# Inventory Management

CHAPTER 12

CHAPTER OUTLINE



Alaska Airlines

**10 OM STRATEGY DECISIONS**

- Design of Goods and Services
- Managing Quality
- Process Strategy
- Location Strategies
- Layout Strategies
- Human Resources
- Supply-Chain Management
- *Inventory Management*
  - ***Independent Demand (Ch. 12)***
  - Dependent Demand (Ch. 14)
  - Lean Operations (Ch. 16)
- Scheduling
- Maintenance

**GLOBAL COMPANY PROFILE**
*Amazon.com*

# Inventory Management Provides Competitive Advantage at Amazon.com

When Jeff Bezos opened his revolutionary business in 1995, Amazon.com was intended to be a "virtual" retailer—no inventory, no warehouses, no overhead—just a bunch of computers taking orders for books and authorizing others to fill them. Things clearly didn't work out that way. Now, Amazon stocks millions of items of inventory, amid hundreds of thousands of bins on shelves in over 150 warehouses around the world. Additionally, Amazon's

Marilyn Newton/Reno Gazette-Journal

**1. You order three items, and a computer in Seattle takes charge.** A computer assigns your order—a book, a game, and a digital camera—to one of Amazon's massive U.S. distribution centers.

**2. The "flow meister" at the distribution center receives your order.** She determines which workers go where to fill your order.

Bernhard Classen/Alamy

**3. Amazon's current system doubles the picking speed of manual operators and drops the error rate to nearly zero.**

Ben Cawthra/Sipa USA/Newscom

**4. Your items are put into crates on moving belts.** Each item goes into a large yellow crate that contains many customers' orders. When full, the crates ride a series of conveyor belts that wind more than 10 miles through the plant at a constant speed of 2.9 feet per second. The bar code on each item is scanned 15 times, by machines and by many of the 600 workers. The goal is to reduce errors to zero—returns are very expensive.

5. All three items converge in a chute and then inside a box. All the crates arrive at a central point where bar codes are matched with order numbers to determine who gets what. Your three items end up in a 3-foot-wide chute—one of several thousand—and are placed into a corrugated box with a new bar code that identifies your order. Picking is sequenced to reduce operator travel.

6. Any gifts you've chosen are wrapped by hand. Amazon trains an elite group of gift wrappers, each of whom processes 30 packages an hour.

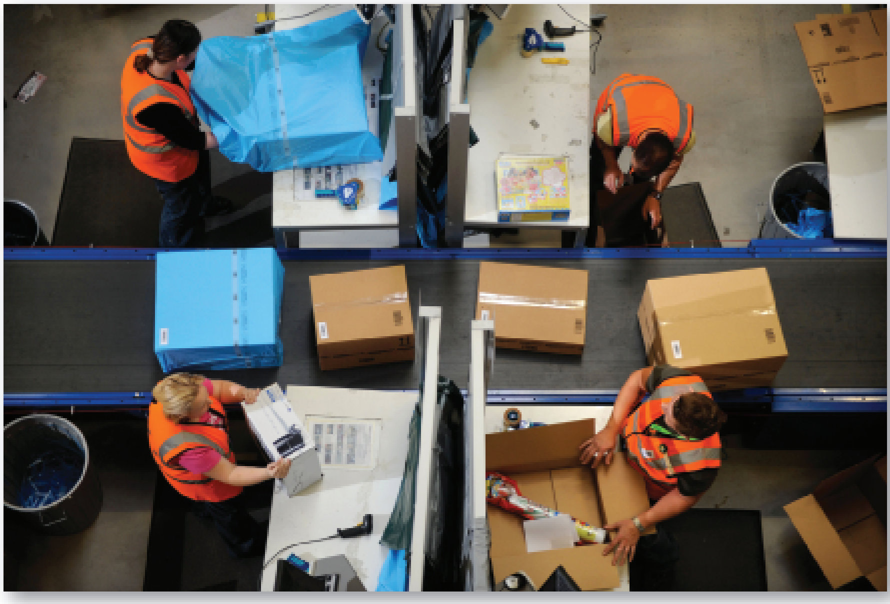
REX/Newscom

software is so good that Amazon sells its order taking, processing, and billing expertise to others. It is estimated that 200 million items are now available via the Amazon Web site.

Bezos expects the customer experience at Amazon to be one that yields the lowest price, the fastest delivery, and an error-free order fulfillment process so no other contact with Amazon is necessary. Exchanges and returns are very expensive.

Managing this massive inventory precisely is the key for Amazon to be the world-class leader in warehouse automation and management. The time to receive, process, and position the stock in storage and to then accurately "pull" and package an order requires a labor investment of less than 3 minutes. And 70% of these orders are multiproduct orders. This underlines the high benchmark that Amazon has achieved. This is world-class performance.

When you place an order with Amazon .com, you are doing business with a company that obtains competitive advantage through inventory management. This *Global Company Profile* shows how Amazon does it.



Adrian Sherratt/Alamy

7. The box is packed, taped, weighed, and labeled before leaving the warehouse in a truck. A typical plant is designed to ship as many as 200,000 pieces a day. About 60% of orders are shipped via the U.S. Postal Service; nearly everything else goes through United Parcel Service.

8. Your order arrives at your doorstep. In 1 or 2 days, your order is delivered.

## LEARNING OBJECTIVES



# The Importance of Inventory

As Amazon.com well knows, inventory is one of the most expensive assets of many companies, representing as much as 50% of total invested capital. Operations managers around the globe have long recognized that good inventory management is crucial. On the one hand, a firm can reduce costs by reducing inventory. On the other hand, production may stop and customers become dissatisfied when an item is out of stock. *The objective of inventory management is to strike a balance between inventory investment and customer service.* You can never achieve a low-cost strategy without good inventory management.

All organizations have some type of inventory planning and control system. A bank has methods to control its inventory of cash. A hospital has methods to control blood supplies and pharmaceuticals. Government agencies, schools, and, of course, virtually every manufacturing and production organization are concerned with inventory planning and control.

In cases involving physical products, the organization must determine whether to produce goods or to purchase them. Once this decision has been made, the next step is to forecast demand, as discussed in Chapter 4. Then operations managers determine the inventory necessary to service that demand. In this chapter, we discuss the functions, types, and management of inventory. We then address two basic inventory issues: how much to order and when to order.

## Functions of Inventory

**VIDEO 12.1**
Managing Inventory at Frito-Lay

Inventory can serve several functions that add flexibility to a firm's operations. The four functions of inventory are:

1. *To provide a selection of goods for anticipated customer demand and to separate the firm from fluctuations in that demand.* Such inventories are typical in retail establishments.
2. To *decouple various parts of the production process*. For example, if a firm's supplies fluctuate, extra inventory may be necessary to decouple the production process from suppliers.
3. To *take advantage of quantity discounts*, because purchases in larger quantities may reduce the cost of goods or their delivery.
4. To *hedge against inflation* and upward price changes.

## Types of Inventory

**Raw material inventory**
Materials that are usually purchased but have yet to enter the manufacturing process.

**Work-in-process (WIP) inventory**
Products or components that are no longer raw materials but have yet to become finished products.

**Maintenance/repair/operating (MRO) inventory**
Maintenance, repair, and operating materials.

To accommodate the functions of inventory, firms maintain four types of inventories: (1) raw material inventory, (2) work-in-process inventory, (3) maintenance/repair/operating supply (MRO) inventory, and (4) finished-goods inventory.

**Raw material inventory** has been purchased but not processed. This inventory can be used to decouple (i.e., separate) suppliers from the production process. However, the preferred approach is to eliminate supplier variability in quality, quantity, or delivery time so that separation is not needed. **Work-in-process (WIP) inventory** is components or raw material that have undergone some change but are not completed. WIP exists because of the time it takes for a product to be made (called *cycle time*). Reducing cycle time reduces inventory. Often this task is not difficult: during most of the time a product is "being made," it is in fact sitting idle. As Figure 12.1 shows, actual work time, or "run" time, is a small portion of the material flow time, perhaps as low as 5%.

**MROs** are inventories devoted to **maintenance/repair/operating** supplies necessary to keep machinery and processes productive. They exist because the need and timing for maintenance and

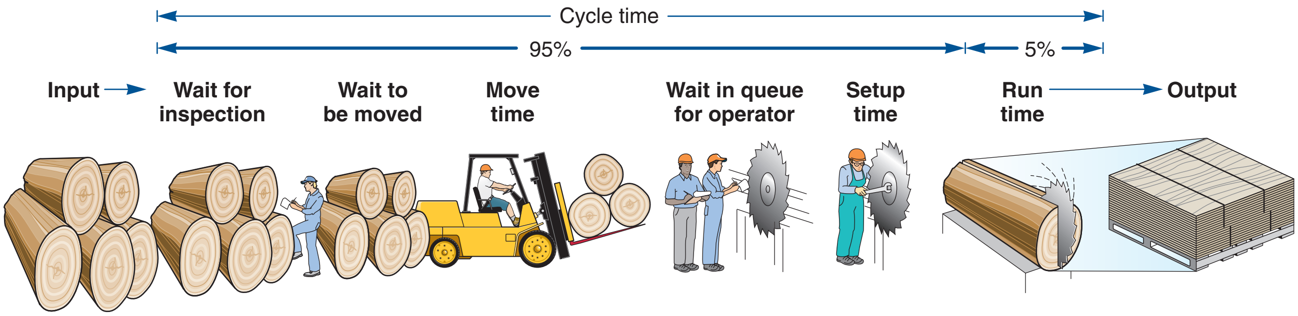



Figure 12.1

**The Material Flow Cycle**

Most of the time that work is in-process (95% of the cycle time) is not productive time.

repair of some equipment are unknown. Although the demand for MRO inventory is often a function of maintenance schedules, other unscheduled MRO demands must be anticipated. **Finished-goods inventory** is completed product awaiting shipment. Finished goods may be inventoried because future customer demands are unknown.

**Finished-goods inventory**
An end item ready to be sold, but still an asset on the company's books.

# Managing Inventory

Operations managers establish systems for managing inventory. In this section, we briefly examine two ingredients of such systems: (1) how inventory items can be classified (called *ABC analysis*) and (2) how accurate inventory records can be maintained. We will then look at inventory control in the service sector.

## ABC Analysis

**ABC analysis** divides on-hand inventory into three classifications on the basis of annual dollar volume. ABC analysis is an inventory application of what is known as the *Pareto principle* (named after Vilfredo Pareto, a 19th-century Italian economist). The Pareto principle states that there are a "critical few and trivial many." The idea is to establish inventory policies that focus resources on the *few critical* inventory parts and not the many trivial ones. It is not realistic to monitor inexpensive items with the same intensity as very expensive items.

**ABC analysis**
A method for dividing on-hand inventory into three classifications based on annual dollar volume.

To determine annual dollar volume for ABC analysis, we measure the *annual demand* of each inventory item times the *cost per unit*. *Class A* items are those on which the annual dollar volume is high. Although such items may represent only about 15% of the total inventory items, they represent 70% to 80% of the total dollar usage. *Class B* items are those inventory items of medium annual dollar volume. These items may represent about 30% of inventory items and 15% to 25% of the total value. Those with low annual dollar volume are *Class C*, which may represent only 5% of the annual dollar volume but about 55% of the total inventory items.

**STUDENT TIP**
A, B, and C categories need not be exact. The idea is to recognize that levels of control should match the risk.

Graphically, the inventory of many organizations would appear as presented in Figure 12.2.

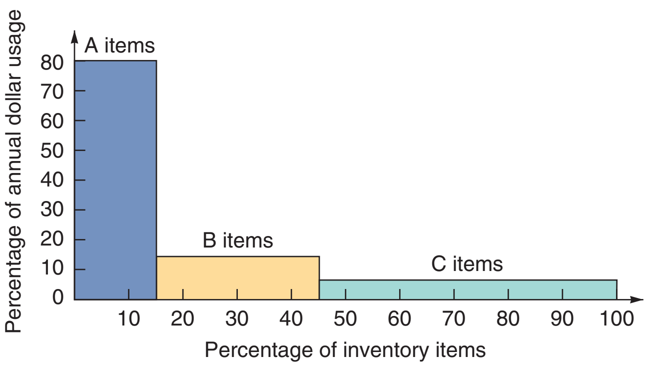



Figure 12.2

**Graphic Representation of ABC Analysis**

An example of the use of ABC analysis is shown in Example 1.

## Example 1

### ABC ANALYSIS FOR A CHIP MANUFACTURER

Silicon Chips, Inc., maker of superfast DRAM chips, wants to categorize its 10 major inventory items using ABC analysis.

**APPROACH ▶** ABC analysis organizes the items on an annual dollar-volume basis. Shown below (in columns 1–4) are the 10 items (identified by stock numbers), their annual demands, and unit costs.

**SOLUTION ▶** Annual dollar volume is computed in column 5, along with the percentage of the total represented by each item in column 6. Column 7 groups the 10 items into A, B, and C categories.

**LO 12.1** *Conduct* an ABC analysis

**ABC Calculation**

| (1) ITEM STOCK NUMBER | (2) PERCENTAGE OF NUMBER OF ITEMS STOCKED | (3) ANNUAL VOLUME (UNITS) | × | (4) UNIT COST | = | (5) ANNUAL DOLLAR VOLUME | (6) PERCENTAGE OF ANNUAL DOLLAR VOLUME | | (7) CLASS |
|---|---|---|---|---|---|---|---|---|---|
| #10286 | 20% | 1,000 | | $ 90.00 | | $ 90,000 | 38.8% | 72% | A |
| #11526 | | 500 | | 154.00 | | 77,000 | 33.2% | | A |
| #12760 | 30% | 1,550 | | 17.00 | | 26,350 | 11.3% | 23% | B |
| #10867 | | 350 | | 42.86 | | 15,001 | 6.4% | | B |
| #10500 | | 1,000 | | 12.50 | | 12,500 | 5.4% | | B |
| #12572 | 50% | 600 | | 14.17 | | 8,502 | 3.7% | 5% | C |
| #14075 | | 2,000 | | .60 | | 1,200 | .5% | | C |
| #01036 | | 100 | | 8.50 | | 850 | .4% | | C |
| #01307 | | 1,200 | | .42 | | 504 | .2% | | C |
| #10572 | | 250 | | .60 | | 150 | .1% | | C |
| | | 8,550 | | | | $232,057 | 100.0% | | |

**INSIGHT ▶** The breakdown into A, B, and C categories is not hard and fast. The objective is to try to separate the "important" from the "unimportant."

**LEARNING EXERCISE ▶** The unit cost for Item #10286 has increased from $90.00 to $120.00. How does this impact the ABC analysis? [Answer: The total annual dollar volume increases by $30,000, to $262,057, and the two A items now comprise 75% of that amount.]

**RELATED PROBLEMS ▶** 12.1, 12.2, 12.3 (12.5–12.6 are available in MyOMLab)

EXCEL OM Data File **Ch12Ex1.xls** can be found in MyOMLab.

Criteria other than annual dollar volume can determine item classification. For instance, high shortage or holding cost, anticipated engineering changes, delivery problems, or quality problems may dictate upgrading items to a higher classification. The advantage of dividing inventory items into classes allows policies and controls to be established for each class.

Policies that may be based on ABC analysis include the following:

1. Purchasing resources expended on supplier development should be much higher for individual A items than for C items.
2. A items, as opposed to B and C items, should have tighter physical inventory control; perhaps they belong in a more secure area, and perhaps the accuracy of inventory records for A items should be verified more frequently.
3. Forecasting A items may warrant more care than forecasting other items.

Better forecasting, physical control, supplier reliability, and an ultimate reduction in inventory can all result from classification systems such as ABC analysis.

## OM in Action Supply Accuracy at Acando

From a logistic point of view we might want to have as few article numbers as possible to simplify the optimization of the supply chain, However, the tendency is clear that the number of article numbers are increasing, the customers are asking for more variations of a product. Fredrik Berndston, management consultant at the Nordic consultant company Acando, has a long experience supporting customers with solutions to problems related to an extended range of products. "The key-word is segmentation, both product and customer segmentation", says Fredrik. "It is necessary to understand what the customer is willing to pay for", he continues. There is a need to differentiate on service level, lead-time, delivery service, etc. "We should aim for transparency in the supply cost, the customers should understand the cost for different service levels".

For the operations management it is necessary to have correct master data and good tools to support this process. In a modern environment it is normal that many systems interact why it is necessary to have strict rules where to keep master data and how to maintain it. Fredrik's experience is that many companies today implements PLM-tools (Product Lifetime Management) to manage the master data. "In a more complex environment, more articles, more possible ways of sourcing the customer, more competition… it is even more necessary to have good tools for optimizing the supply chain," Fredrik states.

## Record Accuracy

Record accuracy is a prerequisite to inventory management, production scheduling, and, ultimately, sales. Accuracy can be maintained by either periodic or perpetual systems. *Periodic systems* require regular (periodic) checks of inventory to determine quantity on hand. Some small retailers and facilities with vendor-managed inventory (the vendor checks quantity on hand and resupplies as necessary) use these systems. However, the downside is lack of control between reviews and the necessity of carrying extra inventory to protect against shortages.

A variation of the periodic system is a *two-bin system.* In practice, a store manager sets up two containers (each with adequate inventory to cover demand during the time required to receive another order) and places an order when the first container is empty.

Alternatively, *perpetual inventory* tracks both receipts and subtractions from inventory on a continuing basis. Receipts are usually noted in the receiving department in some semiautomated way, such as via a bar-code reader, and disbursements are noted as items leave the stockroom or, in retailing establishments, at the point-of-sale (POS) cash register.

Regardless of the inventory system, record accuracy requires good incoming and outgoing record keeping as well as good security. Stockrooms will have limited access, good housekeeping, and storage areas that hold fixed amounts of inventory. In both manufacturing and retail facilities, bins, shelf space, and individual items must be stored and labeled accurately. Meaningful decisions about ordering, scheduling, and shipping, are made only when the firm knows what it has on hand.

Omnicell

In this hospital, these vertically rotating storage carousels provide rapid access to hundreds of critical items and at the same time save floor space. This Omnicell inventory management carousel is also secure and has the added advantage of printing bar code labels.

## Cycle Counting

Even though an organization may have made substantial efforts to record inventory accurately, these records must be verified through a continuing audit. Such audits are known as **cycle counting**. Historically, many firms performed annual physical inventories. This practice often meant shutting down the facility and having inexperienced people count parts and material. Inventory records should instead be verified via cycle counting. Cycle counting uses inventory classifications developed through ABC analysis. With cycle counting procedures, items are counted, records are verified, and inaccuracies are periodically documented. The cause of inaccuracies is then traced and appropriate remedial action taken to ensure integrity of the inventory system. **A** items will be counted frequently, perhaps once a month; **B** items will be counted less frequently, perhaps once a quarter; and **C** items will be counted perhaps once every 6 months. Example 2 illustrates how to compute the number of items of each classification to be counted each day.

**Cycle counting**
A continuing reconciliation of inventory with inventory records.

**LO 12.2** *Explain* and use cycle counting

## Example 2

### CYCLE COUNTING AT COLE'S TRUCKS, INC.

Cole's Trucks, Inc., a builder of high-quality refuse trucks, has about 5,000 items in its inventory. It wants to determine how many items to cycle count each day.

**APPROACH ▶** After hiring Matt Clark, a bright young OM student, for the summer, the firm determined that it has 500 A items, 1,750 B items, and 2,750 C items. Company policy is to count all A items every month (every 20 working days), all B items every quarter (every 60 working days), and all C items every 6 months (every 120 working days). The firm then allocates some items to be counted each day.

**SOLUTION ▶**

| ITEM CLASS | QUANTITY | CYCLE-COUNTING POLICY | NUMBER OF ITEMS COUNTED PER DAY |
|---|---|---|---|
| A | 500 | Each month (20 working days) | 500/20 = 25/day |
| B | 1,750 | Each quarter (60 working days) | 1,750/60 = 29/day |
| C | 2,750 | Every 6 months (120 working days) | 2,750/120 = 23/day |
| | | | 77/day |

Each day, 77 items are counted.

**INSIGHT ▶** This daily audit of 77 items is much more efficient and accurate than conducting a massive inventory count once a year.

**LEARNING EXERCISE ▶** Cole's reclassifies some B and C items so there are now 1,500 B items and 3,000 C items. How does this change the cycle count? [Answer: B and C both change to 25 items each per day, for a total of 75 items per day.]

**RELATED PROBLEM ▶** 12.4

In Example 2, the particular items to be cycle counted can be sequentially or randomly selected each day. Another option is to cycle count items when they are reordered.

Cycle counting also has the following advantages:

**Shrinkage**
Retail inventory that is unaccounted for between receipt and sale.

**Pilferage**
A small amount of theft.

1. Eliminates the shutdown and interruption of production necessary for annual physical inventories.
2. Eliminates annual inventory adjustments.
3. Trained personnel audit the accuracy of inventory.
4. Allows the cause of the errors to be identified and remedial action to be taken.
5. Maintains accurate inventory records.


Robin Nelson/ZUMA Press, Inc./Alamy

Pharmaceutical distributor McKesson Corp., which is one of Arnold Palmer Hospital's main suppliers of surgical materials, makes heavy use of bar-code readers to automate inventory control. The device on the warehouse worker's arm combines a scanner, a computer, and a two-way radio to check orders. With rapid and accurate data, items are easily verified, improving inventory and shipment accuracy.

## Control of Service Inventories

Although we may think of the service sector of our economy as not having inventory, that is seldom the case. Extensive inventory is held in wholesale and retail businesses, making inventory management crucial. In the food-service business, control of inventory is often the difference between success and failure. Moreover, inventory that is in transit or idle in a warehouse is lost value. Similarly, inventory damaged or stolen prior to sale is a loss. In retailing, inventory that is unaccounted for between receipt and time of sale is known as **shrinkage**. Shrinkage occurs from damage and theft as well as from sloppy paperwork. Inventory theft is also known as **pilferage**. Retail inventory loss of 1% of sales is considered good, with losses in many stores exceeding 3%. Because the impact on profitability is substantial, inventory accuracy and control are critical. Applicable techniques include the following:

1. *Good personnel selection, training, and discipline:* These are never easy but very necessary in food-service, wholesale, and retail operations, where employees have access to directly consumable merchandise.

## OM in Action ICC by AGA

"The key asset management challenge for an Industrial & Medical Gas operation is to manage the high-pressure (150 - 300 bar) cylinders that hold the gas. Due to the high pressure - and at times the content - these products are classified as Dangerous goods. Above and beyond regular requirement to manage the stocks - i.e. utilizing the assets, it is crucial to maintain a solid and correct transaction file, since the usage history can have an impact on current performance from safety perspective, " Kenneth Friberg ,Global Head of PGP Investments, Linde Gas.

Already in the mid 90's Swedish gas company AGA (later part of Linde Gas) started developing a software system called ICC to control the cylinders and the movements. Using the system has led to improved stock accuracy and by that an improved service level.

"The key challenge is to maintain a correct transaction file, which implies that all physical cylinder movements between different locations/sites should preferably be monitored, that is,' scanned'. This brings of course an issue of cost, but here we see an opportunity in using an UHF RFID technology providing multi-scanning capability. In short, it's scanning full truck-loads when passing an antenna portal. This could also reduce scanning time/ cost at physical inventories and deliveries at customer."

2. *Tight control of incoming shipments:* This task is being addressed by many firms through the use of Universal Product Code (or bar code) and radio frequency ID (RFID) systems that read every incoming shipment and automatically check tallies against purchase orders. When properly designed, these systems—where each stock keeping unit (SKU; pronounced "skew") has its own identifier—can be very hard to defeat.
3. *Effective control of all goods leaving the facility:* This job is accomplished with bar codes, RFID tags, or magnetic strips on merchandise, and via direct observation. Direct observation can be personnel stationed at exits (as at Costco and Sam's Club wholesale stores) and in potentially high-loss areas or can take the form of one-way mirrors and video surveillance.

A handheld reader can scan RFID tags, aiding control of both incoming and outgoing shipments.

Successful retail operations require very good store-level control with accurate inventory in its proper location. Major retailers lose 10% to 25% of overall profits due to poor or inaccurate inventory records.[1] (See the *OM in Action* box, "ICC by AGA.")

# Inventory Models

**VIDEO 12.2**
Inventory Control at Wheeled Coach Ambulance

We now examine a variety of inventory models and the costs associated with them.

## Independent vs. Dependent Demand

Inventory control models assume that demand for an item is either independent of or dependent on the demand for other items. For example, the demand for refrigerators is *independent* of the demand for toaster ovens. However, the demand for toaster oven components is *dependent* on the requirements of toaster ovens.

This chapter focuses on managing inventory where demand is *independent*. Chapter 14 presents *dependent* demand management.

## Holding, Ordering, and Setup Costs

**Holding costs** are the costs associated with holding or "carrying" inventory over time. Therefore, holding costs also include obsolescence and costs related to storage, such as insurance, extra staffing, and interest payments. Table 12.1 shows the kinds of costs that need to be evaluated to determine holding costs. Many firms fail to include all the inventory holding costs. Consequently, inventory holding costs are often understated.

**Holding cost**
The cost to keep or carry inventory in stock.

**Ordering cost** includes costs of supplies, forms, order processing, purchasing, clerical support, and so forth. When orders are being manufactured, ordering costs also exist, but they are a part

**Ordering cost**
The cost of the ordering process.

**TABLE 12.1 Determining Inventory Holding Costs**

| CATEGORY | COST (AND RANGE) AS A PERCENTAGE OF INVENTORY VALUE |
|---|---|
| **Housing costs** (building rent or depreciation, operating cost, taxes, insurance) | 6% (3–10%) |
| **Material-handling costs** (equipment lease or depreciation, power, operating cost) | 3% (1–3.5%) |
| **Labor cost** (receiving, warehousing, security) | 3% (3–5%) |
| **Investment costs** (borrowing costs, taxes, and insurance on inventory) | 11% (6–24%) |
| **Pilferage, scrap, and obsolescence** (much higher in industries undergoing rapid change like tablets and smart phones) | 3% (2–5%) |
| **Overall carrying cost** | 26% |

*Note:* All numbers are approximate, as they vary substantially depending on the nature of the business, location, and current interest rates.

**STUDENT TIP**
An overall inventory carrying cost of less than 15% is very unlikely, but this cost can exceed 40%, especially in high-tech and fashion industries.

**Setup cost**
The cost to prepare a machine or process for production.

**Setup time**
The time required to prepare a machine or process for production.

of what is called setup costs. **Setup cost** is the cost to prepare a machine or process for manufacturing an order. This includes time and labor to clean and change tools or holders. Operations managers can lower ordering costs by reducing setup costs and by using such efficient procedures as electronic ordering and payment.

In manufacturing environments, setup cost is highly correlated with **setup time**. Setups usually require a substantial amount of work even before a setup is actually performed at the work center. With proper planning, much of the preparation required by a setup can be done prior to shutting down the machine or process. Setup times can thus be reduced substantially. Machines and processes that traditionally have taken hours to set up are now being set up in less than a minute by the more imaginative world-class manufacturers. Reducing setup times is an excellent way to reduce inventory investment and to improve productivity.

# Inventory Models for Independent Demand

In this section, we introduce three inventory models that address two important questions: *when to order* and *how much to order*. These *independent* demand models are:

1. Basic economic order quantity (EOQ) model
2. Production order quantity model
3. Quantity discount model

## The Basic Economic Order Quantity (EOQ) Model

**Economic order quantity (EOQ) model**
An inventory-control technique that minimizes the total of ordering and holding costs.

The **economic order quantity (EOQ) model** is one of the most commonly used inventory-control techniques. This technique is relatively easy to use but is based on several assumptions:

**LO 12.3** *Explain* and use the EOQ model for independent inventory demand

1. Demand for an item is known, reasonably constant, and independent of decisions for other items.
2. Lead time—that is, the time between placement and receipt of the order—is known and consistent.
3. Receipt of inventory is instantaneous and complete. In other words, the inventory from an order arrives in one batch at one time.
4. Quantity discounts are not possible.
5. The only variable costs are the cost of setting up or placing an order (setup or ordering cost) and the cost of holding or storing inventory over time (holding or carrying cost). These costs were discussed in the previous section.
6. Stockouts (shortages) can be completely avoided if orders are placed at the right time.

With these assumptions, the graph of inventory usage over time has a sawtooth shape, as in Figure 12.3. In Figure 12.3, $Q$ represents the amount that is ordered. If this amount is 500 dresses, all 500 dresses arrive at one time (when an order is received). Thus, the inventory

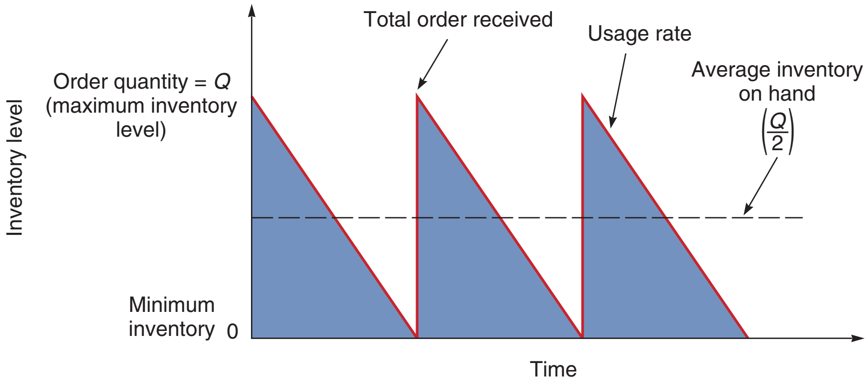



Figure 12.3

**Inventory Usage over Time**

**STUDENT TIP**

If the maximum we can ever have is $Q$ (say, 500 units) and the minimum is zero, then if inventory is used (or sold) at a fairly steady rate, the average $= (Q + 0)/2 = Q/2$.

level jumps from 0 to 500 dresses. In general, an inventory level increases from 0 to $Q$ units when an order arrives.

Because demand is constant over time, inventory drops at a uniform rate over time. (Refer to the sloped lines in Figure 12.3.) Each time the inventory is received, the inventory level again jumps to $Q$ units (represented by the vertical lines). This process continues indefinitely over time.

## Minimizing Costs

The objective of most inventory models is to minimize total costs. With the assumptions just given, significant costs are setup (or ordering) cost and holding (or carrying) cost. All other costs, such as the cost of the inventory itself, are constant. Thus, if we minimize the sum of setup and holding costs, we will also be minimizing total costs. To help you visualize this, in Figure 12.4 we graph total costs as a function of the order quantity, $Q$. The optimal order size, $Q^*$, will be the quantity that minimizes the total costs. As the quantity ordered increases, the total number of orders placed per year will decrease. Thus, as the quantity ordered increases, the annual setup or ordering cost will decrease [Figure 12.4(a)]. But as the order quantity increases, the holding cost will increase due to the larger average inventories that are maintained [Figure 12.4(b)].

**STUDENT TIP**

Figure 12.4 is the heart of EOQ inventory modeling. We want to find the smallest total cost (top curve), which is the sum of the two curves below it.

As we can see in Figure 12.4(c), a reduction in either holding or setup cost will reduce the total cost curve. A reduction in the setup cost curve also reduces the optimal order quantity (lot size). In addition, smaller lot sizes have a positive impact on quality and production flexibility. At Toshiba, the $77 billion Japanese conglomerate, workers can make as few as 10 laptop computers before changing models. This lot-size flexibility has allowed Toshiba to move toward a "build-to-order" mass customization system, an important ability in an industry that has product life cycles measured in months, not years.

You should note that in Figure 12.4(c), the optimal order quantity occurs at the point where the ordering-cost curve and the carrying-cost curve intersect. This was not by chance. With the EOQ model, the optimal order quantity will occur at a point where the total setup cost is equal

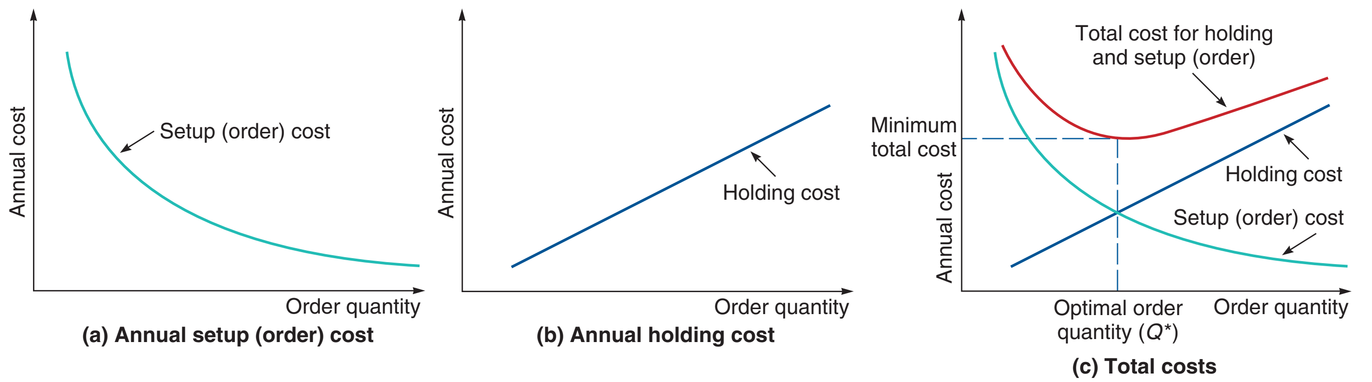



Figure 12.4

**Costs as a Function of Order Quantity**

to the total holding cost.[2] We use this fact to develop equations that solve directly for $Q^*$. The necessary steps are:

1. Develop an expression for setup or ordering cost.
2. Develop an expression for holding cost.
3. Set setup (order) cost equal to holding cost.
4. Solve the equation for the optimal order quantity.

Using the following variables, we can determine setup and holding costs and solve for $Q^*$:

$$\begin{aligned} Q &= \text{Number of units per order} \\ Q^* &= \text{Optimum number of units per order (EOQ)} \\ D &= \text{Annual demand in units for the inventory item} \\ S &= \text{Setup or ordering cost for each order} \\ H &= \text{Holding or carrying cost per unit per year} \end{aligned}$$

1. Annual setup cost = (Number of orders placed per year) × (Setup or order cost per order)

$$= \left(\frac{\text{Annual demand}}{\text{Number of units in each order}}\right)(\text{Setup or order cost per order})$$

$$= \left(\frac{D}{Q}\right)(S) = \frac{D}{Q}S$$

2. Annual holding cost = (Average inventory level) × (Holding cost per unit per year)

$$= \left(\frac{\text{Order quantity}}{2}\right)(\text{Holding cost per unit per year})$$

$$= \left(\frac{Q}{2}\right)(H) = \frac{Q}{2}H$$

3. Optimal order quantity is found when annual setup (order) cost equals annual holding cost, namely:

$$\frac{D}{Q}S = \frac{Q}{2}H$$

4. To solve for $Q^*$, simply cross-multiply terms and isolate $Q$ on the left of the equal sign:

$$2DS = Q^2H$$

$$Q^2 = \frac{2DS}{H}$$

$$Q^* = \sqrt{\frac{2DS}{H}} \tag{12-1}$$

Now that we have derived the equation for the optimal order quantity, $Q^*$, it is possible to solve inventory problems directly, as in Example 3.

## Example 3

**FINDING THE OPTIMAL ORDER SIZE AT SHARP, INC.**

Sharp, Inc., a company that markets painless hypodermic needles to hospitals, would like to reduce its inventory cost by determining the optimal number of hypodermic needles to obtain per order.

**APPROACH ▶** The annual demand is 1,000 units; the setup or ordering cost is \$10 per order; and the holding cost per unit per year is \$.50.

**SOLUTION ▶** Using these figures, we can calculate the optimal number of units per order:

$$Q^* = \sqrt{\frac{2DS}{H}}$$

$$Q^* = \sqrt{\frac{2(1{,}000)(10)}{0.50}} = \sqrt{40{,}000} = 200 \text{ units}$$

**INSIGHT ▶** Sharp, Inc., now knows how many needles to order per order. The firm also has a basis for determining ordering and holding costs for this item, as well as the number of orders to be processed by the receiving and inventory departments.

**LEARNING EXERCISE ▶** If $D$ increases to 1,200 units, what is the new $Q^*$? [Answer: $Q^* = 219$ units.]

**RELATED PROBLEMS ▶** 12.7, 12.8, 12.9, 12.10, 12.11, 12.14, 12.15, 12.17, 12.29 (12.31, 12.32, 12.33a, 12.35a are available in MyOMLab)

**EXCEL OM** Data File **Ch12Ex3.xls** can be found in MyOMLab.

**ACTIVE MODEL 12.1** This example is further illustrated in Active Model 12.1 in MyOMLab.

We can also determine the expected number of orders placed during the year ($N$) and the expected time between orders ($T$), as follows:

$$\text{Expected number of orders} = N = \frac{\text{Demand}}{\text{Order quantity}} = \frac{D}{Q^*} \tag{12-2}$$

$$\text{Expected time between orders} = T = \frac{\text{Number of working days per year}}{N} \tag{12-3}$$

Example 4 illustrates this concept.

## Example 4

### COMPUTING NUMBER OF ORDERS AND TIME BETWEEN ORDERS AT SHARP, INC.

Sharp, Inc. (in Example 3) has a 250-day working year and wants to find the number of orders ($N$) and the expected time between orders ($T$).

**APPROACH ▶** Using Equations (12-2) and (12-3), Sharp enters the data given in Example 3.

**SOLUTION ▶**

$$N = \frac{\text{Demand}}{\text{Order quantity}}$$

$$= \frac{1{,}000}{200} = 5 \text{ orders per year}$$

$$T = \frac{\text{Number of working days per year}}{\text{Expected number of orders}}$$

$$= \frac{250 \text{ working days per year}}{5 \text{ orders}} = 50 \text{ days between orders}$$

**INSIGHT ▶** The company now knows not only how many needles to order per order but that the time between orders is 50 days and that there are five orders per year.

**LEARNING EXERCISE ▶** If $D = 1{,}200$ units instead of 1,000, find $N$ and $T$. [Answer: $N \cong 5.48$, $T = 45.62$.]

**RELATED PROBLEMS ▶** 12.14, 12.15, 12.17 (12.35c,d are available in MyOMLab)

As mentioned earlier in this section, the total annual variable inventory cost is the sum of setup and holding costs:

$$\text{Total annual cost} = \text{Setup (order) cost} + \text{Holding cost} \tag{12-4}$$

In terms of the variables in the model, we can express the total cost $TC$ as:

$$TC = \frac{D}{Q}S + \frac{Q}{2}H \tag{12-5}$$

Example 5 shows how to use this formula.

## Example 5

### COMPUTING COMBINED COST OF ORDERING AND HOLDING

Sharp, Inc. (from Examples 3 and 4) wants to determine the combined annual ordering and holding costs.

**APPROACH ▶** Apply Equation (12-5), using the data in Example 3.

**SOLUTION ▶**

$$\begin{aligned}TC &= \frac{D}{Q}S + \frac{Q}{2}H \\ &= \frac{1{,}000}{200}(\$10) + \frac{200}{2}(\$.50) \\ &= (5)(\$10) + (100)(\$.50) \\ &= \$50 + \$50 = \$100\end{aligned}$$

**INSIGHT ▶** These are the annual setup and holding costs. The \$100 total does not include the actual cost of goods. Notice that in the EOQ model, holding costs always equal setup (order) costs.

**LEARNING EXERCISE ▶** Find the total annual cost if $D = 1{,}200$ units in Example 3. [Answer: \$109.54.]

**RELATED PROBLEMS ▶** 12.11, 12.14, 12.15, 12.16 (12.33b,c; 12.35e; 12.36a,b are available in MyOMLab)

Inventory costs may also be expressed to include the actual cost of the material purchased. If we assume that the annual demand and the price per hypodermic needle are known values (e.g., 1,000 hypodermics per year at $P = \$10$) and total annual cost should include purchase cost, then Equation (12-5) becomes:

$$TC = \frac{D}{Q}S + \frac{Q}{2}H + PD$$

Because material cost does not depend on the particular order policy, we still incur an annual material cost of $D \times P = (1{,}000)(\$10) = \$10{,}000$. (Later in this chapter we will discuss the case in which this may not be true—namely, when a quantity discount is available.)[3]

**Robust**
Giving satisfactory answers even with substantial variation in the parameters.

**Robust Model** A benefit of the EOQ model is that it is robust. By **robust** we mean that it gives satisfactory answers even with substantial variation in its parameters. As we have observed, determining accurate ordering costs and holding costs for inventory is often difficult. Consequently, a robust model is advantageous. The total cost of the EOQ changes little in the neighborhood of the minimum. The curve is very shallow. This means that variations in setup costs, holding costs, demand, or even EOQ make relatively modest differences in total cost. Example 6 shows the robustness of EOQ.

## Example 6

### EOQ IS A ROBUST MODEL

Management in the Sharp, Inc., examples underestimates total annual demand by 50% (say demand is actually 1,500 needles rather than 1,000 needles) while using the same $Q$. How will the annual inventory cost be impacted?

**APPROACH ▶** We will solve for annual costs twice. First, we will apply the wrong EOQ; then we will recompute costs with the correct EOQ.

**SOLUTION ▶** If demand in Example 5 is actually 1,500 needles rather than 1,000, but management uses an order quantity of $Q = 200$ (when it should be $Q = 244.9$ based on $D = 1{,}500$), the sum of holding and ordering cost increases to \$125:

$$\begin{aligned}\text{Annual cost} &= \frac{D}{Q}S + \frac{Q}{2}H \\ &= \frac{1{,}500}{200}(\$10) + \frac{200}{2}(\$.50) \\ &= \$75 + \$50 = \$125\end{aligned}$$

However, had we known that the demand was for 1,500 with an EOQ of 244.9 units, we would have spent \$122.47, as shown:

$$\begin{aligned}\text{Annual cost} &= \frac{1{,}500}{244.9}(\$10) + \frac{244.9}{2}(\$.50)\\ &= 6.125(\$10) + 122.45(\$.50)\\ &= \$61.25 + \$61.22 = \$122.47\end{aligned}$$

**INSIGHT ▶** Note that the expenditure of \$125.00, made with an estimate of demand that was substantially wrong, is only 2% (\$2.52/\$122.47) higher than we would have paid had we known the actual demand and ordered accordingly. Note also that were it not due to rounding, the annual holding costs and ordering costs would be exactly equal.

**LEARNING EXERCISE ▶** Demand at Sharp remains at 1,000, $H$ is still \$.50, and we order 200 needles at a time (as in Example 5). But if the true order cost = $S$ = \$15 (rather than \$10), what is the annual cost? [Answer: Annual order cost increases to \$75, and annual holding cost stays at \$50. So the total cost = \$125.]

**RELATED PROBLEMS ▶** 12.10b, 12.16 (12.36a,b are available in MyOMLab)

We may conclude that the EOQ is indeed robust and that significant errors do not cost us very much. This attribute of the EOQ model is most convenient because our ability to accurately determine demand, holding cost, and ordering cost is limited.

**Lead time**
In purchasing systems, the time between placing an order and receiving it; in production systems, the wait, move, queue, setup, and run times for each component produced.

## Reorder Points

Now that we have decided *how much* to order, we will look at the second inventory question, *when* to order. Simple inventory models assume that receipt of an order is instantaneous. In other words, they assume (1) that a firm will place an order when the inventory level for that particular item reaches zero and (2) that it will receive the ordered items immediately. However, the time between placement and receipt of an order, called **lead time**, or delivery time, can be as short as a few hours or as long as months. Thus, the when-to-order decision is usually expressed in terms of a **reorder point (ROP)**—the inventory level at which an order should be placed (see Figure 12.5).

**Reorder point (ROP)**
The inventory level (point) at which action is taken to replenish the stocked item.

**Safety stock (*ss*)**
Extra stock to allow for uneven demand; a buffer.

The reorder point (ROP) is given as:

$$\begin{aligned}\text{ROP} &= \text{Demand per day} \times\\ &\quad\ \text{Lead time for a new order in days}\\ \text{ROP} &= d \times L \qquad (12\text{-}6)\end{aligned}$$

This equation for ROP *assumes that demand during lead time and lead time itself are constant*. When this is not the case, extra stock, often called **safety stock (*ss*)**, should be added. The reorder point with safety stock then becomes:

$$\text{ROP} = \text{Expected demand during lead time} + \text{Safety stock}$$

The demand per day, $d$, is found by dividing the annual demand, $D$, by the number of working days in a year:

$$d = \frac{D}{\text{Number of working days in a year}}$$

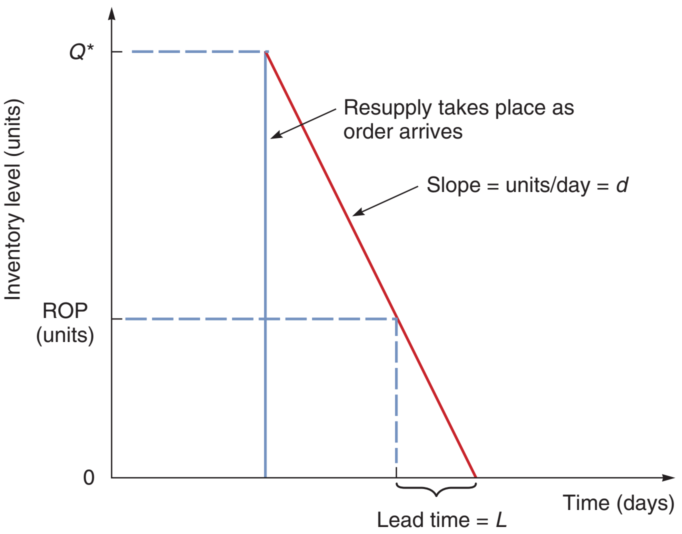



Figure 12.5
**The Reorder Point (ROP)**
$Q^*$ is the optimum order quantity, and lead time represents the time between placing and receiving an order.

Computing the reorder point is demonstrated in Example 7.

## Example 7

### COMPUTING REORDER POINTS (ROP) FOR IPHONES WITH AND WITHOUT SAFETY STOCK

An Apple store has a demand (D) for 8,000 iPhones per year. The firm operates a 250-day working year. On average, delivery of an order takes 3 working days, but has been known to take as long as 4 days. The store wants to calculate the reorder point without a safety stock and then with a one-day safety stock.

**APPROACH ▶** First compute the daily demand and then apply Equation (12-6) for the ROP. Then compute the ROP with safety stock.

**SOLUTION ▶**

**LO 12.4** *Compute* a reorder point and explain safety stock

$$d = \frac{D}{\text{Number of working days in a year}} = \frac{8{,}000}{250} = 32 \text{ units}$$

$$\text{ROP} = \text{Reorder point} = d \times L = 32 \text{ units per day} \times 3 \text{ days} = 96 \text{ units}$$

ROP with safety stock adds 1 day's demand (32 units) to the ROP (for 128 units).

**INSIGHT ▶** When iPhone inventory stock drops to 96 units, an order should be placed. If the safety stock for a possible one-day delay in delivery is added, the ROP is 128 (= 96 + 32).

**LEARNING EXERCISE ▶** If there are only 200 working days per year, what is the correct ROP, without safety stock and with safety stock? [Answer: 120 iPhones without safety stock and 160 with safety stock.]

**RELATED PROBLEMS ▶** 12.11d, 12.12, 12.13, 12.15f (12.33d, 12.34, 12.35f, 12.36c are available in MyOMLab)

When demand is not constant or variability exists in the supply chain, safety stock can be critical. We discuss safety stock in more detail later in this chapter.

## Production Order Quantity Model

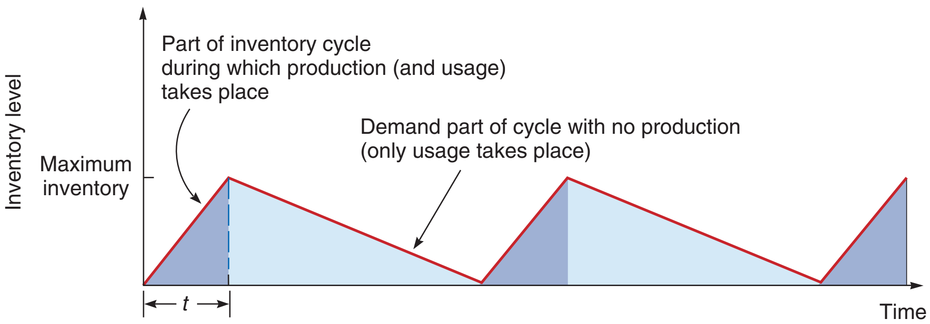
In the previous inventory model, we assumed that the entire inventory order was received at one time. There are times, however, when the firm may receive its inventory over a period of time. Such cases require a different model, one that does not require the instantaneous-receipt assumption. This model is applicable under two situations: (1) when inventory continuously flows or builds up over a period of time after an order has been placed or (2) when units are produced and sold simultaneously. Under these circumstances, we take into account daily production (or inventory-flow) rate and daily demand rate. Figure 12.6 shows inventory levels as a function of time (and inventory dropping to zero between orders).

Because this model is especially suitable for the production environment, it is commonly called the **production order quantity model**. It is useful when inventory continuously builds up over time, and traditional economic order quantity assumptions are valid. We derive this model by setting ordering or setup costs equal to holding costs and solving for optimal order size, $Q^*$. Using the following symbols, we can determine the expression for annual inventory holding cost for the production order quantity model:

**Production order quantity model**
An economic order quantity technique applied to production orders.

Figure **12.6**

**Change in Inventory Levels over Time for the Production Model**



$$\begin{aligned} Q &= \text{Number of units per order} \\ H &= \text{Holding cost per unit per year} \\ p &= \text{Daily production rate} \\ d &= \text{Daily demand rate, or usage rate} \\ t &= \text{Length of the production run in days} \end{aligned}$$

**LO 12.5** *Apply* the production order quantity model

1. $$\begin{pmatrix}\text{Annual inventory} \\ \text{holding cost}\end{pmatrix} = (\text{Average inventory level}) \times \begin{pmatrix}\text{Holding cost} \\ \text{per unit per year}\end{pmatrix}$$

2. $$(\text{Average inventory level}) = (\text{Maximum inventory level})/2$$

3. $$\begin{aligned} \begin{pmatrix}\text{Maximum} \\ \text{inventory level}\end{pmatrix} &= \begin{pmatrix}\text{Total production during} \\ \text{the production run}\end{pmatrix} - \begin{pmatrix}\text{Total used during} \\ \text{the production run}\end{pmatrix} \\ &= pt - dt \end{aligned}$$

However, $Q$ = total produced = $pt$, and thus $t = Q/p$. Therefore:

$$\begin{aligned} \text{Maximum inventory level} &= p\left(\frac{Q}{p}\right) - d\left(\frac{Q}{p}\right) = Q - \frac{d}{p}Q \\ &= Q\left(1 - \frac{d}{p}\right) \end{aligned}$$

**STUDENT TIP**

Note in Figure 12.6 that inventory buildup is not instantaneous but gradual. So the formula reduces the average inventory and thus the holding cost by the ratio of that buildup.

4. Annual inventory holding cost (or simply holding cost) =

$$\frac{\text{Maximum inventory level}}{2}(H) = \frac{Q}{2}\left[1 - \left(\frac{d}{p}\right)\right]H$$

Using this expression for holding cost and the expression for setup cost developed in the basic EOQ model, we solve for the optimal number of pieces per order by equating setup cost and holding cost:

$$\text{Setup cost} = (D/Q)S \qquad\qquad \text{Holding cost} = \tfrac{1}{2}HQ[1 - (d/p)]$$

Set ordering cost equal to holding cost to obtain $Q_p^*$:

$$\begin{aligned} \frac{D}{Q}S &= \tfrac{1}{2}HQ[1 - (d/p)] \\ Q^2 &= \frac{2DS}{H[1 - (d/p)]} \\ Q_p^* &= \sqrt{\frac{2DS}{H[1 - (d/p)]}} \end{aligned} \qquad (12\text{-}7)$$

Dmitry Kalinovsky/Shutterstock

Each order may require a change in the way a machine or process is set up. Reducing setup time usually means a reduction in setup cost, and reductions in setup costs make smaller batches (lots) economical to produce. Increasingly, setup (and operation) is performed by computer-controlled machines, such as this one, operating from previously written programs.

In Example 8, we use the above equation, $Q_p^*$, to solve for the optimum order or production quantity when inventory is consumed as it is produced.

## Example 8

### A PRODUCTION ORDER QUANTITY MODEL

Nathan Manufacturing, Inc., makes and sells specialty hubcaps for the retail automobile aftermarket. Nathan's forecast for its wire-wheel hubcap is 1,000 units next year, with an average daily demand of 4 units. However, the production process is most efficient at 8 units per day. So the company produces 8 per day but uses only 4 per day. The company wants to solve for the optimum number of units per order. (*Note:* This plant schedules production of this hubcap only as needed, during the 250 days per year the shop operates.)

**APPROACH ▶** Gather the cost data and apply Equation (12-7):

$$\begin{aligned} \text{Annual demand} &= D = 1{,}000 \text{ units} \\ \text{Setup costs} &= S = \$10 \\ \text{Holding cost} &= H = \$0.50 \text{ per unit per year} \\ \text{Daily production rate} &= p = 8 \text{ units daily} \\ \text{Daily demand rate} &= d = 4 \text{ units daily} \end{aligned}$$

**SOLUTION ▶**

$$Q_p^* = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$$

$$Q_p^* = \sqrt{\frac{2(1{,}000)(10)}{0.50[1 - (4/8)]}}$$

$$= \sqrt{\frac{20{,}000}{0.50(1/2)}} = \sqrt{80{,}000} = 282.8 \text{ hubcaps, or } 283 \text{ hubcaps}$$

**INSIGHT ▶** The difference between the production order quantity model and the basic EOQ model is that the effective annual holding cost per unit is reduced in the production order quantity model because the entire order does not arrive at once.

**LEARNING EXERCISE ▶** If Nathan can increase its daily production rate from 8 to 10, how does $Q_p^*$ change? [Answer: $Q_p^* = 258$.]

**RELATED PROBLEMS ▶** 12.18, 12.19, 12.20, 12.30 (12.37 is available in MyOMLab)

**EXCEL OM** Data File **Ch12Ex8.xls** can be found in MyOMLab.

**ACTIVE MODEL 12.2** This example is further illustrated in Active Model 12.2 in MyOMLab.

You may want to compare this solution with the answer in Example 3, which had identical $D$, $S$, and $H$ values. Eliminating the instantaneous-receipt assumption, where $p = 8$ and $d = 4$, resulted in an increase in $Q^*$ from 200 in Example 3 to 283 in Example 8. This increase in $Q^*$ occurred because holding cost dropped from \$.50 to [\$.50 × (1 − $d/p$)], making a larger order quantity optimal. Also note that:

$$d = 4 = \frac{D}{\text{Number of days the plant is in operation}} = \frac{1{,}000}{250}$$

We can also calculate $Q_p^*$ when *annual* data are available. When annual data are used, we can express $Q_p^*$ as:

$$Q_p^* = \sqrt{\frac{2DS}{H\left(1 - \dfrac{\text{Annual demand rate}}{\text{Annual production rate}}\right)}} \qquad (12\text{-}8)$$

## Quantity Discount Models

Quantity discounts appear everywhere—you cannot go into a grocery store without seeing them on nearly every shelf. In fact, researchers have found that *most* companies either offer or receive quantity discounts for at least some of the products that they sell or purchase. A **quantity discount** is simply a reduced price ($P$) for an item when it is purchased in larger quantities. A typical quantity discount schedule appears in Table 12.2. As can be seen in the table, the normal price of the item is \$100. When 120 to 1,499 units are ordered at one time, the price per unit drops to \$98; when the quantity ordered at one time is 1,500 units or more, the price is \$96 per unit. The 120 quantity and the 1,500 quantity are called *price-break quantities* because they represent the first order amount that would lead to a new lower price. As always, management must decide when and how much to order. However, given these quantity discounts, how does the operations manager make these decisions?

**Quantity discount**
A reduced price for items purchased in large quantities.

As with other inventory models, the objective is to minimize total cost. Because the unit cost for the second discount in Table 12.2 is the lowest, you may be tempted to order 1,500 units. Placing an order for that quantity, however, even with the greatest discount price, may not minimize total inventory cost. This is because holding cost increases. Thus, the major trade-off when considering quantity discounts is between *reduced product cost* and *increased holding cost*. When we include the cost of the product, the equation for the total annual inventory cost can be calculated as follows:

$$\text{Total annual cost} = \text{Annual setup (ordering) cost} + \text{Annual holding cost} + \text{Annual product cost},$$

or

**LO 12.6** *Explain* and use the quantity discount model

$$TC = \frac{D}{Q}S + \frac{Q}{2}IP + PD \qquad (12\text{-}9)$$

where $Q$ = Quantity ordered
$D$ = Annual demand in units
$S$ = Setup or ordering cost per order
$P$ = Price per unit
$I$ = Holding cost per unit per year expressed as a percent of price $P$

Note that holding cost is $IP$ instead of $H$ as seen in the regular EOQ model. Because the price of the item is a factor in annual holding cost, we do not assume that the holding cost is a constant when the price per unit changes for each quantity discount. Thus, it is common to express the holding cost as a percent ($I$) of unit price ($P$) when evaluating costs of quantity discount schedules.

The EOQ formula (12-1) is modified for the quantity discount problem as follows:

$$Q^* = \sqrt{\frac{2DS}{IP}} \qquad (12\text{-}10)$$

The solution procedure uses the concept of a *feasible EOQ*. An EOQ is feasible if it lies in the quantity range that leads to the same price $P$ used to compute it in Equation (12-10). For example, suppose that $D = 5{,}200$, $S = \$200$, and $I = 28\%$. Using Table 12.2 and Equation (12-10), the EOQ for the \$96 price equals $\sqrt{2(5{,}200)(200)/[(.28)(96)]} = 278$ units. Because $278 < 1{,}500$ (the price-break quantity needed to receive the \$96 price), the EOQ for the \$96 price is *not feasible*. On the other hand, the EOQ for the \$98 price equals 275 units. This amount is *feasible* because if 275 units were actually ordered, the firm would indeed receive the \$98 purchase price.

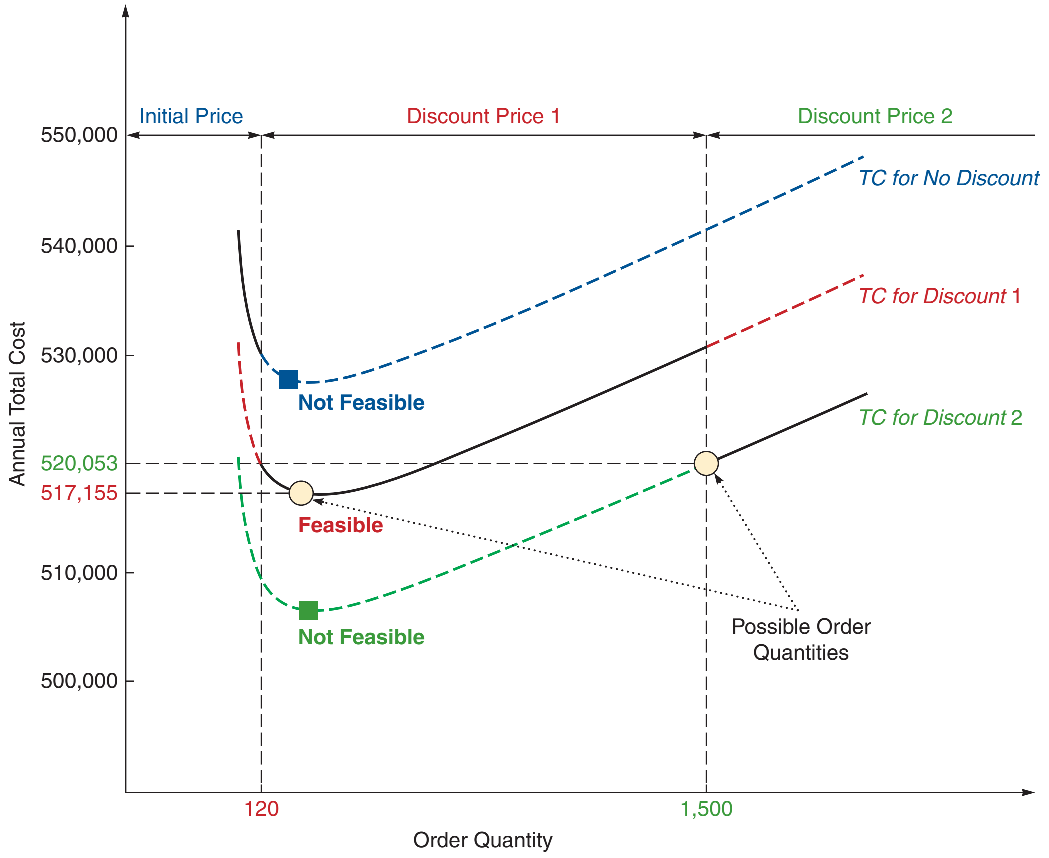
**TABLE 12.2** A Quantity Discount Schedule

| PRICE RANGE | QUANTITY ORDERED | PRICE PER UNIT $P$ |
|---|---|---|
| Initial price | 1–119 | \$100 |
| Discount price 1 | 120–1,499 | \$ 98 |
| Discount price 2 | 1,500 and over | \$ 96 |

Figure 12.7

**EOQs and Possible Best Order Quantities for the Quantity Discount Problem with Three Prices in Table 12.2**

The solid black curves represent the realized total annual setup plus holding plus purchasing cost at the applicable order quantities. The black curve drops to the total cost curve for the next discount level when each price-break quantity is reached.




Now we have to determine the quantity that will minimize the total annual inventory cost. Because there are a few discounts, this process involves two steps. In Step 1, we identify all possible order quantities that could be the best solution. In Step 2, we calculate the total cost of all possible best order quantities, and the least expensive order quantity is selected.

### Solution Procedure

**STEP 1:** Starting with the *lowest* possible purchase price in a quantity discount schedule and working toward the highest price, keep calculating $Q^*$ from Equation (12-10) until the first feasible EOQ is found. The first feasible EOQ is a possible best order quantity, along with all price-break quantities for all *lower* prices.

**STEP 2:** Calculate the total annual cost $TC$ using Equation (12-9) for each of the possible best order quantities determined in Step 1. Select the quantity that has the lowest total cost.

Note that no quantities need to be considered for any prices greater than the first feasible EOQ found in Step 1. This occurs because if an EOQ for a given price is feasible, then the EOQ for any *higher* price *cannot* lead to a lower cost ($TC$ is guaranteed to be higher).

Figure 12.7 provides a graphical illustration of Step 1 using the three price ranges from Table 12.2. In that example, the EOQ for the lowest price is infeasible, but the EOQ for the second-lowest price is feasible. So the EOQ for the second-lowest price, along with the price-break quantity for the lowest price, are the possible best order quantities. Finally, the highest price (no discount) can be ignored because a feasible EOQ has already been found for a lower price.

Example 9 illustrates how the full solution procedure can be applied.

## Example 9

**QUANTITY DISCOUNT MODEL**

Chris Beehner Electronics stocks toy remote control flying drones. Recently, the store has been offered a quantity discount schedule for these drones. This quantity schedule was shown in Table 12.2. Furthermore, setup cost is $200 per order, annual demand is 5,200 units, and annual inventory carrying charge as a percent of cost, $I$, is 28%. What order quantity will minimize the total inventory cost?

**APPROACH ▶** We will follow the two steps just outlined for the quantity discount model.

**SOLUTION ▶** First we calculate the $Q^*$ for the lowest possible price of \$96, as we did earlier:

$$Q^*_{\$96} = \sqrt{\frac{2(5{,}200)(\$200)}{(.28)(\$96)}} = 278 \text{ flying drones per order}$$

Because 278 < 1,500, this EOQ is *infeasible* for the \$96 price. So now we calculate $Q^*$ for the next-higher price of \$98:

$$Q^*_{\$98} = \sqrt{\frac{2(5{,}200)(\$200)}{(.28)(\$98)}} = 275 \text{ flying drones per order}$$

Because 275 is between 120 and 1,499 units, this EOQ is *feasible* for the \$98 price. Thus, the possible best order quantities are 275 (the first feasible EOQ) and 1,500 (the price-break quantity for the lower price of \$96). We need not bother to compute $Q^*$ for the initial price of \$100 because we found a feasible EOQ for a lower price.

Step 2 uses Equation (12-9) to compute the total cost for each of the possible best order quantities. This step is taken with the aid of Table 12.3.

**TABLE 12.3** Total Cost Computations for Chris Beehner Electronics

| ORDER QUANTITY | UNIT PRICE | ANNUAL ORDERING COST | ANNUAL HOLDING COST | ANNUAL PRODUCT COST | TOTAL ANNUAL COST |
|---|---|---|---|---|---|
| 275 | \$98 | \$3,782 | \$ 3,773 | \$509,600 | \$517,155 |
| 1,500 | \$96 | \$ 693 | \$20,160 | \$499,200 | \$520,053 |

Because the total annual cost for 275 units is lower, 275 units should be ordered. The costs for this example are shown in Figure 12.7.

**INSIGHT ▶** Even though Beehner Electronics could save more than \$10,000 in annual product costs, ordering 1,500 units (28.8% of annual demand) at a time would generate even more than that in increased holding costs. So in this example it is not in the store's best interest to order enough to attain the lowest possible purchase price per unit. On the other hand, if the price-break quantity for the \$96 had been 1,000 units rather than 1,500 units, then total annual costs would have been \$513,680, which would have been cheaper than ordering 275 units at \$98.

**LEARNING EXERCISE ▶** Resolve the problem with $D = 2{,}000$, $S = \$5$, $I = 50\%$, discount price 1 = \$99, and discount price 2 = \$98. [Answer: only 20 units should be ordered each time, which is the EOQ at the \$100 price.]

**RELATED PROBLEMS ▶** 12.21–12.28 (12.38–12.40 are available in MyOMLab)

EXCEL OM Data file **Ch12Ex9.xls** can be found in MyOMLab.

In this section we have studied the most popular form of single-purchase quantity discount called the *all-units discount*. In practice, quantity discounts appear in a variety of forms. For example, *incremental quantity discounts* apply only to those units purchased beyond the price-break quantities rather than to all units. *Fixed fees*, such as a fixed shipping and processing cost for a catalog order or a \$5,000 tooling setup cost for any order placed with a manufacturer, encourage buyers to purchase more units at a time. Some discounts are *aggregated* over items or time. *Item aggregation* bases price breaks on total units or dollars purchased. *Time aggregation* applies to total items or dollars spent over a specific time period such as one year. *Truckload discounts*, *buy-one-get-one-free offers*, and *one-time-only sales* also represent types of quantity discounts in that they provide price incentives for buyers to purchase more units at one time. Most purchasing managers deal with some form of quantity discounts on a regular basis.

# Probabilistic Models and Safety Stock

**Probabilistic model**
A statistical model applicable when product demand or any other variable is not known but can be specified by means of a probability distribution.

**Service level**
The probability that demand will not be greater than supply during lead time. It is the complement of the probability of a stockout.

All the inventory models we have discussed so far make the assumption that demand for a product is constant and certain. We now relax this assumption. The following inventory models apply when product demand is not known but can be specified by means of a probability distribution. These types of models are called **probabilistic models**. Probabilistic models are a real-world adjustment because demand and lead time won't always be known and constant.

An important concern of management is maintaining an adequate service level in the face of uncertain demand. The **service level** is the *complement* of the probability of a stockout. For instance, if the probability of a stockout is 0.05, then the service level is .95. Uncertain demand raises the possibility of a stockout. One method of reducing stockouts is to hold extra units in inventory. As we noted earlier such inventory is referred to as safety stock. Safety stock involves adding a number of units as a buffer to the reorder point. As you recall:

$$\text{Reorder point} = \text{ROP} = d \times L$$

where $d$ = Daily demand
$L$ = Order lead time, or number of working days it takes to deliver an order

The inclusion of safety stock ($ss$) changed the expression to:

$$\text{ROP} = d \times L + ss \tag{12-11}$$

The amount of safety stock maintained depends on the cost of incurring a stockout and the cost of holding the extra inventory. Annual stockout cost is computed as follows:

$$\begin{aligned}\text{Annual stockout costs} &= \text{The sum of the units short for each demand level} \\ &\times \text{The probability of that demand level} \times \text{The stockout cost/unit} \\ &\times \text{The number of orders per year}\end{aligned} \tag{12-12}$$

Example 10 illustrates this concept.

## Example 10

### DETERMINING SAFETY STOCK WITH PROBABILISTIC DEMAND AND CONSTANT LEAD TIME

David Rivera Optical has determined that its reorder point for eyeglass frames is 50 ($d \times L$) units. Its carrying cost per frame per year is \$5, and stockout (or lost sale) cost is \$40 per frame. The store has experienced the following probability distribution for inventory demand during the lead time (reorder period). The optimum number of orders per year is six.

| NUMBER OF UNITS | PROBABILITY |
|---|---|
| 30 | .2 |
| 40 | .2 |
| ROP → 50 | .3 |
| 60 | .2 |
| 70 | .1 |
| | 1.0 |

How much safety stock should David Rivera keep on hand?

**APPROACH ▶** The objective is to find the amount of safety stock that minimizes the sum of the additional inventory holding costs and stockout costs. The annual holding cost is simply the holding cost per unit multiplied by the units added to the ROP. For example, a safety stock of 20 frames, which implies that the new ROP, with safety stock, is 70 (= 50 + 20), raises the annual carrying cost by \$5(20) = \$100.

However, computing annual stockout cost is more interesting. For any level of safety stock, stockout cost is the expected cost of stocking out. We can compute it, as in Equation (12-12), by multiplying the number of frames short (Demand – ROP) by the probability of demand at that level, by the stockout cost, by the number of times per year the stockout can occur (which in our case is the number of orders per year). Then we add stockout costs for each possible stockout level for a given ROP.[4]

**SOLUTION ▶** We begin by looking at zero safety stock. For this safety stock, a shortage of 10 frames will occur if demand is 60, and a shortage of 20 frames will occur if the demand is 70. Thus the stockout costs for zero safety stock are:

$$(10 \text{ frames short})(.2)(\$40 \text{ per stockout})(6 \text{ possible stockouts per year}) + (20 \text{ frames short})(.1)(\$40)(6) = \$960$$

The following table summarizes the total costs for each of the three alternatives:

| SAFETY STOCK | ADDITIONAL HOLDING COST | STOCKOUT COST | TOTAL COST |
|---|---|---|---|
| 20 | (20)($5) = $100 | $ 0 | $100 |
| 10 | (10)($5) = $ 50 | (10) (.1) ($40) (6) = $240 | $290 |
| 0 | $ 0 | (10) (.2) ($40) (6) + (20) (.1) ($40) (6) = $960 | $960 |

The safety stock with the lowest total cost is 20 frames. Therefore, this safety stock changes the reorder point to 50 + 20 = 70 frames.

**INSIGHT ▶** The optical company now knows that a safety stock of 20 frames will be the most economical decision.

**LEARNING EXERCISE ▶** David Rivera's holding cost per frame is now estimated to be $20, while the stockout cost is $30 per frame. Does the reorder point change? [Answer: Safety stock = 10 now, with a total cost of $380, which is the lowest of the three. ROP = 60 frames.]

**RELATED PROBLEMS ▶** 12.43, 12.44, 12.45

When it is difficult or impossible to determine the cost of being out of stock, a manager may decide to follow a policy of keeping enough safety stock on hand to meet a prescribed customer service level. For instance, Figure 12.8 shows the use of safety stock when demand (for hospital resuscitation kits) is probabilistic. We see that the safety stock in Figure 12.8 is 16.5 units, and the reorder point is also increased by 16.5.

The manager may want to define the service level as meeting 95% of the demand (or, conversely, having stockouts only 5% of the time). Assuming that demand during lead time (the reorder period) follows a normal curve, only the mean and standard deviation are needed to define the inventory requirements for any given service level. Sales data are usually adequate for computing the mean and standard deviation. Example 11 uses a normal curve with a known mean ($\mu$) and standard deviation ($\sigma$) to determine the reorder point and safety stock necessary

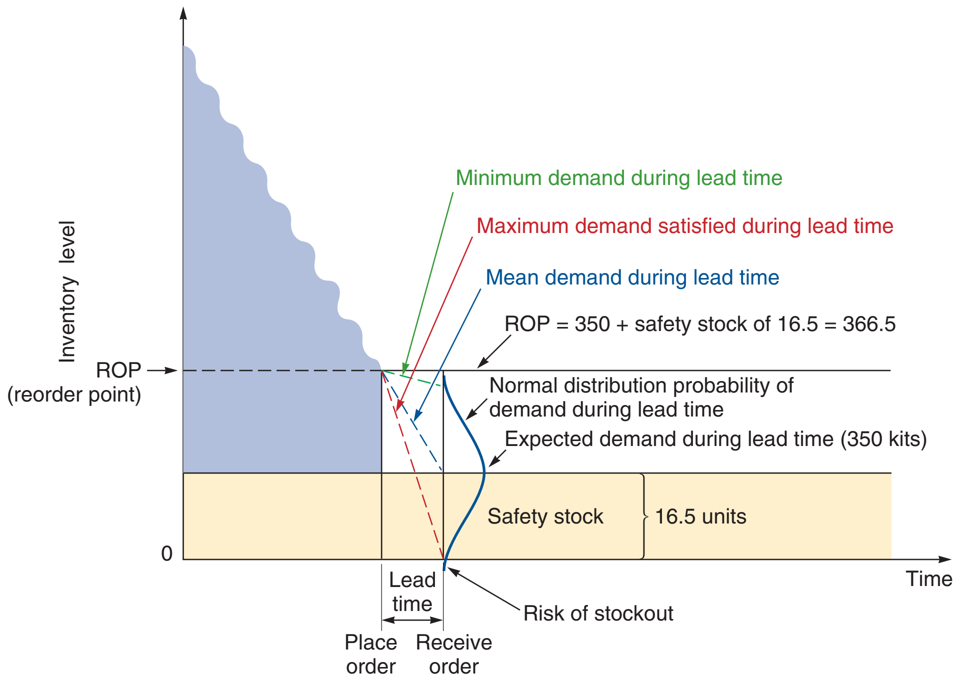



Figure **12.8**

**Probabilistic Demand for a Hospital Item**

Expected number of kits needed during lead time is 350, but for a 95% service level, the reorder point should be raised to 366.5.

for a 95% service level. We use the following formula:

$$ROP = \text{Expected demand during lead time} + Z\sigma_{dLT} \qquad (12\text{-}13)$$

where $Z$ = Number of standard deviations
$\sigma_{dLT}$ = Standard deviation of demand during lead time

## Example 11

### SAFETY STOCK WITH PROBABILISTIC DEMAND

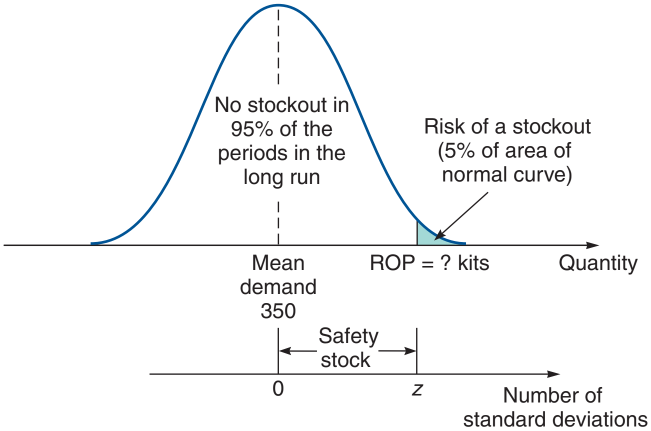
Memphis Regional Hospital stocks a "code blue" resuscitation kit that has a normally distributed demand during the reorder period. The mean (average) demand during the reorder period is 350 kits, and the standard deviation is 10 kits. The hospital administrator wants to follow a policy that results in stockouts only 5% of the time.

(a) What is the appropriate value of $Z$? (b) How much safety stock should the hospital maintain? (c) What reorder point should be used?

**STUDENT TIP** Recall that the service level is 1 minus the risk of a stockout.

**APPROACH ▶** The hospital determines how much inventory is needed to meet the demand 95% of the time. The figure in this example may help you visualize the approach. The data are as follows:

$\mu$ = Mean demand = 350 kits
$\sigma_{dLT}$ = Standard deviation of demand during lead time = 10 kits
$Z$ = Number of standard normal deviations




**SOLUTION ▶**

**a)** We use the properties of a standardized normal curve to get a $Z$-value for an area under the normal curve of .95 (or 1 − .05). Using a normal table (see Appendix I) or the Excel formula =NORMSINV(.95), we find a $Z$-value of 1.645 standard deviations from the mean.

**b)** Because: Safety stock $= x - \mu$

and: $$Z = \frac{x - \mu}{\sigma_{dLT}}$$

then: Safety stock $= Z\sigma_{dLT}$ (12-14)

Solving for safety stock, as in Equation (12-14), gives:

$$\text{Safety stock} = 1.645(10) = 16.5 \text{ kits}$$

This is the situation illustrated in Figure 12.8.

**c)** The reorder point is:

$$ROP = \text{Expected demand during lead time} + \text{Safety stock}$$
$$= 350 \text{ kits} + 16.5 \text{ kits of safety stock} = 366.5, \text{ or } 367 \text{ kits}$$

**INSIGHT ▶** The cost of the inventory policy increases dramatically (exponentially) with an increase in service levels.

**LEARNING EXERCISE ▶** What policy results in stockouts 10% of the time? [Answer: $Z = 1.28$; safety stock = 12.8; ROP = 363 kits.]

**RELATED PROBLEMS ▶** 12.41, 12.42, 12.49 (12.50 is available in MyOMLab)

## Other Probabilistic Models

Equations (12-13) and (12-14) assume that both an estimate of expected demand during lead times and its standard deviation are available. When data on lead time demand are *not* available, the preceding formulas cannot be applied. However, three other models are available. We need to determine which model to use for three situations:

1. Demand is variable and lead time is constant
2. Lead time is variable and demand is constant
3. Both demand and lead time are variable

**LO 12.7** *Understand* service levels and probabilistic inventory models

All three models assume that demand and lead time are independent variables. Note that our examples use days, but weeks can also be used. Let us examine these three situations separately, because a different formula for the ROP is needed for each.

**Demand Is Variable and Lead Time Is Constant** (See Example 12.) When *only the demand is variable*, then:[5]

$$\text{ROP} = (\textit{Average}\text{ daily demand} \times \text{Lead time in days}) + Z\sigma_{dLT} \qquad \text{(12-15)}$$

where $\sigma_{dLT}$ = Standard deviation of demand during lead time = $\sigma_d\sqrt{\text{Lead time}}$
and $\sigma_d$ = Standard deviation of demand per day

### Example 12

**ROP FOR VARIABLE DEMAND AND CONSTANT LEAD TIME**

The *average* daily demand for Lenovo laptop computers at a Circuit Town store is 15, with a standard deviation of 5 units. The lead time is constant at 2 days. Find the reorder point if management wants a 90% service level (i.e., risk stockouts only 10% of the time). How much of this is safety stock?

**APPROACH ▶** Apply Equation (12-15) to the following data:

Average daily demand (normally distributed) = 15

Lead time in days (constant) = 2

Standard deviation of daily demand = $\sigma_d = 5$

Service level = 90%

**SOLUTION ▶** From the normal table (Appendix I) or the Excel formula =NORMSINV(.90), we derive a $Z$-value for 90% of 1.28. Then:

$$\begin{aligned}\text{ROP} &= (15 \text{ units} \times 2 \text{ days}) + Z\sigma_d\sqrt{\text{Lead time}}\\ &= 30 + 1.28(5)(\sqrt{2})\\ &= 30 + 1.28(5)(1.41) = 30 + 9.02 = 39.02 \cong 39\end{aligned}$$

Thus, safety stock is about 9 Lenovo computers.

**INSIGHT ▶** The value of $Z$ depends on the manager's stockout risk level. The smaller the risk, the higher the $Z$.

**LEARNING EXERCISE ▶** If the Circuit Town manager wants a 95% service level, what is the new ROP? [Answer: ROP = 41.63, or 42.]

**RELATED PROBLEM ▶** 12.46

**Lead Time Is Variable and Demand Is Constant** When the demand is constant and *only the lead time is variable*, then:

$$\text{ROP} = (\text{Daily demand} \times \textit{Average}\text{ lead time in days}) + Z \times \text{Daily demand} \times \sigma_{LT} \qquad \text{(12-16)}$$

where $\sigma_{LT}$ = Standard deviation of lead time in days

## Example 13

### ROP FOR CONSTANT DEMAND AND VARIABLE LEAD TIME

The Circuit Town store in Example 12 sells about 10 digital cameras a day (almost a constant quantity). Lead time for camera delivery is normally distributed with a mean time of 6 days and a standard deviation of 1 day. A 98% service level is set. Find the ROP.

**APPROACH ▶** Apply Equation (12-16) to the following data:

Daily demand = 10

Average lead time = 6 days

Standard deviation of lead time = $\sigma_{LT}$ = 1 day

Service level = 98%, so $Z$ (from Appendix I or the Excel formula =NORMSINV(.98)) = 2.055

**SOLUTION ▶** From the equation we get:

$$\begin{aligned} \text{ROP} &= (10 \text{ units} \times 6 \text{ days}) + 2.055(10 \text{ units})(1) \\ &= 60 + 20.55 = 80.55 \end{aligned}$$

The reorder point is about 81 cameras.

**INSIGHT ▶** Note how the very high service level of 98% drives the ROP up.

**LEARNING EXERCISE ▶** If a 90% service level is applied, what does the ROP drop to? [Answer: ROP = 60 + (1.28) (10) (1) = 60 + 12.8 = 72.8 because the $Z$-value is only 1.28.]

**RELATED PROBLEM ▶** 12.47

**Both Demand and Lead Time Are Variable** When both the demand and lead time are variable, the formula for reorder point becomes more complex:[6]

$$\text{ROP} = (\text{Average daily demand} \times \text{Average lead time in days}) + Z\sigma_{dLT} \qquad (12\text{-}17)$$

where

$$\begin{aligned} \sigma_d &= \text{Standard deviation of demand per day} \\ \sigma_{LT} &= \text{Standard deviation of lead time in days} \end{aligned}$$

$$\text{and } \sigma_{dLT} = \sqrt{(\text{Average lead time} \times \sigma_d^2) + (\text{Average daily demand})^2 \sigma_{LT}^2}$$

## Example 14

### ROP FOR VARIABLE DEMAND AND VARIABLE LEAD TIME

The Circuit Town store's most popular item is six-packs of 9-volt batteries. About 150 packs are sold per day, following a normal distribution with a standard deviation of 16 packs. Batteries are ordered from an out-of-state distributor; lead time is normally distributed with an average of 5 days and a standard deviation of 1 day. To maintain a 95% service level, what ROP is appropriate?

**APPROACH ▶** Determine a quantity at which to reorder by applying Equation (12-17) to the following data:

Average daily demand = 150 packs
Standard deviation of demand = $\sigma_d$ = 16 packs
Average lead time = 5 days
Standard deviation of lead time = $\sigma_{LT}$ = 1 day
Service level = 95%, so $Z$ =1.645 (from Appendix I or the Excel formula =NORMSINV(.95))

**SOLUTION ▶** From the equation we compute:

$$\text{ROP} = (150 \text{ packs} \times 5 \text{ days}) + 1.645\,\sigma_{dLT}$$

where $\sigma_{dLT} = \sqrt{(5 \text{ days} \times 16^2) + (150^2 \times 1^2)}$

$$= \sqrt{(5 \times 256) + (22{,}500 \times 1)}$$

$$= \sqrt{1{,}280 + 22{,}500} = \sqrt{23{,}780} \cong 154$$

So ROP = $(150 \times 5) + 1.645(154) \cong 750 + 253 = 1{,}003$ packs

**INSIGHT ▶** When both demand and lead time are variable, the formula looks quite complex. But it is just the result of squaring the standard deviations in Equations (12-15) and (12-16) to get their variances, then summing them, and finally taking the square root.

**LEARNING EXERCISE ▶** For an 80% service level, what is the ROP? [Answer: $Z = .84$ and ROP = 879 packs.]

**RELATED PROBLEM ▶** 12.48

# Single-Period Model

**Single-period inventory model**
A system for ordering items that have little or no value at the end of a sales period (perishables).

A **single-period inventory model** describes a situation in which *one* order is placed for a product. At the end of the sales period, any remaining product has little or no value. This is a typical problem for Christmas trees, seasonal goods, bakery goods, newspapers, and magazines. (Indeed, this inventory issue is often called the "newsstand problem.") In other words, even though items at a newsstand are ordered weekly or daily, they cannot be held over and used as inventory in the next sales period. So our decision is how much to order at the beginning of the period.

Because the exact demand for such seasonal products is never known, we consider a probability distribution related to demand. If the normal distribution is assumed, and we stocked and sold an average (mean) of 100 Christmas trees each season, then there is a 50% chance we would stock out and a 50% chance we would have trees left over. To determine the optimal stocking policy for trees before the season begins, we also need to know the standard deviation and consider these two marginal costs:

$$C_s = \text{Cost of shortage (we underestimated)} = \text{Sales price per unit} - \text{Cost per unit}$$

$$C_o = \text{Cost of overage (we overestimated)} = \text{Cost per unit} - \text{Salvage value per unit (if there is any)}$$

The service level, that is, the probability of *not* stocking out, is set at:

$$\text{Service level} = \frac{C_s}{C_s + C_o} \tag{12-18}$$

Therefore, we should consider increasing our order quantity until the service level is equal to or more than the ratio of $[C_s/(C_s + C_o)]$.

This model, illustrated in Example 15, is used in many service industries, from hotels to airlines to bakeries to clothing retailers.

## Example 15

### SINGLE-PERIOD INVENTORY DECISION

Chris Ellis's newsstand, just outside the Smithsonian subway station in Washington, DC, usually sells 120 copies of the *Washington Post* each day. Chris believes the sale of the *Post* is normally distributed, with a standard deviation of 15 papers. He pays 70 cents for each paper, which sells for \$1.25. The *Post* gives him a 30-cent credit for each unsold paper. He wants to determine how many papers he should order each day and the stockout risk for that quantity.

**APPROACH ▶** Chris's data are as follows:

$$C_s = \text{cost of shortage} = \$1.25 - \$.70 = \$.55$$

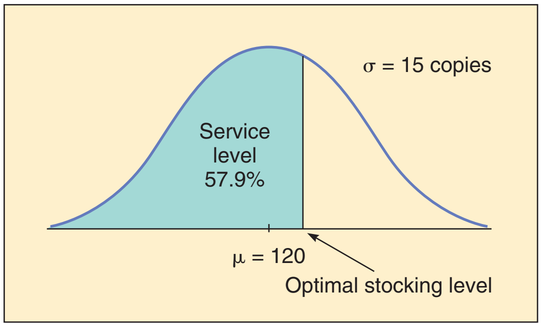
$$C_o = \text{cost of overage} = \$.70 - \$.30 \text{ (salvage value)} = \$.40$$

Chris will apply Equation (12-18) and the normal table, using $\mu = 120$ and $\sigma = 15$.

**SOLUTION ▶**

**a)** Service level $= \dfrac{C_s}{C_s + C_o} = \dfrac{.55}{.55 + .40} = \dfrac{.55}{.95} = .579$

**b)** Chris needs to find the $Z$ score for his normal distribution that yields a probability of .579.




So 57.9% of the area under the normal curve must be to the left of the optimal stocking level.

**c)** Using Appendix I or the Excel formula =NORMSINV(.578), for an area of .578, the $Z$ value ≅ .195.

$$\text{Then, the optimal stocking level} = 120 \text{ copies} + (.195)(\sigma)$$
$$= 120 + (.195)(15) = 120 + 3 = 123 \text{ papers}$$

The stockout risk if Chris orders 123 copies of the *Post* each day is 1 − Service level = 1 − .578 = .422 = 42.2%.

**INSIGHT ▶** If the service level is ever under .50, Chris should order fewer than 120 copies per day.

**LEARNING EXERCISE ▶** How does Chris's decision change if the *Post* changes its policy and offers *no credit* for unsold papers, a policy many publishers are adopting?

[Answer: Service level = .44, Z = −.15. Therefore, stock 120 + (−.15)(15) = 117.75, or 118 papers.]

**RELATED PROBLEMS ▶** 12.51, 12.52, 12.53

# Fixed-Period (*P*) Systems

**Fixed-quantity (*Q*) system**
An ordering system with the same order amount each time.

**Perpetual inventory system**
A system that keeps track of each withdrawal or addition to inventory continuously, so records are always current.

**Fixed-period (*P*) system**
A system in which inventory orders are made at regular time intervals.

The inventory models that we have considered so far are **fixed-quantity**, or ***Q*, systems**. That is, the same fixed amount is added to inventory every time an order for an item is placed. We saw that orders are event triggered. When inventory decreases to the reorder point (ROP), a new order for $Q$ units is placed.

To use the fixed-quantity model, inventory must be continuously monitored.[7] This requires a **perpetual inventory system**. Every time an item is added to or withdrawn from inventory, records must be updated to determine whether the ROP has been reached. In a **fixed-period system** (also called a periodic review, or ***P* system**), on the other hand, inventory is ordered at the end of a given period. Then, and only then, is on-hand inventory counted. Only the amount necessary to bring total inventory up to a prespecified target level ($T$) is ordered. Figure 12.9 illustrates this concept.

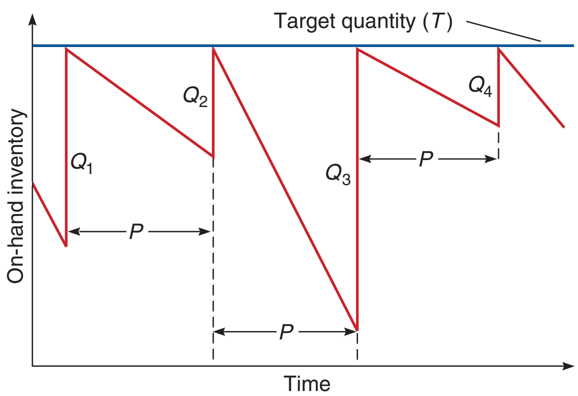



Figure **12.9**

**Inventory Level in a Fixed-Period (*P*) System**

Various amounts ($Q_1$, $Q_2$, $Q_3$, etc.) are ordered at regular time intervals ($P$) based on the quantity necessary to bring inventory up to the target quantity ($T$).

Fixed-period systems have several of the same assumptions as the basic EOQ fixed-quantity system:

- The only relevant costs are the ordering and holding costs.
- Lead times are known and constant.
- Items are independent of one another.

The downward-sloped lines in Figure 12.9 again represent on-hand inventory levels. But now, when the time between orders ($P$) passes, we place an order to raise inventory up to the target quantity ($T$).

The amount ordered during the first period may be $Q_1$, the second period $Q_2$, and so on. The $Q_i$ value is the difference between current on-hand inventory and the target inventory level.

**STUDENT TIP** A fixed-period model potentially orders a different quantity each time.

The advantage of the fixed-period system is that there is no physical count of inventory items after an item is withdrawn—this occurs only when the time for the next review comes up. This procedure is also convenient administratively.

A fixed-period ($P$) system is appropriate when vendors make routine (i.e., at fixed-time interval) visits to customers to take fresh orders or when purchasers want to combine orders to save ordering and transportation costs (therefore, they will have the same review period for similar inventory items). For example, a vending machine company may come to refill its machines every Tuesday. This is also the case at Anheuser-Busch, whose sales reps may visit a store every 5 days.

The disadvantage of the $P$ system is that because there is no tally of inventory during the review period, there is the possibility of a stockout during this time. This scenario is possible if a large order draws the inventory level down to zero right after an order is placed. Therefore, a higher level of safety stock (as compared to a fixed-quantity system) needs to be maintained to provide protection against stockout during both the time between reviews and the lead time.

# Summary

Inventory represents a major investment for many firms. This investment is often larger than it should be because firms find it easier to have "just-in-case" inventory rather than "just-in-time" inventory. Inventories are of four types: Raw material and purchased components, Work-in-process, Maintenance, repair, and operating (MRO), and Finished goods.

In this chapter, we discussed independent inventory, ABC analysis, record accuracy, cycle counting, and inventory models used to control independent demands. The EOQ model, production order quantity model, and quantity discount model can all be solved using Excel, Excel OM, or POM for Windows software.

## Key Terms

Raw material inventory (p. 528)
Work-in-process (WIP) inventory (p. 528)
Maintenance/repair/operating (MRO) inventory (p. 528)
Finished-goods inventory (p. 529)
ABC analysis (p. 529)
Cycle counting (p. 531)
Shrinkage (p. 532)
Pilferage (p. 532)
Holding cost (p. 533)
Ordering cost (p. 533)
Setup cost (p. 534)
Setup time (p. 534)
Economic order quantity (EOQ) model (p. 534)
Robust (p. 538)
Lead time (p. 539)
Reorder point (ROP) (p. 539)
Safety stock (*ss*) (p. 539)
Production order quantity model (p. 540)
Quantity discount (p. 543)
Probabilistic model (p. 546)
Service level (p. 546)
Single-period inventory model (p. 551)
Fixed-quantity ($Q$) system (p. 552)
Perpetual inventory system (p. 552)
Fixed-period ($P$) system (p. 552)

## Ethical Dilemma

In cancer therapy some of the new drugs are extremely expensive, in addition those drugs might also have a short shelf lifetime. Even though the numbers of patients that can be treated with those drugs are quite few, the cost is quite significant.

The lead-time from order to delivery can be long and in worst cases life might be lost whilst waiting for the drug. The University Hospital in Gothenburg is struggling with a limited budget and fast increasing costs for drugs and has a need for a strategy for those new drugs.

As operational responsible at the University Hospital in Gothenburg your responsibility is to set the stock level for all products. How would you discuss and act? Is this an operational decision?

## Discussion Questions

1. Describe the four types of inventory.
2. With the advent of low-cost computing, do you see alternatives to the popular ABC classifications?
3. What is the implication of the Pareto principle?
4. Identify and explain the types of costs that are involved in an inventory system.
5. Explain the major assumptions of the basic EOQ model.
6. Which part of the cost in the in the EOQ is a linear function of the order quantity?
7. Explain why it is not necessary to include product cost (price or price times quantity) in the EOQ model, but the quantity discount model requires this information.

8. What are the advantages of cycle counting?
9. Even if we have substantial uncertainty in the parameters in the EOQ-model, it is still quite a useful model. Why?
10. What shall be considered if we shall increase the order quantity from the optimal to a discount point?
11. What is meant by "service level"?
12. Explain the following: All things being equal, the production order quantity will be larger than the economic order quantity.
13. Suggest how we could set the time between deliveries in a fixed period system
14. Explain what is meant by the expression "robust model." Specifically, what would you tell a manager who exclaimed, "Uh-oh, we're in trouble! The calculated EOQ is wrong; actual demand is 10% greater than estimated."
15. What is "safety stock"? What does safety stock provide safety against?
16. When demand is not constant, the reorder point is a function of what four parameters?
17. How are inventory levels monitored in retail stores?
18. Why would it be advantageous to supply a regional warehouse from the central warehouse using a fixed period system?

## Using Software to Solve Inventory Problems

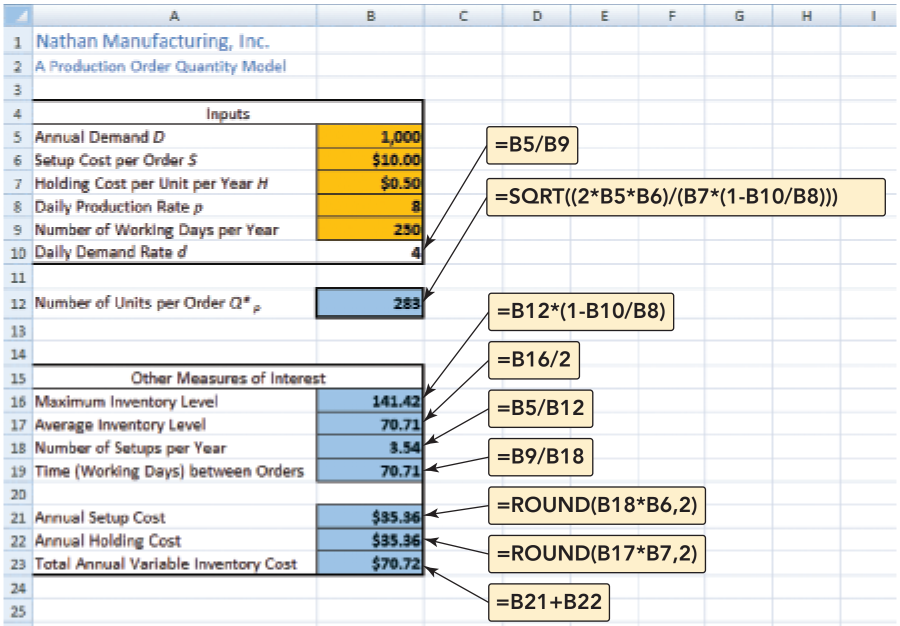
This section presents three ways to solve inventory problems with computer software. First, you can create your own Excel spreadsheets. Second, you can use the Excel OM software that comes free with this text. Third, POM for Windows, also free with this text, can solve all problems marked with a **P**.

### CREATING YOUR OWN EXCEL SPREADSHEETS

Program 12.1 illustrates how you can make an Excel model to solve Example 8, which is a production order quantity model.

Program **12.1**

**Using Excel for a Production Model, with Data from Example 8**

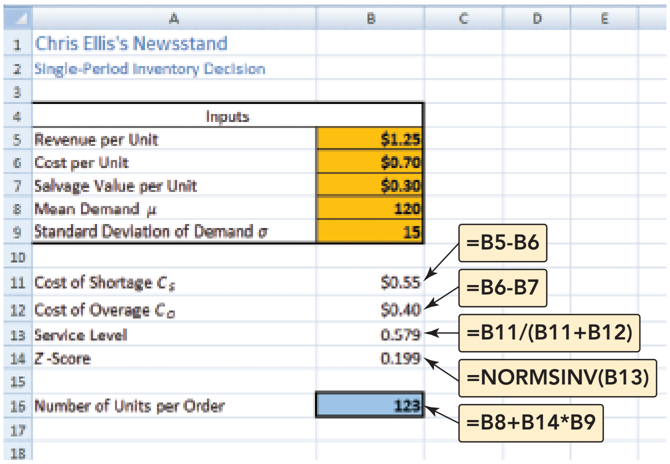


| | A | B | |
|---|---|---|---|
| 1 | Nathan Manufacturing, Inc. | | |
| 2 | A Production Order Quantity Model | | |
| 3 | | | |
| 4 | Inputs | | |
| 5 | Annual Demand $D$ | 1,000 | |
| 6 | Setup Cost per Order $S$ | $10.00 | |
| 7 | Holding Cost per Unit per Year $H$ | $0.50 | |
| 8 | Daily Production Rate $p$ | 8 | |
| 9 | Number of Working Days per Year | 250 | |
| 10 | Daily Demand Rate $d$ | 4 | =B5/B9 |
| 11 | | | |
| 12 | Number of Units per Order $Q^*_p$ | 283 | =SQRT((2*B5*B6)/(B7*(1-B10/B8))) |
| 13 | | | |
| 14 | | | |
| 15 | Other Measures of Interest | | |
| 16 | Maximum Inventory Level | 141.42 | =B12*(1-B10/B8) |
| 17 | Average Inventory Level | 70.71 | =B16/2 |
| 18 | Number of Setups per Year | 3.54 | =B5/B12 |
| 19 | Time (Working Days) between Orders | 70.71 | =B9/B18 |
| 20 | | | |
| 21 | Annual Setup Cost | $35.36 | =ROUND(B18*B6,2) |
| 22 | Annual Holding Cost | $35.36 | =ROUND(B17*B7,2) |
| 23 | Total Annual Variable Inventory Cost | $70.72 | =B21+B22 |
| 24 | | | |
| 25 | | | |

Program 12.2 illustrates how you can make an Excel model to solve Example 15, which is a single-period inventory model.

Program **12.2**

**Using Excel for a Single-Period Inventory Model, with Data from Example 15**

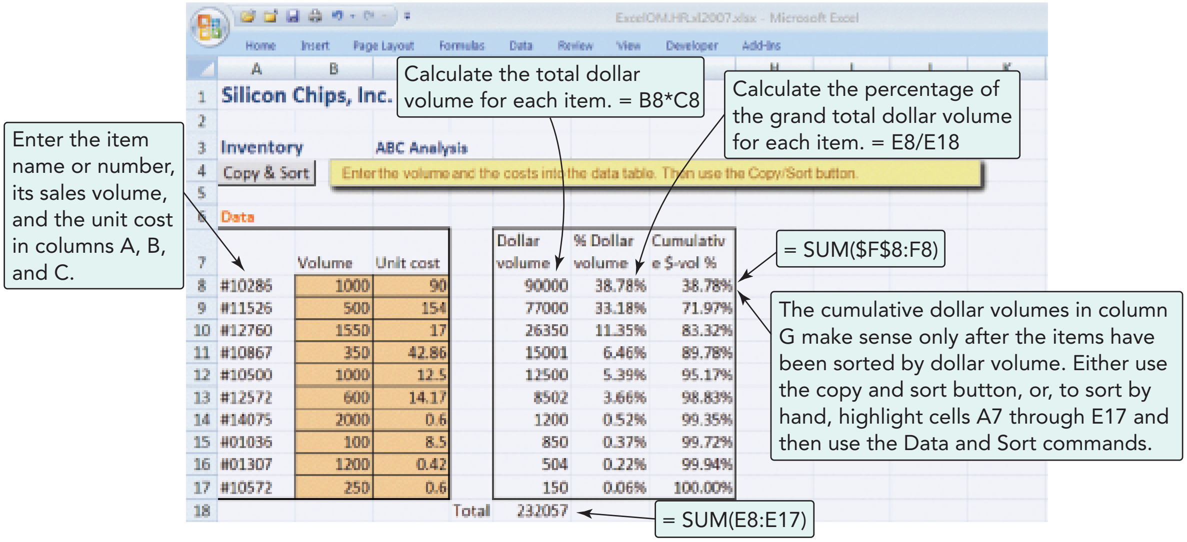


| | A | B | |
|---|---|---|---|
| 1 | Chris Ellis's Newsstand | | |
| 2 | Single-Period Inventory Decision | | |
| 3 | | | |
| 4 | Inputs | | |
| 5 | Revenue per Unit | $1.25 | |
| 6 | Cost per Unit | $0.70 | |
| 7 | Salvage Value per Unit | $0.30 | |
| 8 | Mean Demand $\mu$ | 120 | |
| 9 | Standard Deviation of Demand $\sigma$ | 15 | |
| 10 | | | |
| 11 | Cost of Shortage $C_s$ | $0.55 | =B5-B6 |
| 12 | Cost of Overage $C_o$ | $0.40 | =B6-B7 |
| 13 | Service Level | 0.579 | =B11/(B11+B12) |
| 14 | $Z$-Score | 0.199 | =NORMSINV(B13) |
| 15 | | | |
| 16 | Number of Units per Order | 123 | =B8+B14*B9 |
| 17 | | | |
| 18 | | | |

### USING EXCEL OM

Excel OM allows us to easily model inventory problems ranging from ABC analysis, to the basic EOQ model, to the production model, to quantity discount situations.

Program 12.3 shows the input data, selected formulas, and results for an ABC analysis, using data from Example 1. After the data are entered, we use the *Data* and *Sort* Excel commands to rank the items from largest to smallest dollar volumes.




**Program 12.3**

**Using Excel OM for an ABC Analysis, with Data from Example 1**

### USING POM FOR WINDOWS

The POM for Windows Inventory module can also solve the entire EOQ family of problems. Please refer to Appendix IV for further details.

## Solved Problems

Virtual Office Hours help is available in MyOMLab.

### SOLVED PROBLEM 12.1

David Alexander has compiled the following table of six items in inventory at Angelo Products, along with the unit cost and the annual demand in units:

| IDENTIFICATION CODE | UNIT COST ($) | ANNUAL DEMAND (UNITS) |
|---|---|---|
| XX1 | 5.84 | 1,200 |
| B66 | 5.40 | 1,110 |
| 3CPO | 1.12 | 896 |
| 33CP | 74.54 | 1,104 |
| R2D2 | 2.00 | 1,110 |
| RMS | 2.08 | 961 |

Use ABC analysis to determine which item(s) should be carefully controlled using a quantitative inventory technique and which item(s) should not be closely controlled.

### SOLUTION

The item that needs strict control is 33CP, so it is an A item. Items that do not need to be strictly controlled are 3CPO, R2D2, and RMS; these are C items. The B items will be XX1 and B66.

| CODE | ANNUAL DOLLAR VOLUME = UNIT COST × DEMAND |
|---|---|
| XX1 | $ 7,008.00 |
| B66 | $ 5,994.00 |
| 3CPO | $ 1,003.52 |
| 33CP | $82,292.16 |
| R2D2 | $ 2,220.00 |
| RMS | $ 1,998.88 |

Total cost = $100,516.56
70% of total cost = $70,347.92

### SOLVED PROBLEM 12.2

The Warren W. Fisher Computer Corporation purchases 8,000 transistors each year as components in minicomputers. The unit cost of each transistor is $10, and the cost of carrying one transistor in inventory for a year is $3. Ordering cost is $30 per order.

What are (a) the optimal order quantity, (b) the expected number of orders placed each year, and (c) the expected time between orders? Assume that Fisher operates on a 200-day working year.

**SOLUTION**

a) $Q^* = \sqrt{\dfrac{2DS}{H}} = \sqrt{\dfrac{2(8{,}000)(30)}{3}} = 400$ units

b) $N = \dfrac{D}{Q^*} = \dfrac{8{,}000}{400} = 20$ orders

c) Time between orders $= T = \dfrac{\text{Number of working days}}{N} = \dfrac{200}{20} = 10$ working days

With 20 orders placed each year, an order for 400 transistors is placed every 10 working days.

## SOLVED PROBLEM 12.3

Annual demand for notebook binders at Meyer's Stationery Shop is 10,000 units. Brad Meyer operates his business 300 days per year and finds that deliveries from his supplier generally take 5 working days. Calculate the reorder point for the notebook binders.

**SOLUTION**

$$L = 5 \text{ days}$$

$$d = \frac{10{,}000}{300} = 33.3 \text{ units per day}$$

$$\text{ROP} = d \times L = (33.3 \text{ units per day})(5 \text{ days}) = 166.7 \text{ units}$$

Thus, Brad should reorder when his stock reaches 167 units.

## SOLVED PROBLEM 12.4

Leonard Presby, Inc., has an annual demand rate of 1,000 units but can produce at an average production rate of 2,000 units. Setup cost is \$10; carrying cost is \$1. What is the optimal number of units to be produced each time?

**SOLUTION**

$$Q_p^* = \sqrt{\frac{2DS}{H\left(1 - \dfrac{\text{Annual demand rate}}{\text{Annual production rate}}\right)}} = \sqrt{\frac{2(1{,}000)(10)}{1[1 - (1{,}000/2{,}000)]}}$$

$$= \sqrt{\frac{20{,}000}{1/2}} = \sqrt{40{,}000} = 200 \text{ units}$$

## SOLVED PROBLEM 12.5

Whole Nature Foods sells a gluten-free product for which the annual demand is 5,000 boxes. At the moment, it is paying \$6.40 for each box; carrying cost is 25% of the unit cost; ordering costs are \$25. A new supplier has offered to sell the same item for \$6.00 if Whole Nature Foods buys at least 3,000 boxes per order. Should the firm stick with the old supplier, or take advantage of the new quantity discount?

**SOLUTION**

Step 1, under the lowest possible price of \$6.00 per box:

Economic order quantity, using Equation (12-10):

$$Q^*_{\$6.00} = \sqrt{\frac{2(5{,}000)(25)}{(0.25)(6.00)}}$$

$$= 408.25, \text{ or } 408 \text{ boxes}$$

Because 408 < 3,000, this EOQ is *infeasible* for the \$6.00 price. So now we calculate $Q^*$ for the next-higher price of \$6.40, which equals 395 boxes (and is feasible). Thus, the best possible order quantities are 395 (the first feasible EOQ) and 3,000 (the price-break quantity for the lower price of \$6.00).

Step 2 uses Equation (12-9) to compute the total cost for both of the possible best order quantities:

$$TC_{395} = \frac{5{,}000}{395}(\$25) + \frac{395}{2}(0.25)(\$6.40) + \$6.40(5{,}000)$$
$$= \$316 + \$316 + \$32{,}000$$
$$= \$32{,}632$$

And under the quantity discount price of \$6.00 per box:

$$TC_{3{,}000} = \frac{5{,}000}{3{,}000}(\$25) + \frac{3{,}000}{2}(0.25)(\$6.00) + \$6.00(5{,}000)$$
$$= \$42 + \$2{,}250 + \$30{,}000$$
$$= \$32{,}292$$

Therefore, the new supplier with which Whole Nature Foods would incur a total cost of \$32,292 is preferable, but not by a large amount. If buying 3,000 boxes at a time raises problems of storage or freshness, the company may very well wish to stay with the current supplier.

## SOLVED PROBLEM 12.6

Children's art sets are ordered once each year by Ashok Kumar, Inc., and the reorder point, without safety stock ($dL$), is 100 art sets. Inventory carrying cost is \$10 per set per year, and the cost of a stockout is \$50 per set per year. Given the following demand probabilities during the lead time, how much safety stock should be carried?

| DEMAND DURING LEAD TIME | PROBABILITY |
|---|---|
| 0 | .1 |
| 50 | .2 |
| ROP → 100 | .4 |
| 150 | .2 |
| 200 | .1 |
| | 1.0 |

**SOLUTION**

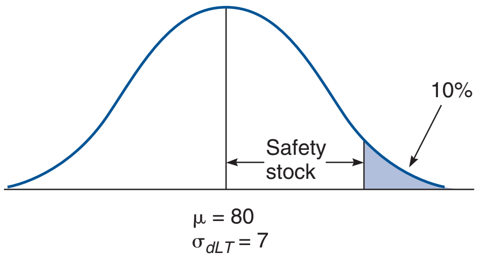
| | INCREMENTAL COSTS | | |
|---|---|---|---|
| SAFETY STOCK | CARRYING COST | STOCKOUT COST | TOTAL COST |
| 0 | 0 | $50 \times (50 \times 0.2 + 100 \times 0.1) = 1{,}000$ | \$1,000 |
| 50 | $50 \times 10 = 500$ | $50 \times (0.1 \times 50) = 250$ | 750 |
| 100 | $100 \times 10 = 1{,}000$ | 0 | 1,000 |

The safety stock that minimizes total incremental cost is 50 sets. The reorder point then becomes 100 sets + 50 sets, or 150 sets.

## SOLVED PROBLEM 12.7

What safety stock should Ron Satterfield Corporation maintain if mean sales are 80 during the reorder period, the standard deviation is 7, and Ron can tolerate stockouts 10% of the time?

**SOLUTION**




From Appendix I, $Z$ at an area of .9 (or $1 - .10$) = 1.28, and Equation (12-14):

$$\text{Safety stock} = Z\sigma_{dLT}$$
$$= 1.28(7) = 8.96 \text{ units, or } 9 \text{ units}$$

## SOLVED PROBLEM 12.8

The daily demand for 52" flat-screen TVs at Sarah's Discount Emporium is normally distributed, with an average of 5 and a standard deviation of 2 units. The lead time for receiving a shipment of new TVs is 10 days and is fairly constant. Determine the reorder point and safety stock for a 95% service level.

**SOLUTION**

The ROP for this variable demand and constant lead time model uses Equation (12-15):

$$\text{ROP} = (\text{Average daily demand} \times \text{Lead time in days}) + Z\sigma_{dLT}$$

where $\sigma_{dLT} = \sigma_d\sqrt{\text{Lead time}}$

So, with $Z = 1.645$,

$$\text{ROP} = (5 \times 10) + 1.645(2)\sqrt{10}$$
$$= 50 + 10.4 = 60.4 \cong 60 \text{ TVs, or rounded up to 61 TVs}$$

The safety stock is 10.4, which can be rounded up to 11 TVs.

## SOLVED PROBLEM 12.9

The demand at Arnold Palmer Hospital for a specialized surgery pack is 60 per week, virtually every week. The lead time from McKesson, its main supplier, is normally distributed, with a mean of 6 weeks for this product and a standard deviation of 2 weeks. A 90% weekly service level is desired. Find the ROP.

**SOLUTION**

Here the demand is constant and lead time is variable, with data given in weeks, not days. We apply Equation (12-16):

$$\text{ROP} = (\text{Weekly demand} \times \text{Average lead time in weeks}) + Z\,(\text{Weekly demand})\,\sigma_{LT}$$

where $\sigma_{LT}$ = standard deviation of lead time in weeks = 2

So, with $Z = 1.28$, for a 90% service level:

$$\text{ROP} = (60 \times 6) + 1.28(60)(2)$$
$$= 360 + 153.6 = 513.6 \cong 514 \text{ surgery packs}$$

## Problems

*Note:* Px means the problem may be solved with POM for Windows and/or Excel OM.

### Problems 12.1–12.6 relate to Managing Inventory

• • **12.1** L. Houts Plastics is a large manufacturer of injection-molded plastics in North Carolina. An investigation of the company's manufacturing facility in Charlotte yields the information presented in the table below. How would the plant classify these items according to an ABC classification system? Px

**L. Houts Plastics' Charlotte Inventory Levels**

| ITEM CODE # | AVERAGE INVENTORY (UNITS) | VALUE ($/UNIT) |
|---|---|---|
| 1289 | 400 | 3.75 |
| 2347 | 300 | 4.00 |
| 2349 | 120 | 2.50 |
| 2363 | 75 | 1.50 |
| 2394 | 60 | 1.75 |
| 2395 | 30 | 2.00 |
| 6782 | 20 | 1.15 |
| 7844 | 12 | 2.05 |
| 8210 | 8 | 1.80 |
| 8310 | 7 | 2.00 |
| 9111 | 6 | 3.00 |

• • **12.2** Boreki Enterprises has the following 10 items in inventory. Theodore Boreki asks you, a recent OM graduate, to divide these items into ABC classifications.

| ITEM | ANNUAL DEMAND | COST/UNIT |
|---|---|---|
| A2 | 3,000 | $ 50 |
| B8 | 4,000 | 12 |
| C7 | 1,500 | 45 |
| D1 | 6,000 | 10 |
| E9 | 1,000 | 20 |
| F3 | 500 | 500 |
| G2 | 300 | 1,500 |
| H2 | 600 | 20 |
| I5 | 1,750 | 10 |
| J8 | 2,500 | 5 |

**a)** Develop an ABC classification system for the 10 items.
**b)** How can Boreki use this information?
**c)** Boreki reviews the classification and then places item A2 into the A category. Why might he do so? Px

• • **12.3** Jean-Marie Bourjolly's restaurant has the following inventory items that it orders on a weekly basis:

| INVENTORY ITEM | $ VALUE/CASE | # ORDERED/WEEK |
|---|---|---|
| Ribeye steak | 135 | 3 |
| Lobster tail | 245 | 3 |
| Pasta | 23 | 12 |
| Salt | 3 | 2 |
| Napkins | 12 | 2 |
| Tomato sauce | 23 | 11 |
| French fries | 43 | 32 |
| Pepper | 3 | 3 |
| Garlic powder | 11 | 3 |
| Trash can liners | 12 | 3 |
| Table cloths | 32 | 5 |
| Fish filets | 143 | 10 |
| Prime rib roasts | 166 | 6 |
| Oil | 28 | 2 |
| Lettuce (case) | 35 | 24 |
| Chickens | 75 | 14 |
| Order pads | 12 | 2 |
| Eggs (case) | 22 | 7 |
| Bacon | 56 | 5 |
| Sugar | 4 | 2 |

*cont'd*

*(cont'd)*

**a)** Which is the most expensive item, using annual dollar volume?
**b)** Which are C items?
**c)** What is the annual dollar volume for all 20 items? Px

• **12.4** The South African company AFROS sells safety equipment to from their central warehouse in Johannesburg. AFROS apply an annual counting of the inventories, approximately 55.000 articles.

The supply manager is not satisfied with the record accuracy and has decided to increase the stock-counting. Maximum 1.100 articles can be counted per day. 20% of the articles represent 80% of the inventory value, A-items. Her decision is to introduce cyclic counting whereA-items should be counted once per month, B-items once every third month and C-items once per 6 months.

What is maximum number of articles that can be classified as B-items? Assume 20 working days per month.

*Additional problems* **12.5–12.6** *are available in* MyOMLab.

### Problems 12.7–12.40 relate to Inventory Models for Independent Demand

• **12.7** William Beville's computer training school, in Richmond, stocks workbooks with the following characteristics:

$$\text{Demand } D = 19{,}500 \text{ units/year}$$
$$\text{Ordering cost } S = \$25/\text{order}$$
$$\text{Holding cost } H = \$4/\text{unit/year}$$

**a)** Calculate the EOQ for the workbooks.
**b)** What are the annual holding costs for the workbooks?
**c)** What are the annual ordering costs? Px

• **12.8** If $D = 8{,}000$ per month, $S = \$45$ per order, and $H = \$2$ per unit per month,
**a)** What is the economic order quantity?
**b)** How does your answer change if the holding cost doubles?
**c)** What if the holding cost drops in half? Px

• • **12.9** The small boatyard WhiteKayaks are building epoxy-kayaks, last year they built 156. Each of those requires 12 kg of epoxy, purchased in 6 kg barrels at $170 each,freight cost is $18 per shipment plus $1 per kg. The annual carrying cost is estimated to 20% of the value.
**a)** If the purchase policy is to buy 20 barrels each time, how much time can be spent on administrative work if the order-quantity should be considered optimal.The boat-builder costs $25/h
**b)** If time spent is 30 minutes, what would be the optimal order-quantity?

• **12.10** Matthew Liotine's Dream Store sells beds and assorted supplies. His best-selling bed has an annual demand of 400 units. Ordering cost is $40; holding cost is $5 per unit per year.

**a)** To minimize the total cost, how many units should be ordered each time an order is placed?
**b)** If the holding cost per unit was $6 instead of $5, what would be the optimal order quantity? PX

• **12.11** Southeastern Bell stocks a certain switch connector at its central warehouse for supplying field service offices. The yearly demand for these connectors is 15,000 units. Southeastern estimates its annual holding cost for this item to be $25 per unit. The cost to place and process an order from the supplier is $75. The company operates 300 days per year, and the lead time to receive an order from the supplier is 2 working days.
**a)** Find the economic order quantity.
**b)** Find the annual holding costs.
**c)** Find the annual ordering costs.
**d)** What is the reorder point? PX

• **12.12** Lead time for one of your fastest-moving products is 21 days. Demand during this period averages 100 units per day.
**a)** What would be an appropriate reorder point?
**b)** How does your answer change if demand during lead time doubles?
**c)** How does your answer change if demand during lead time drops in half?

• **12.13** Atech Service sells professional cleaning machines and scouring agents. They deliver the scouring agents to customers on a fixed weekly schedule.

The annual demand is 1500 bottles; each with a value of $10. The fixed cost for each order placed at the supplier is $100. The leadtime is 2 weeks.
**a)** Assuming that the holding cost is $10/year and bottle, calculate the economical ordersize?
**b)** Calculate the reorder point

•• **12.14** Thomas Kratzer is the purchasing manager for the headquarters of a large insurance company chain with a central inventory operation. Thomas's fastest-moving inventory item has a demand of 6,000 units per year. The cost of each unit is $100, and the inventory carrying cost is $10 per unit per year. The average ordering cost is $30 per order. It takes about 5 days for an order to arrive, and the demand for 1 week is 120 units. (This is a corporate operation, and there are 250 working days per year.)
**a)** What is the EOQ?
**b)** What is the average inventory if the EOQ is used?
**c)** What is the optimal number of orders per year?
**d)** What is the optimal number of days in between any two orders?
**e)** What is the annual cost of ordering and holding inventory?
**f)** What is the total annual inventory cost, including the cost of the 6,000 units? PX

•• **12.15** Joe Henry's machine shop uses 2,500 brackets during the course of a year. These brackets are purchased from a supplier 90 miles away. The following information is known about the brackets:

| | |
|---|---|
| Annual demand: | 2,500 |
| Holding cost per bracket per year: | $1.50 |
| Order cost per order: | $18.75 |
| Lead time: | 2 days |
| Working days per year: | 250 |

**a)** Given the above information, what would be the economic order quantity (EOQ)?
**b)** Given the EOQ, what would be the average inventory? What would be the annual inventory holding cost?
**c)** Given the EOQ, how many orders would be made each year? What would be the annual order cost?
**d)** Given the EOQ, what is the total annual cost of managing the inventory?
**e)** What is the time between orders?
**f)** What is the reorder point (ROP)? PX

•• **12.16** Abey Kuruvilla, of Parkside Plumbing, uses 1,200 of a certain spare part that costs $25 for each order, with an annual holding cost of $24.
**a)** Calculate the total cost for order sizes of 25, 40, 50, 60, and 100.
**b)** Identify the economic order quantity and consider the implications for making an error in calculating economic order quantity. PX

••• **12.17** M. Cotteleer Electronics supplies microcomputer circuitry to a company that incorporates microprocessors into refrigerators and other home appliances. One of the components has an annual demand of 250 units, and this is constant throughout the year. Carrying cost is estimated to be $1 per unit per year, and the ordering (setup) cost is $20 per order.
**a)** To minimize cost, how many units should be ordered each time an order is placed?
**b)** How many orders per year are needed with the optimal policy?
**c)** What is the average inventory if costs are minimized?
**d)** Suppose that the ordering (setup) cost is not $20, and Cotteleer has been ordering 150 units each time an order is placed. For this order policy (of $Q = 150$) to be optimal, determine what the ordering (setup) cost would have to be. PX

•• **12.18** Race One Motors is an Indonesian car manufacturer. At its largest manufacturing facility, in Jakarta, the company produces subcomponents at a rate of 300 per day, and it uses these subcomponents at a rate of 12,500 per year (of 250 working days). Holding costs are $2 per item per year, and ordering (setup) costs are $30 per order.
**a)** What is the economic production quantity?
**b)** How many production runs per year will be made?
**c)** What will be the maximum inventory level?
**d)** What percentage of time will the facility be producing components?
**e)** What is the annual cost of ordering and holding inventory? PX

•• **12.19** Radovilsky Manufacturing Company, in Hayward, California, makes flashing lights for toys. The company operates its production facility 300 days per year. It has orders for about 12,000 flashing lights per year and has the capability of producing 100 per day. Setting up the light production costs $50. The cost of each light is $1. The holding cost is $0.10 per light per year.
**a)** What is the optimal size of the production run?
**b)** What is the average holding cost per year?
**c)** What is the average setup cost per year?
**d)** What is the total cost per year, including the cost of the lights? PX

•• **12.20** Arthur Meiners is the production manager of Wheel-Rite, a small producer of metal parts. Wheel-Rite supplies Cal-Tex, a larger assembly company, with 10,000 wheel bearings each year. This order has been stable for some time. Setup cost for Wheel-Rite is $40, and holding cost is $.60 per wheel bearing per year. Wheel-Rite can produce 500 wheel bearings per day. Cal-Tex is a just-in-time manufacturer and requires that 50 bearings be shipped to it each business day.
**a)** What is the optimum production quantity?
**b)** What is the maximum number of wheel bearings that will be in inventory at Wheel-Rite?

**c)** How many production runs of wheel bearings will Wheel-Rite have in a year?
**d)** What is the total setup + holding cost for Wheel-Rite? Px

•• **12.21** Cesar Rego Computers, a Mississippi chain of computer hardware and software retail outlets, supplies both educational and commercial customers with memory and storage devices. It currently faces the following ordering decision relating to purchases of very high-density disks:

$$D = 36{,}000 \text{ disks}$$
$$S = \$25$$
$$H = \$0.45$$
$$\text{Purchase price} = \$.85$$
$$\text{Discount price} = \$0.82$$
$$\text{Quantity needed to qualify for the discount} = 6{,}000 \text{ disks}$$

Should the discount be taken? Px

•• **12.22** Bell Computers purchases integrated chips at \$350 per chip. The holding cost is \$35 per unit per year, the ordering cost is \$120 per order, and sales are steady, at 400 per month. The company's supplier, Rich Blue Chip Manufacturing, Inc., decides to offer price concessions in order to attract larger orders. The price structure is shown below.

**Rich Blue Chip's Price Structure**

| QUANTITY PURCHASED | PRICE/UNIT |
|---|---|
| 1–99 units | \$350 |
| 100–199 units | \$325 |
| 200 or more units | \$300 |

**a)** What is the optimal order quantity and the minimum annual cost for Bell Computers to order, purchase, and hold these integrated chips?
**b)** Bell Computers wishes to use a 10% holding cost rather than the fixed \$35 holding cost in (a). What is the optimal order quantity, and what is the optimal annual cost? Px

•• **12.23** Wang Distributors has an annual demand for an airport metal detector of 1,400 units. The cost of a typical detector to Wang is \$400. Carrying cost is estimated to be 20% of the unit cost, and the ordering cost is \$25 per order. If Ping Wang, the owner, orders in quantities of 300 or more, he can get a 5% discount on the cost of the detectors. Should Wang take the quantity discount? Px

•• **12.24** The catering manager of La Vista Hotel, Lisa Ferguson, is disturbed by the amount of silverware she is losing every week. Last Friday night, when her crew tried to set up for a banquet for 500 people, they did not have enough knives. She decides she needs to order some more silverware, but wants to take advantage of any quantity discounts her vendor will offer.

For a small order (2,000 or fewer pieces), her vendor quotes a price of \$1.80/piece.

If she orders 2,001–5,000 pieces, the price drops to \$1.60/piece. 5,001–10,000 pieces brings the price to \$1.40/piece, and 10,001 and above reduces the price to \$1.25.

Lisa's order costs are \$200 per order, her annual holding costs are 5%, and the annual demand is 45,000 pieces. For the best option:
**a)** What is the optimal order quantity?
**b)** What is the annual holding cost?
**c)** What is the annual ordering (setup) cost?
**d)** What are the annual costs of the silverware itself with an optimal order quantity?
**e)** What is the total annual cost, including ordering, holding, and purchasing the silverware? Px

•• **12.25** Rocky Mountain Tire Center sells 20,000 go-cart tires per year. The ordering cost for each order is \$40, and the holding cost is 20% of the purchase price of the tires per year. The purchase price is \$20 per tire if fewer than 500 tires are ordered, \$18 per tire if 500 or more—but fewer than 1,000—tires are ordered, and \$17 per tire if 1,000 or more tires are ordered.
**a)** How many tires should Rocky Mountain order each time it places an order?
**b)** What is the total cost of this policy? Px

•• **12.26** M. P. VanOyen Manufacturing has gone out on bid for a regulator component. Expected demand is 700 units per month. The item can be purchased from either Allen Manufacturing or Baker Manufacturing. Their price lists are shown in the table. Ordering cost is \$50, and annual holding cost per unit is \$5.

| *ALLEN MFG.* | | *BAKER MFG.* | |
|---|---|---|---|
| QUANTITY | UNIT PRICE | QUANTITY | UNIT PRICE |
| 1–499 | \$16.00 | 1–399 | \$16.10 |
| 500–999 | 15.50 | 400–799 | 15.60 |
| 1,000+ | 15.00 | 800+ | 15.10 |

**a)** What is the economic order quantity?
**b)** Which supplier should be used? Why?
**c)** What is the optimal order quantity and total annual cost of ordering, purchasing, and holding the component? Px

••• **12.27** Chris Sandvig Irrigation, Inc., has summarized the price list from four potential suppliers of an underground control valve. See the accompanying table. Annual usage is 2,400 valves; order cost is \$10 per order; and annual inventory holding costs are \$3.33 per unit.

Which vendor should be selected and what order quantity is best if Sandvig Irrigation wants to minimize total cost? Px

| *VENDOR A* | | *VENDOR B* | |
|---|---|---|---|
| QUANTITY | PRICE | QUANTITY | PRICE |
| 1–49 | \$35.00 | 1–74 | \$34.75 |
| 50–74 | 34.75 | 75–149 | 34.00 |
| 75–149 | 33.55 | 150–299 | 32.80 |
| 150–299 | 32.35 | 300–499 | 31.60 |
| 300–499 | 31.15 | 500+ | 30.50 |
| 500+ | 30.75 | | |
| *VENDOR C* | | *VENDOR D* | |
| QUANTITY | PRICE | QUANTITY | PRICE |
| 1–99 | \$34.50 | 1–199 | \$34.25 |
| 100–199 | 33.75 | 200–399 | 33.00 |
| 200–399 | 32.50 | 400+ | 31.00 |
| 400+ | 31.10 | | |

••• **12.28** Emery Pharmaceutical uses an unstable chemical compound that must be kept in an environment where both temperature and humidity can be controlled. Emery uses 800 pounds per month of the chemical, estimates the holding cost to be 50% of the purchase price (because of spoilage), and estimates order costs to be \$50 per order. The cost schedules of two suppliers are as follows:

| VENDOR 1 | | VENDOR 2 | |
|---|---|---|---|
| QUANTITY | PRICE/LB | QUANTITY | PRICE/LB |
| 1–499 | $17.00 | 1–399 | $17.10 |
| 500–999 | 16.75 | 400–799 | 16.85 |
| 1,000+ | 16.50 | 800–1,199 | 16.60 |
| | | 1,200+ | 16.25 |

**a)** What is the economic order quantity for each supplier?
**b)** What quantity should be ordered, and which supplier should be used?
**c)** What is the total cost for the most economic order size?
**d)** What factor(s) should be considered besides total cost? PX

••• **12.29** The oar-maker Collars produces high quality oars shipped all over the world. The oars are equipped with bronze oarlocks when sold to customers. Those oarlocks are purchased from an external supplier. In order to simplify the administrative process, and at the same time making sure not running out of oarlocks, Collars have decided to use a two-bin system for controlling oarlock inventory, i.e. they keep two boxes of oarlocks, when one box is empty they order a new one from the supplier.

The supplier has told Collars that they can deliver any quantities and that the lead-time will be 10 working days. They have also offered Collars a fixed price, $200for the freight regardless of orderquantities.

In order to secure a high quality Collars test 5 oarlocks per each shipment, this process takes 60 minutes for one person to do.

Collars sell 5000 oars per year and the plant is open 250 days per year. The value of a pair of oarlocks is $50 and the cost for a working hour is $20. The annual holding cost is set to 15% of the items value
**a)** Calculate the most economical quantity to order
**b)** What would be the minimum allowed quantity to order

•••• **12.30** Emarpy Appliance is a company that produces all kinds of major appliances. Bud Banis, the president of Emarpy, is concerned about the production policy for the company's best-selling refrigerator. The annual demand has been about 8,000 units each year, and this demand has been constant throughout the year. The production capacity is 200 units per day. Each time production starts, it costs the company $120 to move materials into place, reset the assembly line, and clean the equipment. The holding cost of a refrigerator is $50 per year. The current production plan calls for 400 refrigerators to be produced in each production run. Assume there are 250 working days per year.
**a)** What is the daily demand of this product?
**b)** If the company were to continue to produce 400 units each time production starts, how many days would production continue?
**c)** Under the current policy, how many production runs per year would be required? What would the annual setup cost be?
**d)** If the current policy continues, how many refrigerators would be in inventory when production stops? What would the average inventory level be?
**e)** If the company produces 400 refrigerators at a time, what would the total annual setup cost and holding cost be?
**f)** If Bud Banis wants to minimize the total annual inventory cost, how many refrigerators should be produced in each production run? How much would this save the company in inventory costs compared to the current policy of producing 400 in each production run? PX

*Additional problems* **12.31–12.40** *are available in* MyOMLab.

**Problems 12.41–12.50 relate to Probabilistic Models and Safety Stock**

•• **12.41** Barbara Flynn is in charge of maintaining hospital supplies at General Hospital. During the past year, the mean lead time demand for bandage BX-5 was 60 (and was normally distributed). Furthermore, the standard deviation for BX-5 was 7. Ms. Flynn would like to maintain a 90% service level.
**a)** What safety stock level do you recommend for BX-5?
**b)** What is the appropriate reorder point? PX

•• **12.42** Based on available information, lead time demand for PC jump drives averages 50 units (normally distributed), with a standard deviation of 5 drives. Management wants a 97% service level.
**a)** What value of $Z$ should be applied?
**b)** How many drives should be carried as safety stock?
**c)** What is the appropriate reorder point? PX

••• **12.43** Authentic Thai rattan chairs (shown in the photo) are delivered to Gary Schwartz's chain of retail stores, called The Kathmandu Shop, once a year. The reorder point, without safety stock, is 200 chairs. Carrying cost is $30 per unit per year, and the cost of a stockout is $70 per chair per year. Given the following demand probabilities during the lead time, how much safety stock should be carried?

| DEMAND DURING LEAD TIME | PROBABILITY |
|---|---|
| 0 | 0.2 |
| 100 | 0.2 |
| 200 | 0.2 |
| 300 | 0.2 |
| 400 | 0.2 |

•• **12.44** Tobacco is shipped from North Carolina to a cigarette manufacturer in Cambodia once a year. The reorder point, without safety stock, is 200 kilos. The carrying cost is $15 per kilo per year, and the cost of a stockout is $70 per kilo per year. Given the following demand probabilities during the lead time, how much safety stock should be carried?

| DEMAND DURING LEAD TIME (KILOS) | PROBABILITY |
|---|---|
| 0 | 0.1 |
| 100 | 0.1 |
| 200 | 0.2 |
| 300 | 0.4 |
| 400 | 0.2 |

••• **12.45** Mr. Beautiful, an organization that sells weight training sets, has an ordering cost of $40 for the BB-1 set. (BB-1 stands for Body Beautiful Number 1.) The carrying cost for BB-1 is $5 per set per year. To meet demand, Mr. Beautiful orders large quantities of BB-1 seven times a year. The stockout cost for BB-1 is estimated to be $50 per set. Over the past several years, Mr. Beautiful has observed the following demand during the lead time for BB-1:

| DEMAND DURING LEAD TIME | PROBABILITY |
|---|---|
| 40 | .1 |
| 50 | .2 |
| 60 | .2 |
| 70 | .2 |
| 80 | .2 |
| 90 | .1 |
| | 1.0 |

The reorder point for BB-1 is 60 sets. What level of safety stock should be maintained for BB-1? PX

•• **12.46** Chicago's Hard Rock Hotel distributes a mean of 1,000 bath towels per day to guests at the pool and in their rooms. This demand is normally distributed with a standard deviation of 100 towels per day, based on occupancy. The laundry firm that has the linens contract requires a 2-day lead time. The hotel expects a 98% service level to satisfy high guest expectations.

**a)** What is the safety stock?

**b)** What is the ROP? PX

•• **12.47** First Printing has contracts with legal firms in San Francisco to copy their court documents. Daily demand is almost constant at 12,500 pages of documents. The lead time for paper delivery is normally distributed with a mean of 4 days and a standard deviation of 1 day. A 97% service level is expected. Compute First's ROP. PX

••• **12.48** Gainesville Cigar stocks Cuban cigars that have variable lead times because of the difficulty in importing the product: lead time is normally distributed with an average of 6 weeks and a standard deviation of 2 weeks. Demand is also a variable and normally distributed with a mean of 200 cigars per week and a standard deviation of 25 cigars.

**a)** For a 90% service level, what is the ROP?

**b)** What is the ROP for a 95% service level?

**c)** Explain what these two service levels mean. Which is preferable? PX

•••• **12.49** The propane reseller GasPan is during peak season selling in average 71.9 propane cylinders per day, operating 300 days per year. The reseller is sourced with propane cylinder from the main plant in Winterthur, from where cylinders are always delivered one day after order has been placed. For each delivery the reseller is charged $50. Each cylinder has a value of $64

Analysis of the sales shows that the standard deviation of the sales per day, $\sigma_D$, equals 17.6

**a)** Calculate the optimal order quantity, assume a annual holding cost of 15% of hold inventory

**b)** Gas Pan wants to have a service level of 99%, recommend a inventory control strategy that will support the strategy

GasPan is always carefully analyzing its supply chain and the associated costs, and has found that they in order to improve efficiency all orders delivered to the resellers must be delivered in full pallets where each pallet contains 12 cylinders. It has also been decided to use another transport contractor, in this new contract the lead-time has been increased to two days.

**c)** Calculate the percentage increase of the holding and ordering cost during the peak season due to the new policy.

*Additional problem* **12.50** *is available in* MyOMLab.

**Problems 12.51–12.53 relate to Single-Period Model**

•• **12.51** Franz Beckmann is selling sausages at the local soccer games in Pullach. History has shown that in average 190 are sold, with a standarddeviation of 34. He purchases the sausages for $0.25 and sells them for $2 each. Sausages not sold will be given away for free to the players. How many sausages should he purchase in order to maximize profit?

•• **12.52** Henrique Correa's bakery prepares all its cakes between 4 A.M. and 6 A.M. so they will be fresh when customers arrive. Day-old cakes are virtually always sold, but at a 50% discount off the regular $10 price. The cost of baking a cake is $6, and demand is estimated to be normally distributed, with a mean of 25 and a standard deviation of 4. What is the optimal stocking level?

••• **12.53** The Swedish company Rossons Sport sells skis end ski-equipment. They purchase skis at a price of $260 per pair and sales price is $420. Skis not sold this season can be sold early next season at a discounted price, $200. In addition they estimate the stock holding cost between seasons to be $25 per pair of skis.

Past sales shows that demand per season is normal distributed with a mean of 459 pairs and a standard deviation of 56.

**a)** Calculate the shortage and overage cost per pair

**b)** Advice Rossons Sport how many pair of skis they should purchase

**c)** What is the service level?

# CASE STUDIES

## AGA-Fano Gas

AGA-Fano Gas supplies industrial gases to resellers and customers in Paraguay. One of the reseller is Acitelena S.A. owned by Javier Bermudez. The current supply contract with AGA-Fano states that if Acitelena places an order before 12:00, the items will be delivered the next day. AGA-Fana charges Acitelena $20 per each product ordered, regardless of ordersize.

Acitelena is quite a small reseller and has only 10 different gas-cylinders in its assortment. The weekly demand per product is shown in the table below.

| PRODUCT | PRICE ($) | WEEKLY DEMAND | STANDARD DEVIATION |
|---|---|---|---|
| Industrial Oxygen 20 L | 50 | 70 | 15 |
| Industrial Oxygen 50 L | 80 | 176 | 22 |
| Argon 20 L | 62 | 40 | 17 |
| Argon-Mixture 20 L | 73 | 52 | 19 |
| Acetylen 50 L | 102 | 197 | 43 |
| Acetylen 15 L | 26 | 87 | 34 |
| Mison 20 L | 67 | 40 | 12 |
| Mison 50 L | 96 | 110 | 19 |
| Hydrogen 50 L | 167 | 40 | 6 |
| Medical Oxygen 2 L | 14 | 87 | 12 |
| Medical Oxygen 5 L | 23 | 113 | 14 |

Javier's daughter Lena has recently graduated from university with logistics management as her main subject. She has now started working together with her father in the family business. Her first task was to revise her father's replenishment strategy. In order simplify the work he had decided to place orders every

Thursday, he would then order exactly what had been sold since last time he placed an order. In addition he had also decided to keep a certain safety stock per product, since he knew that demand does vary between weeks.

His safety stock is shown in below table

| PRODUCT | JAVIER'S SAFETY STOCK |
|---|---|
| Industrial Oxygen 20 L | 70 |
| Industrial Oxygen 50 L | 100 |
| Argon 20 L | 25 |
| Argon-Mixture 20 L | 20 |
| Acetylen 50 L | 150 |
| Acetylen 15 L | 40 |
| Mison 20 L | 20 |
| Mison 50 L | 50 |
| Hydrogen 50 L | 10 |
| Medical Oxygen 2 L | 10 |
| Medical Oxygen 5 L | 10 |

Lena realizes that it should be possible to save quite some money if she would optimize the replenishment strategy. Her first insight is that they should employ a traditional re-order-point-system. After some discussions with her father they also came to the conclusion that the annual holding cost per cylinder is 15% of the value. She decides also to apply a 98% service level, instead of her fathers old safety stock.

Short after Lena has introduced the new replenishment strategy;Kenneth Freeman, supply manager at AGA-Fano, contacted her. Kenneth is not happy with the development and wants to discuss a new supply contract even though the contract is valid for another 2.5 years.

### Discussion Questions

1. How much would Lena's new strategy save for the Acitelenacompanyon an annual basis? Assume 52 weeks per year and 5 days per week.
2. Why would Kenneth not be happy with the new strategy?
3. When Kenneth suggests Lena that they should go back to weekly deliveries, Lena accepts this as long as the fixed cost is per the whole delivery (not per product). What would be the highest acceptable level on the fixed cost for Lena, if she should accept to change the terms of the contract?

## Parker Hi-Fi Systems

Parker Hi-Fi Systems, located in Wellesley, Massachusetts, a Boston suburb, assembles and sells the very finest home theater systems. The systems are assembled with components from the best manufacturers worldwide. Although most of the components are procured from wholesalers on the East Coast, some critical items, such as LCD screens, come directly from their manufacturer. For instance, the LCD screens are shipped via air from Foxy, Ltd., in Taiwan, to Boston's Logan airport, and the top-of-the-line speakers are purchased from the world-renowned U.S. manufacturer Boss.

Parker's purchasing agent, Raktim Pal, submits an order release for LCD screens once every 4 weeks. The company's annual requirements total 500 units (2 per working day), and Parker's per unit cost is $1,500. (Because of Parker's relatively low volume and the quality focus—rather than volume focus—of many of Parker's suppliers, Parker is seldom able to obtain quantity discounts.) Because Foxy promises delivery within 1 week following receipt of an order release, Parker has never had a shortage of LCDs. (Total time between date of the release and date of receipt is 1 week or 5 working days.)

Parker's activity-based costing system has generated the following inventory-related costs. Procurement costs, which amount to $500 per order, include the actual labor costs involved in ordering, customs inspection, arranging for airport pickup, delivery to the plant, maintaining inventory records, and arranging for the bank to issue a check. Parker's holding costs take into account storage, damage, insurance, taxes, and so forth on a square-foot basis. These costs equal $150 per LCD per year.

With added emphasis being placed on efficiencies in the supply chain, Parker's president has asked Raktim to seriously evaluate the purchase of the LCDs. One area to be closely scrutinized for possible cost savings is inventory procurement.

### Discussion Questions

1. What is the optimal order number of LCDs that should be placed in each order?
2. What is the optimal reorder point (ROP) for LCDs?
3. What cost savings will Parker realize if it implements an order plan based on EOQ?

## Managing Inventory at Frito-Lay

Video Case

Frito-Lay has flourished since its origin—the 1931 purchase of a small San Antonio firm for $100 that included a recipe, 19 retail accounts, and a hand-operated potato ricer. The multi-billion-dollar company, headquartered in Dallas, now has 41 products—15 with sales of over $100 million per year and 7 at over $1 billion in sales. Production takes place in 36 product-focused plants in the U.S. and Canada, with 48,000 employees.

Inventory is a major investment and an expensive asset in most firms. Holding costs often exceed 25% of product value, but in Frito-Lay's prepared food industry, holding cost can be much higher because the raw materials are perishable. In the food industry, inventory spoils. So poor inventory management is not only expensive but can also yield an unsatisfactory product that in the extreme can also ruin market acceptance.

Major ingredients at Frito-Lay are corn meal, corn, potatoes, oil, and seasoning. Using potato chips to illustrate rapid inventory flow: potatoes are moved via truck from farm, to regional plants for processing, to warehouse, to the retail store. This happens in a matter of hours—not days or weeks. This keeps freshness high and holding costs low.

Frequent deliveries of the main ingredients at the Florida plant, for example, take several forms:

- Potatoes are delivered in 10 truckloads per day, with 150,000 lbs consumed in one shift: the entire potato storage area will only hold 7½ hours' worth of potatoes.
- Oil inventory arrives by rail car, which lasts only 4½ days.
- Corn meal arrives from various farms in the Midwest, and inventory typically averages 4 days' production.
- Seasoning inventory averages 7 days.
- Packaging inventory averages 8 to 10 days.

Frito-Lay's product-focused facility represents a major capital investment. That investment must achieve high utilization to be efficient. The capital cost must be spread over a substantial volume to drive down total cost of the snack foods produced. This demand for high utilization requires reliable equipment and tight schedules. Reliable machinery requires an inventory of critical components: this is known as MRO, or maintenance, repair, and operating supplies. MRO inventory of motors, switches, gears, bearings, and other critical specialized components can be costly but is necessary.

Frito-Lay's non-MRO inventory moves rapidly. Raw material quickly becomes work-in-process, moving through the system and out the door as a bag of chips in about $1\frac{1}{2}$ shifts. Packaged finished products move from production to the distribution chain in less than 1.4 days.

### Discussion Questions*

1. How does the mix of Frito-Lay's inventory differ from those at a machine or cabinet shop (a process-focused facility)?
2. What are the major inventory items at Frito-Lay, and how rapidly do they move through the process?
3. What are the four types of inventory? Give an example of each at Frito-Lay.
4. How would you rank the dollar investment in each of the four types (from the most investment to the least investment)?
5. Why does inventory flow so quickly through a Frito-Lay plant?
6. Why does the company keep so many plants open?
7. Why doesn't Frito-Lay make all its 41 products at each of its plants?

*You may wish to view the video that accompanies this case before addressing these questions.

## Inventory Control at Wheeled Coach

**Video Case**

Controlling inventory is one of Wheeled Coach's toughest problems. Operating according to a strategy of mass customization and responsiveness, management knows that success is dependent on tight inventory control. Anything else results in an inability to deliver promptly, chaos on the assembly line, and a huge inventory investment. Wheeled Coach finds that almost 50% of the cost of every ambulance it manufactures is purchased materials. A large proportion of that 50% is in chassis (purchased from Ford), aluminum (from Reynolds Metal), and plywood used for flooring and cabinetry construction (from local suppliers). Wheeled Coach tracks these A inventory items quite carefully, maintaining tight security/control and ordering carefully so as to maximize quantity discounts while minimizing on-hand stock. Because of long lead times and scheduling needs at Reynolds, aluminum must actually be ordered as much as 8 months in advance.

In a crowded ambulance industry in which it is the only giant, its 45 competitors don't have the purchasing power to draw the same discounts as Wheeled Coach. But this competitive cost advantage cannot be taken lightly, according to President Bob Collins. "Cycle counting in our stockrooms is critical. No part can leave the locked stockrooms without appearing on a bill of materials."

Accurate bills of material (BOM) are a requirement if products are going to be built on time. Additionally, because of the custom nature of each vehicle, most orders are won only after a bidding process. Accurate BOMs are critical to cost estimation and the resulting bid. For these reasons, Collins was emphatic that Wheeled Coach maintain outstanding inventory control. The *Global Company Profile* featuring Wheeled Coach (which opens Chapter 14) provides further details about the ambulance inventory control and production process.

### Discussion Questions*

1. Explain how Wheeled Coach implements ABC analysis.
2. If you were to take over as inventory control manager at Wheeled Coach, what additional policies and techniques would you initiate to ensure accurate inventory records?
3. How would you go about implementing these suggestions?

*You may wish to view the video that accompanies this case before addressing these questions.

- **Additional Case Studies:** Visit MyOMLab for these free case studies:
  **Southwestern University (F):** The university must decide how many football day programs to order, and from whom.
  **LaPlace Power and Light:** This utility company is evaluating its current inventory policies.

## Endnotes

1. See E. Malykhina, "Retailers Take Stock," *Information Week* (February 7, 2005): 20–22, and A. Raman, N. DeHoratius, and Z. Ton, "Execution: The Missing Link in Retail Operations," *California Management Review* 43, no. 3 (Spring 2001): 136–141.
2. This is the case when holding costs are linear and begin at the origin—that is, when inventory costs do not decline (or they increase) as inventory volume increases and all holding costs are in small increments. In addition, there is probably some learning each time a setup (or order) is executed—a fact that lowers subsequent setup costs. Consequently, the EOQ model is probably a special case. However, we abide by the conventional wisdom that this model is a reasonable approximation.
3. The formula for the economic order quantity ($Q^*$) can also be determined by finding where the total cost curve is at a minimum (i.e., where the slope of the total cost curve is zero). Using calculus, we set the derivative of the total cost with respect to $Q^*$ equal to 0. The calculations for finding the minimum of
$$TC = \frac{D}{Q}S + \frac{Q}{2}H + PD$$
are $\dfrac{d(TC)}{dQ} = \left(\dfrac{-DS}{Q^2}\right) + \dfrac{H}{2} + 0 = 0$
Thus, $Q^* = \sqrt{\dfrac{2DS}{H}}$
4. The number of units short, Demand–ROP, is true only when Demand–ROP is non-negative.
5. Equations (12-15), (12-16), and (12-17) are expressed in days; however, they could equivalently be expressed in weeks, months, or even years. Just be consistent, and use the same time units for all terms in the equations.
6. Note that Equation (12-17) can also be expressed as:
$$\text{ROP} = \text{Average daily demand} \times \text{Average lead time} + Z\sqrt{(\text{Average lead time} \times \sigma_d^2) + \bar{d}^2\sigma_{LT}^2}.$$
7. OM managers also call these *continuous review systems*.

# Chapter 12 *Rapid Review*



| Main Heading | Review Material | MyOMLab |
|---|---|---|
| **THE IMPORTANCE OF INVENTORY** (pp. 490–491) | Inventory is one of the most expensive assets of many companies.<br>*The objective of inventory management is to strike a balance between inventory investment and customer service.*<br>The two basic inventory issues are how much to order and when to order.<br>■ **Raw material inventory**—Materials that are usually purchased but have yet to enter the manufacturing process.<br>■ **Work-in-process (WIP) inventory**—Products or components that are no longer raw materials but have yet to become finished products.<br>■ **MRO inventory**—Maintenance, repair, and operating materials.<br>■ **Finished-goods inventory**—An end item ready to be sold but still an asset on the company's books. | Concept Questions: 1.1–1.4<br>**VIDEO 12.1** Managing Inventory at Frito-Lay |
| **MANAGING INVENTORY** (pp. 491–495) | ■ **ABC analysis**—A method for dividing on-hand inventory into three classifications based on annual dollar volume.<br>■ **Cycle counting**—A continuing reconciliation of inventory with inventory records.<br>■ **Shrinkage**—Retail inventory that is unaccounted for between receipt and sale.<br>■ **Pilferage**—A small amount of theft. | Concept Questions: 2.1–2.4<br>Problems: 12.1–12.6<br>Virtual Office Hours for Solved Problem: 12.1 |
| **INVENTORY MODELS** (pp. 495–496) | ■ **Holding cost**—The cost to keep or carry inventory in stock.<br>■ **Ordering cost**—The cost of the ordering process.<br>■ **Setup cost**—The cost to prepare a machine or process for production.<br>■ **Setup time**—The time required to prepare a machine or process for production. | Concept Questions: 3.1–3.4<br>**VIDEO 12.2** Inventory Control at Wheeled Coach Ambulance |
| **INVENTORY MODELS FOR INDEPENDENT DEMAND** (pp. 496–507) | ■ **Economic order quantity (EOQ) model**—An inventory-control technique that minimizes the total of ordering and holding costs:<br>$Q^* = \sqrt{\dfrac{2DS}{H}}$ (12-1)<br>Expected number of orders $= N = \dfrac{\text{Demand}}{\text{Order quantity}} = \dfrac{D}{Q^*}$ (12-2)<br>Expected time between orders $= T = \dfrac{\text{Number of working days per year}}{N}$ (12-3)<br>Total annual cost = Setup (order) cost + Holding cost (12-4)<br>$TC = \dfrac{D}{Q}S + \dfrac{Q}{2}H$ (12-5)<br>■ **Robust**—Giving satisfactory answers even with substantial variation in the parameters.<br>■ **Lead time**—In purchasing systems, the time between placing an order and receiving it; in production systems, the wait, move, queue, setup, and run times for each component produced.<br>■ **Reorder point (ROP)**—The inventory level (point) at which action is taken to replenish the stocked item.<br>*ROP for known demand:*<br>ROP = Demand per day × Lead time for a new order in days $= d \times L$ (12-6)<br>■ **Safety stock (*ss*)**—Extra stock to allow for uneven demand; a buffer.<br>■ **Production order quantity model**—An economic order quantity technique applied to production orders:<br>$Q_p^* = \sqrt{\dfrac{2DS}{H[1 - (d/p)]}}$ (12-7)<br>$Q_p^* = \sqrt{\dfrac{2DS}{H\left(1 - \dfrac{\text{Annual demand rate}}{\text{Annual production rate}}\right)}}$ (12-8)<br>■ **Quantity discount**—A reduced price for items purchased in large quantities:<br>$TC = \dfrac{D}{Q}S + \dfrac{Q}{2}H + PD$ (12-9)<br>$Q^* = \sqrt{\dfrac{2DS}{IP}}$ (12-10) | Concept Questions: 4.1–4.4<br>Problems: 12.7–12.40<br>Virtual Office Hours for Solved Problems: 12.2–12.5<br>**ACTIVE MODELS 12.1, 12.2** |

## Chapter 12 *Rapid Review* continued

| Main Heading | Review Material | MyOMLab |
|---|---|---|
| **PROBABILISTIC MODELS AND SAFETY STOCK** (pp. 508–513) | ■ **Probabilistic model**—A statistical model applicable when product demand or any other variable is not known but can be specified by means of a probability distribution.<br>■ **Service level**—The complement of the probability of a stockout.<br>*ROP for unknown demand:*<br>$\text{ROP} = d \times L + ss$ (12-11)<br>Annual stockout costs = The sum of the units short for each demand level × The probability of that demand level × The stockout cost/unit × The number of orders per year (12-12)<br>*ROP for unknown demand and given service level:*<br>$\text{ROP} = \text{Expected demand during lead time} + Z\sigma_{dLT}$ (12-13)<br>$\text{Safety stock} = Z\sigma_{dLT}$ (12-14)<br>*ROP for variable demand and constant lead time:*<br>$\text{ROP} = (\text{Average daily demand} \times \text{Lead time in days}) + Z\sigma_{dLT}$ (12-15)<br>*ROP for constant demand and variable lead time:*<br>$\text{ROP} = (\text{Daily demand} \times \text{Average lead time in days}) + Z \times \text{Daily demand} \times \sigma_{LT}$ (12-16)<br>*ROP for variable demand and variable lead time:*<br>$\text{ROP} = (\text{Average daily demand} \times \text{Average lead time in days}) + Z\sigma_{dLT}$ (12-17)<br>In each case, $\sigma_{dLT} = \sqrt{(\text{Average lead time} \times \sigma_d^2) + \bar{d}^2\sigma_{LT}^2}$<br>but under constant demand: $\sigma_d^2 = 0$,<br>and under constant lead time: $\sigma_{LT}^2 = 0$. | Concept Questions: 5.1–5.4<br>Problems: 12.41–12.50<br>Virtual Office Hours for Solved Problems: 12.6–12.9 |
| **SINGLE-PERIOD MODEL** (pp. 513–514 ) | ■ **Single-period inventory model**—A system for ordering items that have little or no value at the end of the sales period:<br>$\text{Service level} = \dfrac{C_s}{C_s + C_o}$ (12-18) | Concept Questions: 6.1–6.4<br>Problems: 12.51–12.53 |
| **FIXED-PERIOD (*P*) SYSTEMS** (pp. 514–515) | ■ **Fixed-quantity (*Q*) system**—An ordering system with the same order amount each time.<br>■ **Perpetual inventory system**—A system that keeps track of each withdrawal or addition to inventory continuously, so records are always current.<br>■ **Fixed-period (*P*) system**—A system in which inventory orders are made at regular time intervals. | Concept Questions: 7.1–7.4 |

## *Self* Test

■ **Before taking the self-test,** refer to the learning objectives listed at the beginning of the chapter and the key terms listed at the end of the chapter.

**LO 12.1** ABC analysis divides on-hand inventory into three classes, based on:
- **a)** unit price.
- **b)** the number of units on hand.
- **c)** annual demand.
- **d)** annual dollar values.

**LO 12.2** Cycle counting
- **a)** means that we count all articles after a specified time period
- **b)** leads us to count A items more frequent then B items
- **c)** require all A items to be counted the same day
- **d)** d) is a legal requirement

**LO 12.3** The two most important inventory-based questions answered by the typical inventory model are:
- **a)** when to place an order and the cost of the order.
- **b)** when to place an order and how much of an item to order.
- **c)** how much of an item to order and the cost of the order.
- **d)** how much of an item to order and with whom the order should be placed.

**LO 12.4** Extra units in inventory to help reduce stockouts are called:
- **a)** reorder point.
- **b)** safety stock.
- **c)** just-in-time inventory.
- **d)** all of the above.

**LO 12.5** The difference(s) between the basic EOQ model and the production order quantity model is(are) that:
- **a)** the production order quantity model does not require the assumption of known, constant demand.
- **b)** the EOQ model does not require the assumption of negligible lead time.
- **c)** the production order quantity model does not require the assumption of instantaneous delivery.
- **d)** all of the above.

**LO 12.6** The EOQ model is
- **a)** not very useful in practice since the assumptions are unrealistic
- **b)** finds the optimal safety stock
- **c)** is quite useful, due to the fact that the optimum is quite flat
- **d)** depending on the standard deviation of the demand
- **e)** has an impact of the re-order-point

**LO 12.7** The safety stock in re-order point system:
- **a)** shall with a certain probability cover demand exceeding the expected during the lead-time
- **b)** is a non-linear function of lead-time
- **c)** is not depending on the order-quantity
- **d)** all of the above

Answers: LO 12.1. d; LO 12.2. b; LO 12.3. b; LO 12.4. b; LO 12.5. c; LO 12.6. c; LO 12.7. d.