

Chapter 9—Fundamentals of Hypothesis Testing: One-Sample Tests

- 9.1 Decision rule: Reject H_0 if $Z_{STAT} < -1.96$ or $Z_{STAT} > +1.96$.
Decision: Since $Z_{STAT} = -0.76$ is in between the two critical values, do not reject H_0 .
- 9.2 Decision rule: Reject H_0 if $Z_{STAT} < -1.96$ or $Z_{STAT} > +1.96$.
Decision: Since $Z_{STAT} = +2.21$ is greater than the upper critical value of $+1.96$, reject H_0 .
- 9.3 Decision rule: Reject H_0 if $Z_{STAT} < -1.645$ or $Z_{STAT} > +1.645$.
- 9.4 Decision rule: Reject H_0 if $Z_{STAT} < -2.58$ or $Z_{STAT} > +2.58$.
- 9.5 Decision: Since $Z_{STAT} = -2.61$ is less than the lower critical value of -2.58 , reject H_0 .
- 9.6 $p\text{-value} = 2(1 - .9772) = 0.0456$
- 9.7 Since the p -value of 0.0456 is less than the 0.10 level of significance, the statistical decision is to reject the null hypothesis.
- 9.8 $p\text{-value} = 0.1676$
- 9.9 α is the probability of incorrectly convicting the defendant when he is innocent. β is the probability of incorrectly failing to convict the defendant when he is guilty.
- 9.10 Under the French judicial system, unlike ours in the United States, the null hypothesis assumes the defendant is guilty, the alternative hypothesis assumes the defendant is innocent. A Type I error would be not convicting a guilty person and a Type II error would be convicting an innocent person.
- 9.11 (a) A Type I error is the mistake of approving an unsafe drug. A Type II error is not approving a safe drug.
(b) The consumer groups are trying to avoid a Type I error.
(c) The industry lobbyists are trying to avoid a Type II error.
(d) To lower both Type I and Type II errors, the FDA can require more information and evidence in the form of more rigorous testing. This can easily translate into longer time to approve a new drug.
- 9.12 $H_0: \mu = 20$ minutes. 20 minutes is adequate travel time between classes.
 $H_1: \mu \neq 20$ minutes. 20 minutes is not adequate travel time between classes.
- 9.13 (a) $H_0: \mu = 14.6$ hours
 $H_1: \mu \neq 14.6$ hours
(b) A Type I error is the mistake of concluding that the mean number of hours studied at your school is different from the 14.6 hour benchmark reported by *Business Week* when in fact it is not any different.

- 9.13 (c) A Type II error is the mistake of not concluding that the mean number of hours studied at your school is different from the 14.6 hour benchmark reported by *Business Week* when it is in fact different.

- 9.14 (a) PHStat output:

Data	
Null Hypothesis $\mu =$	375
Level of Significance	0.05
Population Standard Deviation	100
Sample Size	64
Sample Mean	350
Intermediate Calculations	
Standard Error of the Mean	12.5
Z Test Statistic	-2
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	0.045500264
Reject the null hypothesis	

$H_0: \mu = 375$. The mean life of a large shipment of light bulbs is equal to 375 hours.

$H_1: \mu \neq 375$. The mean life of a large shipment of light bulbs differs from 375 hours.

Decision rule: Reject H_0 if $|Z_{STAT}| > 1.96$

$$\text{Test statistic: } Z_{STAT} = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{350 - 375}{100 / \sqrt{64}} = -2$$

Decision: Since $|Z_{STAT}| > 1.96$, reject H_0 . There is enough evidence to conclude that the mean life of a large shipment of light bulbs differs from 375 hours.

- (b) p-value = 0.0455. If the population mean life of a large shipment of light bulbs is indeed equal to 375 hours, the probability of obtaining a test statistic that is more than 2 units away from 0 is 0.0455.
- (c) PHStat output:

Data	
Population Standard Deviation	100
Sample Mean	350
Sample Size	64
Confidence Level	95%
Intermediate Calculations	
Standard Error of the Mean	12.5
Z Value	-1.95996398
Interval Half Width	24.49954981
Confidence Interval	
Interval Lower Limit	325.5004502
Interval Upper Limit	374.4995498

$$9.14 \quad (c) \quad \bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 350 \pm 1.96 \frac{100}{\sqrt{64}} \quad 325.5005 \leq \mu \leq 374.4995$$

- cont. (d) You are 95% confident that the population mean life of a large shipment of light bulbs is somewhere between 325.5005 and 374.4995 hours.
 Since the 95% confidence interval does not contain the hypothesized value of 375, you will reject H_0 . The conclusions are the same.

- 9.15 (a) (a) PHStat output:

Z Test of Hypothesis for the Mean	
Data	
Null Hypothesis $\mu =$	375
Level of Significance	0.05
Population Standard Deviation	120
Sample Size	64
Sample Mean	350
Intermediate Calculations	
Standard Error of the Mean	15
Z Test Statistic	-1.66666667
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	0.095580705
Do not reject the null hypothesis	

$H_0: \mu = 375$. The mean life of a large shipment of light bulbs is equal to 375 hours.

$H_1: \mu \neq 375$. The mean life of a large shipment of light bulbs differs from 375 hours.

Decision rule: Reject H_0 if $|Z_{STAT}| > 1.96$

$$\text{Test statistic: } Z_{STAT} = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{350 - 375}{120 / \sqrt{64}} = -1.6667$$

Decision: Since $|Z_{STAT}| < 1.96$, do not reject H_0 . There is not enough evidence to conclude that the mean life of a large shipment of light bulbs differs from 375 hours.

- (b) p -value = 0.0956. If the population mean life of a large shipment of light bulbs is indeed equal to 375 hours, the probability of obtaining a test statistic that is more than 1.6667 units away from 0 is 0.0956.

- 9.15 (a) (c) PHStat output:
cont.

Confidence Interval Estimate for the Mean	
Data	
Population Standard Deviation	120
Sample Mean	350
Sample Size	64
Confidence Level	95%
Intermediate Calculations	
Standard Error of the Mean	15
Z Value	-1.95996398
Interval Half Width	29.39945977
Confidence Interval	
Interval Lower Limit	320.6005402
Interval Upper Limit	379.3994598

- 9.15 (a) (c) $\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 350 \pm 1.96 \frac{120}{\sqrt{64}} \quad 320.6005 \leq \mu \leq 379.3995$
cont. (d) You are 95% confident that the population mean life of a large shipment of light bulbs is somewhere between 320.6005 and 379.3995 hours. Since the 95% confidence interval contains the hypothesized value of 375, you will not reject H_0 . The conclusions are the same.
- (b) The larger population standard deviation results in a larger standard error of the Z test and, hence, larger p -value. Instead of rejecting H_0 as in Problem 9.14, you end up not rejecting H_0 in the problem.

- 9.16 (a) PHStat output:

Data	
Null Hypothesis $\mu =$	1
Level of Significance	0.01
Population Standard Deviation	0.02
Sample Size	50
Sample Mean	0.995
Intermediate Calculations	
Standard Error of the Mean	0.002828427
Z Test Statistic	-1.767766953
Two-Tail Test	
Lower Critical Value	-2.575829304
Upper Critical Value	2.575829304
p-Value	0.077099872
Do not reject the null hypothesis	

H_0 : $\mu = 1$. The mean amount of paint is 1 gallon.

H_1 : $\mu \neq 1$. The mean amount of paint differs from 1 gallon.

Decision rule: Reject H_0 if $|Z_{STAT}| > 2.5758$

9.16 (a) Test statistic: $Z_{STAT} = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{.995 - 1}{.02 / \sqrt{50}} = -1.7678$

cont. Decision: Since $|Z_{STAT}| < 2.5758$, do not reject H_0 . There is not enough evidence to conclude that the mean amount of paint contained in 1-gallon cans purchased from a nationally known manufacturer is different from 1 gallon.

(b) p -value = 0.0771. If the population mean amount of paint contained in 1-gallon cans purchased from a nationally known manufacturer is actually 1 gallon, the probability of obtaining a test statistic that is more than 1.7678 standard error units away from 0 is 0.0771.

(c) PHStat output:

Data	
Population Standard Deviation	0.02
Sample Mean	0.995
Sample Size	50
Confidence Level	99%
Intermediate Calculations	
Standard Error of the Mean	0.002828427
Z Value	-2.5758293
Interval Half Width	0.007285545
Confidence Interval	
Interval Lower Limit	0.987714455
Interval Upper Limit	1.002285545

$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = .995 \pm 2.5758 \frac{.02}{\sqrt{50}} \quad 0.9877 \leq \mu \leq 1.0023$$

You are 99% confident that population mean amount of paint contained in 1-gallon cans purchased from a nationally known manufacturer is somewhere between 0.9877 and 1.0023 gallons.

(d) Since the 99% confidence interval does not contain the hypothesized value of 1, you will reject H_0 . The conclusions are the same.

9.17 (a) (a) PHStat output:

Z Test of Hypothesis for the Mean	
Data	
Null Hypothesis	m= 1
Level of Significance	0.01
Population Standard Deviation	0.012
Sample Size	50
Sample Mean	0.995
Intermediate Calculations	
Standard Error of the Mean	0.001697056
Z Test Statistic	-2.946278255
Two-Tail Test	
Lower Critical Value	-2.575829304
Upper Critical Value	2.575829304
p-Value	0.003216229
Reject the null hypothesis	

- 9.17 (a) (a) $H_0: \mu = 1$. The mean amount of paint is 1 gallon.
 cont. $H_1: \mu \neq 1$. The mean amount of paint differs from 1 gallon.

Decision rule: Reject H_0 if $|Z_{STAT}| > 2.5758$

$$\text{Test statistic: } Z_{STAT} = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{.995 - 1}{.012 / \sqrt{50}} = -2.9463$$

Decision: Since $|Z_{STAT}| > 2.5758$, reject H_0 . There is enough evidence to conclude that the mean amount of paint contained in 1-gallon cans purchased from a nationally known manufacturer is different from 1 gallon.

- (b) $p\text{-value} = 0.0032$. If the population mean amount of paint contained in 1-gallon cans purchased from a nationally known manufacturer is actually 1 gallon, the probability of obtaining a test statistic that is more than 2.9463 standard error units away from 0 is 0.0032.
- (c) PHStat output:

Confidence Interval Estimate for the Mean	
Data	
Population Standard Deviation	0.012
Sample Mean	0.995
Sample Size	50
Confidence Level	99%
Intermediate Calculations	
Standard Error of the Mean	0.001697056
Z Value	-2.5758293
Interval Half Width	0.004371327
Confidence Interval	
Interval Lower Limit	0.990628673
Interval Upper Limit	0.999371327

$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = .995 \pm 2.5758 \frac{.012}{\sqrt{50}} \quad 0.9906 \leq \mu \leq 0.9994$$

You are 99% confident that population mean amount of paint contained in 1-gallon cans purchased from a nationally known manufacturer is somewhere between 0.9906 and 0.9994 gallon.

- (d) Since the 99% confidence interval does contain the hypothesized value of 1, you will not reject H_0 . The conclusions are the same.
- (b) The smaller population standard deviation results in a smaller standard error of the Z test and, hence, smaller $p\text{-value}$. Instead of not rejecting H_0 as in Problem 9.16, you end up rejecting H_0 in this problem.

$$9.18 \quad t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{56 - 50}{12 / \sqrt{16}} = 2.00$$

$$9.19 \quad d.f. = n - 1 = 16 - 1 = 15$$

500 Chapter 9: Fundamentals of Hypothesis Testing: One-Sample Tests

- 9.20 For a two-tailed test with a 0.05 level of confidence, the critical values are ± 2.1315 .
- 9.21 Since $t_{STAT} = 2.00$ is between the critical bounds of ± 2.1315 , do not reject H_0 .
- 9.22 No, you should not use the t test to test the null hypothesis that $\mu = 60$ on a population that is left-skewed because the sample size ($n = 16$) is less than 30. The t test assumes that, if the underlying population is not normally distributed, the sample size is sufficiently large to enable the test to be valid. If sample sizes are small ($n < 30$), the t test should not be used because the sampling distribution does not meet the requirements of the Central Limit Theorem.
- 9.23 Yes, you may use the t test to test the null hypothesis that $\mu = 60$ even though the population is left-skewed because the sample size is sufficiently large ($n = 160$). The t test assumes that, if the underlying population is not normally distributed, the sample size is sufficiently large to enable the test statistic t to be influenced by the Central Limit Theorem.

9.24 PHStat output:

t Test for Hypothesis of the Mean	
Data	
Null Hypothesis $\mu =$	3.7
Level of Significance	0.05
Sample Size	64
Sample Mean	3.57
Sample Standard Deviation	0.8
Intermediate Calculations	
Standard Error of the Mean	0.1
Degrees of Freedom	63
t Test Statistic	-1.3
Two-Tail Test	
Lower Critical Value	-1.9983405
Upper Critical Value	1.9983405
p-Value	0.1983372
Do not reject the null hypothesis	

(a) $H_0 : \mu = 3.7 \quad H_1 : \mu \neq 3.7$

Decision rule: Reject H_0 if $|t_{STAT}| > 1.9983 \quad d.f. = 63$

Test statistic: $t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{3.57 - 3.7}{0.8 / \sqrt{64}} = -1.3$

Decision: Since $|t_{STAT}| < 1.9983$, do not reject H_0 . There is not enough evidence to conclude that the population mean waiting time is different from 3.7 minutes at the 0.05 level of significance.

- (b) The sample size of 64 is large enough to apply the Central Limit Theorem and, hence, you do not need to be concerned about the shape of the population distribution when conducting the t -test in (a). In general, the t test is appropriate for this sample size except for the case where the population is extremely skewed or bimodal.

9.25 (a) $H_0 : \mu = 8.17$ $H_1 : \mu \neq 8.17$

Decision rule: Reject H_0 if $|t_{STAT}| > 2.0096$ or $p\text{-value} < 0.05$ $d.f. = 49$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{8.159 - 8.17}{0.051 / \sqrt{50}} = -1.5251$$

$$p\text{-value} = 0.1337$$

Decision: Since $|t_{STAT}| < 2.0096$ and the $p\text{-value}$ of $0.1337 > 0.05$, do not reject H_0 . There is not enough evidence to conclude that the mean amount is different from 8.17 ounces.

- (b) The $p\text{-value}$ is 0.1337. If the population mean is indeed 8.17 ounces, the probability of obtaining a sample mean that is more than 0.011 ounces away from 8.17 ounces is 0.1337.

9.26 PHStat output:

t Test for Hypothesis of the Mean	
Data	
Null Hypothesis $\mu =$	2.5
Level of Significance	0.05
Sample Size	100
Sample Mean	2.55
Sample Standard Deviation	0.44
Intermediate Calculations	
Standard Error of the Mean	0.044
Degrees of Freedom	99
t Test Statistic	1.136363636
Two-Tail Test	
Lower Critical Value	-1.9842169
Upper Critical Value	1.9842169
p-Value	0.258547677
Do not reject the null hypothesis	

(a) $H_0 : \mu = \$2.5$ $H_1 : \mu \neq \$2.5$

Decision rule: Reject H_0 if $|t_{STAT}| > 1.9842$ or $p\text{-value} < 0.05$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{2.55 - 2.5}{.44 / \sqrt{100}} = 1.1364$$

$$p\text{-value} = 0.2585$$

Decision: Since $|t_{STAT}| < 1.9842$ and the $p\text{-value}$ of $0.2585 > 0.05$, do not reject H_0 . There is not enough evidence to conclude that the population mean retail value of the greeting cards is different from \$2.50.

- (b) The $p\text{-value}$ is 0.2585. If the population mean is indeed \$2.5, the probability of obtaining a test statistic that is more than 1.1364 standard error units away from 0 in either direction is 0.2585.

9.27 PHStat output:

t Test for Hypothesis of the Mean	
Data	
Null Hypothesis $\mu =$	200
Level of Significance	0.05
Sample Size	18
Sample Mean	195.3
Sample Standard Deviation	21.4
Intermediate Calculations	
Standard Error of the Mean	5.044028372
Degrees of Freedom	17
t Test Statistic	-0.931794917
Two-Tail Test	
Lower Critical Value	-2.109815559
Upper Critical Value	2.109815559
p-Value	0.364487846
Do not reject the null hypothesis	

(a) $H_0 : \mu = 200$ $H_1 : \mu \neq 200$

Decision rule: Reject H_0 if $|t_{STAT}| > 2.1098$ or $p\text{-value} < 0.05$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{195.3 - 200}{21.4 / \sqrt{18}} = -0.9318$$

$$p\text{-value} = 0.3645$$

Decision: Since $|t_{STAT}| < 2.1098$ and the $p\text{-value}$ of $0.3645 > 0.05$, do not reject H_0 .

There is not enough evidence to conclude that the population mean tread wear index is different from 200.

- (b) The $p\text{-value}$ is 0.3645. If the population mean is indeed 200, the probability of obtaining a test statistic that is more than 0.9318 standard error units away from 0 in either direction is 0.3645.

9.28 PHStat output:

t Test for Hypothesis of the Mean	
Data	
Null Hypothesis $\mu =$	6.5
Level of Significance	0.05
Sample Size	9
Sample Mean	7.03
Sample Standard Deviation	1.812487548
Intermediate Calculations	
Standard Error of the Mean	0.604162516
Degrees of Freedom	8
t Test Statistic	0.875408313
Two-Tail Test	
Lower Critical Value	-2.306004133
Upper Critical Value	2.306004133
p-Value	0.406866177
Do not reject the null hypothesis	

(a) $H_0 : \mu = \$6.50 \quad H_1 : \mu \neq \6.50

Decision rule: Reject H_0 if $|t_{STAT}| > 2.3060$ or $p\text{-value} < 0.05$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{7.03 - 6.5}{1.8125 / \sqrt{9}} = 0.8754$$

Decision: Since $|t_{STAT}| < 2.3060$, do not reject H_0 . There is not enough evidence to conclude that the mean amount spent for lunch is different from \$6.50.

- (b) The p-value is 0.4069. If the population mean is indeed \$6.50, the probability of obtaining a test statistic that is more than 0.8754 standard error units away from 0 in either direction is 0.4069.
- (c) That the distribution of the amount spent on lunch is normally distributed.
- (d) With a small sample size, it is difficult to evaluate the assumption of normality. However, the distribution may be symmetric since the mean and the median are close in value.

9.29 (a) $H_0 : \mu = 45$ $H_1 : \mu \neq 45$

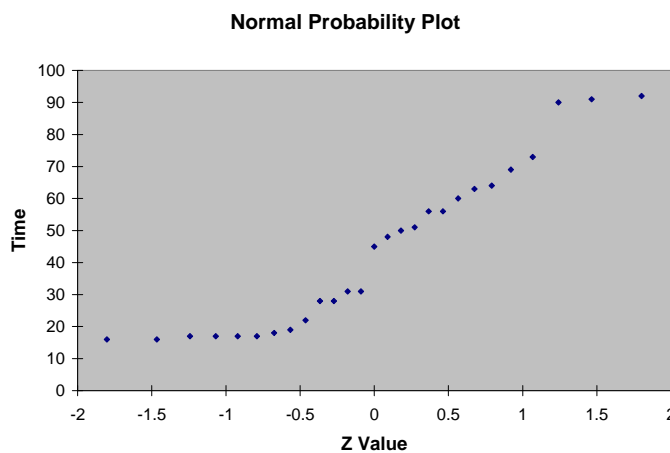
Decision rule: Reject H_0 if $|t_{STAT}| > 2.0555$ $d.f. = 26$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{43.8889 - 45}{25.2835 / \sqrt{27}} = -0.2284$$

Decision: Since $|t_{STAT}| < 2.0555$, do not reject H_0 . There is not enough evidence to conclude that the mean processing time has changed from 45 days.

(b) The population distribution needs to be normal.

(c)



(d) The normal probability plot indicates that the distribution is not normal and is skewed to the right. The assumption in (b) has been violated.

9.30 (a) $H_0 : \mu = 2$ $H_1 : \mu \neq 2$ $d.f. = 49$

Decision rule: Reject H_0 if $|t_{STAT}| > 2.0096$

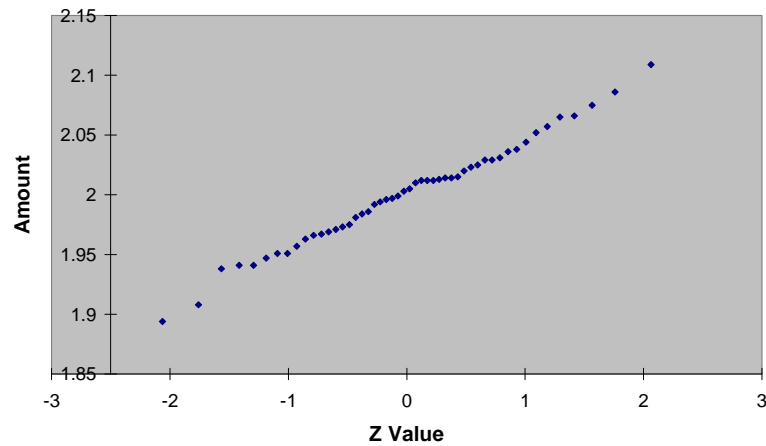
$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{2.0007 - 2}{0.0446 / \sqrt{50}} = 0.1143$$

Decision: Since $|t_{STAT}| < 2.0096$, do not reject H_0 . There is not enough evidence to conclude that the mean amount of soft drink filled is different from 2.0 liters.

(b) $p\text{-value} = 0.9095$. If the population mean amount of soft drink filled is indeed 2.0 liters, the probability of observing a sample of 50 soft drinks that will result in a sample mean amount of fill more different from 2.0 liters is 0.9095.

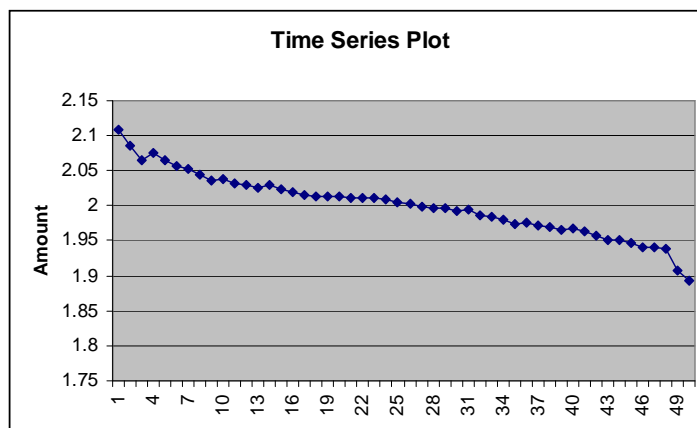
9.30 (c)
cont.

Normal Probability Plot



- (d) The normal probability plot suggests that the data are rather normally distributed. Hence, the results in (a) are valid in terms of the normality assumption.
- (e)

Time Series Plot



The time series plot of the data reveals that there is a downward trend in the amount of soft drink filled. This violates the assumption that data are drawn independently from a normal population distribution because the amount of fill in consecutive bottles appears to be closely related. As a result, the t test in (a) becomes invalid.

9.31 (a) PHStat output:

Data	
Null Hypothesis $\mu =$	20
Level of Significance	0.05
Sample Size	50
Sample Mean	43.04
Sample Standard Deviation	41.92605736
Intermediate Calculations	
Standard Error of the Mean	5.929239893
Degrees of Freedom	49
t Test Statistic	3.885826921
Two-Tail Test	
Lower Critical Value	-2.009575199
Upper Critical Value	2.009575199
p-Value	0.000306263
Reject the null hypothesis	

$$H_0 : \mu = 20 \quad H_1 : \mu \neq 20$$

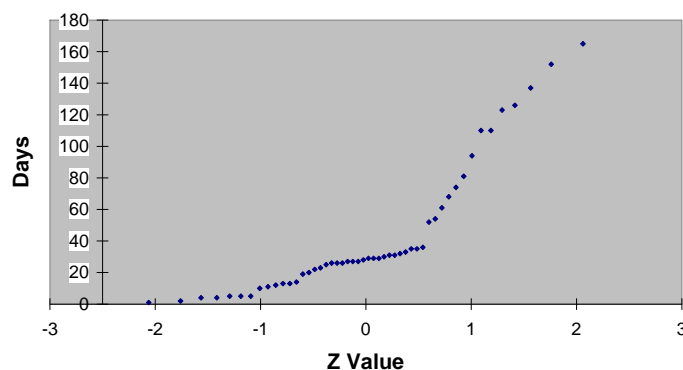
Decision rule: Reject H_0 if $t_{STAT} > 2.0096$ $d.f. = 49$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{43.04 - 20}{41.9261 / \sqrt{50}} = 3.8858$$

Decision: Since $t_{STAT} > 2.0096$, reject H_0 . There is enough evidence to conclude that the mean number of days is different from 20.

- (b) The population distribution needs to be normal.
 (c)

Normal Probability Plot



- (d) The normal probability plot indicates that the distribution is skewed to the right. Even though the population distribution is probably not normally distributed, the result obtained in (a) should still be valid due to the Central Limit Theorem as a result of the relatively large sample size of 50.

9.32 (a) $H_0 : \mu = 8.46$ $H_1 : \mu \neq 8.46$

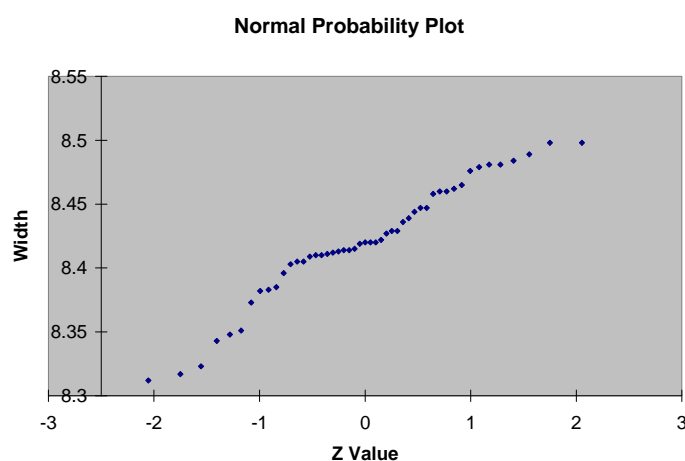
Decision rule: Reject H_0 if $|t_{STAT}| > 2.0106$ $d.f. = 48$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{8.4209 - 8.46}{0.0461 / \sqrt{49}} = -5.9355$$

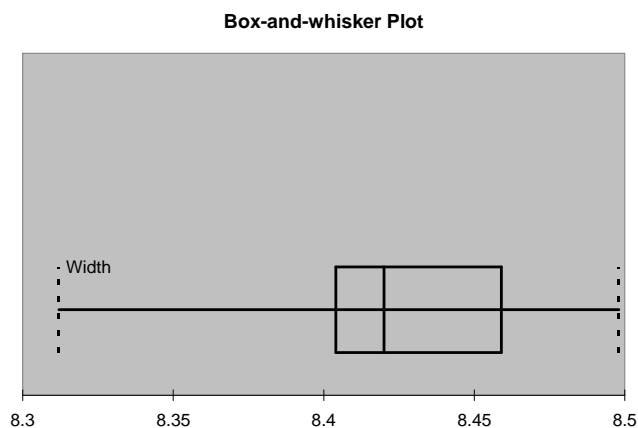
Decision: Since $|t_{STAT}| > 2.0106$, reject H_0 . There is enough evidence to conclude that mean widths of the troughs is different from 8.46 inches.

(b) The population distribution needs to be normal.

(c)



(c)



- (d) The normal probability plot and the boxplot indicate that the distribution is skewed to the left. Even though the population distribution is not normally distributed, the result obtained in (a) should still be valid due to the Central Limit Theorem as a result of the relatively large sample size of 49.

9.33 (a) $H_0 : \mu = 0 \quad H_1 : \mu \neq 0$

Decision rule: Reject H_0 if $|t_{STAT}| > 1.9842$ $d.f. = 99$

Test statistic: $t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{-0.00023}{0.00170 / \sqrt{100}} = -1.3563$

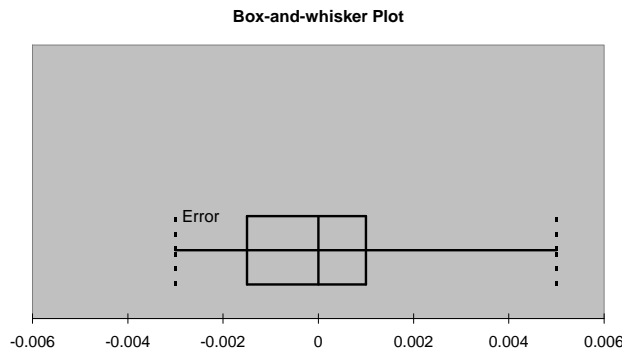
Decision: Since $|t_{STAT}| < 1.9842$, do not reject H_0 . There is not enough evidence to conclude that the mean difference is different from 0.0 inches.

(b) $\bar{X} \pm t \frac{S}{\sqrt{n}} = -0.00023 \pm 1.9842 \left(\frac{0.001696}{\sqrt{100}} \right) \quad -0.0005665 \leq \mu \leq 0.0001065$

You are 95% confident that the mean difference is somewhere between -0.0005665 and 0.0001065 inches.

(c) Since the 95% confidence interval does not contain 0, you do not reject the null hypothesis in part (a). Hence, you will make the same decision and arrive at the same conclusion as in (a).

(d) In order for the t test to be valid, the data are assumed to be independently drawn from a population that is normally distributed. Since the sample size is 100, which is considered quite large, the t distribution will provide a good approximation to the sampling distribution of the mean as long as the population distribution is not very skewed.



The boxplot suggests that the data has a distribution that is skewed slightly to the right. Given the relatively large sample size of 100 observations, the t distribution should still provide a good approximation to the sampling distribution of the mean.

9.34 (a) $H_0 : \mu = 5.5 \quad H_1 : \mu \neq 5.5$

Decision rule: Reject H_0 if $|t_{STAT}| > 2.680$ $d.f. = 49$

Test statistic: $t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{5.5014 - 5.5}{0.1058 / \sqrt{50}} = 0.0935$

Decision: Since $|t_{STAT}| < 2.680$, do not reject H_0 . There is not enough evidence to conclude that the mean amount of tea per bag is different from 5.5 grams.

$$9.34 \quad (b) \quad \bar{X} \pm t \cdot \frac{s}{\sqrt{n}} = 5.5014 \pm 2.6800 \cdot \frac{0.1058}{\sqrt{50}} \quad 5.46 < \mu < 5.54$$

cont. With 99% confidence, you can conclude that the population mean amount of tea per bag is somewhere between 5.46 and 5.54 grams.

(c) The conclusions are the same.

9.35 (a) PHStat output:

Data	
Null Hypothesis $\mu =$	40
Level of Significance	0.05
Sample Size	25
Sample Mean	45.628
Sample Standard Deviation	6.958658875
Intermediate Calculations	
Standard Error of the Mean	1.391731775
Degrees of Freedom	24
t Test Statistic	4.043882666
Two-Tail Test	
Lower Critical Value	-2.063898547
Upper Critical Value	2.063898547
p-Value	0.000471576
Reject the null hypothesis	

$$H_0 : \mu = 40 \quad H_1 : \mu \neq 40$$

Decision rule: Reject H_0 if $|t_{STAT}| > 2.0639$ $d.f. = 24$

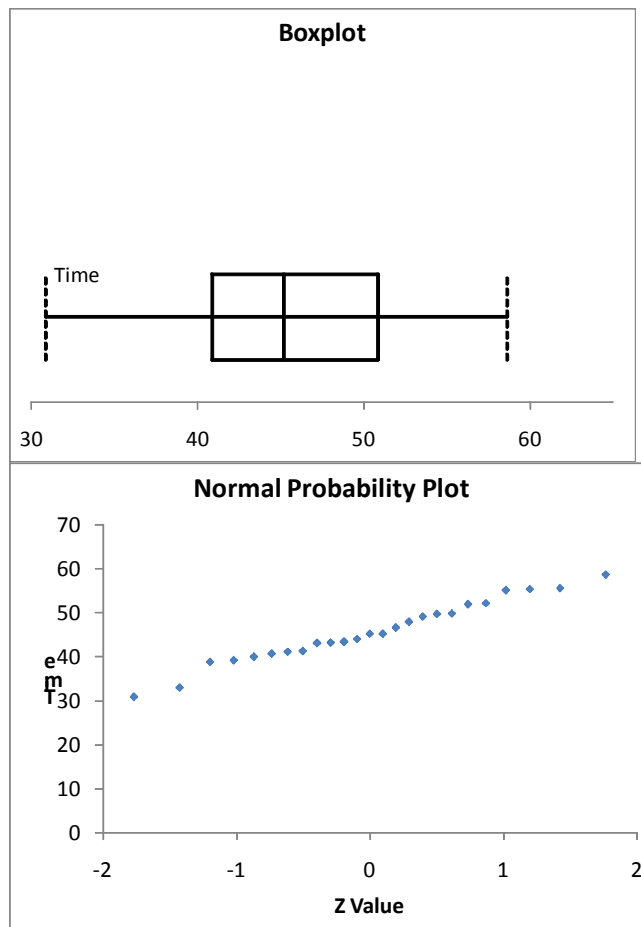
$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{45.628 - 40}{\frac{6.9587}{\sqrt{25}}} = 4.0439$$

Decision: Since $p\text{-value} = 0.0005 < 0.05$, reject H_0 . There is enough evidence to conclude that the population mean time to prepare and cook dinner is different from 40 minutes.

(b) The population distribution needs to be normal.

(c) You could construct charts and observe their appearance. For small- or moderate-sized data sets, construct a stem-and-leaf display or a boxplot. For large data sets, plot a histogram or polygon. You could also compute descriptive numerical measures and compare the characteristics of the data with the theoretical properties of the normal distribution. Compare the mean and median and see if the interquartile range is approximately 1.33 times the standard deviation and if the range is approximately 6 times the standard deviation. Evaluate how the values in the data are distributed. Determine whether approximately two-thirds of the values lie between the mean and ± 1 standard deviation. Determine whether approximately four-fifths of the values lie between the mean and ± 1.28 standard deviations. Determine whether approximately 19 out of every 20 values lie between the mean ± 2 standard deviations.

9.35 (d)
cont.



Both the boxplot and the normal probability plot indicate that the distribution is quite normal. Hence, the t -test in (a) is valid.

9.36 $p\text{-value} = 1 - 0.9772 = 0.0228$

9.37 Since the $p\text{-value} = 0.0228$ is less than $\alpha = 0.05$, reject H_0 .

9.38 $p\text{-value} = 0.0838$

9.39 Since the $p\text{-value} = 0.0838$ is greater than $\alpha = 0.01$, do not reject H_0 .

9.40 $p\text{-value} = P(Z < 1.38) = 0.9162$

9.41 Since the $p\text{-value} = 0.9162 > 0.01$, do not reject the null hypothesis.

9.42 $t = 2.7638$

9.43 Since $t_{STAT} = 2.39 < 2.7638$, do not reject H_0 .

9.44 $t = -2.5280$

9.45 Since $t_{STAT} = -1.15 > -2.5280$, do not reject H_0 .

9.46 (a) $H_0 : \mu \geq 36.5$ $H_1 : \mu < 36.5$

Decision rule: Reject H_0 if $t_{STAT} < -1.6604$ or $p\text{-value} < 0.05$ $d.f. = 99$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{34.5 - 36.5}{11.7 / \sqrt{100}} = -1.7094$$

$$p\text{-value} = 0.0453$$

Decision: Since $t_{STAT} < -1.6604$ and the $p\text{-value}$ of $0.0453 < 0.05$, reject H_0 . There is enough evidence to conclude that the population mean amount is different from 36.5 hours.

(b) The $p\text{-value}$ is 0.0453. If the population mean is indeed at least 36.5 hours, the probability of obtaining a test statistic that is less than -1.7094 is 0.0453.

9.47 (a) PHStat output:

Data	
Null Hypothesis $\mu =$	25.3
Level of Significance	0.05
Sample Size	100
Sample Mean	22.3
Sample Standard Deviation	8.3
Intermediate Calculations	
Standard Error of the Mean	0.83
Degrees of Freedom	99
t Test Statistic	-3.614457831
Lower-Tail Test	
Lower Critical Value	-1.660391157
$p\text{-Value}$	0.000237649
Reject the null hypothesis	

$$H_0 : \mu \geq 25.3 \quad H_1 : \mu < 25.3$$

Decision rule: Reject H_0 if $t_{STAT} < -1.6604$ or $p\text{-value} < 0.05$ $d.f. = 99$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{22.3 - 25.3}{8.3 / \sqrt{100}} = -3.6145$$

$$p\text{-value} = 0.0002$$

Decision: Since $t_{STAT} < -1.6604$ and the $p\text{-value}$ of $0.0002 < 0.05$, reject H_0 . There is enough evidence to conclude that the population mean amount is different from 25.3 hours.

(b) The $p\text{-value}$ is 0.0002. If the population mean is indeed 25.3 hours, the probability of obtaining a test statistic that is less than -3.6145 standard error units away from 0 in either direction is 0.0002.

9.48

PHStat output:

t Test for Hypothesis of the Mean	
Data	
Null Hypothesis $\mu =$	68
Level of Significance	0.01
Sample Size	50
Sample Mean	32
Sample Standard Deviation	9
Intermediate Calculations	
Standard Error of the Mean	1.272792206
Degrees of Freedom	49
t Test Statistic	-28.28427125
Lower-Tail Test	
Lower Critical Value	-2.40489175
p-Value	2.61548E-32
Reject the null hypothesis	

- (a)
- $H_0: \mu \geq 68$
- $H_1: \mu < 68$

Decision rule: If $t_{STAT} < -2.4049$ or p -value < 0.01 , reject H_0 .

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{32 - 68}{9/\sqrt{50}} = -28.2843$$

Decision: Since $t_{STAT} = -28.2843$ is less than -2.4049 , reject H_0 . There is enough evidence to conclude the new process has reduced turnaround time.

- (b) The probability of obtaining a sample whose mean is 32 hours or less when the null hypothesis is true is essentially zero.

9.49 $H_0: \mu \geq 25$ min. The mean delivery time is not less than 25 minutes. $H_1: \mu < 25$ min. The mean delivery time is less than 25 minutes.

- (a) Decision rule: If
- $t_{STAT} < -1.6896$
- , reject
- H_0
- .

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{22.4 - 25}{6/\sqrt{36}} = -2.6$$

Decision: Since $t_{STAT} = -2.6$ is less than -1.6896 , reject H_0 . There is enough evidence to conclude the population mean delivery time has been reduced below the previous value of 25 minutes.

- (b) Decision rule: If
- p
- value
- < 0.05
- , reject
- H_0
- .

 p value = 0.0068Decision: Since p value = 0.0068 is less than $\alpha = 0.05$, reject H_0 . There is enough evidence to conclude the population mean delivery time has been reduced below the previous value of 25 minutes.

- (c) The probability of obtaining a sample whose mean is 22.4 minutes or less when the null hypothesis is true is 0.0068.

- (d) The conclusions are the same.

9.50 PHStat output:

t Test for Hypothesis of the Mean	
Data	
Null Hypothesis $\mu =$	900
Level of Significance	0.01
Sample Size	34
Sample Mean	974
Sample Standard Deviation	96
Intermediate Calculations	
Standard Error of the Mean	16.46386417
Degrees of Freedom	33
t Test Statistic	4.494692086
Upper-Tail Test	
Upper Critical Value	2.444794184
p-Value	4.05368E-05
Reject the null hypothesis	

(a) $H_0: \mu \leq 900$

The mean customer count is not more than 900.

 $H_1: \mu > 900$

The mean customer count is more than 900.

(b) A Type I error occurs when you conclude the mean customer count is more than 900 when in fact the mean number is not more than 900.

A Type II error occurs when you conclude the mean customer count is not more than 900 when in fact the mean number is more than 900.

(c) Decision rule: If $t_{STAT} > 2.4448$ or when the p -value < 0.01 , reject H_0 .

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{974 - 900}{96 / \sqrt{34}} = 4.4947$$

 p -value is virtually 0.Decision: Since $t_{STAT} = 4.4947$ is greater than 2.4448 or the p -value is less than 0.01, reject H_0 . There is enough evidence to conclude that reducing coffee prices is a good strategy for increasing the mean customer count.

(d) When the null hypothesis is true, the probability of obtaining a sample whose mean is 974 or more is virtually 0.

514 Chapter 9: Fundamentals of Hypothesis Testing: One-Sample Tests

- 9.51 (a) $H_0: \mu \geq 10.73$ min. The mean waiting time is not less than 10.73 minutes.
 $H_1: \mu < 10.73$ min. The mean waiting time is less than 10.73 minutes.
 Decision rule: If $t_{STAT} < -1.6604$, reject H_0 .
 Test statistic: $t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{9.52 - 10.73}{5.8/\sqrt{100}} = -2.0862$
 Decision: Since $t_{STAT} = -2.0862$ is less than -1.6604 , reject H_0 . There is enough evidence to conclude the population mean waiting time is less than 10.73 minutes.
- (b) Decision rule: If p value < 0.05 , reject H_0 .
 p value $= 0.0198$
 Decision: Since p value $= 0.0198$ is less than $\alpha = 0.05$, reject H_0 . There is enough evidence to conclude the population mean waiting time is less than 10.73 minutes.
- (c) The probability of obtaining a sample whose mean is 9.52 minutes or less when the null hypothesis is true is 0.0198.
- (d) The conclusions are the same.

9.52 $p = \frac{X}{n} = \frac{88}{400} = 0.22$

9.53 $Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.22 - 0.20}{\sqrt{\frac{0.20(0.8)}{400}}} = 1.00$
 or $Z_{STAT} = \frac{X - n\pi}{\sqrt{n\pi(1-\pi)}} = \frac{88 - 400(0.20)}{\sqrt{400(0.20)(0.80)}} = 1.00$

- 9.54 $H_0: \pi = 0.20$
 $H_1: \pi \neq 0.20$
 Decision rule: If $Z < -1.96$ or $Z > 1.96$, reject H_0 .
 Test statistic: $Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.22 - 0.20}{\sqrt{\frac{0.20(0.8)}{400}}} = 1.00$
 Decision: Since $Z = 1.00$ is between the critical bounds of ± 1.96 , do not reject H_0 .

9.55 (a) PHStat output:

Data	
Null Hypothesis $\pi =$	0.46
Level of Significance	0.05
Number of Items of Interest	29
Sample Size	60
Intermediate Calculations	
Sample Proportion	0.483333333
Standard Error	0.064342832
Z Test Statistic	0.362640759
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	0.716873259
Do not reject the null hypothesis	

$$H_0: \pi = 0.46$$

$$H_1: \pi \neq 0.46$$

Decision rule: $p\text{-value} < 0.05$, reject H_0 .

$$\text{Test statistic: } Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.4833 - 0.46}{\sqrt{\frac{0.46(0.46)}{60}}} = 0.3626$$

Decision: Since $p\text{-value} = 0.7169 > 0.05$, do not reject H_0 . There is not enough evidence that the proportion of full-time students at Miami University is different from the national norm of 0.46.

516 Chapter 9: Fundamentals of Hypothesis Testing: One-Sample Tests

9.55 (b) PHStat output:
cont.

Data	
Null Hypothesis $\pi =$	0.46
Level of Significance	0.05
Number of Items of Interest	36
Sample Size	60
Intermediate Calculations	
Sample Proportion	0.6
Standard Error	0.064342832
Z Test Statistic	2.175844553
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	0.029566886
Reject the null hypothesis	

$$H_0: \pi = 0.46$$

$$H_1: \pi \neq 0.46$$

Decision rule: $p\text{-value} < 0.05$, reject H_0 .

$$\text{Test statistic: } Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.6 - 0.46}{\sqrt{\frac{0.46(0.46)}{60}}} = 2.1758$$

Decision: Since $p\text{-value} = 0.0296 < 0.05$, reject H_0 . There is enough evidence that the proportion of full-time students at Miami University is different from the national norm of 0.46.

9.56 (a) PHStat output:

Z Test of Hypothesis for the Proportion	
Data	
Null Hypothesis $\pi =$	0.5
Level of Significance	0.05
Number of Successes	3586
Sample Size	6403
Intermediate Calculations	
Sample Proportion	0.560049977
Standard Error	0.006248536
Z Test Statistic	9.610247862
Upper-Tail Test	
Upper Critical Value	1.644853627
p-Value	0
Reject the null hypothesis	

9.56
cont.

$$H_0: \pi \leq 0.5 \quad H_1: \pi > 0.5$$

Decision rule: If p -value < 0.05 , reject H_0 .

$$\text{Test statistic: } Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.56 - 0.5}{\sqrt{\frac{0.5(0.5)}{6403}}} = 9.6102,$$

p -value is essentially zero.

Decision: Since p -value < 0.05 , reject H_0 . There is enough evidence that more than half of all the readers of online magazines have linked to an advertiser's Web site.

(b) PHStat output:

Z Test of Hypothesis for the Proportion	
Data	
Null Hypothesis $\pi =$	0.5
Level of Significance	0.05
Number of Successes	56
Sample Size	100
Intermediate Calculations	
Sample Proportion	0.56
Standard Error	0.05
Z Test Statistic	1.2
Upper-Tail Test	
Upper Critical Value	1.644853627
p-Value	0.11506967
Do not reject the null hypothesis	

$$H_0: \pi \leq 0.5 \quad H_1: \pi > 0.5$$

Decision rule: If p -value < 0.05 , reject H_0 .

$$\text{Test statistic: } Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.56 - 0.5}{\sqrt{\frac{0.5(0.5)}{100}}} = 1.2,$$

p -value = 0.1151.

Decision: Since p -value > 0.05 , do not reject H_0 . There is not enough evidence that more than half of all the readers of online magazines have linked to an advertiser's Web site.

- (c) A larger sample size reduces the standard error (variation) of the sample proportion and, hence, reduces the p -value and makes it easier to reject H_0 holding everything else constant.
- (d) You would be very unlikely to reject the null hypothesis with a sample of 20.

9.57 PHStat output:

Z Test of Hypothesis for the Proportion	
Data	
Null Hypothesis $\pi =$	0.55
Level of Significance	0.05
Number of Successes	9
Sample Size	17
Intermediate Calculations	
Sample Proportion	0.529411765
Standard Error	0.12065995
Z Test Statistic	-0.170630232
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	0.864514525
Do not reject the null hypothesis	

 $H_0: \pi = 0.55 \quad H_1: \pi \neq 0.55$
Decision rule: If $Z_{STAT} < -1.96$ or $Z_{STAT} > 1.96$, reject H_0 .

$$\text{Test statistic: } Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.5294 - 0.55}{\sqrt{\frac{0.55(1-0.55)}{17}}} = -0.1706$$

Decision: Since $Z_{STAT} = -0.1706$ is between the two critical bounds, do not reject H_0 . There is not enough evidence that the proportion of females in this position at this medical center is different from what would be expected in the general workforce.

9.58 PHStat output:

Z Test of Hypothesis for the Proportion	
Data	
Null Hypothesis $\pi =$	0.6
Level of Significance	0.05
Number of Successes	650
Sample Size	1000
Intermediate Calculations	
Sample Proportion	0.65
Standard Error	0.015491933
Z Test Statistic	3.227486122
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	0.001248831
Reject the null hypothesis	

9.58
cont.

$$H_0: \pi = 0.60$$

$$H_1: \pi \neq 0.60$$

Decision rule: If $Z_{STAT} < -1.96$ or $Z_{STAT} > 1.96$, reject H_0 .

$$\text{Test statistic: } Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.65 - 0.60}{\sqrt{\frac{0.60(1-0.60)}{1000}}} = 3.2275$$

Decision: Since $Z_{STAT} = 3.2275$ is greater than the upper critical bound of 1.96, reject H_0 . You conclude that there is enough evidence that the proportion of all young jobseekers aged 24 to 35 who preferred to “look for a job in a place where I would like to live” rather than “look for the best job I can find, the place where I live is secondary” is different from 60%.

9.59 PHStat output:

Z Test of Hypothesis for the Proportion	
Data	
Null Hypothesis $\pi =$	0.2
Level of Significance	0.05
Number of Items of Interest	135
Sample Size	500
Intermediate Calculations	
Sample Proportion	0.27
Standard Error	0.017888544
Z Test Statistic	3.913118961
Upper-Tail Test	
Upper Critical Value	1.644853627
p-Value	4.55558E-05
Reject the null hypothesis	

(a) $H_0: \pi \leq 0.2$ $H_1: \pi > 0.2$

Decision rule: If $Z_{STAT} > 1.6449$ or $p\text{-value} < 0.05$, reject H_0 .

$$\text{Test statistic: } Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.27 - 0.2}{\sqrt{\frac{0.2(1-0.2)}{500}}} = 3.9131$$

Decision: Since $Z_{STAT} = 3.9131$ is larger than the critical bound of 1.6449, reject H_0 . There is enough evidence to conclude that more than 20% of the customers would purchase the additional telephone line.

- (b) The manager in charge of promotional programs concerning residential customers can use the results in (a) to try to convince potential customers to purchase the additional telephone line since more than 20% of all potential customers will do so based on the conclusion in (a).

520 Chapter 9: Fundamentals of Hypothesis Testing: One-Sample Tests

- 9.60 (a) $H_0: \pi \leq 0.08$ No more than 8% of students at your school are Omnivores
 $H_1: \pi > 0.08$ More than 8% of students at your school are Omnivores
 (b) Decision rule: If $p\text{-value} < 0.05$, reject H_0 .
 Test statistic: $Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} = \frac{0.15 - 0.08}{\sqrt{\frac{0.08(1 - 0.08)}{200}}} = 3.6490$, $p\text{-value} = 0.0001316$
 Decision: Since $p\text{-value} = 0.0001316 < 0.05$, reject H_0 . There is enough evidence to show that the percentage of Omnivores at your school is greater than 8%.
- 9.61 The null hypothesis represents the status quo or the hypothesis that is to be disproved. The null hypothesis includes an equal sign in its definition of a parameter of interest. The alternative hypothesis is the opposite of the null hypothesis and usually represents taking an action. The alternative hypothesis includes either a less than sign, a not equal to sign, or a greater than sign in its definition of a parameter of interest.
- 9.62 A Type I error represents rejecting a true null hypothesis, while a Type II error represents not rejecting a false null hypothesis.
- 9.63 The power of a test is the probability that the null hypothesis will be rejected when the null hypothesis is false.
- 9.64 In a one-tailed test for a mean or proportion, the entire rejection region is contained in one tail of the distribution. In a two-tailed test, the rejection region is split into two equal parts, one in the lower tail of the distribution, and the other in the upper tail.
- 9.65 The $p\text{-value}$ is the probability of obtaining a test statistic equal to or more extreme than the result obtained from the sample data, given that the null hypothesis is true.
- 9.66 Assuming a two-tailed test is used, if the hypothesized value for the parameter does not fall into the confidence interval, then the null hypothesis can be rejected.
- 9.67 The following are the 6-step critical value approach to hypothesis testing: (1) State the null hypothesis H_0 . State the alternative hypothesis H_1 . (2) Choose the level of significance α . Choose the sample size n . (3) Determine the appropriate statistical technique and corresponding test statistic to use. (4) Set up the critical values that divide the rejection and nonrejection regions. (5) Collect the data and compute the sample value of the appropriate test statistic. (6) Determine whether the test statistic has fallen into the rejection or the nonrejection region. The computed value of the test statistic is compared with the critical values for the appropriate sampling distribution to determine whether it falls into the rejection or nonrejection region. Make the statistical decision. If the test statistic falls into the nonrejection region, the null hypothesis H_0 cannot be rejected. If the test statistic falls into the rejection region, the null hypothesis is rejected. Express the statistical decision in terms of a particular situation.

- 9.68 The following are the 5-step p -value approach to hypothesis testing: (1) State the null hypothesis, H_0 , and the alternative hypothesis, H_1 . (2) Choose the level of significance, α , and the sample size, n . (3) Determine the appropriate test statistic and the sampling distribution. (4) Collect the sample data, compute the value of the test statistic, and compute the p -value. (5) Make the statistical decision and state the managerial conclusion. If the p -value is greater than or equal to α , you do not reject the null hypothesis, H_0 . If the p -value is less than α , you reject the null hypothesis.
- 9.69 (a) $H_0 : \pi = 0.5$ $H_1 : \pi \neq 0.5$
- (b) The level of significance is the probability of committing a Type I error, which is the probability of concluding the proportion of customers who prefer product 1 over product 2 is not 50% when in fact 50% of customers prefer product 1 over product 2. The risk associated with Type II error is the probability of not rejecting the claim that 50% of customers prefer product 1 over product 2 when it should be rejected.
- (c) If you reject the null hypothesis for a p -value of 0.22, there is a 22% probability that you may have incorrectly concluded that the proportion of customers preferring product 1 is not 50% when in fact the correct proportion is 50%.
- (d) The article suggests raising the level of significance because the consequences of incorrectly concluding the proportion is not 50% are not very severe.
- (e) Before raising the level of significance of a test, you have to genuinely evaluate whether the cost of committing a Type I error is really not as bad as you have thought.
- (f) If the p -value is actually 0.12, you will be more confident about rejecting the null hypothesis. If the p -value is 0.06, you will be even more confident that a Type I error is much less likely to occur.
- 9.70 (a) La Quinta Motor Inns commits a Type I error when it purchases a site that is not profitable.
- (b) Type II error occurs when La Quinta Motor Inns fails to purchase a profitable site. The cost to the Inns when a Type II error is committed is the loss on the amount of profit the site could have generated had the Inns decided to purchase the site.
- (c) The executives at La Quinta Motor Inns are trying to avoid a Type I error by adopting a very stringent decision criterion. Only sites that are classified as capable of generating high profit will be purchased.
- (d) If the executives adopt a less stringent rejection criterion by buying sites for which the computer model predicts moderate or large profit, the probability of committing a Type I error will increase. Many more of the sites the computer model predicts that will generate moderate profit may end up not being profitable at all. On the other hand, the less stringent rejection criterion will lower the probability of committing a Type II error since more potentially profitable sites will be purchased.

9.71 (a) PHStat output:

Data	
Null Hypothesis $\pi =$	0.5
Level of Significance	0.05
Number of Items of Interest	588
Sample Size	1132
Intermediate Calculations	
Sample Proportion	0.52
Standard Error	0.014860957
Z Test Statistic	1.345808307
Upper-Tail Test	
Upper Critical Value	1.644853627
p-Value	0.089182173
Do not reject the null hypothesis	

(a) $H_0: \pi \leq 0.5$ $H_1: \pi > 0.5$ Decision rule: If p -value < 0.05 , reject H_0 .

$$\text{Test statistic: } Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.52 - 0.5}{\sqrt{\frac{0.5(0.5)}{1132}}} = 1.3458,$$

 p -value = 0.0892

Decision: Since p -value = 0.0892 > 0.05 , do not reject H_0 . There is not enough evidence to show that more than half of all mobile phone users are now using the mobile Internet.

(b) The claim by the authors is invalid.

(c) PHStat output:

Data	
Null Hypothesis $\pi =$	0.5
Level of Significance	0.05
Number of Items of Interest	600
Sample Size	1132
Intermediate Calculations	
Sample Proportion	0.53
Standard Error	0.014860957
Z Test Statistic	2.018712461
Upper-Tail Test	
Upper Critical Value	1.644853627
p-Value	0.021758557
Reject the null hypothesis	

- 9.71 (c) (a) $H_0: \pi \leq 0.5$ $H_1: \pi > 0.5$
 cont. Decision rule: If $p\text{-value} < 0.05$, reject H_0 .
 Test statistic: $Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.53 - 0.5}{\sqrt{\frac{0.5(0.5)}{1132}}} = 2.0187$,
 $p\text{-value} = 0.0218$
 Decision: Since $p\text{-value} = 0.0218 < 0.05$, reject H_0 . There is enough evidence to show that more than half of all mobile phone users are now using the mobile Internet.
- (b) The claim by the authors is valid.
- (d) With the same level of significance, 53% is considered as far enough above 50% to reject the null hypothesis of no more than half of all mobile phone users are now using the mobile Internet while 52% is not considered as far enough above 0.5.
- 9.72 (a) $H_0: \mu = 10.0$ gallons. The mean gasoline purchase is equal to 10 gallons.
 $H_1: \mu \neq 10.0$ gallons. The mean gasoline purchase differs from 10 gallons.
 Decision rule: $d.f. = 59$. If $t_{STAT} < -2.0010$ or $t_{STAT} > 2.0010$, reject H_0 .
 Test statistic: $t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{11.3 - 10.0}{3.1/\sqrt{60}} = 3.2483$
 Decision: Since $t_{STAT} = 3.2483$ is greater than the upper critical value of 2.0010, reject H_0 . There is enough evidence to conclude that the mean gasoline purchase differs from 10 gallons.
- (b) $p\text{-value} = 0.0019$.
 Note: The p value was found using Excel.
- (c) $H_0: \pi \geq 0.20$. At least 20% of the motorists purchase premium-grade gasoline.
 $H_1: \pi < 0.20$. Less than 20% of the motorists purchase premium-grade gasoline.
 Decision rule: If $Z_{STAT} < -1.645$, reject H_0 .
 Test statistic: $Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.1833 - 0.20}{\sqrt{\frac{0.20(1-0.20)}{60}}} = -0.32$
 Decision: Since $Z_{STAT} = -0.32$ is greater than the critical bound of -1.645 , do not reject H_0 . There is not sufficient evidence to conclude that less than 20% of the motorists purchase premium-grade gasoline.
- (d) $H_0: \mu = 10.0$ gallons. The mean gasoline purchase is equal to 10 gallons.
 $H_1: \mu \neq 10.0$ gallons. The mean gasoline purchase differs from 10 gallons.
 Decision rule: $d.f. = 59$. If $t_{STAT} < -2.0010$ or $t_{STAT} > 2.0010$, reject H_0 .
 Test statistic: $t = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{10.3 - 10.0}{3.1/\sqrt{60}} = 0.7496$
 Decision: Since the test statistic of $t_{STAT} = 0.7496$ is between the critical bounds of ± 2.0010 , do not reject H_0 . There is not enough evidence to conclude that the mean gasoline purchase differs from 10 gallons.

- 9.72 (e) $H_0: \pi \geq 0.20$. At least 20% of the motorists purchase premium-grade gasoline.
 cont.. $H_1: \pi < 0.20$. Less than 20% of the motorists purchase premium-grade gasoline.
 Decision rule: If $Z_{STAT} < -1.645$, reject H_0 .

$$\text{Test statistic: } Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.1167 - 0.20}{\sqrt{\frac{0.20(1-0.20)}{60}}} = -1.61$$

Decision: Since $Z_{STAT} = -1.61$ is greater than the critical bound of -1.645 , do not reject H_0 . There is not sufficient evidence to conclude that less than 20% of the motorists purchase premium-grade gasoline.

- 9.73 (a) $H_0: \mu \geq \$100$ $H_1: \mu < \$100$
 Decision rule: $d.f. = 74$. If $t_{STAT} < -1.6657$, reject H_0 .

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{\$93.70 - \$100}{\$34.55/\sqrt{75}} = -1.5791$$

Decision: Since the test statistic of $t_{STAT} = -1.5791$ is greater than the critical bound of

-1.6657 , do not reject H_0 . There is not enough evidence to conclude that the mean reimbursement for office visits to doctors paid by Medicare is less than \$100.

- (b) $H_0: \pi \leq 0.10$. At most 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.
 $H_1: \pi > 0.10$. More than 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.
 Decision rule: If $Z_{STAT} > 1.645$, reject H_0 .

$$\text{Test statistic: } Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.16 - 0.10}{\sqrt{\frac{0.10(1-0.10)}{75}}} = 1.73$$

Decision: Since $Z_{STAT} = 1.73$ is greater than the critical bound of 1.645 , reject H_0 . There is sufficient evidence to conclude that more than 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.

- (c) To perform the t -test on the population mean, you must assume that the observed sequence in which the data were collected is random and that the data are approximately normally distributed.

- (d) $H_0: \mu \geq \$100$. The mean reimbursement for office visits to doctors paid by Medicare is at least \$100.
 $H_1: \mu < \$100$. The mean reimbursement for office visits to doctors paid by Medicare is less than \$100.

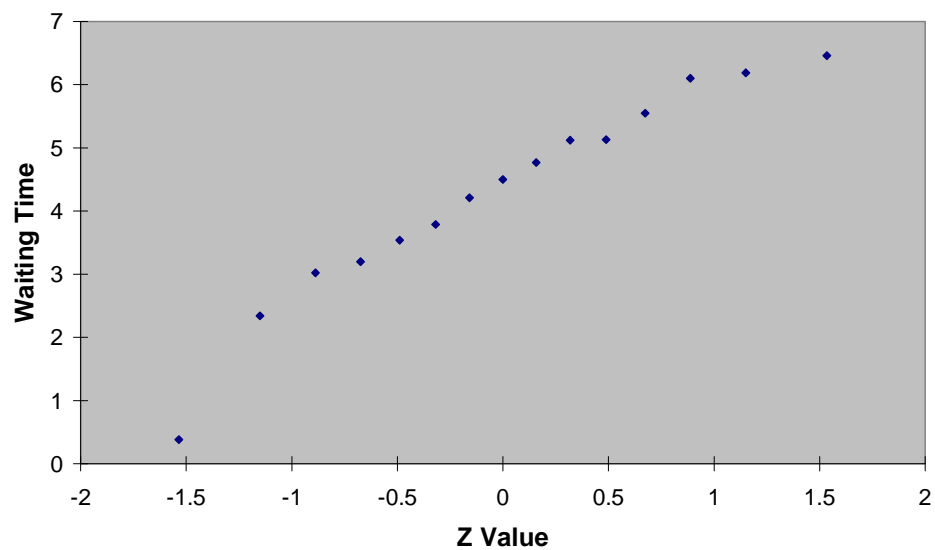
Decision rule: $d.f. = 74$. If $t_{STAT} < -1.6657$, reject H_0 .

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{\$90 - \$100}{\$34.55/\sqrt{75}} = -2.5066$$

Decision: Since $t_{STAT} = -2.5066$ is less than the critical bound of -1.6657 , reject H_0 . There is enough evidence to conclude that the mean reimbursement for office visits to doctors paid by Medicare is less than \$100.

- 9.73 (e) $H_0: \pi \leq 0.10$. At most 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.
 $H_1: \pi > 0.10$. More than 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.
 Decision rule: If $Z_{STAT} > 1.645$, reject H_0 .
 Test statistic: $Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.20 - 0.10}{\sqrt{\frac{0.10(1-0.10)}{75}}} = 2.89$
 Decision: Since $Z_{STAT} = 2.89$ is greater than the critical bound of 1.645, reject H_0 . There is sufficient evidence to conclude that more than 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.
- 9.74 (a) $H_0: \mu \geq 5$ minutes. The mean waiting time at a bank branch in a commercial district of the city is at least 5 minutes during the 12:00 p.m. to 1 p.m. peak lunch period.
 $H_1: \mu < 5$ minutes. The mean waiting time at a bank branch in a commercial district of the city is less than 5 minutes during the 12:00 p.m. to 1 p.m. peak lunch period.
 Decision rule: $d.f. = 14$. If $t_{STAT} < -1.7613$, reject H_0 .
 Test statistic: $t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{4.2866 - 5.0}{1.637985/\sqrt{15}} = -1.6867$
 Decision: Since $t_{STAT} = -1.6867$ is greater than the critical bound of -1.7613 , do not reject H_0 . There is not enough evidence to conclude that the mean waiting time at a bank branch in a commercial district of the city is less than 5 minutes during the 12:00 p.m. to 1 p.m. peak lunch period.
- (b) To perform the t -test on the population mean, you must assume that the observed sequence in which the data were collected is random and that the data are approximately normally distributed.
- (c) Normal probability plot:

Normal Probability Plot



526 Chapter 9: Fundamentals of Hypothesis Testing: One-Sample Tests

- 9.74 (d) With the exception of one extreme point, the data are approximately normally distributed.
 cont. (e) Based on the results of (a), the manager does not have enough evidence to make that statement.

- 9.75 (a) $H_0 : \mu \leq 1500$ $H_1 : \mu > 1500$

Decision rule: Reject H_0 if $t_{STAT} > 1.6991$ $df = 29$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{1723.4 - 1500}{89.55 / \sqrt{30}} = 13.664$$

Decision: Since $t_{STAT} = 13.664 > 1.6991$, reject H_0 . There is enough evidence to conclude that the mean force is greater than 1500 pounds.

- (b) In order for the t test to be valid, the data are assumed to be independently drawn from a population that is normally distributed.
 (c)



The boxplot suggests that the distribution of the data is skewed only slightly to the left. With a sample size of 30, the t distribution will provide a good approximation to the sampling distribution of the sample mean.

- (d) Based on (a), you can conclude that the mean force is greater than 1500 pounds, especially in light of p -value = 0.0.
- 9.76 (a) $H_0 : \mu \geq 0.35$ $H_1 : \mu < 0.35$

Decision rule: Reject H_0 if $t_{STAT} < -1.690$ $df = 35$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{0.3167 - 0.35}{0.1357 / \sqrt{36}} = -1.4735$$

Decision: Since $t_{STAT} > -1.690$, do not reject H_0 . There is not enough evidence to conclude that the mean moisture content for Boston shingles is less than 0.35 pounds per 100 square feet.

- (b) p -value = 0.0748. If the population mean moisture content is in fact no less than 0.35 pounds per 100 square feet, the probability of observing a sample of 36 shingles that will result in a sample mean moisture content of 0.3167 pounds per 100 square feet or less is .0748.

9.76 (c) $H_0 : \mu \geq 0.35$ $H_1 : \mu < 0.35$

cont. Decision rule: Reject H_0 if $t_{STAT} < -1.6973$ $d.f. = 30$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{0.2735 - 0.35}{0.1373/\sqrt{31}} = -3.1003$$

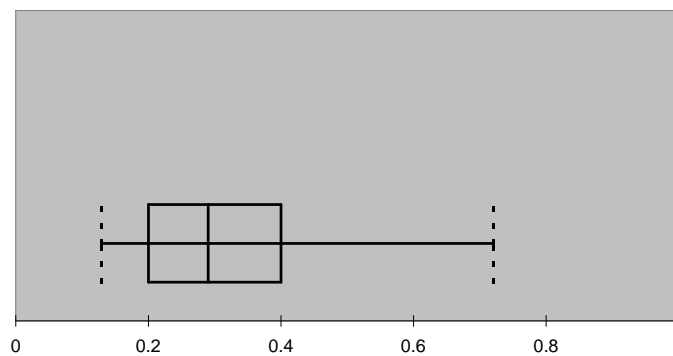
Decision: Since $t_{STAT} < -1.6973$, reject H_0 . There is enough evidence to conclude that the mean moisture content for Vermont shingles is less than 0.35 pounds per 100 square feet.

(d) $p\text{-value} = 0.0021$. If the population mean moisture content is in fact no less than 0.35 pounds per 100 square feet, the probability of observing a sample of 31 shingles that will result in a sample mean moisture content of 0.2735 pounds per 100 square feet or less is .0021.

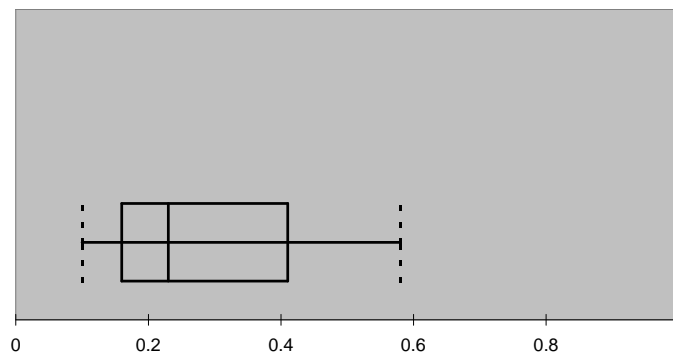
(e) In order for the t test to be valid, the data are assumed to be independently drawn from a population that is normally distributed. Since the sample sizes are 36 and 31, respectively, which are considered quite large, the t distribution will provide a good approximation to the sampling distribution of the mean as long as the population distribution is not very skewed.

(f)

Box-and-whisker Plot (Boston)



Box-and-whisker Plot (Vermont)



Both boxplots suggest that the data are skewed slightly to the right, more so for the Boston shingles. However, the very large sample sizes mean that the results of the t test are relatively insensitive to the departure from normality.

9.77 (a) $H_0 : \mu = 3150$ $H_1 : \mu \neq 3150$

Decision rule: Reject H_0 if $|t_{STAT}| > 1.9665$ $d.f. = 367$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{3124.2147 - 3150}{34.713/\sqrt{368}} = -14.2497$$

Decision: Since $t_{STAT} < -1.9665$, reject H_0 . There is enough evidence to conclude that the mean weight for Boston shingles is different from 3150 pounds.

- (b) p -value is virtually zero. If the population mean weight is in fact 3150 pounds, the probability of observing a sample of 368 shingles that will yield a test statistic more extreme than -14.2497 is virtually zero.

(c) $H_0 : \mu = 3700$ $H_1 : \mu \neq 3700$

Decision rule: Reject H_0 if $|t_{STAT}| > 1.967$ $d.f. = 329$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{3704.0424 - 3700}{46.7443/\sqrt{330}} = 1.571$$

Decision: Since $t_{STAT} < 1.967$, do not reject H_0 . There is not enough evidence to conclude that the mean weight for Vermont shingles is different from 3700 pounds.

- (d) p -value = .1171. The probability of observing a sample of 330 shingles that will yield a test statistic more extreme than 1.571 is .1171 if the population mean weight is in fact 3700 pounds.

- (e) In order for the t test to be valid, the data are assumed to be independently drawn from a population that is normally distributed. Since the sample sizes are 368 and 330, respectively, which are considered large enough, the t distribution will provide a good approximation to the sampling distribution of the mean even if the population is not normally distributed.

9.78 (a) $H_0 : \mu = 0.5$ $H_1 : \mu \neq 0.5$

Decision rule: Reject H_0 if $|t_{STAT}| > 1.9741$ $d.f. = 169$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{0.2641 - 0.5}{0.1424/\sqrt{170}} = -21.6059$$

Decision: Since $t_{STAT} < -1.9741$, reject H_0 . There is enough evidence to conclude that the mean granule loss for Boston shingles is different from 0.5 grams.

- (b) p -value is virtually zero. If the population mean granule loss is in fact 0.5 grams, the probability of observing a sample of 170 shingles that will yield a test statistic more extreme than -21.6059 is virtually zero.

9.78 (c) $H_0 : \mu = 0.5$ $H_1 : \mu \neq 0.5$

cont. Decision rule: Reject H_0 if $|t_{STAT}| > 1.977$ $d.f. = 139$

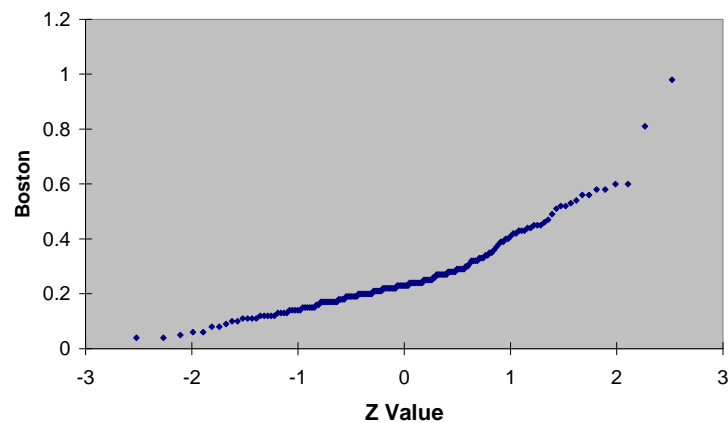
$$\text{Test statistic: } t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{0.218 - 0.5}{0.1227/\sqrt{140}} = -27.1940$$

Decision: Since $t_{STAT} < -1.977$, reject H_0 . There is enough evidence to conclude that the mean granule loss for Vermont shingles is different from 0.5 grams.

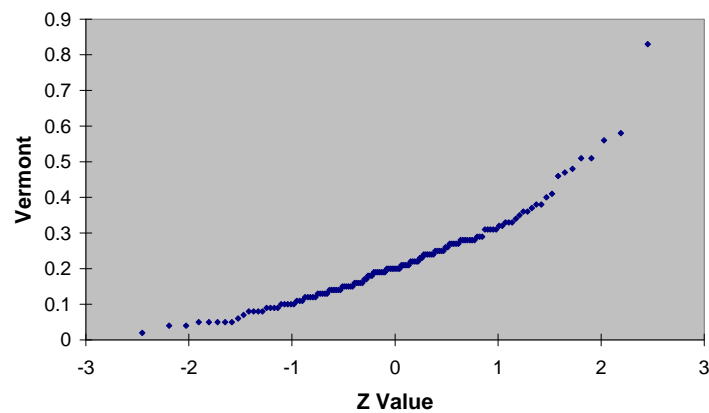
(d) p -value is virtually zero. The probability of observing a sample of 140 shingles that will yield a test statistic more extreme than -27.1940 is virtually zero if the population mean granule loss is in fact 0.5 grams.

(e) In order for the t test to be valid, the data are assumed to be independently drawn from a population that is normally distributed. Both normal probability plots indicate that the data are slightly right-skewed. Since the sample sizes are 170 and 140, respectively, which are considered large enough, the t distribution will provide a good approximation to the sampling distribution of the mean even if the population is not normally distributed.

Normal Probability Plot



Normal Probability Plot



530 Chapter 9: Fundamentals of Hypothesis Testing: One-Sample Tests

9.80 $H_0: \mu \geq 7, H_1: \mu < 7, \alpha = 0.05, n = 16, \sigma = 0.2$

Lower critical value: $Z_L = -1.6449, \bar{X}_L = \mu + Z_L \left(\frac{\sigma}{\sqrt{n}} \right) = 7 - 1.6449 \left(\frac{.2}{\sqrt{16}} \right) = 6.9178$

(a)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{6.9178 - 6.9}{\frac{.2}{\sqrt{16}}} = 0.3551$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < 0.3551) = 0.6388$

$\beta = 1 - 0.6388 = 0.3612$

(b)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{6.9178 - 6.8}{\frac{.2}{\sqrt{16}}} = 2.3551$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < 2.3551) = 0.9907$

$\beta = 1 - 0.9907 = 0.0093$

9.81 $H_0: \mu \geq 7, H_1: \mu < 7, \alpha = 0.01, n = 16, \sigma = 0.2$

Lower critical value: $Z_L = -2.3263, \bar{X}_L = \mu + Z_L \left(\frac{\sigma}{\sqrt{n}} \right) = 7 - 2.3263 \left(\frac{.2}{\sqrt{16}} \right) = 6.8837$

(a)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{6.8837 - 6.9}{\frac{.2}{\sqrt{16}}} = -0.3263$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < -0.3263) = 0.3721$

$\beta = 1 - 0.3721 = 0.6279$

(b)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{6.8837 - 6.8}{\frac{.2}{\sqrt{16}}} = 1.6737$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < 1.6737) = 0.9529$

$\beta = 1 - 0.9529 = 0.0471$

- 9.81 (c) Holding everything else constant, the greater the distance between the true mean and the hypothesized mean, the higher the power of the test will be and the lower the probability of committing a Type II error will be. Holding everything else constant, the smaller the level of significance, the lower the power of the test will be and the higher the probability of committing a Type II error will be.

9.82 $H_0: \mu \geq 7$, $H_1: \mu < 7$, $\alpha = 0.05$, $n = 25$, $\sigma = 0.2$

Lower critical value: $Z_L = -1.6449$, $\bar{X}_L = \mu + Z_L \left(\frac{\sigma}{\sqrt{n}} \right) = 7 - 1.6449 \left(\frac{.2}{\sqrt{25}} \right) = 6.9342$

(a)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{6.9342 - 6.9}{\frac{.2}{\sqrt{25}}} = 0.8551$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < 0.8551) = 0.8038$
 $\beta = 1 - 0.8038 = 0.1962$

(b)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{6.9342 - 6.8}{\frac{.2}{\sqrt{25}}} = 3.3551$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < 3.3551) = 0.9996$
 $\beta = 1 - 0.9996 = 0.0004$

- (c) Holding everything else constant, the larger the sample size, the higher the power of the test will be and the lower the probability of committing a Type II error will be.

9.83 $H_0: \mu \geq 25,000$, $H_1: \mu < 25,000$, $\alpha = 0.05$, $n = 100$, $\sigma = 3500$

Lower critical value: $Z_L = -1.6449$,

$$\bar{X}_L = \mu + Z_L \left(\frac{\sigma}{\sqrt{n}} \right) = 25,000 - 1.6449 \left(\frac{3,500}{\sqrt{100}} \right) = 24,424.3013$$

(a)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{24,424.3012 - 24,000}{\frac{3500}{\sqrt{100}}} = 1.2123$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < 1.2123) = 0.8873$
 $\beta = 1 - 0.8873 = 0.1127$

(b)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{24,424.3012 - 24,900}{\frac{3500}{\sqrt{100}}} = -1.3591$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < -1.3591) = 0.0871$
 $\beta = 1 - 0.0871 = 0.9129$

532 Chapter 9: Fundamentals of Hypothesis Testing: One-Sample Tests

9.84 $H_0 : \mu \geq 25,000$ vs. $H_1 : \mu < 25,000$, $\alpha = 0.01$, $n = 100$, $\sigma = 3500$

Lower critical value: $Z_L = -2.3263$,

$$\bar{X}_L = \mu + Z_L \left(\frac{\sigma}{\sqrt{n}} \right) = 25,000 - 2.3263 \left(\frac{3,500}{\sqrt{100}} \right) = 24,185.7786$$

(a)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{24,185.7786 - 24,000}{\frac{3500}{\sqrt{100}}} = 0.5308$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < 0.5308) = 0.7022$

$\beta = 1 - 0.7022 = 0.2978$

(b)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{24,185.7786 - 24,900}{\frac{3500}{\sqrt{100}}} = -2.0406$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < -2.0406) = 0.0206$

$\beta = 1 - 0.0206 = 0.9794$

- (c) Holding everything else constant, the greater the distance between the true mean and the hypothesized mean, the higher the power of the test will be and the lower the probability of committing a Type II error will be. Holding everything else constant, the smaller the level of significance, the lower the power of the test will be and the higher the probability of committing a Type II error will be.

9.85 $H_0 : \mu \geq 25,000$, $H_1 : \mu < 25,000$, $\alpha = 0.05$, $n = 25$, $\sigma = 3500$

Lower critical value: $Z_L = -1.6449$,

$$\bar{X}_L = \mu + Z_L \left(\frac{\sigma}{\sqrt{n}} \right) = 25,000 - 1.6449 \left(\frac{3,500}{\sqrt{25}} \right) = 23,848.6026$$

(a)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{23,848.6026 - 24,000}{\frac{3500}{\sqrt{25}}} = -0.2163$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < -0.2163) = 0.4144$

$\beta = 1 - 0.4144 = 0.5856$

(b)
$$Z_{STAT} = \frac{\bar{X}_L - \mu_1}{\frac{\sigma}{\sqrt{n}}} = \frac{23,848.6026 - 24,900}{\frac{3500}{\sqrt{25}}} = -1.5020$$

power = $1 - \beta = P(\bar{X} < \bar{X}_L) = P(Z < -1.5020) = 0.0665$

$\beta = 1 - 0.0665 = 0.9335$

- (c) Holding everything else constant, the larger the sample size, the higher the power of the test will be and the lower the probability of committing a Type II error will be.

9.86 $H_0 : \mu = 25,000, H_1 : \mu \neq 25,000, \alpha = 0.05, n = 100, \sigma = 3500$

Critical values: $Z_L = -1.960, Z_U = 1.960$

$$\bar{X}_L = \mu + Z_L \left(\frac{\sigma}{\sqrt{n}} \right) = 25,000 - 1.960 \left(\frac{3,500}{\sqrt{100}} \right) = 24,314.0130$$

$$\bar{X}_U = \mu + Z_U \left(\frac{\sigma}{\sqrt{n}} \right) = 25,000 + 1.960 \left(\frac{3,500}{\sqrt{100}} \right) = 25,685.9870$$

(a) $\beta = P(\bar{X}_L < \bar{X} < \bar{X}_U) = P(0.8972 < Z < 4.8171) = 0.1848$

$$\text{power} = 1 - \beta = 1 - 0.1848 = 0.8152$$

(b) $\beta = P(\bar{X}_L < \bar{X} < \bar{X}_U) = P(-1.6742 < Z < 2.2457) = 0.9406$

$$\text{power} = 1 - \beta = 1 - 0.9406 = 0.0594$$

(c) A one-tail test is more powerful than a two-tail test, holding everything else constant.