Statistical Process Control

SUPPLEMENT 6

PowerPoint presentation to accompany
Heizer and Render
Operations Management, Global Edition, Eleventh Edition
Principles of Operations Management, Global Edition, Ninth Edition

PowerPoint slides by Jeff Heyl

Outline

- Statistical Process Control
- Process Capability
- Acceptance Sampling

Learning Objectives

When you complete this supplement you should be able to :

1. **Explain** the purpose of a control chart
2. **Explain** the role of the central limit theorem in SPC
3. **Build** $\overline{x}$-charts and R-charts
4. **List** the five steps involved in building control charts

Learning Objectives

When you complete this supplement you should be able to :

5. **Build** *p*-charts and *c*-charts
6. **Explain** process capability and compute C_p and C_{pk}
7. **Explain** acceptance sampling

Statistical Process Control

The objective of a process control system is to provide a statistical signal when assignable causes of variation are present

Statistical Process Control (SPC)

- Variability is inherent in every process
  - Natural or common causes
  - Special or assignable causes

- Provides a statistical signal when assignable causes are present
- Detect and eliminate assignable causes of variation

Natural Variations

- Also called common causes
- Affect virtually all production processes
- Expected amount of variation
- Output measures follow a probability distribution
- For any distribution there is a measure of central tendency and dispersion
- If the distribution of outputs falls within acceptable limits, the process is said to be "in control"

Assignable Variations

- Also called special causes of variation
  - Generally this is some change in the process
- Variations that can be traced to a specific reason
- The objective is to discover when assignable causes are present
  - Eliminate the bad causes
  - Incorporate the good causes

Samples

To measure the process, we take samples and analyze the sample statistics following these steps

(a) Samples of the product, say five boxes of cereal taken off the filling machine line, vary from each other in weight

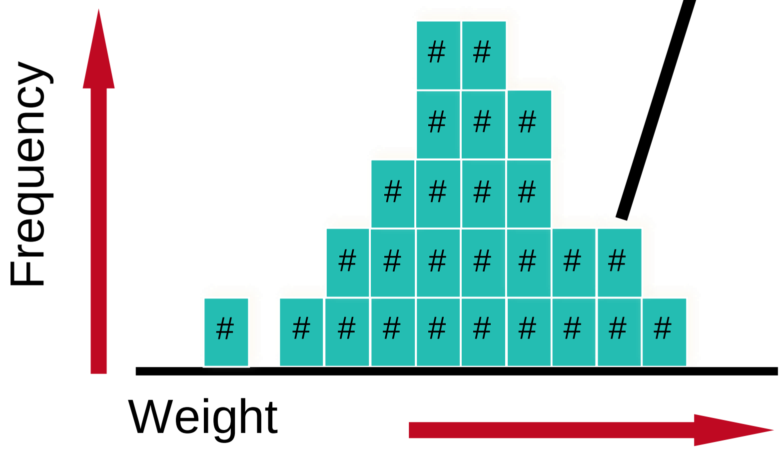


Figure S6.1

Samples

To measure the process, we take samples and analyze the sample statistics following these steps

(b) After enough samples are taken from a stable process, they form a pattern called a distribution

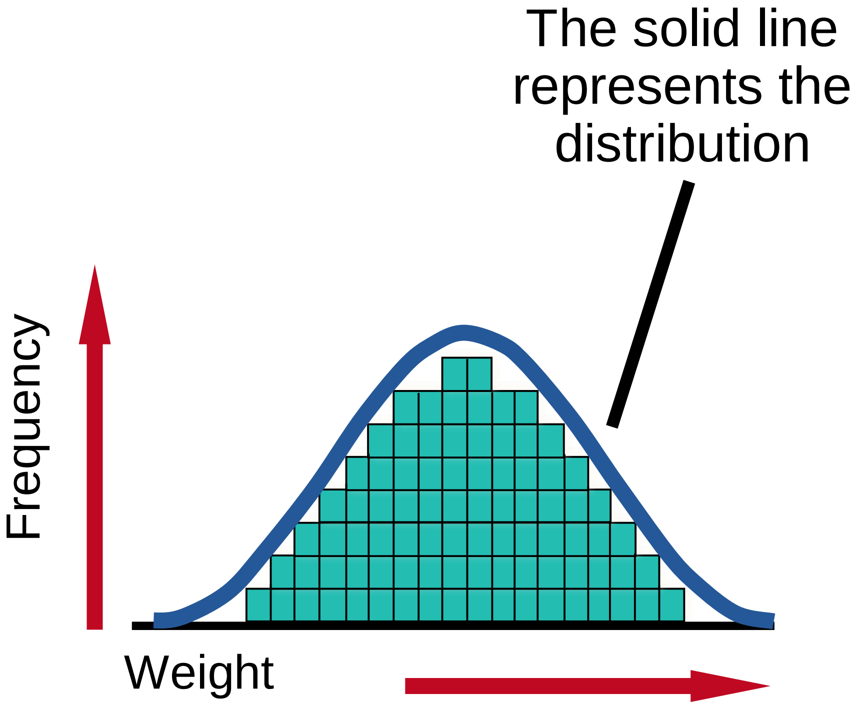


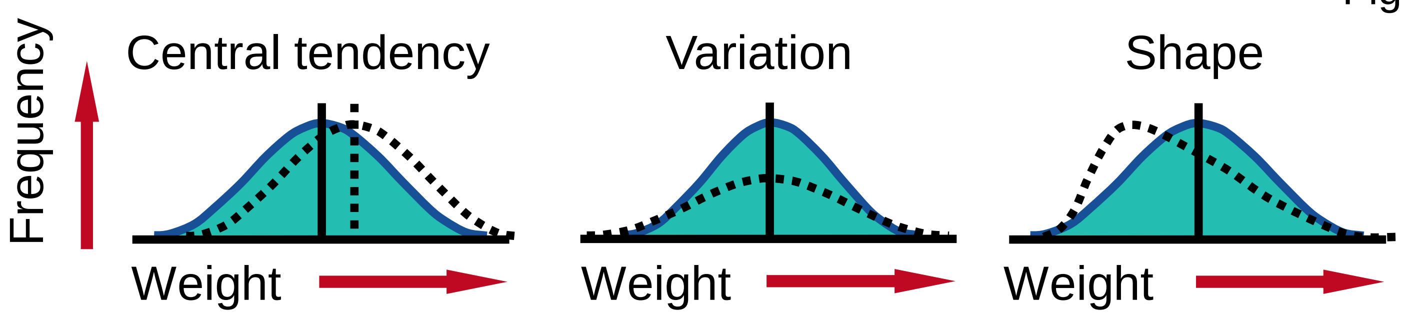
Figure S6.1

Samples

To measure the process, we take samples and analyze the sample statistics following these steps

(c) There are many types of distributions, including the normal (bell-shaped) distribution, but distributions do differ in terms of central tendency (mean), standard deviation or variance, and shape

Figure S6.1



Samples

To measure the process, we take samples and analyze the sample statistics following these steps

(d) If only natural causes of variation are present, the output of a process forms a distribution that is stable over time and is predictable

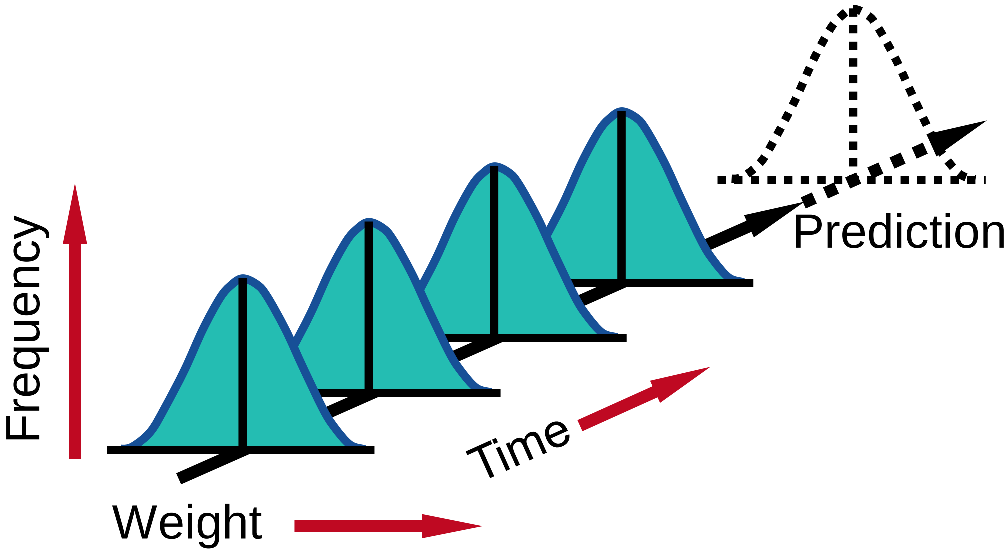


Figure S6.1

Samples

To measure the process, we take samples and analyze the sample statistics following these steps

(e) If assignable causes are present, the process output is not stable over time and is not predicable

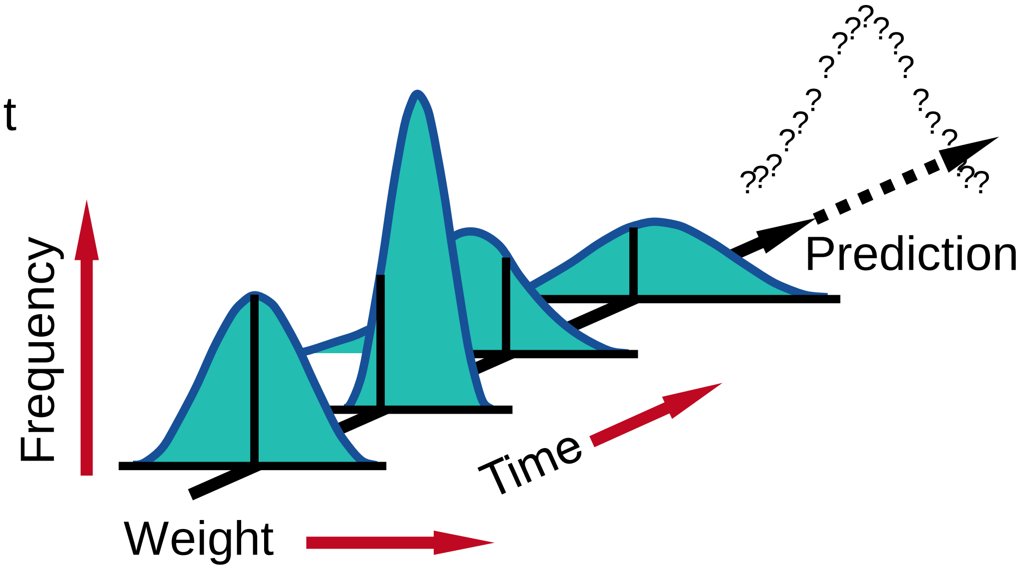
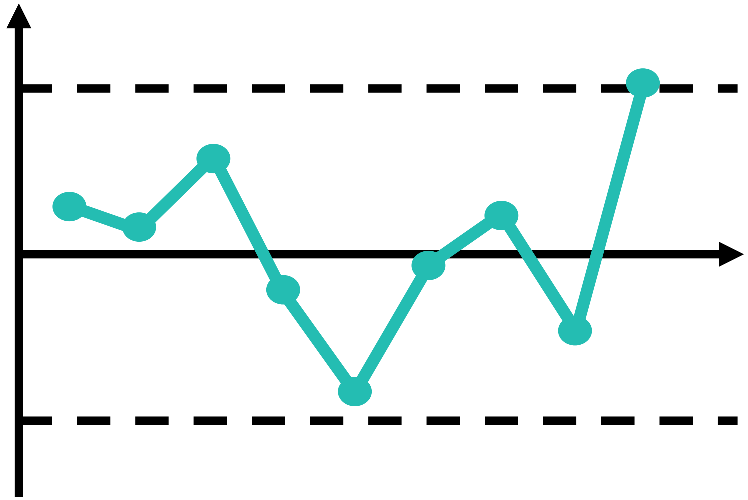


Figure S6.1

Control Charts

Constructed from historical data, the purpose of control charts is to help distinguish between natural variations and variations due to assignable causes



Process Control

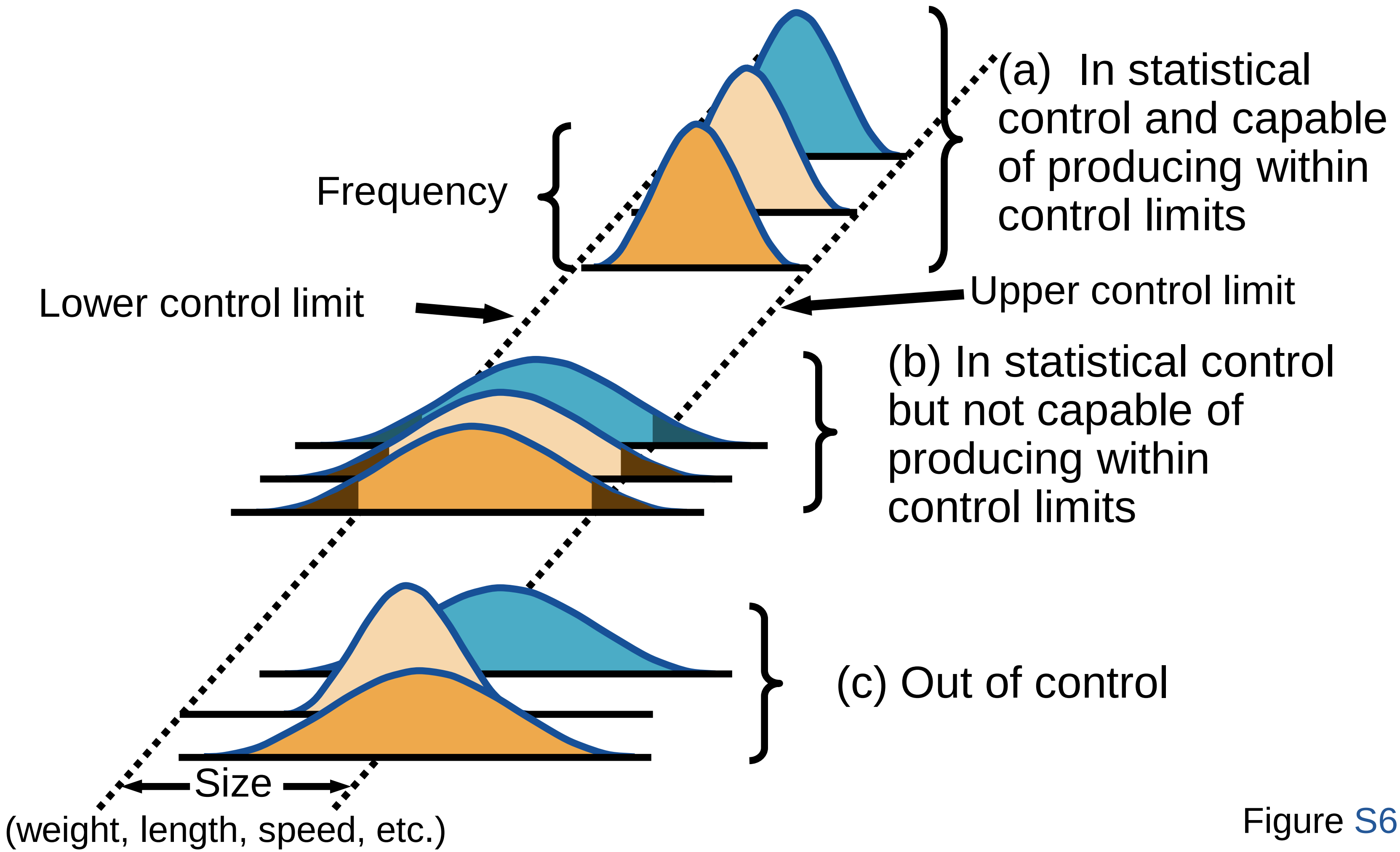


Figure S6.2

Control Charts for Variables

- Characteristics that can take any real value
- May be in whole or in fractional numbers
- Continuous random variables

$\bar{x}$-chart tracks changes in the central tendency

R-chart indicates a gain
dispersion

These two charts must be used together

Central Limit Theorem

Regardless of the distribution of the population, the distribution of sample means drawn from the population will tend to follow a normal curve

1) The mean of the sampling distribution will be the same as the population mean μ

$$\bar{\bar{x}} = n$$

2) The standard deviation of the sampling distribution ($s_{\bar{x}}$) will equal the population standard deviation (σ) divided by the square root of the sample size, n

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Population and Sampling Distributions

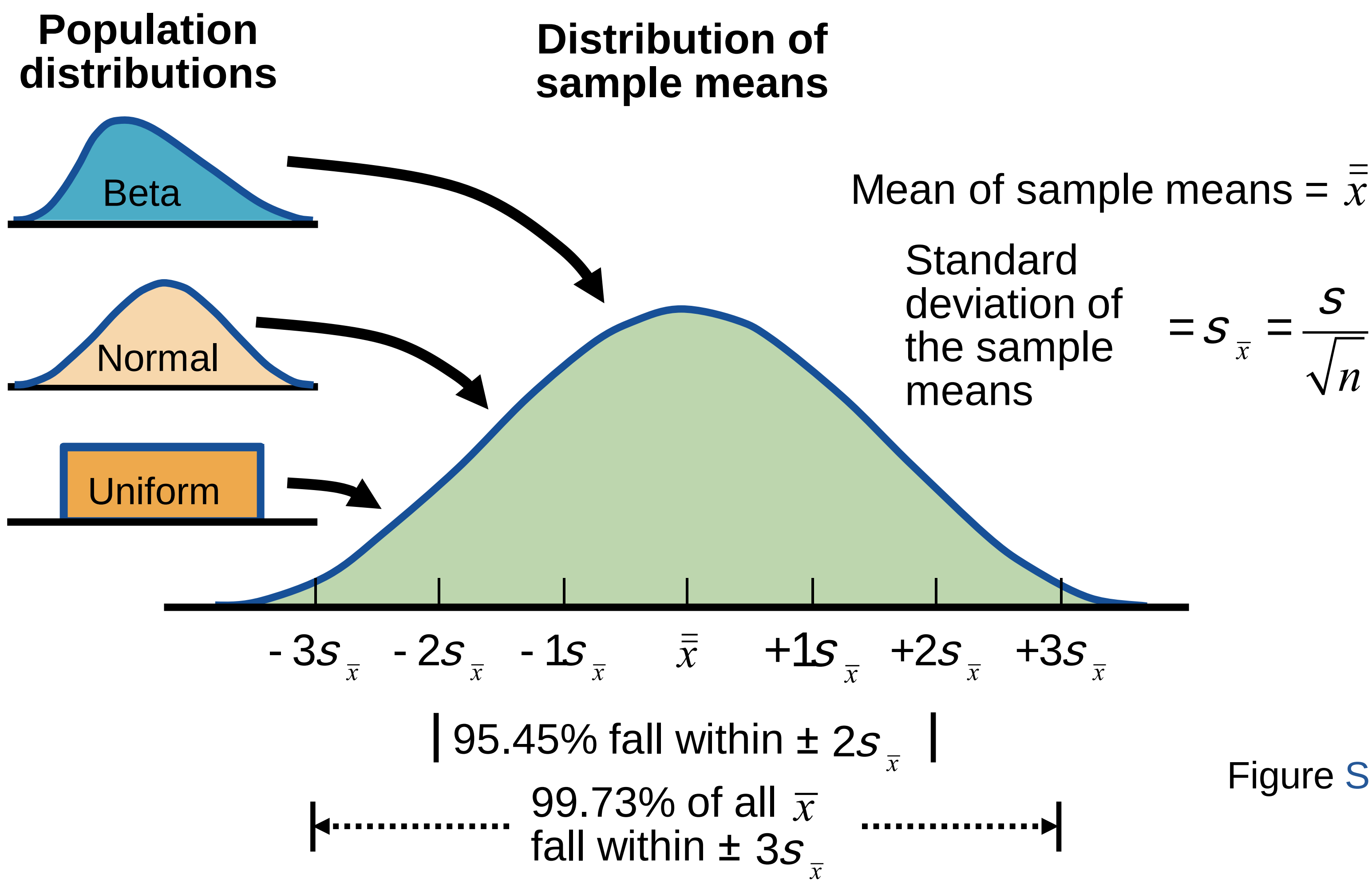
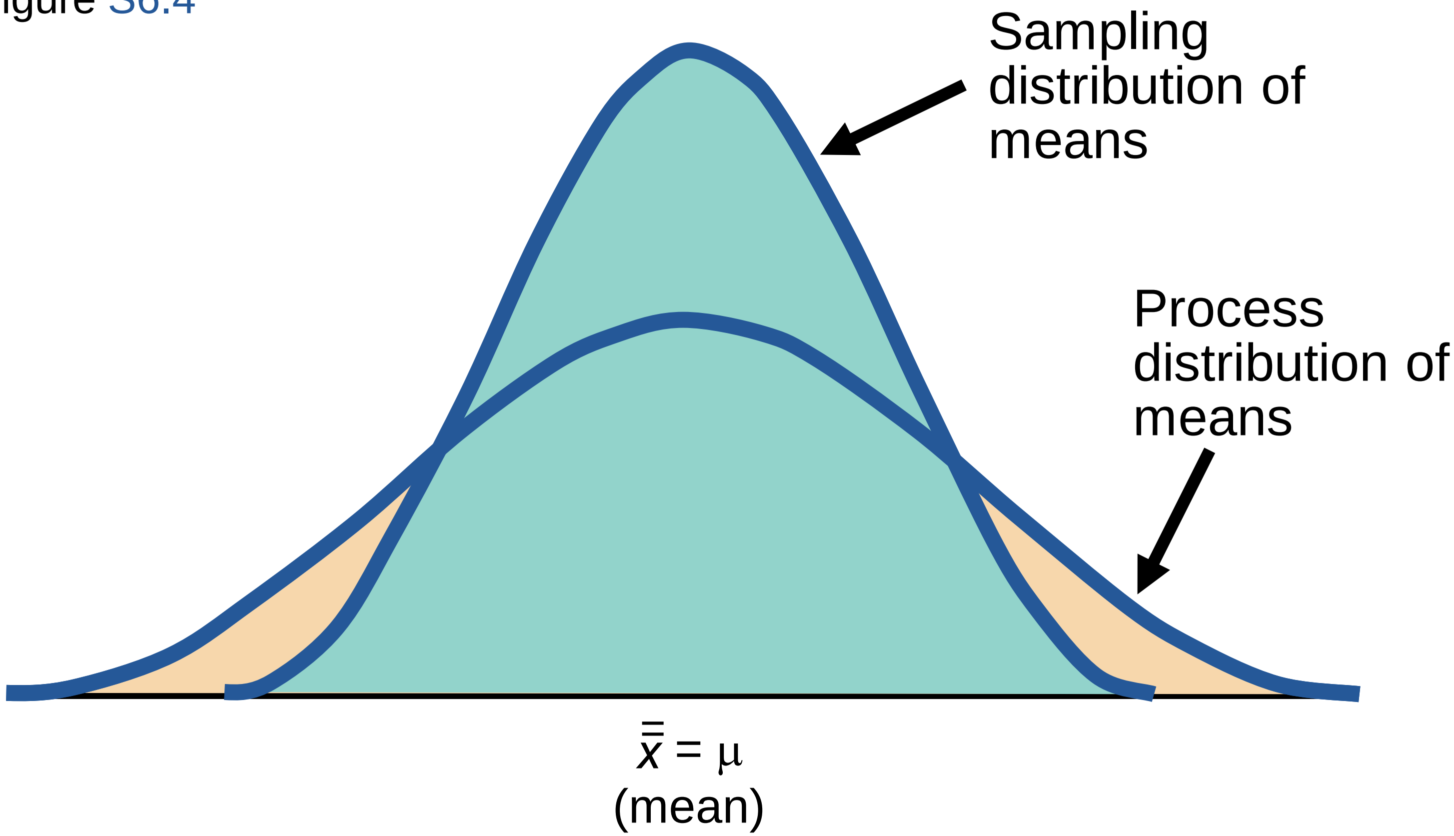


Figure S6.3

Sampling Distribution

Figure S6.4



Setting Chart Limits

For $\bar{x}$-Charts when we know σ

$$\text{Lower control limit (UCL)} = \bar{\bar{x}} - z s_{\bar{x}}$$

$$\text{Upper control limit (UCL)} = \bar{\bar{x}} + z s_{\bar{x}}$$

Where

- $\bar{\bar{x}}$ = mean of the sample means or a target value set for the process
- z = number of normal standard deviations
- σ_x = standard deviation of the sample means $= s/\sqrt{n}$
- σ = population (process) standard deviation
- n = sample size

Setting Control Limits

- Randomly select and weigh nine (n = 9) boxes each hour

$$\text{Average weight in the first sample} = \frac{17+13+16+18+17+16+15+17+16}{9} = 16.1 \text{ ounces}$$

WEIGHT OF SAMPLE		WEIGHT OF SAMPLE		WEIGHT OF SAMPLE	
HOUR	(AVG. OF 9 BOXES)	HOUR	(AVG. OF 9 BOXES)	HOUR	(AVG. OF 9 BOXES)
1	16.1	5	16.5	9	16.3
2	16.8	6	16.4	10	14.8
3	15.5	7	15.2	11	14.2
4	16.5	8	16.4	12	17.3

Setting Control Limits

$$\text{Average mean of 12 samples} = \bar{\bar{x}} = \left[\frac{\sum_{i=1}^{12} \left(\text{Avg of 9 boxes}\right)}{12} \right]$$

$$\bar{\bar{x}} = 16 \text{ ounces}$$

$$n = 9$$

$$z = 3$$

$$\sigma = 1 \text{ ounce}$$

Setting Control Limits

Average mean of 12 samples $= \left[\bar{\bar{x}} = \dfrac{\sum_{i=1}^{12} (\text{Avg of 9 boxes})}{12} \right]$

$\bar{\bar{x}} = 16$ ounces

$n = 9$

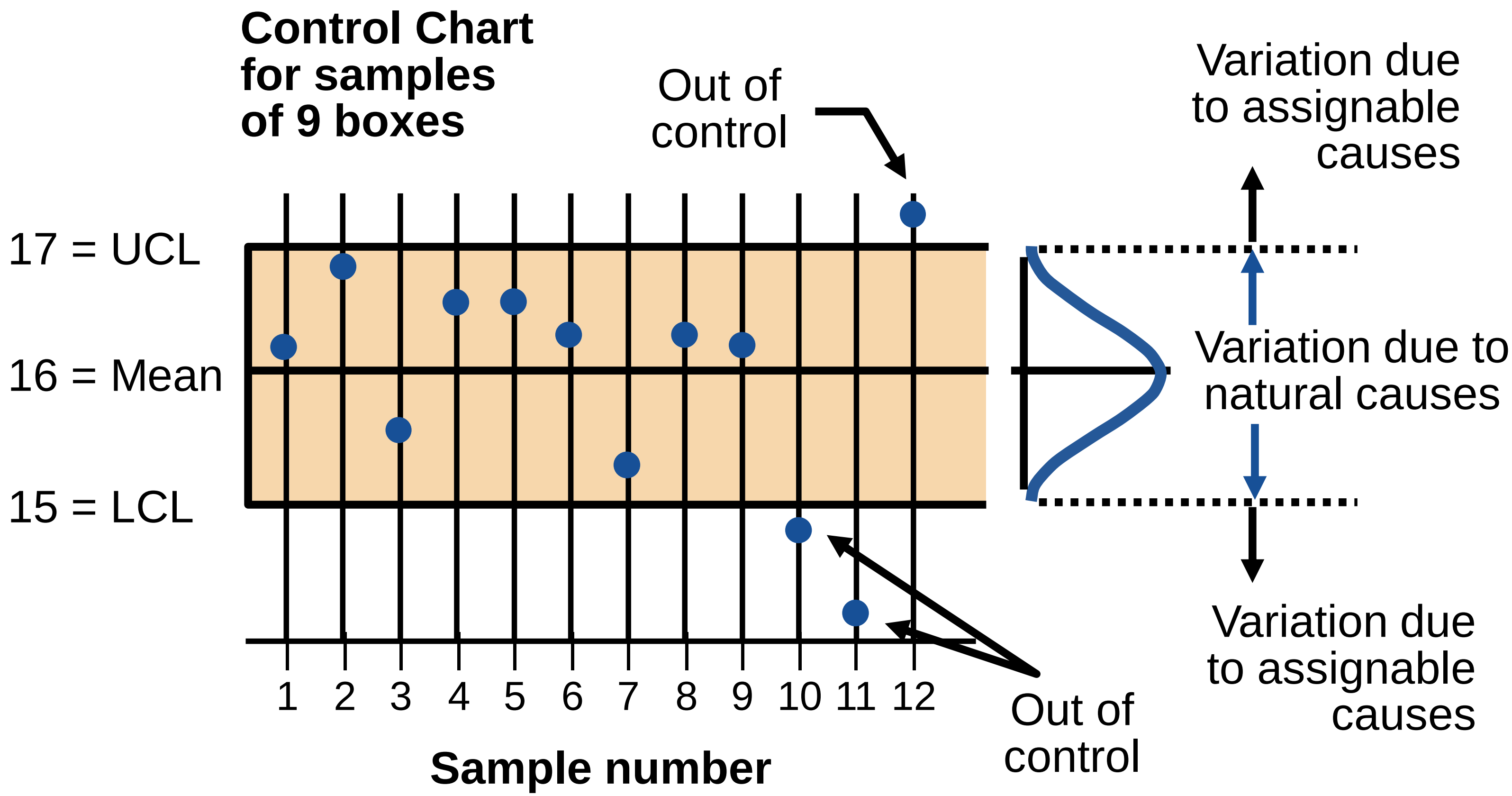
$z = 3$

$\sigma = 1$ ounce

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + z\sigma_{\bar{x}} = 16 + 3\left(\frac{1}{\sqrt{9}}\right) = 16 + 3\left(\frac{1}{3}\right) = 17 \text{ ounces}$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - z\sigma_{\bar{x}} = 16 - 3\left(\frac{1}{\sqrt{9}}\right) = 16 - 3\left(\frac{1}{3}\right) = 15 \text{ ounces}$$

Setting Control Limits



Setting Chart Limits

For $\bar{x}$-Charts when we don't know σ

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$$

where $\bar{R} = \dfrac{\sum_{i=1}^{n} R_i}{n}$ = average range of the samples

A_2 = control chart factor found in Table S6.1

$\bar{\bar{x}}$ = mean of the sample means

Control Chart Factors

TABLE S6.1	Factors for Computing Control Chart Limits (3 sigma)		
SAMPLE SIZE, n	**MEAN FACTOR, A_2**	**UPPER RANGE, D_4**	**LOWER RANGE, D_3**
2	1.880	3.268	0
3	1.023	2.574	0
4	.729	2.282	0
5	.577	2.115	0
6	.483	2.004	0
7	.419	1.924	0.076
8	.373	1.864	0.136
9	.337	1.816	0.184
10	.308	1.777	0.223
12	.266	1.716	0.284

Setting Control Limits

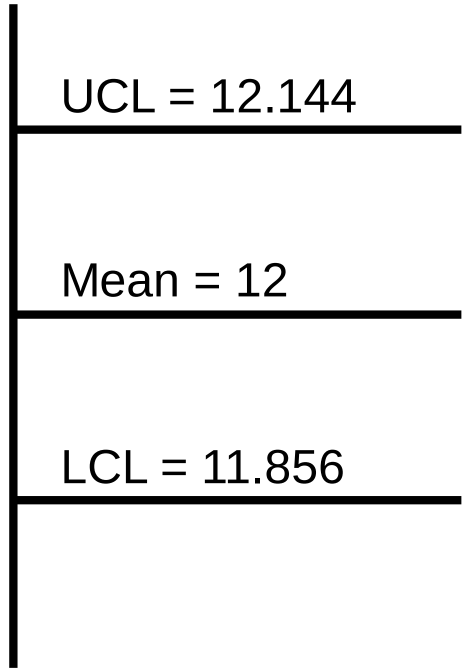
Super Cola Example
Labeled as “net weight 12 ounces”

Process average = 12 ounces
Average range = .25 ounce
Sample size = 5

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}$$
$$= 12 + (.577)(.25)$$
$$= 12 + .144$$
$$= 12.144 \text{ ounces}$$

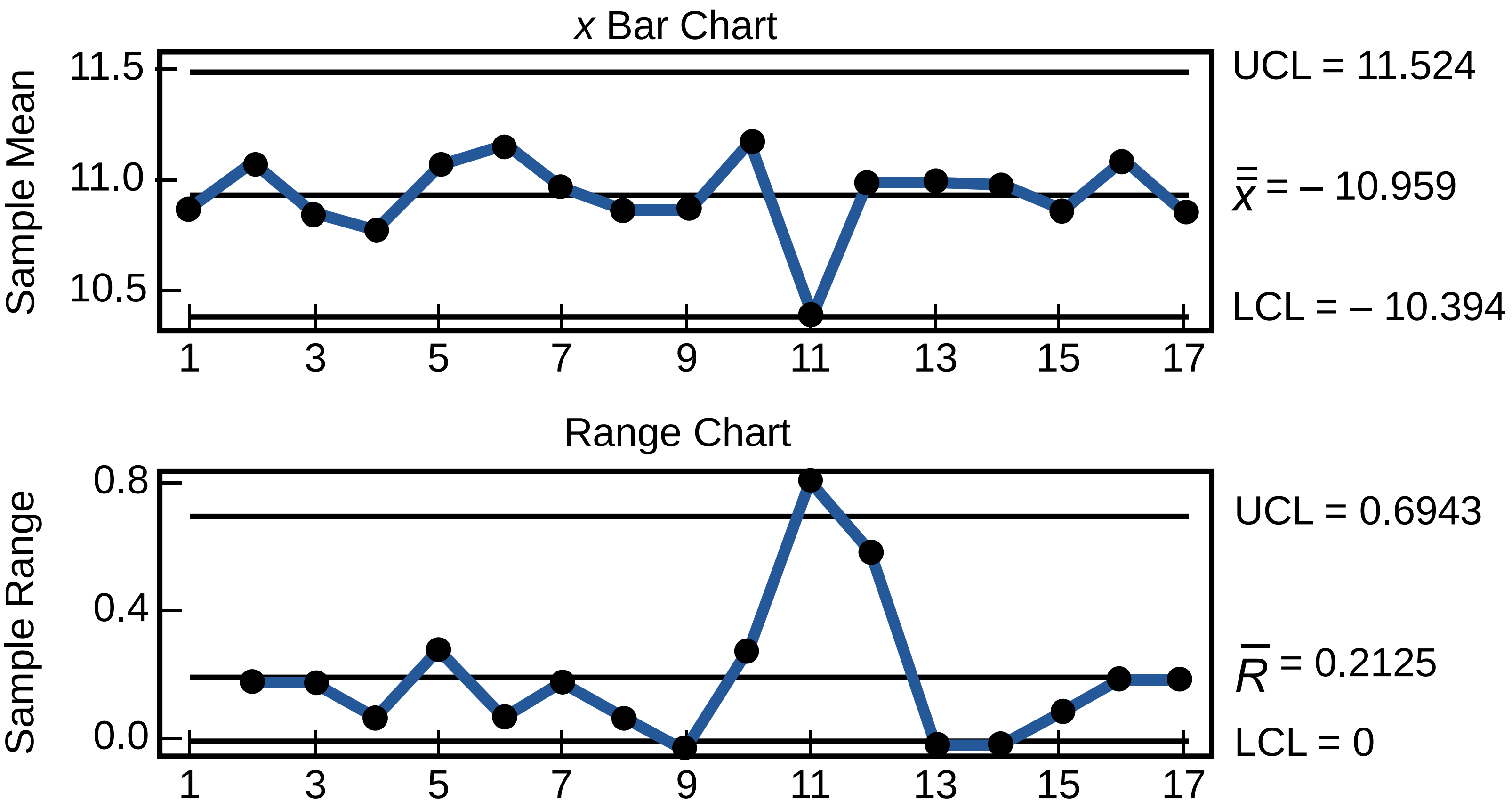
From Table S6.1

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$$
$$= 12 - .144$$
$$= 11.856 \text{ ounces}$$



Restaurant Control Limits

For salmon filets at Darden Restaurants



R – Chart

- Type of variables control chart
- Shows sample ranges over time
  - Difference between smallest and largest values in sample
- Monitors process variability
- Independent from process mean

Setting Chart Limits

For *R*-Charts

$$\text{Upper control limit } (\text{UCL}_R) = D_4\bar{R}$$

$$\text{Lower control limit } (\text{LCL}_R) = D_3\bar{R}$$

where

$\text{UCL}_{\bar{R}}$ = upper control chart limit for the range

$\text{LCL}_{\bar{R}}$ = lower control chart limit for the range

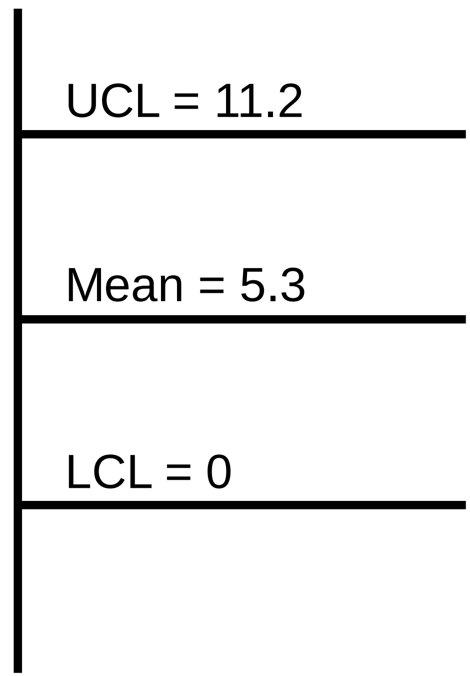
D_4 and D_3 = values from Table S6.1

Setting Control Limits

Average range = 5.3 pounds
Sample size = 5
From Table S6.1 D_4 = 2.115, D_3 = 0

$$\text{UCL}_R = D_4\overline{R} = (2.115)(5.3) = 11.2 \text{ pounds}$$

$$\text{LCL}_R = D_3\overline{R} = (0)(5.3) = 0 \text{ pounds}$$

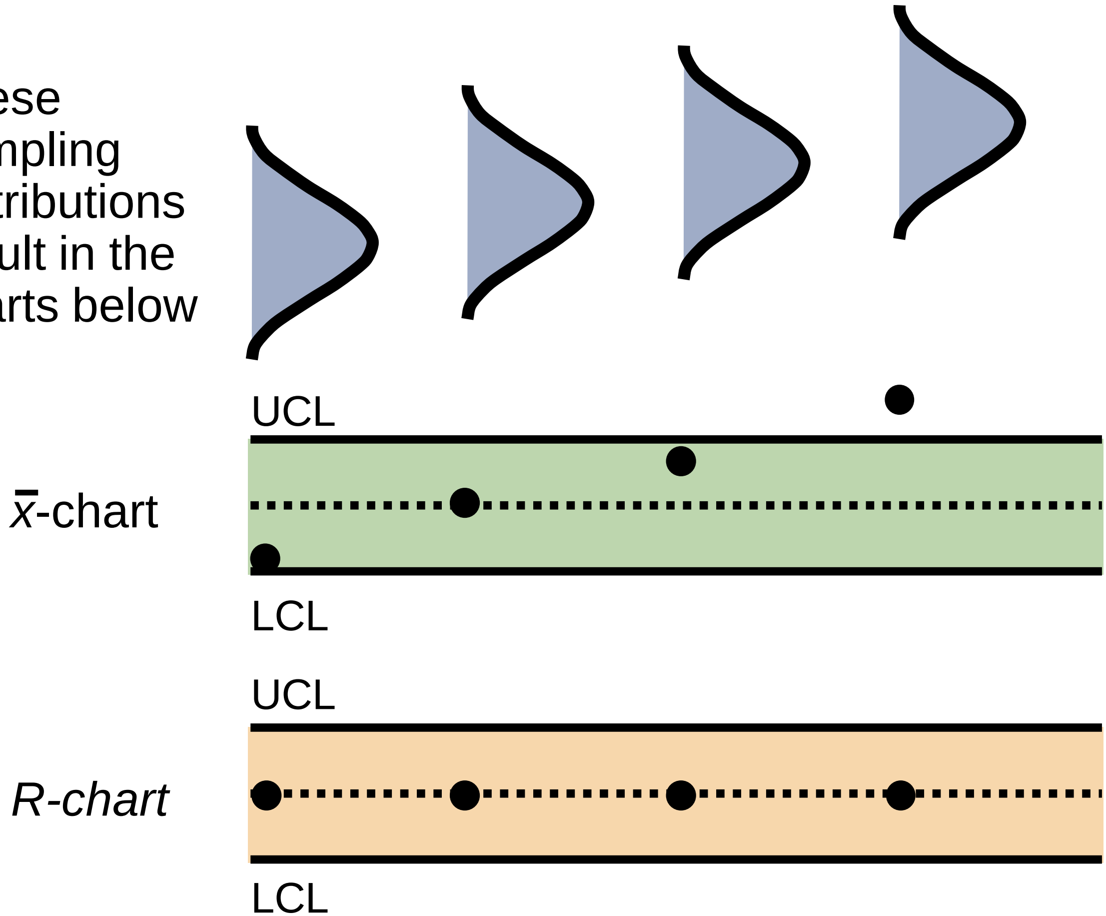


Mean and Range Charts

(a)

These sampling distributions result in the charts below

(Sampling mean is shifting upward, but range is consistent)



($\bar{x}$-chart detects shift in central tendency)

(*R*-chart does not detect change in mean)

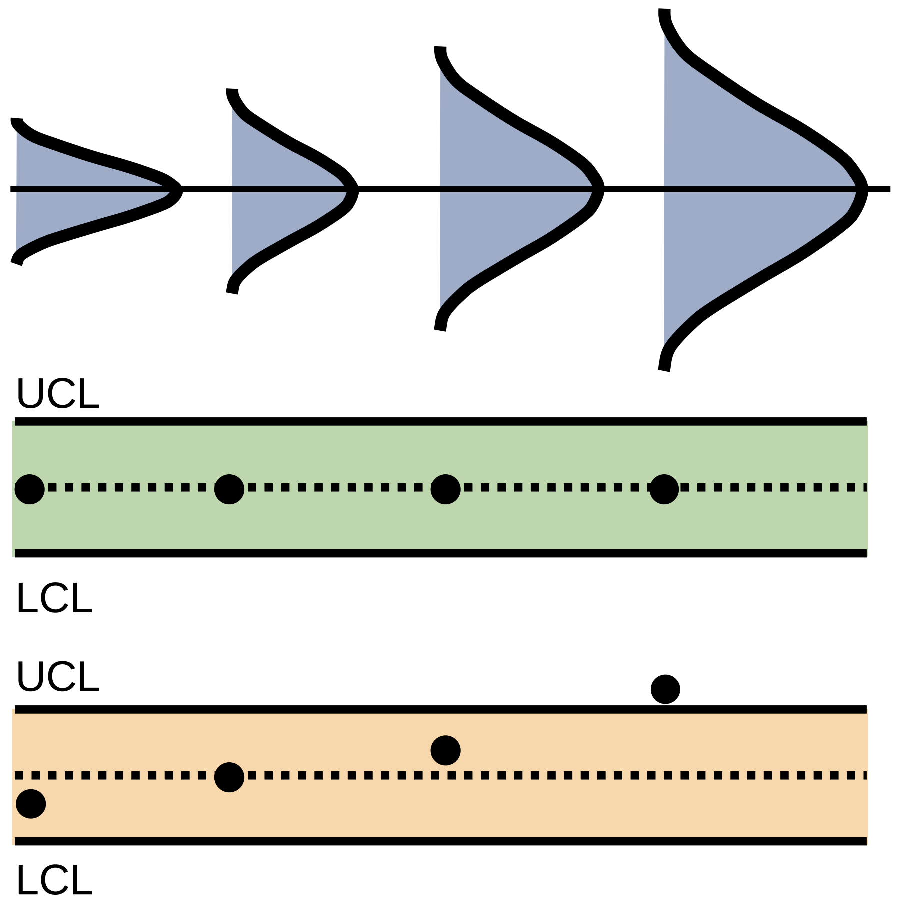
Figure S6.5

Mean and Range Charts

(b)

These sampling distributions result in the charts below

(Sampling mean is constant, but dispersion is increasing)



$\bar{x}$-chart

($\bar{x}$-chart indicates no change in central tendency)

R-chart

(*R*-chart detects increase in dispersion)

Figure S6.5

Steps In Creating Control Charts

1. Collect 20 to 25 samples, often of $n = 4$ or $n = 5$ observations each, from a stable process and compute the mean and range of each
2. Compute the overall means ($\bar{\bar{x}}$ and $\bar{R}$), set appropriate control limits, usually at the 99.73% level, and calculate the preliminary upper and lower control limits
   - If the process is not currently stable and in control, use the desired mean, μ, instead of $\bar{\bar{x}}$ to calculate limits.

Steps In Creating Control Charts

3. Graph the sample means and ranges on their respective control charts and determine whether they fall outside the acceptable limits
4. Investigate points or patterns that indicate the process is out of control – try to assign causes for the variation, address the causes, and then resume the process
5. Collect additional samples and, if necessary, revalidate the control limits using the new data

Setting Other Control Limits

TABLE S6.2	Common *z* Values
DESIRED CONTROL LIMIT (%)	*Z*-VALUE (STANDARD DEVIATION REQUIRED FOR DESIRED LEVEL OF CONFIDENCE
90.0	1.65
95.0	1.96
95.45	2.00
99.0	2.58
99.73	3.00

Control Charts for Attributes

- For variables that are categorical
  - Defective/nondefective, good/bad, yes/no, acceptable/unacceptable
- Measurement is typically counting defectives
- Charts may measure
  1. *Percent* defective (p-chart)
  2. *Number* of defects (c-chart)

Control Limits for *p*-Charts

Population will be a binomial distribution, but applying the Central Limit Theorem allows us to assume a normal distribution for the sample statistics

$$UCL_p = \bar{p} + z\sigma_{\hat{p}}$$

$$LCL_p = \bar{p} - z\sigma_{\hat{p}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where

$\bar{p}$ = mean fraction (percent) defective in the samples

z = number of standard deviations

$\sigma_{\hat{p}}$ = standard deviation of the sampling distribution

n = number of observations in *each* sample

p-Chart for Data Entry

SAMPLE NUMBER	NUMBER OF ERRORS	FRACTION DEFECTIVE	SAMPLE NUMBER	NUMBER OF ERRORS	FRACTION DEFECTIVE
1	6	.06	11	6	.06
2	5	.05	12	1	.01
3	0	.00	13	8	.08
4	1	.01	14	7	.07
5	4	.04	15	5	.05
6	2	.02	16	4	.04
7	5	.05	17	11	.11
8	3	.03	18	3	.03
9	3	.03	19	0	.00
10	2	.02	20	4	.04
				80	

p-Chart for Data Entry

$$\bar{p} = \frac{\text{Total number of errors}}{\text{Total number of records examined}} = \frac{80}{(100)(20)} = .04$$

$$s_{\hat{p}} = \sqrt{\frac{(.04)(1-.04)}{100}} = .02 \text{ (rounded up from .0196)}$$

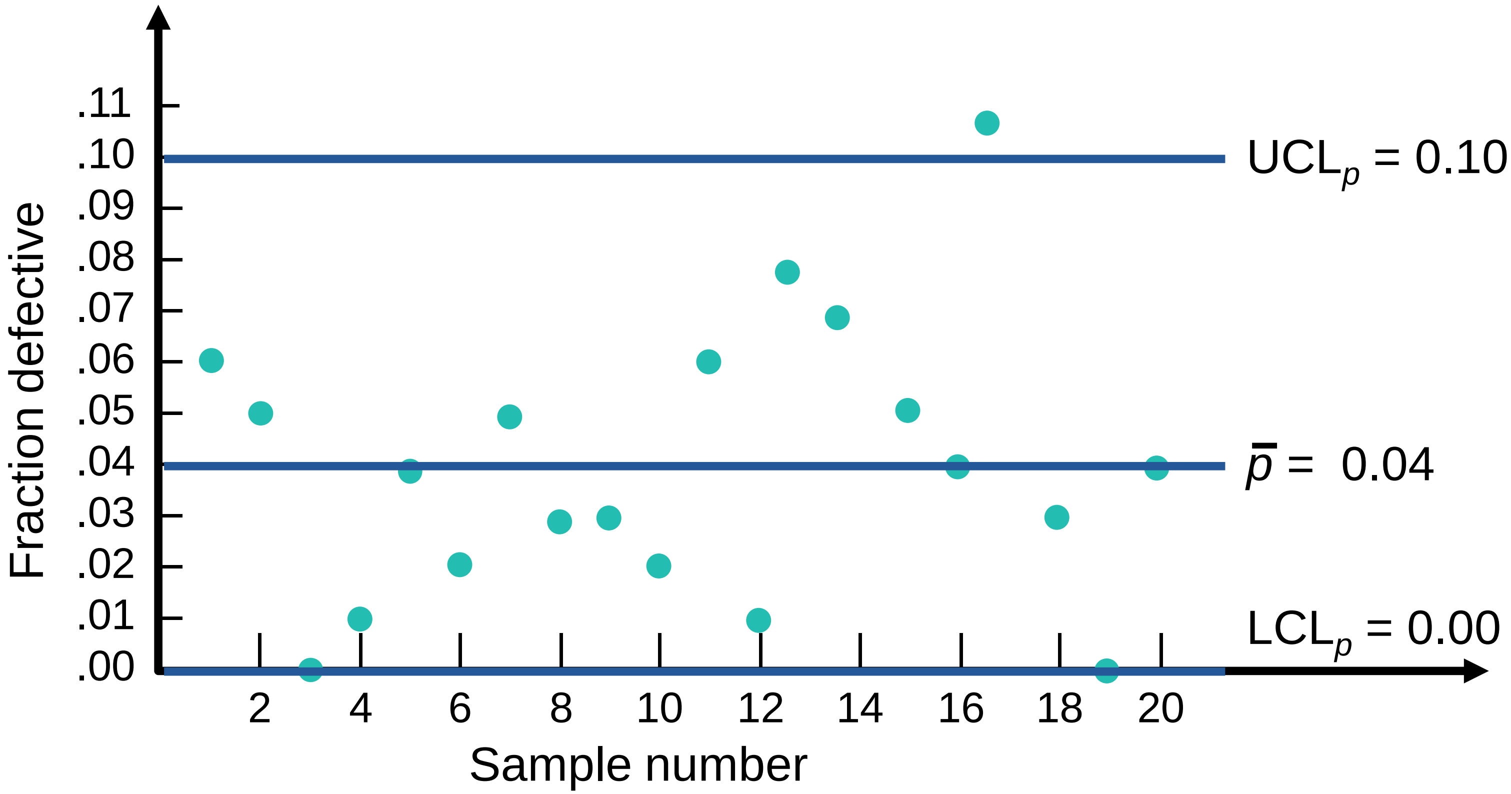
$$\text{UCL}_p = \bar{p} + zs_{\hat{p}} = .04 + 3(.02) = .10$$

$$\text{LCL}_p = \bar{p} - zs_{\hat{p}} = .04 - 3(.02) = 0$$

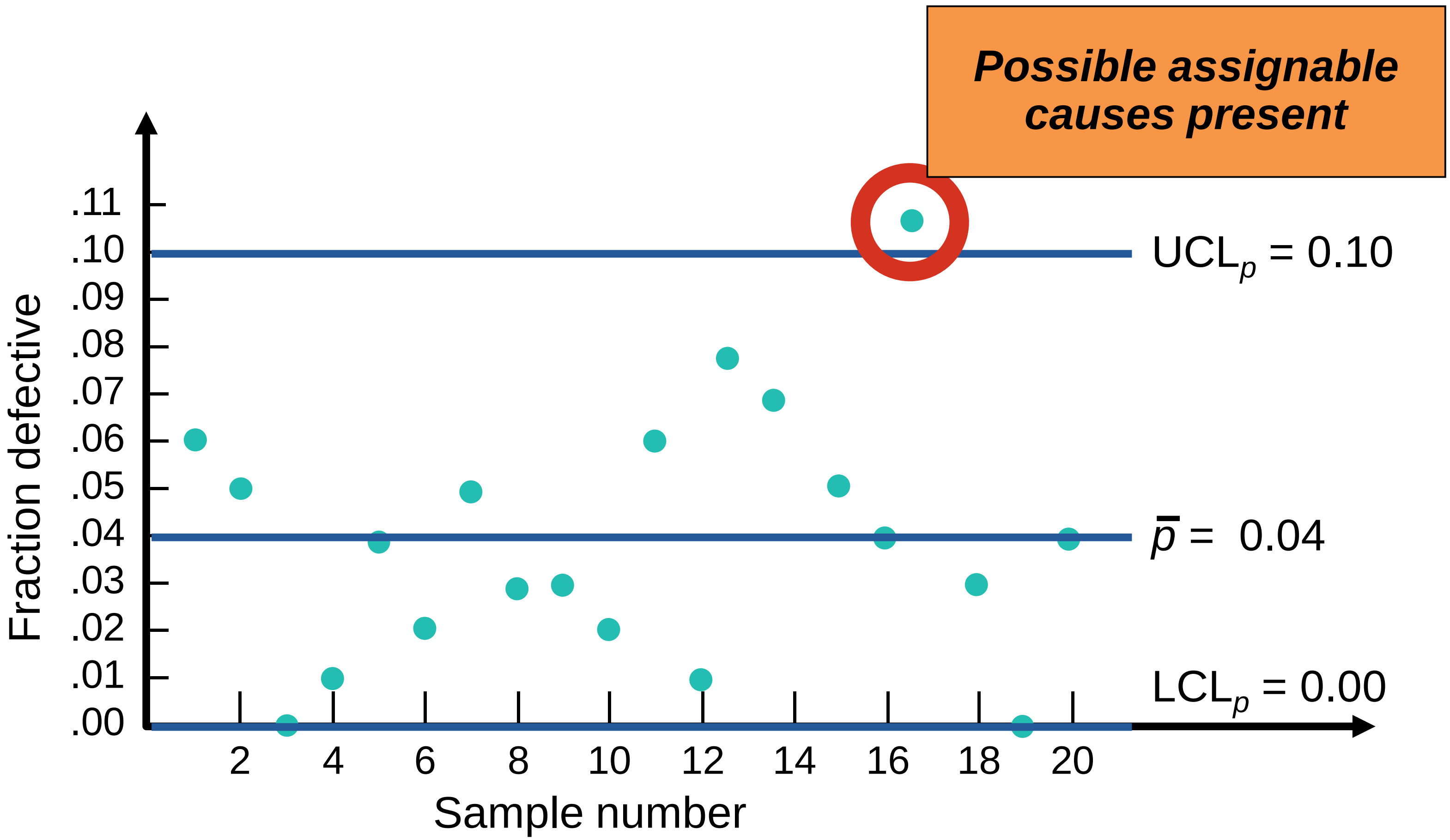
(because we cannot have a negative percent defective)

10	2	.02	20	

p-Chart for Data Entry



p-Chart for Data Entry



Control Limits for *c*-Charts

Population will be a Poisson distribution, but applying the Central Limit Theorem allows us to assume a normal distribution for the sample statistics

$\bar{c}$ = mean number of defects per unit

$\sqrt{\bar{c}}$ = standard deviation of defects per unit

$$\text{Control limits (99.73\%)} = \bar{c} \pm 3\sqrt{\bar{c}}$$

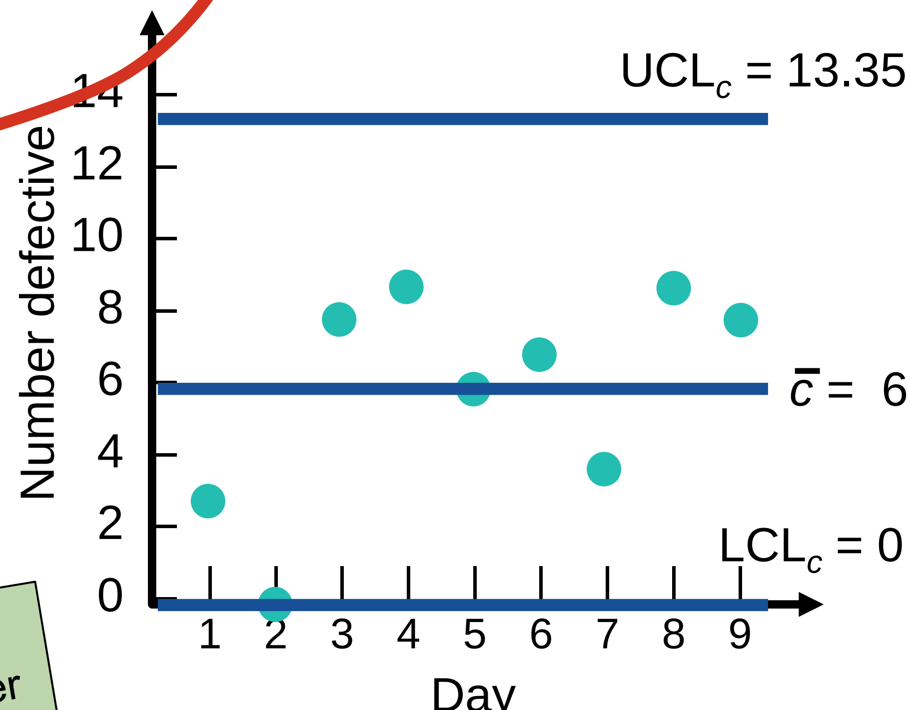
c-Chart for Cab Company

$\bar{c}$ = 54 complaints/9 days = 6 complaints/day

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$$
$$= 6 + 3\sqrt{6}$$
$$= 13.35$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$
$$= 6 - 3\sqrt{6}$$
$$= 0$$

Cannot be a negative number



Managerial Issues and Control Charts

Three major management decisions:

- Select points in the processes that need SPC
- Determine the appropriate charting technique
- Set clear policies and procedures

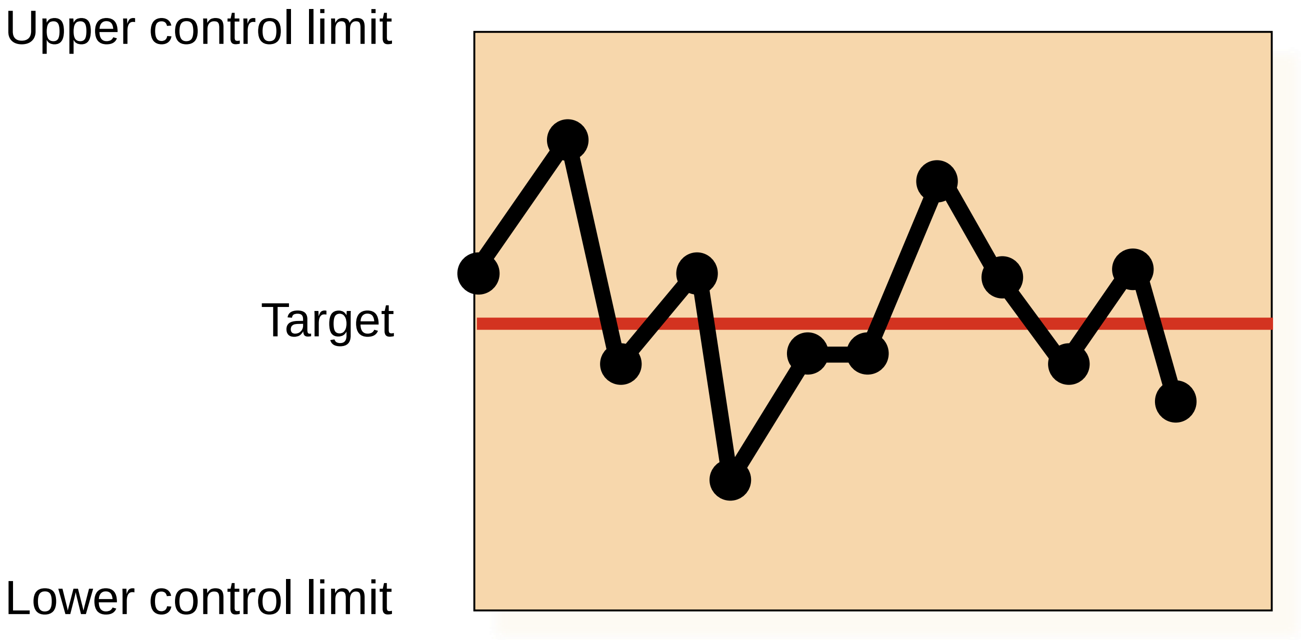
Which Control Chart to Use

TABLE S6.3	Helping You Decide Which Control Chart to Use
VARIABLE DATA **USING AN $\bar{x}$-CHART AND *R*-CHART**	
1. Observations are *variables* 2. Collect 20 - 25 samples of n = 4, or n = 5, or more, each from a stable process and compute the mean for the $\bar{x}$-chart and range for the R-chart 3. Track samples of n observations	

Which Control Chart to Use

TABLE S6.3	Helping You Decide Which Control Chart to Use
ATTRIBUTE DATA USING A *P*-CHART	
1. Observations are *attributes* that can be categorized as good or bad (or pass–fail, or functional–broken), that is, in two states 2. We deal with fraction, proportion, or percent defectives 3. There are several samples, with many observations in each	
ATTRIBUTE DATA USING A *C*-CHART	
1. Observations are *attributes* whose defects per unit of output can be counted 2. We deal with the number counted, which is a small part of the possible occurrences 3. Defects may be: number of blemishes on a desk; crimes in a year; broken seats in a stadium; typos in a chapter of this text; flaws in a bolt of cloth	

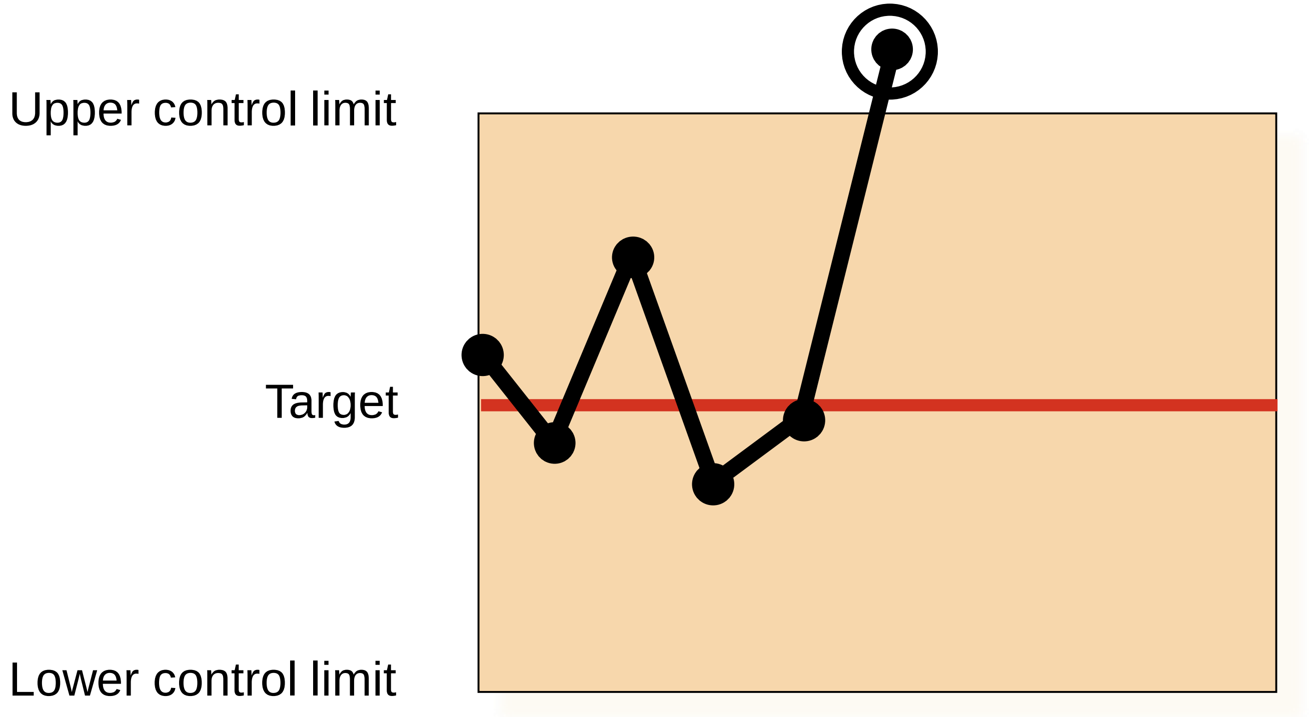
Patterns in Control Charts



Normal behavior. Process is "in control."

Figure S6.7

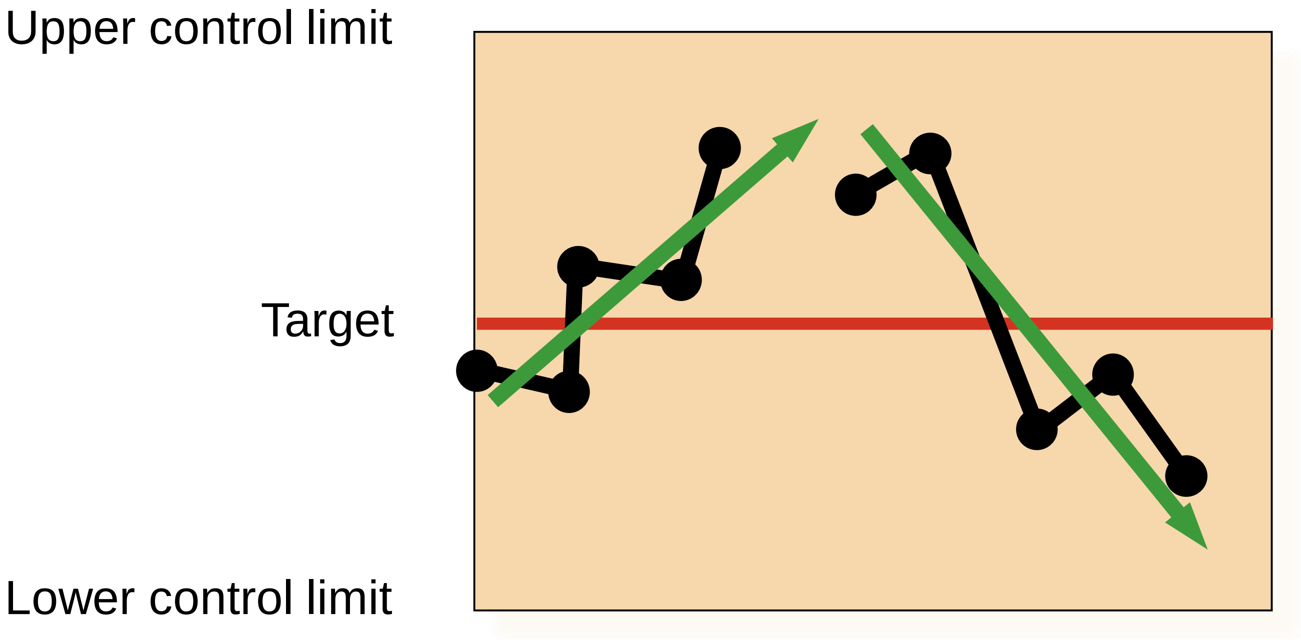
Patterns in Control Charts



One plot out above (or below). Investigate for cause. Process is "out of control."

Figure S6.7

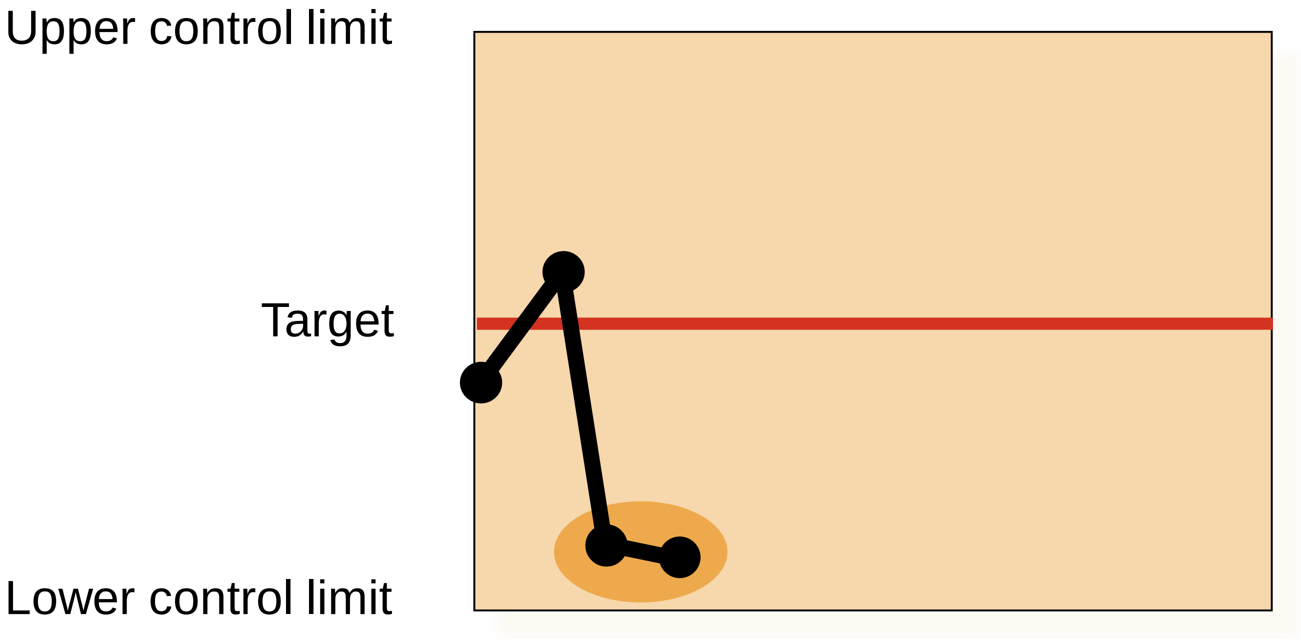
Patterns in Control Charts



Trends in either direction, 5 plots.
Investigate for cause of
progressive change.

Figure S6.7

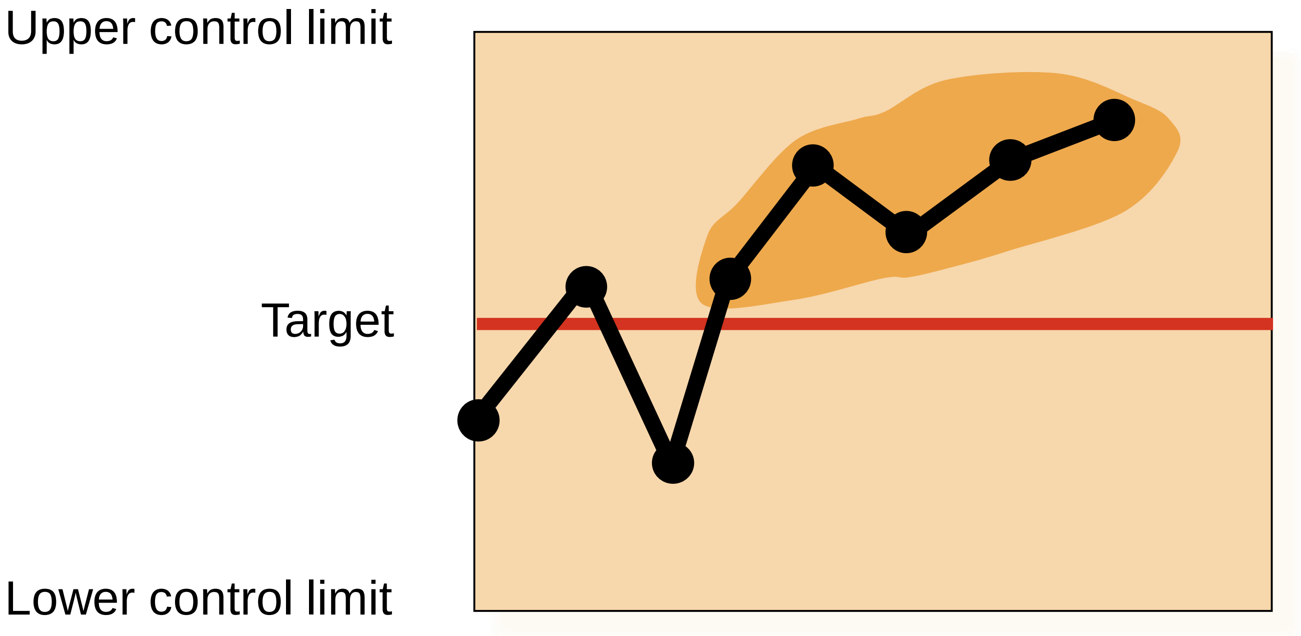
Patterns in Control Charts



Two plots very near lower (or upper) control. Investigate for cause.

Figure S6.7

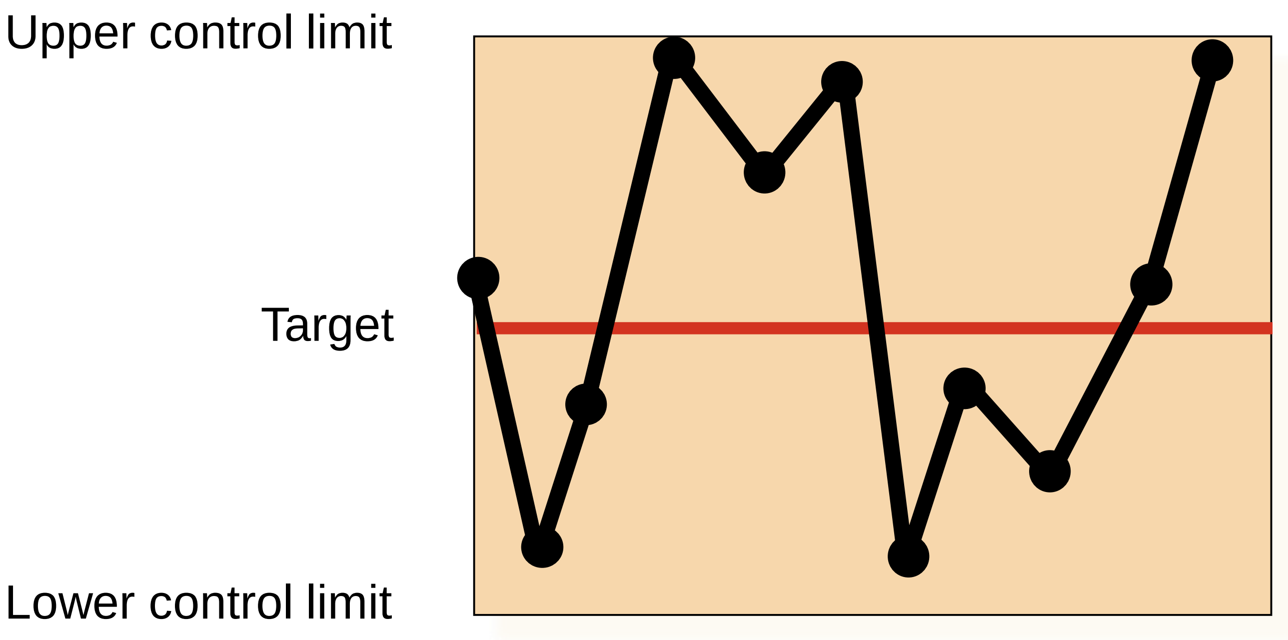
Patterns in Control Charts



Run of 5 above (or below) central line. Investigate for cause.

Figure S6.7

Patterns in Control Charts



Erratic behavior. Investigate.

Figure S6.7

Process Capability

- The natural variation of a process should be small enough to produce products that meet the standards required
- A process in statistical control does not necessarily meet the design specifications
- **Process capability** is a measure of the relationship between the natural variation of the process and the design specifications

Process Capability Ratio

$$C_p = \frac{\text{Upper Specification} - \text{Lower Specification}}{6\sigma}$$

- A capable process must have a C_p of at least 1.0
- Does not look at how well the process is centered in the specification range
- Often a target value of $C_p = 1.33$ is used to allow for off-center processes
- Six Sigma quality requires a $C_p = 2.0$

Process Capability Ratio

Insurance claims process

Process mean $\bar{x}$ = 210.0 minutes
Process standard deviation σ = .516 minutes
Design specification = 210 ± 3 minutes

$$C_p = \frac{\text{Upper Specification - Lower Specification}}{6\sigma}$$

Process Capability Ratio

Insurance claims process

Process mean $\bar{x}$ = 210.0 minutes
Process standard deviation σ = .516 minutes
Design specification = 210 ± 3 minutes

$$C_p = \frac{\text{Upper Specification - Lower Specification}}{6\sigma}$$

$$= \frac{213 - 207}{6(.516)} = 1.938$$

Process Capability Ratio

Insurance claims process

Process mean $\bar{x}$ = 210.0 minutes
Process standard deviation σ = .516 minutes
Design specification = 210 ± 3 minutes

$$C_p = \frac{\text{Upper Specification - Lower Specification}}{6\sigma}$$

$$= \frac{213 - 207}{6(.516)} = 1.938$$

Process is capable

Process Capability Index

$$C_{pk} = \text{minimum of } \left[\frac{\text{Upper Specification Limit} - \bar{x}}{3\sigma}, \frac{\bar{x} - \text{Lower Specification Limit}}{3\sigma} \right]$$

- A capable process must have a C_{pk} of at least 1.0
- A capable process is not necessarily in the center of the specification, but it falls within the specification limit at both extremes

Process Capability Index

New Cutting Machine

New process mean $\bar{x}$ = .250 inches
Process standard deviation σ = .0005 inches
Upper Specification Limit = .251 inches
Lower Specification Limit = .249 inches

Process Capability Index

New Cutting Machine

New process mean $\bar{x}$ = .250 inches
Process standard deviation σ = .0005 inches
Upper Specification Limit = .251 inches
Lower Specification Limit = .249 inches

$$C_{pk} = \text{minimum of} \left[\frac{(.251) - .250}{(3).0005} \right]$$

Process Capability Index

New Cutting Machine

New process mean $\bar{x}$ = .250 inches
Process standard deviation σ = .0005 inches
Upper Specification Limit = .251 inches
Lower Specification Limit = .249 inches

$$C_{pk} = \text{minimum of} \left[\frac{(.251) - .250}{(3).0005}\right], \left[\frac{.250 - (.249)}{(3).0005}\right]$$

Process Capability Index

New Cutting Machine

New process mean $\bar{x}$ = .250 inches
Process standard deviation σ = .0005 inches
Upper Specification Limit = .251 inches
Lower Specification Limit = .249 inches

$$C_{pk} = \text{minimum of} \left[\frac{(.251) - .250}{(3).0005} \right], \left[\frac{.250 - (.249)}{(3).0005} \right]$$

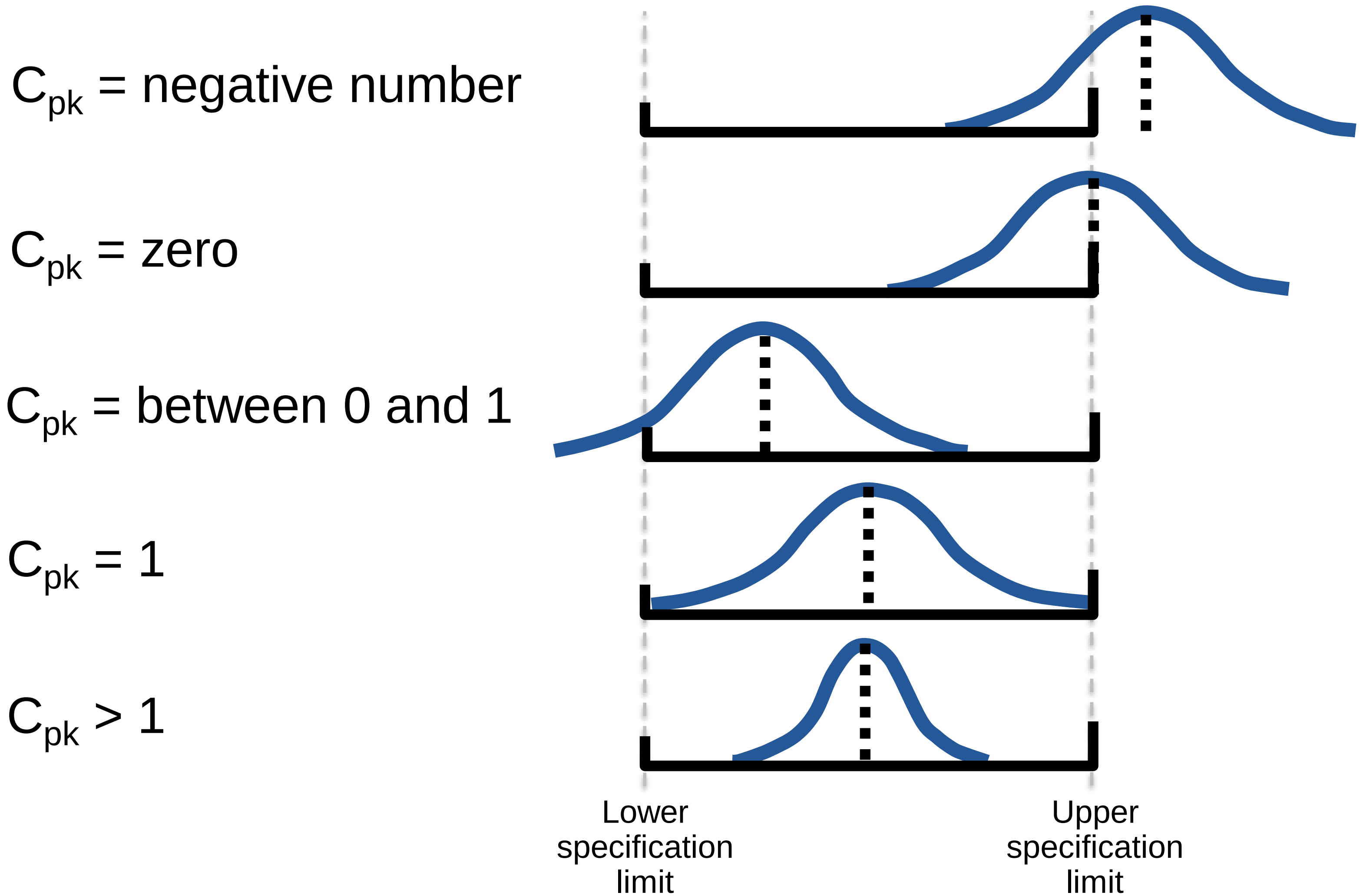
Both calculations result in

$$C_{pk} = \frac{.001}{.0015} = 0.67$$

New machine is NOT capable

Interpreting C_{pk}

Figure S6.8



Acceptance Sampling

- Form of quality testing used for incoming materials or finished goods
  - Take samples at random from a lot (shipment) of items
  - Inspect each of the items in the sample
  - Decide whether to reject the whole lot based on the inspection results
- Only screens lots; does not drive quality improvement efforts

Acceptance Sampling

- Form of quality testing used for incoming materials or finis
  - Take samples (shipment) of
  - Inspect each
  - Decide whethe based on the i
- Only screens lo improvement ef

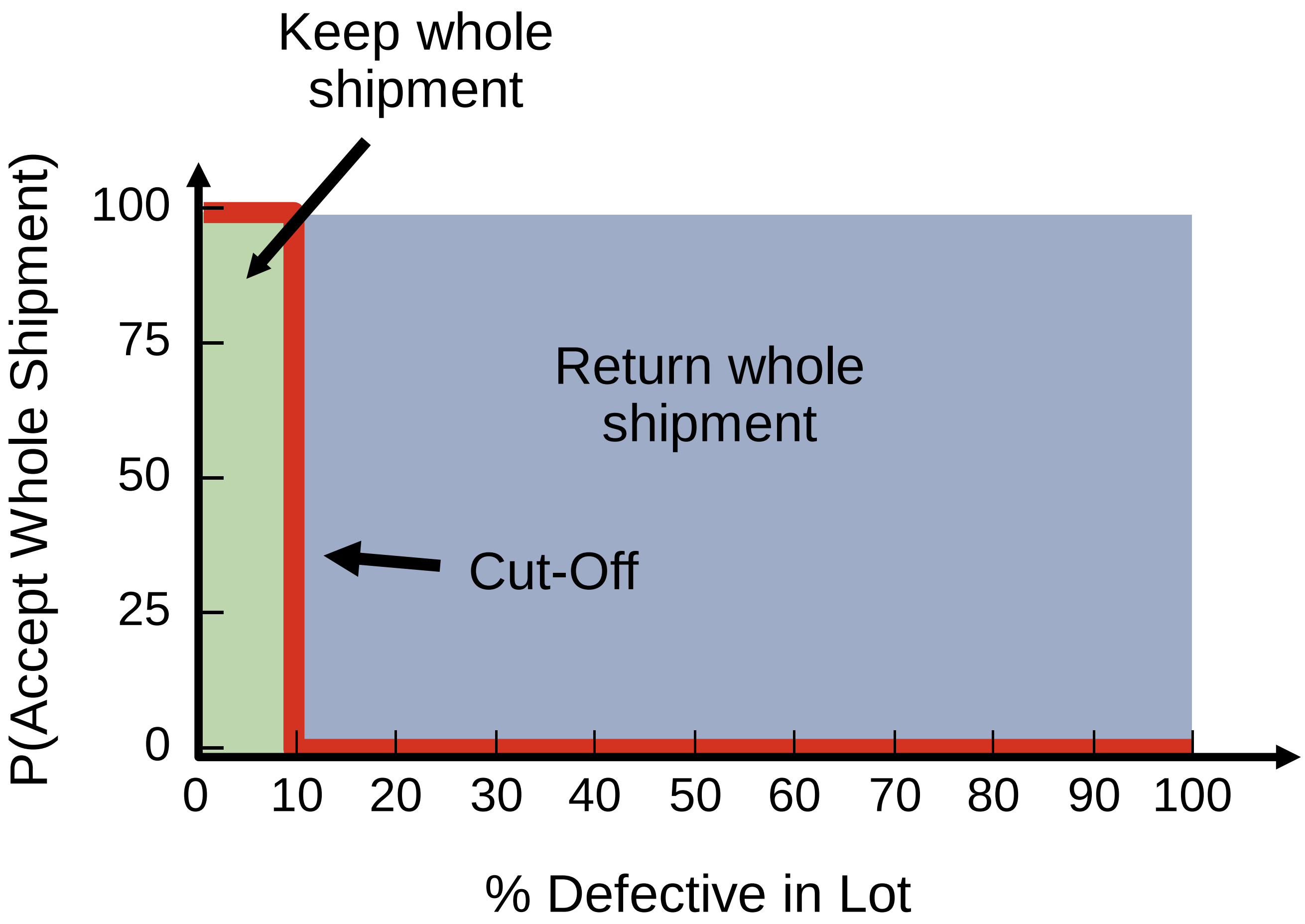
Rejected lots can be:

1. Returned to the supplier
2. Culled for defectives (100% inspection)
3. May be re-graded to a lower specification

Operating Characteristic Curve

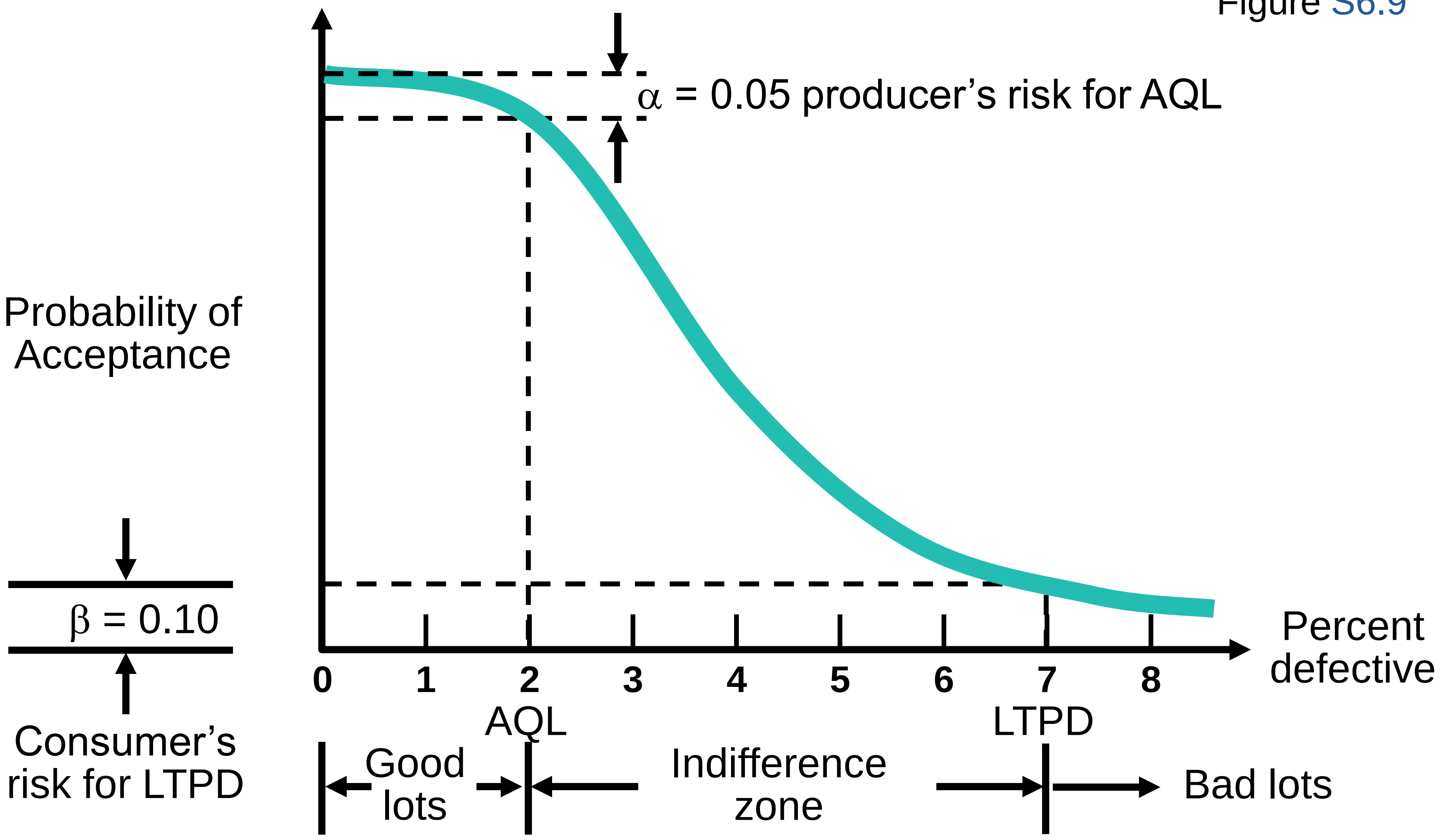
- Shows how well a sampling plan discriminates between good and bad lots (shipments)
- Shows the relationship between the probability of accepting a lot and its quality level

The “Perfect” OC Curve



An OC Curve

Figure S6.9



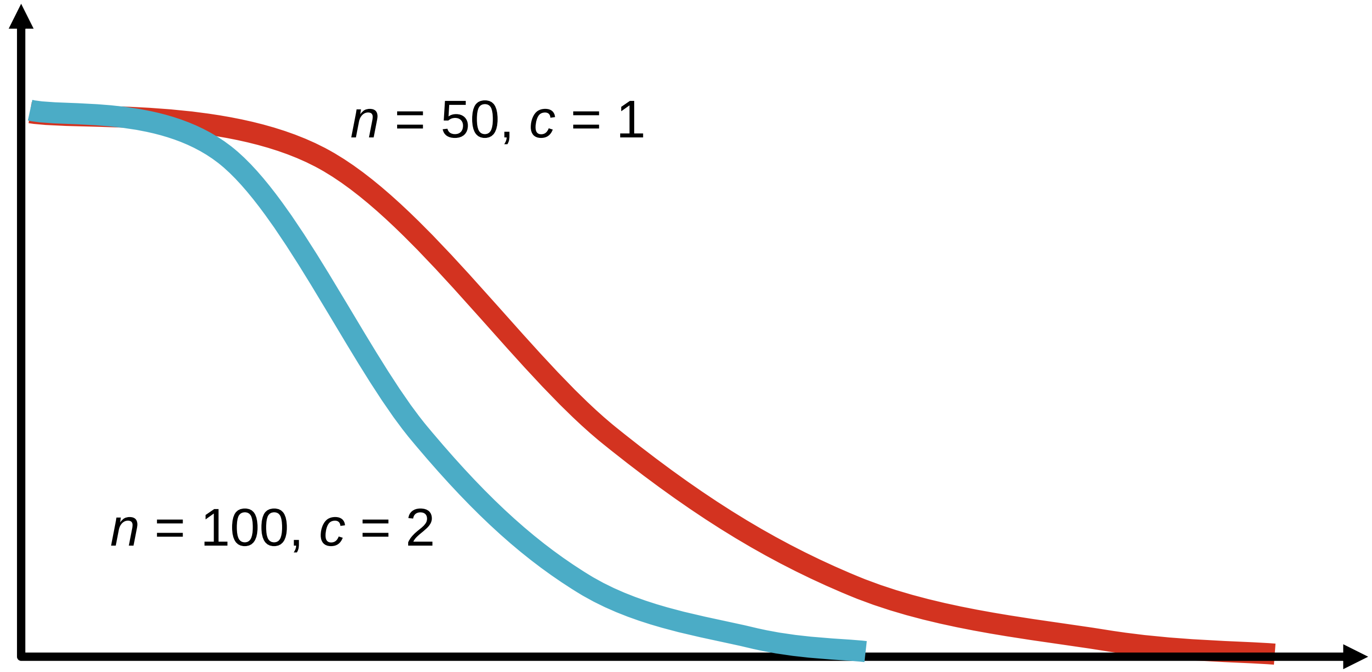
AQL and LTPD

- Acceptable Quality Level (AQL)
  - Poorest level of quality we are willing to accept
- Lot Tolerance Percent Defective (LTPD)
  - Quality level we consider bad
  - Consumer (buyer) does not want to accept lots with more defects than LTPD

Producer's and Consumer's Risks

- Producer's risk (α)
  - Probability of rejecting a good lot
  - Probability of rejecting a lot when the fraction defective is at or above the AQL
- Consumer's risk (β)
  - Probability of accepting a bad lot
  - Probability of accepting a lot when fraction defective is below the LTPD

OC Curves for Different Sampling Plans



Average Outgoing Quality

$$\text{AOQ} = \frac{(P_d)(P_a)(N - n)}{N}$$

where

P_d = true percent defective of the lot

P_a = probability of accepting the lot

N = number of items in the lot

n = number of items in the sample

Average Outgoing Quality

1. If a sampling plan replaces all defectives
2. If we know the incoming percent defective for the lot

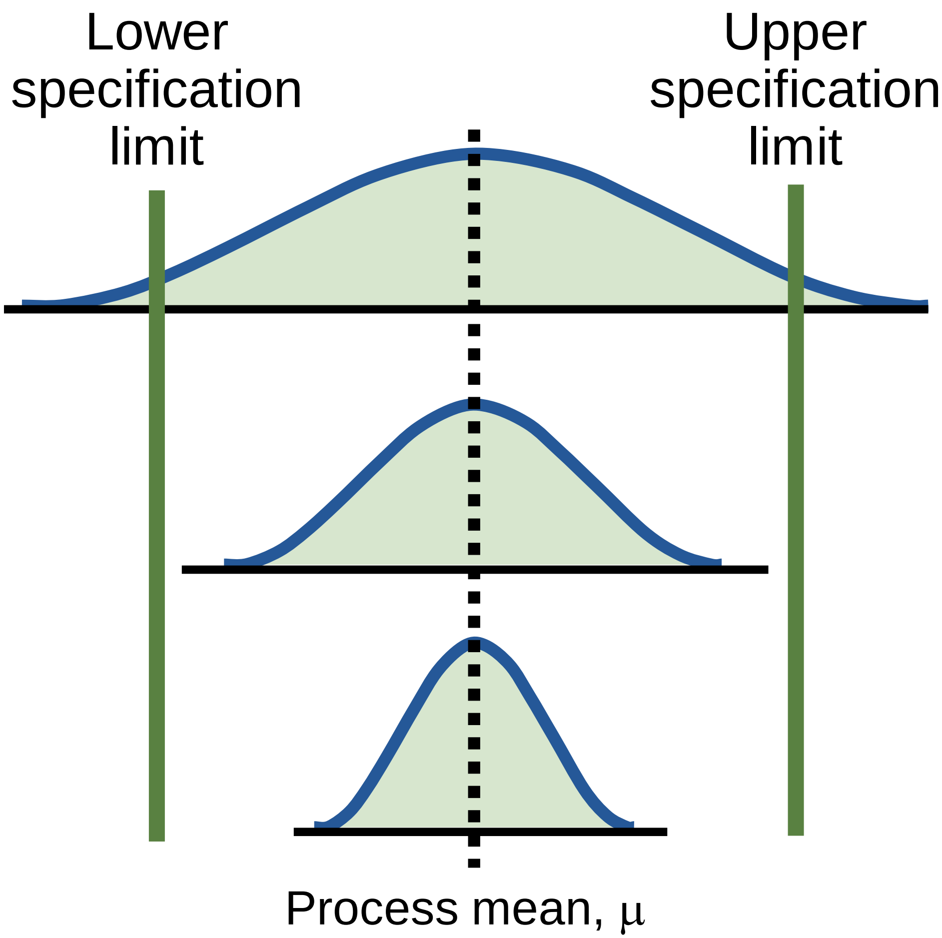
We can compute the average outgoing quality (AOQ) in percent defective

The maximum AOQ is the highest percent defective or the lowest average quality and is called the *average outgoing quality limit* (AOQL)

Automated Inspection

- Modern technologies allow virtually 100% inspection at minimal costs
- Not suitable for all situations

SPC and Process Variability



(a) Acceptance sampling (Some bad units accepted; the "lot" is good or bad)

(b) Statistical process control (Keep the process "in control")

(c) C_{pk} > 1 (Design a process that is in within specification)

Figure S6.10