

## Chapter 10—Two-Sample Tests

$$10.1 \quad df = n_1 + n_2 - 2 = 12 + 15 - 2 = 25$$

$$10.2 \quad (a) \quad S_p^2 = \frac{(n_1 - 1) \cdot S_1^2 + (n_2 - 1) \cdot S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(7) \cdot 4^2 + (14) \cdot 5^2}{7 + 14} = 22$$

$$t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{(42 - 34) - 0}{\sqrt{22 \left( \frac{1}{8} + \frac{1}{15} \right)}} = 3.8959$$

$$(b) \quad d.f. = (n_1 - 1) + (n_2 - 1) = 7 + 14 = 21$$

(c) Decision rule:  $d.f. = 21$ . If  $t_{STAT} > 2.5177$ , reject  $H_0$ .

(d) Decision: Since  $t = 3.8959$  is greater than the critical bound of 2.5177, reject  $H_0$ . There is enough evidence to conclude that the first population mean is larger than the second population mean.

10.3 Assume that you are sampling from two independent normal distributions having equal variances.

10.4

$$(\bar{X}_1 - \bar{X}_2) \pm t \sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} = (42 - 34) \pm 2.0796 \sqrt{22 \left( \frac{1}{8} + \frac{1}{15} \right)}$$

$$3.7296 \leq \mu_1 - \mu_2 \leq 12.2704$$

$$10.5 \quad df = n_1 + n_2 - 2 = 5 + 4 - 2 = 7$$

10.6 PHStat output:

<b>Data</b>	
Hypothesized Difference	<b>0</b>
Level of Significance	<b>0.01</b>
<b>Population 1 Sample</b>	
Sample Size	<b>5</b>
Sample Mean	<b>42</b>
Sample Standard Deviation	<b>4</b>
<b>Population 2 Sample</b>	
Sample Size	<b>4</b>
Sample Mean	<b>34</b>
Sample Standard Deviation	<b>5</b>

<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	<b>4</b>
Population 2 Sample Degrees of Freedom	<b>3</b>
Total Degrees of Freedom	<b>7</b>
Pooled Variance	<b>19.85714</b>
Difference in Sample Means	<b>8</b>
<b>t Test Statistic</b>	<b>2.676242</b>

<b>Upper-Tail Test</b>	
Upper Critical Value	<b>2.997949</b>
p-Value	<b>0.015856</b>
<b>Do not reject the null hypothesis</b>	

$$H_0: \mu_1 \leq \mu_2$$

$$H_1: \mu_1 > \mu_2$$

$$\text{Test statistic: } t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} = 2.6762$$

Decision: Since  $t_{STAT} = 2.6762$  is smaller than the upper critical bounds of 2.9979, do not reject  $H_0$ . There is not enough evidence of a difference in the means of the two populations.

- 10.7 (a)  $H_0: \mu_1 \leq \mu_2$  The mean amount spent is no higher for men than women.  
 $H_1: \mu_1 > \mu_2$  The mean amount spent is higher for men than women.
- (b) Type I error is the error made in concluding that the mean amount spent is higher for men than women when the mean amount spent is in fact no higher for men than women.
- (c) Type II error is the error made in concluding that the mean amount spent is no higher for men than women when the mean amount spent is in fact higher for men than women.
- (d) PHStat output:

<b>Separate-Variances t Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.01</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>600</b>
<b>Sample Mean</b>	<b>2401</b>
<b>Sample Standard Deviation</b>	<b>1200</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>700</b>
<b>Sample Mean</b>	<b>1527</b>
<b>Sample Standard Deviation</b>	<b>1000</b>
<b>Intermediate Calculations</b>	
Numerator of Degrees of Freedom	14657959.1837
Denominator of Degrees of Freedom	12535.6495
Total Degrees of Freedom	1169.3019
Degrees of Freedom	1169
Separate Variance Denominator	61.8755
Difference in Sample Means	874
<b>Separate-Variance t Test Statistic</b>	<b>14.1251</b>
<b>Upper-Tail Test</b>	
<b>Upper Critical Value</b>	<b>2.3263</b>
<b>p-Value</b>	<b>0.0000</b>
<b>Reject the null hypothesis</b>	

$$t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = 14.1251$$

Decision: Since  $t_{STAT} = 14.1251$  is greater than the critical bound of 2.3263, reject  $H_0$ . There is evidence that the mean amount spent is higher for men than for women.

10.8 (a) PHStat output:

Pooled-Variance <i>t</i> Test for the Difference Between Two Means			
(assumes equal population variances)			
<b>Data</b>		<b>Confidence Interval Estimate for the Difference Between Two Means</b>	
Hypothesized Difference	0	<b>Data</b>	
Level of Significance	0.05	Confidence Level	95%
<b>Population 1 Sample</b>		<b>Intermediate Calculations</b>	
Sample Size	59	Degrees of Freedom	116
Sample Mean	28.5	<i>t</i> Value	1.980625937
Sample Standard Deviation	8.6	Interval Half Width	3.01117417
<b>Population 2 Sample</b>		<b>Confidence Interval</b>	
Sample Size	59	Interval Lower Limit	5.78882583
Sample Mean	19.7	Interval Upper Limit	11.81117417
Sample Standard Deviation	7.9		
<b>Intermediate Calculations</b>			
Population 1 Sample Degrees of Freedom	58		
Population 2 Sample Degrees of Freedom	58		
Total Degrees of Freedom	116		
Pooled Variance	68.185		
Difference in Sample Means	8.8		
<i>t</i> Test Statistic	5.788276		
<b>Upper-Tail Test</b>			
Upper Critical Value	1.658096		
<i>p</i> -Value	3.09E-08		
<b>Reject the null hypothesis</b>			

$$H_0: \mu_1 \leq \mu_2$$

$$H_1: \mu_1 > \mu_2$$

where population 1 = children who watched food ads

population 2 = children who do not watch food ads

Decision rule: If *p*-value < 0.05, reject  $H_0$ .

$$\text{Test statistic: } t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} = 5.7883, df = 116$$

*p*-value is virtually 0.

Decision: Since the *p*-value is smaller than 0.05, reject  $H_0$ . There is enough evidence that the mean amount of Goldfish crackers eaten was significantly higher for the children who watched food ads.

$$(b) \quad (\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} \sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} = (28.5 - 19.7) \pm 1.9806 \sqrt{68.185 \left( \frac{1}{59} + \frac{1}{59} \right)}$$

$$5.7888 \leq \mu_1 - \mu_2 \leq 11.8112$$

- (c) The results cannot be compared because (a) is a one-tail test and (b) is a confidence interval that is comparable only to the results of a two-tail test.

- 10.9 (a)  $H_0 : \mu_1 = \mu_2$  Mean times to clear problems at Office I and Office II are the same.  
 $H_1 : \mu_1 \neq \mu_2$  Mean times to clear problems at Office I and Office II are different.

PHStat output:

t Test for Differences in Two Means	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	20
Sample Mean	2.214
Sample Standard Deviation	1.718039
Population 2 Sample	
Sample Size	20
Sample Mean	2.0115
Sample Standard Deviation	1.891706
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	19
Population 2 Sample Degrees of Freedom	19
Total Degrees of Freedom	38
Pooled Variance	3.265105
Difference in Sample Means	0.2025
t-Test Statistic	0.354386
Two-Tailed Test	
Lower Critical Value	-2.02439
Upper Critical Value	2.024394
p-Value	0.725009
Do not reject the null hypothesis	

Since the  $p$ -value of 0.725 is greater than the 5% level of significance, do not reject the null hypothesis. There is not enough evidence to conclude that the mean time to clear problems in the two offices is different.

- (b)  $p$ -value = 0.725. The probability of obtaining a sample that will yield a  $t$  test statistic more extreme than 0.3544 is 0.725 if, in fact, the mean waiting times between Office 1 and Office 2 are the same.
- (c) We need to assume that the two populations are normally distributed.

$$(d) \quad (\bar{X}_1 - \bar{X}_2) + t \sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} = (2.214 - 2.0115) + 2.0244 \sqrt{3.2651 \left( \frac{1}{20} + \frac{1}{20} \right)}$$

$$-0.9543 \leq \mu_1 - \mu_2 \leq 1.3593$$

Since the Confidence Interval contains 0, we cannot claim that there's a difference between the two means.

- 10.10 (a)  $H_0: \mu_1 = \mu_2$  where Populations: 1 = Males, 2 = Females  
 Mean computer anxiety experienced by males and females is the same.  
 $H_1: \mu_1 \neq \mu_2$   
 Mean computer anxiety experienced by males and females is different.

Decision rule:  $d.f. = 170$ . If  $t_{STAT} < -1.974$  or  $t_{STAT} > 1.974$ , reject  $H_0$ .

Test statistic:

$$S_p^2 = \frac{(n_1 - 1) \cdot S_1^2 + (n_2 - 1) \cdot S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(99) \cdot 13.35^2 + (71) \cdot 9.42^2}{99 + 71} = 140.8489$$

$$t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} = 1.859$$

Decision: Since  $t_{STAT} = 1.859$  is between the lower and upper critical bound of  $-1.974$  and  $1.974$ , do not reject  $H_0$ . There is not enough evidence to conclude that the mean computer anxiety experienced by males and females is different.

- (b) Using PHStat, the  $p$ -value = 0.0648. The probability of obtaining a sample that yields a  $t$  test statistic farther away from 0 in either direction is 0.0648 if there is no difference in the mean computer anxiety experienced by males and females.
- (c) In order to use the pooled-variance  $t$  test, you need to assume that the populations are normally distributed with equal variances.

10.11 (a) PHStat output:

Pooled-Variance <i>t</i> Test for the Difference Between Two Means			
(assumes equal population variances)			
Data		Confidence Interval Estimate	
Hypothesized Difference	0	for the Difference Between Two Means	
Level of Significance	0.05		
Population 1 Sample		Data	
Sample Size	29	Confidence Level	95%
Sample Mean	234.6552		
Sample Standard Deviation	49.20676	Intermediate Calculations	
Population 2 Sample		Degrees of Freedom	43
Sample Size	16	<i>t</i> Value	2.016692173
Sample Mean	303.125	Interval Half Width	48.45314711
Sample Standard Deviation	111.9952		
		Confidence Interval	
Intermediate Calculations		Interval Lower Limit	-116.9229747
Population 1 Sample Degrees of Freedom	28	Interval Upper Limit	-20.01668047
Population 2 Sample Degrees of Freedom	15		
Total Degrees of Freedom	43		
Pooled Variance	5952.1		
Difference in Sample Means	-68.4698		
<i>t</i> Test Statistic	-2.84982		
Two-Tail Test			
Lower Critical Value	-2.01669		
Upper Critical Value	2.016692		
<i>p</i> -Value	0.00669		
Reject the null hypothesis			

 $H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$  where Populations: 1 = subcompact, 2 = compact
Decision rule:  $d.f. = 43$ . If  $|t_{STAT}| > 2.0167$ , reject  $H_0$ .

$$t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = -2.8498$$

Decision: Since  $t_{STAT} = -2.8498$  is smaller the lower critical value of  $-2.0167$ , reject  $H_0$ . There is enough evidence of a difference in the mean battery life between the two types of digital cameras.

- (b)  $p$ -value = 0.0067. The probability of obtaining a sample that yields a  $t$  test statistic farther away from 0 in either direction is 0.0067 if there is no difference in the mean battery life between the two types of digital cameras.

10.11 (c)

$$\text{cont. } (\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} \sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} = (234.6552 - 303.125) \pm 2.0167 \sqrt{5952.1 \left( \frac{1}{29} + \frac{1}{16} \right)}$$

$$-116.9230 \leq \mu_1 - \mu_2 \leq -20.0167$$

You are 95% confident that the difference between the population mean battery life of the two types of digital cameras is somewhere between -116.9230 and -20.0167.

- 10.12 (a)  $H_0: \mu_1 = \mu_2$  Mean waiting times of Bank 1 and Bank 2 are the same.  
 $H_1: \mu_1 \neq \mu_2$  Mean waiting times of Bank 1 and Bank 2 are different.

PHStat output:

t Test for Differences in Two Means	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	15
Sample Mean	4.286667
Sample Standard Deviation	1.637985
Population 2 Sample	
Sample Size	15
Sample Mean	7.114667
Sample Standard Deviation	2.082189
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	14
Population 2 Sample Degrees of Freedom	14
Total Degrees of Freedom	28
Pooled Variance	3.509254
Difference in Sample Means	-2.828
t-Test Statistic	-4.13431
Two-Tailed Test	
Lower Critical Value	-2.04841
Upper Critical Value	2.048409
p-Value	0.000293
Reject the null hypothesis	

Since the  $p$ -value of 0.000293 is less than the 5% level of significance, reject the null hypothesis. There is enough evidence to conclude that the mean waiting time is different in the two banks.

- (b)  $p$ -value = 0.000293. The probability of obtaining a sample that will yield a  $t$  test statistic more extreme than -4.13431 is 0.000293 if, in fact, the mean waiting times of Bank 1 and Bank 2 are the same.
- (c) We need to assume that the two populations are normally distributed.

$$(d) (\bar{X}_1 - \bar{X}_2) \pm t \sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} = (4.2867 - 7.1147) \pm 2.0484 \sqrt{3.5093 \left( \frac{1}{15} + \frac{1}{15} \right)}$$

$$-4.2292 \leq \mu_1 - \mu_2 \leq -1.4268$$

You are 95% confident that the difference in mean waiting time between Bank 1 and Bank 2 is between -4.2292 and -1.4268 minutes.



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10.13  $H_0 : \mu_1 = \mu_2$  Mean waiting times of Bank 1 and Bank 2 are the same.

$H_1 : \mu_1 \neq \mu_2$  Mean waiting times of Bank 1 and Bank 2 are different.

PHStat output:

<b>Separate-Variances <i>t</i> Test for the Difference Between Two Means</b>	
(assumes unequal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>15</b>
<b>Sample Mean</b>	<b>4.286666667</b>
<b>Sample Standard Deviation</b>	<b>1.637985115</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>15</b>
<b>Sample Mean</b>	<b>7.114666667</b>
<b>Sample Standard Deviation</b>	<b>2.082189324</b>
<b>Intermediate Calculations</b>	
Numerator of Degrees of Freedom	0.2189
Denominator of Degrees of Freedom	0.0083
Total Degrees of Freedom	26.5293
Degrees of Freedom	26
Separate Variance Denominator	0.6840
Difference in Sample Means	-2.828
<b>Separate-Variance <i>t</i> Test Statistic</b>	<b>-4.1343</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.0555</b>
<b>Upper Critical Value</b>	<b>2.0555</b>
<b><i>p</i>-Value</b>	<b>0.0003</b>
<b>Reject the null hypothesis</b>	

Since the *p*-value of 0.00031 is less than the 5% level of significance, reject the null hypothesis. There is enough evidence to conclude that the mean waiting times are different in the two banks.

Both *t* tests yield the same conclusion.

10.14 (a)  $H_0: \mu_1 = \mu_2$   $H_1: \mu_1 \neq \mu_2$

Excel output:

	<i>Untreated</i>	<i>Treated</i>
t-Test: Two-Sample Assuming Equal Variances		
Mean	165.0948	155.7314
Variance	41.6934168	62.4141
Observations	20	20
Pooled Variance	52.05375826	
Hypothesized Mean Difference	0	
df	38	
t Stat	4.104023608	
P(T<=t) one-tail	0.000103572	
t Critical one-tail	1.685953066	
P(T<=t) two-tail	0.000207144	
t Critical two-tail	2.024394234	

Decision: Since  $t_{STAT} = 4.104$  is greater than the upper critical bound of 2.024, reject  $H_0$ . There is evidence that the mean surface hardness of untreated steel plates is different from the mean surface hardness of treated steel plates.

- (b)  $p$ -value = 0.0002. The probability of obtaining two samples with a mean difference of 9.3634 or more is 0.0002 if the mean surface hardness of untreated steel plates is not different from the mean surface hardness of treated steel plates
- (c) Since both sample sizes are smaller than 30, you need to assume that the population of hardness of both untreated and treated steel plates is normally distributed.
- (d)

$$(\bar{X}_1 - \bar{X}_2) + t \sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$$

$$= (165.0948 - 155.7314) + 2.0244 \sqrt{52.0538 \left( \frac{1}{20} + \frac{1}{20} \right)}$$

$$4.7447 \leq \mu_1 - \mu_2 \leq 13.9821$$

You are 95% confident that the difference in the mean surface hardness between untreated and treated steel plates is between 4.7447 and 13.9821.

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10.15  $H_0: \mu_1 = \mu_2$   $H_1: \mu_1 \neq \mu_2$

PHStat output:

<b>Separate-Variances t Test for the Difference Between Two Means</b>	
(assumes unequal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>20</b>
<b>Sample Mean</b>	<b>165.0948</b>
<b>Sample Standard Deviation</b>	<b>6.457043968</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>20</b>
<b>Sample Mean</b>	<b>155.73135</b>
<b>Sample Standard Deviation</b>	<b>7.900259471</b>
<b>Intermediate Calculations</b>	
Numerator of Degrees of Freedom	27.0959
Denominator of Degrees of Freedom	0.7413
Total Degrees of Freedom	36.5520
Degrees of Freedom	36
Separate Variance Denominator	2.2815
Difference in Sample Means	9.36345
<b>Separate-Variance t Test Statistic</b>	<b>4.1040</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.0281</b>
<b>Upper Critical Value</b>	<b>2.0281</b>
<b>p-Value</b>	<b>0.0002</b>
<b>Reject the null hypothesis</b>	

Decision: Since  $t_{STAT} = 4.104$  is greater than the upper critical bounds 2.0281, reject  $H_0$ . There is evidence of a difference in the mean surface hardness between untreated and treated steel plates.

The value of pooled-variance  $t$  test statistic and the separate-variance  $t$  test statistic are almost identical while the critical bound of the pooled-variance  $t$  test is slightly smaller than that of the separate-variance  $t$  test because the degrees of freedom of the pooled-variance  $t$  test is two more than that of the separate-variance  $t$  test.

10.16 (a) PHStat output:

Pooled-Variance <i>t</i> Test for the Difference Between Two Means			
(assumes equal population variances)			
<b>Data</b>		<b>Confidence Interval Estimate for the Difference Between Two Means</b>	
Hypothesized Difference	0		
Level of Significance	0.05		
<b>Population 1 Sample</b>		<b>Data</b>	
Sample Size	50	Confidence Level	95%
Sample Mean	137		
Sample Standard Deviation	51.7	Intermediate Calculations	
<b>Population 2 Sample</b>		Degrees of Freedom	98
Sample Size	50	<i>t</i> Value	1.984467404
Sample Mean	231	Interval Half Width	23.88403599
Sample Standard Deviation	67.6		
		<b>Confidence Interval</b>	
Intermediate Calculations		Interval Lower Limit	-117.884036
Population 1 Sample Degrees of Freedom	49	Interval Upper Limit	-70.11596401
Population 2 Sample Degrees of Freedom	49		
Total Degrees of Freedom	98		
Pooled Variance	3621.325		
Difference in Sample Means	-94		
<i>t</i> Test Statistic	-7.81024		
<b>Two-Tail Test</b>			
Lower Critical Value	-1.98447		
Upper Critical Value	1.984467		
<i>p</i> -Value	6.43E-12		
<b>Reject the null hypothesis</b>			

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

where Populations: 1 = users under 12 years of age; 2 = users 13 to 17 years of age

Decision rule:  $d.f. = 98$ . If  $t_{STAT} < -1.9845$  or  $t_{STAT} > 1.9845$ , reject  $H_0$ .

Test statistic:

$$t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{(137 - 231) - 0}{\sqrt{3621.325 \left( \frac{1}{50} + \frac{1}{50} \right)}} = -7.8102$$

Decision: Since  $t_{STAT} = -7.8102$  is below the lower critical bound of -1.9845, reject  $H_0$ . There is enough evidence of a difference in the mean cellphone usage between cellphone users under 12 years of age and cellphone users 13 to 17 years of age.

- (b) You must assume that each of the two independent populations is normally distributed.

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10.17 (a)  $H_0: \mu_1 \geq \mu_2$  where Populations: 1 = unflawed, 2 = flawed

$$H_1: \mu_1 < \mu_2$$

Decision rule:  $d.f. = 56$ . If  $t < -1.6725$ , reject  $H_0$ .

Test statistic:

$$S_p^2 = \frac{(n_1 - 1) \cdot S_1^2 + (n_2 - 1) \cdot S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(17) \cdot 0.0219^2 + (39) \cdot 0.0840^2}{17 + 39} = 0.0051$$

$$t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} = -2.9047$$

Decision: Since  $t_{STAT} = -2.9047$  is less than the lower critical bound of  $-1.6725$ , reject  $H_0$ . There is enough evidence to conclude that the mean crack size is smaller for the unflawed specimens than for the flawed specimens.

(b)  $H_0: \mu_1 \geq \mu_2$  where Populations: 1 = unflawed, 2 = flawed

$$H_1: \mu_1 < \mu_2$$

Decision rule:  $d.f. = 49$ . If  $t < -1.6766$ , reject  $H_0$ .

t-Test: Two-Sample Assuming Unequal Variances

	Unflawed	flawed
Mean	0.035944	0.0946
Variance	0.000481	0.007059
Observations	18	40
Hypothesized Mean Difference	0	
df	49	
t Stat	-4.11472	
P(T<=t) one-tail	7.4E-05	
t Critical one-tail	1.676551	
P(T<=t) two-tail	0.000148	
t Critical two-tail	2.009574	

Since  $p$ -value is virtually zero and is smaller than 0.05, reject  $H_0$ . There is enough evidence to conclude that the mean crack size is lower for the unflawed specimens than for the flawed specimens.

(c) The conclusions in (a) and (b) are the same. Since the sample variance of the flawed sample is almost 15 times as big as that of the unflawed sample, the test in (b) is the appropriate test to perform assuming that both samples are drawn from normally distributed populations.

10.18  $d.f. = n - 1 = 20 - 1 = 19$ , where  $n$  = number of pairs of data

10.19  $d.f. = n - 1 = 15 - 1 = 14$ , where  $n$  = number of pairs of data

- 10.20 Excel output:  
t-Test: Paired Two Sample for Means

	A	B
Mean	24	25.55555556
Variance	7	3.527777778
Observations	9	9
Pearson Correlation	0.85524255	
Hypothesized Mean Difference	0	
Df	8	
t Stat	-3.277152121	
P(T<=t) one-tail	0.00561775	
t Critical one-tail	1.859548033	
P(T<=t) two-tail	0.011235501	
t Critical two-tail	2.306004133	

- (a) Define the difference in summated rating as the rating on brand A minus the rating on brand B.

$$H_0 : \mu_D = 0 \quad \text{vs.} \quad H_1 : \mu_D \neq 0$$

$$\text{Test statistic: } t_{STAT} = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}} = -3.2772, p\text{-value} = 0.0112$$

Decision: Since the  $p\text{-value} = 0.0112 < 0.05$ , reject  $H_0$ . There is enough evidence of a difference in the mean summated ratings between the two brands.

- (b) You must assume that the distribution of the differences between the two ratings is approximately normal.
- (c)  $p\text{-value}$  is 0.0112. The probability of obtaining a mean difference in ratings that gives rise to a test statistic that deviates from 0 by 3.2772 or more in either direction is 0.0112 if there is no difference in the mean summated ratings between the two brands.

(d)  $\bar{D} \pm t_{\frac{\alpha}{2}} \frac{S_D}{\sqrt{n}} = -1.5556 \pm 2.3060 \frac{1.4240}{\sqrt{9}} \quad -2.6501 \leq \mu_D \leq -0.4610$

You are 95% confident that the mean difference in summated ratings between brand A and brand B is somewhere between -2.6501 and -0.4610.

10.21 (a)  $H_0: \mu_D = 0$  vs.  $H_1: \mu_D \neq 0$

Excel Output:

t-Test: Paired Two Sample for Means

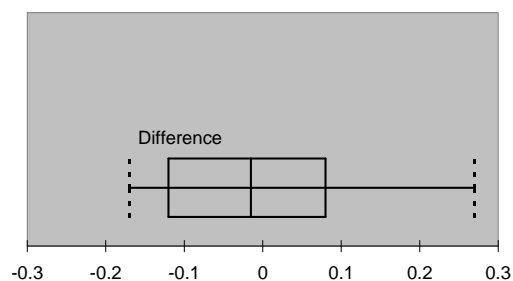
	<i>In-Line</i>	<i>Analytical lab</i>
Mean	6.490833	6.49375
Variance	1.179912	1.247928804
Observations	24	24
Pearson Correlation	0.994239	
Hypothesized Mean Difference	0	
df	23	
t Stat	-0.11692	
P(T<=t) one-tail	0.453968	
t Critical one-tail	1.71387	
P(T<=t) two-tail	0.907937	
t Critical two-tail	2.068655	

Test statistic:  $t_{STAT} = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}} = -0.1169$

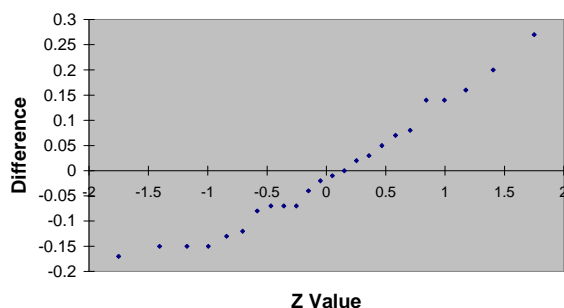
Decision: Since  $t_{STAT} = -0.1169$  falls between the lower and upper critical values  $\pm 2.0687$ , do not reject  $H_0$ . There is not enough evidence to conclude that there is a difference in the mean measurements in-line and from an analytical lab.

- (b) You must assume that the distribution of the differences between the mean measurements is approximately normal.
- (c)

Box-and-whisker Plot



Normal Probability Plot



The distributions appear to be right skewed.

10.21 (d)

$$\text{cont.} \quad \bar{D} \pm t \frac{S_D}{\sqrt{n}} = -0.0029 \pm 2.0687 \frac{0.1222}{\sqrt{24}} \quad -0.0545 \leq \mu_D \leq 0.0487$$

You are 95% confident that the difference in the mean measurements in-line and from an analytical lab is somewhere between -0.0545 and 0.0487.

10.22 (a)

Define the difference in price as the price of Costco minus the price of store-brands.

PHStat output:

t-Test: Paired Two Sample for Means

	Costco	Store Brand
Mean	6.049	6.025
Variance	34.17881	28.75756111
Observations	10	10
Pearson Correlation	0.920273316	
Hypothesized Mean Difference	0	
df	9	
t Stat	0.033176962	
P(T<=t) one-tail	0.487128804	
t Critical one-tail	1.833112923	
P(T<=t) two-tail	0.974257608	
t Critical two-tail	2.262157158	

$H_0: \mu_{\bar{D}} = 0$  There is no difference between the mean price of Costco purchases and store brand purchases.

$H_1: \mu_{\bar{D}} \neq 0$  There is a difference between the mean price of Costco purchases and store brand purchases.

Decision rule: If  $t_{STAT} < -2.2622$  or  $t_{STAT} > 2.2622$ , reject  $H_0$ .

$$\text{Test statistic: } t_{STAT} = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}} = 0.0332$$

Decision: Since  $t_{STAT} = 0.0332$  falls between the lower and upper critical bound, do not reject  $H_0$ . There is not enough evidence to conclude that there is a difference between the mean price of Costco purchases and store brand purchases.

(b) You must assume that the distribution of the differences between the price of Costco and price of store brand is approximately normal.

$$(c) \quad \bar{D} \pm t_{\alpha/2} \frac{S_D}{\sqrt{n}} = 0.02 \pm 2.2622 \left( \frac{2.2876}{\sqrt{10}} \right) \quad \$ -1.61 \leq \mu_D \leq \$ 1.66$$

You are 95% confident that the mean difference between the prices is between \$-1.61 and \$1.66.



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- 10.22 (d) cont. The results in (a) and (c) are the same. The hypothesized value of 0 for the difference between the mean price of Costco purchases and store brand purchases is not inside the 95% confidence interval and, hence, the null hypothesis that there is no difference between the mean price of Costco purchases and store brand purchases should not be rejected.

- 10.23 (a) Define the difference to be the number of pages devoted to advertisements in May 2008 minus the number of pages devoted to advertisement in May 2009.

$$H_0: \mu_D \leq 0 \quad \text{vs.} \quad H_1: \mu_D > 0$$

Excel output:

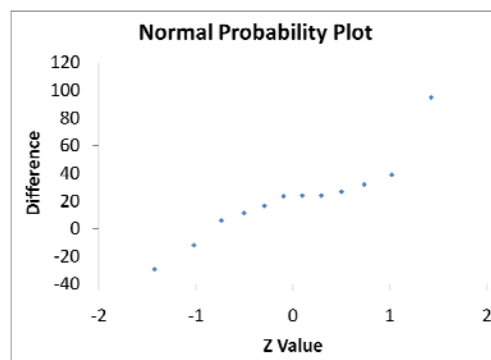
t-Test: Paired Two Sample for Means		
	2008	2009
Mean	107.1975	85.90583333
Variance	3199.168366	4145.798154
Observations	12	12
Pearson Correlation	0.884928463	
Hypothesized Mean Difference	0	
df	11	
t Stat	2.459363538	
P(T<=t) one-tail	0.015857275	
t Critical one-tail	1.795884814	
P(T<=t) two-tail	0.031714549	
t Critical two-tail	2.200985159	

$$\text{Test statistic: } t_{STAT} = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}} = 2.4594$$

Decision: Since  $t_{STAT} = 2.4594$  is greater than the critical value of 1.7959, reject  $H_0$ .

There is enough evidence to conclude that the mean number of pages devoted to advertisements in Men's Magazines is higher in May 2008 than in May 2009.

- (b) The differences are assumed to be normally distributed.  
(c)



The normal probability plot does not indicate severe departure from normality.

10.23 (d)  $\bar{D} \pm t_{\alpha/2} \frac{S_D}{\sqrt{n}} = 21.2917 \pm 2.2010 \frac{29.9901}{\sqrt{12}}$  2.24  $\leq \mu_D \leq$  40.35

cont. You are 95% confident that the difference in the mean number of pages devoted to advertisements in Men's Magazines between May 2008 and May 2009 is somewhere between 2.24 and 40.35.

- 10.24 (a) Define the difference in bone marrow microvessel density as the density before the transplant minus the density after the transplant and assume that the difference in density is normally distributed.

$$H_0: \mu_D \leq 0 \quad \text{vs.} \quad H_1: \mu_D > 0$$

Excel output:

t-Test: Paired Two Sample for Means

	Before	After
Mean	312.1429	226
Variance	15513.14	4971
Observations	7	7
Pearson Correlation	0.295069	
Hypothesized Mean Difference	0	
df	6	
t Stat	1.842455	
P(T<=t) one-tail	0.057493	
t Critical one-tail	1.943181	
P(T<=t) two-tail	0.114986	
t Critical two-tail	2.446914	

Test statistic:  $t_{STAT} = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}} = 1.8425$

Decision: Since  $t_{STAT} = 1.8425$  is less than the critical value of 1.943, do not reject  $H_0$ . There is not enough evidence to conclude that the mean bone marrow microvessel density is higher before the stem cell transplant than after the stem cell transplant.

- (b)  $p\text{-value} = 0.0575$ . The probability of obtaining a mean difference in density that gives rise to a  $t$  test statistic that deviates from 0 by 1.8425 or more is .0575 if the mean density is not higher before the stem cell transplant than after the stem cell transplant.

(c)  $\bar{D} \pm t \frac{S_D}{\sqrt{n}} = 86.1429 \pm 2.4469 \frac{123.7005}{\sqrt{7}}$   $-28.26 \leq \mu_D \leq 200.55$

You are 95% confident that the mean difference in bone marrow microvessel density before and after the stem cell transplant is somewhere between -28.26 and 200.55.

- (d) You must assume that the distribution of differences between the mean density of before and after stem cell transplant is approximately normal.

10.25 From the descriptive statistics provided in the Microsoft Excel output there does not seem to be any violation of the assumption of normality. The mean and median are similar and the skewness value is near 0. Without observing other graphical devices such as a stem-and-leaf display, boxplot, or normal probability plot, the fact that the sample size ( $n = 35$ ) is not very small enables us to assume that the paired  $t$  test is appropriate here. The Microsoft Excel output for the paired  $t$  test indicates that a significant improvement in mean performance ratings has occurred. The calculated  $t$  statistic of  $-2.699$  falls far below the one-tailed critical value of  $-1.6909$  using a  $0.05$  level of significance. The  $p$ -value is  $0.005376$ .

10.26 (a)  $H_0: \mu_{\bar{D}} \geq 0$

$H_1: \mu_{\bar{D}} < 0$

Decision rule:  $d.f. = 39$ . If  $t_{STAT} < -2.4258$ , reject  $H_0$ .

Test statistic:  $t_{STAT} = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}} = -9.372$

Decision: Since  $t_{STAT} = -9.372$  is less than the critical bound of  $-2.4258$ , reject  $H_0$ . There is enough evidence to conclude that the mean strength is lower at two days than at seven days.

(b) You must assume that the distribution of the differences between the mean strength of the concrete is approximately normal.

(c)  $p$ -value is virtually 0. The probability of obtaining a mean difference that gives rise to a test statistic that is  $-9.372$  or less when the null hypothesis is true is virtually 0.

10.27 (a)  $p_1 = \frac{X_1}{n_1} = \frac{50}{100} = 0.50, \quad p_2 = \frac{X_2}{n_2} = \frac{30}{100} = 0.30,$

and  $\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{50 + 30}{100 + 100} = 0.40$

$H_0: \pi_1 = \pi_2 \quad H_1: \pi_1 \neq \pi_2$

Decision rule: If  $Z_{STAT} < -1.96$  or  $Z_{STAT} > 1.96$ , reject  $H_0$ .

Test statistic:  $Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \frac{(0.50 - 0.30) - 0}{\sqrt{0.40(1 - 0.40)\left(\frac{1}{100} + \frac{1}{100}\right)}} = 2.89$

Decision: Since  $Z_{STAT} = 2.89$  is above the critical bound of  $1.96$ , reject  $H_0$ . There is sufficient evidence to conclude that the population proportions differ for group 1 and group 2.

(b)  $(p_1 - p_2) \pm Z \sqrt{\left(\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}\right)} = 0.2 \pm 1.96 \sqrt{\left(\frac{.5(.5)}{100} + \frac{.3(.7)}{100}\right)}$   
 $0.0671 \leq \pi_1 - \pi_2 \leq 0.3329$

$$10.28 \quad (a) \quad p_1 = \frac{X_1}{n_1} = \frac{45}{100} = 0.45, \quad p_2 = \frac{X_2}{n_2} = \frac{25}{50} = 0.50,$$

$$\text{and } \bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{45 + 25}{100 + 50} = 0.467$$

$$H_0: \pi_1 = \pi_2 \quad H_1: \pi_1 \neq \pi_2$$

Decision rule: If  $Z < -2.58$  or  $Z > 2.58$ , reject  $H_0$ .

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \frac{(0.45 - 0.50) - 0}{\sqrt{0.467(1 - 0.467)\left(\frac{1}{100} + \frac{1}{50}\right)}} = -0.58$$

Decision: Since  $Z_{STAT} = -0.58$  is between the critical bound of  $\pm 2.58$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the population proportion differs for group 1 and group 2.

$$(b) \quad (p_1 - p_2) \pm Z \sqrt{\left(\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)} = -0.05 \pm 2.5758 \sqrt{\left(\frac{.45(.55)}{100} + \frac{.5(.5)}{50}\right)}$$

$$-0.2727 \leq \pi_1 - \pi_2 \leq 0.1727$$

$$10.29 \quad (a) \quad p_1 = \frac{X_1}{n_1} = \frac{136}{240} = 0.567, \quad p_2 = \frac{X_2}{n_2} = \frac{224}{260} = 0.862,$$

$$\text{and } \bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{136 + 224}{240 + 260} = 0.72$$

$$H_0: \pi_1 = \pi_2 \quad H_1: \pi_1 \neq \pi_2 \quad \text{where Populations: 1 = males, 2 = females}$$

Decision rule: If  $Z_{STAT} < -2.58$  or  $Z_{STAT} > 2.58$ , reject  $H_0$ .

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \frac{(0.567 - 0.862) - 0}{\sqrt{0.72(1 - 0.72)\left(\frac{1}{240} + \frac{1}{260}\right)}} = -7.34$$

Decision: Since  $Z_{STAT} = -7.34$  is well below the lower critical bound of  $-2.58$ , reject  $H_0$ . There is sufficient evidence to conclude that a significant difference exists in the proportion of males and females who enjoy shopping for clothing.

(b)  $p$ -value = virtually zero. The probability of obtaining a difference in two sample proportions of 0.295 or more when the null hypothesis is true is virtually zero.

$$(c) \quad (p_1 - p_2) \pm Z \sqrt{\left(\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)} = -0.2949 \pm 2.5758 \sqrt{\left(\frac{.5667(.4333)}{240} + \frac{.8615(.1385)}{260}\right)}$$

$$-0.3940 \leq \pi_1 - \pi_2 \leq -0.1957$$

You are 99% confident that the difference in the proportions of males and females who enjoy shopping for clothing is between  $-0.3940$  and  $-0.1957$ .

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10.29 (d) (a)

cont.  $p_1 = \frac{X_1}{n_1} = \frac{206}{240} = 0.858, p_2 = \frac{X_2}{n_2} = \frac{224}{260} = 0.862, \bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = 0.86$

$H_0: \pi_1 = \pi_2 \quad H_1: \pi_1 \neq \pi_2$

Decision rule: If  $Z_{STAT} < -2.58$  or  $Z_{STAT} > 2.58$ , reject  $H_0$ .

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \frac{(0.858 - 0.862) - 0}{\sqrt{0.86(1 - 0.86)\left(\frac{1}{240} + \frac{1}{260}\right)}} = -0.103$$

Decision: Since  $Z_{STAT} = -0.103$  is between the critical bounds of  $\pm 2.58$ , do not reject  $H_0$ . There is insufficient evidence to conclude that a significant difference exists in the proportion of males and females who enjoy shopping for clothing.

(b)  $p$ -value = 0.9178. The probability of obtaining a difference in two sample proportions of 0.004 or more when the null hypothesis is true is 0.9178

(c)  $(p_1 - p_2) \pm Z \sqrt{\left(\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}\right)}$

$$= -0.0032 \pm 2.5758 \sqrt{\left(\frac{.8583(.1417)}{240} + \frac{.8615(.1385)}{260}\right)}$$

$$-0.0832 \leq \pi_1 - \pi_2 \leq 0.0768$$

You are 99% confident that the difference in the proportions of males and females who enjoy shopping for clothing is between  $-0.0832$  and  $0.0768$ .

10.30 (a)  $H_0: \pi_1 \leq \pi_2 \quad H_1: \pi_1 > \pi_2$

Population 1 = 2009, 2 = 2008

(b) PHStat output:

Z Test for Differences in Two Proportions	
<b>Data</b>	
Hypothesized Difference	0
Level of Significance	0.05
<b>Group 1</b>	
Number of Items of Interest	39
Sample Size	100
<b>Group 2</b>	
Number of Items of Interest	7
Sample Size	100
Intermediate Calculations	
Group 1 Proportion	0.39
Group 2 Proportion	0.07
Difference in Two Proportions	0.32
Average Proportion	0.23
Z Test Statistic	5.376822502
<b>Upper-Tail Test</b>	
Upper Critical Value	1.644853627
p-Value	3.79059E-08
Reject the null hypothesis	

- 10.30 (b) Decision rule: If  $Z_{STAT} > 1.6449$ , reject  $H_0$ .  
cont. Test statistic:

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{39 + 7}{100 + 100} = 0.23$$

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \frac{(0.39 - 0.07) - 0}{\sqrt{0.23(1 - 0.23)\left(\frac{1}{100} + \frac{1}{100}\right)}} = 5.3768$$

Decision: Since  $Z_{STAT} = 5.3768$  is greater than the critical bound of 1.6449, reject  $H_0$ . There is sufficient evidence to conclude that it takes more time to be removed from an e-mail list than it used to.

- (c) Yes, the result in (b) makes it appropriate to claim that it takes more time to be removed from an email list than it used to.

- 10.31 (a) PHStat output:

<b>Z Test for Differences in Two Proportions</b>	
<b>Data</b>	
Hypothesized Difference	<b>0</b>
Level of Significance	<b>0.05</b>
<b>Group 1</b>	
Number of Items of Interest	<b>22</b>
Sample Size	<b>50</b>
<b>Group 2</b>	
Number of Items of Interest	<b>3</b>
Sample Size	<b>50</b>
<b>Intermediate Calculations</b>	
Group 1 Proportion	0.44
Group 2 Proportion	0.06
Difference in Two Proportions	0.38
Average Proportion	0.25
<b>Z Test Statistic</b>	<b>4.387862046</b>
<b>Two-Tail Test</b>	
Lower Critical Value	<b>-1.959963985</b>
Upper Critical Value	<b>1.959963985</b>
<b>p-Value</b>	<b>1.1447E-05</b>
<b>Reject the null hypothesis</b>	

- (a)  $p_1 = \frac{22}{50} = 0.44$   
(b)  $p_2 = \frac{3}{50} = 0.06$

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- 10.31 (c)  $H_0: \pi_1 = \pi_2$   $H_1: \pi_1 \neq \pi_2$  where Populations: 1 = Concert, 2 = PDA  
cont. Decision rule: If  $Z_{STAT} < -1.96$  or  $Z_{STAT} > 1.96$ , reject  $H_0$ .

Test statistic:

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.44 - 0.06) - 0}{\sqrt{0.25(1 - 0.25)\left(\frac{1}{50} + \frac{1}{50}\right)}} = 4.3879$$

Decision: Since  $Z_{STAT} = 4.3879$  is greater than the upper critical bound of 1.96, reject  $H_0$ . There is evidence of a significant difference in the proportion willing to delay the date of the concert and the proportion willing to delay receipt of a new PDA.

- 10.32 (a) PHStat output:

Z Test for Differences in Two Proportions	
Data	
Hypothesized Difference	0
Level of Significance	0.01
Group 1	
Number of Items of Interest	707
Sample Size	1000
Group 2	
Number of Items of Interest	536
Sample Size	1000
Intermediate Calculations	
Group 1 Proportion	0.707
Group 2 Proportion	0.536
Difference in Two Proportions	0.171
Average Proportion	0.6215
Z Test Statistic	7.883654882
Two-Tail Test	
Lower Critical Value	-2.575829304
Upper Critical Value	2.575829304
p-Value	3.10862E-15
Reject the null hypothesis	

$$H_0: \pi_1 = \pi_2 \quad H_1: \pi_1 \neq \pi_2$$

Population: 1 = users over 70 years of age; 2 = users 12 to 50 years of age.

Decision rule: If  $|Z_{STAT}| > 2.5758$ , reject  $H_0$ .

Test statistic:

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.707 - 0.536) - 0}{\sqrt{0.6215(1 - 0.6215)\left(\frac{1}{1000} + \frac{1}{1000}\right)}} = 7.8837$$

Decision: Since  $Z_{STAT} = 7.8837$  is greater than the upper critical bound of 2.5758, reject  $H_0$ . There is sufficient evidence of a significant difference between the two age groups who believe that e-mail messages should be answered quickly.

- 10.32 (b)  $p$ -value is virtually 0. The probability of obtaining a difference in proportions that gives rise to a test statistic that deviates from 0 by 7.8847 or more in either direction is virtually 0 if there is no difference between the two age groups who believe that e-mail messages should be answered quickly.

- 10.33 (a) PHStat output:

Z Test for Differences in Two Proportions	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Group 1	
Number of Items of Interest	273
Sample Size	665
Group 2	
Number of Items of Interest	252
Sample Size	500
Intermediate Calculations	
Group 1 Proportion	0.410526316
Group 2 Proportion	0.504
Difference in Two Proportions	-0.093473684
Average Proportion	0.450643777
Z Test Statistic	-3.173792352
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
$p$ -Value	0.001504613
Reject the null hypothesis	

$$H_0: \pi_1 = \pi_2 \quad H_1: \pi_1 \neq \pi_2$$

Decision rule: If  $|Z_{STAT}| > 1.96$ , reject  $H_0$ .

Test statistic:

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{273 + 252}{665 + 500} = 0.4506$$

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.4105 - 0.504) - 0}{\sqrt{0.4506(1 - 0.4506)\left(\frac{1}{665} + \frac{1}{500}\right)}} = -3.1738$$

Decision: Since  $|Z_{STAT}| = 3.1738$  is larger than 1.96, reject  $H_0$ . There is sufficient evidence of a difference between consumer magazines and newspapers in the proportion of online-only content that is copy-edited as rigorously as print content.

- (b)  $p$ -value = 0.0015. The probability of obtaining a difference in proportions that gives rise to a test statistic that is 3.1738 or more away from 0 in either direction is 0.0015 if there is not a difference between consumer magazines and newspapers in the proportion of online-only content that is copy-edited as rigorously as print content.



10.33 (c) PHStat output:  
cont.

Z Test for Differences in Two Proportions	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Group 1	
Number of Items of Interest	379
Sample Size	665
Group 2	
Number of Items of Interest	296
Sample Size	500
Intermediate Calculations	
Group 1 Proportion	0.569924812
Group 2 Proportion	0.592
Difference in Two Proportions	-0.022075188
Average Proportion	0.579399142
Z Test Statistic	-0.755463126
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	0.449971149
Do not reject the null hypothesis	

$$H_0: \pi_1 = \pi_2 \quad H_1: \pi_1 \neq \pi_2$$

Decision rule: If  $|Z_{STAT}| > 1.96$ , reject  $H_0$ .

Test statistic:

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{379 + 296}{665 + 500} = 0.5794$$

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.5699 - 0.592) - 0}{\sqrt{0.5794(1 - 0.5794)\left(\frac{1}{665} + \frac{1}{500}\right)}} = -0.7555$$

Decision: Since  $|Z_{STAT}| = 0.7555$  is smaller than 1.96, do not reject  $H_0$ . There is not sufficient evidence of a difference between consumer magazines and newspapers in the proportion of online-only content that is fact-checked as rigorously as print content.

- (d)  $p$ -value = 0.45. The probability of obtaining a difference in proportions that gives rise to a test statistic that is 0.7555 or more away from 0 in either direction is 0.45 if there is not a difference between consumer magazines and newspapers in the proportion of online-only content that is fact-checked as rigorously as print content.

- 10.34 (a)
- $H_0: \pi_1 = \pi_2$
- $H_1: \pi_1 \neq \pi_2$

Z Test for Differences in Two Proportions	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Group 1	
Number of Items of Interest	670
Sample Size	1000
Group 2	
Number of Items of Interest	510
Sample Size	1000
Intermediate Calculations	
Group 1 Proportion	0.67
Group 2 Proportion	0.51
Difference in Two Proportions	0.16
Average Proportion	0.59
Z Test Statistic	7.274230368
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	3.48388E-13
Reject the null hypothesis	

Decision rule: If  $|Z_{STAT}| > 1.96$ , reject  $H_0$ .

Test statistic:

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 7.2742$$

Decision: Since  $|Z_{STAT}| = 7.2742$  is greater than 1.96, reject  $H_0$ . There is sufficient evidence of a difference between adult Internet users and Internet users age 12 – 17 in the proportion who oppose ads.

- (b)  $p$ -value is virtually 0. The probability of obtaining a difference in proportions that gives rise to a test statistic that deviates from 0 by 7.2742 or more in either direction is virtually 0 if there is not a difference between adult Internet users and Internet users age 12 – 17 in the proportion who oppose ads.

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- 10.35 (a)  $H_0: \pi_1 = \pi_2$  where Populations: 1 = under age 50, 2 = age above 50  
 $H_1: \pi_1 \neq \pi_2$

Z Test for Differences in Two Proportions	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Group 1	
Number of Items of Interest	470
Sample Size	1000
Group 2	
Number of Items of Interest	134
Sample Size	891
Intermediate Calculations	
Group 1 Proportion	0.47
Group 2 Proportion	0.150392817
Difference in Two Proportions	0.319607183
Average Proportion	0.319407721
Z Test Statistic	14.87967361
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	0
Reject the null hypothesis	

Decision rule: If  $|Z_{STAT}| > 1.96$ , reject  $H_0$ .

Test statistic:

$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 14.8797$$

Decision: Since  $|Z_{STAT}| = 14.8797$  is greater than 1.96, reject  $H_0$ . There is sufficient evidence of a significant difference in the proportion of users under age 50 and users 50 years and older that accessed the news on their cellphones.

- (b)  $p$ -value is virtually 0. The probability of obtaining a difference in proportions that gives rise to a test statistic that deviates from 0 in either direction by 14.8797 or more in either direction is virtually 0 if there is no difference in the proportion of users under age 50 and users 50 years and older that accessed the news on their cellphones.

$$(c) \quad (p_1 - p_2) \pm Z \sqrt{\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}}$$

$$= (0.47 - 0.1504) \pm 1.96 \sqrt{\frac{0.47(1 - 0.47)}{1000} + \frac{0.1504(1 - 0.1504)}{891}}$$

$$0.2808 \leq \pi_1 - \pi_2 \leq 0.3584$$

10.36 (a)  $\alpha=0.10, n_1=16, n_2=21, F_{0.10/2} = 2.20$

(b)  $\alpha=0.05, n_1=16, n_2=21, F_{0.05/2} = 2.57$

(c)  $\alpha=0.01, n_1=16, n_2=21, F_{0.01/2} = 3.50$

10.37 (a)  $\alpha=0.05, n_1=16, n_2=21, F_{0.05} = 2.20$

(b)  $\alpha=0.01, n_1=16, n_2=21, F_{0.01} = 3.09$

10.38 (a) You place the larger sample variance  $S^2 = 25$  in the numerator of  $F_{STAT}$ .

(b) 
$$F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{25}{16} = 1.5625$$

10.39 
$$F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{161.9}{133.7} = 1.2109$$

10.40 The degrees of freedom for the numerator is 24 and for the denominator is 24.

10.41  $\alpha=0.05, n_1=25, n_2=25, F_{0.05/2} = 2.27$

10.42 Since  $F_{STAT} = 1.2109$  is lower than  $F_{0.05/2} = 2.27$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different.

10.43 In testing the equality of two population variances, the  $F$ -test statistic is very sensitive to the assumption of normality for each population. If the populations are very right-skewed, the  $F$ -test should not be used.

10.44 (a)  $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

Decision rule: If  $F_{STAT} > 3.18$  reject  $H_0$ .

Test statistic: 
$$F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{47.3}{36.4} = 1.2995$$

Decision: Since  $F_{STAT} = 1.2995$  is less than  $F_{\alpha/2} = 3.18$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different.

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10.44 (b)  $H_0: \sigma_1^2 \leq \sigma_2^2$  The variance for population 1 is less than or equal to the variance for population 2.

$H_1: \sigma_1^2 > \sigma_2^2$  The variance for population 1 is greater than the variance for population 2.

Decision rule: If  $F_{STAT} > 2.62$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{47.3}{36.4} = 1.2995$$

Decision: Since  $F_{STAT} = 1.2995$  is less than the critical bound of  $F_\alpha = 2.62$ , do not reject  $H_0$ . There is not enough evidence to conclude that the variance for population 1 is greater than the variance for population 2.

10.45 (a) PHStat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	11
Sample Standard Deviation	0.012
Smaller-Variance Sample	
Sample Size	16
Sample Standard Deviation	0.005
Intermediate Calculations	
F Test Statistic	5.76
Population 1 Sample Degrees of Freedom	10
Population 2 Sample Degrees of Freedom	15
Upper-Tail Test	
Upper Critical Value	2.543719
p-Value	0.001333
Reject the null hypothesis	

$H_0: \sigma_1^2 \leq \sigma_2^2$  where Populations: 1 = Line A, 2 = Line B

$H_1: \sigma_1^2 > \sigma_2^2$

Decision rule: If  $F_{STAT} > 2.5437$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{0.012^2}{0.005^2} = 5.76$$

Decision: Since  $F_{STAT} = 5.76$  is greater than the critical bound of  $F_\alpha = 2.5437$ , reject  $H_0$ . There is enough evidence that the variance in line A is greater than the variance in line B.

(b)  $p\text{-value} = 0.001333$

The probability of obtaining a test statistic of 5.76 or larger is 0.001333 when the null hypothesis is true.

(c) The test assumes that the two populations are both normally distributed.

10.46 (a)  $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

Decision rule: If  $F_{STAT} > 1.556$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{13.35^2}{9.42^2} = 2.008$$

Decision: Since  $F_{STAT} = 2.008$  is greater than  $F_{\alpha/2} = 1.556$ , reject  $H_0$ . There is enough evidence to conclude that the two population variances are different.

(b)  $p\text{-value} = 0.0022$ . The probability of obtaining a sample that yields a test statistic more extreme than 2.008 is 0.0022 if the null hypothesis that there is no difference in the two population variances is true.

(c) The test assumes that the two populations are both normally distributed.

(d) Based on (a) and (b), a separate variance  $t$  test should be used.

10.47 (a)  $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

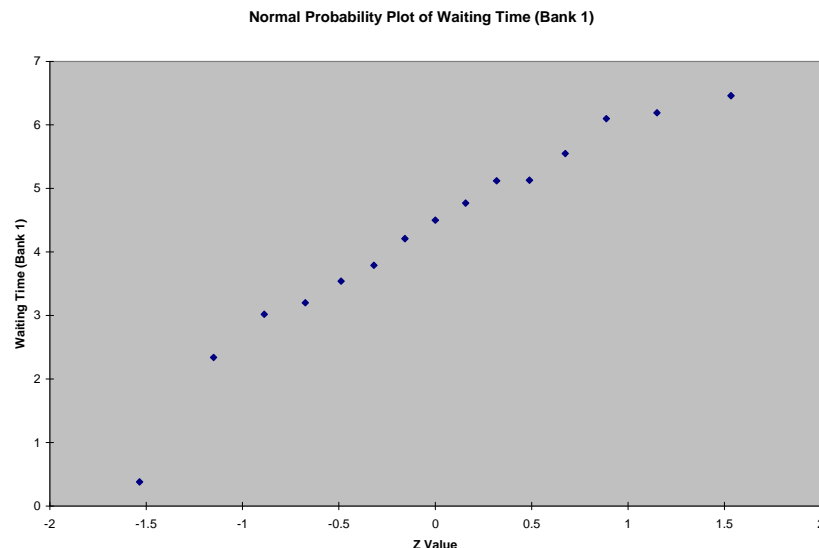
Decision rule: If  $F_{STAT} > 2.9786$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{2.0822^2}{1.6380^2} = 1.6159$$

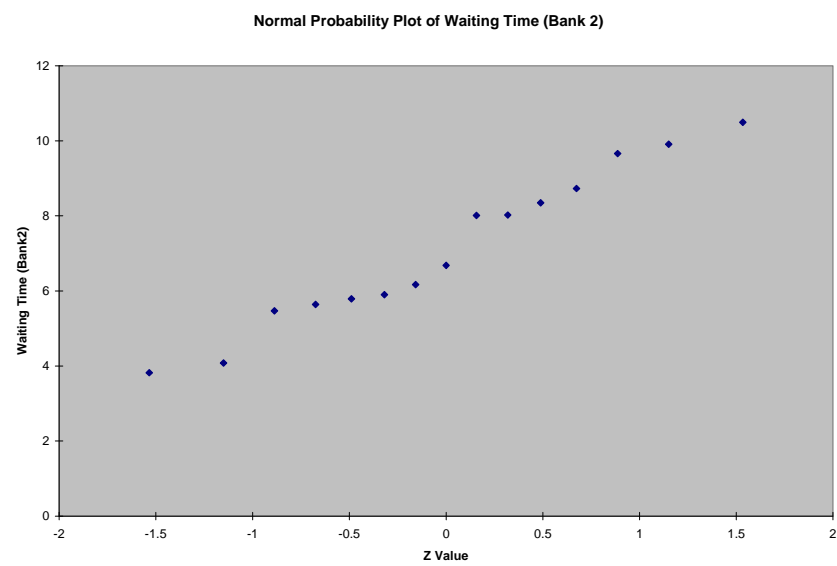
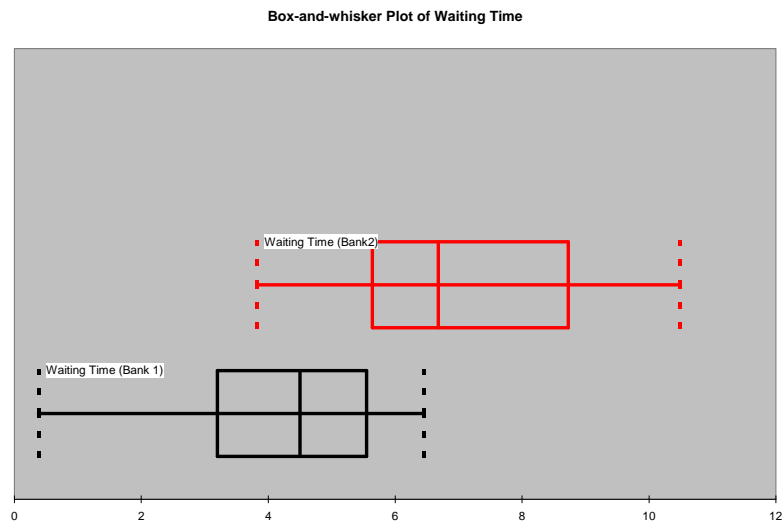
Decision: Since  $F_{STAT} = 1.6159$  is below the upper critical bound of  $F_{\alpha/2} = 2.9786$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different.

(b)  $p\text{-value} = 0.715$ . The probability of obtaining a sample that yields a test statistic more extreme than 1.6159 is 0.715 if the null hypothesis that there is no difference in the two population variances is true.

(c) The test assumes that the two populations are both normally distributed.



10.47 (c)  
cont.



10.47 (c)  
cont.

	<i>Waiting Time (Bank 1)</i>	<i>Waiting Time (Bank2)</i>
Mean	4.286666667	7.114666667
Standard Error	0.422925938	0.537618972
Median	4.5	6.68
Mode	#N/A	#N/A
Standard Deviation	1.637985115	2.082189324
Sample Variance	2.682995238	4.335512381
Kurtosis	0.832925217	-1.056273871
Skewness	-0.832946775	0.072493057
Range	6.08	6.67
Minimum	0.38	3.82
Maximum	6.46	10.49
Sum	64.3	106.72
Count	15	15
Interquartile range	2.35	3.09
1.33 * std dev	2.178520203	2.769311801
Range	6.08	6.67
6 * std dev	9.827910692	12.49313594

Both the normal probability plots and the boxplots suggest that the waiting times for both branches do not appear to be normally distributed. Waiting times for Bank 1 appear to be skewed to the left while the waiting times for Bank 2 are slightly skewed to the right. Hence, the  $F$  test for the difference in variances, which is sensitive to departure from the normality assumption, should not be used to test the equality of two variances. From the boxplots and the summary statistics, the two samples appear to have about the same amount of dispersion. Since the pooled-variance  $t$  test is robust to departure from the normality assumption, it can be used to test for the difference in means.

- (d) Based on the results of (a), it is appropriate to use the pooled-variance  $t$ -test to compare the means of the two branches.



10.48 (a) PHStat output:

<b>F Test for Differences in Two Variances</b>	
<b>Data</b>	
<b>Level of Significance</b>	<b>0.05</b>
<b>Larger-Variance Sample</b>	
<b>Sample Size</b>	<b>16</b>
<b>Sample Standard Deviation</b>	<b>111.9952</b>
<b>Smaller-Variance Sample</b>	
<b>Sample Size</b>	<b>29</b>
<b>Sample Standard Deviation</b>	<b>49.20676</b>
<b>Intermediate Calculations</b>	
<b>F Test Statistic</b>	<b>5.180229</b>
Population 1 Sample Degrees of Freedom	15
Population 2 Sample Degrees of Freedom	28
<b>Two-Tail Test</b>	
<b>Upper Critical Value</b>	<b>2.343847</b>
<b>p-Value</b>	<b>0.000181</b>
<b>Reject the null hypothesis</b>	

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

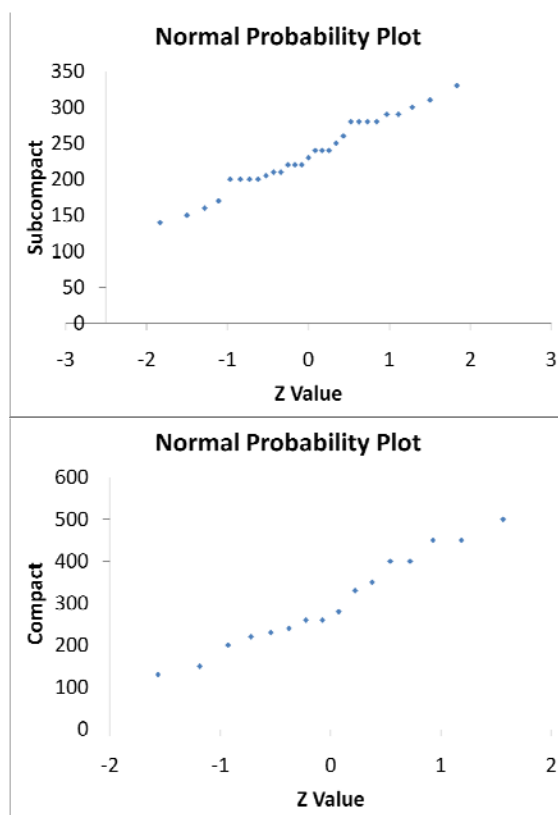
Decision rule: If  $F_{STAT} > 2.3438$ , reject  $H_0$ .

Test statistic:  $F_{STAT} = \frac{S_1^2}{S_2^2} = 5.1803$

Decision: Since  $F_{STAT} = 5.1803$  is greater than the upper critical value of 2.3438, reject  $H_0$ . There is enough evidence of a difference in the variability of the battery life between the two types of digital cameras.

(b)  $p$ -value = 0.0002. The probability of obtaining a sample that yields a test statistic greater than 5.1803 is 0.0002 if the null hypothesis that there is no difference in the two population variances is true.

- 10.48 (c) The test assumes that the two populations are both normally distributed.  
cont.



- The probability plots do not indicate any departure from the normality assumption.  
(d) Based on (a), a separate-variance  $t$  test should be used.

- 10.49 (a) PHStat output:

Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	50
Sample Standard Deviation	67.6
Smaller-Variance Sample	
Sample Size	50
Sample Standard Deviation	51.7
Intermediate Calculations	
F Test Statistic	1.70967
Population 1 Sample Degrees of Freedom	49
Population 2 Sample Degrees of Freedom	49
Two-Tail Test	
Upper Critical Value	1.762189
p-Value	0.063376
Do not reject the null hypothesis	

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- 10.49 (a)  $H_0: \sigma_1^2 = \sigma_2^2$  where Populations: 1 = users 13 to 17 years of age,  
cont. 2 = users under 12 years of age

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

Decision rule: If  $F_{STAT} > 1.7622$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{67.6^2}{51.7^2} = 1.7097$$

Decision: Since  $F_{STAT} = 1.7097$  is smaller than the upper critical bound of  $F_{\alpha/2} = 1.7622$ , do not reject  $H_0$ . There is not enough evidence of a difference between the variances in cellphone usage between cellphone users under 12 years of age and cellphone users 13 to 17 years of age.

- (b) Assuming the underlying normality in the two populations is met, based on the results obtained in part (a), it is more appropriate to use the pooled-variance  $t$ -test.

10.50

PHStat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	25
Sample Standard Deviation	0.315053
Smaller-Variance Sample	
Sample Size	25
Sample Standard Deviation	0.189352
Intermediate Calculations	
F Test Statistic	2.768403
Population 1 Sample Degrees of Freedom	24
Population 2 Sample Degrees of Freedom	24
Two-Tail Test	
Upper Critical Value	2.269277
p-Value	0.015584
Reject the null hypothesis	

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

Decision rule: If  $F_{STAT} > 2.2693$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = 2.7684$$

Decision: Since  $F_{STAT} = 2.7684$  is larger than the upper critical value of 2.2693, reject  $H_0$ . There is evidence of a difference in the variance of the yield of five-year CDs on March 29, 2010 and August 23, 2010.

- 10.51 Among the criteria to be used in selecting a particular hypothesis test are the type of data, whether the samples are independent or paired, whether the test involves central tendency or variation, whether the assumption of normality is valid, and whether the variances in the two populations are equal.
- 10.52 The pooled variance  $t$ -test should be used when the populations are approximately normally distributed and the variances of the two populations are equal.
- 10.53 The  $F$  test can be used to examine differences in two variances when each of the two populations is assumed to be normally distributed.
- 10.54 With independent populations, the outcomes in one population do not depend on the outcomes in the second population. With two related populations, either repeated measurements are obtained on the same set of items or individuals, or items or individuals are paired or matched according to some characteristic.
- 10.55 Repeated measurements represent two measurements on the same items or individuals, while paired measurements involve matching items according to a characteristic of interest.
- 10.56 They are two different ways of investigating the concern of whether there is significant difference between the means of two independent populations. If the hypothesized value of 0 for the difference in two population means is not in the confidence interval, then, assuming a two-tailed test is used, the null hypothesis of no difference in the two population means can be rejected.
- 10.57 When you have obtained data from either repeated measurements or paired data.
- 10.58 (a) **Stores that priced the small coffee at \$0.59**  
 $H_0 : \mu \leq 900$  vs.  $H_1 : \mu > 900$

t Test for Hypothesis of the Mean	
Data	
Null Hypothesis $\mu =$	900
Level of Significance	0.05
Sample Size	15
Sample Mean	964
Sample Standard Deviation	88
Intermediate Calculations	
Standard Error of the Mean	22.7215023
Degrees of Freedom	14
$t$ Test Statistic	2.816715161
Upper-Tail Test	
Upper Critical Value	1.761310115
$p$ -Value	0.006860614
Reject the null hypothesis	

Since the  $p$ -value = 0.0069 < 0.05, reject  $H_0$ . There is evidence that reducing the price of a small coffee to \$0.59 increases per store average daily customer count.

*B*

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- 10.58 (a) **Stores that priced the small coffee at \$0.79**  
 cont.  $H_0 : \mu \leq 900$  vs.  $H_1 : \mu > 900$

t Test for Hypothesis of the Mean	
Data	
Null Hypothesis $\mu =$	900
Level of Significance	0.05
Sample Size	15
Sample Mean	941
Sample Standard Deviation	76
Intermediate Calculations	
Standard Error of the Mean	19.62311562
Degrees of Freedom	14
t Test Statistic	2.089372595
Upper-Tail Test	
Upper Critical Value	1.761310115
p-Value	0.027705582
Reject the null hypothesis	

Since the  $p$ -value = 0.0277 < 0.05, reject  $H_0$ . There is evidence that reducing the price of a small coffee to \$0.79 increases per store average daily customer count.

- (b)  $H_0 : \sigma_1^2 = \sigma_2^2$  vs.  $H_1 : \sigma_1^2 \neq \sigma_2^2$

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	15
Sample Standard Deviation	88
Smaller-Variance Sample	
Sample Size	15
Sample Standard Deviation	76
Intermediate Calculations	
F Test Statistic	1.34072
Population 1 Sample Degrees of Freedom	14
Population 2 Sample Degrees of Freedom	14
Two-Tail Test	
Upper Critical Value	2.978588
p-Value	0.590648
Do not reject the null hypothesis	

Since the  $p$ -value = 0.5906 > 0.05, do not reject  $H_0$ . There is not enough evidence that the two variances are different. Hence, a pooled-variance  $t$  test is appropriate.

- 10.58 (a)  $H_0 : \mu_1 = \mu_2$  vs.  $H_1 : \mu_1 \neq \mu_2$   
cont.

<b>Pooled-Variance <math>t</math> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>15</b>
<b>Sample Mean</b>	<b>964</b>
<b>Sample Standard Deviation</b>	<b>88</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>15</b>
<b>Sample Mean</b>	<b>941</b>
<b>Sample Standard Deviation</b>	<b>76</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	14
Population 2 Sample Degrees of Freedom	14
Total Degrees of Freedom	28
Pooled Variance	6760
Difference in Sample Means	23
<b><math>t</math> Test Statistic</b>	<b>0.766099</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.04841</b>
<b>Upper Critical Value</b>	<b>2.048407</b>
<b><math>p</math>-Value</b>	<b>0.450028</b>
<b>Do not reject the null hypothesis</b>	

Since the  $p$ -value = 0.45 > 0.05, do not reject  $H_0$ . There is not enough evidence of a difference in the per store daily customer count between stores in which a small coffee was priced at \$0.59 and stores in which a small coffee was priced at \$0.79 for a 12 ounce cup.

- (c) Since there is not enough evidence of a difference in the per store mean daily customer count between stores in which a small coffee was priced at \$0.59 and stores in which a small coffee was priced at \$0.79 for a 12 ounce cup, you will recommend that a small coffee should be priced at \$0.79 since that will bring in more profit per cup.

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- 10.59 (a)  $H_0 : \pi_1 \leq \pi_2$   $H_1 : \pi_1 > \pi_2$   
 Population: 1 = Democrats, 2 = Republicans  
 (b) Type I Error: Rejecting the null hypothesis that the proportion of Democrats trusting the government more than business is no greater than the proportion of Republicans trusting the government more than business when the proportion of Democrats trusting the government more than business is indeed no greater than the proportion of Republicans trusting the government more than business.  
 (c) Type II Error: Failing to reject the null hypothesis that the proportion of Democrats trusting the government more than business is no greater than the proportion of Republicans trusting the government more than business when the proportion of Democrats trusting the government more than business is indeed greater than the proportion of Republicans trusting the government more than business.

- 10.60 (a)  $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.  
 $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHStat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	219
Sample Standard Deviation	20955
Smaller-Variance Sample	
Sample Size	34
Sample Standard Deviation	18137
Intermediate Calculations	
F Test Statistic	1.334887
Population 1 Sample Degrees of Freedom	218
Population 2 Sample Degrees of Freedom	33
Two-Tail Test	
Upper Critical Value	1.778739
p-Value	0.32362
Do not reject the null hypothesis	

Decision rule: If  $F_{STAT} > 1.7787$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = 1.3349$$

Decision: Since  $F_{STAT} = 1.3349$  is smaller than the upper critical bound of 1.7787, do not reject  $H_0$ . There is not enough evidence of any difference in the variability of salaries between green belt and black belts.

- (b) Since there is not enough evidence of any difference in the variability of salaries between green belts and black belts, a pooled-variance  $t$  test should be used.

- 10.60 (c)  $H_0: \mu_1 = \mu_2$   $H_1: \mu_1 \neq \mu_2$   
cont.

<b>Pooled-Variance <math>t</math> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>219</b>
<b>Sample Mean</b>	<b>87342</b>
<b>Sample Standard Deviation</b>	<b>20955</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>34</b>
<b>Sample Mean</b>	<b>65679</b>
<b>Sample Standard Deviation</b>	<b>18137</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	218
Population 2 Sample Degrees of Freedom	33
Total Degrees of Freedom	251
Pooled Variance	4.25E+08
Difference in Sample Means	21663
<b><math>t</math> Test Statistic</b>	<b>5.703155</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-1.96946</b>
<b>Upper Critical Value</b>	<b>1.96946</b>
<b><math>p</math>-Value</b>	<b>3.3E-08</b>
<b>Reject the null hypothesis</b>	

Decision rule: If  $|t_{STAT}| > 1.9695$ , reject  $H_0$ .

Decision: Since  $|t_{STAT}| = 5.7032$  is greater than 1.9695, reject  $H_0$ . There is enough evidence of a difference between the mean salary of green belt and the mean salary of black belts.



- 10.61 (a)  $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.  
 $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

<b>F Test for Differences in Two Variances</b>	
<b>Data</b>	
<b>Level of Significance</b>	<b>0.05</b>
<b>Larger-Variance Sample</b>	
<b>Sample Size</b>	<b>20</b>
<b>Sample Standard Deviation</b>	<b>5.714421</b>
<b>Smaller-Variance Sample</b>	
<b>Sample Size</b>	<b>38</b>
<b>Sample Standard Deviation</b>	<b>5.406387</b>
<b>Intermediate Calculations</b>	
<b>F Test Statistic</b>	<b>1.117198</b>
Population 1 Sample Degrees of Freedom	19
Population 2 Sample Degrees of Freedom	37
<b>Two-Tail Test</b>	
<b>Upper Critical Value</b>	<b>2.11685</b>
<b>p-Value</b>	<b>0.749246</b>
<b>Do not reject the null hypothesis</b>	

Decision rule: If  $F_{STAT} > 2.1169$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{(5.7144)^2}{(5.4064)^2} = 1.1172$$

Decision: Since  $F_{STAT} = 1.1172$  is smaller than the upper critical bound of 2.1169, do not reject  $H_0$ . There is not enough evidence of any difference in the variance of the study time for male students and female students.

- (b) Since there is not enough evidence of any difference in the variance of the study time for male students and female students, a pooled-variance  $t$  test should be used.

10.61 (c)  $H_0: \mu_1 = \mu_2$   $H_1: \mu_1 \neq \mu_2$

cont.

Decision rule:  $d.f. = 56$ . If  $t_{STAT} < -2.0032$  or  $t_{STAT} > 2.0032$ , reject  $H_0$ .

<b>Pooled-Variance <math>t</math> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>20</b>
<b>Sample Mean</b>	<b>16.625</b>
<b>Sample Standard Deviation</b>	<b>5.714421</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>38</b>
<b>Sample Mean</b>	<b>11.02632</b>
<b>Sample Standard Deviation</b>	<b>5.406387</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	19
Population 2 Sample Degrees of Freedom	37
Total Degrees of Freedom	56
Pooled Variance	30.39127
Difference in Sample Means	5.598684
<b><math>t</math> Test Statistic</b>	<b>3.676244</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.00324</b>
<b>Upper Critical Value</b>	<b>2.003241</b>
<b><math>p</math>-Value</b>	<b>0.000532</b>
<b>Reject the null hypothesis</b>	

Decision: Since  $t_{STAT} = 3.6762$  is larger than the upper critical bound of 2.0032, reject  $H_0$ .

- (d) There is enough evidence of a difference in the mean study time for male and female students.

10.62 (a)  $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

Decision rule: If  $F_{STAT} > 1.6275$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{(6.29)^2}{(1.32)^2} = 22.7067$$

Decision: Since  $F_{STAT} = 22.7067$  is greater than the upper critical bound of 1.6275, reject  $H_0$ . There is enough evidence to conclude that there is a difference between the variances in age of students at the Western school and at the Eastern school.

(b) Since there is a difference between the variances in age of students at the Western school and at the Eastern school, schools should take that into account when designing their curriculum to accommodate the larger variance in age of students in the state university in the "Western" U.S.

(c) It is more appropriate to use a separate-variance  $t$  test.

(d)  $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

Decision rule: If  $F_{STAT} > 1.6275$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{(2.4)^2}{(2.1)^2} = 1.3061$$

Decision: Since  $F_{STAT} = 1.3061$  is lower than the upper critical bound 1.6275, do not reject  $H_0$ . There is not enough evidence to conclude that there is a difference between the variances in years of spreadsheet usage of students at the Western school and at the Eastern school.

(e)  $H_0: \mu_1 = \mu_2$   $H_1: \mu_1 \neq \mu_2$

Decision rule:  $d.f. = 226$ . If  $t_{STAT} < -2.5978$  or  $t_{STAT} > 2.5978$ , reject  $H_0$ .

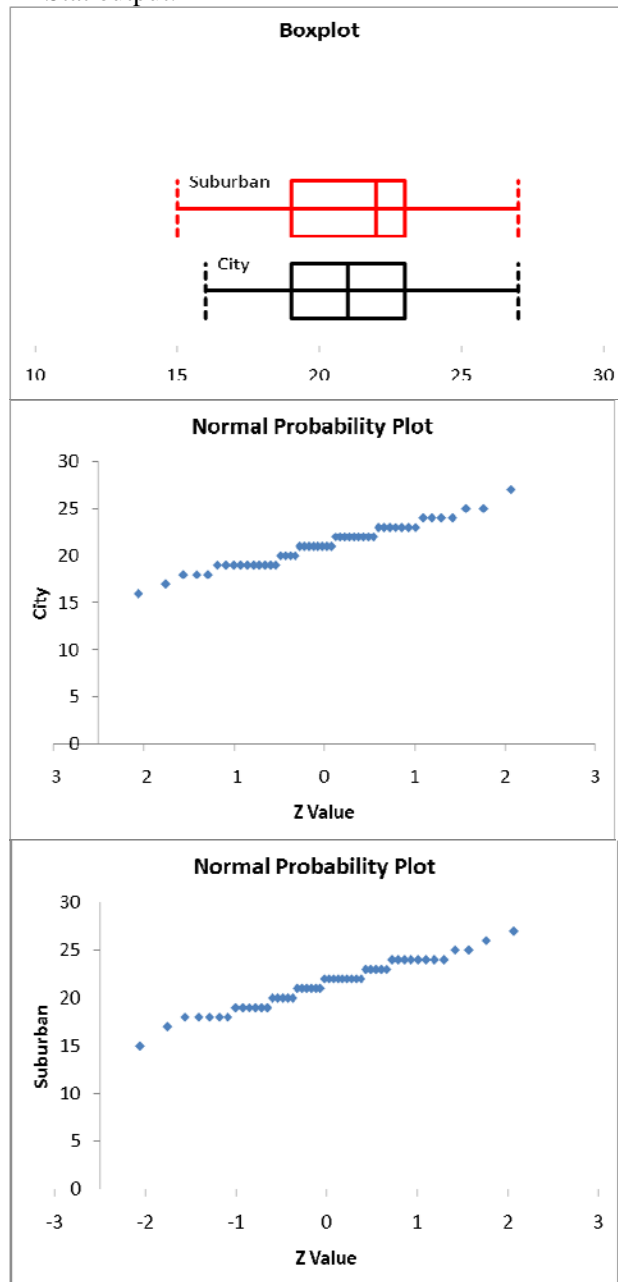
Test statistic:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(93 - 1)(2.4)^2 + (135 - 1)(2.1)^2}{(93 - 1) + (135 - 1)} = 4.9596$$

$$t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{(2.6 - 4) - 0}{\sqrt{4.9596 \left( \frac{1}{93} + \frac{1}{135} \right)}} = -4.6650$$

Decision: Since  $t_{STAT} = -4.6650$  is smaller than the lower critical bound of  $-2.5978$ , reject  $H_0$ . There is enough evidence of a difference in the mean years of spreadsheet usage of students at the Western school and at the Eastern school.

10.63 **Food:**  
PHStat output:



From the boxplots and normal probability plots, you saw that the distribution of the food rating was quite normal for the suburban restaurants and the city restaurants. Hence, the results from the  $F$  test on the difference in variances to determine whether the pooled-variance  $t$  test or separate variance  $t$  test is more appropriate for the difference in means is reliable.

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10.63  $H_0: \sigma_1^2 = \sigma_2^2$   $H_1: \sigma_1^2 \neq \sigma_2^2$  Let Population 1 = suburban, 2 = city  
cont.

<b>F Test for Differences in Two Variances</b>	
<b>Data</b>	
<b>Level of Significance</b>	<b>0.05</b>
<b>Larger-Variance Sample</b>	
<b>Sample Size</b>	<b>50</b>
<b>Sample Standard Deviation</b>	<b>2.52336</b>
<b>Smaller-Variance Sample</b>	
<b>Sample Size</b>	<b>50</b>
<b>Sample Standard Deviation</b>	<b>2.267877</b>
<b>Intermediate Calculations</b>	
<b>F Test Statistic</b>	<b>1.237997</b>
Population 1 Sample Degrees of Freedom	49
Population 2 Sample Degrees of Freedom	49
<b>Two-Tail Test</b>	
<b>Upper Critical Value</b>	<b>1.762189</b>
<b>p-Value</b>	<b>0.457646</b>
<b>Do not reject the null hypothesis</b>	

Since the  $p$ -value = 0.45 > 0.05, do not reject  $H_0$ . At 5% level of significance, there is insufficient evidence to conclude that the two variances are not the same. Hence, a pooled variance  $t$  test is more appropriate.

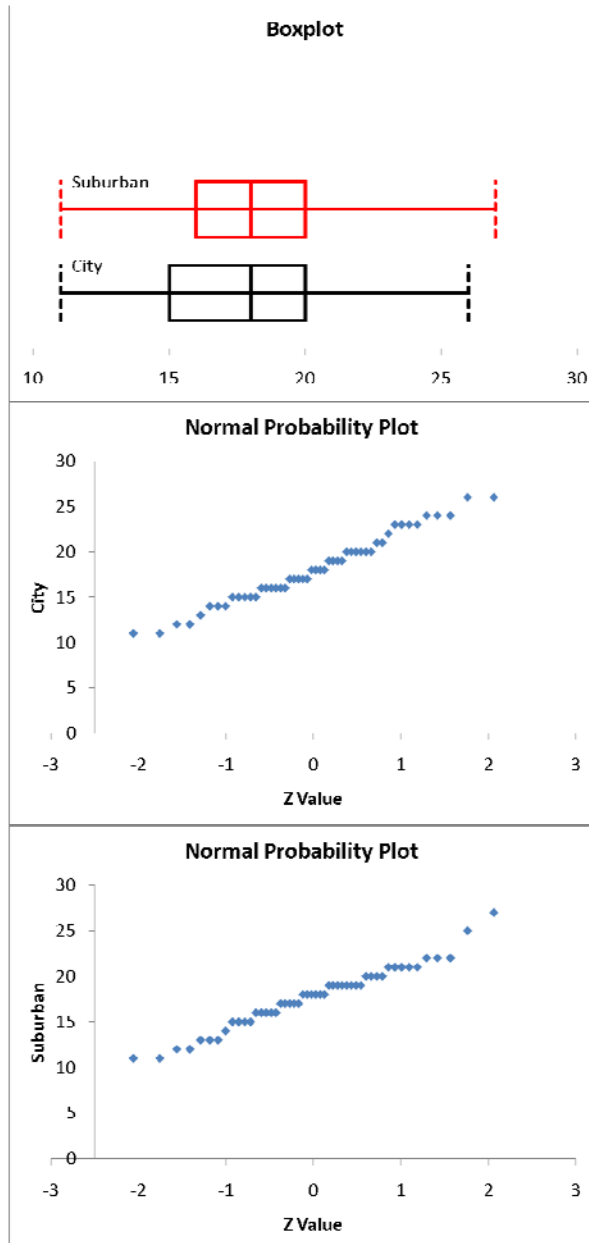
10.63  $H_0: \mu_1 = \mu_2$      $H_1: \mu_1 \neq \mu_2$

cont.

<b>Pooled-Variance <math>t</math> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>50</b>
<b>Sample Mean</b>	<b>21.4</b>
<b>Sample Standard Deviation</b>	<b>2.52336</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>50</b>
<b>Sample Mean</b>	<b>21.14</b>
<b>Sample Standard Deviation</b>	<b>2.267877</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	49
Population 2 Sample Degrees of Freedom	49
Total Degrees of Freedom	98
Pooled Variance	5.755306
Difference in Sample Means	0.26
<b><math>t</math> Test Statistic</b>	<b>0.541888</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-1.98447</b>
<b>Upper Critical Value</b>	<b>1.984467</b>
<b><math>p</math>-Value</b>	<b>0.589126</b>
<b>Do not reject the null hypothesis</b>	

- 10.63 Since the  $p$ -value = 0.5891 is greater than the 5% level of significance, do not reject  $H_0$ .
- cont. There is not enough evidence to conclude that the mean food rating between suburban and city restaurants is different.

**Décor:**



From the boxplots and normal probability plots, you saw that the distribution of the décor rating was quite normal for both the suburban restaurants and the city restaurants. Hence, it is appropriate to perform the  $F$  test on the difference in variances to determine whether the pooled-variance  $t$  test or separate variance  $t$  test is more appropriate for the difference in means.

- 10.63  $H_0: \sigma_1^2 = \sigma_2^2$   $H_1: \sigma_1^2 \neq \sigma_2^2$  Let Population 1 = city, 2 = suburban  
cont.

<b>F Test for Differences in Two Variances</b>	
<b>Data</b>	
Level of Significance	<b>0.05</b>
<b>Larger-Variance Sample</b>	
Sample Size	<b>50</b>
Sample Standard Deviation	<b>3.81859</b>
<b>Smaller-Variance Sample</b>	
Sample Size	<b>50</b>
Sample Standard Deviation	<b>3.397538</b>
<b>Intermediate Calculations</b>	
<b>F Test Statistic</b>	<b>1.263216</b>
Population 1 Sample Degrees of Freedom	49
Population 2 Sample Degrees of Freedom	49
<b>Two-Tail Test</b>	
<b>Upper Critical Value</b>	<b>1.762189</b>
<b>p-Value</b>	<b>0.416372</b>
<b>Do not reject the null hypothesis</b>	

Since the  $p$ -value = 0.4164 is greater than the 5% level of significance, do not reject  $H_0$ . There is not enough evidence to conclude that the variances are different. Hence, a pooled-variance  $t$  test is appropriate.



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10.63  $H_0: \mu_1 = \mu_2$      $H_1: \mu_1 \neq \mu_2$

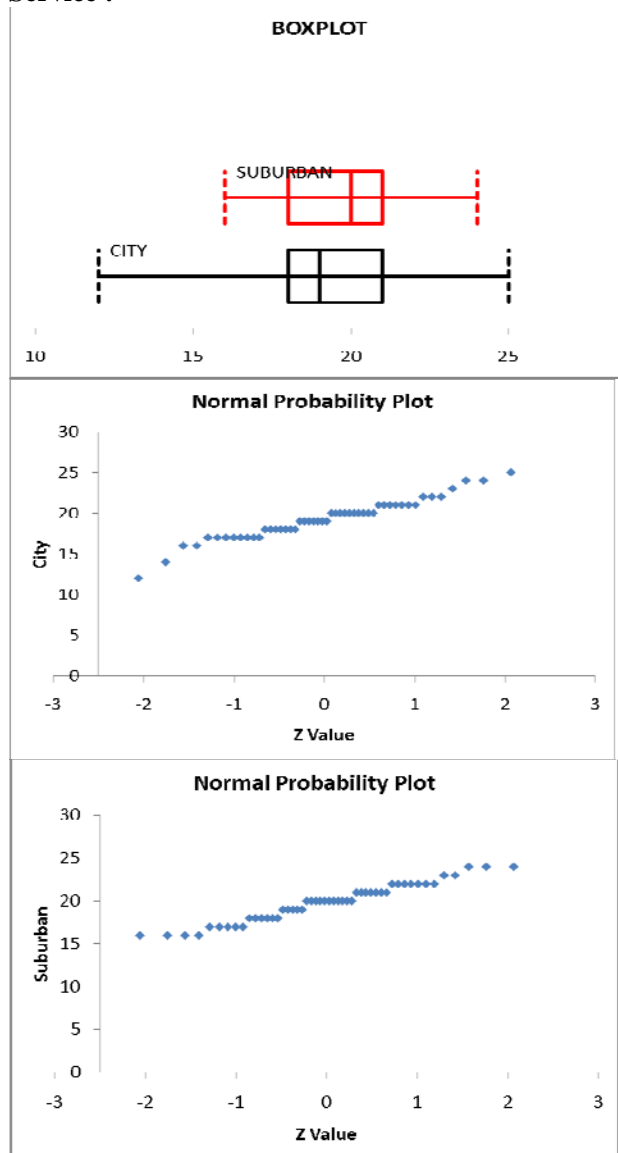
cont.

<b>Pooled-Variance <math>t</math> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>50</b>
<b>Sample Mean</b>	<b>18.1</b>
<b>Sample Standard Deviation</b>	<b>3.81859</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>50</b>
<b>Sample Mean</b>	<b>17.74</b>
<b>Sample Standard Deviation</b>	<b>3.397538</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	49
Population 2 Sample Degrees of Freedom	49
Total Degrees of Freedom	98
Pooled Variance	13.06245
Difference in Sample Means	0.36
<b><math>t</math> Test Statistic</b>	<b>0.498035</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-1.98447</b>
<b>Upper Critical Value</b>	<b>1.984467</b>
<b><math>p</math>-Value</b>	<b>0.619575</b>
<b>Do not reject the null hypothesis</b>	

10.63  
cont.

Since the  $p$ -value = 0.6196 is greater than the 5% level of significance, do not reject  $H_0$ . There is not enough evidence to conclude that the mean décor rating between city restaurants and suburban restaurants is different.

**Service :**



From the boxplots and normal probability plots, you saw that the distribution of the service rating was quite normal for both the suburban restaurants and the city restaurants. Hence, it is appropriate to perform the  $F$  test on the difference in variances to determine whether the pooled-variance  $t$  test or separate variance  $t$  test is more appropriate for the difference in means.

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10.63  $H_0: \sigma_1^2 = \sigma_2^2$   $H_1: \sigma_1^2 \neq \sigma_2^2$  Let Population 1 = city, 2 = suburban  
cont.

<b>F Test for Differences in Two Variances</b>	
<b>Data</b>	
<b>Level of Significance</b>	<b>0.05</b>
<b>Larger-Variance Sample</b>	
<b>Sample Size</b>	<b>50</b>
<b>Sample Standard Deviation</b>	<b>2.454151</b>
<b>Smaller-Variance Sample</b>	
<b>Sample Size</b>	<b>50</b>
<b>Sample Standard Deviation</b>	<b>2.20102</b>
<b>Intermediate Calculations</b>	
<b>F Test Statistic</b>	<b>1.243239</b>
Population 1 Sample Degrees of Freedom	49
Population 2 Sample Degrees of Freedom	49
<b>Two-Tail Test</b>	
<b>Upper Critical Value</b>	<b>1.762189</b>
<b>p-Value</b>	<b>0.448811</b>
<b>Do not reject the null hypothesis</b>	

Since the  $p$ -value = 0.4488 is greater than the 5% level of significance, do not reject  $H_0$ . There is not enough evidence to conclude that the variances are different. Hence, a pooled-variance  $t$  test is appropriate.

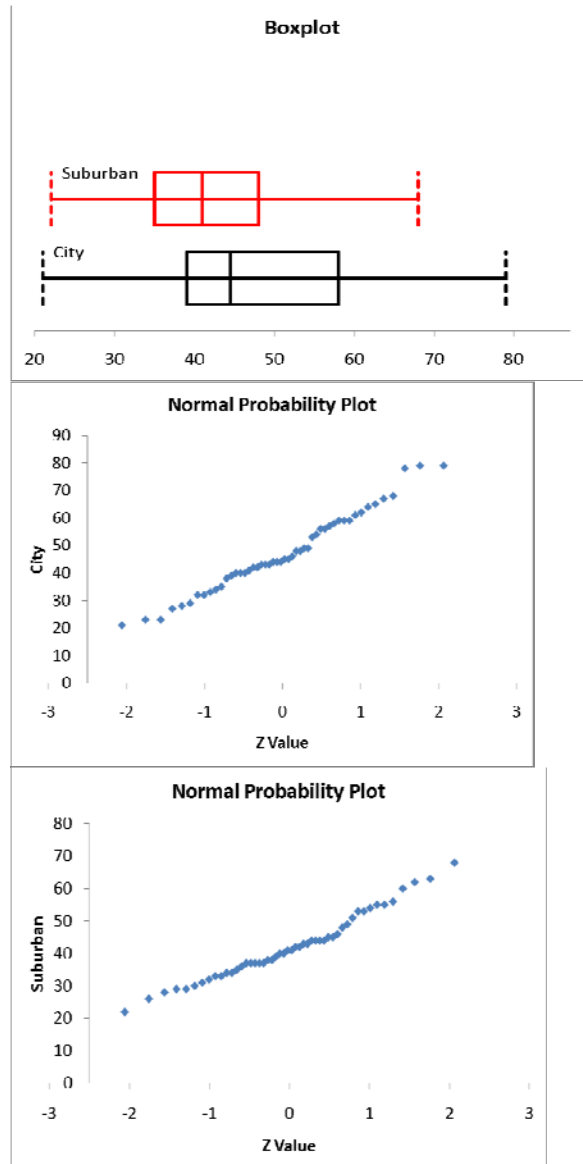
10.63  $H_0: \mu_1 = \mu_2$      $H_1: \mu_1 \neq \mu_2$

cont.

<b>Pooled-Variance <math>t</math> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>50</b>
<b>Sample Mean</b>	<b>19.24</b>
<b>Sample Standard Deviation</b>	<b>2.454151</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>50</b>
<b>Sample Mean</b>	<b>19.82</b>
<b>Sample Standard Deviation</b>	<b>2.20102</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	49
Population 2 Sample Degrees of Freedom	49
Total Degrees of Freedom	98
Pooled Variance	5.433673
Difference in Sample Means	-0.58
<b><math>t</math> Test Statistic</b>	<b>-1.24409</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-1.98447</b>
<b>Upper Critical Value</b>	<b>1.984467</b>
<b><math>p</math>-Value</b>	<b>0.216434</b>
<b>Do not reject the null hypothesis</b>	

- 10.63 Since the  $p$ -value = .2164 is larger than the 5% level of significance, do not reject  $H_0$ .  
 cont. There is not enough evidence to conclude that the mean service rating between city restaurants and suburban restaurants is different.

**Cost:**



From the boxplots and normal probability plots, you saw that the distribution of the price was quite normal for both the suburban restaurants and the city restaurants. Hence, it is appropriate to perform the  $F$  test on the difference in variances to determine whether the pooled-variance  $t$  test or separate variance  $t$  test is more appropriate for the difference in means.

- 10.63  $H_0: \sigma_1^2 = \sigma_2^2$      $H_1: \sigma_1^2 \neq \sigma_2^2$     Let Population 1 = city, 2 = suburban  
cont.

<b>F Test for Differences in Two Variances</b>	
<b>Data</b>	
Level of Significance	<b>0.05</b>
<b>Larger-Variance Sample</b>	
Sample Size	<b>50</b>
Sample Standard Deviation	<b>14.37407</b>
<b>Smaller-Variance Sample</b>	
Sample Size	<b>50</b>
Sample Standard Deviation	<b>10.18685</b>
<b>Intermediate Calculations</b>	
<b>F Test Statistic</b>	<b>1.99104</b>
Population 1 Sample Degrees of Freedom	49
Population 2 Sample Degrees of Freedom	49
<b>Two-Tail Test</b>	
<b>Upper Critical Value</b>	<b>1.762189</b>
<b>p-Value</b>	<b>0.017546</b>
<b>Reject the null hypothesis</b>	

Since the  $p$ -value = 0.0175 is smaller than the 5% level of significance, reject  $H_0$ . There is enough evidence to conclude that the variances are different. Hence, a separate-variance  $t$  test is appropriate.

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10.63  $H_0: \mu_1 = \mu_2$      $H_1: \mu_1 \neq \mu_2$

cont.

Separate-Variances $t$ Test for the Difference Between Two Means	
(assumes unequal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	50
Sample Mean	47.28
Sample Standard Deviation	14.37406962
Population 2 Sample	
Sample Size	50
Sample Mean	42.06
Sample Standard Deviation	10.18684626
Intermediate Calculations	
Numerator of Degrees of Freedom	38.5357
Denominator of Degrees of Freedom	0.4364
Total Degrees of Freedom	88.3055
Degrees of Freedom	88
Separate Variance Denominator	2.4915
Difference in Sample Means	5.22
Separate-Variance $t$ Test Statistic	2.0951
Two-Tail Test	
Lower Critical Value	-1.9873
Upper Critical Value	1.9873
$p$ -Value	0.0390
Reject the null hypothesis	

Since the  $p$ -value = 0.039 is smaller than the 5% level of significance, reject  $H_0$ .

There is enough evidence to conclude that the mean price between city restaurants and suburban restaurants is different.

- 10.64 (a)  $H_0: \mu \leq 10$  minutes. Introductory computer students required no more than a mean of 10 minutes to write and run a program in Visual Basic.  
 $H_1: \mu > 10$  minutes. Introductory computer students required more than a mean of 10 minutes to write and run a program in Visual Basic.  
 Decision rule:  $d.f. = 8$ . If  $t_{STAT} > 1.8595$ , reject  $H_0$ .  
 Test statistic:  $t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{12 - 10}{1.8028/\sqrt{9}} = 3.3282$   
 Decision: Since  $t_{STAT} = 3.3282$  is greater than the critical bound of 1.8595, reject  $H_0$ . There is enough evidence to conclude that the introductory computer students required more than a mean of 10 minutes to write and run a program in Visual Basic.
- (b)  $H_0: \mu \leq 10$  minutes. Introductory computer students required no more than a mean of 10 minutes to write and run a program in Visual Basic.  
 $H_1: \mu > 10$  minutes. Introductory computer students required more than a mean of 10 minutes to write and run a program in Visual Basic.  
 Decision rule:  $d.f. = 8$ . If  $t_{STAT} > 1.8595$ , reject  $H_0$ .  
 Test statistic:  $t_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{16 - 10}{13.2004/\sqrt{9}} = 1.3636$   
 Decision: Since  $t_{STAT} = 1.3636$  is less than the critical bound of 1.8595, do not reject  $H_0$ . There is not enough evidence to conclude that the introductory computer students required more than a mean of 10 minutes to write and run a program in Visual Basic.
- (c) Although the mean time necessary to complete the assignment increased from 12 to 16 minutes as a result of the increase in one data value, the standard deviation went from 1.8 to 13.2, which in turn brought the  $t$ -value down because of the increased denominator.
- (d)  $H_0: \sigma_{IC}^2 = \sigma_{CS}^2$        $H_1: \sigma_{IC}^2 \neq \sigma_{CS}^2$   
 Decision rule: If  $F_{STAT} > 3.8549$ , reject  $H_0$ .  
 Test statistic:  $F_{STAT} = \frac{S_{IC}^2}{S_{CS}^2} = \frac{2.0^2}{1.8028^2} = 1.2307$   
 Decision: Since  $F_{STAT} = 1.2307$  is lower than the critical bound 3.8549, do not reject  $H_0$ . There is not enough evidence to conclude that the population variances are different for the Introduction to Computers students and computer majors. Hence, the pooled variance  $t$  test is a valid test to see whether computer majors can write a Visual Basic program (on average) in less time than introductory students, assuming that the distributions of the time needed to write a Visual Basic program for both the Introduction to Computers students and the computer majors are approximately normal.



10.64 (d)  $H_0: \mu_{IC} \leq \mu_{CS}$  The mean amount of time needed by Introduction to Computers students is not greater than the mean amount of time needed by computer majors.

$H_1: \mu_{IC} > \mu_{CS}$  The mean amount of time needed by Introduction to Computers students is greater than the mean amount of time needed by computer majors.

PHStat output:

Pooled-Variance <i>t</i> Test for the Difference Between Two Means	
(assumes equal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	9
Sample Mean	12
Sample Standard Deviation	1.802776
Population 2 Sample	
Sample Size	11
Sample Mean	8.5
Sample Standard Deviation	2
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	8
Population 2 Sample Degrees of Freedom	10
Total Degrees of Freedom	18
Pooled Variance	3.666667
Difference in Sample Means	3.5
<i>t</i> Test Statistic	4.066633
Upper-Tail Test	
Upper Critical Value	1.734064
<i>p</i> -Value	0.000362
Reject the null hypothesis	

Decision rule:  $d.f. = 18$ . If  $t_{STAT} > 1.7341$ , reject  $H_0$ .

Test statistic:

$$S_p^2 = \frac{(n_{IC} - 1) \cdot S_{IC}^2 + (n_{CS} - 1) \cdot S_{CS}^2}{(n_{IC} - 1) + (n_{CS} - 1)} = \frac{9 \cdot 1.8028^2 + 11 \cdot 2.0^2}{8 + 10} = 3.6667$$

$$t_{STAT} = \frac{(\bar{X}_{IC} - \bar{X}_{CS}) - (\mu_{IC} - \mu_{CS})}{\sqrt{S_p^2 \left( \frac{1}{n_{IC}} + \frac{1}{n_{CS}} \right)}} = \frac{12.0 - 8.5}{\sqrt{3.6667 \left( \frac{1}{9} + \frac{1}{11} \right)}} = 4.0666$$

Decision: Since  $t_{STAT} = 4.0666$  is greater than 1.7341, reject  $H_0$ . There is enough evidence to support a conclusion that the mean time is higher for Introduction to Computers students than for computer majors.

- 10.64 (e)  $p$ -value = 0.0052. If the true population mean amount of time needed for Introduction to Computer students to write a Visual Basic program is indeed no more than 10 minutes, the probability for observing a sample mean greater than the 12 minutes in the current sample is 0.0052, which means it will be a quite unlikely event. Hence, at a 95% level of confidence, you can conclude that the population mean amount of time needed for Introduction to Computer students to write a Visual Basic program is more than 10 minutes.
- As illustrated in part (d) in which there is not enough evidence to conclude that the population variances are different for the Introduction to Computers students and computer majors, the pooled variance  $t$  test performed is a valid test to determine whether computer majors can write a Visual Basic program in less time than in introductory students, assuming that the distributions of the time needed to write a Visual Basic program for both the Introduction to Computers students and the computer majors are approximately normal.

- 10.65 Population 1 = 18 to 24 year olds, 2 = 45 to 54 year olds

- (a)  $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.  
 $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

F Test for Differences in Two Variances	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	100
Sample Standard Deviation	100
Smaller-Variance Sample	
Sample Size	100
Sample Standard Deviation	90
Intermediate Calculations	
F Test Statistic	1.234568
Population 1 Sample Degrees of Freedom	99
Population 2 Sample Degrees of Freedom	99
Two-Tail Test	
Upper Critical Value	1.486234
p-Value	0.296154
Do not reject the null hypothesis	

Since the  $p$ -value = 0.296154 is greater than the 5% level of significance, do not reject  $H_0$ . There is not enough evidence to conclude that the variances for phone calls are different. Hence, a pooled-variance  $t$  test is appropriate.

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- 10.65 (b)  $H_0: \mu_1 = \mu_2$   $H_1: \mu_1 \neq \mu_2$   
cont.

<b>Pooled-Variance <math>t</math> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>100</b>
<b>Sample Mean</b>	<b>290</b>
<b>Sample Standard Deviation</b>	<b>100</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>100</b>
<b>Sample Mean</b>	<b>194</b>
<b>Sample Standard Deviation</b>	<b>90</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	99
Population 2 Sample Degrees of Freedom	99
Total Degrees of Freedom	198
Pooled Variance	9050
Difference in Sample Means	96
<b><math>t</math> Test Statistic</b>	<b>7.135624</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-1.97202</b>
<b>Upper Critical Value</b>	<b>1.972017</b>
<b><math>p</math>-Value</b>	<b>1.78E-11</b>
<b>Reject the null hypothesis</b>	

- (b) Since the  $p$ -value is virtually zero, reject  $H_0$ . There is enough evidence of a difference in the mean number of cell phone calls per month for 18 to 24 year olds and 45 to 54 year olds.

$$(c) \quad (\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} \sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} = (290 - 194) \pm 1.9720 \sqrt{9050 \left( \frac{1}{100} + \frac{1}{100} \right)}$$

$$69.4692 \leq \mu_1 - \mu_2 \leq 122.5308$$

- 10.65 (d)  $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.  
 cont.  $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	100
Sample Standard Deviation	90
Smaller-Variance Sample	
Sample Size	100
Sample Standard Deviation	77
Intermediate Calculations	
F Test Statistic	1.366166
Population 1 Sample Degrees of Freedom	99
Population 2 Sample Degrees of Freedom	99
Two-Tail Test	
Upper Critical Value	1.486234
p-Value	0.1223
Do not reject the null hypothesis	

Since the  $p$ -value = 0.1223 is greater than the 5% level of significance, do not reject  $H_0$ . There is not enough evidence to conclude that the variances for text messages are different. Hence, a pooled-variance  $t$  test is appropriate.

- (e)  $H_0: \mu_1 = \mu_2$   $H_1: \mu_1 \neq \mu_2$

Pooled-Variance $t$ Test for the Difference Between Two Means	
(assumes equal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	100
Sample Mean	290
Sample Standard Deviation	90
Population 2 Sample	
Sample Size	100
Sample Mean	57
Sample Standard Deviation	77
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	99
Population 2 Sample Degrees of Freedom	99
Total Degrees of Freedom	198
Pooled Variance	7014.5
Difference in Sample Means	233
$t$ Test Statistic	19.67173
Two-Tail Test	
Lower Critical Value	-1.97202
Upper Critical Value	1.972017
p-Value	1.84E-48
Reject the null hypothesis	

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- 10.65 (e) Since the  $p$ -value is virtually zero, reject  $H_0$ . There is enough evidence of a difference in the mean number of text messages per month for 18 to 24 year olds and 45 to 54 year olds.

$$(f) \quad (\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} \sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} = (290 - 57) \pm 1.9720 \sqrt{7014.5 \left( \frac{1}{100} + \frac{1}{100} \right)}$$

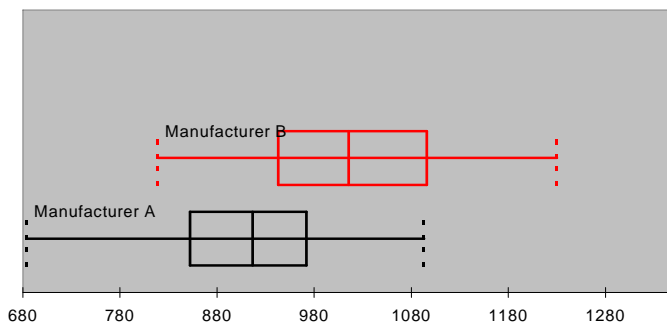
$$209.6426 \leq \mu_1 - \mu_2 \leq 256.3574$$

- (g) There is enough evidence of a difference in the mean number of cell phone calls per month for 18 to 24 year olds and 45 to 54 year olds. There is also enough evidence of a difference in the mean number of text messages per month for 18 to 24 year olds and 45 to 54 year olds. You are 95% confident that the difference in the mean number of cell phone calls per month for 18 to 24 year olds and 45 to 54 year olds is somewhere between 69 and 123 while the difference in the mean number of text messages per month for 18 to 24 year olds and 45 to 54 year olds is somewhere between 210 and 256.

10.66

	Manufacturer A	Manufacturer B
Minimum	684	819
First Quartile	852	943
Median	916.5	1015.5
Third Quartile	972	1096
Interquartile Range	120	153
Maximum	1093	1230
Range	409	411
Mean	909.65	1018.35
Median	916.5	1015.5
Mode	926	1077
Standard Deviation	94.3052	96.9014
Sample Variance	8893.4641	9389.8744
Count	40	40

Box-and-whisker Plot



From the box plot and the summary statistics, both data seem to have come from rather symmetrical distributions that are quite normally distributed.

The following  $F$  test for any evidence of difference between two population variances suggests that there is insufficient evidence to conclude that the two population variances are significantly different at 5% level of significance.

10.66

cont. PHStat output:

<b>F Test for Differences in Two Variances</b>	
<b>Data</b>	
<b>Level of Significance</b>	<b>0.05</b>
<b>Larger-Variance Sample</b>	
<b>Sample Size</b>	<b>40</b>
<b>Sample Standard Deviation</b>	<b>96.90136</b>
<b>Smaller-Variance Sample</b>	
<b>Sample Size</b>	<b>40</b>
<b>Sample Standard Deviation</b>	<b>94.30516</b>
<b>Intermediate Calculations</b>	
<b>F Test Statistic</b>	<b>1.055817</b>
Population 1 Sample Degrees of Freedom	39
Population 2 Sample Degrees of Freedom	39
<b>Two-Tail Test</b>	
<b>Upper Critical Value</b>	<b>1.890719</b>
<b>p-Value</b>	<b>0.866186</b>
<b>Do not reject the null hypothesis</b>	

Since both data are drawn from independent populations, the most appropriate test for any difference in the life of the bulbs between the two manufacturers is the pooled-variance  $t$  test.

PHStat output:

<b>Pooled Variance t Test for Differences in Two Means</b>	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>40</b>
<b>Sample Mean</b>	<b>909.65</b>
<b>Sample Standard Deviation</b>	<b>94.3052</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>40</b>
<b>Sample Mean</b>	<b>1018.35</b>
<b>Sample Standard Deviation</b>	<b>96.9014</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	39
Population 2 Sample Degrees of Freedom	39
Total Degrees of Freedom	78
Pooled Variance	9141.676
Difference in Sample Means	-108.7
t-Test Statistic	-5.08431
<b>Two-Tailed Test</b>	
<b>Lower Critical Value</b>	<b>-1.99085</b>
<b>Upper Critical Value</b>	<b>1.990848</b>
<b>p-Value</b>	<b>2.47E-06</b>
<b>Reject the null hypothesis</b>	

Since the p-value is virtually zero, at the 5% level of significance, there is sufficient evidence to reject the null hypothesis of no difference in the mean life of the bulbs between the two manufacturers. You can conclude that there is significant difference in the mean life of the bulbs between the two manufacturers.

Based on the above analyses, you can conclude that there is significant difference in the life of the bulbs between the two manufacturers.

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10.67 Population 1 = Wing A, 2 = Wing B

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

Decision rule: If  $F_{STAT} > 2.5265$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{(1.4172)^2}{(1.3700)^2} = 1.0701$$

Decision: Since  $F_{STAT} = 1.0701$  is lower than the critical bound of  $F_{\alpha/2} = 2.5265$ , do not reject  $H_0$ . There is not enough evidence to conclude that there is a difference between the variances in Wing A and Wing B. Hence, a pooled-variance  $t$  test is more appropriate for determining whether there is a difference in the mean delivery time in the two wings of the hotel.

$H_0: \mu_1 = \mu_2$      $H_1: \mu_1 \neq \mu_2$

Decision rule:  $d.f. = 38$ . If  $t_{STAT} < -2.0244$  or  $t_{STAT} > 2.0244$ , reject  $H_0$ .

Test statistic:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(20 - 1)(1.3700)^2 + (20 - 1)(1.4172)^2}{(20 - 1) + (20 - 1)} = 1.9427$$

$$t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{(10.40 - 8.12) - 0}{\sqrt{1.9427 \left( \frac{1}{20} + \frac{1}{20} \right)}} = 5.1615$$

Decision: Since  $t_{STAT} = 5.1615$  is greater than the upper critical bound of 2.0244, reject  $H_0$ . There is enough evidence of a difference in the mean delivery time in the two wings of the hotel.

10.68  $H_0: \pi_1 = \pi_2$   $H_1: \pi_1 \neq \pi_2$  where Populations: 1 = Men, 2 = Women

Decision rule: If  $p$ -value  $< 0.05$ , reject  $H_0$ .

**Played a game on a video game system:**

PHStat output:

Z Test for Differences in Two Proportions	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Group 1	
Number of Items of Interest	498
Sample Size	600
Group 2	
Number of Items of Interest	243
Sample Size	623
Intermediate Calculations	
Group 1 Proportion	0.83
Group 2 Proportion	0.390048154
Difference in Two Proportions	0.439951846
Average Proportion	0.605887163
Z Test Statistic	15.74002429
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	0
Reject the null hypothesis	

Test statistic: 
$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 15.74, p\text{-value is virtually } 0.$$

Decision: Since the  $p$ -value is smaller than 0.05, reject  $H_0$ . There is enough evidence that there is a difference between boys and girls in the proportion who played a game on a video game system.



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10.68 Read a book for fun:

cont. PHStat output:

f

Z Test for Differences in Two Proportions	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Group 1	
Number of Items of Interest	276
Sample Size	600
Group 2	
Number of Items of Interest	324
Sample Size	623
Intermediate Calculations	
Group 1 Proportion	0.46
Group 2 Proportion	0.520064205
Difference in Two Proportions	-0.060064205
Average Proportion	0.490596893
Z Test Statistic	-2.10053037
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	0.035682212
Reject the null hypothesis	

Test statistic: 
$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = -2.1005, p\text{-value} = 0.0357$$

Decision: Since the  $p$ -value = 0.0357 is smaller than 0.05, reject  $H_0$ . There is enough evidence that there is a difference between boys and girls in the proportion who read a book for fun.

10.68 Gave product advice to parents:

cont. PHstat output:

Z Test for Differences in Two Proportions	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Group 1	
Number of Items of Interest	186
Sample Size	600
Group 2	
Number of Items of Interest	181
Sample Size	623
Intermediate Calculations	
Group 1 Proportion	0.31
Group 2 Proportion	0.290529695
Difference in Two Proportions	0.019470305
Average Proportion	0.300081766
Z Test Statistic	0.742738125
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	0.457640243
Do not reject the null hypothesis	

Test statistic: 
$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 0.7427, p\text{-value} = 0.4576$$

Decision: Since the  $p\text{-value} = 0.4576$  is larger than 0.05, do not reject  $H_0$ . There is not enough evidence that there is a difference between boys and girls in the proportion who gave product advice to parents.

10.68 **Shopped at a mall:**

cont. PHStat output:

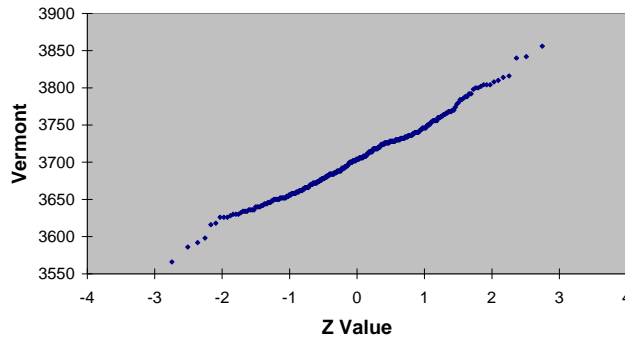
Z Test for Differences in Two Proportions	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Group 1	
Number of Items of Interest	144
Sample Size	600
Group 2	
Number of Items of Interest	262
Sample Size	623
Intermediate Calculations	
Group 1 Proportion	0.24
Group 2 Proportion	0.420545746
Difference in Two Proportions	-0.180545746
Average Proportion	0.331970564
Z Test Statistic	-6.702643071
Two-Tail Test	
Lower Critical Value	-1.959963985
Upper Critical Value	1.959963985
p-Value	2.04683E-11
Reject the null hypothesis	

Test statistic: 
$$Z_{STAT} = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = -6.7026, p\text{-value is virtually } 0.$$

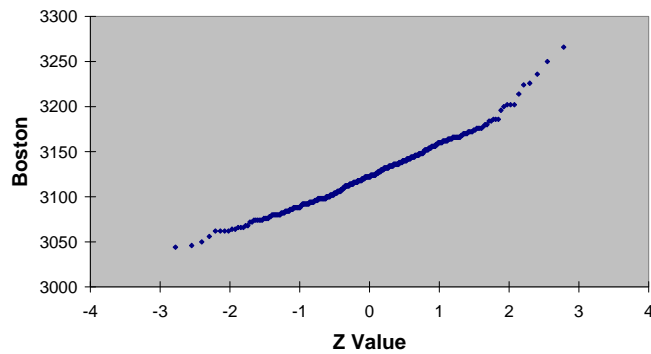
Decision: Since the  $p$ -value is smaller than 0.05, reject  $H_0$ . There is enough evidence that there is a difference between boys and girls in the proportion who shopped at a mall.

10.69

Normal Probability Plot



Normal Probability Plot



The normal probability plots suggest that the two populations are not normally distributed so an  $F$  test is inappropriate for testing the difference in two variances. The sample variances for Boston and Vermont shingles are 1204.992 and 2185.032, respectively. It appears that a separate variance  $t$  test is more appropriate for testing the difference in means.

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10.69  $H_0 : \mu_B = \mu_V$  Mean weights of Boston and Vermont shingles are the same.

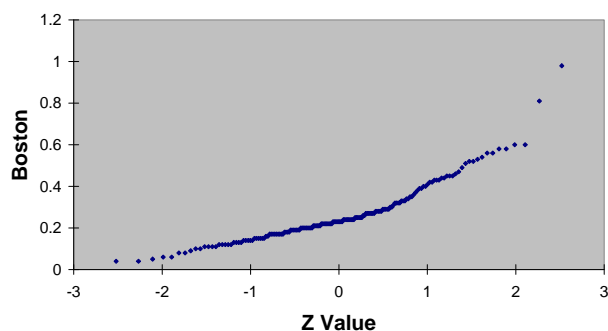
cont.  $H_1 : \mu_B \neq \mu_V$  Mean weights of Boston and Vermont shingles are different.

<b>Separate-Variances <i>t</i> Test for the Difference Between Two Means</b>	
(assumes unequal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>368</b>
<b>Sample Mean</b>	<b>3124.214674</b>
<b>Sample Standard Deviation</b>	<b>34.71299377</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>330</b>
<b>Sample Mean</b>	<b>3704.042424</b>
<b>Sample Standard Deviation</b>	<b>46.74432189</b>
<b>Intermediate Calculations</b>	
Numerator of Degrees of Freedom	97.9257
Denominator of Degrees of Freedom	0.1625
Total Degrees of Freedom	602.7216
Degrees of Freedom	602
Separate Variance Denominator	3.1457
Difference in Sample Means	-579.8277503
<b>Separate-Variance <i>t</i> Test Statistic</b>	<b>-184.3210</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-1.9639</b>
<b>Upper Critical Value</b>	<b>1.9639</b>
<b><i>p</i>-Value</b>	<b>0.0000</b>
<b>Reject the null hypothesis</b>	

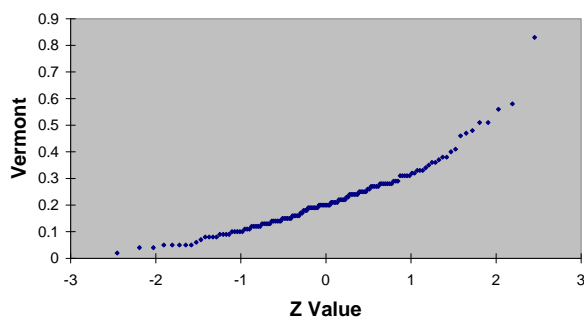
Since the *p*-value is essentially zero, reject  $H_0$ . There is sufficient evidence to conclude that the mean weights of Boston and Vermont shingles are different.

10.70

Normal Probability Plot



Normal Probability Plot



The normal probability plots suggest that the two populations are not normally distributed so an  $F$  test is inappropriate for testing the difference in two variances. The sample variances for Boston and Vermont shingles are 0.0203 and 0.015, respectively, which are not very different. It appears that a pooled-variance  $t$  test is appropriate for testing the difference in means.

$H_0 : \mu_B = \mu_V$  Mean granule loss of Boston and Vermont shingles are the same.

$H_1 : \mu_B \neq \mu_V$  Mean granule loss of Boston and Vermont shingles are different.

Excel output:

t-Test: Two-Sample Assuming Equal Variances

	<i>Boston</i>	<i>Vermont</i>
Mean	0.264059	0.218
Variance	0.020273	0.015055
Observations	170	140
Pooled Variance	0.017918	
Hypothesized Mean Difference	0	
Df	308	
t Stat	3.014921	
P(T<=t) one-tail	0.001392	
t Critical one-tail	1.649817	
P(T<=t) two-tail	0.002784	
t Critical two-tail	1.967696	

Since the  $p$ -value = 0.0028 is less than the 5% level of significance, reject  $H_0$ . There is sufficient evidence to conclude that there is a difference in the mean granule loss of Boston and Vermont shingles.

- 10.71 Since the sample size is small, you have to assume that the 3-year return, 5-year return, 10-year return and expense ratio are all normally distributed to perform the following tests.

**3-year return:**

Populations: 1 = large cap value, 2 = large cap growth

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	10
Sample Standard Deviation	3.569454
Smaller-Variance Sample	
Sample Size	10
Sample Standard Deviation	1.27349
Intermediate Calculations	
F Test Statistic	7.856193
Population 1 Sample Degrees of Freedom	9
Population 2 Sample Degrees of Freedom	9
Two-Tail Test	
Upper Critical Value	4.025994
p-Value	0.005161
Reject the null hypothesis	

Decision: Since  $p\text{-value} < 0.05$ , reject  $H_0$ . There is enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the separate-variance  $t$  test.

10.71

cont.

Populations: 1 = large cap value, 2 = large cap growth

$$H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$$

PHStat output:

<b>Separate Variances t Test for the Difference Between T</b>	
(assumes unequal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>10</b>
<b>Sample Mean</b>	<b>2.11</b>
<b>Sample Standard Deviation</b>	<b>3.56945374</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>10</b>
<b>Sample Mean</b>	<b>0.42</b>
<b>Sample Standard Deviation</b>	<b>1.273490392</b>
<b>Intermediate Calculations</b>	
Numerator of Degrees of Freedom	2.0629
Denominator of Degrees of Freedom	0.1833
Total Degrees of Freedom	11.2547
Degrees of Freedom	11
Separate Variance Denominator	1.1984
Difference in Sample Means	1.69
<b>Separate-Variance t Test Statistic</b>	<b>1.4102</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.2010</b>
<b>Upper Critical Value</b>	<b>2.2010</b>
<b>p-Value</b>	<b>0.1861</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since the  $p$ -value = 0.1861 is larger than 0.05, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean 3-year return is different between the large cap value and large cap growth mutual funds.



## 10.71 5-year return:

cont.

Populations: 1 = large cap growth, 2 = large cap value

 $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same. $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	10
Sample Standard Deviation	2.08444
Smaller-Variance Sample	
Sample Size	10
Sample Standard Deviation	1.863837
Intermediate Calculations	
F Test Statistic	1.250728
Population 1 Sample Degrees of Freedom	9
Population 2 Sample Degrees of Freedom	9
Two-Tail Test	
Upper Critical Value	4.025994
p-Value	0.744361
Do not reject the null hypothesis	

Decision: Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

10.71

cont.

Populations: 1 = large cap growth, 2 = large cap value

 $H_0: \mu_1 = \mu_2$      $H_1: \mu_1 \neq \mu_2$ 

PHStat output:

<b>Pooled-Variance t Test for the Difference Between</b> (assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>10</b>
<b>Sample Mean</b>	<b>4.64</b>
<b>Sample Standard Deviation</b>	<b>2.08444</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>10</b>
<b>Sample Mean</b>	<b>4.95</b>
<b>Sample Standard Deviation</b>	<b>1.863837</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	9
Population 2 Sample Degrees of Freedom	9
Total Degrees of Freedom	18
Pooled Variance	3.909389
Difference in Sample Means	-0.31
<b>t Test Statistic</b>	<b>-0.35058</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.10092</b>
<b>Upper Critical Value</b>	<b>2.100922</b>
<b>p-Value</b>	<b>0.72997</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean 5-year return is different between the large cap value and large cap growth mutual funds.

## 10.71 10-year return:

cont.

Populations: 1 = large cap value, 2 = large cap growth

 $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same. $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	10
Sample Standard Deviation	2.897585
Smaller-Variance Sample	
Sample Size	10
Sample Standard Deviation	1.262317
Intermediate Calculations	
F Test Statistic	5.269089
Population 1 Sample Degrees of Freedom	9
Population 2 Sample Degrees of Freedom	9
Two-Tail Test	
Upper Critical Value	4.025994
p-Value	0.021064
Reject the null hypothesis	

Decision: Since  $p\text{-value} < 0.05$ , reject  $H_0$ . There is enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the separate-variance  $t$  test.

10.71

cont.

Populations: 1 = large cap value, 2 = large cap growth

$$H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$$

PHStat output:

<b>Separate Variances t Test for the Difference Between T</b>	
(assumes unequal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	10
Sample Mean	6.06
Sample Standard Deviation	2.897585202
Population 2 Sample	
Sample Size	10
Sample Mean	2.37
Sample Standard Deviation	1.262317093
Intermediate Calculations	
Numerator of Degrees of Freedom	0.9979
Denominator of Degrees of Freedom	0.0811
Total Degrees of Freedom	12.2974
Degrees of Freedom	12
Separate Variance Denominator	0.9995
Difference in Sample Means	3.69
<b>Separate Variance t Test Statistic</b>	<b>3.6919</b>
Two-Tail Test	
Lower Critical Value	-2.1788
Upper Critical Value	2.1788
<b>p-Value</b>	<b>0.0031</b>
Reject the null hypothesis	

Decision: Since the  $p$ -value < than 0.05, reject  $H_0$ . There is sufficient evidence to conclude that the mean 10-year return is different between the large cap value and large cap growth mutual funds.

## 610 Chapter 10: Two-Sample Tests

### 10.71 Expense ratio:

cont.

Populations: 1 = large cap value, 2 = large cap growth

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	10
Sample Standard Deviation	0.400866
Smaller-Variance Sample	
Sample Size	10
Sample Standard Deviation	0.239574
Intermediate Calculations	
F Test Statistic	2.799752
Population 1 Sample Degrees of Freedom	9
Population 2 Sample Degrees of Freedom	9
Two-Tail Test	
Upper Critical Value	4.025994
p-Value	0.141114
Do not reject the null hypothesis	

Decision: Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

10.71

cont.

Populations: 1 = large cap value, 2 = large cap growth

$$H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$$

PHStat output:

<b>Pooled-Variance t Test for the Difference Between</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>10</b>
<b>Sample Mean</b>	<b>1.206</b>
<b>Sample Standard Deviation</b>	<b>0.400866</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>10</b>
<b>Sample Mean</b>	<b>0.972</b>
<b>Sample Standard Deviation</b>	<b>0.239574</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	9
Population 2 Sample Degrees of Freedom	9
Total Degrees of Freedom	18
Pooled Variance	0.109044
Difference in Sample Means	0.234
<b>t Test Statistic</b>	<b>1.584525</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.10092</b>
<b>Upper Critical Value</b>	<b>2.100922</b>
<b>p-Value</b>	<b>0.130486</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean expense ratio is different between the large cap value and large cap growth mutual funds.

## 612 Chapter 10: Two-Sample Tests

- 10.73 It is reasonable to think that those bond funds with fees will have to be compensated with higher mean return. Hence, you will like to find out if there is evidence that bond funds with fees have a higher mean return than those without fees.

### 2009 return:

Populations: 1 = with fees, 2 = without fees

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	54
Sample Standard Deviation	6.912137
Smaller-Variance Sample	
Sample Size	130
Sample Standard Deviation	5.741261
Intermediate Calculations	
F Test Statistic	1.449473
Population 1 Sample Degrees of Freedom	53
Population 2 Sample Degrees of Freedom	129
Two-Tail Test	
Upper Critical Value	1.543727
p-Value	0.094045
Do not reject the null hypothesis	

Decision: Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

10.73

cont.

Populations: 1 = with fees, 2 = without fees

$$H_0: \mu_1 \leq \mu_2 \quad H_1: \mu_1 > \mu_2$$

PHStat output:

<b>Pooled-Variance t Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>54</b>
<b>Sample Mean</b>	<b>6.916667</b>
<b>Sample Standard Deviation</b>	<b>6.912137</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>130</b>
<b>Sample Mean</b>	<b>7.266923</b>
<b>Sample Standard Deviation</b>	<b>5.741261</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	53
Population 2 Sample Degrees of Freedom	129
Total Degrees of Freedom	182
Pooled Variance	37.2765
Difference in Sample Means	-0.35026
<b>t Test Statistic</b>	<b>-0.35435</b>
<b>Lower-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-1.65327</b>
<b>p-Value</b>	<b>0.361745</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since the  $p$ -value is larger than 0.05, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean 2009 return is higher for bond funds with fees than those without fees.



## 614 Chapter 10: Two-Sample Tests

### 10.73 3-year return:

cont.

Populations: 1 = without fees, 2 = with fees

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHStat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	130
Sample Standard Deviation	2.618265
Smaller-Variance Sample	
Sample Size	54
Sample Standard Deviation	2.269859
Intermediate Calculations	
F Test Statistic	1.330544
Population 1 Sample Degrees of Freedom	129
Population 2 Sample Degrees of Freedom	53
Two-Tail Test	
Upper Critical Value	1.612929
p-Value	0.239508
Do not reject the null hypothesis	

Decision: Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

10.73

cont.

Populations: 1 = without fees, 2 = with fees

$$H_0: \mu_1 \geq \mu_2 \quad H_1: \mu_1 < \mu_2$$

PHStat output:

<b>Pooled-Variance t Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>130</b>
<b>Sample Mean</b>	<b>4.606154</b>
<b>Sample Standard Deviation</b>	<b>2.618265</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>54</b>
<b>Sample Mean</b>	<b>4.798148</b>
<b>Sample Standard Deviation</b>	<b>2.269859</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	129
Population 2 Sample Degrees of Freedom	53
Total Degrees of Freedom	182
Pooled Variance	6.359368
Difference in Sample Means	-0.19199
<b>t Test Statistic</b>	<b>-0.47026</b>
<b>Lower-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-1.65327</b>
<b>p-Value</b>	<b>0.319365</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since  $p$ -value is larger than 0.05, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean 3-year return is higher for bond funds with fees than those without fees.

## 616 Chapter 10: Two-Sample Tests

### 10.73 5-year return:

cont.

Populations: 1 = without fees, 2 = with fees

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHStat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	130
Sample Standard Deviation	1.566466
Smaller-Variance Sample	
Sample Size	54
Sample Standard Deviation	1.282251
Intermediate Calculations	
F Test Statistic	1.492436
Population 1 Sample Degrees of Freedom	129
Population 2 Sample Degrees of Freedom	53
Two-Tail Test	
Upper Critical Value	1.612929
p-Value	0.099584
Do not reject the null hypothesis	

Decision: Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

10.73

cont.

Populations: 1 = without fees, 2 = with fees

$$H_0: \mu_1 \geq \mu_2 \quad H_1: \mu_1 < \mu_2$$

PHStat output:

<b>Pooled-Variance t Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>130</b>
<b>Sample Mean</b>	<b>3.985385</b>
<b>Sample Standard Deviation</b>	<b>1.566466</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>54</b>
<b>Sample Mean</b>	<b>3.987037</b>
<b>Sample Standard Deviation</b>	<b>1.282251</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	129
Population 2 Sample Degrees of Freedom	53
Total Degrees of Freedom	182
Pooled Variance	2.218039
Difference in Sample Means	-0.00165
<b>t Test Statistic</b>	<b>-0.00685</b>
<b>Lower-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-1.65327</b>
<b>p-Value</b>	<b>0.49727</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean 5-year return is higher for bond funds with fees than those without fees.

## 618 Chapter 10: Two-Sample Tests

### 10.73 Expense Ratio:

cont.

Populations: 1 = without fees, 2 = with fees

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHStat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	130
Sample Standard Deviation	0.243789
Smaller-Variance Sample	
Sample Size	54
Sample Standard Deviation	0.14039
Intermediate Calculations	
F Test Statistic	3.015452
Population 1 Sample Degrees of Freedom	129
Population 2 Sample Degrees of Freedom	53
Two-Tail Test	
Upper Critical Value	1.612929
p-Value	1.52E-05
Reject the null hypothesis	

Decision: Since  $p\text{-value} < 0.05$ , reject  $H_0$ . There is enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the separate-variance  $t$  test.

10.73

cont.

Populations: 1 = without fees, 2 = with fees

$$H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$$

PHStat output:

<b>Separate-Variances t Test for the Difference Between Two Means</b>	
(assumes unequal population variances)	
<b>Data</b>	
Hypothesized Difference	0
Level of Significance	0.05
<b>Population 1 Sample</b>	
Sample Size	130
Sample Mean	0.625307692
Sample Standard Deviation	0.243788536
<b>Population 2 Sample</b>	
Sample Size	54
Sample Mean	0.92
Sample Standard Deviation	0.140390292
<b>Intermediate Calculations</b>	
Numerator of Degrees of Freedom	0.0000
Denominator of Degrees of Freedom	0.0000
Total Degrees of Freedom	163.5206
Degrees of Freedom	163
Separate Variance Denominator	0.0287
Difference in Sample Means	-0.294692308
<b>Separate-Variance t Test Statistic</b>	<b>-10.2775</b>
<b>Two-Tail Test</b>	
Lower Critical Value	-1.9746
Upper Critical Value	1.9746
<b>p-Value</b>	<b>0.0000</b>
<b>Reject the null hypothesis</b>	

Decision: Since the  $p$ -value is virtually zero, reject  $H_0$ . There is sufficient evidence to conclude that the mean expense ratio is different between bonds funds with fees and those without fees.

10.74

**2009 return:**

Populations: 1 = short term corporate, 2 = intermediate government

 $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same. $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	97
Sample Standard Deviation	5.686734
Smaller-Variance Sample	
Sample Size	87
Sample Standard Deviation	5.360641
Intermediate Calculations	
F Test Statistic	1.125362
Population 1 Sample Degrees of Freedom	96
Population 2 Sample Degrees of Freedom	86
Two-Tail Test	
Upper Critical Value	1.516688
p-Value	0.577999
Do not reject the null hypothesis	

Decision: Since  $p\text{-value} > 0.05$ , reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

10.74

cont.

Populations: 1 = short term corporate, 2 = intermediate government

 $H_0: \mu_1 = \mu_2$  $H_1: \mu_1 \neq \mu_2$ 

PHStat output:

<b>Pooled-Variance t Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	97
Sample Mean	9.595876
Sample Standard Deviation	5.686734
Population 2 Sample	
Sample Size	87
Sample Mean	4.452874
Sample Standard Deviation	5.360641
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	96
Population 2 Sample Degrees of Freedom	86
Total Degrees of Freedom	182
Pooled Variance	30.63668
Difference in Sample Means	5.143003
t Test Statistic	6.292635
Two-Tail Test	
Lower Critical Value	-1.97308
Upper Critical Value	1.973084
p-Value	2.27E-09
Reject the null hypothesis	

Decision: Since the  $p$ -value is virtually zero, reject  $H_0$ . There is sufficient evidence to conclude that the mean 2009 return is different between the short term corporate bond funds and the intermediate government bond funds.



## 10.74 3-year return:

cont.

Populations: 1 = short term corporate, 2 = intermediate government

 $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same. $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	97
Sample Standard Deviation	2.888542
Smaller-Variance Sample	
Sample Size	87
Sample Standard Deviation	1.570289
Intermediate Calculations	
F Test Statistic	3.383748
Population 1 Sample Degrees of Freedom	96
Population 2 Sample Degrees of Freedom	86
Two-Tail Test	
Upper Critical Value	1.516688
p-Value	2.64E-08
Reject the null hypothesis	

Decision: Since  $p\text{-value} < 0.05$ , reject  $H_0$ . There is enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the separate-variance  $t$  test.

10.74

cont.

Populations: 1 = short term corporate, 2 = intermediate government

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

PHStat output:

Separate-Variances <i>t</i> Test for the Difference Between Two Means	
(assumes unequal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	97
Sample Mean	3.819587629
Sample Standard Deviation	2.88854199
Population 2 Sample	
Sample Size	87
Sample Mean	5.602298851
Sample Standard Deviation	1.570289339
Intermediate Calculations	
Numerator of Degrees of Freedom	0.0131
Denominator of Degrees of Freedom	0.0001
Total Degrees of Freedom	151.3445
Degrees of Freedom	151
Separate Variance Denominator	0.3382
Difference in Sample Means	-1.782711222
Separate-Variance <i>t</i> Test Statistic	-5.2716
Two-Tail Test	
Lower Critical Value	-1.9758
Upper Critical Value	1.9758
<i>p</i> -Value	0.0000
Reject the null hypothesis	

Decision: Since the *p*-value is virtually zero, reject  $H_0$ . There is sufficient evidence to conclude that the mean 3-year return is different between the short term corporate bond funds and the intermediate government bond funds.

## 624 Chapter 10: Two-Sample Tests

### 10.74 5-year return:

cont.

Populations: 1 = short term corporate, 2 = intermediate government

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	97
Sample Standard Deviation	1.668427
Smaller-Variance Sample	
Sample Size	87
Sample Standard Deviation	0.979634
Intermediate Calculations	
F Test Statistic	2.900596
Population 1 Sample Degrees of Freedom	96
Population 2 Sample Degrees of Freedom	86
Two-Tail Test	
Upper Critical Value	1.516688
p-Value	9.72E-07
Reject the null hypothesis	

Decision: Since  $p\text{-value} < 0.05$ , reject  $H_0$ . There is enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the separate-variance  $t$  test.

10.74

cont.

Populations: 1 = short term corporate, 2 = intermediate government

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

Separate-Variances <i>t</i> Test for the Difference Between Two Means (assumes unequal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	97
Sample Mean	3.473195876
Sample Standard Deviation	1.66842712
Population 2 Sample	
Sample Size	87
Sample Mean	4.557471264
Sample Standard Deviation	0.979633556
Intermediate Calculations	
Numerator of Degrees of Freedom	0.0016
Denominator of Degrees of Freedom	0.0000
Total Degrees of Freedom	157.9370
Degrees of Freedom	157
Separate Variance Denominator	0.1993
Difference in Sample Means	-1.084275388
Separate-Variance <i>t</i> Test Statistic	-5.4399
Two-Tail Test	
Lower Critical Value	-1.9752
Upper Critical Value	1.9752
<i>p</i> -Value	0.0000
Reject the null hypothesis	

Decision: Since the *p*-value is virtually zero, reject  $H_0$ . There is sufficient evidence to conclude that the mean 5-year return is different between the short term corporate bond funds and the intermediate government bond funds.

## 10.74 Expense Ratio:

cont.

Populations: 1 = intermediate government, 2 = short term corporate

 $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same. $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHStat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	87
Sample Standard Deviation	0.300741
Smaller-Variance Sample	
Sample Size	97
Sample Standard Deviation	0.201476
Intermediate Calculations	
F Test Statistic	2.228129
Population 1 Sample Degrees of Freedom	86
Population 2 Sample Degrees of Freedom	96
Two-Tail Test	
Upper Critical Value	1.509473
p-Value	0.000153
Reject the null hypothesis	

Decision: Since  $p\text{-value} < 0.05$ , reject  $H_0$ . There is enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the separate-variance  $t$  test.

10.74

cont.

Populations: 1 = short term corporate, 2 = intermediate government

 $H_0: \mu_1 = \mu_2$      $H_1: \mu_1 \neq \mu_2$ 

PHStat output:

Separate-Variances <i>t</i> Test for the Difference Between Two Means (assumes unequal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	97
Sample Mean	0.670515464
Sample Standard Deviation	0.201475656
Population 2 Sample	
Sample Size	87
Sample Mean	0.757816092
Sample Standard Deviation	0.300741043
Intermediate Calculations	
Numerator of Degrees of Freedom	0.0000
Denominator of Degrees of Freedom	0.0000
Total Degrees of Freedom	147.7279
Degrees of Freedom	147
Separate Variance Denominator	0.0382
Difference in Sample Means	-0.087300628
Separate-Variance <i>t</i> Test Statistic	-2.2863
Two-Tail Test	
Lower Critical Value	-1.9762
Upper Critical Value	1.9762
<i>p</i> -Value	0.0237
Reject the null hypothesis	

Decision: Since  $p\text{-value} < 0.05$ , reject  $H_0$ . There is sufficient evidence to conclude that the mean expense ratio is different between the short term corporate bond funds and the intermediate government bond funds.

10.75 (a) **GPA:**

Population 1 = males, 2 = females

 $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same. $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	29
Sample Standard Deviation	1.536902
Smaller-Variance Sample	
Sample Size	33
Sample Standard Deviation	1.354706
Intermediate Calculations	
F Test Statistic	1.287072
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Two-Tail Test	
Upper Critical Value	2.058973
p-Value	0.488273
Do not reject the null hypothesis	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = males, 2 = females

 $H_0: \mu_1 = \mu_2$  $H_1: \mu_1 \neq \mu_2$

10.75 (a) PHStat output:  
cont.

<b>Pooled-Variance <i>t</i> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>29</b>
<b>Sample Mean</b>	<b>21.17241</b>
<b>Sample Standard Deviation</b>	<b>1.536902</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>33</b>
<b>Sample Mean</b>	<b>21.09091</b>
<b>Sample Standard Deviation</b>	<b>1.354706</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Total Degrees of Freedom	60
Pooled Variance	2.081087
Difference in Sample Means	0.081505
<b><i>t</i> Test Statistic</b>	<b>0.221972</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.0003</b>
<b>Upper Critical Value</b>	<b>2.000298</b>
<b><i>p</i>-Value</b>	<b>0.82509</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean GPA is different between males and females.

### Expected Salary:

Population 1 = females, 2 = males

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.



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10.75 (a) PHstat output:  
cont.

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	33
Sample Standard Deviation	13.2724
Smaller-Variance Sample	
Sample Size	29
Sample Standard Deviation	10.79317
Intermediate Calculations	
F Test Statistic	1.512171
Population 1 Sample Degrees of Freedom	32
Population 2 Sample Degrees of Freedom	28
Two-Tail Test	
Upper Critical Value	2.096283
p-Value	0.269804
Do not reject the null hypothesis	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = males, 2 = females

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

10.75 (a) PHStat output:  
cont.

Pooled-Variance <i>t</i> Test for the Difference Between Two Means (assumes equal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	29
Sample Mean	48.27586
Sample Standard Deviation	10.79317
Population 2 Sample	
Sample Size	33
Sample Mean	48.78788
Sample Standard Deviation	13.2724
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Total Degrees of Freedom	60
Pooled Variance	148.3135
Difference in Sample Means	-0.51202
<i>t</i> Test Statistic	-0.16518
Two-Tail Test	
Lower Critical Value	-2.0003
Upper Critical Value	2.000298
<i>p</i> -Value	0.869359
Do not reject the null hypothesis	

Since the *p*-value > 0.05, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean expected salary is different between males and females.

#### Social Networking:

Population 1 = males, 2 = females

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

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10.75 (a) PHstat output:  
cont.

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	29
Sample Standard Deviation	0.941647
Smaller-Variance Sample	
Sample Size	33
Sample Standard Deviation	0.751262
Intermediate Calculations	
F Test Statistic	1.571065
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Two-Tail Test	
Upper Critical Value	2.058973
p-Value	0.217459
Do not reject the null hypothesis	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = males, 2 = females

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

10.75 (a) PHStat output:  
cont.

<b>Pooled-Variance <i>t</i> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>29</b>
<b>Sample Mean</b>	<b>1.62069</b>
<b>Sample Standard Deviation</b>	<b>0.941647</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>33</b>
<b>Sample Mean</b>	<b>1.424242</b>
<b>Sample Standard Deviation</b>	<b>0.751262</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Total Degrees of Freedom	60
Pooled Variance	0.714803
Difference in Sample Means	0.196447
<b><i>t</i> Test Statistic</b>	<b>0.912878</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.0003</b>
<b>Upper Critical Value</b>	<b>2.000298</b>
<b><i>p</i>-Value</b>	<b>0.36496</b>
<b>Do not reject the null hypothesis</b>	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean number of social networking sites registered for is different between males and females.

**Age:**

Population 1 = males, 2 = females

$H_0$ :  $\sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1$ :  $\sigma_1^2 \neq \sigma_2^2$  The population variances are different.

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10.75 (a) PHstat output:  
cont.

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	29
Sample Standard Deviation	1.536902
Smaller-Variance Sample	
Sample Size	33
Sample Standard Deviation	1.354706
Intermediate Calculations	
F Test Statistic	1.287072
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Two-Tail Test	
Upper Critical Value	2.058973
p-Value	0.488273
Do not reject the null hypothesis	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = males, 2 = females

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

10.75 (a) PHStat output:  
cont.

<b>Pooled-Variance <i>t</i> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>29</b>
<b>Sample Mean</b>	<b>21.17241</b>
<b>Sample Standard Deviation</b>	<b>1.536902</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>33</b>
<b>Sample Mean</b>	<b>21.09091</b>
<b>Sample Standard Deviation</b>	<b>1.354706</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Total Degrees of Freedom	60
Pooled Variance	2.081087
Difference in Sample Means	0.081505
<b><i>t</i> Test Statistic</b>	<b>0.221972</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.0003</b>
<b>Upper Critical Value</b>	<b>2.000298</b>
<b><i>p</i>-Value</b>	<b>0.82509</b>
<b>Do not reject the null hypothesis</b>	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean age is different between males and females.

**Spending:**

Population 1 = males, 2 = females

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

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10.75 (a) PHStat output:  
cont.

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	29
Sample Standard Deviation	239.245
Smaller-Variance Sample	
Sample Size	33
Sample Standard Deviation	204.5843
Intermediate Calculations	
F Test Statistic	1.367544
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Two-Tail Test	
Upper Critical Value	2.058973
p-Value	0.391107
Do not reject the null hypothesis	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = males, 2 = females

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

PHStat output:

Pooled-Variance t Test for the Difference Between Two Means (assumes equal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	29
Sample Mean	516.0345
Sample Standard Deviation	239.245
Population 2 Sample	
Sample Size	33
Sample Mean	452.1212
Sample Standard Deviation	204.5843
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Total Degrees of Freedom	60
Pooled Variance	49033.67
Difference in Sample Means	63.91327
t Test Statistic	1.133976
Two-Tail Test	
Lower Critical Value	-2.0003
Upper Critical Value	2.000298
p-Value	0.261315
Do not reject the null hypothesis	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean spending is different between males and females.

10.75 (a)  
cont.

**Text Messages:**

Population 1 = males, 2 = females

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	29
Sample Standard Deviation	219.0672
Smaller-Variance Sample	
Sample Size	33
Sample Standard Deviation	213.348
Intermediate Calculations	
F Test Statistic	1.054333
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Two-Tail Test	
Upper Critical Value	2.058973
p-Value	0.879489
Do not reject the null hypothesis	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = males, 2 = females

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$



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10.75 (a) PHStat output:  
cont.

Pooled-Variance <i>t</i> Test for the Difference Between Two Means (assumes equal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	29
Sample Mean	256.2069
Sample Standard Deviation	219.0672
Population 2 Sample	
Sample Size	33
Sample Mean	237.4242
Sample Standard Deviation	213.348
Intermediate Calculations	
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Total Degrees of Freedom	60
Pooled Variance	46671.48
Difference in Sample Means	18.78265
<i>t</i> Test Statistic	0.341579
Two-Tail Test	
Lower Critical Value	-2.0003
Upper Critical Value	2.000298
<i>p</i> -Value	0.733861
Do not reject the null hypothesis	

Since the *p*-value > 0.05, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean text messages sent in a week is different between males and females.

### Wealth:

Population 1 = males, 2 = females

$H_0$ :  $\sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1$ :  $\sigma_1^2 \neq \sigma_2^2$  The population variances are different.

10.75 (a) PHstat output:  
cont.

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	29
Sample Standard Deviation	27.34992
Smaller-Variance Sample	
Sample Size	33
Sample Standard Deviation	5.360431
Intermediate Calculations	
F Test Statistic	26.0323
Population 1 Sample Degrees of Freedom	28
Population 2 Sample Degrees of Freedom	32
Two-Tail Test	
Upper Critical Value	2.058973
p-Value	7.69E-15
Reject the null hypothesis	

Since the  $p$ -value  $< 0.05$ , reject  $H_0$ . There is enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the separate-variance  $t$  test.

Population 1 = males, 2 = females

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

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10.75 (a) PHStat output:  
cont.

<b>Separate-Variances <i>t</i> Test for the Difference Between Two Means</b>	
(assumes unequal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>29</b>
<b>Sample Mean</b>	<b>12.55344828</b>
<b>Sample Standard Deviation</b>	<b>27.34991816</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>33</b>
<b>Sample Mean</b>	<b>2.403787879</b>
<b>Sample Standard Deviation</b>	<b>5.360431278</b>
<b>Intermediate Calculations</b>	
Numerator of Degrees of Freedom	710.9934
Denominator of Degrees of Freedom	23.7850
Total Degrees of Freedom	29.8925
Degrees of Freedom	29
Separate Variance Denominator	5.1638
Difference in Sample Means	10.1496604
<b>Separate-Variance <i>t</i> Test Statistic</b>	<b>1.9656</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.0452</b>
<b>Upper Critical Value</b>	<b>2.0452</b>
<b><i>p</i>-Value</b>	<b>0.0590</b>
<b>Do not reject the null hypothesis</b>	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean wealth needed to feel rich is different between males and females.

10.75 (b)  
cont.

**GPA:**

Population 1 = do not plan to go to graduate school, 2 = plan to go to graduate school

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	12
Sample Standard Deviation	0.425245
Smaller-Variance Sample	
Sample Size	28
Sample Standard Deviation	0.379252
Intermediate Calculations	
F Test Statistic	1.257251
Population 1 Sample Degrees of Freedom	11
Population 2 Sample Degrees of Freedom	27
Two-Tail Test	
Upper Critical Value	2.514294
p-Value	0.599929
Do not reject the null hypothesis	

Decision rule: If  $F_{STAT} > 2.5143$ , reject  $H_0$ .

Test statistic:  $F_{STAT} = \frac{S_1^2}{S_2^2} = 1.2573$

Decision: Since  $F_{STAT} = 1.2573$  is lower than 2.5143, do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = do not plan to go to graduate school, 2 = plan to go to graduate school

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

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10.75 (b) PHStat output:  
cont.

<b>Pooled-Variance t Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>12</b>
<b>Sample Mean</b>	<b>3.141667</b>
<b>Sample Standard Deviation</b>	<b>0.425245</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>28</b>
<b>Sample Mean</b>	<b>3.077857</b>
<b>Sample Standard Deviation</b>	<b>0.379252</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	11
Population 2 Sample Degrees of Freedom	27
Total Degrees of Freedom	38
Pooled Variance	0.154543
Difference in Sample Means	0.06381
<b>t Test Statistic</b>	<b>0.470436</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.02439</b>
<b>Upper Critical Value</b>	<b>2.024394</b>
<b>p-Value</b>	<b>0.640733</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since  $t_{STAT} = 0.4704$  is in between the upper critical bound of 2.0244 and the lower critical bound of -2.0244, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean GPA is different between those students who plan to go to graduate school and those students who do not plan to go to graduate school.

### Expected Salary:

Population 1 = do not plan to go to graduate school, 2 = plan to go to graduate school

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

10.75 (b) PHstat output:  
cont.

<b>F Test for Differences in Two Variances</b>	
<b>Data</b>	
Level of Significance	<b>0.05</b>
<b>Larger-Variance Sample</b>	
Sample Size	<b>12</b>
Sample Standard Deviation	<b>12.05386</b>
<b>Smaller-Variance Sample</b>	
Sample Size	<b>28</b>
Sample Standard Deviation	<b>10.13942</b>
<b>Intermediate Calculations</b>	
<b>F Test Statistic</b>	<b>1.413272</b>
Population 1 Sample Degrees of Freedom	<b>11</b>
Population 2 Sample Degrees of Freedom	<b>27</b>
<b>Two-Tail Test</b>	
<b>Upper Critical Value</b>	<b>2.514294</b>
<b>p-Value</b>	<b>0.446767</b>
<b>Do not reject the null hypothesis</b>	

Decision rule: If  $F_{STAT} > 2.5143$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = 1.4133$$

Decision: Since  $F_{STAT} = 1.4133$  is lower than 2.5143, do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

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10.75 (b) PHStat output:  
cont.

<b>Pooled-Variance t Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>12</b>
<b>Sample Mean</b>	<b>42.25</b>
<b>Sample Standard Deviation</b>	<b>12.05386</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>28</b>
<b>Sample Mean</b>	<b>48.125</b>
<b>Sample Standard Deviation</b>	<b>10.13942</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	11
Population 2 Sample Degrees of Freedom	27
Total Degrees of Freedom	38
Pooled Variance	115.1069
Difference in Sample Means	-5.875
<b>t Test Statistic</b>	<b>-1.58707</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.02439</b>
<b>Upper Critical Value</b>	<b>2.024394</b>
<b>p-Value</b>	<b>0.120784</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since  $t_{STAT} = -1.5871$  is between the lower critical bound of -2.0244 and the upper critical bound of 2.0244, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean expected salary is different between those students who plan to go to graduate school and those students who do not plan to go to graduate school.

### Number of Social Networking:

Population 1 = plan to go to graduate school, 2 = do not plan to go to graduate school

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

10.75 (b) PHstat output:  
cont.

<b>F Test for Differences in Two Variances</b>	
<b>Data</b>	
<b>Level of Significance</b>	<b>0.05</b>
<b>Larger-Variance Sample</b>	
<b>Sample Size</b>	<b>28</b>
<b>Sample Standard Deviation</b>	<b>0.922958</b>
<b>Smaller-Variance Sample</b>	
<b>Sample Size</b>	<b>12</b>
<b>Sample Standard Deviation</b>	<b>0.514929</b>
<b>Intermediate Calculations</b>	
<b>F Test Statistic</b>	<b>3.212698</b>
Population 1 Sample Degrees of Freedom	27
Population 2 Sample Degrees of Freedom	11
<b>Two-Tail Test</b>	
<b>Upper Critical Value</b>	<b>3.142182</b>
<b>p-Value</b>	<b>0.046019</b>
<b>Reject the null hypothesis</b>	

Decision rule: If  $F_{STAT} > 3.1422$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = 3.2127$$

Decision: Since  $F_{STAT} = 3.2127$  is larger than the upper critical bound of 3.1422, reject  $H_0$ . There is enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the separate-variance  $t$  test.

Population 1 = plan to go to graduate school, 2 = do not plan to go to graduate school  
 $H_0: \mu_1 = \mu_2$                        $H_1: \mu_1 \neq \mu_2$



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10..75 (b) Excel output:  
cont.

<b>Separate-Variances <i>t</i> Test for the Difference Between Two Means</b>	
(assumes unequal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>28</b>
<b>Sample Mean</b>	<b>1.5</b>
<b>Sample Standard Deviation</b>	<b>0.922958207</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>12</b>
<b>Sample Mean</b>	<b>1.416666667</b>
<b>Sample Standard Deviation</b>	<b>0.514928651</b>
<b>Intermediate Calculations</b>	
Numerator of Degrees of Freedom	0.0028
Denominator of Degrees of Freedom	0.0001
Total Degrees of Freedom	35.0634
Degrees of Freedom	35
Separate Variance Denominator	0.2292
Difference in Sample Means	0.083333333
<b>Separate-Variance <i>t</i> Test Statistic</b>	<b>0.3636</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.0301</b>
<b>Upper Critical Value</b>	<b>2.0301</b>
<b><i>p</i>-Value</b>	<b>0.7183</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since  $t_{STAT} = 0.3636$  is between the lower critical bound of -2.0301 and the upper critical bound of 2.0301, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean number of social networking sites registered for is different between those students who plan to go to graduate school and those students who do not plan to go to graduate school.

10.75 (b)  
cont. **Age:**

Population 1 = plan to go to graduate school, 2 = do not plan to go to graduate school

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

<b>F Test for Differences in Two Variances</b>	
<b>Data</b>	
Level of Significance	<b>0.05</b>
<b>Larger-Variance Sample</b>	
Sample Size	<b>28</b>
Sample Standard Deviation	<b>1.634208</b>
<b>Smaller-Variance Sample</b>	
Sample Size	<b>12</b>
Sample Standard Deviation	<b>0.792961</b>
<b>Intermediate Calculations</b>	
<b>F Test Statistic</b>	<b>4.247275</b>
Population 1 Sample Degrees of Freedom	27
Population 2 Sample Degrees of Freedom	11
<b>Two-Tail Test</b>	
<b>Upper Critical Value</b>	<b>3.142182</b>
<b>p-Value</b>	<b>0.015281</b>
<b>Reject the null hypothesis</b>	

Decision rule: If  $F_{STAT} > 3.1422$ , reject  $H_0$ .

Test statistic:  $F_{STAT} = \frac{S_1^2}{S_2^2} = 4.2473$

Decision: Since  $F_{STAT} = 4.2473$  is greater than 3.1422, reject  $H_0$ . There is enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the separate-variance  $t$  test.

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- 10.75 (b) Population 1 = plan to go to graduate school, 2 = do not plan to go to graduate school  
 cont.  $H_0: \mu_1 = \mu_2$   $H_1: \mu_1 \neq \mu_2$

Excel output:

Separate-Variances <i>t</i> Test for the Difference Between Two Means	
(assumes unequal population variances)	
Data	
Hypothesized Difference	0
Level of Significance	0.05
Population 1 Sample	
Sample Size	28
Sample Mean	21.32142857
Sample Standard Deviation	1.634207735
Population 2 Sample	
Sample Size	12
Sample Mean	20.91666667
Sample Standard Deviation	0.792961461
Intermediate Calculations	
Numerator of Degrees of Freedom	0.0218
Denominator of Degrees of Freedom	0.0006
Total Degrees of Freedom	37.2327
Degrees of Freedom	37
Separate Variance Denominator	0.3844
Difference in Sample Means	0.404761905
Separate-Variance <i>t</i> Test Statistic	1.0529
Two-Tail Test	
Lower Critical Value	-2.0262
Upper Critical Value	2.0262
<i>p</i> -Value	0.2992
Do not reject the null hypothesis	

10.75 (b)  
cont.

Decision: Since  $t_{STAT} = 1.0529$  is between the lower critical bound of -2.0262 and the upper critical bound of 2.0262, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean age is different between those students who plan to go to graduate school and those students who do not plan to go to graduate school.

**Spending:**

Population 1 = plan to go to graduate school, 2 = do not plan to go to graduate school

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	28
Sample Standard Deviation	269.1621
Smaller-Variance Sample	
Sample Size	12
Sample Standard Deviation	228.7565
Intermediate Calculations	
F Test Statistic	1.384462
Population 1 Sample Degrees of Freedom	27
Population 2 Sample Degrees of Freedom	11
Two-Tail Test	
Upper Critical Value	3.142182
p-Value	0.584208
Do not reject the null hypothesis	

Decision rule: If  $F_{STAT} > 3.1422$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{152.5941^2}{75.7109^2} = 1.3845$$

Decision: Since  $F_{STAT} = 1.3845$  is smaller than 3.1422, do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = plan to go to graduate school, 2 = do not plan to go to graduate school

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

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10.75 (b) Excel output:  
cont.

<b>Pooled-Variance t Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>28</b>
<b>Sample Mean</b>	<b>500.8929</b>
<b>Sample Standard Deviation</b>	<b>269.1621</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>12</b>
<b>Sample Mean</b>	<b>487.5</b>
<b>Sample Standard Deviation</b>	<b>228.7565</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	27
Population 2 Sample Degrees of Freedom	11
Total Degrees of Freedom	38
Pooled Variance	66624.41
Difference in Sample Means	13.39286
<b>t Test Statistic</b>	<b>0.150382</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.02439</b>
<b>Upper Critical Value</b>	<b>2.024394</b>
<b>p-Value</b>	<b>0.881259</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since  $t_{STAT} = 0.1504$  is in between the lower critical bound of -2.0244 and the upper critical bound of 2.0244, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean spending is different between those students who plan to go to graduate school and those students who do not plan to go to graduate school.

10.75 (b)

cont. **Text messages sent:**

Population 1 = do not plan to go to graduate school, 2 = plan to go to graduate school

 $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same. $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	12
Sample Standard Deviation	253.776
Smaller-Variance Sample	
Sample Size	28
Sample Standard Deviation	207.5808
Intermediate Calculations	
F Test Statistic	1.494607
Population 1 Sample Degrees of Freedom	11
Population 2 Sample Degrees of Freedom	27
Two-Tail Test	
Upper Critical Value	2.514294
p-Value	0.38163
Do not reject the null hypothesis	

Decision rule: If  $F_{STAT} > 2.5143$ , reject  $H_0$ .Test statistic:  $F_{STAT} = \frac{S_1^2}{S_2^2} = 1.4946$ 

Decision: Since  $F_{STAT} = 1.4946$  is lower than 2.5143, do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = do not plan to go to graduate school, 2 = plan to go to graduate school

 $H_0: \mu_1 = \mu_2$  $H_1: \mu_1 \neq \mu_2$

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10.75 (b) PHStat output:  
cont.

<b>Pooled-Variance t Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>12</b>
<b>Sample Mean</b>	<b>292.5</b>
<b>Sample Standard Deviation</b>	<b>253.776</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>28</b>
<b>Sample Mean</b>	<b>252.6786</b>
<b>Sample Standard Deviation</b>	<b>207.5808</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	11
Population 2 Sample Degrees of Freedom	27
Total Degrees of Freedom	38
Pooled Variance	49259.19
Difference in Sample Means	39.82143
<b>t Test Statistic</b>	<b>0.520011</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.02439</b>
<b>Upper Critical Value</b>	<b>2.024394</b>
<b>p-Value</b>	<b>0.606072</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since  $t_{STAT} = 0.5200$  is in between the upper critical bound of 2.0244 and the lower critical bound of -2.0244, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean text messages sent in a week is different between those students who plan to go to graduate school and those students who do not plan to go to graduate school.

### Wealth:

Population 1 = plan to go to graduate school, 2 = do not plan to go to graduate school

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

10.75 (b) PHstat output:  
cont.

<b>F Test for Differences in Two Variances</b>	
<b>Data</b>	
Level of Significance	<b>0.05</b>
<b>Larger-Variance Sample</b>	
Sample Size	<b>28</b>
Sample Standard Deviation	<b>28.27505</b>
<b>Smaller-Variance Sample</b>	
Sample Size	<b>12</b>
Sample Standard Deviation	<b>1.578165</b>
<b>Intermediate Calculations</b>	
<b>F Test Statistic</b>	<b>320.9975</b>
Population 1 Sample Degrees of Freedom	<b>27</b>
Population 2 Sample Degrees of Freedom	<b>11</b>
<b>Two-Tail Test</b>	
<b>Upper Critical Value</b>	<b>3.142182</b>
<b>p-Value</b>	<b>2.99E-12</b>
<b>Reject the null hypothesis</b>	

Decision rule: If  $F_{STAT} > 3.1422$ , reject  $H_0$ .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = 320.9975$$

Decision: Since  $F_{STAT} = 320.9975$  is greater than 3.1422, reject  $H_0$ . There is enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the separate-variance  $t$  test.

Population 1 = plan to go to graduate school, 2 = do not plan to go to graduate school

$$H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$$



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10.75 (b) Excel output:  
cont.

<b>Separate-Variances <i>t</i> Test for the Difference Between Two Means</b>	
(assumes unequal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>28</b>
<b>Sample Mean</b>	<b>12.70535714</b>
<b>Sample Standard Deviation</b>	<b>28.27504573</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>12</b>
<b>Sample Mean</b>	<b>1.716666667</b>
<b>Sample Standard Deviation</b>	<b>1.57816541</b>
<b>Intermediate Calculations</b>	
Numerator of Degrees of Freedom	827.1574
Denominator of Degrees of Freedom	30.1988
Total Degrees of Freedom	27.3904
Degrees of Freedom	27
Separate Variance Denominator	5.3629
Difference in Sample Means	10.98869048
<b>Separate-Variance <i>t</i> Test Statistic</b>	<b>2.0490</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.0518</b>
<b>Upper Critical Value</b>	<b>2.0518</b>
<b><i>p</i>-Value</b>	<b>0.0503</b>
<b>Do not reject the null hypothesis</b>	

Decision: Since  $t_{STAT} = 2.0490$  is between the lower critical bound of -2.0518 and the upper critical bound of 2.0518, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean wealth needed to feel rich is different between those students who plan to go to graduate school and those students who do not plan to go to graduate school.

## 10.77 Undergrad GPA:

Population 1 = Females, 2 = Males

 $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same. $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	19
Sample Standard Deviation	0.316597
Smaller-Variance Sample	
Sample Size	25
Sample Standard Deviation	0.308869
Intermediate Calculations	
F Test Statistic	1.05067
Population 1 Sample Degrees of Freedom	18
Population 2 Sample Degrees of Freedom	24
Two-Tail Test	
Upper Critical Value	2.364797
p-Value	0.894969
Do not reject the null hypothesis	

Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = Males, 2 = females

 $H_0: \mu_1 = \mu_2$  $H_1: \mu_1 \neq \mu_2$

10.77  
cont.

PHStat output:

<b>Pooled-Variance <i>t</i> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>19</b>
<b>Sample Mean</b>	<b>3.463158</b>
<b>Sample Standard Deviation</b>	<b>0.316597</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>25</b>
<b>Sample Mean</b>	<b>3.296</b>
<b>Sample Standard Deviation</b>	<b>0.308869</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	18
Population 2 Sample Degrees of Freedom	24
Total Degrees of Freedom	42
Pooled Variance	0.097472
Difference in Sample Means	0.167158
<b><i>t</i> Test Statistic</b>	<b>1.759171</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.01808</b>
<b>Upper Critical Value</b>	<b>2.018082</b>
<b><i>p</i>-Value</b>	<b>0.085832</b>
<b>Do not reject the null hypothesis</b>	

Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean undergraduate GPA is different between males and females.

#### Graduate GPA:

Population 1 = Females, 2 = Males

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

10.77  
cont.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	19
Sample Standard Deviation	0.419412
Smaller-Variance Sample	
Sample Size	25
Sample Standard Deviation	0.399917
Intermediate Calculations	
F Test Statistic	1.099873
Population 1 Sample Degrees of Freedom	18
Population 2 Sample Degrees of Freedom	24
Two-Tail Test	
Upper Critical Value	2.364797
p-Value	0.814295
Do not reject the null hypothesis	

Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = Males, 2 = females

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

10.77  
cont.

PHStat output:

<b>Pooled-Variance <i>t</i> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
Hypothesized Difference	0
Level of Significance	0.05
<b>Population 1 Sample</b>	
Sample Size	25
Sample Mean	3.308
Sample Standard Deviation	0.399917
<b>Population 2 Sample</b>	
Sample Size	19
Sample Mean	3.357895
Sample Standard Deviation	0.419412
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	24
Population 2 Sample Degrees of Freedom	18
Total Degrees of Freedom	42
Pooled Variance	0.166779
Difference in Sample Means	-0.04989
<b><i>t</i> Test Statistic</b>	<b>-0.40143</b>
<b>Two-Tail Test</b>	
Lower Critical Value	-2.01808
Upper Critical Value	2.018082
<b><i>p</i>-Value</b>	<b>0.690142</b>
<b>Do not reject the null hypothesis</b>	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean graduate GPA is different between males and females.

**Age:**

Population 1 = Females, 2 = Males

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

10.77  
cont.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	19
Sample Standard Deviation	5.919044
Smaller-Variance Sample	
Sample Size	25
Sample Standard Deviation	4.982302
Intermediate Calculations	
F Test Statistic	1.411377
Population 1 Sample Degrees of Freedom	18
Population 2 Sample Degrees of Freedom	24
Two-Tail Test	
Upper Critical Value	2.364797
p-Value	0.425058
Do not reject the null hypothesis	

Since the  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = Males, 2 = females

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

10.77  
cont.

PHStat output:

<b>Pooled-Variance <i>t</i> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>25</b>
<b>Sample Mean</b>	<b>26.36</b>
<b>Sample Standard Deviation</b>	<b>4.982302</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>19</b>
<b>Sample Mean</b>	<b>26.57895</b>
<b>Sample Standard Deviation</b>	<b>5.919044</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	24
Population 2 Sample Degrees of Freedom	18
Total Degrees of Freedom	42
Pooled Variance	29.1998
Difference in Sample Means	-0.21895
<b><i>t</i> Test Statistic</b>	<b>-0.13313</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.01808</b>
<b>Upper Critical Value</b>	<b>2.018082</b>
<b><i>p</i>-Value</b>	<b>0.894728</b>
<b>Do not reject the null hypothesis</b>	

Since the *p*-value > 0.05, do not reject  $H_0$ . There is insufficient evidence to conclude that the mean age is different between males and females.

### Expected Salary:

Population 1 = Males, 2 = females

$H_0$ :  $\sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1$ :  $\sigma_1^2 \neq \sigma_2^2$  The population variances are different.

10.77  
cont.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	25
Sample Standard Deviation	51.06574
Smaller-Variance Sample	
Sample Size	19
Sample Standard Deviation	34.6125
Intermediate Calculations	
F Test Statistic	2.176674
Population 1 Sample Degrees of Freedom	24
Population 2 Sample Degrees of Freedom	18
Two-Tail Test	
Upper Critical Value	2.502697
p-Value	0.094729
Do not reject the null hypothesis	

Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = Males, 2 = females

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$



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10.77  
cont.

PHStat output:

<b>Pooled-Variance <i>t</i> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>25</b>
<b>Sample Mean</b>	<b>86.72</b>
<b>Sample Standard Deviation</b>	<b>51.06574</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>19</b>
<b>Sample Mean</b>	<b>78.05263</b>
<b>Sample Standard Deviation</b>	<b>34.6125</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	24
Population 2 Sample Degrees of Freedom	18
Total Degrees of Freedom	42
Pooled Variance	2003.559
Difference in Sample Means	8.667368
<b><i>t</i> Test Statistic</b>	<b>0.636219</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.01808</b>
<b>Upper Critical Value</b>	<b>2.018082</b>
<b><i>p</i>-Value</b>	<b>0.528086</b>
<b>Do not reject the null hypothesis</b>	

Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean expected salary is different between males and females.

## 10.77 Spending:

cont.

Population 1 = Males, 2 = Females

 $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same. $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	25
Sample Standard Deviation	407.3448
Smaller-Variance Sample	
Sample Size	19
Sample Standard Deviation	296.0216
Intermediate Calculations	
F Test Statistic	1.893553
Population 1 Sample Degrees of Freedom	24
Population 2 Sample Degrees of Freedom	18
Two-Tail Test	
Upper Critical Value	2.502697
p-Value	0.168423
Do not reject the null hypothesis	

Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = Males, 2 = females

 $H_0: \mu_1 = \mu_2$  $H_1: \mu_1 \neq \mu_2$

10.77  
cont.

PHStat output:

<b>Pooled-Variance <math>t</math> Test for the Difference Between Two Means</b> (assumes equal population variances)	
<b>Data</b>	
Hypothesized Difference	0
Level of Significance	0.05
<b>Population 1 Sample</b>	
Sample Size	25
Sample Mean	361.2
Sample Standard Deviation	407.3448
<b>Population 2 Sample</b>	
Sample Size	19
Sample Mean	367.3684
Sample Standard Deviation	296.0216
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	24
Population 2 Sample Degrees of Freedom	18
Total Degrees of Freedom	42
Pooled Variance	132372.2
Difference in Sample Means	-6.16842
<b><math>t</math> Test Statistic</b>	<b>-0.05571</b>
<b>Two-Tail Test</b>	
Lower Critical Value	-2.01808
Upper Critical Value	2.018082
<b><math>p</math>-Value</b>	<b>0.955841</b>
<b>Do not reject the null hypothesis</b>	

Since  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean spending is different between males and females.

## 10.77 Text Messages:

cont.

Population 1 = Females, 2 = Males

 $H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same. $H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	19
Sample Standard Deviation	323.4155
Smaller-Variance Sample	
Sample Size	25
Sample Standard Deviation	278.349
Intermediate Calculations	
F Test Statistic	1.350027
Population 1 Sample Degrees of Freedom	18
Population 2 Sample Degrees of Freedom	24
Two-Tail Test	
Upper Critical Value	2.364797
p-Value	0.485676
Do not reject the null hypothesis	

Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = Males, 2 = females

 $H_0: \mu_1 = \mu_2$  $H_1: \mu_1 \neq \mu_2$

10.77  
cont.

PHStat output:

<b>Pooled-Variance <i>t</i> Test for the Difference Between Two Means</b> (assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>25</b>
<b>Sample Mean</b>	<b>203.6</b>
<b>Sample Standard Deviation</b>	<b>278.349</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>19</b>
<b>Sample Mean</b>	<b>270.4211</b>
<b>Sample Standard Deviation</b>	<b>323.4155</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	24
Population 2 Sample Degrees of Freedom	18
Total Degrees of Freedom	42
Pooled Variance	89100.78
Difference in Sample Means	-66.8211
<b><i>t</i> Test Statistic</b>	<b>-0.73552</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.01808</b>
<b>Upper Critical Value</b>	<b>2.018082</b>
<b><i>p</i>-Value</b>	<b>0.466112</b>
<b>Do not reject the null hypothesis</b>	

Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean number of text messages sent in a week is different between males and females.

**Wealth:**

Population 1 = Females, 2 = Males

$H_0: \sigma_1^2 = \sigma_2^2$  The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$  The population variances are different.

10.77  
cont.

PHstat output:

F Test for Differences in Two Variances	
Data	
Level of Significance	0.05
Larger-Variance Sample	
Sample Size	19
Sample Standard Deviation	24.26602
Smaller-Variance Sample	
Sample Size	25
Sample Standard Deviation	21.2986
Intermediate Calculations	
F Test Statistic	1.298061
Population 1 Sample Degrees of Freedom	18
Population 2 Sample Degrees of Freedom	24
Two-Tail Test	
Upper Critical Value	2.364797
p-Value	0.542857
Do not reject the null hypothesis	

Since  $p\text{-value} > 0.05$ , do not reject  $H_0$ . There is not enough evidence to conclude that the two population variances are different. Hence, the appropriate test for the difference in two means is the pooled-variance  $t$  test.

Population 1 = Males, 2 = females

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

10.77  
cont.

PHStat output:

<b>Pooled-Variance <math>t</math> Test for the Difference Between Two Means</b>	
(assumes equal population variances)	
<b>Data</b>	
<b>Hypothesized Difference</b>	<b>0</b>
<b>Level of Significance</b>	<b>0.05</b>
<b>Population 1 Sample</b>	
<b>Sample Size</b>	<b>25</b>
<b>Sample Mean</b>	<b>10.576</b>
<b>Sample Standard Deviation</b>	<b>21.2986</b>
<b>Population 2 Sample</b>	
<b>Sample Size</b>	<b>19</b>
<b>Sample Mean</b>	<b>10.72632</b>
<b>Sample Standard Deviation</b>	<b>24.26602</b>
<b>Intermediate Calculations</b>	
Population 1 Sample Degrees of Freedom	24
Population 2 Sample Degrees of Freedom	18
Total Degrees of Freedom	42
Pooled Variance	511.5772
Difference in Sample Means	-0.15032
<b><math>t</math> Test Statistic</b>	<b>-0.02184</b>
<b>Two-Tail Test</b>	
<b>Lower Critical Value</b>	<b>-2.01808</b>
<b>Upper Critical Value</b>	<b>2.018082</b>
<b><math>p</math>-Value</b>	<b>0.982682</b>
<b>Do not reject the null hypothesis</b>	

Since  $p$ -value  $> 0.05$ , do not reject  $H_0$ . There is insufficient evidence to conclude that the mean wealth needed to feel rich is different between males and females.