KFUPM BUSINESS SCHOOL

كلية الأعمال
جامعة الملك فهد للبترول والمعادن

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

BUSINESS SCHOOL

DEPARTMENT OF INFORMATION SYSTEM & OPERATIONS MANAGEMENT

MANAGEMENT SCIENCE OM 511

03 – Sensitivity Analysis

Dr. Igor Barahona

DHAHRAN, SAUDI ARABIA

Sensitivity analysis (SA)

SA is about how the changes on objective function or constrains affect the optimal solution. **Two questions emerge from above**

1. How a change in the ***coefficients of the objective function will*** affect the optimal solution? (reduced cost)

2. How a change in the ***right-hand-side value for a constraint*** **will** affect the optimal solution? (shadow price)

Sensitivity analysis

Why Sensitivity Analysis is important?

- Prices of raw materials change
- Product demand changes
- Companies purchase new machinery,
- Storage prices fluctuate
- Employee turnover occurs

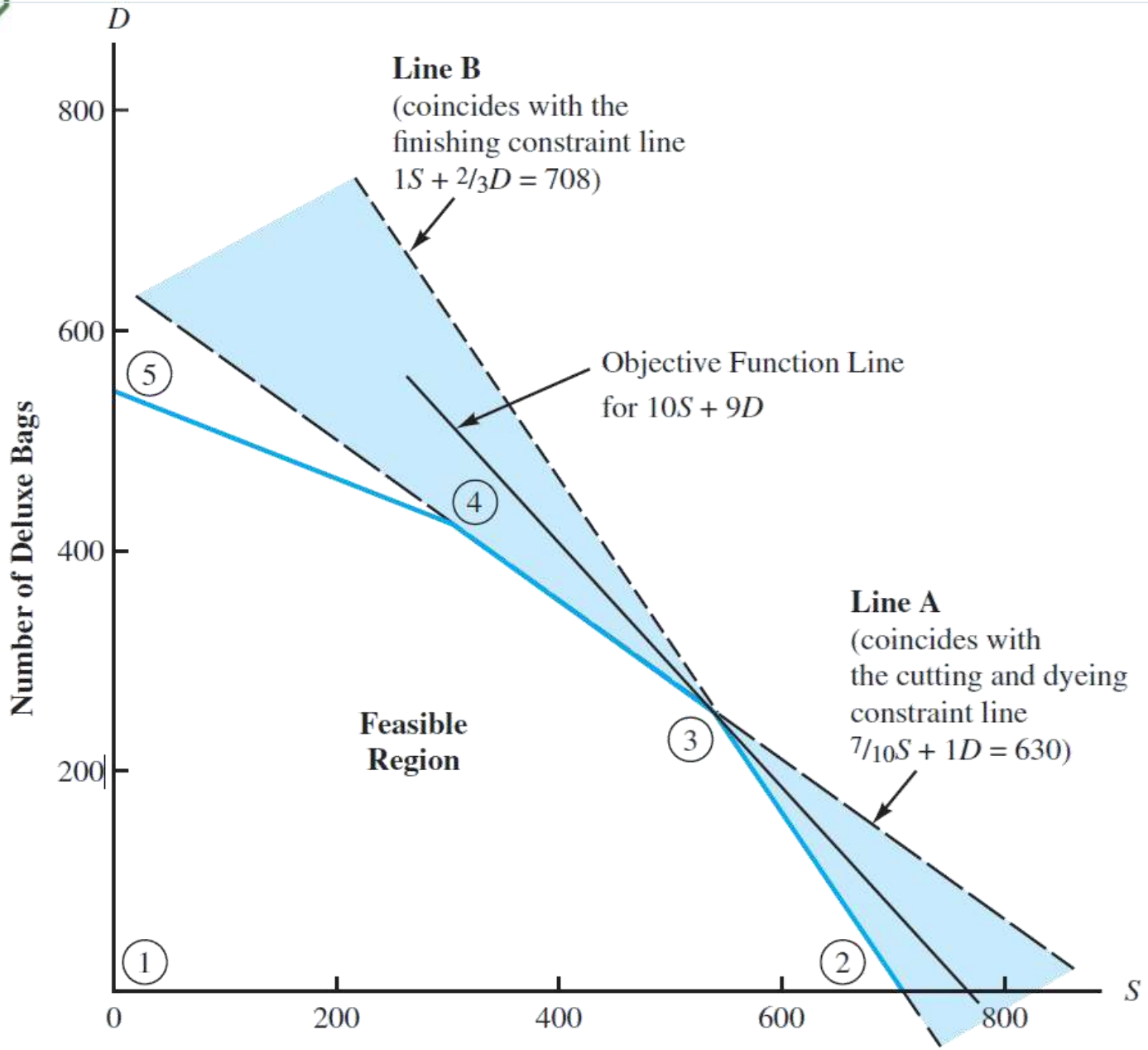
Sensitivity analysis

The **range of optimality** comprises the domain on which coefficient values (both: objective function and constrains) throw an optimal solution

Attention should be focused on coefficients that

- Have a **narrow range** of optimality
- **Near to end** points of the range

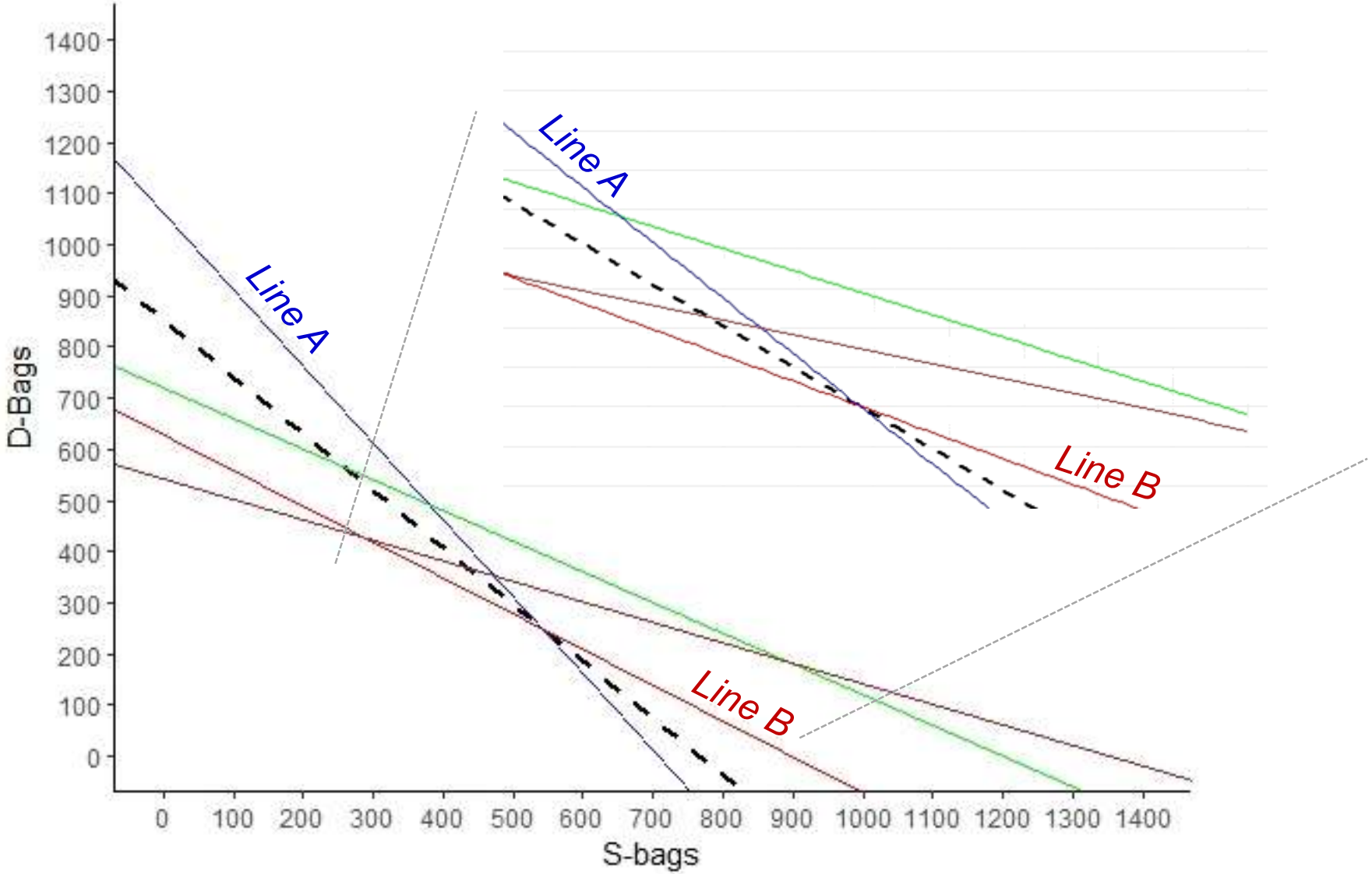
Sensitivity analysis



Sensitivity analysis

- As long as the slope of the Objective Function (OF) is between the slope of lines A and B point 3 will be optimal (S=540 and D=252)
- Changing an OF coefficients will cause the slope change
- Rotating the OF **clockwise** causes the slope to become **more negative**
- Slope of Line A provides an upper limit for the slope of the OF

Sensitivity analysis



Sensitivity analysis

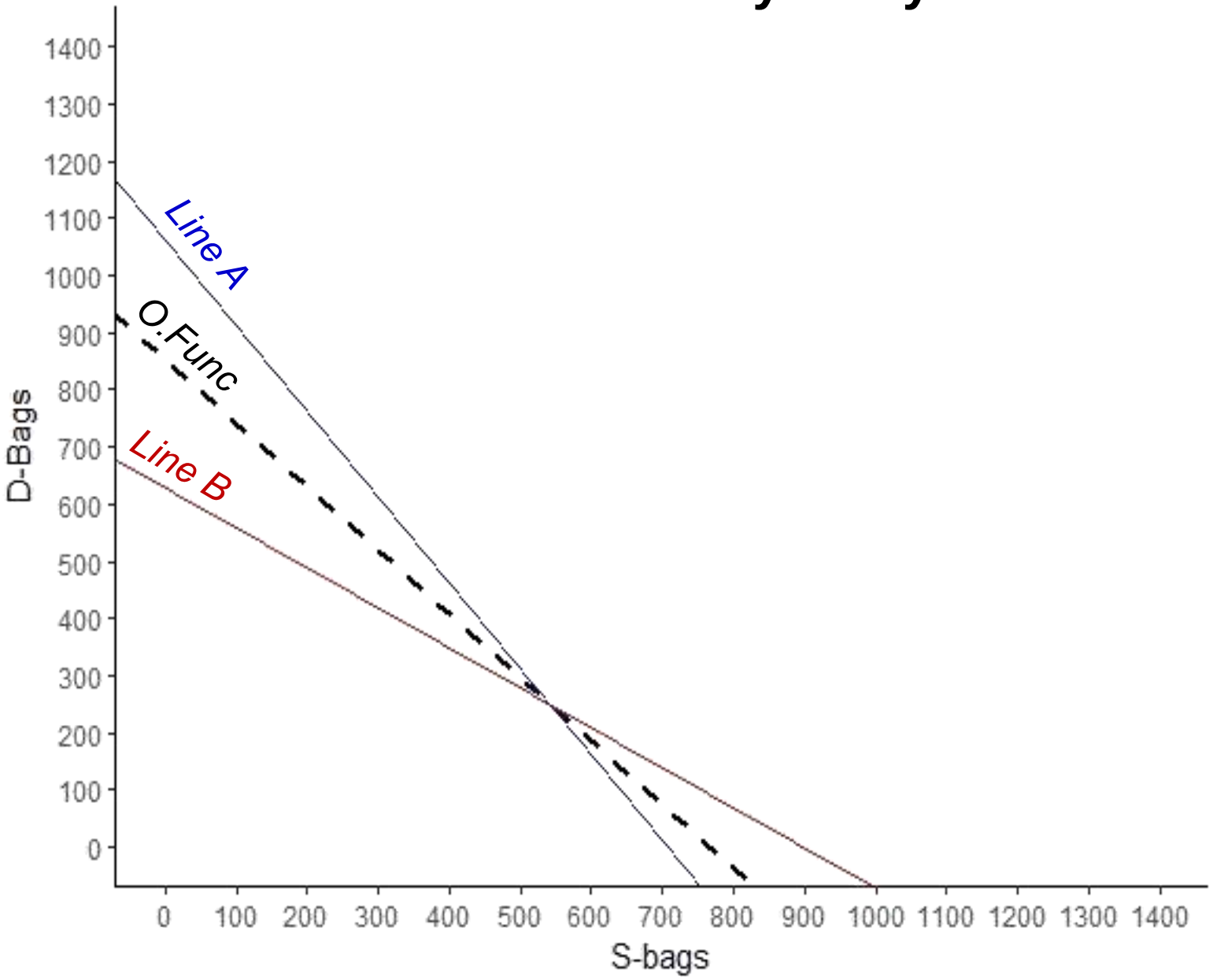
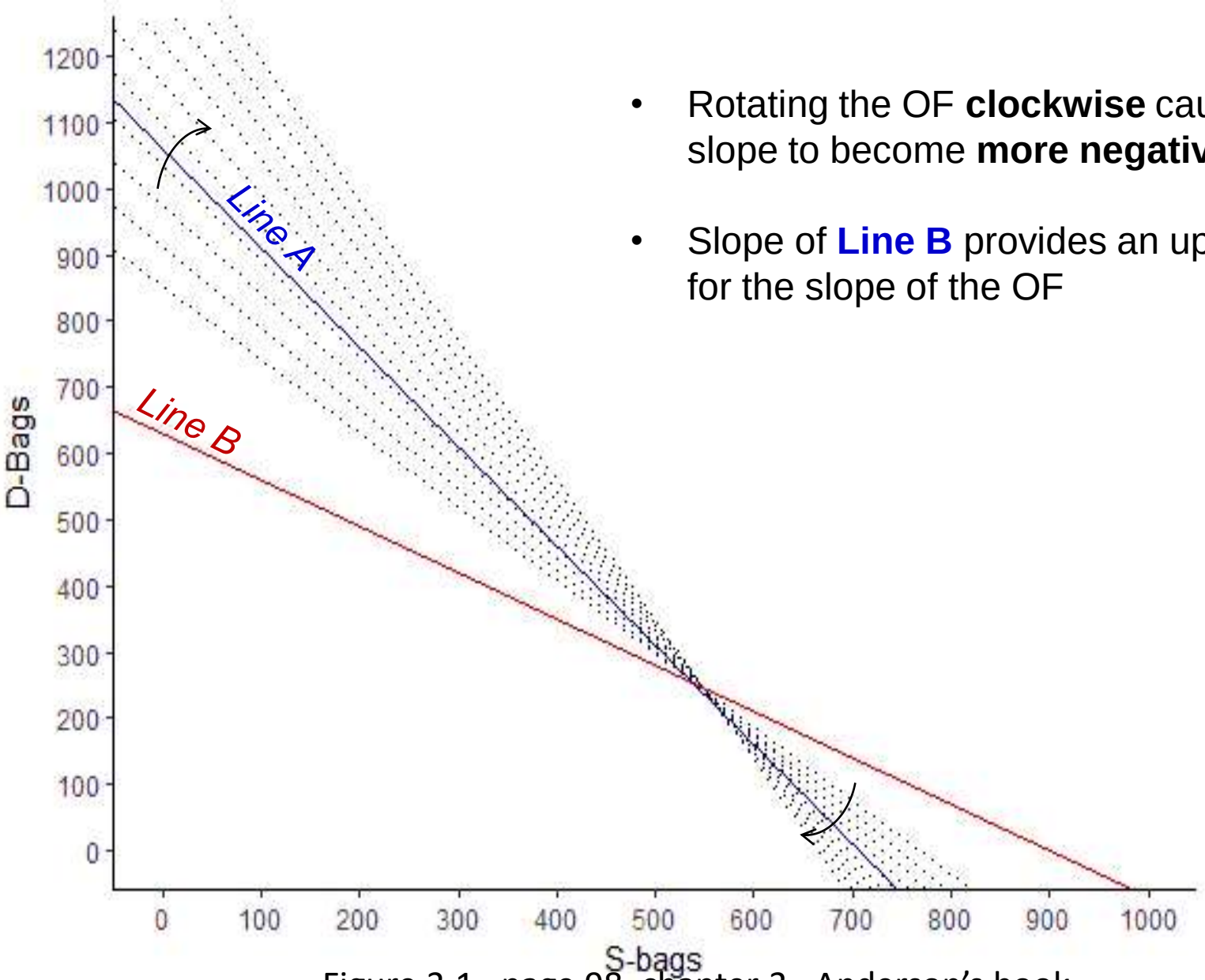


Figure 3.1, page 98, chapter 3, Anderson's book



- Rotating the OF **clockwise** causes the slope to become **more negative**
- Slope of **Line B** provides an upper limit for the slope of the OF

Figure 3.1, page 98, chapter 3, Anderson's book

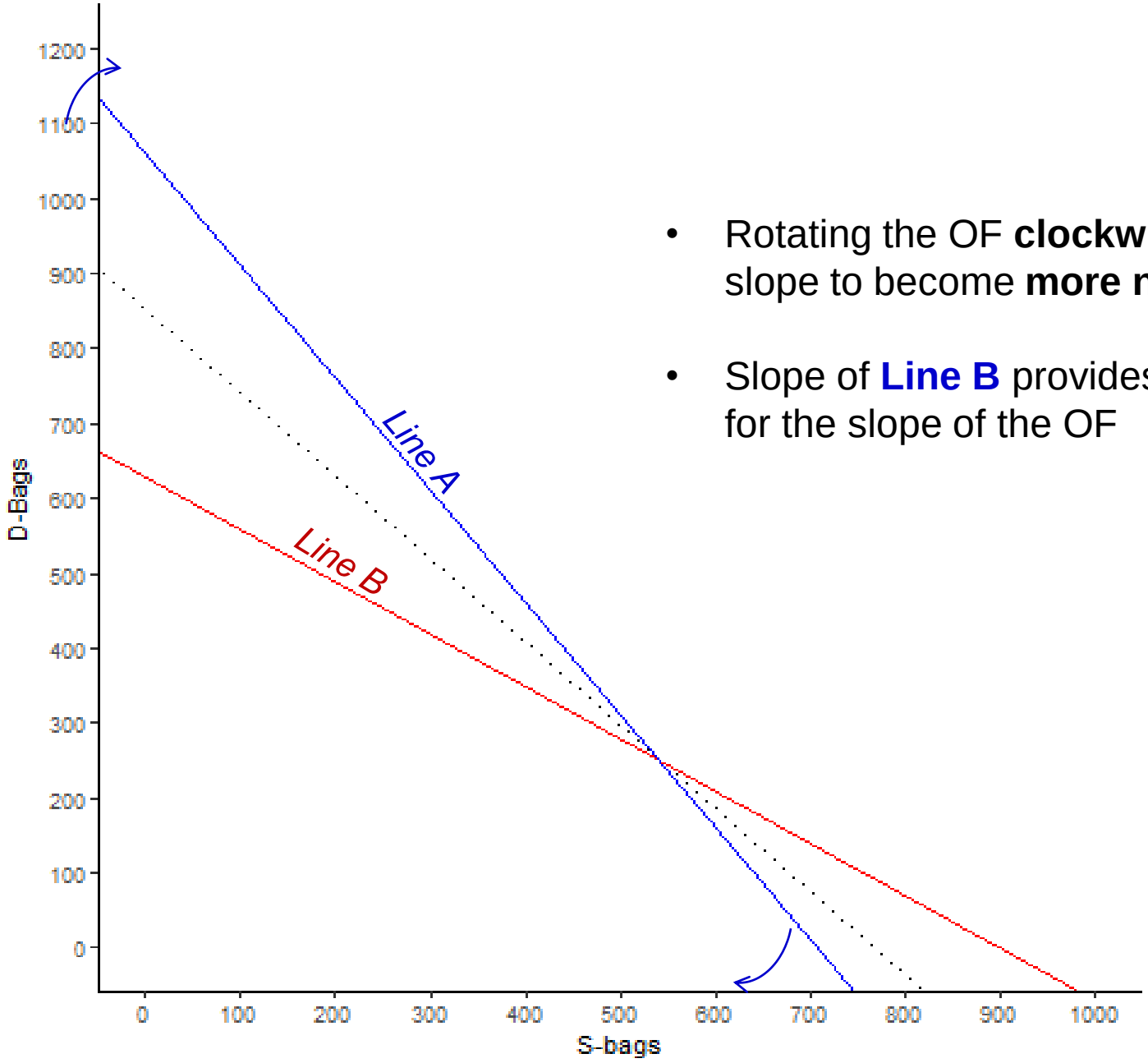


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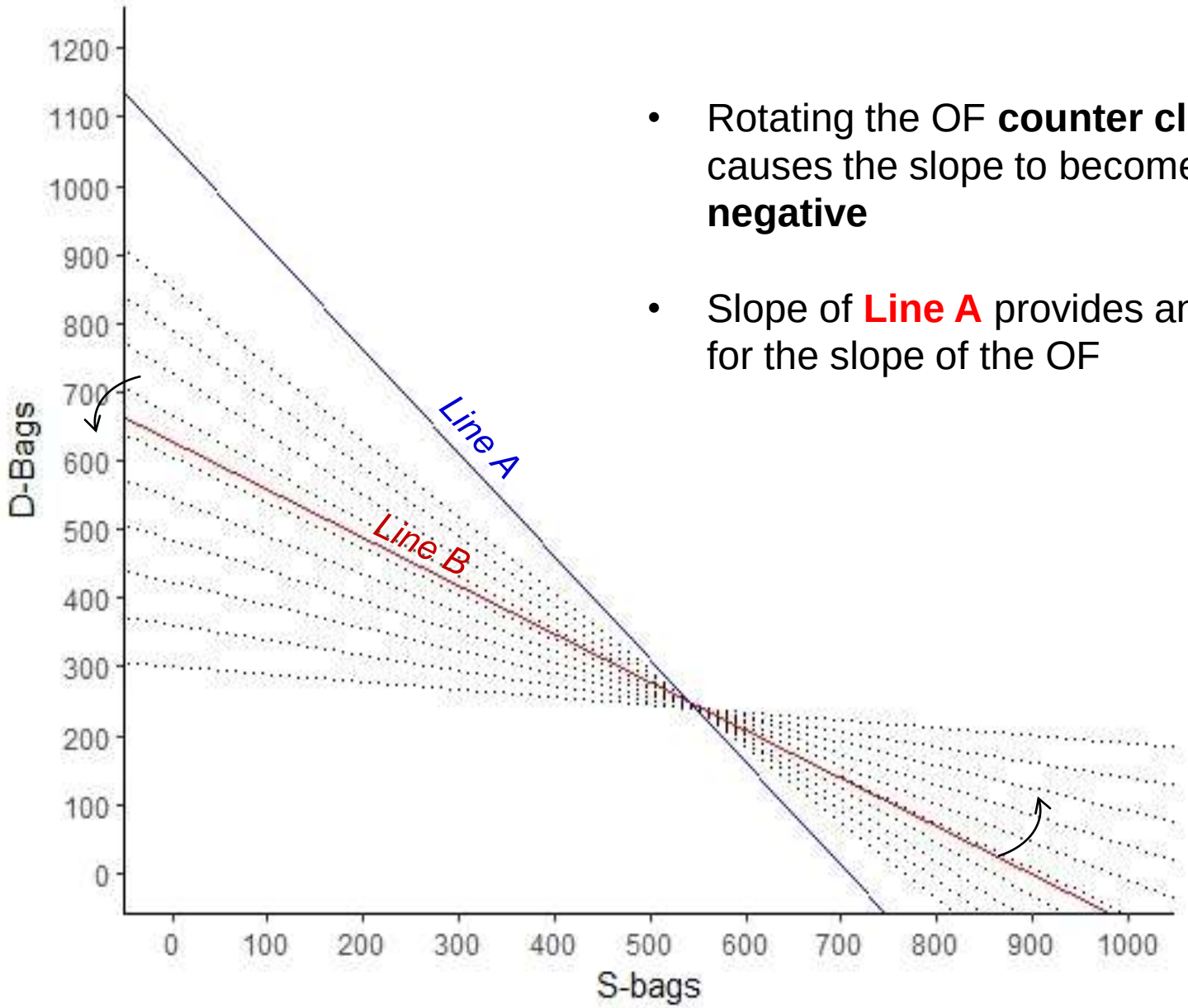
Figure 3.1, page 98, chapter 3, Anderson's book



- Rotating the OF **counter clockwise** causes the slope to become **less negative**
- Slope of **Line A** provides an lower limit for the slope of the OF

Figure 3.1, page 98, chapter 3, Anderson's book

- Rotating the OF **counter clockwise** causes the slope to become **less negative**
- Slope of **Line A** provides an upper limit for the slope of the OF

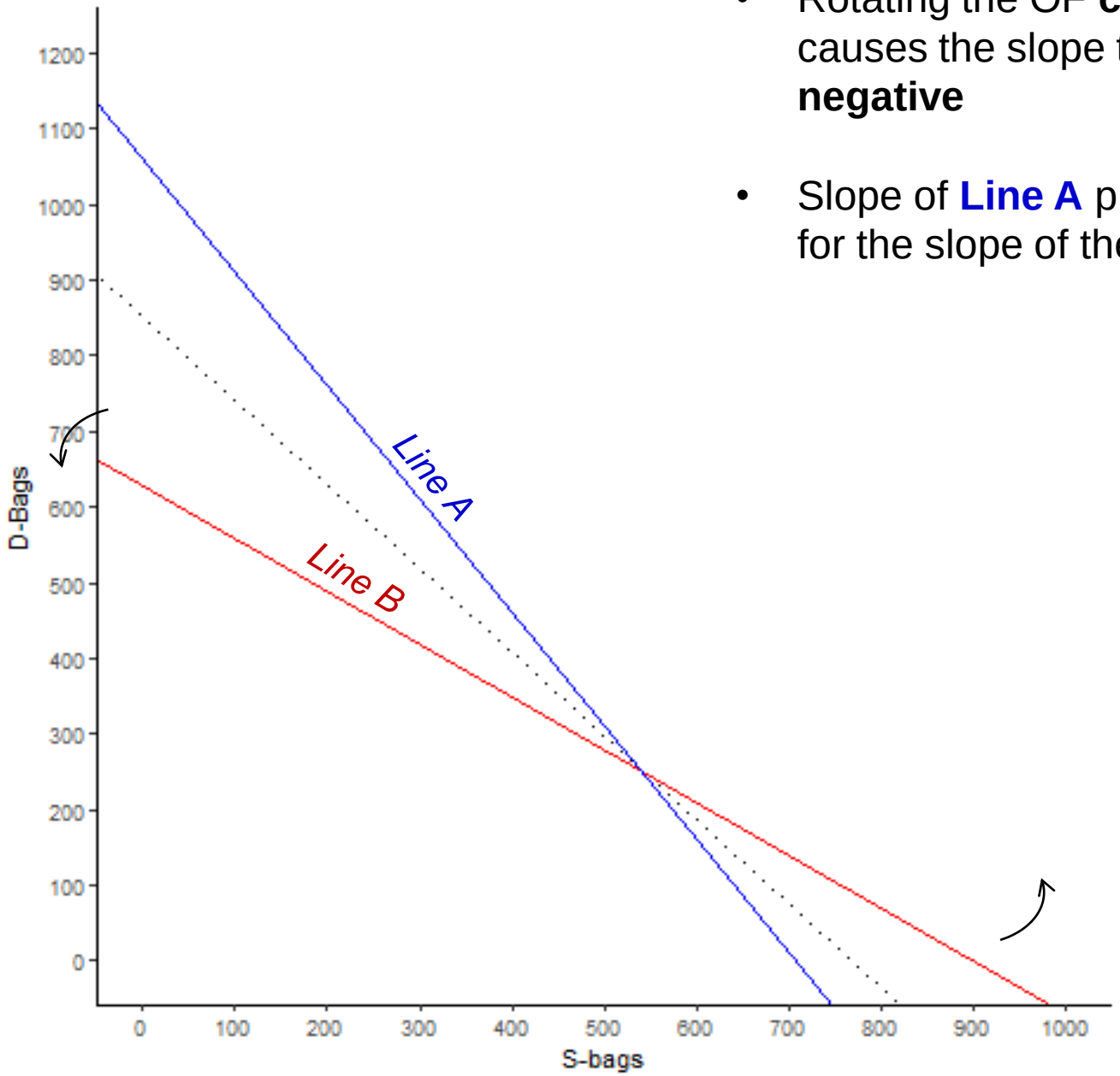


Figure 3.1, page 98, chapter 3, Anderson's book

Sensitivity analysis

From previous charts we conclude that :

- **Slope of Line A** <= **Slope Objective function** <= **Slope of Line B**
- As long as O.Func is between **Line A** and Line B the optimal solution remains the same.
- When final value is equal to RH constrains, then there is not slack. All available resources are being used completely
- **Shadow price:** How objective function changes while units in constrains are added or removed ¿How optimal solution changes by adding / removing recourses?

Practical example

$$\text{Max} \quad 10S + 9D$$

subject to (s.t.)

$$\begin{aligned}
\tfrac{7}{10}S + 1D &\leq 630 && \text{Cutting and dyeing} \\
\tfrac{1}{2}S + \tfrac{5}{6}D &\leq 600 && \text{Sewing} \\
1S + \tfrac{2}{3}D &\leq 708 && \text{Finishing} \\
\tfrac{1}{10}S + \tfrac{1}{4}D &\leq 135 && \text{Inspection and packaging} \\
S, D &\geq 0
\end{aligned}$$

Sensitivity analysis Excel Solver

SA report consist into two parts

1) Cell Variables
2) Constraints section

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B14	Quantity Produced Jordanelle	5.25	0	50	1E+30	6.66666667
C14	Quantity Produced Deercrest	10.5	0	65	10	90

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D15	Fabrication Hours Used	60.38	0	84	1E+30	23.625
D16	Finishing Hours Used	21	45	21	8.2173913	21
D19	Market mixture Excess Deercrest	0	-2.5	0	14	42

Sensitivity analysis Excel Solver

The **reduced cost** value indicates on what extend **optimal value (Z)** is changing for a unit increase in that **decision variables**

Positive reduced-cost shows that a unit increase on **decision variables** will result in a increase on the **optimal value (Z)**.

A **negative reduced-cost** indicates that a unit increase in **decision variables** will result in a decrease on the **optimal value (Z)**.

Sensitivity analysis Excel Solver

Allowable increase and **allowable decrease** refers to the intervals inside of which changes on **Coefficients** (Objective Function) don't affect the **Optimal Solution**.

When changes on **Coefficients** (Objective Function) are either higher or lower than allowable intervals, then **Optimal Solution** is not valid.

Sensitivity analysis Excel Solver

Shadow prices refers to what extent the **optimal value (Z)** will change as result of adding or removing resources on constrains.

Shadow price equal to **zero** indicates that one additional unit of that constrain will not affect the **optimal value (Z)** (e.g. no additional profits)

Negative shadow prices indicate that changing the constrain value in one unit, will decrease the **optimal value (Z)**

A negative shadow price indicates that loosening (or increasing the right-hand side) of a constraint will decrease the objective function value. Tightening (or decreasing the right-hand side) will increase the objective function value.

Sensitivity analysis Excel Solver

If the objective is to **maximize profit** and a resource constraint has a **shadow price of $10**, it means that increasing the availability of that resource by **one unit** would increase the profit by $10**.**

If the objective is to **minimize cost** and a constraint has a shadow **price of $5**, relaxing the constraint by **one unit** would reduce the cost by $5.

Sensitivity analysis Excel Solver

Allowable increase and **allowable decrease values** refers to the extend the **Right Side Constrain** can change before the **optimal value (Z)** in not further valid.

If RS constraint increases by an **amount greater than the allowable increase** the shadow price will not be valid any more.

On the other hand, If RS constrain **decreases by an amount more than the allowable**, the shadow price changes will not be valid any more.

SA report consist into two parts

1) Cell Variables
2) Constraints section

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	N	560	0	1	0.25	0.10714286
C4	R	240	0	1.25	0.15	0.25

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D6	Meat Left Side	4480	0.125	4480	1120	160
D7	Spicy sauce Left Side	1920	0	2080	1E+30	160
D8	Vegetables Left Side	1600	0.1875	1600	40	320

Refer to the Kelson Sporting Equipment problem (Chapter 2, Problem 24).
Letting
R= number of regular gloves
C= number of catcher's mitts

Objective Function
5R + 8C >> MAX
s.t.

R + 3/2*C <= 900
½*R + 1/3*C <= 300
1/8*R + ¼*C <= 100
R,C => 0

1. Find the optimal solution
2. Which constraints are binding?
3. What are the dual values for the resources? Interpret each.
4. If overtime can be scheduled in one of the departments, where would you recommend doing so?

Prepare your solution either using Excel Solver or Pulp python

15. Seastrand Oil Company produces two grades of gasoline: regular and high octane. Both gasolines are produced by blending two types of crude oil. Although both types of crude oil contain the two important ingredients required to produce both gasolines, the percentage of important ingredients in each type of crude oil differs, as does the cost per gallon. The percentage of ingredients A and B in each type of crude oil and the cost per gallon are shown.

Crude Oil	Cost	Ingredient A	Ingredient B
1	\$0.10	20%	60%
2	\$0.15	50%	30%

Crude oil 1 is 60% ingredient B

Each gallon of regular gasoline must contain at least 40% of ingredient A, whereas each gallon of high octane can contain at most 50% of ingredient B. Daily demand for regular and high-octane gasoline is 800,000 and 500,000 gallons, respectively. How many gallons of each type of crude oil should be used in the two gasolines to satisfy daily demand at a minimum cost?

Exercise in class

1. Arrange teams up to 5 participants

2. Based on the SeaStrand case of study, your team will find the following

   2.1 Settle the linear programming model

   2.2 Write the mathematical formulation

   2.3 Calculate how many gallons of each type of crude satisfy the minimal cost by using Excel Solver

Exercise in class

KFUPM Investment Club is working on designing a strategy for allocating founds among six possible alternatives, as it is illustrated on the Table.

Decision makers have collected information about return rate and risk associated to each alternative. The risk is measured in an index between 0 and 1, on which the higher the index, the higher the volatility and therefore more uncertainty.

Alternative	Annual Return Rate	Price / Stock	Risk
NVIDIA	168.3%	$120.30	0.17
Apple Inc	34.1%	$256.16	0.32
Amazon	45.3%	$183.55	0.41
Tesla	3.9%	$256.23	0.48
Google (Alphabet Inc Class A)	23.8%	$166.23	0.22
Cash / Savings	2.0%	----	0.0

Exercise in class

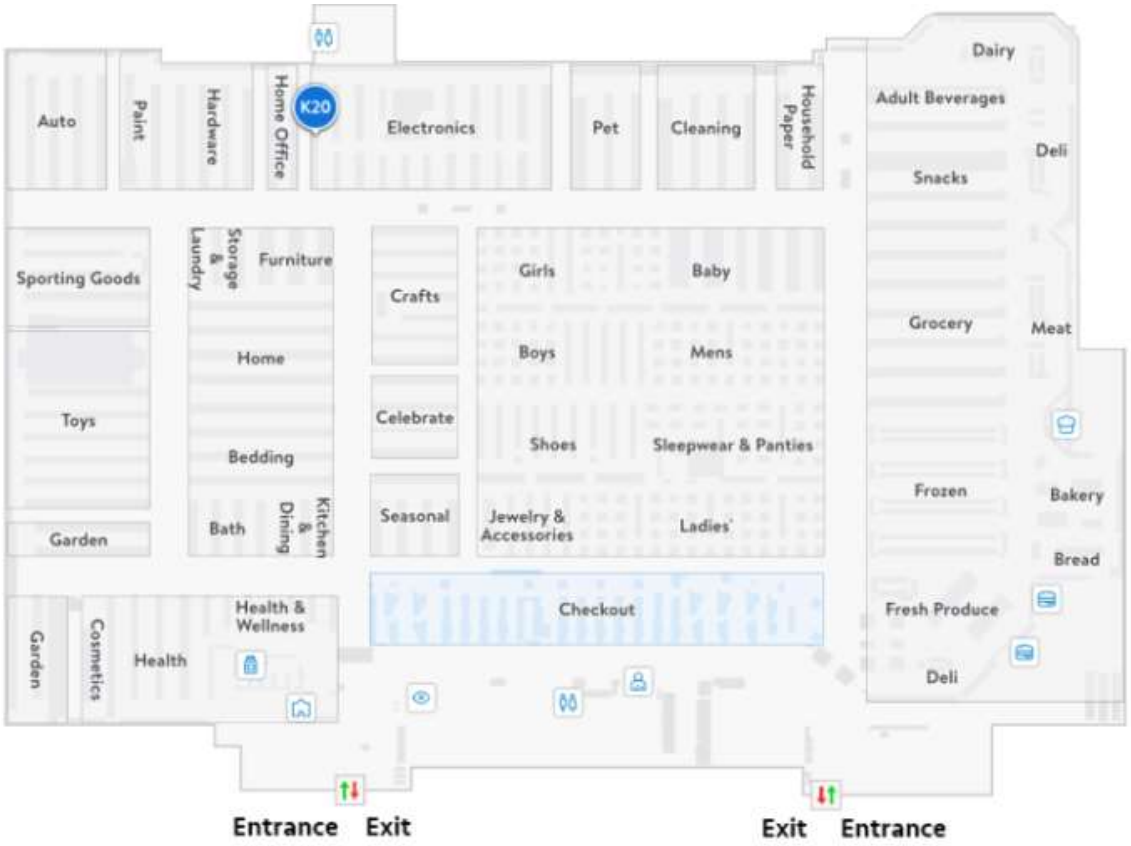
- A **conservative** strategy implies the average risk for the overall portfolio is lower or equal to 0.25.
- A **moderate strategy** comprises that the average risk for the overall portfolio should be between 0.26 and 0.39
- An **aggressive strategy** comprises an average risk higher or equal to 0.40

The considerations for the Club members are:

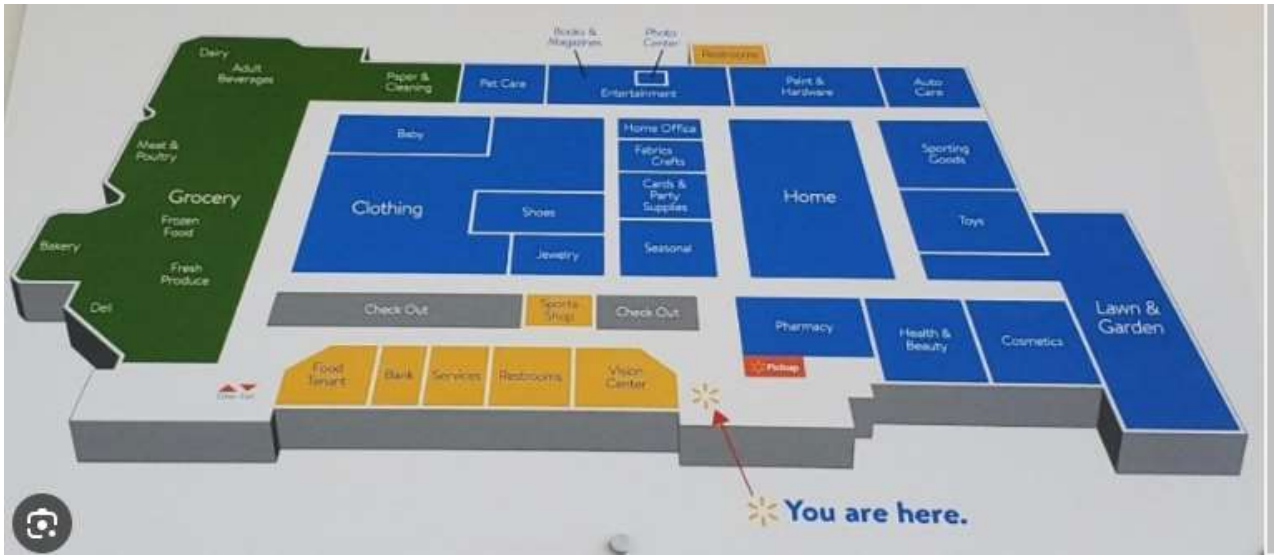
- No more than 50% of the total investment may be in Google (Alphabet Inc Class A)
- The amount invested on Apple should be at least the amount invested on Amazon.
- At least the 5% of the investments, should be kept on cash
- The plan comprises to allocate 100,000 USD based on the proposed model.

What is the optimal portfolio allocation for a conservative, moderate and aggressive strategy given the previous considerations?

Jarir Book store is planning to open a new store in Damman. The architects calculated that the store has **100,000** square meters of area. Besides the business analytics department made the following projections based on the current demand.



Department	Investments sf	Risk (% of invested)	Minimum sf	Maximum st	Expected profit per st
Video games X1	100	24	7,000	30,000	13.0
Office supplies X2	50	12	11,000	30,000	7.0
Arabic books X3	30	5	3,000	5,000	3.0
Smart TVs x4	600	10	2,000	42,000	30.0
Laptops X5	900	14	1,000	11,000	20.0
School X6	50	2	1,000	7,000	2.0
Smart phones X7	400	3	12,000	40,000	14.0



Jarir Book store is planning to allocate 200 million SAR for the new store. The risk column refers to the associated risk for investing on the floor based on historical data. The amount of risk should be no more than 10% of the total investment.

a. Develop a linear optimization model to maximize profit.

b. If the chain obtains another $1 million of investment capital for stock, what would the new solution be?

Exercise in class

1. Arrange teams up to 5 participants

2. Based on the Jarir Bookstore case of study, your team will find the following

   2.1. Configure the linear programming model

   2.2. Write the mathematical formulation this this model.

   2.3. Calculate how many cost by using Excel Solver

10. An investment advisor at Shore Financial Services wants to develop a model that can be used to allocate investment funds among four alternatives: stocks, bonds, mutual funds, and cash. For the coming investment period, the company developed estimates of the annual rate of return and the associated risk for each alternative. Risk is measured using an index between 0 and 1, with higher risk values denoting more volatility and thus more uncertainty.

Investment	Annual Rate of Return (%)	Risk
Stocks	10	0.8
Bonds	3	0.2
Mutual funds	4	0.3
Cash	1	0.0