Forecasting Demand

4

PowerPoint presentation to accompany
Heizer and Render
Operations Management, Global Edition, Eleventh Edition
Principles of Operations Management, Global Edition, Ninth Edition

PowerPoint slides by Jeff Heyl

Learning Objectives

When you complete this chapter you should be able to :

1. **Understand** the three time horizons and which models apply for each use
2. **Explain** when to use each of the four qualitative models
3. **Apply** the naive, moving average, exponential smoothing, and trend methods

Learning Objectives

When you complete this chapter you should be able to :

4. **Compute** three measures of forecast accuracy
5. **Develop** seasonal indices
6. **Conduct** a regression and correlation analysis
7. **Use** a tracking signal

What is Forecasting?

- Process of predicting a future event
- Underlying basis of all business decisions
  - Production
  - Inventory
  - Personnel
  - Facilities

Forecasting Time Horizons

1. *Short-range forecast*
   - Up to 1 year, generally less than 3 months
   - Purchasing, job scheduling, workforce levels, job assignments, production levels
2. *Medium-range forecast*
   - 3 months to 3 years
   - Sales and production planning, budgeting
3. *Long-range forecast*
   - 3^+ years
   - New product planning, facility location, research and development

Distinguishing Differences

1. Medium/long range forecasts deal with more comprehensive issues and support management decisions regarding planning and products, plants and processes
2. Short-term forecasting usually employs different methodologies than longer-term forecasting
3. Short-term forecasts tend to be more accurate than longer-term forecasts

Influence of Product Life Cycle

Introduction – Growth – Maturity – Decline

- Introduction and growth require longer forecasts than maturity and decline
- As product passes through life cycle, forecasts are useful in projecting
    - Staffing levels
    - Inventory levels
    - Factory capacity

Product Life Cycle

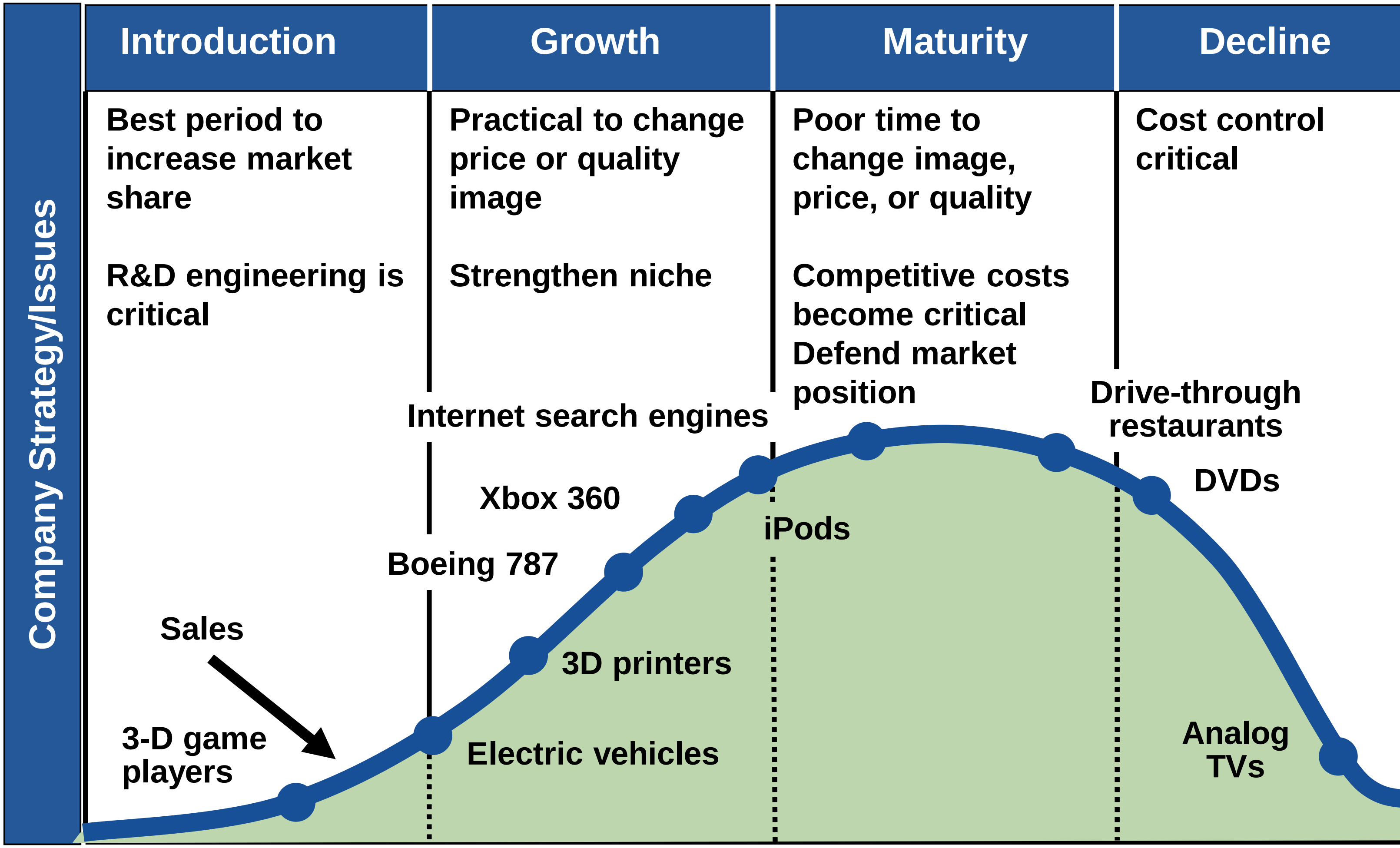


Figure 2.5

Product Life Cycle

	Introduction	Growth	Maturity	Decline
OM Strategy/Issues	Product design and development critical Frequent product and process design changes Short production runs High production costs Limited models Attention to quality	Forecasting critical Product and process reliability Competitive product improvements and options Increase capacity Shift toward product focus Enhance distribution	Standardization Fewer product changes, more minor changes Optimum capacity Increasing stability of process Long production runs Product improvement and cost cutting	Little product differentiation Cost minimization Overcapacity in the industry Prune line to eliminate items not returning good margin Reduce capacity

Figure 2.5

Types of Forecasts

1. Economic forecasts
   - Address business cycle – inflation rate, money supply, housing starts, etc.
2. Technological forecasts
   - Predict rate of technological progress
   - Impacts development of new products
3. Demand forecasts
   - Predict sales of existing products and services

Strategic Importance of Forecasting

- Supply-Chain Management – Good supplier relations, advantages in product innovation, cost and speed to market
- Human Resources – Hiring, training, laying off workers
- Capacity – Capacity shortages can result in undependable delivery, loss of customers, loss of market share

Seven Steps in Forecasting

1. Determine the use of the forecast
2. Select the items to be forecasted
3. Determine the time horizon of the forecast
4. Select the forecasting model(s)
5. Gather the data needed to make the forecast
6. Make the forecast
7. Validate and implement results

The Realities!

- Forecasts are seldom perfect, unpredictable outside factors may impact the forecast
- Most techniques assume an underlying stability in the system
- Product family and aggregated forecasts are more accurate than individual product forecasts

Forecasting Approaches

Qualitative Methods

- Used when situation is vague and little data exist
  - New products
  - New technology
- Involves intuition, experience
  - e.g., forecasting sales on Internet

Forecasting Approaches

Quantitative Methods

- Used when situation is ‘stable’ and historical data exist
  - Existing products
  - Current technology
- Involves mathematical techniques
  - e.g., forecasting sales of color televisions

Overview of Qualitative Methods

1. Jury of executive opinion
   - Pool opinions of high-level experts, sometimes augment by statistical models
2. Delphi method
   - Panel of experts, queried iteratively

Overview of Qualitative Methods

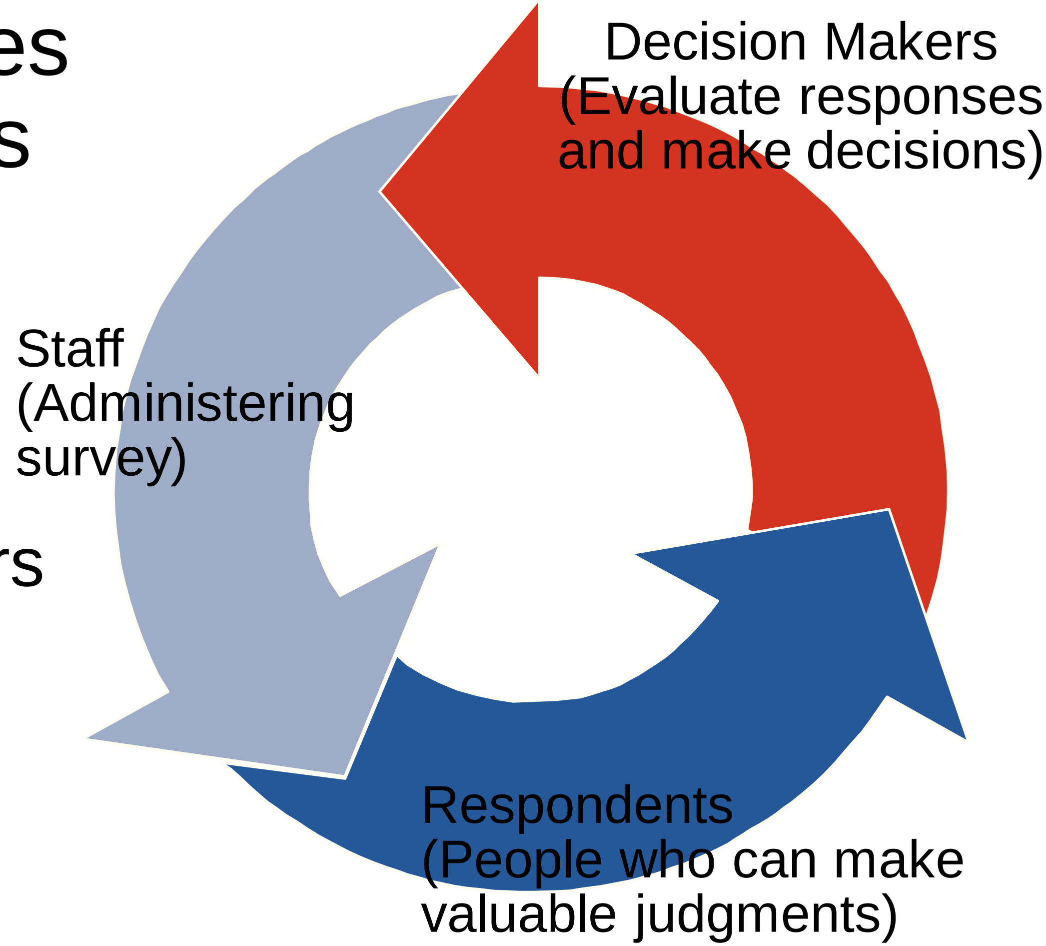
3. Sales force composite
   - Estimates from individual salespersons are reviewed for reasonableness, then aggregated
4. Market Survey
   - Ask the customer

Jury of Executive Opinion

- Involves small group of high-level experts and managers
- Group estimates demand by working together
- Combines managerial experience with statistical models
- Relatively quick
- ‘Group-think’ disadvantage

Delphi Method

- Iterative group process, continues until consensus is reached
- 3 types of participants
  - Decision makers
  - Staff
  - Respondents



Sales Force Composite

- Each salesperson projects his or her sales
- Combined at district and national levels
- Sales reps know customers' wants
- May be overly optimistic

Market Survey

- Ask customers about purchasing plans
- Useful for demand and product design and planning
- What consumers say, and what they actually do may be different
- May be overly optimistic

Overview of Quantitative Approaches

1. Naive approach
2. Moving averages
3. Exponential smoothing
4. Trend projection
5. Linear regression

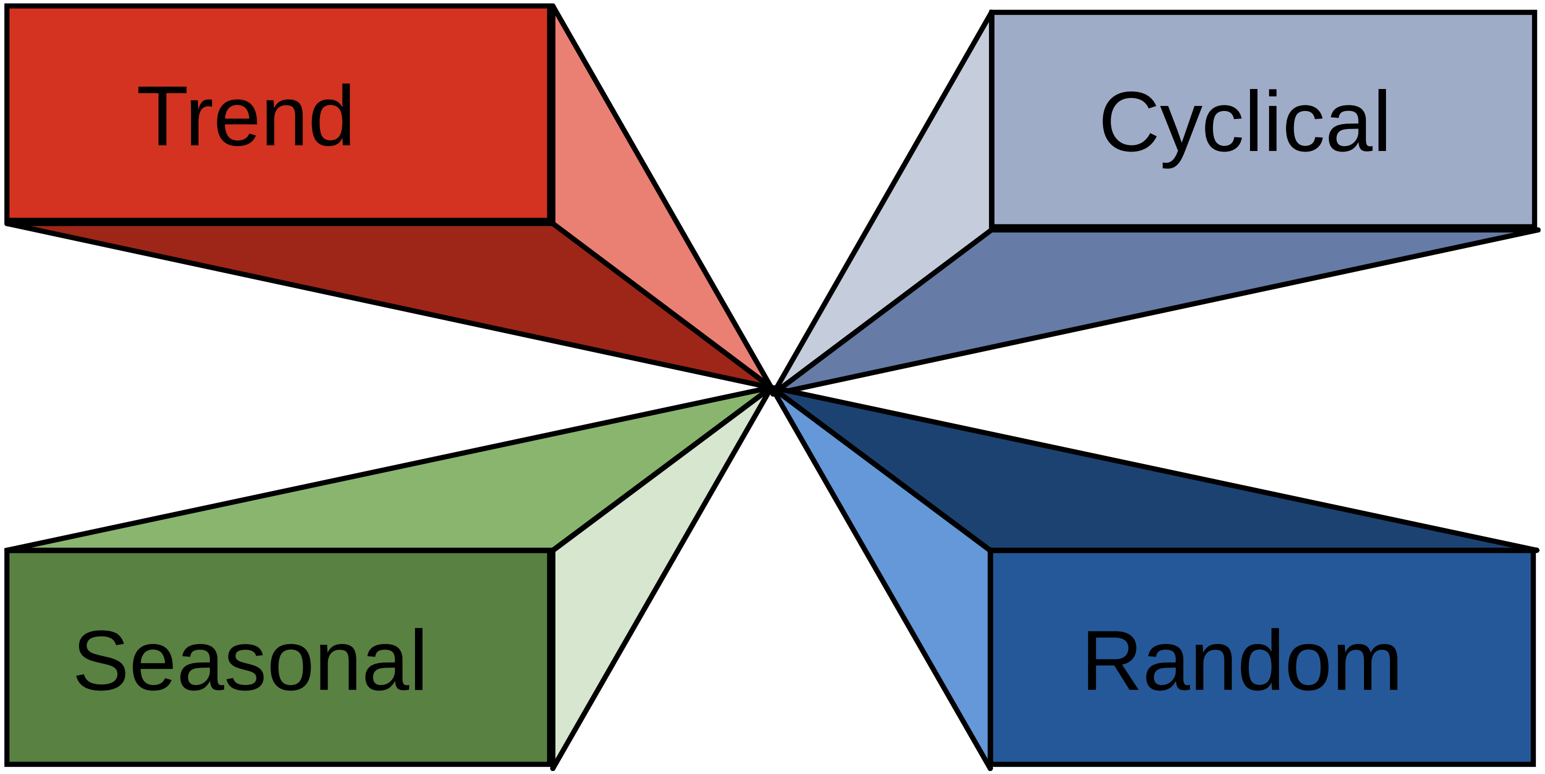
Time-series models (1–4)

Associative model (5)

Time-Series Forecasting

- Set of evenly spaced numerical data
  - Obtained by observing response variable at regular time periods
- Forecast based only on past values, no other variables important
  - Assumes that factors influencing past and present will continue influence in future

Time-Series Components



Components of Demand

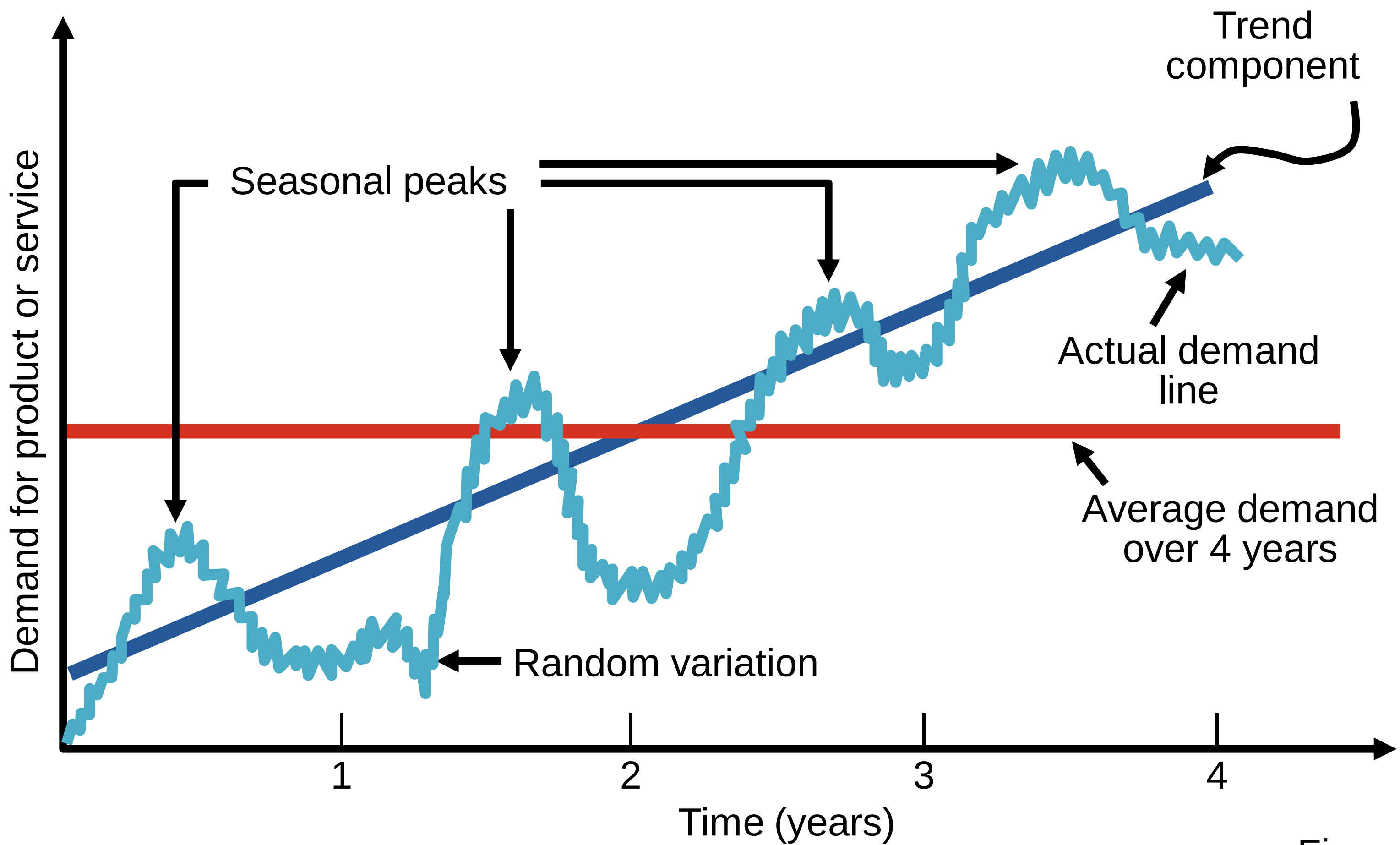
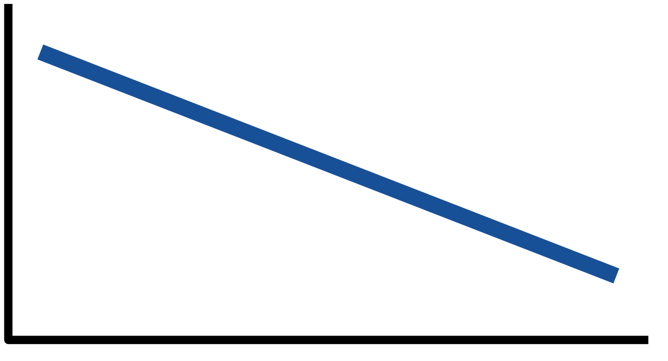


Figure 4.1

Trend Component

- Persistent, overall upward or downward pattern
- Changes due to population, technology, age, culture, etc.
- Typically several years duration



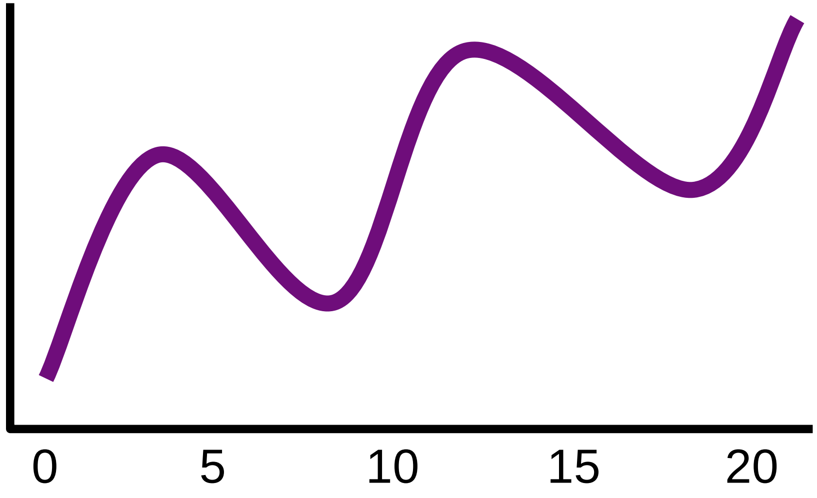
Seasonal Component

- Regular pattern of up and down fluctuations
- Due to weather, customs, etc.
- Occurs within a single year

PERIOD LENGTH	"SEASON" LENGTH	NUMBER OF "SEASONS" IN PATTERN
Week	Day	7
Month	Week	4 – 4.5
Month	Day	28 – 31
Year	Quarter	4
Year	Month	12
Year	Week	52

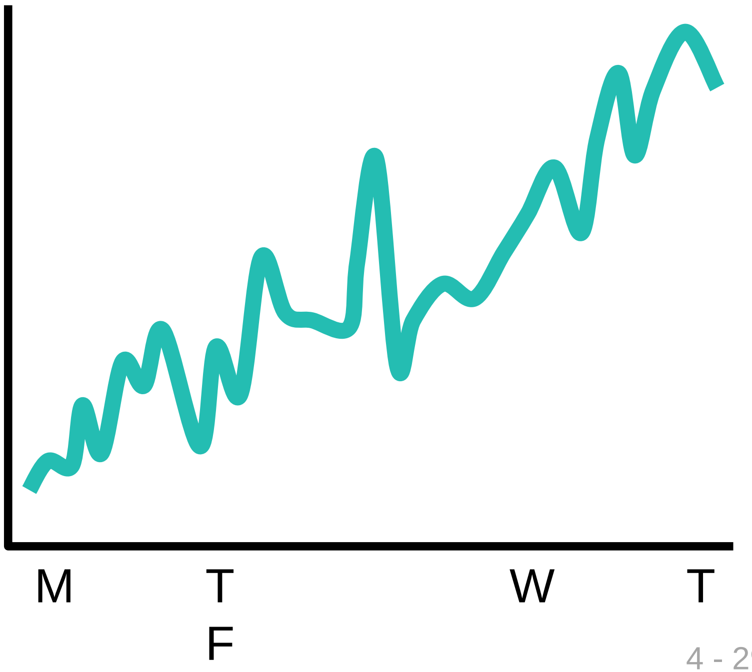
Cyclical Component

- Repeating up and down movements
- Affected by business cycle, political, and economic factors
- Multiple years duration
- Often causal or associative relationships



Random Component

- Erratic, unsystematic, ‘residual’ fluctuations
- Due to random variation or unforeseen events
- Short duration and nonrepeating



Naive Approach

- Assumes demand in next period is the same as demand in most recent period
  - e.g., If January sales were 68, then February sales will be 68
- Sometimes cost effective and efficient
- Can be good starting point

Moving Average Method

- MA is a series of arithmetic means
- Used if little or no trend
- Used often for smoothing
  - Provides overall impression of data over time

$$\text{Moving average} = \frac{\sum \text{demand in previous } n \text{ periods}}{n}$$

Moving Average Example

MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11\ ^2/_3$
May	19	$(12 + 13 + 16)/3 = 13\ ^2/_3$
June	23	$(13 + 16 + 19)/3 = 16$
July	26	$(16 + 19 + 23)/3 = 19\ ^1/_3$
August	30	$(19 + 23 + 26)/3 = 22\ ^2/_3$
September	28	$(23 + 26 + 30)/3 = 26\ ^1/_3$
October	18	$(29 + 30 + 28)/3 = 28$
November	16	$(30 + 28 + 18)/3 = 25\ ^1/_3$
December	14	$(28 + 18 + 16)/3 = 20\ ^2/_3$

Weighted Moving Average

- Used when some trend might be present
  - Older data usually less important
- Weights based on experience and intuition

$$\text{Weighted moving average} = \frac{\sum \left(\left(\text{Weight for period } n \right) \left(\text{Demand in period } n \right) \right)}{\sum \text{Weights}}$$

Weighted Moving Average

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	[(3 x 13) + (2 x 12) + (10)]/6 = 12 $^{1}/_{6}$
May	19	
June		
July		
August		
September		
October		
November		
December		

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago
6	Sum of the weights

Forecast for this month =

$$\frac{3 \text{ x Sales last mo.} + 2 \text{ x Sales 2 mos. ago} + 1 \text{ x Sales 3 mos. ago}}{\text{Sum of the weights}}$$

Weighted Moving Average

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$[(3 \times 13) + (2 \times 12) + (10)]/6 = 12\ ^1/_6$
May	19	$[(3 \times 16) + (2 \times 13) + (12)]/6 = 14\ ^1/_3$
June	23	$[(3 \times 19) + (2 \times 16) + (13)]/6 = 17$
July	26	$[(3 \times 23) + (2 \times 19) + (16)]/6 = 20\ ^1/_2$
August	30	$[(3 \times 26) + (2 \times 23) + (19)]/6 = 23\ ^5/_6$
September	28	$[(3 \times 30) + (2 \times 26) + (23)]/6 = 27\ ^1/_2$
October	18	$[(3 \times 28) + (2 \times 30) + (26)]/6 = 28\ ^1/_3$
November	16	$[(3 \times 18) + (2 \times 28) + (30)]/6 = 23\ ^1/_3$
December	14	$[(3 \times 16) + (2 \times 18) + (28)]/6 = 18\ ^2/_3$

Potential Problems With Moving Average

- Increasing *n* smooths the forecast but makes it less sensitive to changes
- Does not forecast trends well
- Requires extensive historical data

Graph of Moving Averages

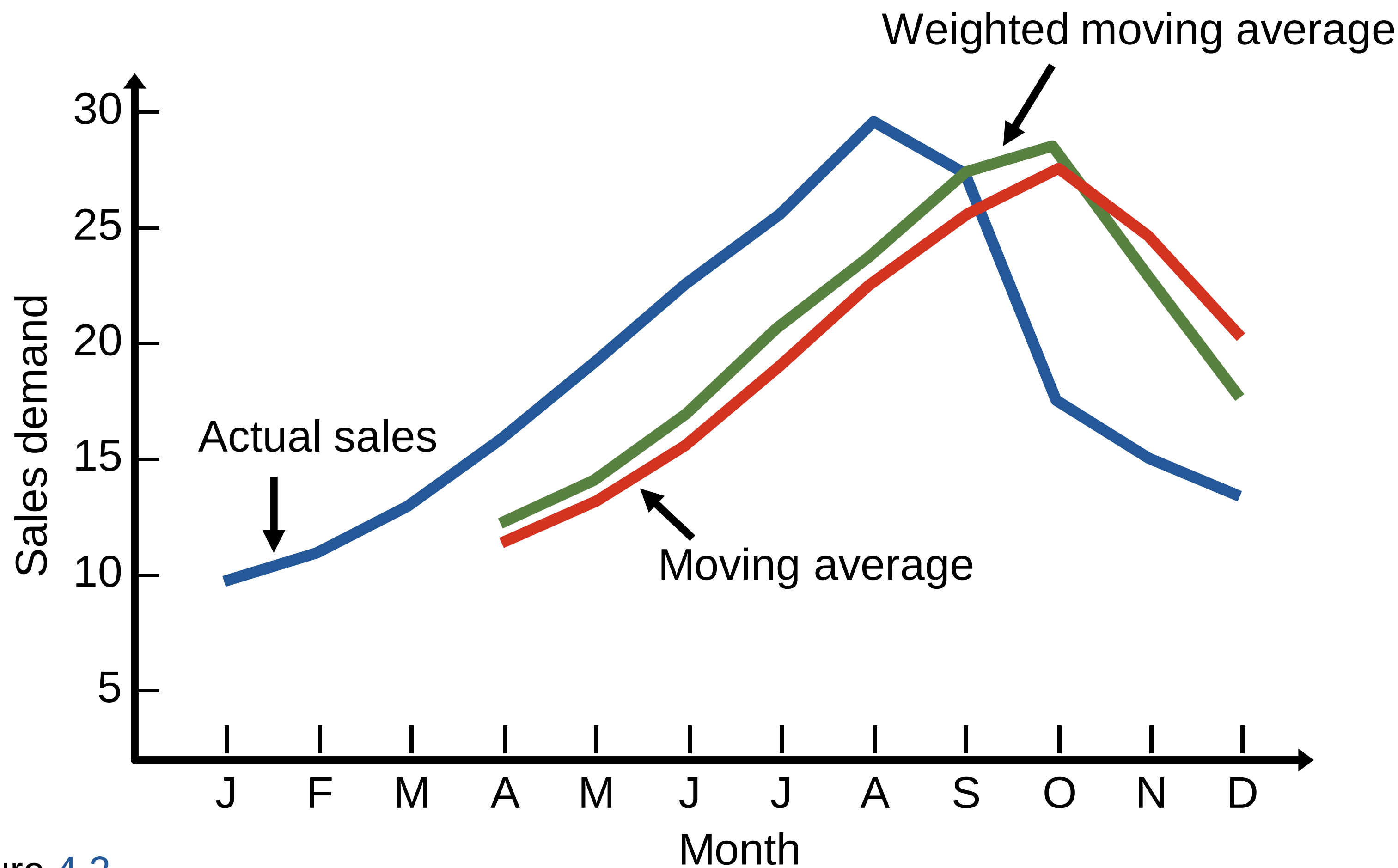


Figure 4.2

Exponential Smoothing

- Form of weighted moving average
  - Weights decline exponentially
  - Most recent data weighted most
- Requires smoothing constant (α)
  - Ranges from 0 to 1
  - Subjectively chosen
- Involves little record keeping of past data

Exponential Smoothing

New forecast = Last period's forecast
+ α (Last period's actual demand
– Last period's forecast)

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

where

F_t = new forecast

F_{t-1} = previous period's forecast

α = smoothing (or weighting) constant ($0 \le \alpha \le 1$)

A_{t-1} = previous period's actual demand

Exponential Smoothing Example

Predicted demand = 142 Ford Mustangs
Actual demand = 153
Smoothing constant α = .20

Exponential Smoothing Example

Predicted demand = 142 Ford Mustangs

Actual demand = 153

Smoothing constant α = .20

New forecast = 142 + .2(153 – 142)

Exponential Smoothing Example

Predicted demand = 142 Ford Mustangs
Actual demand = 153
Smoothing constant α = .20

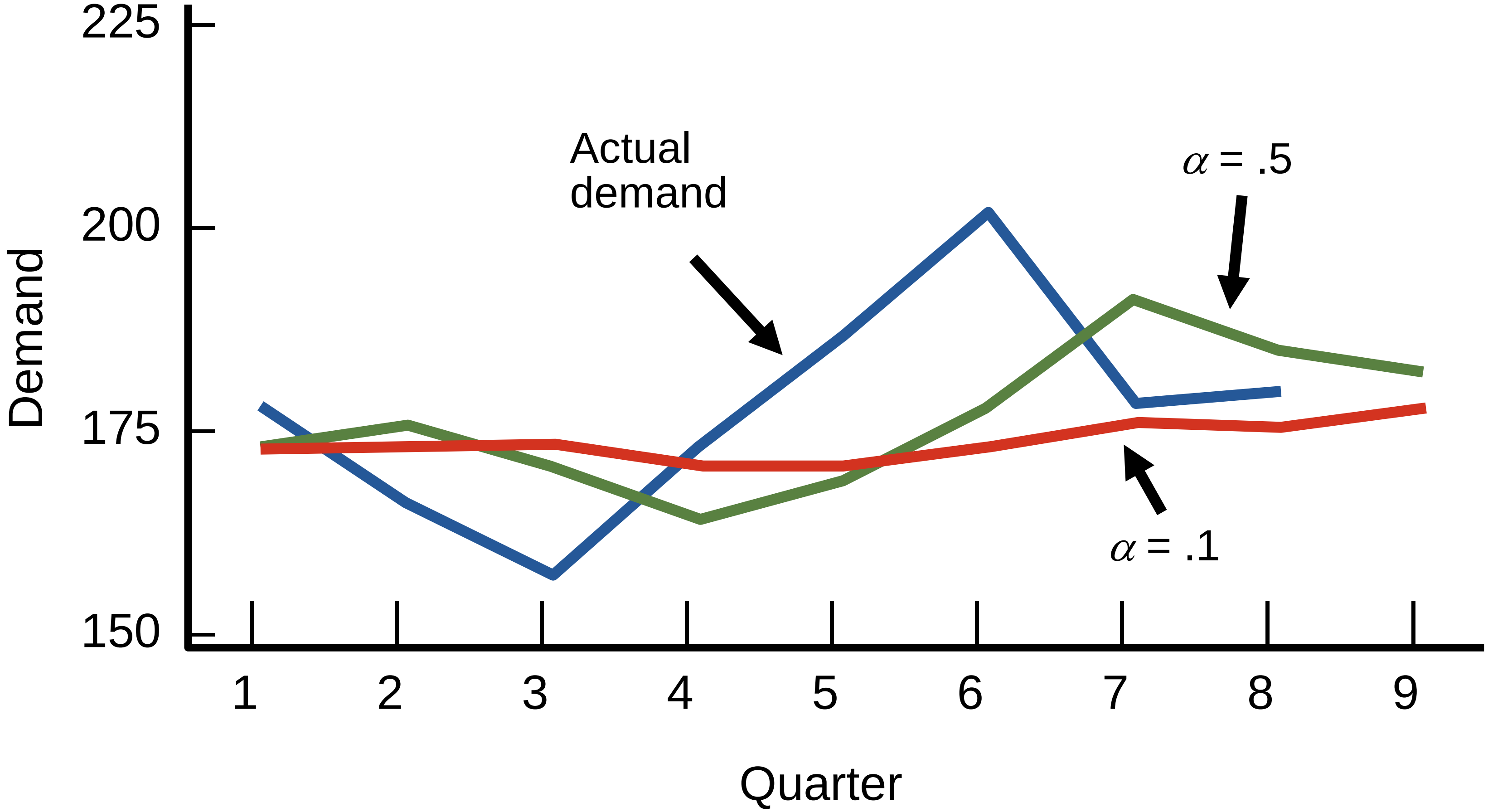
New forecast $= 142 + .2(153 – 142)$
$= 142 + 2.2$
$= 144.2 \approx 144$ cars

Effect of Smoothing Constants

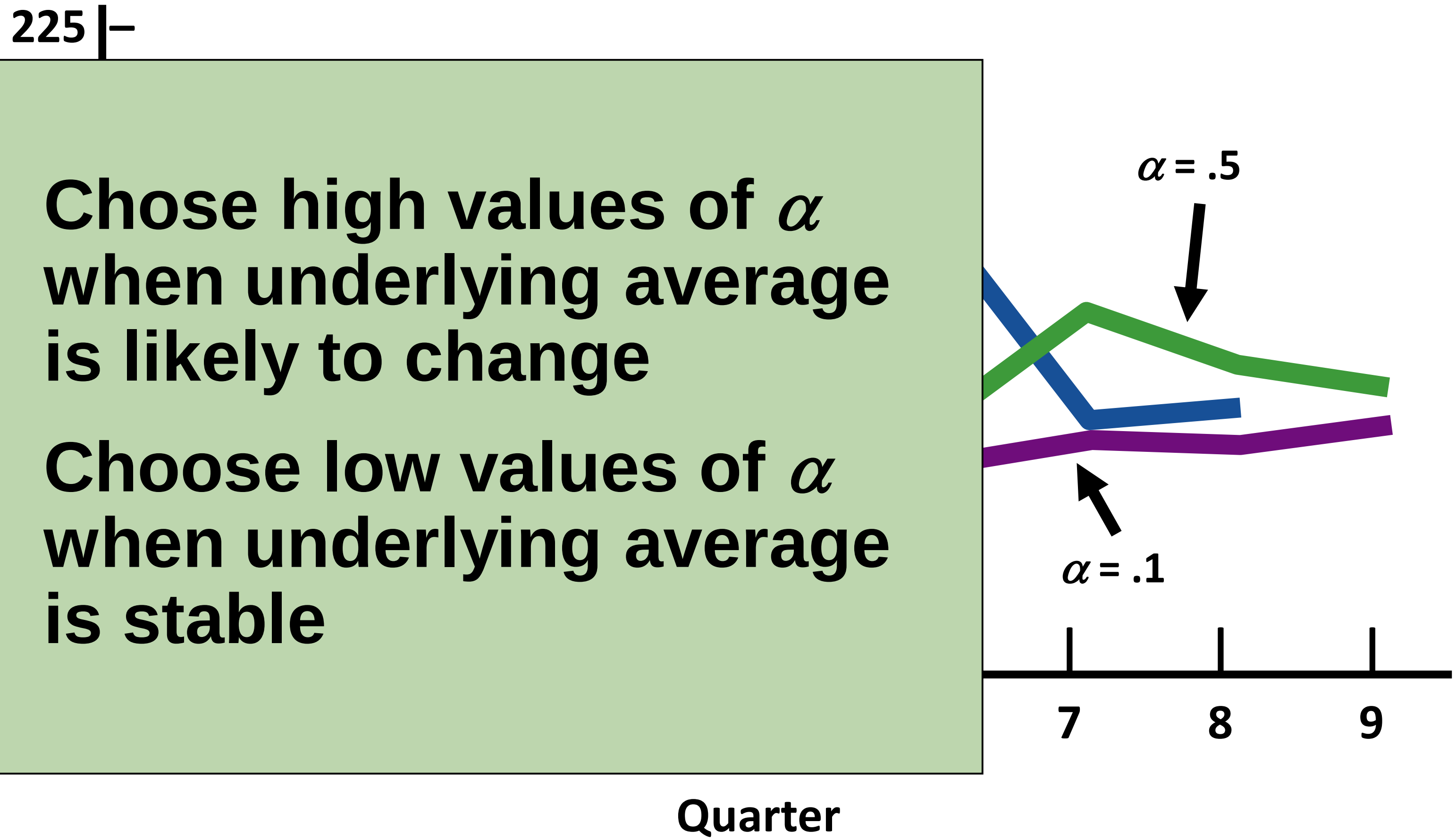
- Smoothing constant generally .05 ≤ α ≤ .50
- As α increases, older values become less significant

	WEIGHT ASSIGNED TO				
SMOOTHING CONSTANT	MOST RECENT PERIOD (α)	2ND MOST RECENT PERIOD $\alpha(1-\alpha)$	3RD MOST RECENT PERIOD $\alpha(1-\alpha)^2$	4th MOST RECENT PERIOD $\alpha(1-\alpha)^3$	5th MOST RECENT PERIOD $\alpha(1-\alpha)^4$
α = .1	.1	.09	.081	.073	.066
α = .5	.5	.25	.125	.063	.031

Impact of Different α



Impact of Different α



- Chose high values of α when underlying average is likely to change
- Choose low values of α when underlying average is stable

Choosing α

The objective is to obtain the most accurate forecast no matter the technique

We generally do this by selecting the model that gives us the lowest forecast error

Forecast error = Actual demand – Forecast value

$$= A_t - F_t$$

Common Measures of Error

Mean Absolute Deviation (MAD)

$$\text{MAD} = \frac{\sum \left| \text{Actual} - \text{Forecast} \right|}{n}$$

Determining the MAD

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = .10$	FORECAST WITH $\alpha = .50$
1	**180**	175	175
2	168	175.50 = **175.00** + .10(**180** – **175**)	177.50
3	159	174.75 = 175.50 + .10(168 – 175.50)	172.75
4	175	173.18 = 174.75 + .10(159 – 174.75)	165.88
5	190	173.36 = 173.18 + .10(175 – 173.18)	170.44
6	205	175.02 = 173.36 + .10(190 – 173.36)	180.22
7	180	178.02 = 175.02 + .10(205 – 175.02)	192.61
8	182	178.22 = 178.02 + .10(180 – 178.02)	186.30
9	?	178.59 = 178.22 + .10(182 – 178.22)	184.15

Determining the MAD

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = .10$	ABSOLUTE DEVIATION FOR a = .10	FORECAST WITH $\alpha = .50$	ABSOLUTE DEVIATION FOR a = .50
1	180	175	5.00	175	5.00
2	168	175.50	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
Sum of absolute deviations:			82.45		98.62
		$\text{MAD} = \dfrac{\Sigma\lvert\text{Deviations}\rvert}{n}$	10.31		12.33

Common Measures of Error

Mean Squared Error (MSE)

$$\text{MSE} = \frac{\sum(\text{Forecast errors})^2}{n}$$

Determining the MSE

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	$(\text{ERROR})^2$
1	180	175	$5^2 = 25$
2	168	175.50	$(-7.5)^2 = 56.25$
3	159	174.75	$(-15.75)^2 = 248.06$
4	175	173.18	$(1.82)^2 = 3.31$
5	190	173.36	$(16.64)^2 = 276.89$
6	205	175.02	$(29.98)^2 = 898.80$
7	180	178.02	$(1.98)^2 = 3.92$
8	182	178.22	$(3.78)^2 = 14.29$
			Sum of errors squared = 1,526.52

$$\text{MSE} = \frac{\sum (\text{Forecast errors})^2}{n} = 1{,}526.52 / 8 = 190.8$$

Common Measures of Error

Mean Absolute Percent Error (MAPE)

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100 \left| \text{Actual}_i - \text{Forecast}_i \right| / \text{Actual}_i}{n}$$

Determining the MAPE

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	ABSOLUTE PERCENT ERROR 100(ERROR/ACTUAL)
1	180	175.00	100(5/180) = 2.78%
2	168	175.50	100(7.5/168) = 4.46%
3	159	174.75	100(15.75/159) = 9.90%
4	175	173.18	100(1.82/175) = 1.05%
5	190	173.36	100(16.64/190) = 8.76%
6	205	175.02	100(29.98/205) = 14.62%
7	180	178.02	100(1.98/180) = 1.10%
8	182	178.22	100(3.78/182) = 2.08%
			Sum of % errors = 44.75%

$$\text{MAPE} = \frac{\sum \text{absolute percent error}}{n} = \frac{44.75\%}{8} = 5.59\%$$

Comparison of Forecast Error

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
1	180	175	5.00	175	5.00
2	168	175.5	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
			82.45		98.62

Comparison of Forecast Error

$$\text{MAD} = \frac{\sum |\text{deviations}|}{n}$$

For $\alpha = .10$

$$= 82.45/8 = 10.31$$

For $\alpha = .50$

$$= 98.62/8 = 12.33$$

				Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
				175	5.00
				177.50	9.50
				172.75	13.75
				165.88	9.12
				170.44	19.56
				180.22	24.78
				192.61	12.61
8	182	178.22	3.78	186.30	4.30
			82.45		98.62

Comparison of Forecast Error

$$\text{MSE} = \frac{\sum (\text{forecast errors})^2}{n}$$

For $\alpha = .10$

$$= 1{,}526.54/8 = 190.82$$

For $\alpha = .50$

$$= 1{,}561.91/8 = 195.24$$

				Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
				175	5.00
				177.50	9.50
				172.75	13.75
				165.88	9.12
				170.44	19.56
				180.22	24.78
				192.61	12.61
8	182	178.22	3.78	186.30	4.30
			82.45		98.62
		MAD	10.31		12.33

Comparison of Forecast Error

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100|\text{deviation}_i|/\text{actual}_i}{n}$$

For $\alpha = .10$

$$= 44.75/8 = 5.59\%$$

For $\alpha = .50$

$$= 54.05/8 = 6.76\%$$

					Absolute Deviation for $\alpha = .50$
					5.00
					9.50
					13.75
					9.12
					19.56
					24.78
					12.61
8	182	178.22	3.78	180.50	4.30
			82.45		98.62
		MAD	10.31		12.33
		MSE	190.82		195.24

Comparison of Forecast Error

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
1	180	175	5.00	175	5.00
2	168	175.5	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
			82.45		98.62
		MAD	10.31		12.33
		MSE	190.82		195.24
		MAPE	5.59%		6.76%

Exponential Smoothing with Trend Adjustment

When a trend is present, exponential smoothing must be modified

MONTH	ACTUAL DEMAND	FORECAST (F_t) FOR MONTHS 1 – 5
1	100	F_t = 100 (given)
2	200	$F_t = F_1 + \alpha(A_1 - F_1) = 100 + .4(100 - 100) = 100$
3	300	$F_t = F_2 + \alpha(A_2 - F_2) = 100 + .4(200 - 100) = 140$
4	400	$F_t = F_3 + \alpha(A_3 - F_3) = 140 + .4(300 - 140) = 204$
5	500	$F_t = F_4 + \alpha(A_4 - F_4) = 204 + .4(400 - 204) = 282$

Exponential Smoothing with Trend Adjustment

Forecast including trend	$(FIT_t) =$	Exponentially smoothed forecast	(F_t)	$+$	Exponentially smoothed trend	(T_t)

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1})$$

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1}$$

where
F_t = exponentially smoothed forecast average
T_t = exponentially smoothed trend
A_t = actual demand
α = smoothing constant for average $(0 \leq \alpha \leq 1)$
β = smoothing constant for trend $(0 \leq \beta \leq 1)$

Exponential Smoothing with Trend Adjustment

Step 1: Compute F_t

Step 2: Compute T_t

Step 3: Calculate the forecast $FIT_t = F_t + T_t$

Exponential Smoothing with Trend Adjustment Example

MONTH (t)	ACTUAL DEMAND (A_t)	MONTH (t)	ACTUAL DEMAND (A_t)
1	12	6	21
2	17	7	31
3	20	8	28
4	19	9	36
5	24	10	?

$\alpha = .2$ $\beta = .4$

Exponential Smoothing with Trend Adjustment Example

TABLE 4.1 Forecast with α - .2 and β = .4

MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80		
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 1: Average for Month 2

$$F_2 = \alpha A_1 + (1 - \alpha)(F_1 + T_1)$$
$$F_2 = (.2)(12) + (1 - .2)(11 + 2)$$
$$= 2.4 + (.8)(13) = 2.4 + 10.4$$
$$= 12.8 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example

TABLE 4.1	Forecast with α - .2 and β = .4			
MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 2: Trend for Month 2

$$T_2 = \beta(F_2 - F_1) + (1 - b)T_1$$

$$T_2 = (.4)(12.8 - 11) + (1 - .4)(2)$$

$$= .72 + 1.2 = 1.92 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example

TABLE 4.1	Forecast with α - .2 and β = .4			
MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10	—			

Step 3: Calculate *FIT* for Month 2

$$FIT_2 = F_2 + T_2$$

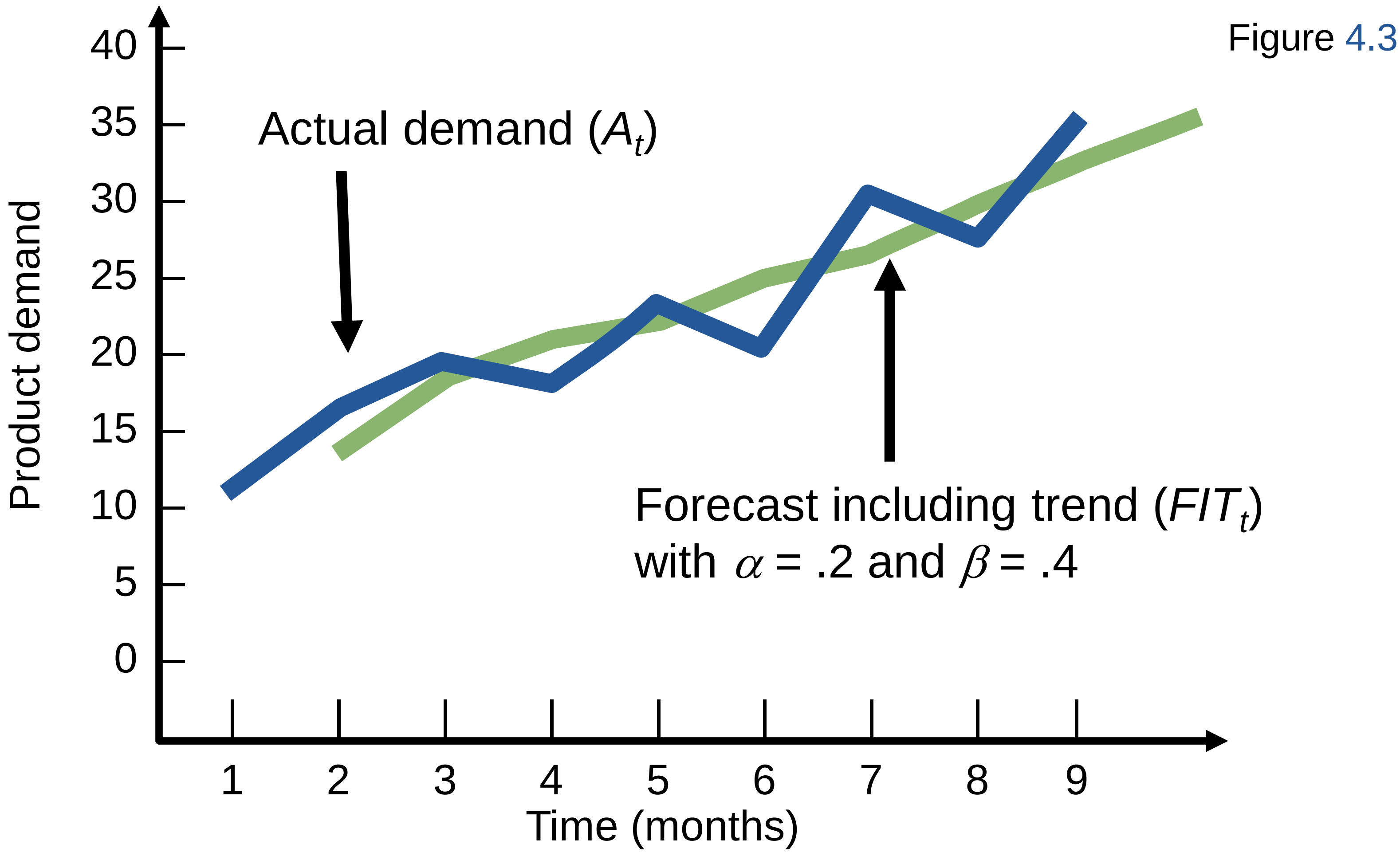
$$FIT_2 = 12.8 + 1.92$$

$$= 14.72 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example

TABLE 4.1	Forecast with α - .2 and β = .4			
MONTH	ACTUAL DEMAND	SMOOTHED FORECAST AVERAGE, F_t	SMOOTHED TREND, T_t	FORECAST INCLUDING TREND, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20	15.18	2.10	17.28
4	19	17.82	2.32	20.14
5	24	19.91	2.23	22.14
6	21	22.51	2.38	24.89
7	31	24.11	2.07	26.18
8	28	27.14	2.45	29.59
9	36	29.28	2.32	31.60
10	—	32.48	2.68	35.16

Exponential Smoothing with Trend Adjustment Example



Trend Projections

Fitting a trend line to historical data points to project into the medium to long-range

Linear trends can be found using the least squares technique

$$\hat{y} = a + bx$$

where $\hat{y}$ = computed value of the variable to be predicted (dependent variable)

a = y-axis intercept

b = slope of the regression line

x = the independent variable

Least Squares Method

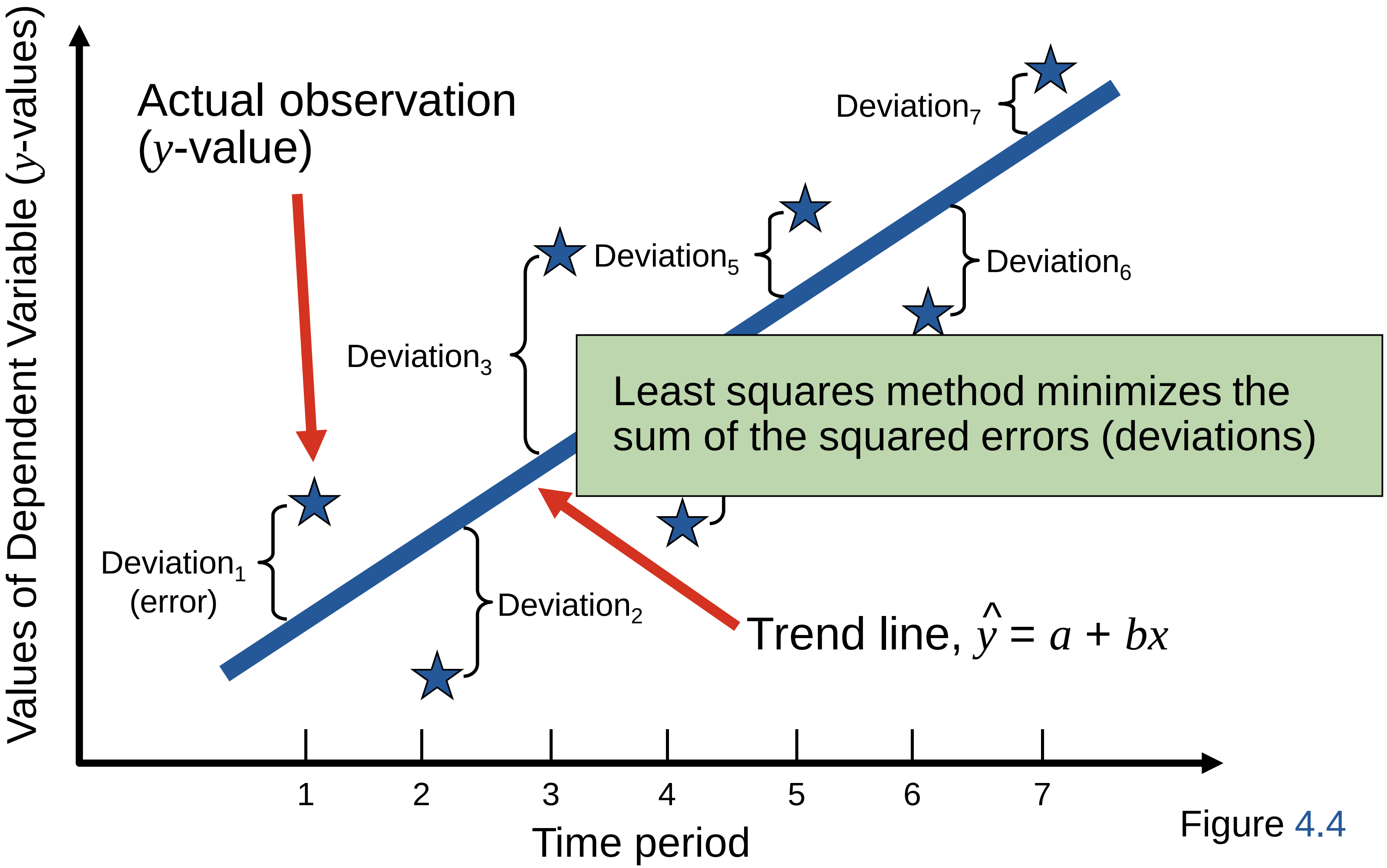


Figure 4.4

Least Squares Method

Equations to calculate the regression variables

$$\hat{y} = a + bx$$

$$b = \frac{\sum xy - n\overline{xy}}{\sum x^2 - n\overline{x}^2}$$

$$a = \overline{y} - b\overline{x}$$

Least Squares Example

YEAR	ELECTRICAL POWER DEMAND	YEAR	ELECTRICAL POWER DEMAND
1	74	5	105
2	79	6	142
3	80	7	122
4	90		

Least Squares Example

YEAR (x)	ELECTRICAL POWER DEMAND (y)	x^2	xy
1	74	1	74
2	79	4	158
3	80	9	240
4	90	16	360
5	105	25	525
6	142	36	852
7	122	49	854
$\Sigma x = 28$	$\Sigma y = 692$	$\Sigma x^2 = 140$	$\Sigma xy = 3{,}063$

$$\bar{x} = \frac{\sum x}{n} = \frac{28}{7} = 4 \qquad \bar{y} = \frac{\sum y}{n} = \frac{692}{7} = 98.86$$

Least Squares Example

Y			
7	122	49	854
$\Sigma x = 28$	$\Sigma y = 692$	$\Sigma x^2 = 140$	$\Sigma xy = 3{,}063$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{3{,}063 - (7)(4)(98.86)}{140 - (7)(4^2)} = \frac{295}{28} = 10.54$$

$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$

Thus, $\hat{y} = 56.70 + 10.54x$

Demand in year 8 = 56.70 + 10.54(8)
= 141.02, or 141 megawatts

Least Squares Example

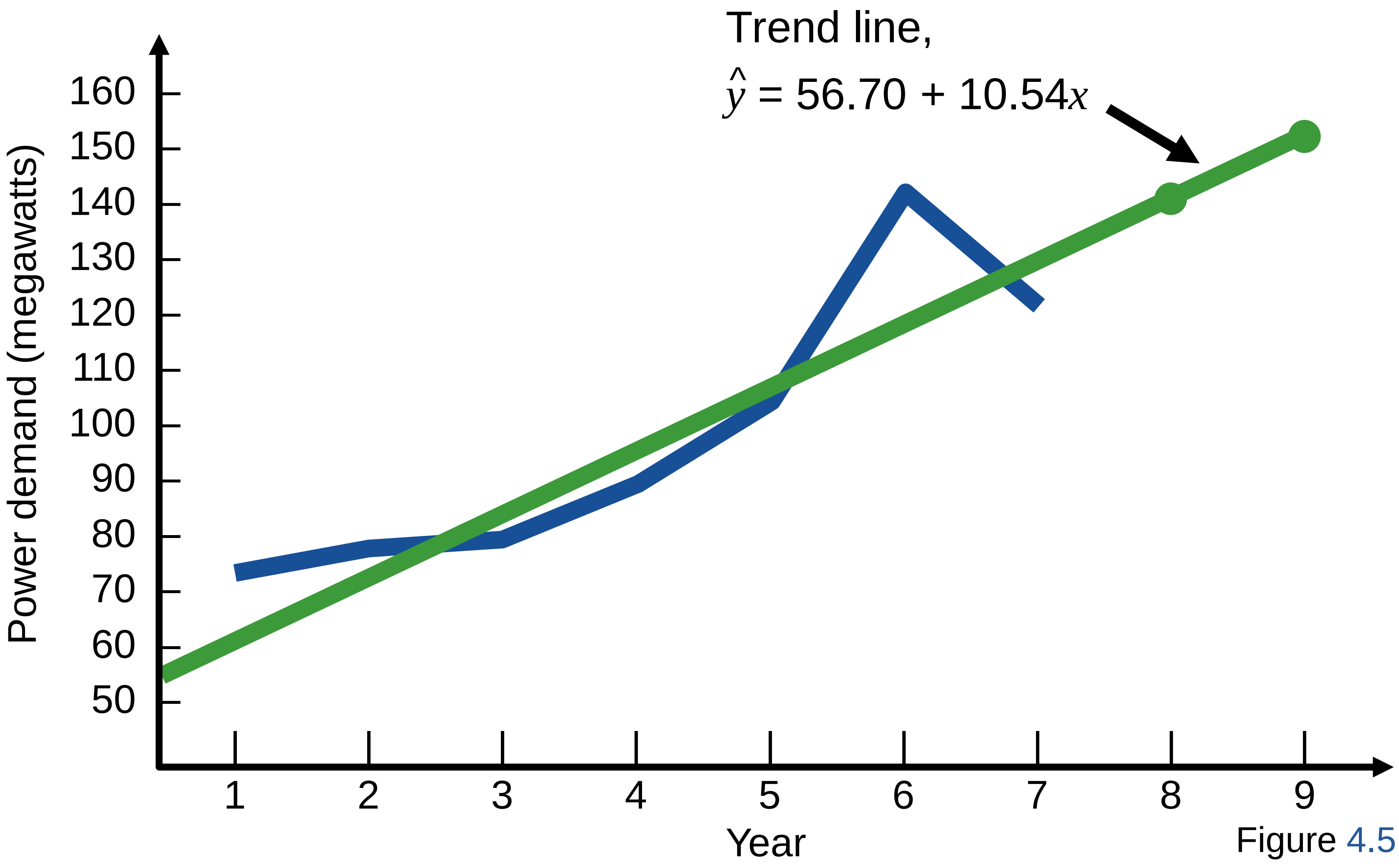


Figure 4.5

Least Squares Requirements

1. We always plot the data to insure a linear relationship
2. We do not predict time periods far beyond the database
3. Deviations around the least squares line are assumed to be random

Seasonal Variations In Data

The multiplicative seasonal model can adjust trend data for seasonal variations in demand

Seasonal Variations In Data

Steps in the process for monthly seasons:

1. Find average historical demand for each month
2. Compute the average demand over all months
3. Compute a seasonal index for each month
4. Estimate next year's total demand
5. Divide this estimate of total demand by the number of months, then multiply it by the seasonal index for that month

Seasonal Index Example

DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE YEARLY DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90		
Feb	70	85	85	80		
Mar	80	93	82	85		
Apr	90	95	115	100		
May	113	125	131	123		
June	110	115	120	115		
July	100	102	113	105		
Aug	88	102	110	100		
Sept	85	90	95	90		
Oct	77	78	85	80		
Nov	75	82	83	80		
Dec	82	78	80	80		
		Total average annual demand =		1,128		

Seasonal Index Example

DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE YEARLY DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	
Feb	70	85	85	80	94	
Mar	[illegible]	[illegible]	[illegible]	[illegible]	94	
Apr	[illegible]	[illegible]	[illegible]	[illegible]	94	
May	[illegible]	[illegible]	[illegible]	[illegible]	94	
June	[illegible]	[illegible]	[illegible]	[illegible]	94	
July	100	102	113	105	94	
Aug	88	102	110	100	94	
Sept	85	90	95	90	94	
Oct	77	78	85	80	94	
Nov	75	82	83	80	94	
Dec	82	78	80	80	94	
		Total average annual demand =		1,128		

$$\text{Average monthly demand} = \frac{1{,}128}{12 \text{ months}} = 94$$

Seasonal Index Example

DEMAND						
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE YEARLY DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	.957(= 90/94)
Feb	70	85	85	80	94	
Mar	80	93	82	85	94	
Apr	90	95	115	100	94	
Sept	85	90	95	90	94	
Oct	77	78	85	80	94	
Nov	75	82	83	80	94	
Dec	82	78	80	80	94	
Total average annual demand =				1,128		

$$\text{Seasonal index} = \frac{\text{Average monthly demand for past 3 years}}{\text{Average monthly demand}}$$

Seasonal Index Example

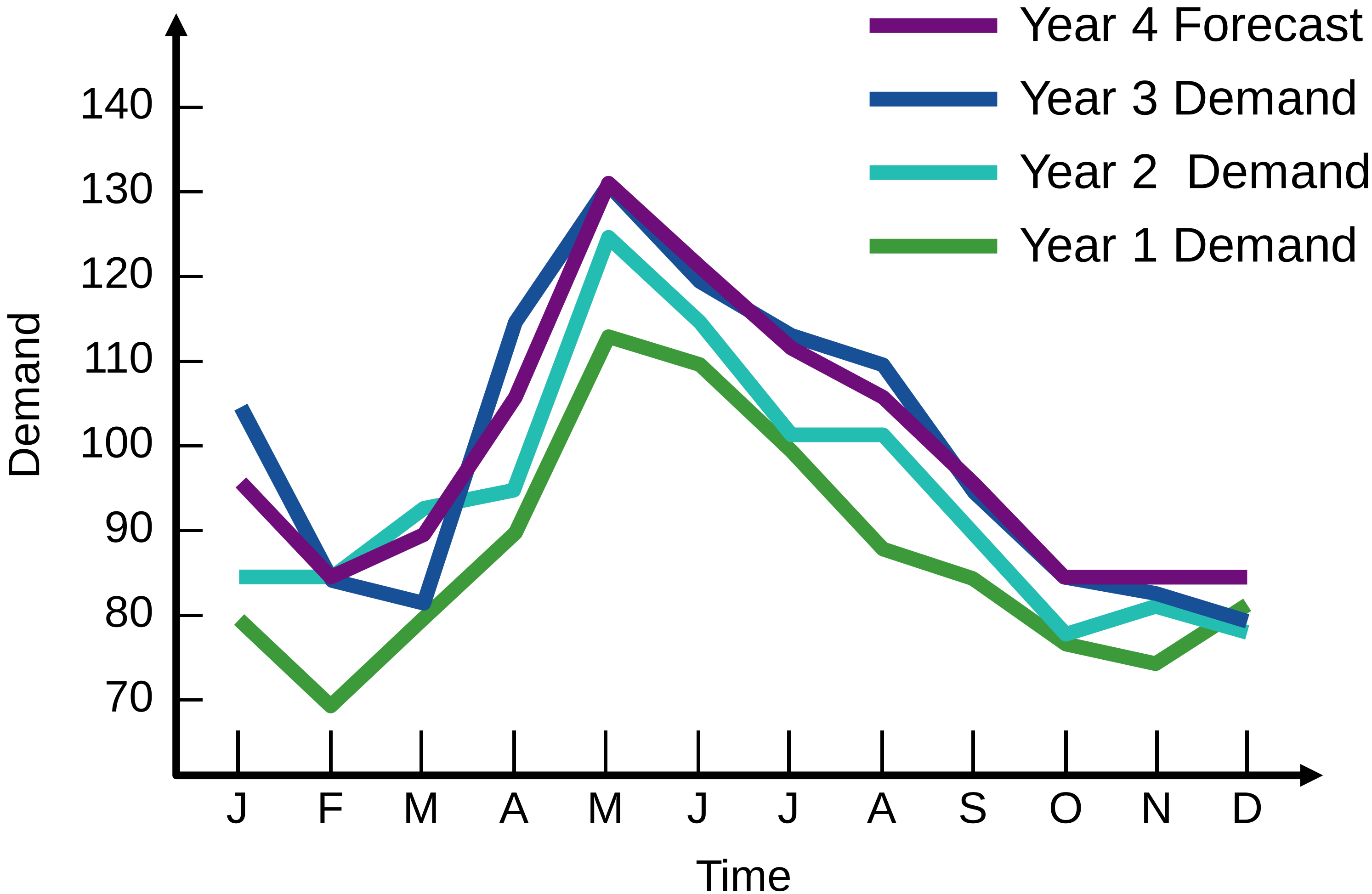
	DEMAND					
MONTH	YEAR 1	YEAR 2	YEAR 3	AVERAGE YEARLY DEMAND	AVERAGE MONTHLY DEMAND	SEASONAL INDEX
Jan	80	85	105	90	94	.957(= 90/94)
Feb	70	85	85	80	94	.851(= 80/94)
Mar	80	93	82	85	94	.904(= 85/94)
Apr	90	95	115	100	94	1.064(= 100/94)
May	113	125	131	123	94	1.309(= 123/94)
June	110	115	120	115	94	1.223(= 115/94)
July	100	102	113	105	94	1.117(= 105/94)
Aug	88	102	110	100	94	1.064(= 100/94)
Sept	85	90	95	90	94	.957(= 90/94)
Oct	77	78	85	80	94	.851(= 80/94)
Nov	75	82	83	80	94	.851(= 80/94)
Dec	82	78	80	80	94	.851(= 80/94)
Total average annual demand =				1,128		

Seasonal Index Example

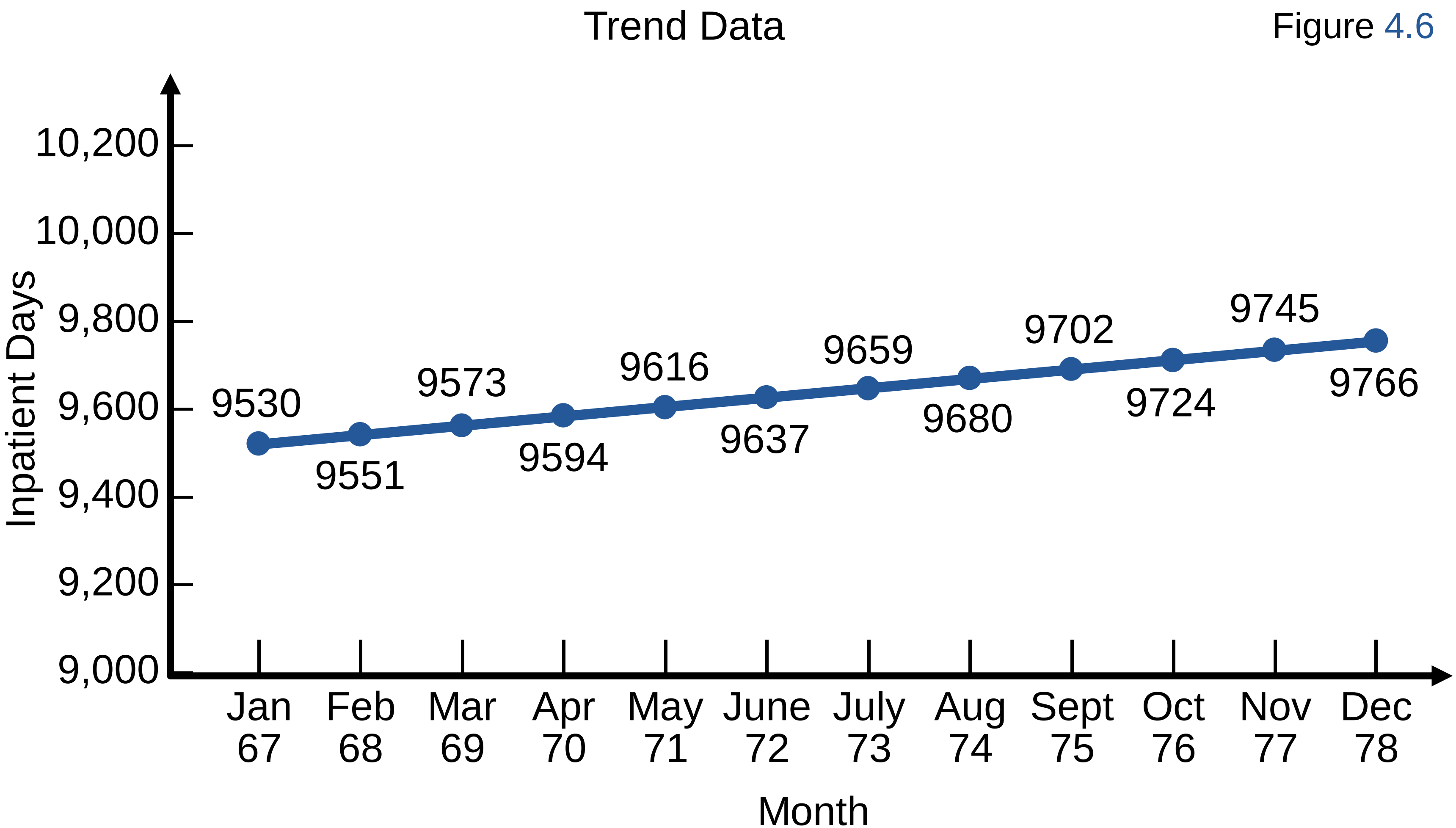
Seasonal forecast for Year 4

MONTH	DEMAND	MONTH	DEMAND
Jan	$\frac{1{,}200}{12}$ x .957 = 96	July	$\frac{1{,}200}{12}$ x 1.117 = 112
Feb	$\frac{1{,}200}{12}$ x .851 = 85	Aug	$\frac{1{,}200}{12}$ x 1.064 = 106
Mar	$\frac{1{,}200}{12}$ x .904 = 90	Sept	$\frac{1{,}200}{12}$ x .957 = 96
Apr	$\frac{1{,}200}{12}$ x 1.064 = 106	Oct	$\frac{1{,}200}{12}$ x .851 = 85
May	$\frac{1{,}200}{12}$ x 1.309 = 131	Nov	$\frac{1{,}200}{12}$ x .851 = 85
June	$\frac{1{,}200}{12}$ x 1.223 = 122	Dec	$\frac{1{,}200}{12}$ x .851 = 85

Seasonal Index Example



San Diego Hospital



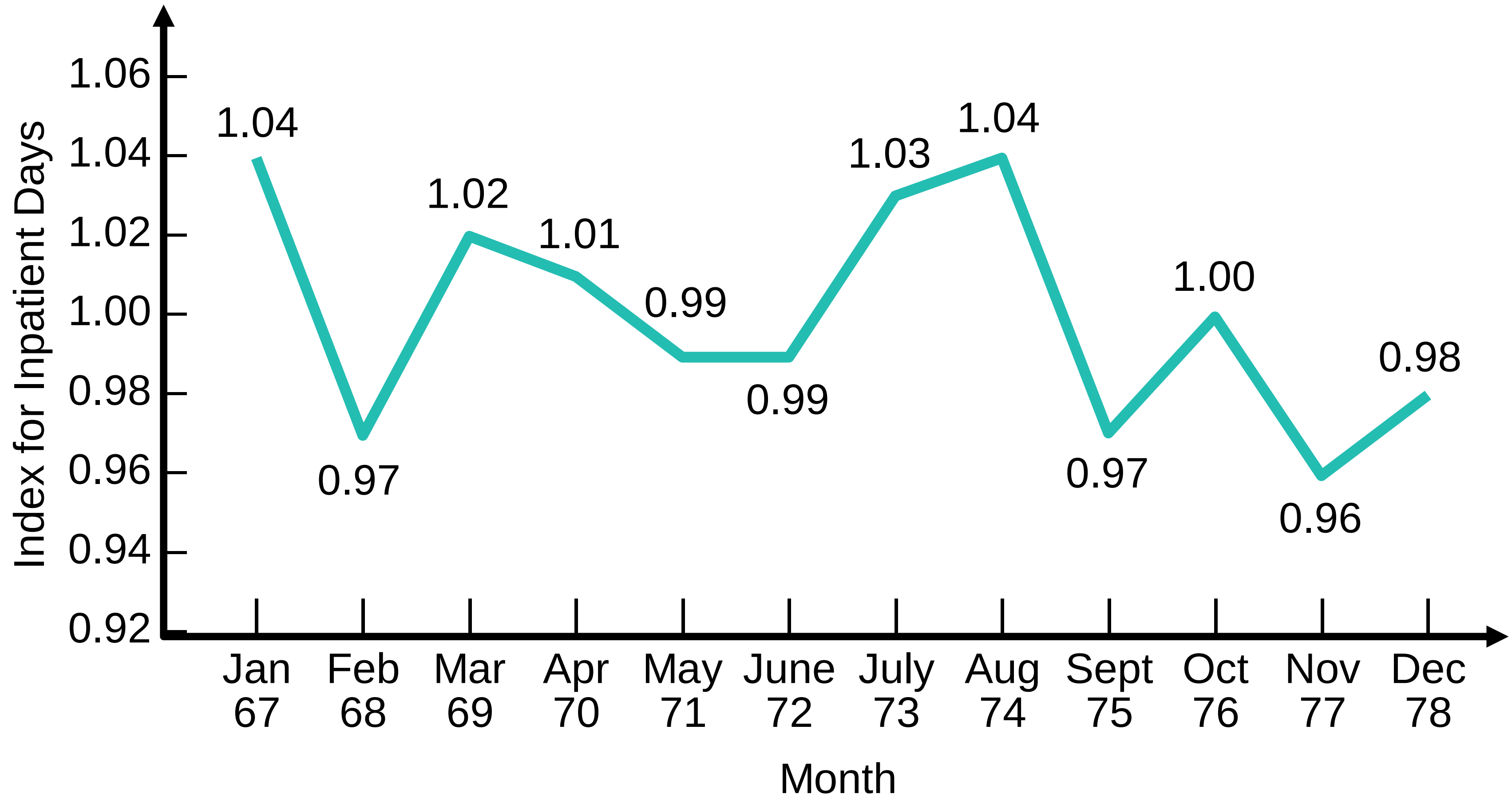
San Diego Hospital

	Seasonality Indices for Adult Inpatient Days at San Diego Hospital		
MONTH	**SEASONALITY INDEX**	**MONTH**	**SEASONALITY INDEX**
January	1.04	July	1.03
February	0.97	August	1.04
March	1.02	September	0.97
April	1.01	October	1.00
May	0.99	November	0.96
June	0.99	December	0.98

San Diego Hospital

Seasonal Indices

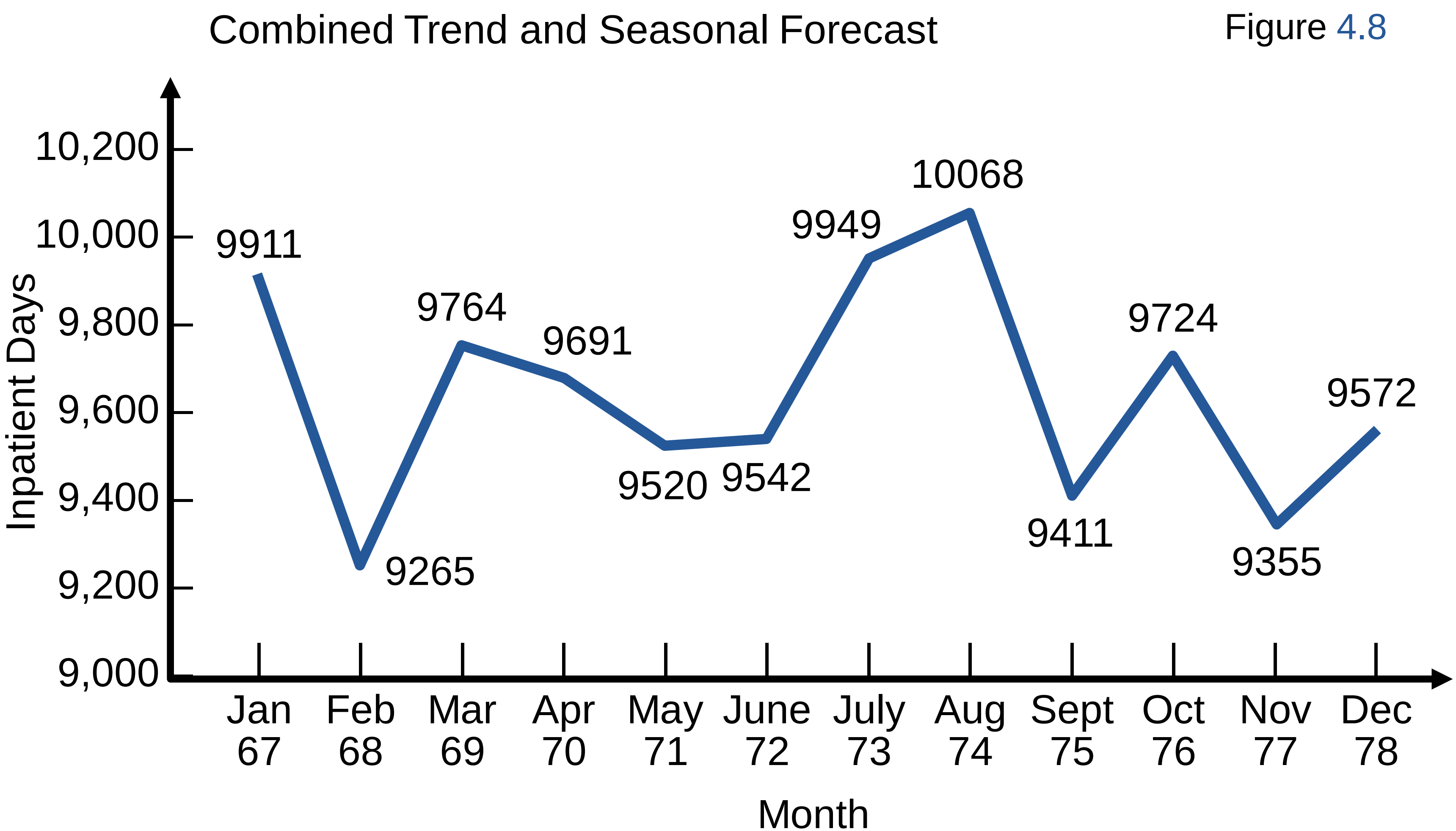
Figure 4.7



San Diego Hospital

Period	67	68	69	70	71	72
Month	Jan	Feb	Mar	Apr	May	June
Forecast with Trend & Seasonality	9,911	9,265	9,164	9,691	9,520	9,542
Period	73	74	75	76	77	78
Month	July	Aug	Sept	Oct	Nov	Dec
Forecast with Trend & Seasonality	9,949	10,068	9,411	9,724	9,355	9,572

San Diego Hospital



Adjusting Trend Data

$$\hat{y}_{\text{seasonal}} = \text{Index} \times \hat{y}_{\text{trend forecast}}$$

Quarter I: $\hat{y}_{\text{I}} = (1.30)(\$100{,}000) = \$130{,}000$

Quarter II: $\hat{y}_{\text{II}} = (.90)(\$120{,}000) = \$108{,}000$

Quarter III: $\hat{y}_{\text{III}} = (.70)(\$140{,}000) = \$98{,}000$

Quarter IV: $\hat{y}_{\text{IV}} = (1.10)(\$160{,}000) = \$176{,}000$