2	51.7
8:00	50.6	49.6	51.4	8:00	47.4	44.0	48.7
9:00	50.8	50.2	51.8	9:00	47.0	44.2	48.9
10:00	49.9	49.2	52.3	10:00	47.2	46.6	50.2
11:00	50.3	48.6	51.7	11:00	48.6	47.0	50.0
12 noon	48.6	46.2	50.4	12 midnight	49.8	48.2	50.4
1:00 P.M.	49.0	46.4	50.0	1:00 A.M.	49.6	48.4	51.7
2:00	49.0	46.0	50.6	2:00	50.0	49.0	52.2
3:00	49.8	48.2	50.8	3:00	50.0	49.2	50.0
4:00	50.3	49.2	52.7	4:00	47.2	46.3	50.5
5:00	51.4	50.0	55.3	5:00	47.0	44.1	49.7
6:00	51.6	49.2	54.7	6:00	48.4	45.0	49.0
7:00	51.8	50.0	55.6	7:00	48.8	44.8	49.7
8:00	51.0	48.6	53.2	8:00	49.6	48.0	51.8
9:00	50.5	49.4	52.4	9:00	50.0	48.1	52.7
10:00	49.2	46.1	50.7	10:00	51.0	48.1	55.2
11:00	49.0	46.3	50.8	11:00	50.4	49.5	54.1
12 midnight	48.4	45.4	50.2	12 noon	50.0	48.7	50.9
1:00 A.M.	47.6	44.3	49.7	1:00 P.M.	48.9	47.6	51.2
2:00	47.4	44.1	49.6	2:00	49.8	48.4	51.0
3:00	48.2	45.2	49.0	3:00	49.8	48.8	50.8
4:00	48.0	45.5	49.1	4:00	50.0	49.1	50.6
5:00	48.4	47.1	49.6	5:00	47.8	45.2	51.2

(cont'd)

		RANGE				*RANGE*	
TIME	AVERAGE WEIGHT (POUNDS)	SMALLEST	LARGEST	TIME	AVERAGE WEIGHT (POUNDS)	SMALLEST	LARGEST
6:00 A.M.	48.6	47.4	52.0	6:00 P.M.	46.4	44.0	49.7
7:00	50.0	49.2	52.2	7:00	46.4	44.4	50.0
8:00	49.8	49.0	52.4	8:00	47.2	46.6	48.9
9:00	50.3	49.4	51.7	9:00	48.4	47.2	49.5
10:00	50.2	49.6	51.8	10:00	49.2	48.1	50.7
11:00	50.0	49.0	52.3	11:00	48.4	47.0	50.8
12 noon	50.0	48.8	52.4	12 midnight	47.2	46.4	49.2
1:00 P.M.	50.1	49.4	53.6	1:00 A.M.	47.4	46.8	49.0
2:00	49.7	48.6	51.0	2:00	48.8	47.2	51.4
3:00	48.4	47.2	51.7	3:00	49.6	49.0	50.6
4:00	47.2	45.3	50.9	4:00	51.0	50.5	51.5
5:00	46.8	44.1	49.0	5:00	50.5	50.0	51.9

The additional night-shift bagging crew was staffed entirely by new employees. The most experienced foremen were temporarily assigned to supervise the night-shift employees. Most emphasis was placed on increasing the output of bags to meet ever-increasing demand. It was suspected that only occasional reminders were made to double-check the bag weight-feeder. (A double-check is performed by systematically weighing a bag on a scale to determine if the proper weight is being loaded by the weight-feeder. If there is significant deviation from 50 pounds, corrective adjustments are made to the weight-release mechanism.)

To verify this expectation, the quality control staff randomly sampled the bag output and prepared the chart on the previous page. Six bags were sampled and weighed each hour.

Discussion Questions

1. What control charts should be used to determine whether the process is in control or out of control?
2. Develop a control chart for each shift. Does there appear to be a difference between the shifts?
3. Cecil Rice Export is planning on providing incentives for senior personnel to rotate into other shifts to better mix personnel among the shifts. If the same control charts were used for all three shifts, what would be the control limits of these charts?

Source: Dr. Ian M. Langella, Shippensburg University, USA.

Frito-Lay's Quality-Controlled Potato Chips

Video Case

Frito-Lay, the multi-billion-dollar snack food giant, produces billions of pounds of product every year at its dozens of U.S. and Canadian plants. From the farming of potatoes—in Florida, North Carolina, and Michigan—to factory and to retail stores, the ingredients and final product of Lay's chips, for example, are inspected at least 11 times: in the field, before unloading at the plant, after washing and peeling, at the sizing station, at the fryer, after seasoning, when bagged (for weight), at carton filling, in the warehouse, and as they are placed on the store shelf by Frito-Lay personnel. Similar inspections take place for its other famous products, including Cheetos, Fritos, Ruffles, and Tostitos.

In addition to these employee inspections, the firm uses proprietary vision systems to look for defective potato chips. Chips are pulled off the high-speed line and checked twice if the vision system senses them to be too brown.

The company follows the very strict standards of the American Institute of Baking (AIB), standards that are much tougher than those of the U.S. Food and Drug Administration. Two unannounced AIB site visits per year keep Frito-Lay's plants on their toes. Scores, consistently in the "excellent" range, are posted, and every employee knows exactly how the plant is doing.

There are two key metrics in Frito-Lay's continuous improvement quality program: (1) total customer complaints (measured on a complaints per million bag basis) and (2) hourly or daily statistical process control scores (for oil, moisture, seasoning, and salt content, for chip thickness, for fryer temperature, and for weight).

In the Florida plant, Angela McCormack, who holds engineering and MBA degrees, oversees a 15-member quality assurance staff. They watch all aspects of quality, including training employees on the factory floor, monitoring automated processing equipment, and developing and updating statistical process control (SPC) charts. The upper and lower control limits for one checkpoint, salt content in Lay's chips, are 2.22% and 1.98%, respectively. To see exactly how these limits are created using SPC, watch the video that accompanies this case.

Discussion Questions*

1. Angela is now going to evaluate a new salt process delivery system and wants to know if the upper and lower control limits at 3 standard deviations for the new system will meet the upper and lower control specifications noted earlier.

   The data (in percents) from the initial trial samples are:

   Sample 1: 1.98, 2.11, 2.15, 2.06
   Sample 2: 1.99, 2.0, 2.08, 1.99
   Sample 3: 2.20, 2.10. 2.20, 2.05
   Sample 4: 2.18, 2.01, 2.23, 1.98
   Sample 5: 2.01, 2.08, 2.14, 2.16

   Provide the report to Angela.
2. What are the advantages and disadvantages of Frito-Lay drivers stocking their customers' shelves?
3. Why is quality a critical function at Frito-Lay?

*You may wish to view the video that accompanies this case before answering these questions.

Farm to Fork: Quality at Darden Restaurants

Video Case

Darden Restaurants, the $6.3 billion owner of such popular brands as Olive Garden, Seasons 52, and Bahama Breeze, serves more than 320 million meals annually in its 1,500 restaurants across the U.S. and Canada. Before any one of these meals is placed before a guest, the ingredients for each recipe must pass quality control inspections at the source, ranging from measurement and weighing to tasting, touching, or lab testing. Darden has differentiated itself from its restaurant peers by developing the gold standard in continuous improvement.

To assure both customers and the company that quality expectations are met, Darden uses a rigorous inspection process, employing statistical process control (SPC) as part of its "Farm to Fork" program. More than 50 food scientists, microbiologists, and public health professionals report to Ana Hooper, vice president of quality assurance.

As part of Darden's Point Source program, Hooper's team, based in Southeast Asia (in China, Thailand, and Singapore) and Latin America (in Equador, Honduras, and Chile), approves and inspects—and works with Darden buyers to purchase—more than 50 million pounds of seafood each year for restaurant use. Darden used to build quality in at the end by inspecting shipments as they reached U.S. distribution centers. Now, thanks to coaching and partnering with vendors abroad, Darden needs but a few domestic inspection labs to verify compliance to its exacting standards. Food vendors in source countries know that when supplying Darden, they are subject to regular audits that are stricter than U.S. Food and Drug Administration (FDA) standards.

Two Quality Success Stories

Quality specialists' jobs include raising the bar and improving quality and safety at all plants in their geographic area. The Thai quality representative, for example, worked closely with several of Darden's largest shrimp vendors to convert them to a production-line-integrated quality assurance program. The vendors were able to improve the quality of shrimp supplied and reduce the percentage of defects by 19%.

Likewise, when the Darden quality teams visited fields of growers/shippers in Mexico recently, it identified challenges such as low employee hygiene standards, field food safety problems, lack of portable toilets, child labor, and poor working conditions. Darden addressed these concerns and hired third-party independent food safety verification firms to ensure continued compliance to standards.

SPC Charts

SPC charts, such as the one shown on page 253 in this supplement, are particularly important. These charts document precooked food weights; meat, seafood and poultry temperatures; blemishes on produce; and bacteria counts on shrimp—just to name a few. Quality assurance is part of a much bigger process that is key to Darden's success—its supply chain (see Chapters 2 and 11 for discussion and case studies on this topic). That's because quality comes from the source and flows through distribution to the restaurant and guests.

Discussion Questions*

1. How does Darden build quality into the supply chain?
2. Select two potential problems—one in the Darden supply chain and one in a restaurant—that can be analyzed with a fish-bone chart. Draw a complete chart to deal with each problem.
3. Darden applies SPC in many product attributes. Identify where these are probably used.
4. The SPC chart on page 253 illustrates Darden's use of control charts to monitor the weight of salmon filets. Given these data, what conclusion do you, as a Darden quality control inspector, draw? What report do you issue to your supervisor? How do you respond to the salmon vendor?

*You might want to view the video that accompanies this case before answering these questions.

- **Additional Case Study:** Visit MyOMLab for this free case study:
  Green River Chemical Company: Involves a company that needs to set up a control chart to monitor sulfate content because of customer complaints.

Endnotes

1. Removing assignable causes is work. Quality expert W. Edwards Deming observed that a state of statistical control is not a natural state for a manufacturing process. Deming instead viewed it as an achievement, arrived at by elimination, one by one, by determined effort, of special causes of excessive variation.
2. The standard deviation is easily calculated as

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n - 1}}$$

. For a good review of this and other statistical terms, refer to Tutorial 1, "Statistical Review for Managers," in MyOMLab.
3. Lower control limits cannot take negative values in control charts. So the LCL = max $(0, \bar{\bar{x}} - z\sigma_{\bar{x}})$.
4. If the sample sizes are not the same, other techniques must be used.
5. A Poisson probability distribution is a discrete distribution commonly used when the items of interest (in this case, defects) are infrequent or occur in time or space.
6. This is because a C_p of 1.0 has 99.73% of outputs within specifications. So 1.00 − .9973 = .0027; with 1,000 parts, there are .0027 = 1,000 = 2.7 defects.
   For a C_p of 2.0, 99.99966% of outputs are "within spec." So 1.00 − .9999966 = .0000034; with 1 million parts, there are 3.4 defects.
7. Refer to Tutorial 2 in MyOMLab for an extended discussion of acceptance sampling.
8. Note that sampling always runs the danger of leading to an erroneous conclusion. Let us say that in one company the total population under scrutiny is a load of 1,000 computer chips, of which in reality only 30 (or 3%) are defective. This means that we would want to accept the shipment of chips, because for this particular firm 4% is the allowable defect rate. However, if a random sample of $n = 50$ chips was drawn, we could conceivably end up with 0 defects and accept that shipment (that is, it is okay), or we could find all 30 defects in the sample. If the latter happened, we could wrongly conclude that the whole population was 60% defective and reject them all.

Supplement 6 *Rapid* Review

Main Heading	Review Material	MyOMLab

STATISTICAL PROCESS CONTROL (SPC)

(pp. 246–260)

- **Statistical process control (SPC)**—A process used to monitor standards by taking measurements and corrective action as a product or service is being produced.
- **Control chart**—A graphical presentation of process data over time.

A process is said to be operating *in statistical control* when the only source of variation is common (natural) causes. The process must first be brought into statistical control by detecting and eliminating special (assignable) causes of variation.

The objective of a process control system is to provide a statistical signal when assignable causes of variation are present.

- **Natural variations**—The variability that affects every production process to some degree and is to be expected; also known as common cause.

When natural variations form a *normal distribution,* they are characterized by two parameters:

- Mean, μ (the measure of central tendency—in this case, the average value)
- Standard deviation, σ (the measure of dispersion)

As long as the distribution (output measurements) remains within specified limits, the process is said to be "in control," and natural variations are tolerated.

- **Assignable variation**—Variation in a production process that can be traced to specific causes.

Control charts for the mean, $\bar{x}$, and the range, R, are used to monitor *variables* (outputs with continuous dimensions), such as weight, speed, length, or strength.

- **$\bar{x}$-chart**—A quality control chart for variables that indicates when changes occur in the central tendency of a production process.
- ***R*-chart**—A control chart that tracks the range within a sample; it indicates that a gain or loss in uniformity has occurred in dispersion of a production process.
- **Central limit theorem**—The theoretical foundation for $\bar{x}$-charts, which states that regardless of the distribution of the population of all parts or services, the $\bar{x}$ distribution will tend to follow a normal curve as the number of samples increases:

$$\bar{\bar{x}} = \mu \qquad \text{(S6-1)}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \qquad \text{(S6-2)}$$

The $\bar{x}$-chart limits, if we know the true standard deviation σ of the process population, are:

$$\text{Upper control limit (UCL)} = \bar{\bar{x}} + z\sigma_{\bar{x}} \qquad \text{(S6-3)}$$

$$\text{Lower control limit (LCL)} = \bar{\bar{x}} - z\sigma_{\bar{x}} \qquad \text{(S6-4)}$$

where z = confidence level selected (e.g., $z = 3$ is 99.73% confidence).

The *range*, R, of a sample is defined as the difference between the largest and smallest items. If we do not know the true standard deviation, σ, of the population, the $\bar{x}$-chart limits are:

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} \qquad \text{(S6-5)}$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} \qquad \text{(S6-6)}$$

In addition to being concerned with the process average, operations managers are interested in the process dispersion, or range. The R-chart control limits for the range of a process are:

$$\text{UCL}_R = D_4\bar{R} \qquad \text{(S6-7)}$$

$$\text{LCL}_R = D_3\bar{R} \qquad \text{(S6-8)}$$

Attributes are typically classified as *defective* or *nondefective*. The two attribute charts are (1) p-charts (which measure the *percent* defective in a sample), and (2) c-charts (which *count* the number of defects in a sample).

- ***p*-chart**—A quality control chart that is used to control attributes:

$$\text{UCL}_p = \bar{p} + z\sigma_p \qquad \text{(S6-9)}$$

$$\text{LCL}_p = \bar{p} - z\sigma_p \qquad \text{(S6-10)}$$

$$\hat{\sigma}_p = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} \qquad \text{(S6-11)}$$

- ***c*-chart**—A quality control chart used to control the number of defects per unit of output. The Poisson distribution is the basis for c-charts, whose 99.73% limits are computed as:

$$\text{Control limits} = \bar{c} \pm 3\sqrt{\bar{c}} \qquad \text{(S6-12)}$$

- **Run test**—A test used to examine the points in a control chart to determine whether nonrandom variation is present.

MyOMLab

Concept Questions: 1.1–1.4

Problems: S6.1–S6.39

VIDEO S6.1
Farm to Fork: Quality at Darden Restaurants

Virtual Office Hours for Solved Problems: S6.1–S6.3

ACTIVE MODELS S6.1 and S6.2

VIDEO S6.2
Frito-Lay's Quality-Controlled Potato Chips

Virtual Office Hours for Solved Problem: S6.5

Supplement 6 *Rapid Review* continued

Main Heading	Review Material	MyOMLab
PROCESS CAPABILITY (pp. 260–262)	■ **Process capability**—The ability to meet design specifications. ■ $\mathbf{C_p}$—A ratio for determining whether a process meets design specifications. $$C_p = \frac{(\text{Upper specification} - \text{Lower specification})}{6\sigma} \quad \text{(S6-13)}$$ ■ $\mathbf{C_{pk}}$—A proportion of variation (3σ) between the center of the process and the nearest specification limit: $$C_{pk} = \text{Minimum of}\left[\frac{\text{Upper spec limit} - \overline{X}}{3\sigma}, \frac{\overline{X} - \text{Lower spec limit}}{3\sigma}\right] \quad \text{(S6-14)}$$	Concept Questions: 2.1–2.4 Problems: S6.40-S6.50 Virtual Office Hours for Solved Problems: S6.4 ACTIVE MODEL S6.3
ACCEPTANCE SAMPLING (pp. 262–265)	■ **Acceptance sampling**—A method of measuring random samples of lots or batches of products against predetermined standards. ■ **Operating characteristic (OC) curve**—A graph that describes how well an acceptance plan discriminates between good and bad lots. ■ **Producer's risk**—The mistake of having a producer's good lot rejected through sampling. ■ **Consumer's risk**—The mistake of a customer's acceptance of a bad lot overlooked through sampling. ■ **Acceptable quality level (AQL)**—The quality level of a lot considered good. ■ **Lot tolerance percent defective (LTPD)**—The quality level of a lot considered bad. ■ **Type I error**—Statistically, the probability of rejecting a good lot. ■ **Type II error**—Statistically, the probability of accepting a bad lot. ■ **Average outgoing quality (AOQ)**—The percent defective in an average lot of goods inspected through acceptance sampling: $$\text{AOQ} = \frac{(P_d)(P_a)(N - n)}{N} \quad \text{(S6-15)}$$	Concept Questions: 3.1–3.4 Problems: S6.51–S6.55

Self Test

■ **Before taking the self-test,** refer to the learning objectives listed at the beginning of the supplement and the key terms listed at the end of the supplement.

LO S6.1 If the mean of a particular sample is within control limits and the range of that sample is not within control limits:
- **a)** the process is in control, with only assignable causes of variation.
- **b)** the process is not producing within the established control limits.
- **c)** the process is producing within the established control limits, with only natural causes of variation.
- **d)** the process has both natural and assignable causes of variation.

LO S6.2 The central limit theorem:
- **a)** is the theoretical foundation of the *c*-chart.
- **b)** states that the average of assignable variations is zero.
- **c)** allows managers to use the normal distribution as the basis for building some control charts.
- **d)** states that the average range can be used as a proxy for the standard deviation.
- **e)** controls the steepness of an operating characteristic curve.

LO S6.3 The type of chart used to control the central tendency of variables with continuous dimensions is:
- **a)** $\bar{x}$-chart.
- **b)** *R*-chart.
- **c)** *p*-chart.
- **d)** *c*-chart.
- **e)** none of the above.

LO S6.4 If parts in a sample are measured and the mean of the sample measurement is outside the control limits:
- **a)** the process is out of control, and the cause should be established.
- **b)** the process is in control but not capable of producing within the established control limits.
- **c)** the process is within the established control limits, with only natural causes of variation.
- **d)** all of the above are true.

LO S6.5 Control charts for attributes are:
- **a)** *p*-charts.
- **b)** *c*-charts.
- **c)** *R*-charts.
- **d)** $\bar{x}$-charts.
- **e)** both a and b.

LO S6.6 The ability of a process to meet design specifications is called:
- **a)** Taguchi.
- **b)** process capability.
- **c)** capability index.
- **d)** acceptance sampling.
- **e)** average outgoing quality.

LO S6.7 The _______ risk is the probability that a lot will be rejected despite the quality level exceeding or meeting the _______.

Answers: LO S6.1. b; LO S6.2. c; LO S6.3. a; LO S6.4. a; LO S6.5. e; LO S6.6. b; LO S6.7. producer's, AQL