KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

BUSINESS SCHOOL

DEPARTMENT OF INFORMATION SYSTEM & OPERATIONS MANAGEMENT

MANAGEMENT SCIENCE OM 511

02 – Optimization Modelling – Graphical Method

DHAHRAN, SAUDI ARABIA

amazon logistics

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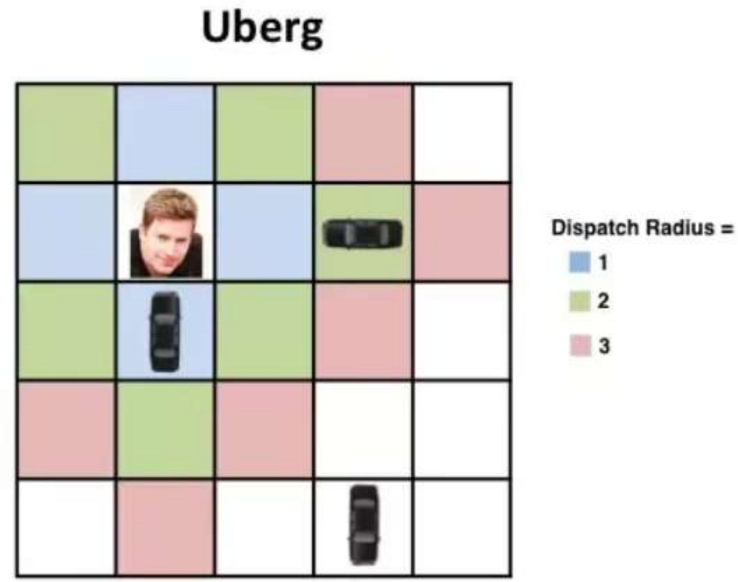
Uber


Uberg
Dispatch Radius =
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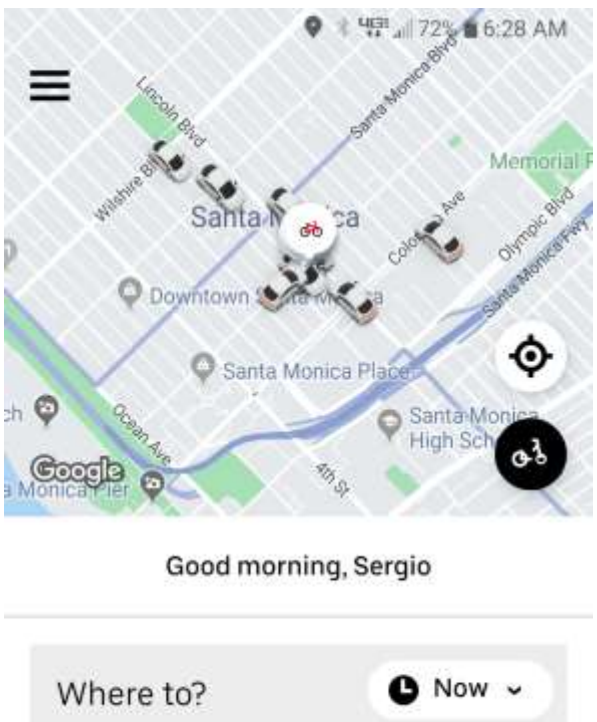
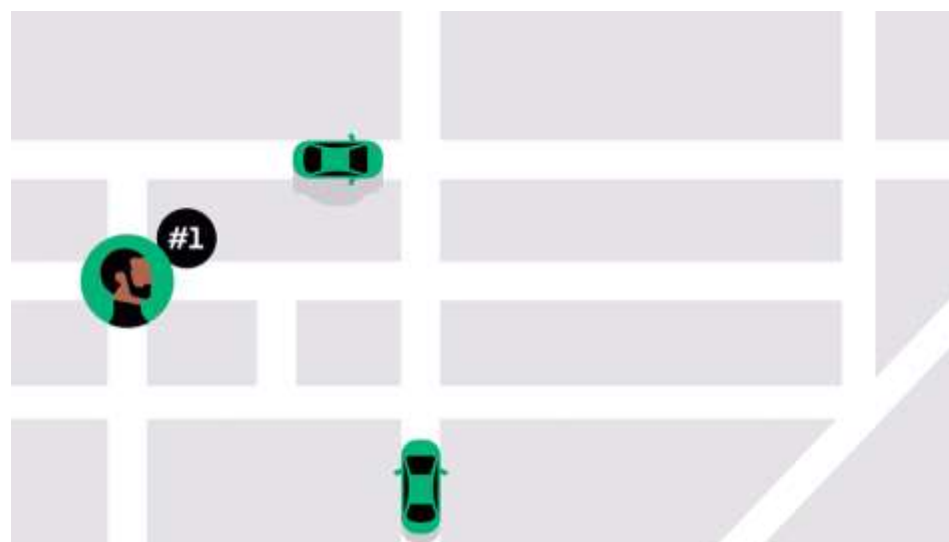
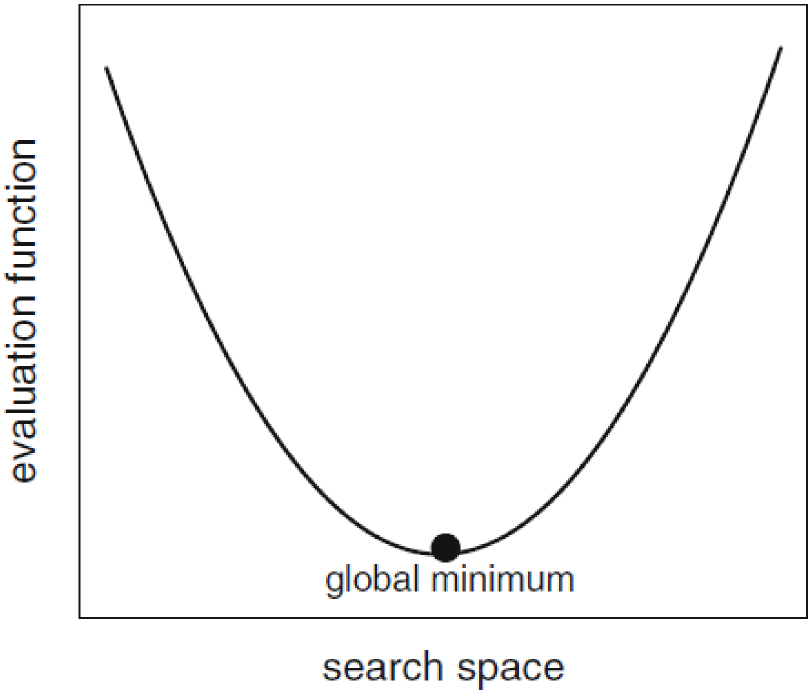
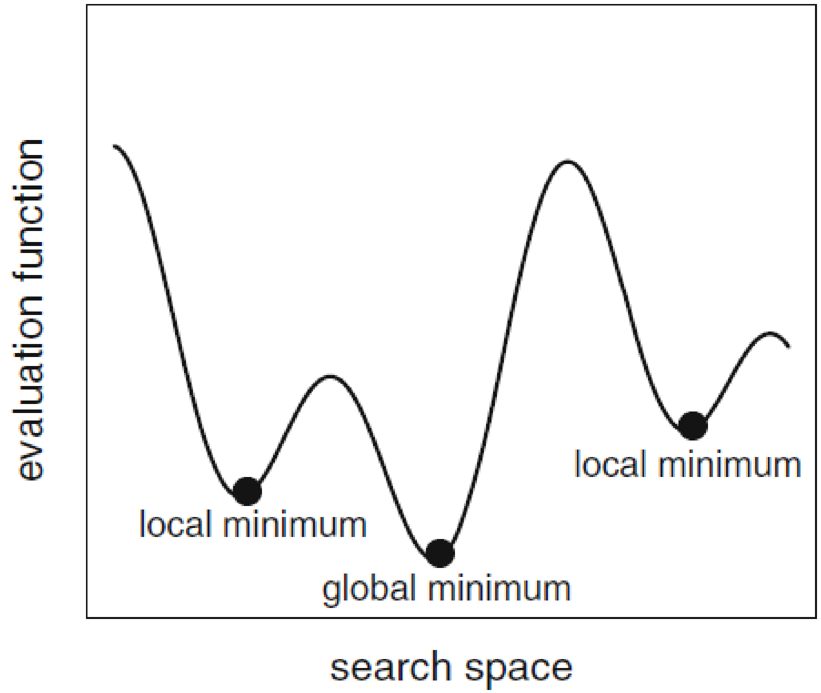

72%
6:28 AM
Lincoln Blvd
Santa Monica Blvd
Downtown
Santa Monica Place
Ocean Ave
4th St
Olympic Blvd
Google
Good morning, Sergio
Where to?
Now


#1

Convex



Non - Convex

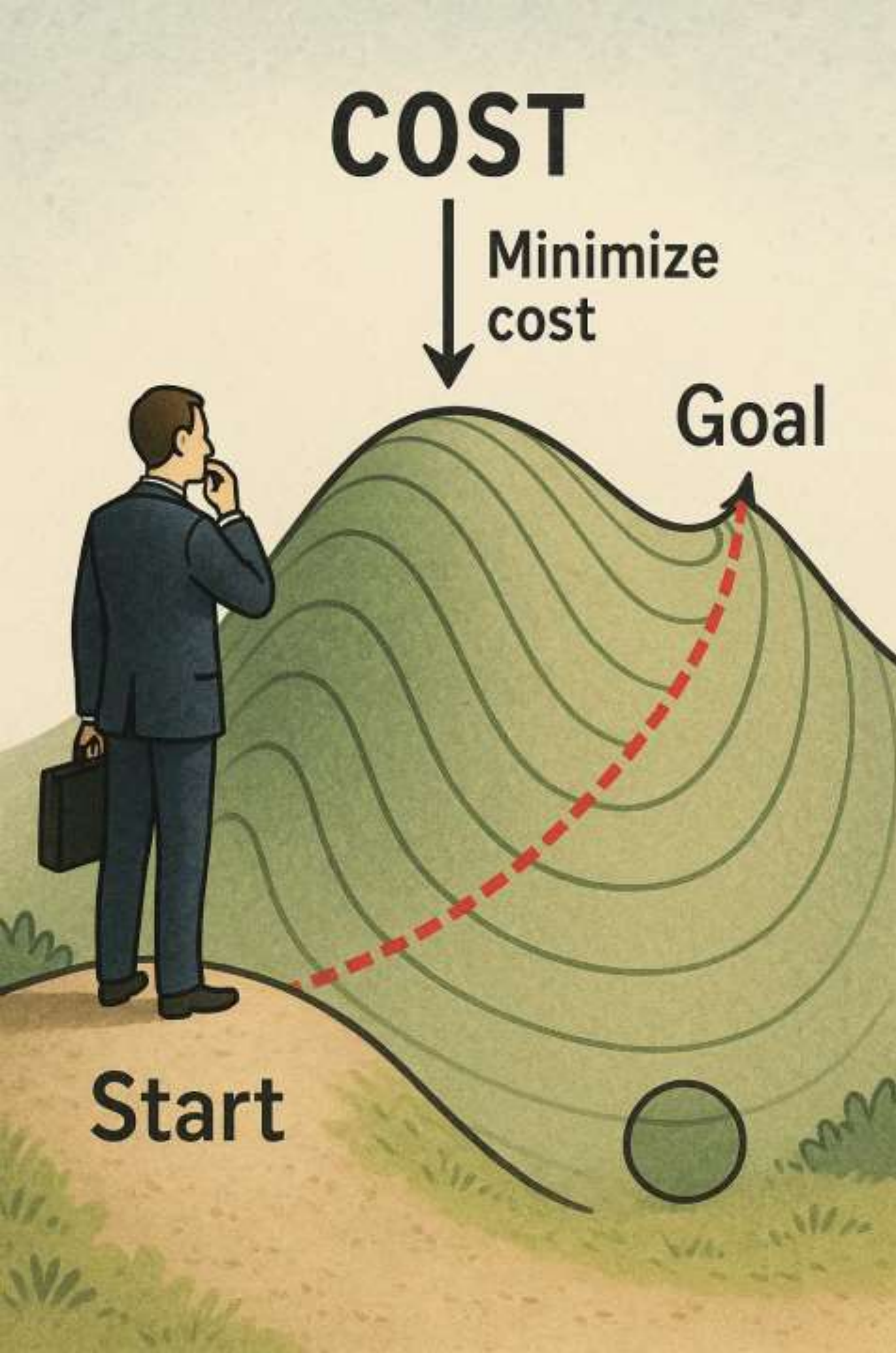


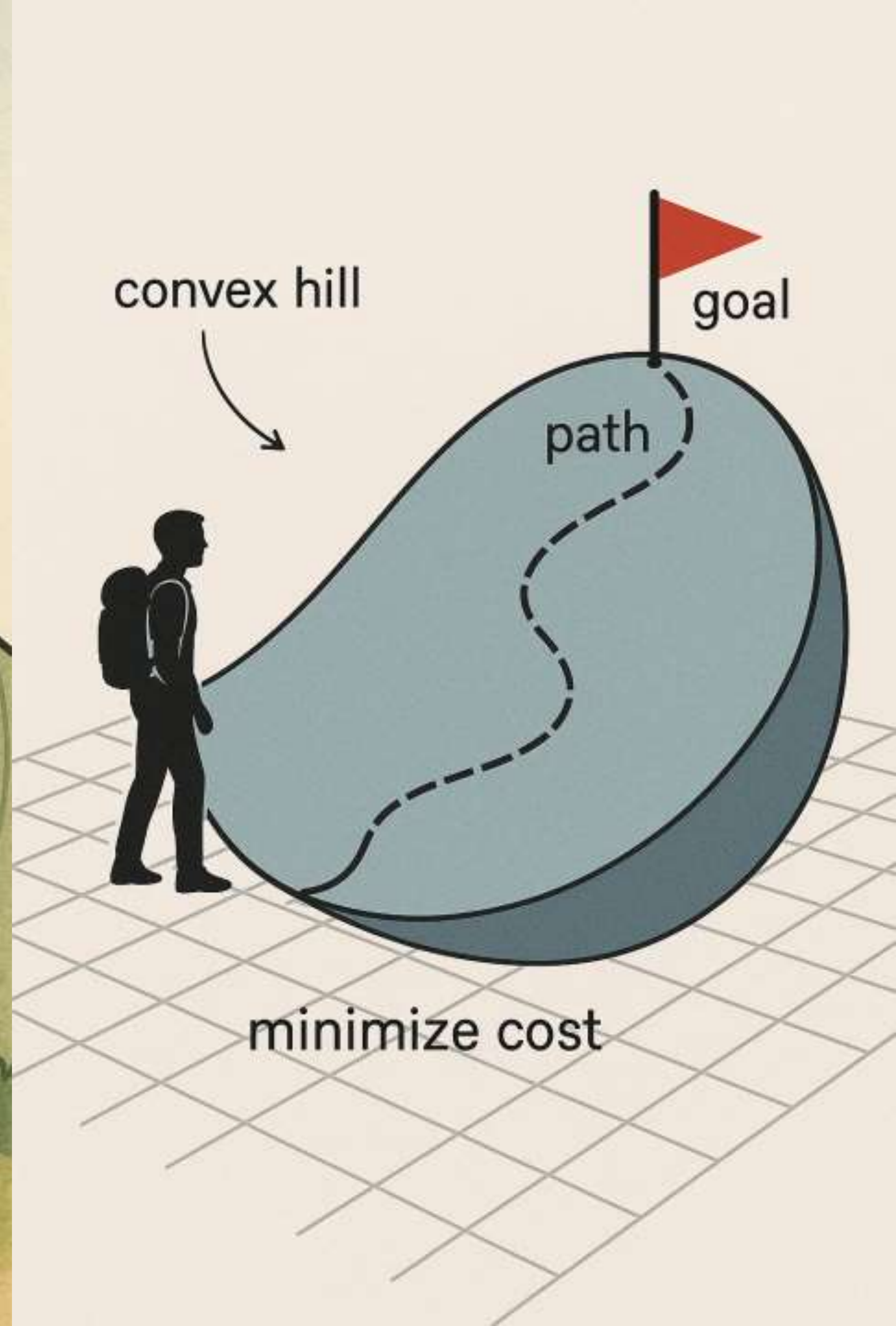
Linear programming

Non - Linear programming

A convex function curves upwards, and any local minimum is also a global minimum.

A non-convex function may have regions where it curves downwards or contains multiple local minima and maxima


COST
Minimize
cost
Goal
Start


convex hill
goal
path
minimize cost

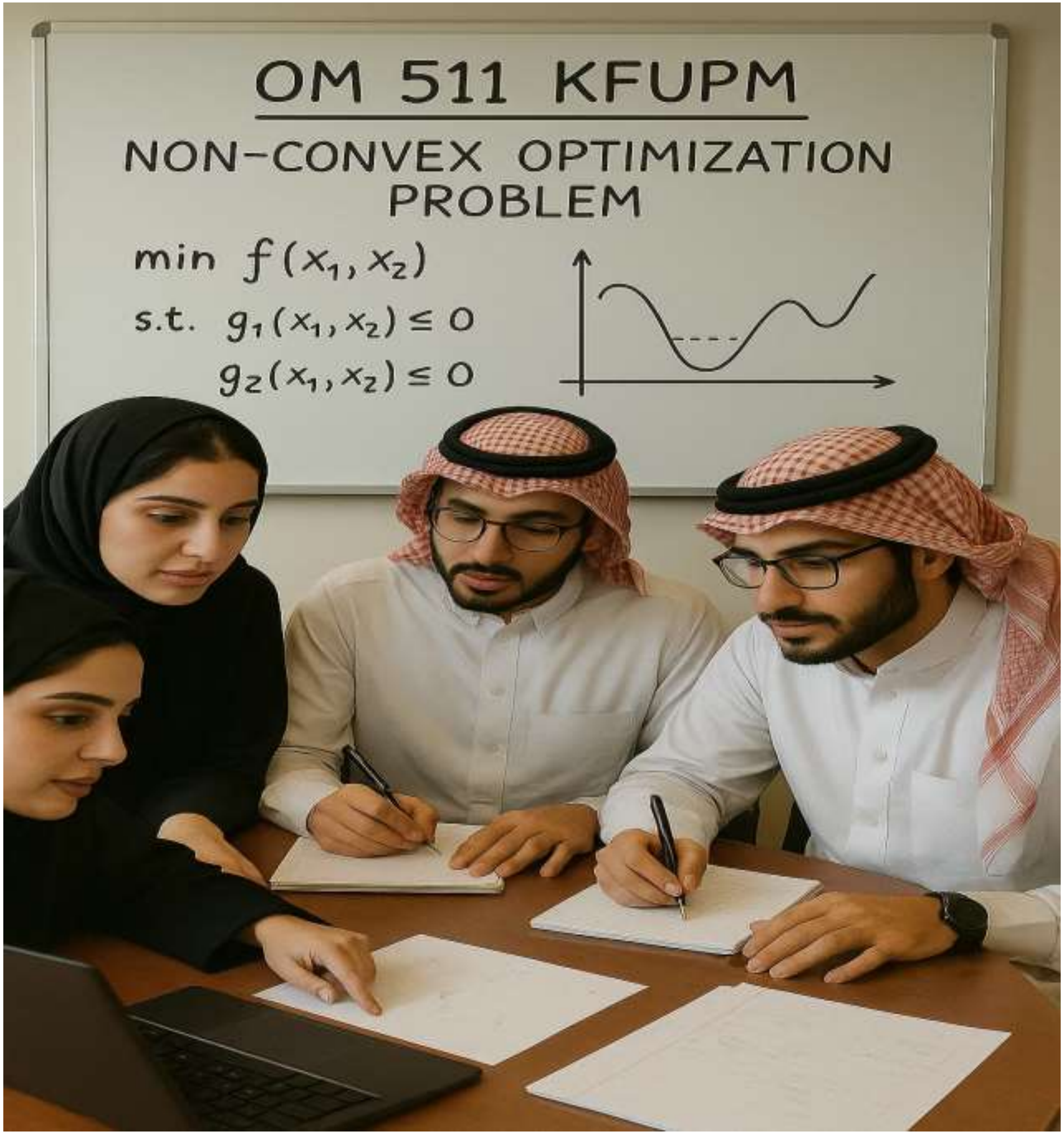
Convex optimization

Economic Dispatch in Power Grids: Minimizing generation cost subject to supply-demand balance and capacity constraints (convex quadratic programming).

Airline Crew Scheduling (relaxed): Convex approximations before integer rounding.

Production Planning: Minimizing convex production and holding costs under linear capacity constraints.

Portfolio Optimization: Minimizing risk (variance) subject to expected return targets.


OM 511 KFUPM
NON-CONVEX OPTIMIZATION PROBLEM
$\min f(x_1, x_2)$
s.t. $g_1(x_1, x_2) \leq 0$
$g_2(x_1, x_2) \leq 0$

Domain	Convex Optimization Problem	Convex Non-Optimization Problem
Energy & Power Systems	Economic dispatch in power grids: minimize (unique) cost of generation subject to demand and capacity constraints.	Feasibility regions for safe operating limits of reactors, turbines, or boilers.
Manufacturing & Production	Model Predictive Control (MPC): convex linear programming (max or min) for process control.	Interval tolerance analysis to ensure parts fit in assembly without misalignment.
Finance & Economics	Portfolio optimization. Minimize risk subject to return constraints.	Calculate a range-interval on which the portfolio is minized.
Chemical, Oil & Gas	Fuel blending. Minimization under quality and composition constraints.	Oil reservoir well locations and drilling paths are non-convex due to geological constraints and nonlinear flow dynamics.
Machine Learning & AI in Industry	SVM classification for fault detection.	Deep Neural Network Training: Loss functions (with ReLU, sigmoid, etc.) are highly non-convex, leading to many local minima/saddle points.
Robotics & Industrial Automation	Convex trajectory optimization for robotic arm movement (SOCP/QP).	Robot Motion Planning in Cluttered Environments: Obstacles create non-convex feasible spaces for trajectories.

Generic algorithm for solving optimization problems

Algorithm 1 Generic modern optimization method

Inputs: f, C ▷ f is the evaluation function, C includes control parameters
$S \leftarrow initialization(C)$ ▷ S is a solution or population
$i \leftarrow 0$ ▷ i is the number of iterations of the method
while not $termination\_criteria(S, f, C, i)$ **do**
    $S' \leftarrow change(S, f, C, i)$ ▷ new solution or population
    $B \leftarrow best(S, S', f, C, i)$ ▷ store the best solution
    $S \leftarrow select(S, S', f, C, i)$ ▷ solution or population for next iteration
    $i \leftarrow i + 1$
end while
Output: B ▷ the best solution

Problem formulation

The process of translating the **verbal statement of a problem into a mathematical statement**.

Formulating models is an art that can only be mastered with practice and experience.

Even though every problem has some unique features, most problems also have common features

Problem formulation

Some practical advices before starting

- Understand the problem thoroughly
- Describe the objective (max, min, punctual?)
- Identify the number of constrains
- Describe each constraint
- Define the Decision Variables
- Write the Objective in Terms of the Decision Variables
- Write the Constraints in Terms of the Decision Variables

Practical example

The director of manufacturing analysed each of the operations and concluded that if the company produces a medium-priced standard model, each bag will require 7/10 hour in the cutting and dyeing department, 1.0 hour in the sewing department, 1.0 hour in the finishing department, and 1/10 hour in the inspection and packaging department. The more expensive deluxe model will require 1.0 hour for cutting and dyeing, 5/6 hour for sewing, 2/3 hour for finishing, and 1/4 hour for inspection and packaging.

	Production Time (hours)	
Department	**Standard Bag**	**Deluxe Bag**
Cutting and Dyeing	7/10	1
Sewing	1/2	5/6
Finishing	1	2/3
Inspection and Packaging	1/10	1/4

Practical example

$$
\begin{aligned}
\text{Max} \quad & 10S + 9D \\
\text{subject to (s.t.)} \quad & \\
& \tfrac{7}{10}S + 1D \le 630 \quad \text{Cutting and dyeing} \\
& \tfrac{1}{2}S + \tfrac{5}{6}D \le 600 \quad \text{Sewing} \\
& 1S + \tfrac{2}{3}D \le 708 \quad \text{Finishing} \\
& \tfrac{1}{10}S + \tfrac{1}{4}D \le 135 \quad \text{Inspection and packaging} \\
& S, D \ge 0
\end{aligned}
$$

Graphical method

- **What is Linear Programming?**
  - A mathematical technique for optimizing (maximizing or minimizing) a linear objective function, subject to a set of linear constraints.

- **Purpose of the Graphical Method:**
  - A visual approach to solving linear programming problems with two decision variables.
  - Helps in finding the optimal solution by visually representing constraints and identifying the feasible region.

Graphical method

Five step approach for solving LP problems through the graphical method

Step 1: Define the Problem
- Identify the objective function and constraints.

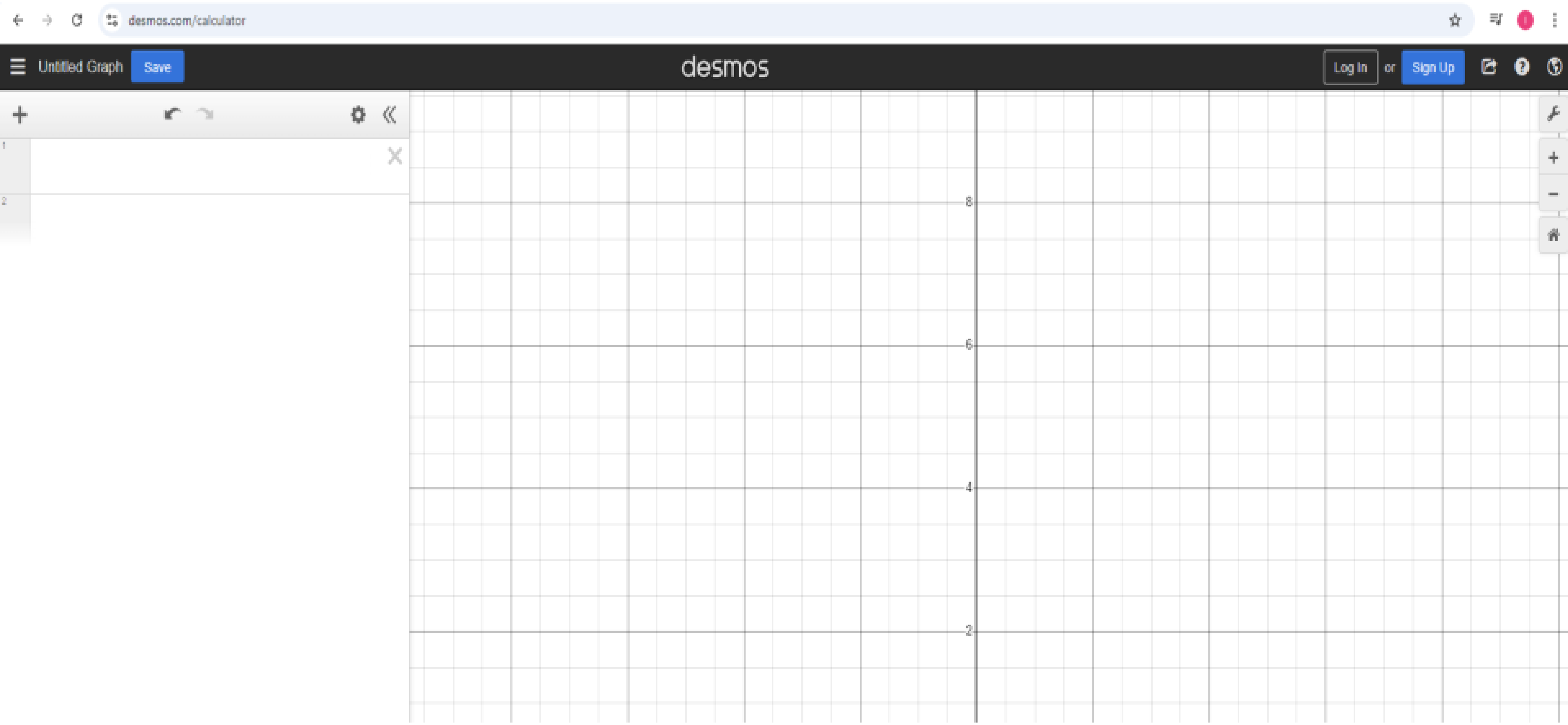
Step 2: Plot the Constraints
- Convert each inequality constraint into an equation.
- Plot each equation on a graph with decision variables

Step 3: Identify the Feasible Region
- The feasible region is where all the constraints overlap.
- Represents all possible solutions that satisfy the constraints.

Step 4: Locate the Corner Points
- The optimal solution lies at one of the corner points (vertices) of the feasible region.
- Find these points by calculating the intersection of the constraint lines.

Step 5: Evaluate the Objective Function
- Calculate the value of the objective function at each corner point.
- The point that gives the highest (or lowest, if minimizing) value is the optimal solution.

Graphical method



https://www.desmos.com/calculator

Graphical method

- **Problem Statement:**
  - Maximize Z=10S + 9D

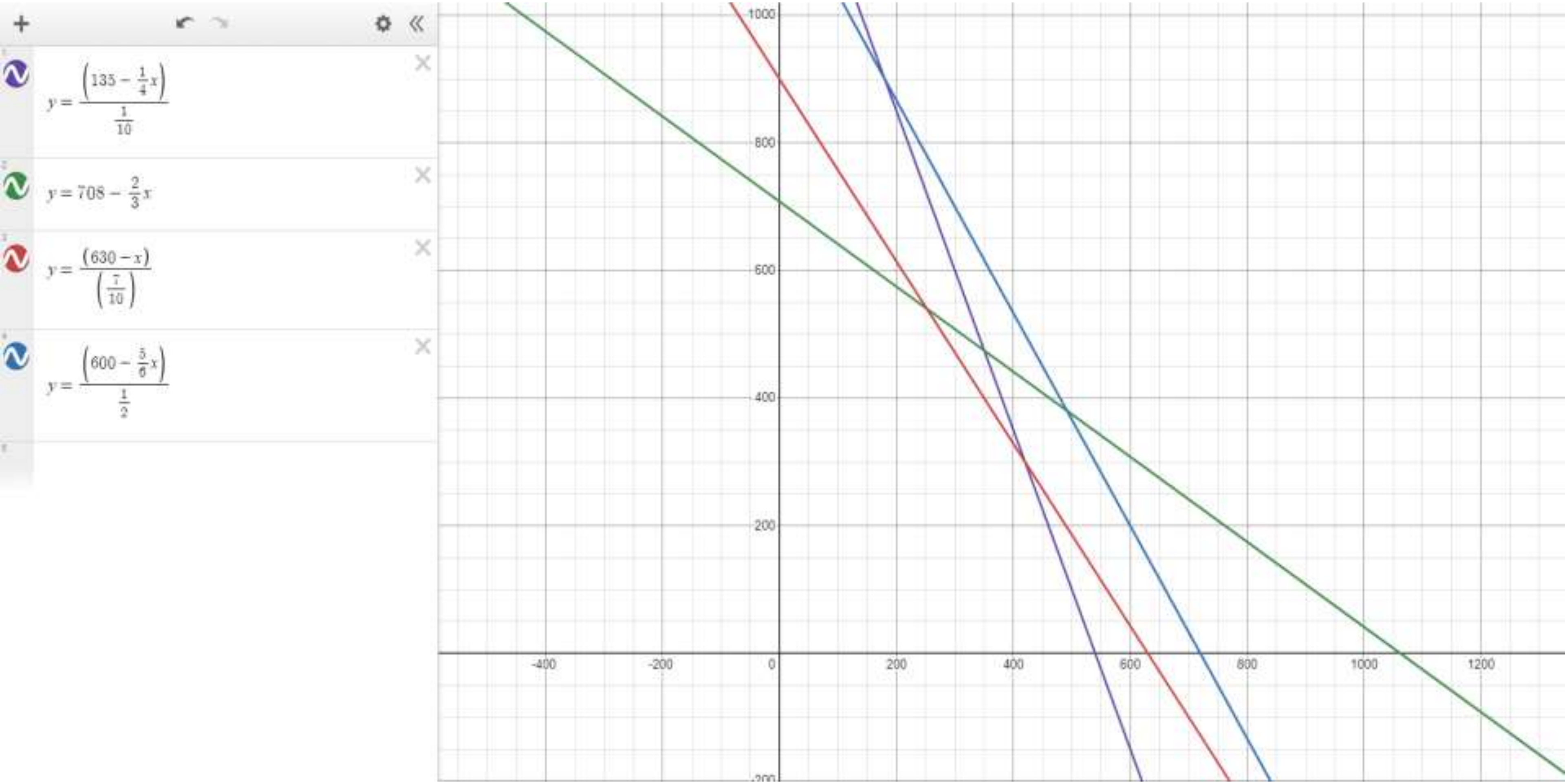
  - Subject to:

$$\tfrac{7}{10}S + 1D \le 630 \quad \text{Cutting and dyeing}$$
$$\tfrac{1}{2}S + \tfrac{5}{6}D \le 600 \quad \text{Sewing}$$
$$1S + \tfrac{2}{3}D \le 708 \quad \text{Finishing}$$
$$\tfrac{1}{10}S + \tfrac{1}{4}D \le 135 \quad \text{Inspection and packaging}$$
$$S, D \ge 0$$

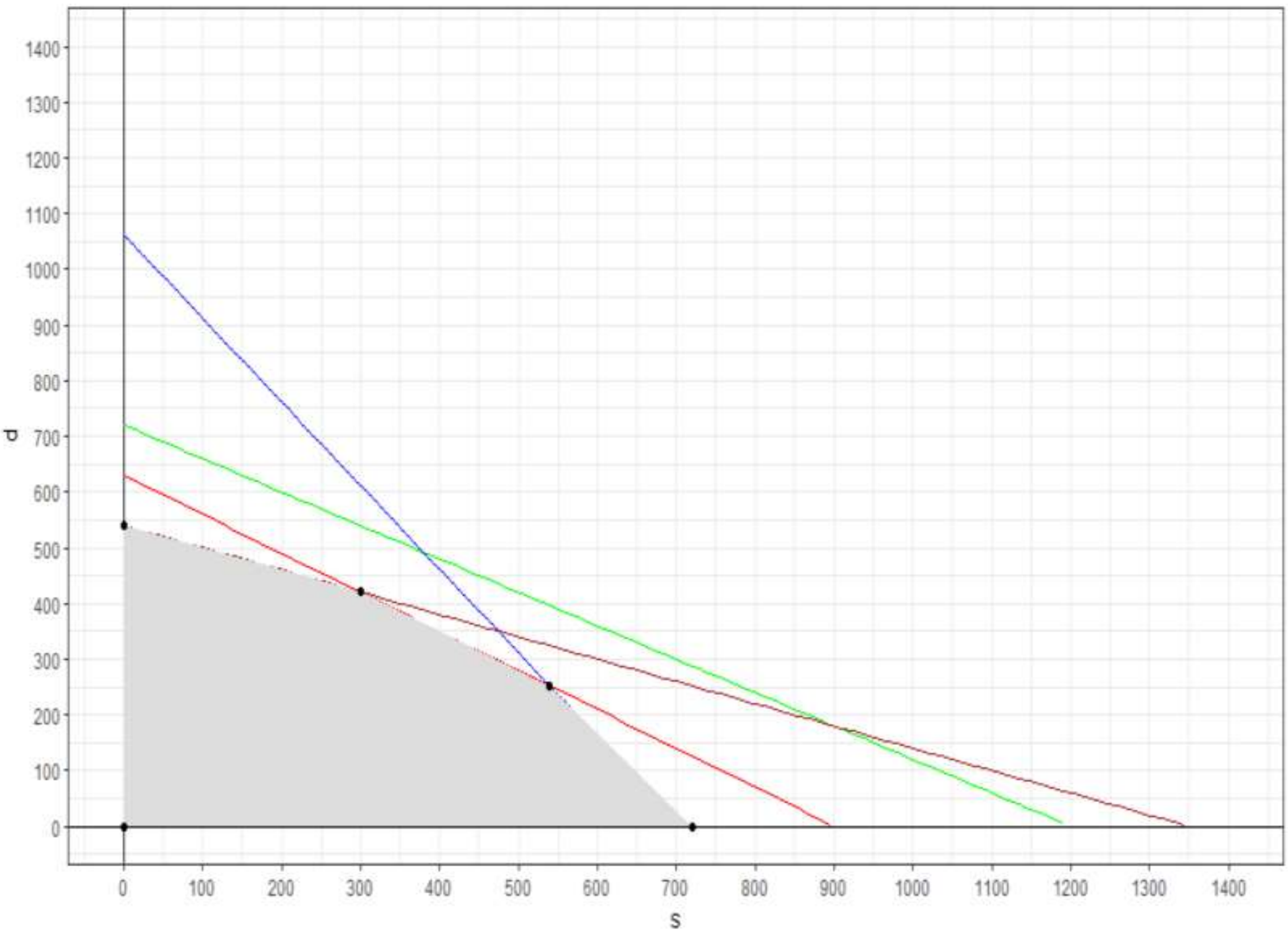
- **Graphical Representation:**
  - Show a simple graph with constraints plotted as lines.
  - Highlight the feasible region where all constraints overlap.

- **Solution:**
  - Indicate the corner points of the feasible region.
  - Show the calculation of the objective function at each point.
  - Identify the optimal solution based on the highest value of ZZZ.

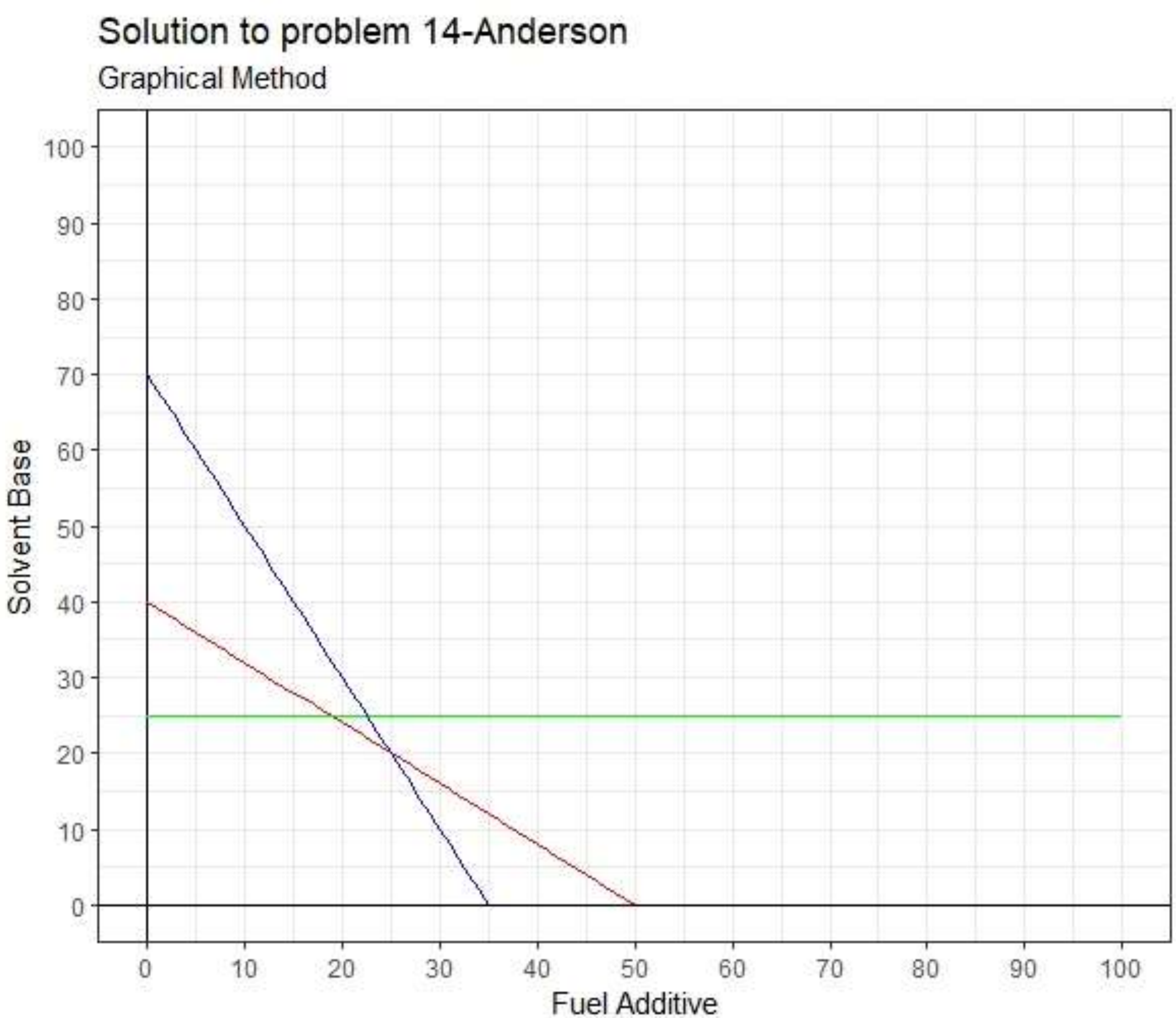
https://www.desmos.com/calculator



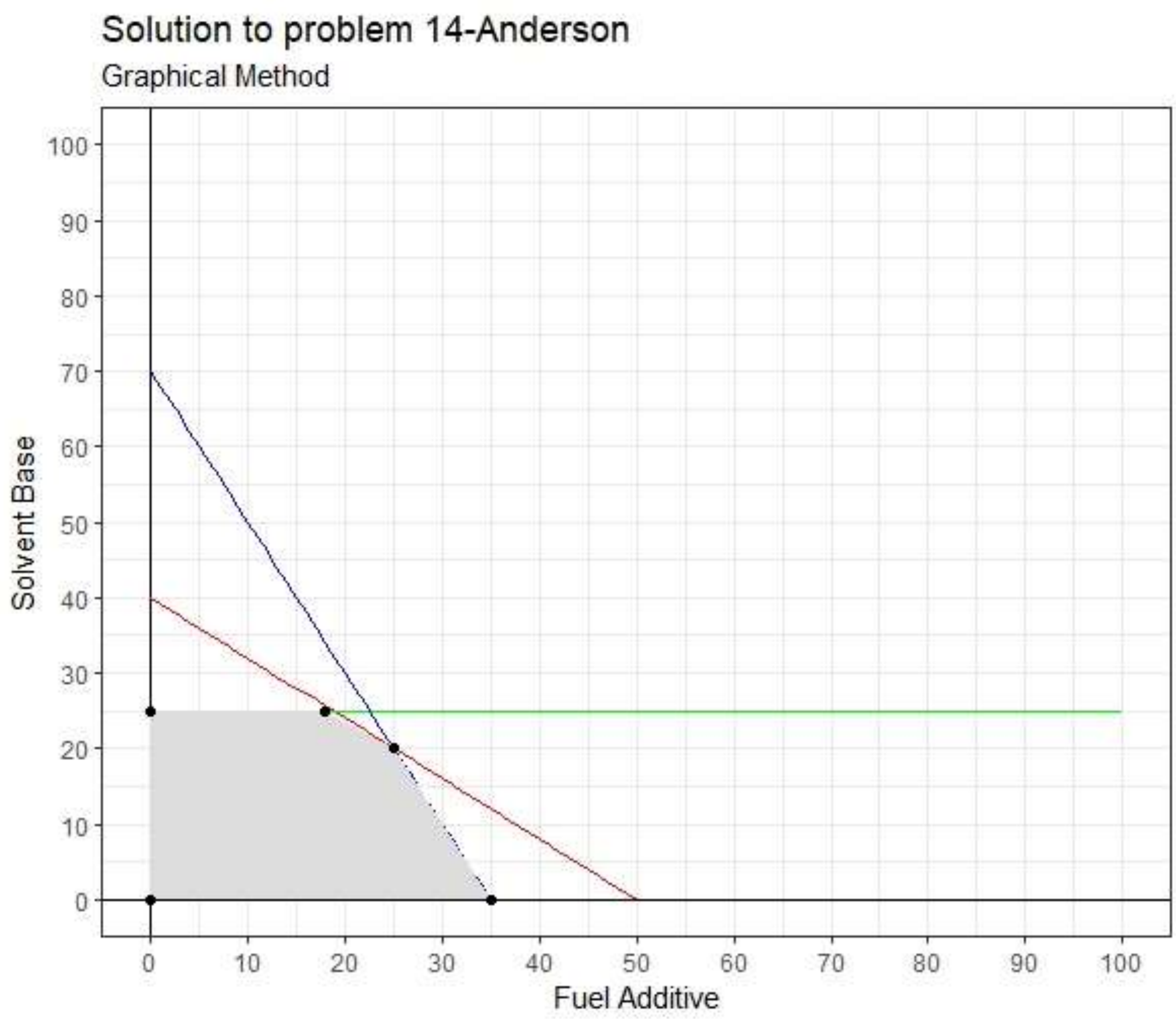
Solution problem in page 33, chapter 2, Anderson book. Graphical method



Solution to problem 14 Anderson book. Graphical Method



Solution to problem 14 Anderson book. Graphical Method



Exercise in classroom

Chapter 2. Anderson. Pp 70 #14

RMC, Inc., is a small firm that produces a variety of chemical products. In a particular production process, three raw materials are blended (mixed together) to produce two products: **a fuel additive (F) and a solvent base (S)**.

Each ton of **fuel additive (F)** is a mixture of 2/5 ton of material 1 and 3/5 of material 3. A ton of **solvent base (S)** is a mixture of 1/2 ton of material 1, 1/5 ton of material 2, and 3/10 ton of material 3.

The profit contribution is $40 for every ton of **fuel additive (F)** produced and $30 for every ton of **solvent base (S)** produced.

RMC's production is constrained by a limited availability of the three raw materials. For the current production period, RMC has available the following quantities of each raw material:

Raw Material	Amount Available for Production
Material 1	20 tons
Material 2	5 tons
Material 3	21 tons

Exercise in classroom

Chapter 2. Anderson. Pp 74 #24

Kelson Sporting Equipment, Inc., makes two different types of baseball gloves: a **regular model (R)** and a **catcher's model (C)**. The firm has 900 hours of production time available in its **cutting and sewing department**, 300 hours available in its **finishing department**, and 100 hours available in its **packaging and shipping department**. The production time requirements and the profit contribution per glove are given in the following table:

	Production Time (hours)			
Model	Cutting and Sewing	Finishing	Packaging and Shipping	Profit/Glove
Regular model	1	$\frac{1}{2}$	$\frac{1}{8}$	$5
Catcher's model	$\frac{3}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$8

Sustainability (vs) Cost

What do you guys might select?

How operations managers make value trade-offs decisions that involve sustainability and cost?

How elicitation methods impact common biases, such as sensitivity?

Are managers capable to make consistent value trade-offs that reflect rational prioritizations?

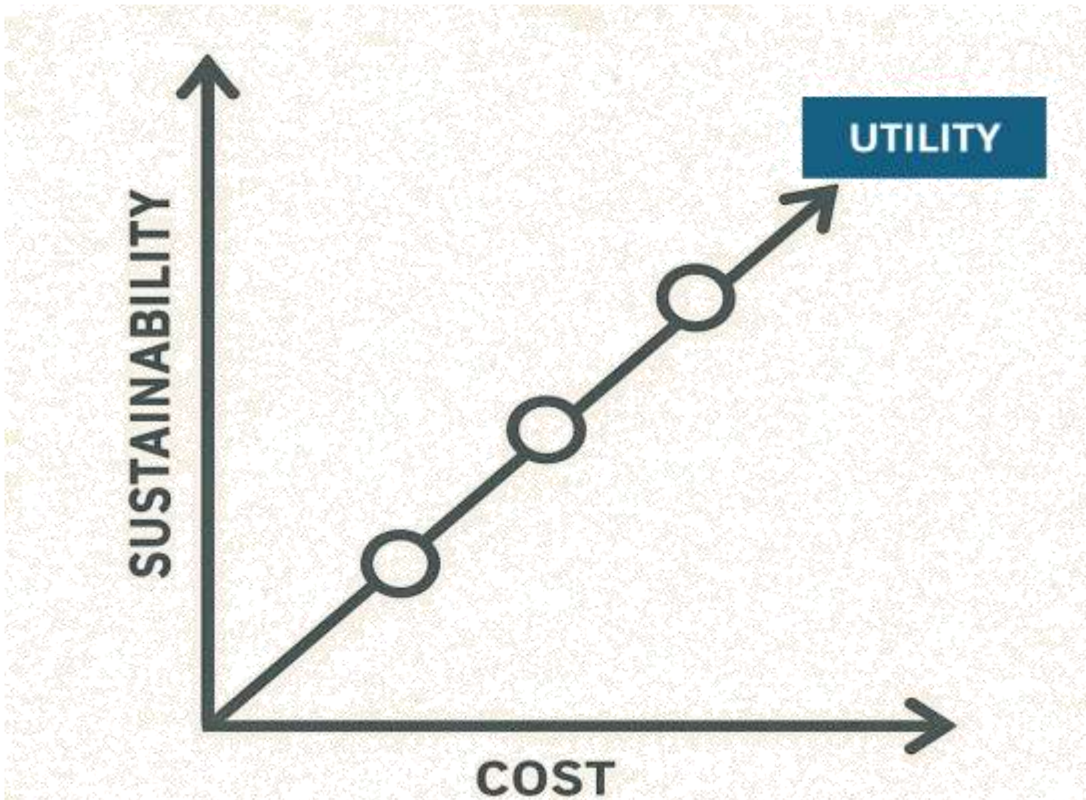
Are importance weights and trade-offs considered consistent when attribute ranges are explicitly specified?

Are managers’ prioritizations between sustainability and cost conditioned on whether:

a) Personal preferences or

b) Ranges of the underlying attributes.

A naïve perspective assumes a linear relationship between **Sustainability** and **Cost**



Our experiments show that the relationship is far from linear