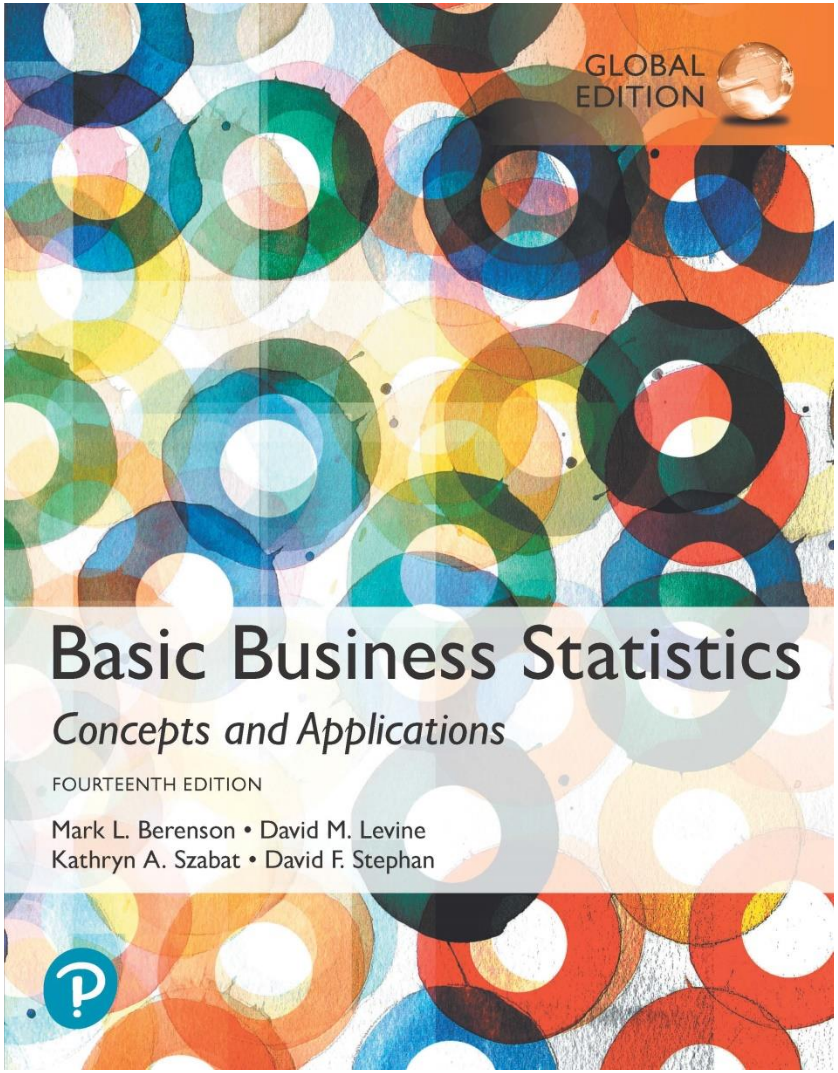


Chapter 10

Two-Sample Tests

Objectives

In this chapter, you learn:

- How to compare the means of two independent populations.
- How to compare the means of two related populations.
- How to compare the proportions of two independent populations.
- How to compare the variances of two independent populations.

Two-Sample Tests

DCOVA

Two-Sample Tests

- Population Means, Independent Samples
- Population Means, Related Samples
- Population Proportions
- Population Variances

Examples:

- Group 1 vs. Group 2
- Same group before vs. after treatment
- Proportion 1 vs. Proportion 2
- Variance 1 vs. Variance 2

Difference Between Two Means

DCOVA

Population means, independent samples *

- σ_1 and σ_2 unknown, assumed equal
- σ_1 and σ_2 unknown, not assumed equal

Goal: Test hypothesis or form a confidence interval for the difference between two population means, $\mu_1 - \mu_2$.

The point estimate for the difference is

$$\bar{X}_1 - \bar{X}_2$$

Difference Between Two Means: Independent Samples

DCOVA

Population means, independent samples *

- Different data sources
  - Unrelated.
  - Independent.
    - Sample selected from one population has no effect on the sample selected from the other population.

σ_1 and σ_2 unknown, assumed equal → Use S_p to estimate unknown σ. Use a **Pooled-Variance t test.**

σ_1 and σ_2 unknown, not assumed equal → Use S_1 and S_2 to estimate unknown σ_1 and σ_2. Use a **Separate-variance t test**

Hypothesis Tests for Two Population Means

DCOVA

Two Population Means, Independent Samples

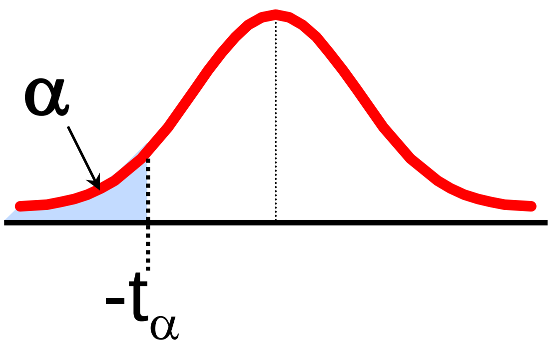
Lower-tail test:	Upper-tail test:	Two-tail test:
$H_0: \mu_1 \geq \mu_2$ $H_1: \mu_1 < \mu_2$	$H_0: \mu_1 \leq \mu_2$ $H_1: \mu_1 > \mu_2$	$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$
i.e.,	i.e.,	i.e.,
$H_0: \mu_1 - \mu_2 \geq 0$ $H_1: \mu_1 - \mu_2 < 0$	$H_0: \mu_1 - \mu_2 \leq 0$ $H_1: \mu_1 - \mu_2 > 0$	$H_0: \mu_1 - \mu_2 = 0$ $H_1: \mu_1 - \mu_2 \neq 0$

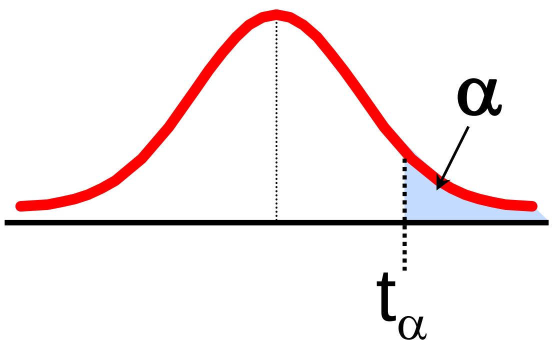
Hypothesis tests for $\mu_1 - \mu_2$

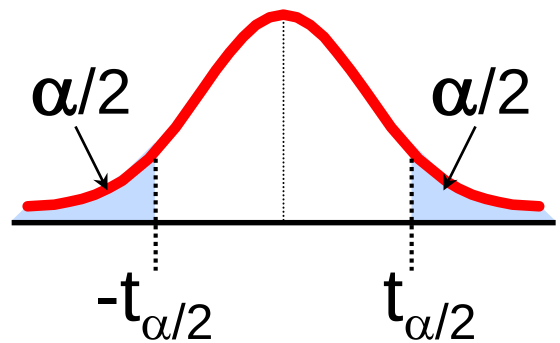
DCOVA

Two Population Means, Independent Samples

Lower-tail test:	Upper-tail test:	Two-tail test:
$H_0: \mu_1 - \mu_2 \geq 0$ $H_1: \mu_1 - \mu_2 < 0$	$H_0: \mu_1 - \mu_2 \leq 0$ $H_1: \mu_1 - \mu_2 > 0$	$H_0: \mu_1 - \mu_2 = 0$ $H_1: \mu_1 - \mu_2 \neq 0$







Reject H_0 if $t_{STAT} < -t_\alpha$

Reject H_0 if $t_{STAT} > t_\alpha$

Reject H_0 if $t_{STAT} < -t_{\alpha/2}$ or $t_{STAT} > t_{\alpha/2}$

Hypothesis tests for μ_1 - μ_2 with σ_1 and σ_2 unknown and assumed equal

DCOVA

Population means, independent samples

σ_1 and σ_2 unknown, assumed equal *

σ_1 and σ_2 unknown, not assumed equal

Assumptions:

- Samples are randomly and independently drawn.
- Populations are normally distributed or both sample sizes are at least 30.
- Population variances are unknown but assumed equal.

Hypothesis tests for μ_1 - μ_2 with σ_1 and σ_2 unknown and assumed equal

(continued)

DCOVA

Population means, independent samples

- σ_1 and σ_2 unknown, assumed equal *
- σ_1 and σ_2 unknown, not assumed equal

- The pooled variance is:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

- The test statistic is:

$$t_{STAT} = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

- Where t_{STAT} has d.f. = $(n_1 + n_2 - 2)$.

Confidence interval for μ_1 - μ_2 with σ_1 and σ_2 unknown and assumed equal

DCOVA

Population means, independent samples

- σ_1 and σ_2 unknown, assumed equal *
- σ_1 and σ_2 unknown, not assumed equal

The confidence interval for $\mu_1 - \mu_2$ is:

$$\left(\bar{X}_1 - \bar{X}_2\right) \pm t_{\alpha/2} \sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Where $t_{\alpha/2}$ has d.f. = $n_1 + n_2 - 2$.

Pooled-Variance t Test Example

DCOVA

You are a financial analyst for a brokerage firm. Is there a difference in dividend yield between stocks listed on the NYSE & NASDAQ? You collect the following data:

	NYSE	NASDAQ
Number	**21**	**25**
Sample mean	**3.27**	**2.53**
Sample std dev	**1.30**	**1.16**

Assuming both populations are approximately normal with equal variances, is there a difference in mean dividend yield ($\alpha = 0.05$)?

Pooled-Variance t Test Example: Calculating the Test Statistic

(continued)

H0: $\mu_1 - \mu_2 = 0$ i.e. ($\mu_1 = \mu_2$)
H1: $\mu_1 - \mu_2 \neq 0$ i.e. ($\mu_1 \neq \mu_2$)

DCOVA

The test statistic is:

$$t = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{\left(3.27 - 2.53\right) - 0}{\sqrt{1.5021\left(\frac{1}{21} + \frac{1}{25}\right)}} = \boxed{2.040}$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(21 - 1)1.30^2 + (25 - 1)1.16^2}{(21 - 1) + (25 - 1)} = 1.5021$$

Pooled-Variance t Test Example: Hypothesis Test Solution

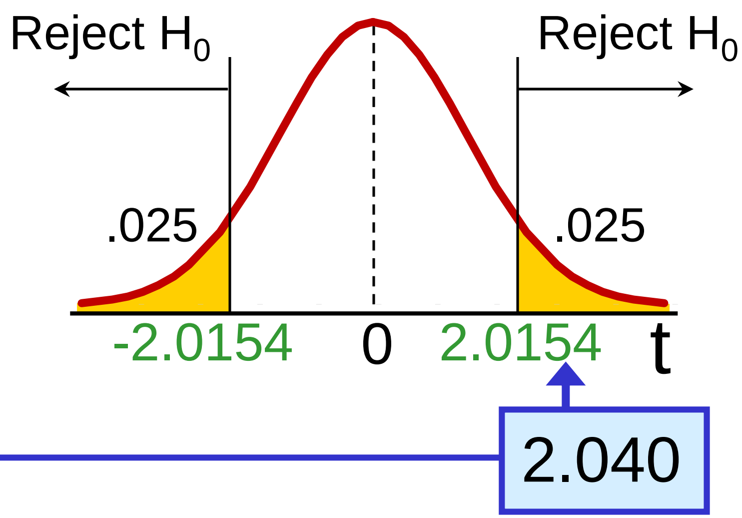
DCOVA

H_0: $\mu_1 - \mu_2 = 0$ i.e. ($\mu_1 = \mu_2$)
H_1: $\mu_1 - \mu_2 \neq 0$ i.e. ($\mu_1 \neq \mu_2$)

α = 0.05

df = 21 + 25 - 2 = 44

Critical Values: t = ± 2.0154



Test Statistic:

$$t = \frac{3.27 - 2.53}{\sqrt{1.5021\left(\frac{1}{21} + \frac{1}{25}\right)}} = \boxed{2.040}$$

Decision:

Reject H_0 at α = 0.05.

Conclusion:

There is evidence of a difference in means.

Pooled-Variance t Test Example: Confidence Interval for $\mu_1 - \mu_2$

DCOVA

Since we rejected H_0 can we be 95% confident that $\mu_{NYSE} > \mu_{NASDAQ}$?

95% Confidence Interval for $\mu_{NYSE} - \mu_{NASDAQ}$:

$$\left(\bar{X}_1 - \bar{X}_2\right) \pm t_{\alpha/2} \sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \mathbf{0.74 \pm 2.0154 \times 0.3628 = (0.009,\ 1.471)}$$

Since 0 is less than the entire interval, we can be 95% confident that $\mu_{NYSE} > \mu_{NASDAQ}$.

Hypothesis tests for $\mu_1 - \mu_2$ with σ_1 and σ_2 unknown, not assumed equal

DCOVA

Population means, independent samples

σ_1 and σ_2 unknown, assumed equal

σ_1 and σ_2 unknown, not assumed equal *

Assumptions:

- Samples are randomly and independently drawn.
- Populations are normally distributed or both sample sizes are at least 30.
- Population variances are unknown and cannot be assumed to be equal.

Hypothesis tests for $\mu_1 - \mu_2$ with σ_1 and σ_2 unknown and not assumed equal

(continued)

DCOVA

Population means, independent samples

σ_1 and σ_2 unknown, assumed equal

σ_1 and σ_2 unknown, not assumed equal *

This test is known at the **separate-variance t test**.

The formulae for this test are not covered in this chapter. See reference 3 for more details.

This test done via software is shown on the next slide.

Case 2: $\sigma_1^2 \neq \sigma_2^2$

In some situations, we cannot reasonably assume that the unknown variances σ_1^2 and σ_2^2 are equal. There is not an exact t-statistic available for testing $H_0 : \mu_1 - \mu_2 = \Delta_0$ in this case. However, an approximate result can be applied.

If $H_0: \mu_1 - \mu_2 = \Delta_0$ is true, the statistic

$$T_0^* = \frac{\overline{X}_1 - \overline{X}_2 - \Delta_0}{\sqrt{\dfrac{S_1^{\,2}}{n_1} + \dfrac{S_2^{\,2}}{n_2}}} \tag{10-15}$$

is distributed approximately as t with degrees of freedom given by

$$v = \frac{\left(\dfrac{s_1^{\,2}}{n_1} + \dfrac{s_2^{\,2}}{n_2}\right)^2}{\dfrac{\left(s_1^{\,2}/n_1\right)^2}{n_1 - 1} + \dfrac{\left(s_2^{\,2}/n_2\right)^2}{n_2 - 1}} \tag{10-16}$$

If v is not an integer, round down to the nearest integer.

Separate-Variance t Test Example

DCOVA

You are a financial analyst for a brokerage firm. Is there a difference in dividend yield between stocks listed on the NYSE & NASDAQ? You collect the following data:

	NYSE	**NASDAQ**
Number	**21**	**25**
Sample mean	**3.27**	**2.53**
Sample std dev	**1.30**	**1.16**

Assuming both populations are approximately normal with unequal variances, is there a difference in mean dividend yield ($\alpha = 0.05$)?

Separate-Variance t Test In Excel, JMP, & Minitab
(continued)

DCOVA

	A	B
1	**Separate-Variances *t* Test**	
2	(assumes unequal population variances)	
3	**Data**	
4	Hypothesized Difference	0
5	Level of Significance	0.05
6	**Population 1 Sample**	
7	Sample Size	21
8	Sample Mean	3.27
9	Sample Standard Deviation	1.3000
10	**Population 2 Sample**	
11	Sample Size	25
12	Sample Mean	2.53
13	Sample Standard Deviation	1.1600
14		
15	Intermediate Calculations	
16	Numerator of Degrees of Freedom	0.0180
17	Denominator of Degrees of Freedom	0.0004
18	Total Degrees of Freedom	40.5744
19	Degrees of Freedom	40
20	Separate Variance Denominator	0.3665
21	Difference in Sample Means	0.7400
22	Separate-Variance *t* Test Statistic	2.0193
23		
24	**Two-Tail Test**	
25	Lower Critical Value	-2.0211
26	Upper Critical Value	2.0211
27	*p*-Value	0.0502
28	Do not reject the null hypothesis	

Test Inputs

Hypothesized Difference Mean 2 - Mean 1)	0
Sample 1 Mean	3.27
Sample 1 Standard Deviation	1.3
Sample 1 Size	21
Sample 2 Mean	2.53
Sample 2 Standard Deviation	1.16
Sample 2 Size	25
Significance Level (alpha)	0.05

Test Results

Result	Value
Observed Difference (Mean 2 - Mean 1)	-0.74
Standard Error of the Difference (Mean 2 - Mean 1)	0.3665
t-score	-2.0193
t critical values	+/- 2.0202
Observed Significance (p-value)	0.0501

Fail to Reject Null Hypothesis

Two-Sample T-Test and CI

```
Sample    N  Mean  StDev  SE Mean
1        21  3.27   1.30     0.28
2        25  2.53   1.16     0.23


Difference = mu (1) - mu (2)
Estimate for difference:  0.740
95% CI for difference:  (-0.001, 1.481)
T-Test of difference = 0 (vs not =): T-Value = 2.02  P-Value = 0.050  DF = 40
```

- Using α=0.05 this test fails to reject the null.
- For this data whether we can assume equal variances or not is important to determine because when we assumed equal variances the null was rejected.
- In Section 10.4 a test to help determine whether this is a reasonable assumption or not is discussed.

Related Populations
The Paired Difference Test

DCOVA

Related samples

Tests Means of 2 Related Populations

- Paired or matched samples.
- Repeated measures (before/after).
- Use difference between paired values:

$$D_i = X_{1i} - X_{2i}$$

- Eliminates Variation Among Subjects.
- Assumptions:
  - Differences are normally distributed.
  - Or, if not Normal, use large samples.

Related Populations The Paired Difference Test

(continued)

DCOVA

Related samples

The i^{th} paired difference is D_i , where

$$D_i = X_{1i} - X_{2i}$$

The point estimate for the paired difference population mean μ_D is $\bar{D}$:

$$\bar{D} = \frac{\sum_{i=1}^{n} D_i}{n}$$

The sample standard deviation is S_D.

$$S_D = \sqrt{\frac{\sum_{i=1}^{n} (D_i - \bar{D})^2}{n-1}}$$

n is the number of pairs in the paired sample

The Paired Difference Test: Finding t_{STAT}

DCOVA

Paired samples

- The test statistic for μ_D is:

$$t_{STAT} = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}}$$

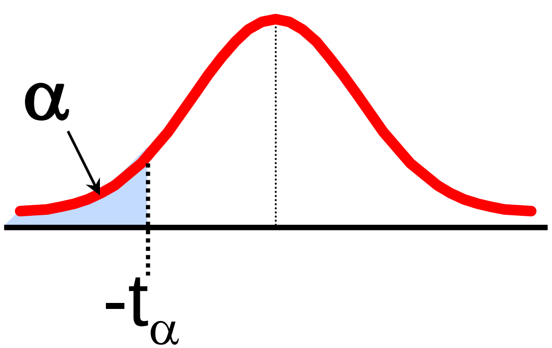
- Where t_{STAT} has $n - 1$ d.f.

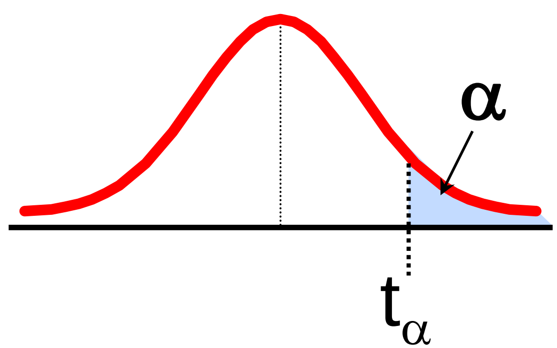
The Paired Difference Test: Possible Hypotheses

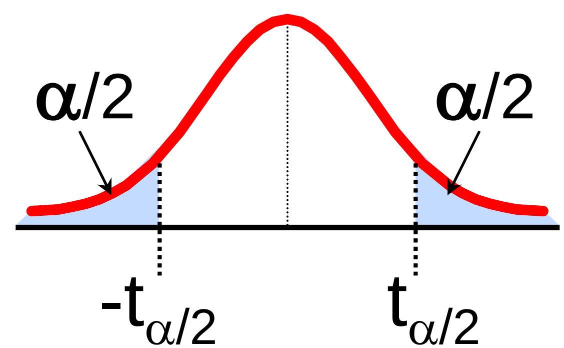
DCOVA

Paired Samples

Lower-tail test:	Upper-tail test:	Two-tail test:
H_0: $\mu_D \geq 0$ H_1: $\mu_D < 0$	H_0: $\mu_D \leq 0$ H_1: $\mu_D > 0$	H_0: $\mu_D = 0$ H_1: $\mu_D \neq 0$







Reject H_0 if $t_{STAT} < -t_{\alpha}$

Reject H_0 if $t_{STAT} > t_{\alpha}$

Reject H_0 if $t_{STAT} < -t_{\alpha/2}$ or $t_{STAT} > t_{\alpha/2}$

Where t_{STAT} has $n - 1$ d.f.

The Paired Difference Confidence Interval

DCOVA

Paired samples

The confidence interval for μ_D is:

$$\bar{D} \pm t_{\alpha/2} \frac{S_D}{\sqrt{n}}$$

where $$S_D = \sqrt{\frac{\sum_{i=1}^{n}(D_i - \bar{D})^2}{n-1}}$$

Paired Difference Test: Example

DCOVA

- Assume you send your salespeople to a "customer service" training workshop. Has the training made a difference in the number of customer complaints? You collect the following data:

Salesperson	Number of Complaints: Before (1)	After (2)	(2) - (1) Difference, D_i
C.B.	6	4	- 2
T.F.	20	6	-14
M.H.	3	2	- 1
R.K.	0	0	0
M.O.	4	0	- 4
			-21

$$\bar{D} = \frac{\sum D_i}{n}$$

$$= -4.2$$

$$S_D = \sqrt{\frac{\sum (D_i - \bar{D})^2}{n-1}}$$

$$= 5.67$$

Paired Difference Test: Solution

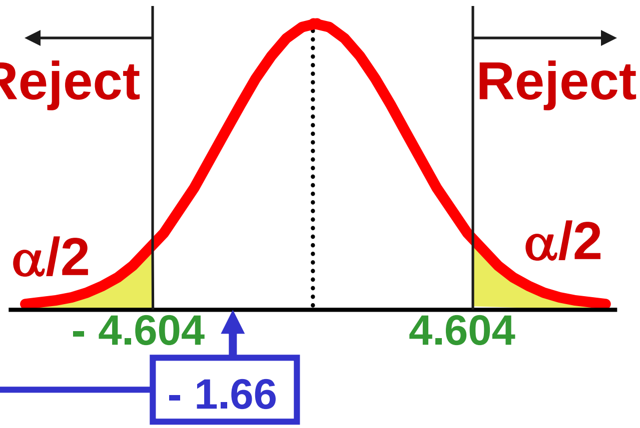
DCOVA

- Has the training made a difference in the number of complaints (at the 0.01 level)?

H_0: $\mu_D = 0$
H_1: $\mu_D \neq 0$

$\alpha = .01 \qquad \bar{D} = -\,4.2$

$t_{0.005} = \pm\, 4.604$
d.f. = n - 1 = 4



Test Statistic:

$$t_{STAT} = \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}} = \frac{-4.2 - 0}{5.67/\sqrt{5}} = \boxed{-1.66}$$

Decision: Do not reject H_0 (t_{stat} is not in the rejection region).

Conclusion: There is insufficient evidence of a change in the number of complaints.

The Paired Difference Confidence Interval -- Example

DCOVA

The confidence interval for μ_D is:

$$\bar{D} \pm t_{\alpha/2} \frac{S_D}{\sqrt{n}}$$

$\bar{D}$ **= -4.2, S_D = 5.67**

$$\mathbf{99\%\ CI\ for\ } \mu_D : -4.2 \pm 4.604 \frac{5.67}{\sqrt{5}}$$

$$= (-15.87, 7.47)$$

The probability this interval contains the true value of μ_D is 99%.

Two Population Proportions

DCOVA

Population proportions

Goal: test a hypothesis or form a confidence interval for the difference between two population proportions,

$$\pi_1 - \pi_2$$

Assumptions:

$n_1 \pi_1 \geq 5$, $n_1(1 - \pi_1) \geq 5$

$n_2 \pi_2 \geq 5$, $n_2(1 - \pi_2) \geq 5$

The point estimate for the difference is $p_1 - p_2$

Two Population Proportions

DCOVA

Population proportions

In the null hypothesis we assume the null hypothesis is true, so we assume $\pi_1 = \pi_2$ and pool the two sample estimates.

The pooled estimate for the overall proportion is:

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2}$$

where X_1 and X_2 are the number of items of interest in samples 1 and 2.

Two Population Proportions

(continued)

DCOVA

Population proportions

The test statistic for

$\pi_1 - \pi_2$ is a Z statistic:

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\bar{p} = \dfrac{X_1 + X_2}{n_1 + n_2}$, $p_1 = \dfrac{X_1}{n_1}$, $p_2 = \dfrac{X_2}{n_2}$

Hypothesis Tests for Two Population Proportions

DCOVA

Population proportions

Lower-tail test:

$H_0: \pi_1 \geq \pi_2$
$H_1: \pi_1 < \pi_2$

i.e.,

$H_0: \pi_1 - \pi_2 \geq 0$
$H_1: \pi_1 - \pi_2 < 0$

Upper-tail test:

$H_0: \pi_1 \leq \pi_2$
$H_1: \pi_1 > \pi_2$

i.e.,

$H_0: \pi_1 - \pi_2 \leq 0$
$H_1: \pi_1 - \pi_2 > 0$

Two-tail test:

$H_0: \pi_1 = \pi_2$
$H_1: \pi_1 \neq \pi_2$

i.e.,

$H_0: \pi_1 - \pi_2 = 0$
$H_1: \pi_1 - \pi_2 \neq 0$

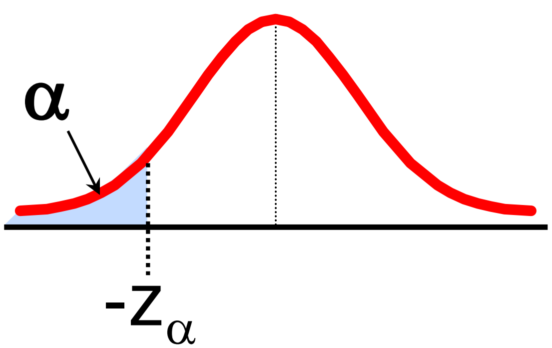
Hypothesis Tests for Two Population Proportions

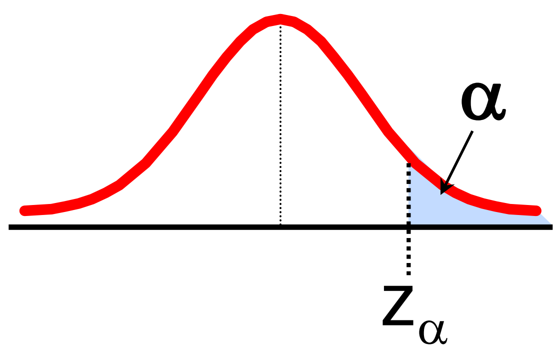
(continued)

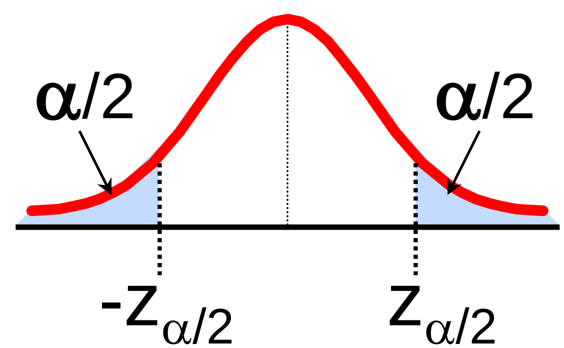
DCOVA

Population proportions

Lower-tail test:	Upper-tail test:	Two-tail test:
$H_0: \pi_1 - \pi_2 \geq 0$ $H_1: \pi_1 - \pi_2 < 0$	$H_0: \pi_1 - \pi_2 \leq 0$ $H_1: \pi_1 - \pi_2 > 0$	$H_0: \pi_1 - \pi_2 = 0$ $H_1: \pi_1 - \pi_2 \neq 0$







Reject H_0 if $Z_{STAT} < -Z_{\alpha}$

Reject H_0 if $Z_{STAT} > Z_{\alpha}$

Reject H_0 if $Z_{STAT} < -Z_{\alpha/2}$ or $Z_{STAT} > Z_{\alpha/2}$

Hypothesis Test Example: Two Population Proportions

DCOVA

Is there a significant difference between the proportion of men and the proportion of women who will vote Yes on Proposition A?

- In a random sample, 36 of 72 men and 35 of 50 women indicated they would vote Yes.
- Test at the .05 level of significance.

Hypothesis Test Example: Two Population Proportions

(continued)

DCOVA

- The hypothesis test is:

H_0: $\pi_1 - \pi_2 = 0$ (the two proportions are equal).
H_1: $\pi_1 - \pi_2 \neq 0$ (there is a significant difference between proportions).

- The sample proportions are:
  - Men: $p_1 = 36/72 = 0.50$
  - Women: $p_2 = 35/50 = 0.70$

- The pooled estimate for the overall proportion is:

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{36 + 35}{72 + 50} = \frac{71}{122} = .582$$

Hypothesis Test Example: Two Population Proportions

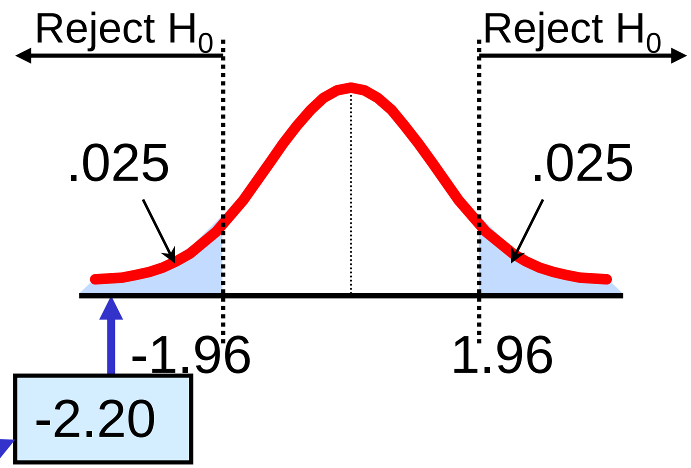
(continued)

DCOVA

The test statistic for $\pi_1 - \pi_2$ is:

$$z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{(.50 - .70) - (0)}{\sqrt{.582(1-.582)\left(\frac{1}{72} + \frac{1}{50}\right)}} = -2.20$$



Decision: Reject H_0.

Conclusion: There is evidence of a significant difference in the proportion of men and women who will vote yes.

Critical Values = ±1.96
For α = .05

Confidence Interval for Two Population Proportions

DCOVA

Population proportions

The confidence interval for $\pi_1 - \pi_2$ is:

$$\left(p_1 - p_2\right) \pm Z_{\alpha/2} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

Confidence Interval for Two Population Proportions -- Example

DCOVA

The 95% confidence interval for $\pi_1 - \pi_2$ is:

$$\left(0.50 - 0.70\right) \pm 1.96\sqrt{\frac{0.50(0.50)}{72} + \frac{0.70(0.30)}{50}}$$

$$= (-0.37, -0.03)$$

Since this interval does not contain 0 can be 95% confident the two proportions are different.

Testing for the Ratio Of Two Population Variances

DCOVA

Tests for Two Population Variances *

F test statistic

Hypotheses	F_{STAT}
$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 \neq \sigma_2^2$	
$H_0: \sigma_1^2 \leq \sigma_2^2$ $H_1: \sigma_1^2 > \sigma_2^2$	S_1^2 / S_2^2

Where:

S_1^2 = Variance of sample 1 (the larger sample variance)

n_1 = sample size of sample 1

S_2^2 = Variance of sample 2 (the smaller sample variance)

n_2 = sample size of sample 2

$n_1 - 1$ = numerator degrees of freedom

$n_2 - 1$ = denominator degrees of freedom

The F Distribution

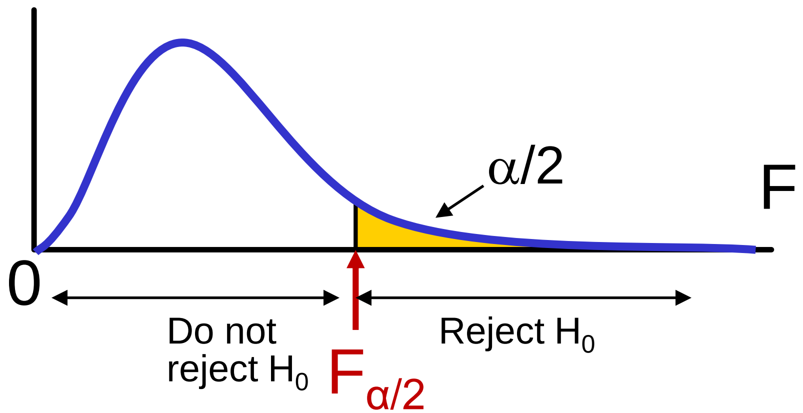
DCOVA

- The F critical value is found from the F table.
- There are two degrees of freedom required: numerator and denominator.
- The larger sample variance is always the numerator.
- When $\boxed{F_{STAT} = \dfrac{S_1^2}{S_2^2}}$ $df_1 = n_1 - 1$; $df_2 = n_2 - 1$.
- In the F table;
  - numerator degrees of freedom determine the column.
  - denominator degrees of freedom determine the row.

Finding the Rejection Region

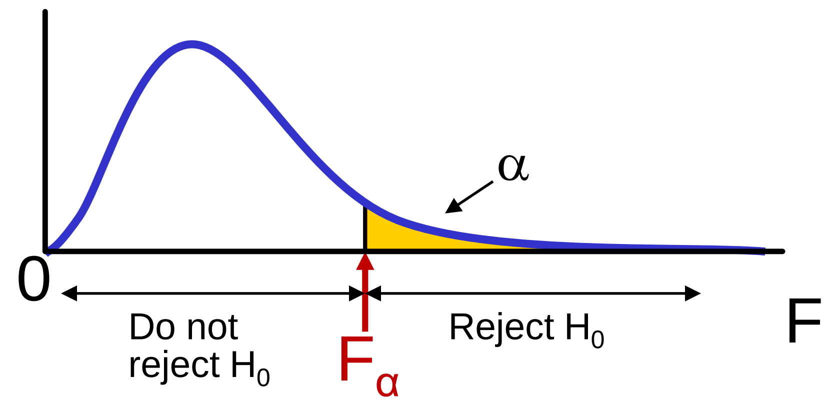
DCOVA

$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \neq \sigma_2^2$



Reject H_0 if $F_{STAT} > F_{\alpha/2}$

$H_0: \sigma_1^2 \leq \sigma_2^2$
$H_1: \sigma_1^2 > \sigma_2^2$



Reject H_0 if $F_{STAT} > F_{\alpha}$

F Test: An Example

DCOVA

You are a financial analyst for a brokerage firm. You want to compare dividend yields between stocks listed on the NYSE & NASDAQ. You collect the following data:

	NYSE	NASDAQ
Number	**21**	**25**
Mean	**3.27**	**2.53**
Std dev	**1.30**	**1.16**

Is there a difference in the variances between the NYSE & NASDAQ at the $\alpha = 0.05$ level?

F Test: Example Solution

DCOVA

- Form the hypothesis test:

  $H_0: \sigma^2_1 = \sigma^2_2$ (there is no difference between variances.)

  $H_1: \sigma^2_1 \neq \sigma^2_2$ (there is a difference between variances.)

- Find the F critical value for $\alpha = 0.05$.

- Numerator d.f. = $n_1 - 1 = 21 - 1 = 20$.

- Denominator d.f. = $n_2 - 1 = 25 - 1 = 24$.

- $F_{\alpha/2} = F_{.025,\ 20,\ 24} = 2.33$.

F Test: Example Solution

DCOVA

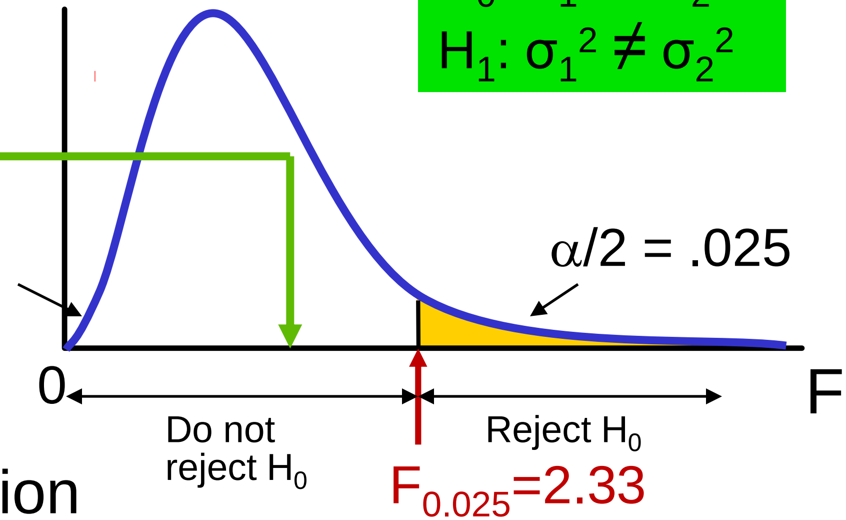
(continued)

- The test statistic is:

$$F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{1.30^2}{1.16^2} = \boxed{1.256}$$

H_0: $\sigma_1^2 = \sigma_2^2$

H_1: $\sigma_1^2 \neq \sigma_2^2$



- F_{STAT} = 1.256 is not in the rejection region, so we do not reject H_0.

- Conclusion: There is not sufficient evidence of a difference in variances at α = .05.

Chapter Summary

In this chapter we discussed:

- Comparing the means of two independent populations.
- Comparing the means of two related populations.
- Comparing the proportions of two independent populations.
- Comparing the variances of two independent populations.