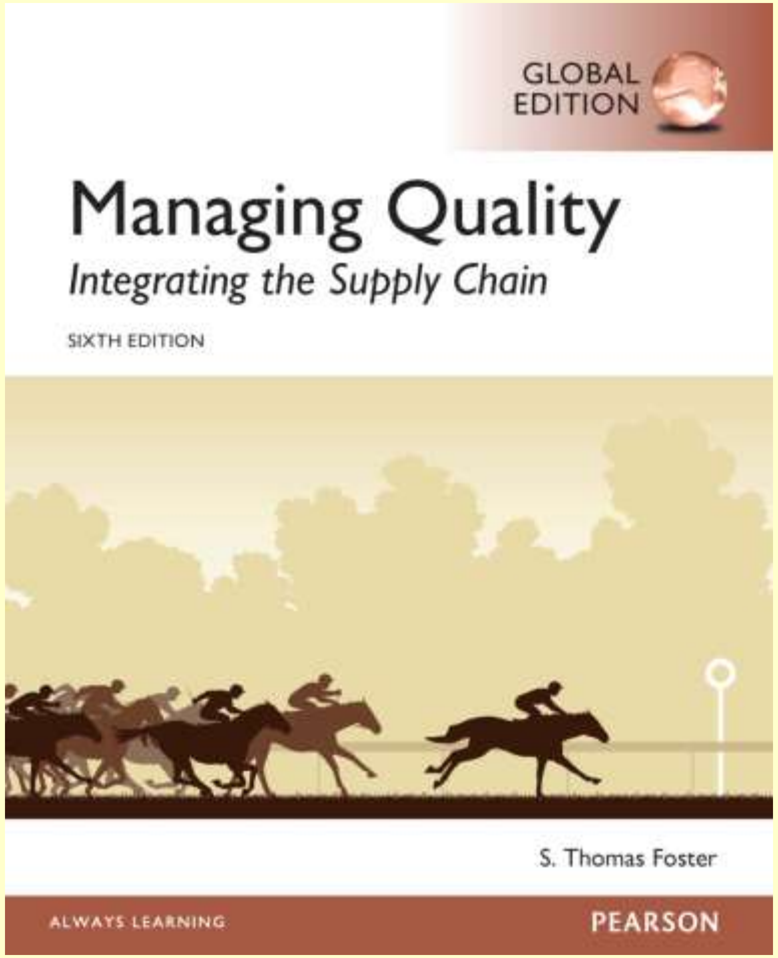


Chapter 11

Statistically Based Quality Improvements for Variables

Chapter Objectives

1. Discuss the basics of process variation and applied statistical methods.
2. Demonstrate the differences between random and non-random variation.
3. Implement $\overline{x}$, *R, X, MR, median,* and *s* charts.
4. Develop control charts using Excel.
5. Interpret control charts.

Statistical Thinking

- **Statistical thinking is a decision-making skill demonstrated by the ability to draw conclusions based on data.**

- **Statistical thinking is based on three concepts:**
  - All work occurs in a system of interconnected processes.
  - All processes have variation (the amount of variation tends to be underestimated).
  - Understanding variation and reducing variation are important keys to success.

Why Do Statistics Sometimes Fail in the Workplace?

- Statistics emphasizing mathematical development rather than application.
- Cultural barriers in a company make the use of statistics for continual improvement difficult. (Blame, structure, real data)
- Statistics are viewed as something to buttress an already-held opinion rather than informing decision-making.
- Statistical specialists have trouble communicating with managerial generalists.
- People have a poor understanding of the scientific method

why statistical decisions can fail in practice even when the math is correct.

The failure here stems from a misunderstanding of risk, not from poor statistics.

Type 1 error

- Producer's risk
- Rejecting a good product or process by mistake.
- Example: A student answers correctly, but the examiner marks it wrong.

Type 2 error

- Consumer's risk
- Probability that a nonconforming product will be available for sale
- Accepting a bad product or process by mistake
- Example: A student answers incorrectly, but still passes the exam.

Understanding Process Variation

- **All processes exhibit variation**
  - Some variation can be managed, and some cannot be managed.
- **Types of process variation:**
  - Random
  - Nonrandom

Random Variation

- Also called *common cause*
- Centered around the mean and occurs with a somewhat consistent amount of dispersion
- Uncontrolled variation
- May be either large or small

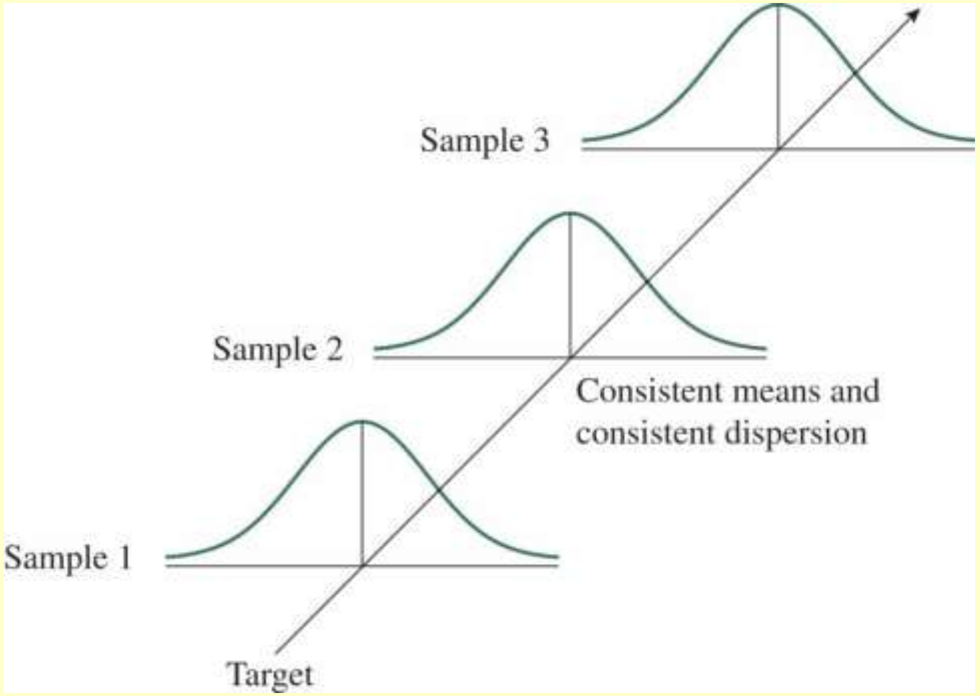


Figure 11-1

Nonrandom Variation

- Also called *special cause variation*
- Results from some event which may be a shift in a process mean or some unexpected occurrence
- Dispersion and average of the process are changing
- Process is not repeatable

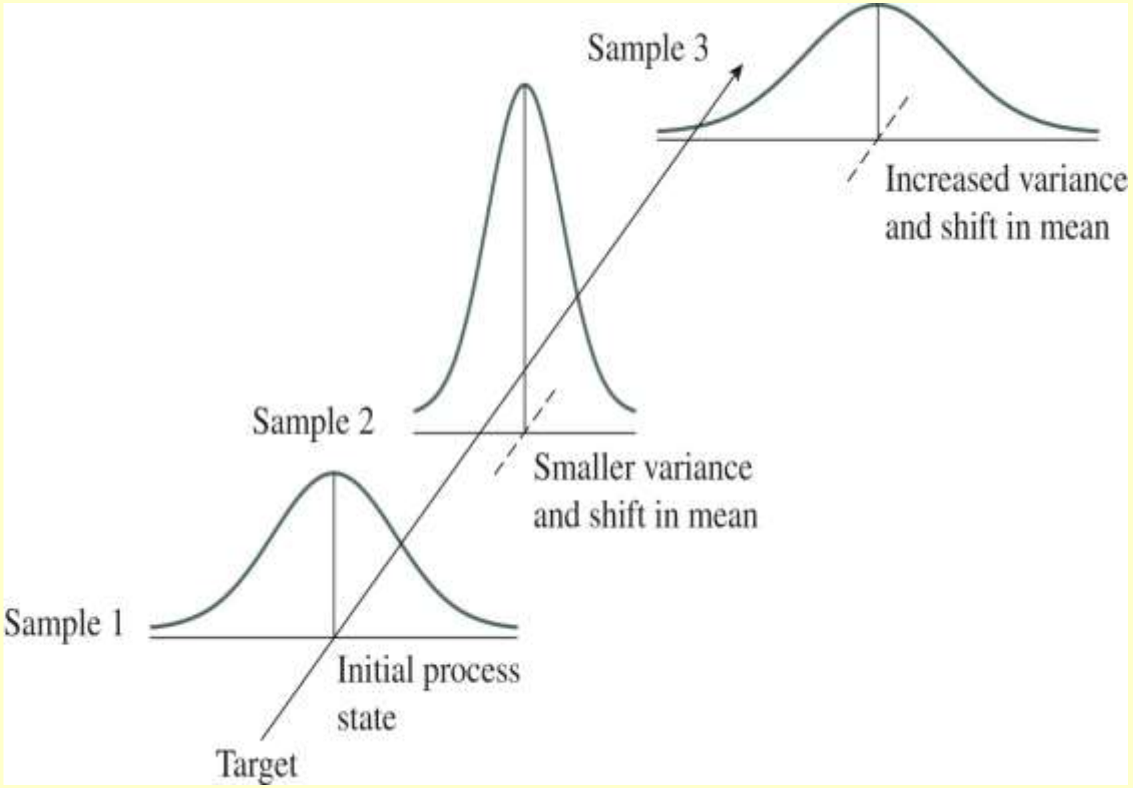


Figure 11-2

Process Stability

- **The variation that we observe in the process is random variation and not nonrandom.**

- **Process charts**
  - Graphs designed to signal process workers when nonrandom variation is occurring in a process

Sampling Methods

Reasons why sampling is used:

- Samples are cheaper, take less time, are less pushy, and allow the user to frame the sample.
- If quality testing is destructive, 100% inspection would be impossible.

Sampling Methods

Reasons why 100% inspection is used:

- When a lot of material has been rejected in the past and materials must be sorted to keep good materials and return defective materials for a refund
- When employees perform their own in-process inspection

Sampling Methods

- **Random samples**
  - To sample in such a way that every piece or product has an equal chance of being selected for inspection
- **Systematic samples**
  - To sample according to time or according to sequence
- **Rational subgroup samples**
  - To sample a group of data that is logically homogenous

Control Plans

- Provide a documented, proactive approach to defining how to respond when process control charts show that a process is out of control

- Required part of an ISO 9000 quality management system (QMS)

Control Plan Sample

Part Number:	Part Name Description:					Plant:			
	Sprinkler Head								
		Characteristics		Methods					
Process Name/ Operation	Machine/ Tools Fixture	Product	Process	Specifications/ Tolerance	Measurement Technique	Sample Size	Sample Freq.	Control Method	Reaction Plan
Plastic Injection Molding	Machine 1	Appearance		Free of Blemishes	Visual Inspection	100%	Continuous	100% Inspection	Notify Supervisor
				Flow Marks	1st Piece Inspection			Check Sheet	Adjust Machine
				Pot Holes	1st Piece Inspection			Check Sheet	Adjust Machine
	Machine 2	Mounting Hole Location		Nozzle Diameter	Vernier Caliper		Rectifying Sample	Check Sheet	Adjust Machine
				15 ± 1 mm			Every 1/2 Hour	$\bar{x}$ & R Chart	Quarantine and Cause-and-Effect Analysis
	Machine 3	Dimensional		6 ± 1 mm	Fixture 2		Sample	Check Sheet	Adjust/Recheck
	Fixture #1	Perimeter		6 ± 1 mm	Check Gap to Fixture and Datum		Every 1/2 Hour	$\bar{x}$ & R Chart	Quarantine and Cause-and-Effect Analysis

-3

Process Control Charts

Statistical process control charts:

- Tools for monitoring process variation

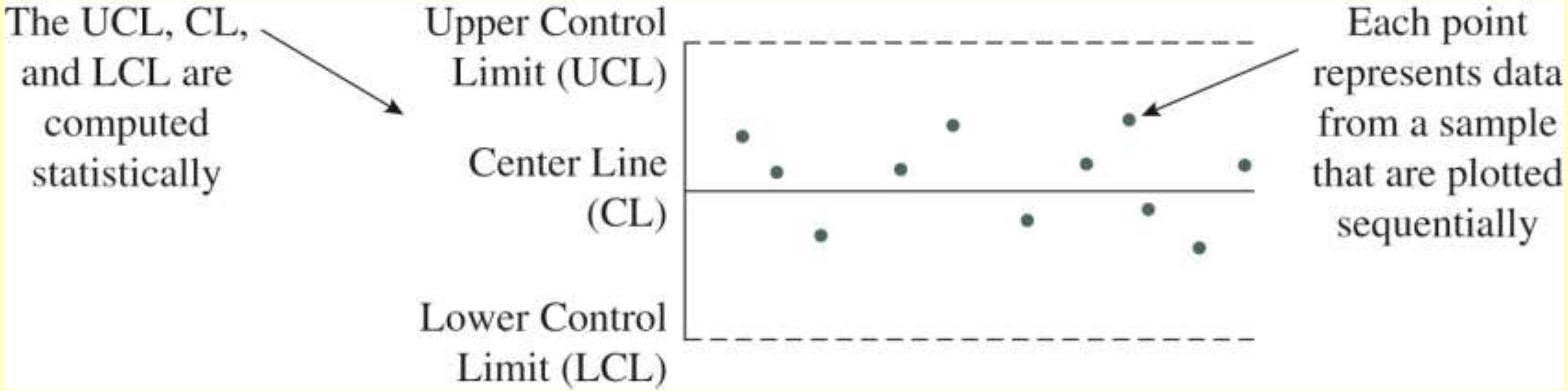


Figure 11-4

Variables and Attributes Control Charts

- **Variable**
  - Continuous measurement such as height, weight, or volume.
- **Attribute**
  - An *either-or* situation, such as a motor starting or not, or a lens being scratched or not

Example: Bottled water production

- **Variable data:** Amount of water in bottle = 500.8 ml
- **Attribute data:** Bottle leaking? Yes / No

Variables and Attributes Control Charts

The most common types of variable and attribute charts

Variables	Attributes
X (process population average)	p (proportion defective)
$\bar{x}$ (mean or average)	np (number defective or number nonconforming)
R (range)	c (number nonconforming in a consistent sample space)
MR (moving range)	
s (standard deviation)	u (number defects per unit)

Table 11-1

Variables and Attributes Control Charts

Central requirements for properly using process charts:

1. You must understand this generic process for implementing process charts.
2. You must know how to interpret process charts.
3. You need to know when different process charts are used.
4. You need to know how to compute limits for the different types of process charts.
5. We treat each of these topics separately.

Understanding Process Charts

- **Process charts are an application of hypothesis testing where the null hypothesis is that the process is stable.**

Sheet Number	Measurement
1	11.0001
2	10.9999
3	10.9998
4	11.0002
5	11.0004
6	11.0020
7	10.9980
8	10.9999
9	10.9870
10	11.0004
Sum	109.9877
Sample mean	10.99877

Understanding Process Charts

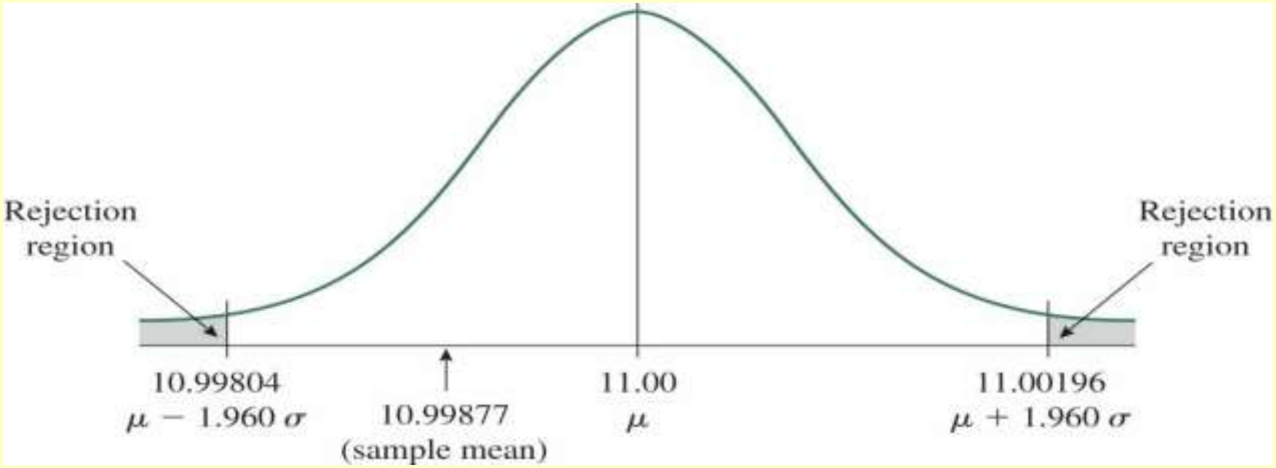
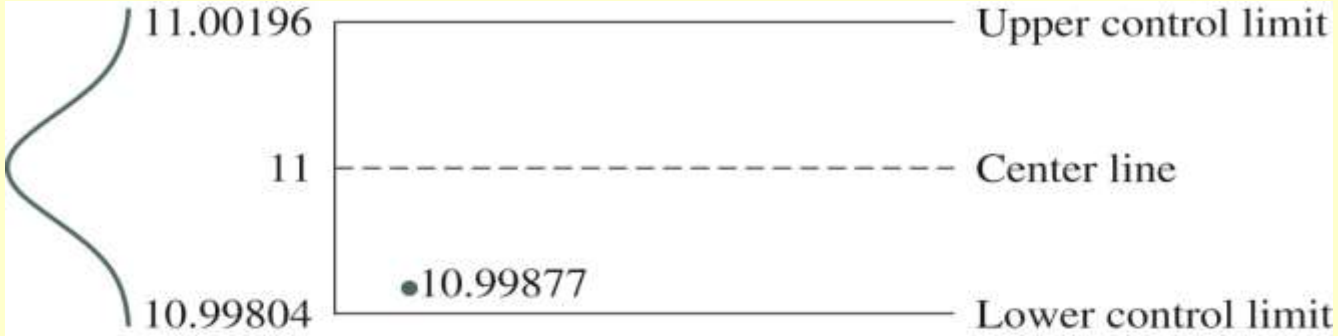


Figure 11-5

Process Chart

Figure 11-6

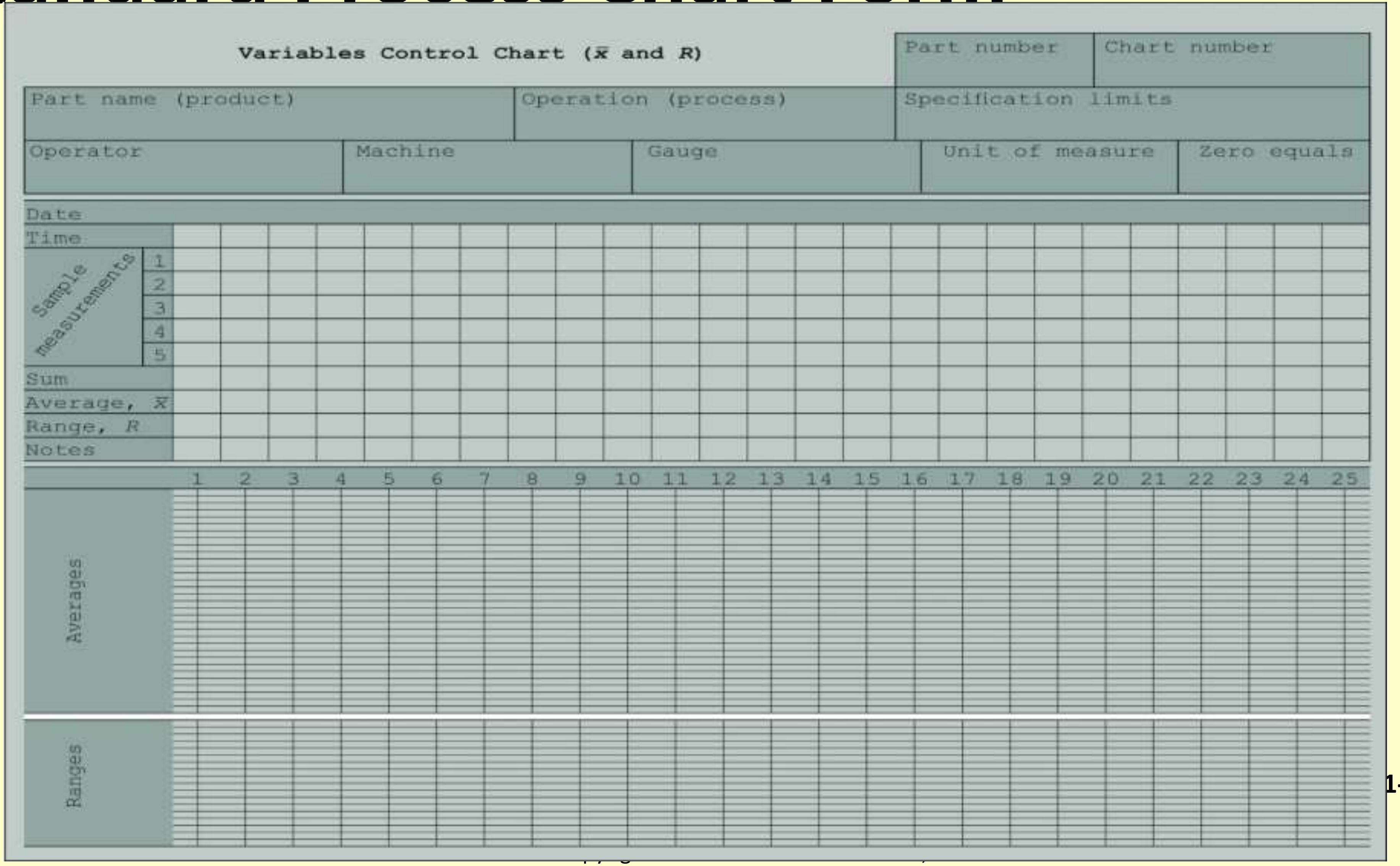


$\overline{x}$ and *R* charts

- **$\overline{x}$ chart**
  - A process chart used to monitor the average of the characteristic being measured.

- ***R* chart**
  - A process chart used to monitor the dispersion of the process

Standard Process Chart Form



1-7

Completed Process Chart Form

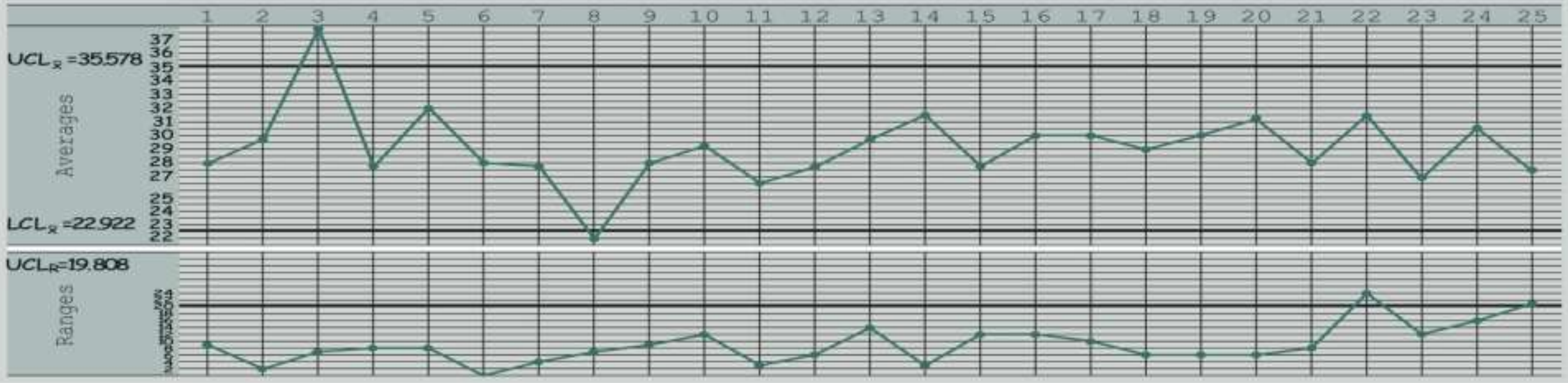
Variables Control Chart ($\bar{x}$ and R)

Part number	Chart number

Part name (product)	Operation (process)	Specification limits
Grommet		24–36

Operator	Machine	Gauge	Unit of measure	Zero equals
			.0001	

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Date																										
Time																										
Sample measurements	1	24	29	38	24	33	28	30	19	24	26	25	28	31	30	26	24	25	26	27	28	24	18	29	20	14
	2	27	30	42	28	36	28	28	26	27	27	26	24	26	31	25	36	35	30	31	33	24	42	30	36	25
	3	28	29	36	27	31	28	27	23	33	38	27	29	38	33	24	24	25	28	29	30	32	36	31	34	36
	4	33	31	35	32	28	28	26	22	28	26	28	30	24	32	36	36	35	32	33	34	32	30	28	32	35
	5																									
Sum		112	119	151	111	128	112	111	90	112	117	106	111	119	126	111	120	120	116	120	125	112	126	118	122	110
Average, $\bar{x}$		28	29.75	37.75	27.75	32	28	27.75	22.5	28	29.25	26.5	27.75	29.75	31.5	27.75	30	30	29	30	31.25	28	31.5	29.5	30.5	27.5
Range, R		9	2	7	8	8	0	4	7	9	12	3	6	14	3	12	12	10	6	6	6	8	24	3	16	22
Notes																										



1-8

x and *R* Charts Calculation Worksheet

Control Limits

Subgroups included ________________ ____________________

$\overline{R} = \frac{\Sigma R}{k}$ = ______________ = _____

$\overline{\overline{X}} = \frac{\Sigma \overline{x}}{k}$ = ______________ = _____

or

$\overline{x}$ (Midspec or std) =

$A_2\overline{R} =$ × = _____

$UCL_{\overline{x}} = \overline{\overline{X}} + A_2\overline{R}$ =

$LCL_{\overline{x}} = \overline{\overline{X}} - A_2\overline{R}$ =

$UCL_{\overline{R}} = D_4\overline{R} =$ × =

Limits for Individuals

Compare with specification or tolerance limits

$\overline{\overline{X}}$ =

$\frac{3}{d_2}\overline{R}$ = × = _____

$UL_x = \overline{x} + \frac{3}{d_2}\overline{R}$ =

$LL_x = \overline{x} - \frac{3}{d_2}\overline{R}$ =

US =

LS = _____

US – LS =

$6\sigma = \frac{6}{d_2}\overline{R}$ =

Modified Control Limits for Averages

Based on specification limits and process capability

Applicable only if US – LS > 6σ

US = LS =

$A_m\overline{R}$ = × = _________ $A_m\overline{R}$ = _____

$URL_{\overline{x}} = US - A_m\overline{R}$ = $LRL_{\overline{X}} = LS + A_m\overline{R}$ =

Factors for Control Limits

n	A_2	D_4	d_2	$\frac{3}{d_2}$	A_m
2	1.880	3.268	1.128	2.659	0.779
3	1.023	2.574	1.693	1.722	0.749
4	0.729	2.282	2.059	1.457	0.728
5	0.577	2.114	2.326	1.290	0.713
6	0.483	2.004	2.534	1.184	0.701

Figure 11-9

Calculations for Figure 11-8 Data

Control Limits
Subgroups included __________ __________

$\overline{R} = \frac{\Sigma R}{k} = \frac{217}{25} = 8.68$ ______

$\overline{\overline{X}} = \frac{\Sigma \overline{x}}{k} = \frac{731.25}{25} = 29.25$ ______

or

$\overline{x}$ (Midspec or std) =

$A_2\overline{R} =$ .729 × 8.68 = 6.328

$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R}$ 29.25+6.328 = 35.578

$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R}$ 29.25-6.328 = 22.922

$UCL_{\overline{R}} = D_4\overline{R} =$ 2.282 × 8.68 = 19.808

Limits for Individuals
Compare with specification or tolerance limits

$\overline{\overline{X}}$ =

$\frac{3}{d_2}\overline{R}$ = × = ______

$UL_x = \overline{x} + \frac{3}{d_2}\overline{R}$ =

$LL_x = \overline{x} - \frac{3}{d_2}\overline{R}$ =

US =

LS = ______

US – LS =

$6\sigma = \frac{6}{d_2}\overline{R}$ =

Modified Control Limits for Averages
Based on specification limits and process capability
Applicable only if US – LS > 6σ

US = LS =

$A_m\overline{R}$ = × = ______ $A_m\overline{R}$ = ______

$URL_{\overline{X}} = US - A_m\overline{R}$ = $LRL_{\overline{X}} = LS + A_m\overline{R}$ =

Factors for Control Limits

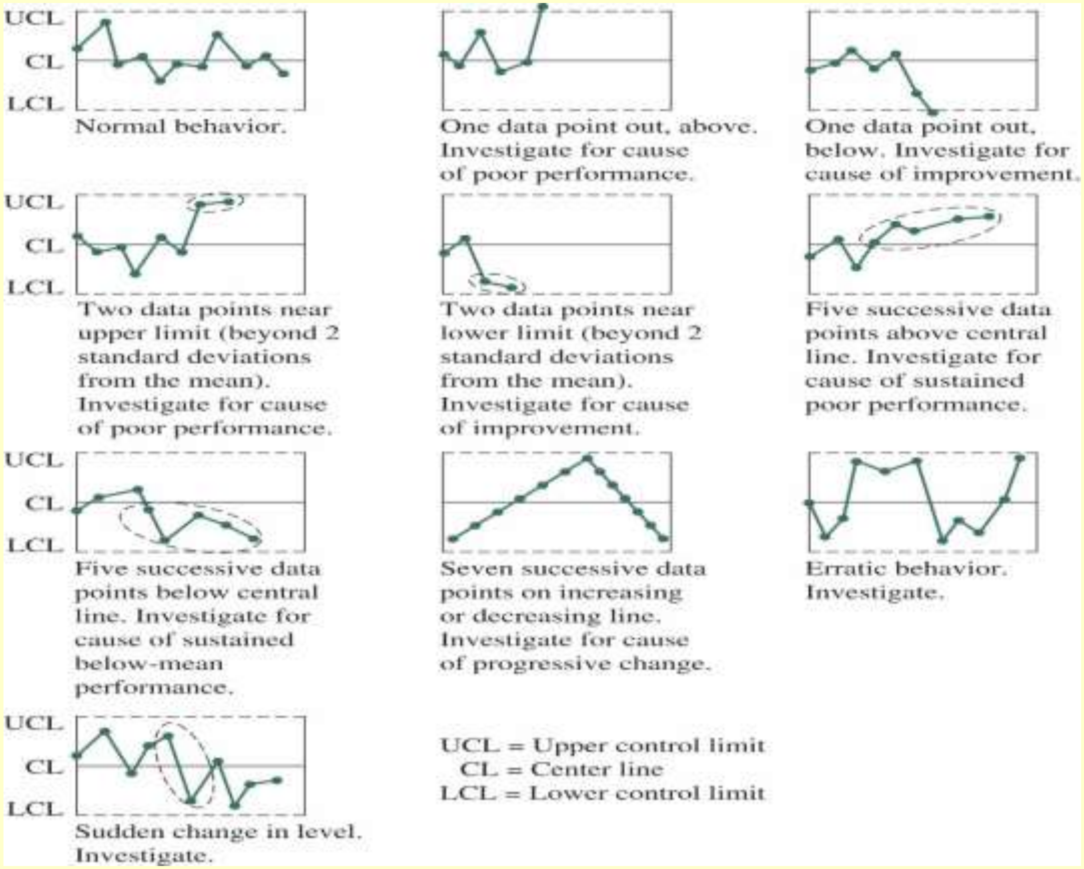
n	A_2	D_4	d_2	$\frac{3}{d_2}$	A_m
2	1.880	3.268	1.128	2.659	0.779
3	1.023	2.574	1.693	1.722	0.749
4	0.729	2.282	2.059	1.457	0.728
5	0.577	2.114	2.326	1.290	0.713
6	0.483	2.004	2.534	1.184	0.701

Figure 11-10

Interpreting Control Charts

Signals for concern sent by a control chart

UCL = Upper control limit
CL = Center line
LCL = Lower control limit



Hansen, Bertrand L. *Quality Control: Theory and Applications*. Upper Saddle River, NJ: Pearson Education (1964). ISBN: 013745208X. ©1964, p.65. Reprinted and Electronically reproduced by permission of Pearson Education, Inc., New York, NY.

Figure 11-11

Interpreting Control Charts

Signals for concern sent by a control chart (cont'd)

UCL = Upper control limit
CL = Center line
LCL = Lower control limit

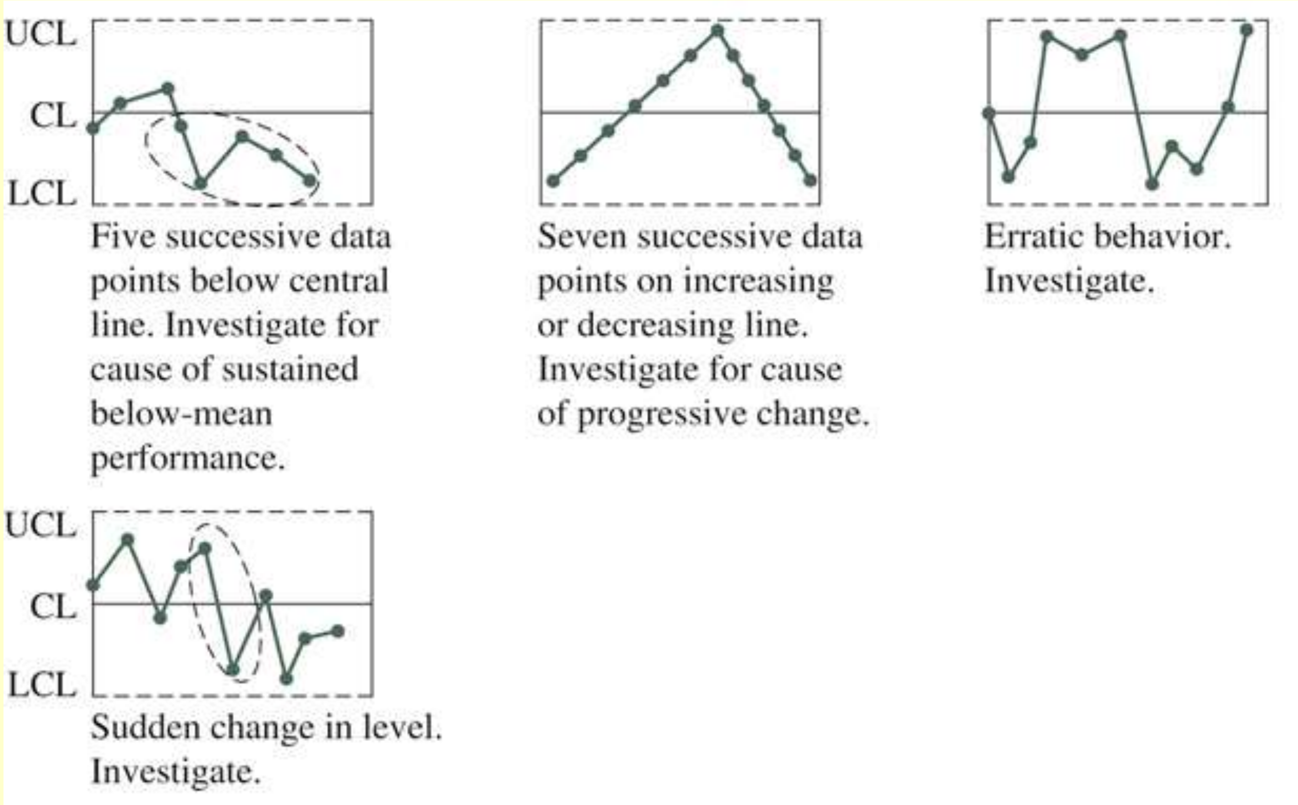


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Figure 11-11

Interpreting Control Charts

Out-of-control situations:

- Two points in succession farther than two standard deviations from the mean
- Process run – Five points in succession either above or below the center line
- Process drift – Seven points, all increasing or decreasing
- Erratic behavior – Large jumps of more than three or four standard deviations

Example 11-1

- **Problem:** The Sampson Company produces high-tech radar that is used in top-secret weapons by the Secret Service and the Green Berets. It has had trouble with a particular round component with a target of 6 centimeters. Samples of size 4 were taken during four successive days.
- The results are in the following table.

Day	*x*				Means	Ranges
1	6	6	5	7	6	2
2	8	6	6	7	6.75	2
3	7	6	6	6	6.25	1
4	6	7	5	4	5.5	3

Example 11-1

- Develop a process chart to determine whether the process is stable. Because these are measurements, use *x* and *R* charts.
- Using the calculation work sheet, Figure 11-12 shows the values for the process control limits.
- The *x* control chart for this problem is shown with the appropriate limits. The *R* chart is also in control. The sample averages were placed on the control chart, and the process was found to be historically in control. Because the averages and ranges fall within the control limits, and no other signals of nonrandom activity are present, we conclude that the process variation is random.
- Note that this example is very simple. Generally, you use 15 to 20 subgroups to establish control charts.

Example 11-1

- $\overline{\overline{x}}$ = 6.125
- $\overline{R}$ = 2

Control Limits

Subgroups included ______ ______

$\overline{R} = \frac{\Sigma R}{k} = \frac{8}{4} = 2$

$\overline{\overline{X}} = \frac{\Sigma \overline{x}}{k} = \frac{24.5}{4} = 6.125$

or

$\overline{x}$ (Midspec or std) =

$A_2\overline{R} = .729 \times 2 = 1.458$

$UCL_{\overline{x}} = \overline{\overline{X}} + A_2\overline{R} = 7.583$

$LCL_{\overline{x}} = \overline{\overline{X}} - A_2\overline{R} = 4.667$

$UCL_{\overline{R}} = D_4\overline{R} = 2.282 \times 2 = 4.564$

Limits for Individuals

Compare with specification or tolerance limits

$\overline{\overline{X}}$ =

$\frac{3}{d_2}\overline{R}$ = × =

$UL_x = \overline{\overline{x}} + \frac{3}{d_2}\overline{R}$ =

$LL_x = \overline{\overline{x}} - \frac{3}{d_2}\overline{R}$ =

US =

LS =

US – LS =

$6\sigma = \frac{6}{d_2}\overline{R}$ =

Modified Control Limits for Averages

Based on specification limits and process capability

Applicable only if US – LS > 6σ

US = LS =

$A_m\overline{R}$ = × = $A_m\overline{R}$ =

$URL_{\overline{x}} = US - A_m\overline{R}$ = $LRL_{\overline{x}} = LS + A_m\overline{R}$ =

Factors for Control Limits

n	A_2	D_4	d_2	$\frac{3}{d_2}$	A_m
2	1.880	3.268	1.128	2.659	0.779
3	1.023	2.574	1.693	1.772	0.749
4	0.729	2.282	2.059	1.457	0.728
5	0.577	2.114	2.326	1.290	0.713
6	0.483	2.004	2.534	1.184	0.701

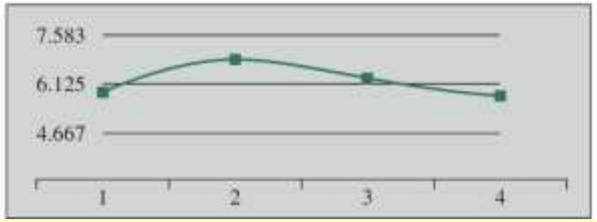


Figure 11-12

Example 11-1

Calculations using Excel

Problem data

=AVERAGE(B2:E2)

Day	x				Means	Cl x-bar	LCL x-bar	UCL x-bar	Sample Range	R-bar	UCL R
1	6	6	5	7	6	6.125	4.667	7.583	2	2	4.564
2	8	6	6	7	6.75	6.125	4.667	7.583	2	2	4.564
3	7	6	6	6	6.25	6.125	4.667	7.583	1	2	4.564
4	6	7	5	4	5.5	6.125	4.667	7.583	3	2	4.564
				Grand Mean	6.125			Mean Range:	2		
		A2:	0.729								
		D4:	2.282								

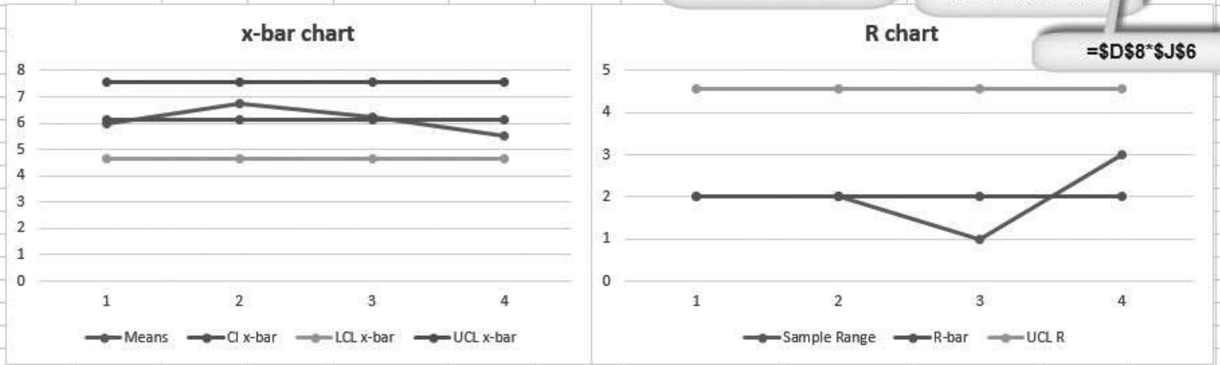
=$F$6

=$F$6-($D$7*$J$6)

=$F$6+$D$7*$J$6

=$K$3

=$D$8*$J$6



To draw an x-bar chart in Excel 2013:
1. Select (highlight) the data in cells F1: I5 (including titles);
2. Click on insert; 2-D line chart with markers (using the left-most option in the drop-down box);
3. A chart will appear;
If the chart doesn't look right, right click on the chart and choose "Change Chart Type." This will allow you to change the aspect of the chart.
4. To add a title to the chart, click on the chart title and type in x-bar chart"

To draw an R chart:
Follow the same process as we did for the x-bar chart only start by selecting the data in cells J1: L5.

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Figure 11-13

X and Moving Range (*MR*) Charts for Population Data

X and MR charts are used if you have a variable measurement that you want to monitor and do not have enough observations to use sampling.

- Central limit theorem does not apply, which may result in the data being non-normally distributed.
- Therefore, there is an increase in the likelihood that you will draw an erroneous conclusion.
- It is best to first make sure that the data are normally distributed.

X and Moving Range (*MR*) Charts for Population Data

- ***X* chart limits**
  - Center line: $\bar{x} = \frac{\Sigma X}{k}$
  - Limits: $\bar{x} \pm E_2(\overline{MR})$

X = a population value
k = the number of values used to compute $\bar{x}$
$E_2 = 2.66$ ($n = 2$) (see Table A-1 in the Appendix)

- ***MR* limits**
  - Same as *R* chart (where n=2),except that the ranges are computed as the differences from one sample to the next

Example 11-2

- **Problem:** The EA Trucking Company of Columbia, Missouri hauls corn from local fields to the SL Processing Plant in Lincoln, Nebraska. Although the trucks generally take 6.5 hours to make the daily trip, recently there seems to be more variability in the arrival times. Mr. Everett, the owner, suspects that one of his drivers, Paul, may be visiting his girlfriend Janice en route in Kansas City. The driver claims that this is not the case and that the increase is simply random variation because of variability in traffic flows. The drivers keep written logs of departure and arrival times.

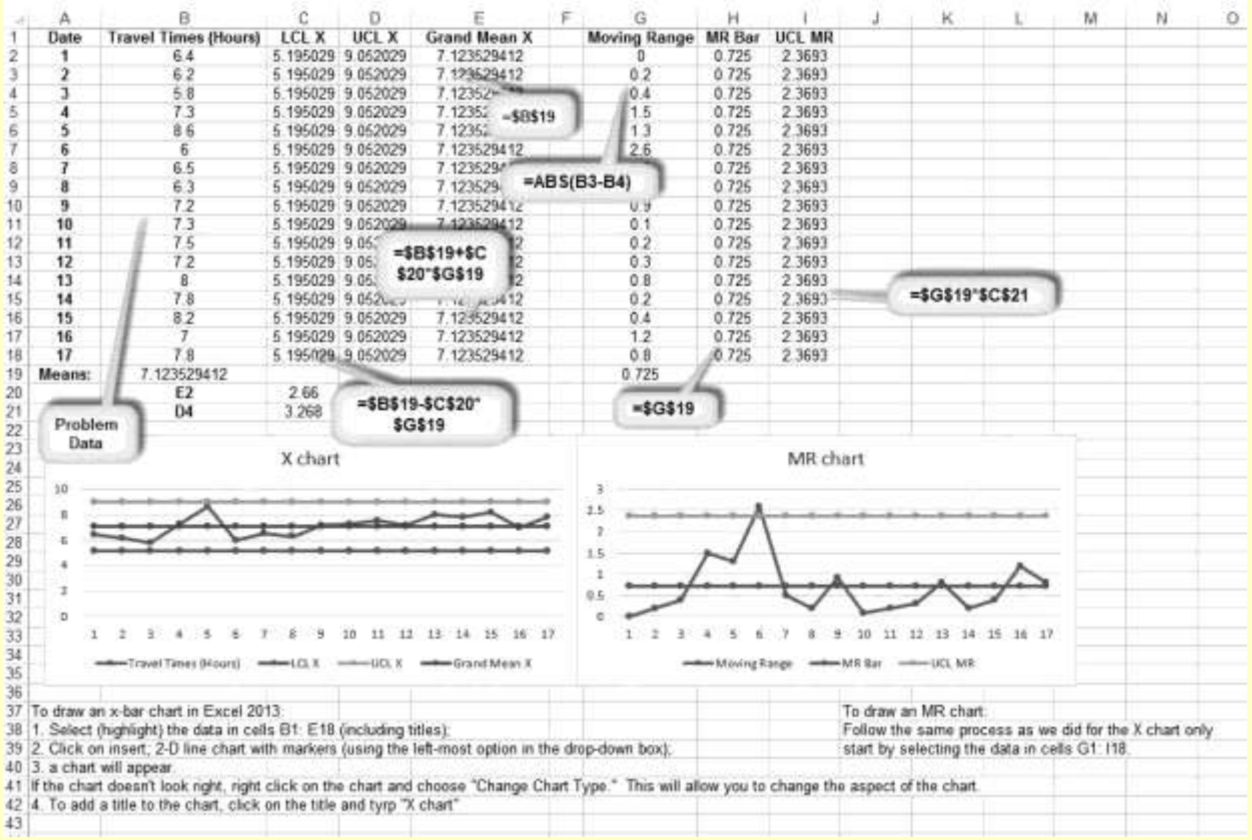
Example 11-2

Mr. Everett has listed these times in the following table. You are chosen as the analyst to investigate this situation. What do you think?

Date	Travel Times (Hrs.)	Moving Range
1	6.4	—
2	6.2	0.2
3	5.8	0.4
4	7.3	1.5
5	8.6	1.3
6	6.0	2.6
7	6.5	0.5
8	6.3	0.2
9	7.2	0.9
10	7.3	0.1
11	7.5	0.2
12	7.2	0.3
13	8.0	0.8
14	7.8	0.2
15	8.2	0.4
16	7.0	1.2
17	7.8	0.8
	$\bar{x} = 7.1235$	$\overline{MR} = .725$

Example 11-2

- **Solution:** You develop an *X* and *MR* process chart to test the hypothesis. The results from Excel are in Figure 11-14.



Date	Travel Times (Hours)	LCL X	UCL X	Grand Mean X		Moving Range	MR Bar	UCL MR
1	6.4	5.195029	9.052029	7.123529412		0	0.725	2.3693
2	6.2	5.195029	9.052029	7.123529412		0.2	0.725	2.3693
3	5.8	5.195029	9.052029	7.12352[illegible]		0.4	0.725	2.3693
4	7.3	5.195029	9.052029	7.12352[illegible]		1.5	0.725	2.3693
5	8.6	5.195029	9.052029	7.12352[illegible]		1.3	0.725	2.3693
6	6	5.195029	9.052029	7.123529412		2.6	0.725	2.3693
7	6.5	5.195029	9.052029	7.123529[illegible]		[illegible]	0.725	2.3693
8	6.3	5.195029	9.052029	7.123529[illegible]		[illegible]	0.725	2.3693
9	7.2	5.195029	9.052029	7.123529412		0.9	0.725	2.3693
10	7.3	5.195029	9.052029	7.123529412		0.1	0.725	2.3693
11	7.5	5.195029	9.05[illegible]	[illegible]		0.2	0.725	2.3693
12	7.2	5.195029	9.05[illegible]	[illegible]		0.3	0.725	2.3693
13	8	5.195029	9.05[illegible]	[illegible]		0.8	0.725	2.3693
14	7.8	5.195029	9.052[illegible]	[illegible]412		0.2	0.725	2.3693
15	8.2	5.195029	9.052029	7.123529412		0.4	0.725	2.3693
16	7	5.195029	9.052029	7.123529412		1.2	0.725	2.3693
17	7.8	5.195029	9.052029	7.123529412		0.8	0.725	2.3693
Means:	7.123529412					0.725		
	E2	2.66						
	D4	3.268						

Microsoft Excel, Microsoft Corporation. Used by permission.

Figure 11-14

Median Charts

- **Median charts may be used if it is too time consuming or inconvenient to compute subgroup averages or you have concerns about the accuracy of computed means.**

- **Need to use an odd sample size, usually 3, 5, or 7**

Median Chart Limits

- **Center line:** Mean of medians $=$ sum of the medians/number of medians $= \overline{\widetilde{x}}$

- **Control limits:**

$$\mathrm{LCL}_{\tilde{x}} = \overline{\widetilde{x}} + \widetilde{A}_2\overline{R}$$
$$\mathrm{UCL}_{\tilde{x}} = \overline{\widetilde{x}} - \widetilde{A}_2\overline{R}$$

Example 11-3

- **Problem:** The Luftig food company has gathered the following data with weights of its new health food product. Because the published weight on the package is 6 ounces, Mr. Luftig wants to know if the company is complying with weight requirements.

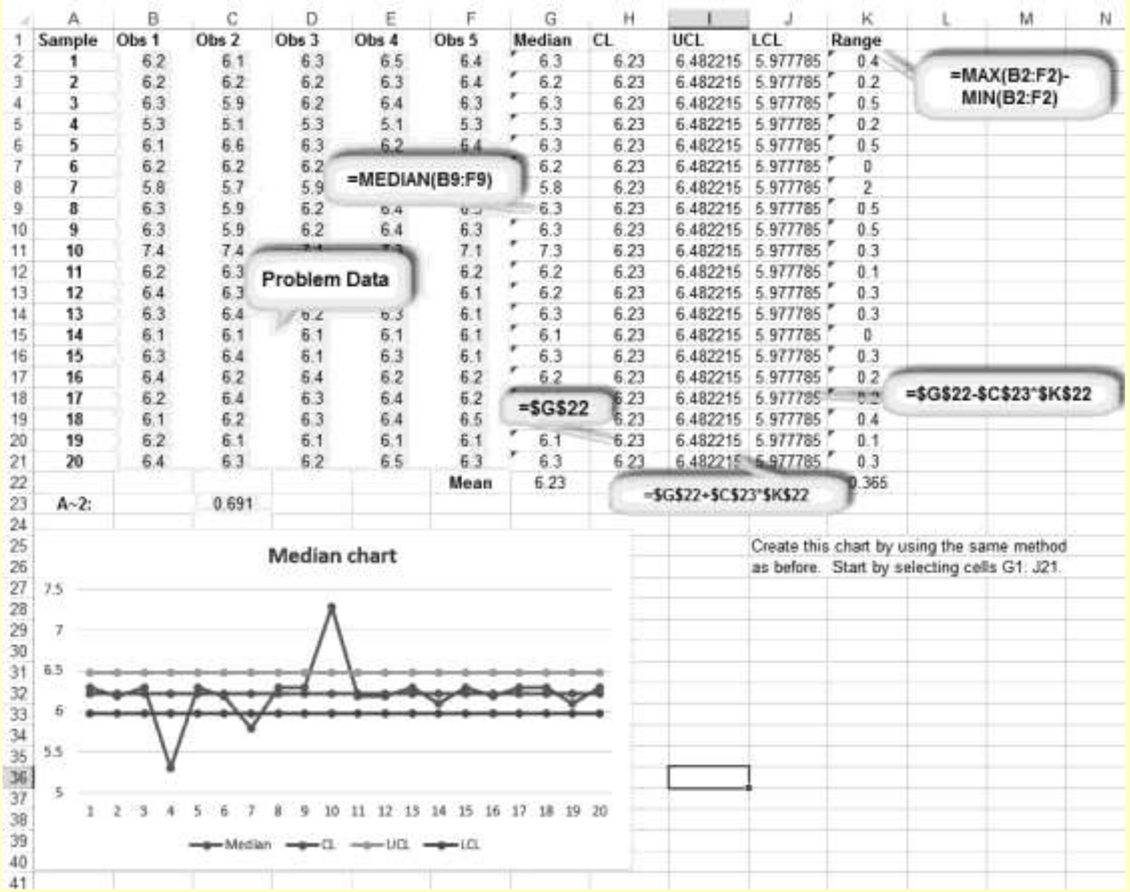
Example 11-3

Twenty samples of size 5 were drawn.

Sample	Observation 1	Observation 2	Observation 3	Observation 4	Observation 5
1	6.2	6.1	6.3	6.5	6.4
2	6.2	6.2	6.2	6.3	6.4
3	6.3	5.9	6.2	6.4	6.3
4	5.3	5.1	5.3	5.1	5.3
5	6.1	6.6	6.3	6.2	6.4
6	6.2	6.2	6.2	6.2	6.2
7	5.8	5.7	5.9	7.2	5.2
8	6.3	5.9	6.2	6.4	6.3
9	6.3	5.9	6.2	6.4	6.3
10	7.4	7.4	7.1	7.3	7.1
11	6.2	6.3	6.2	6.3	6.2
12	6.4	6.3	6.2	6.1	6.1
13	6.3	6.4	6.2	6.3	6.1
14	6.1	6.1	6.1	6.1	6.1
15	6.3	6.4	6.1	6.3	6.1
16	6.4	6.2	6.4	6.2	6.2
17	6.2	6.4	6.3	6.4	6.2
18	6.1	6.2	6.3	6.4	6.5
19	6.2	6.1	6.1	6.1	6.1
20	6.4	6.3	6.2	6.5	6.3

Example 11-3

- **Solution:** Results show that the process is not in control, with an average median of 6.23. The median process chart does show that some product is being made that is below 6 ounces. It also shows that points 4, 7, and 10 are out of control.



	A	B	C	D	E	F	G	H	I	J	K
1	Sample	Obs 1	Obs 2	Obs 3	Obs 4	Obs 5	Median	CL	UCL	LCL	Range
2	1	6.2	6.1	6.3	6.5	6.4	6.3	6.23	6.482215	5.977785	0.4
3	2	6.2	6.2	6.2	6.3	6.4	6.2	6.23	6.482215	5.977785	0.2
4	3	6.3	5.9	6.2	6.4	6.3	6.3	6.23	6.482215	5.977785	0.5
5	4	5.3	5.1	5.3	5.1	5.3	5.3	6.23	6.482215	5.977785	0.2
6	5	6.1	6.6	6.3	6.2	6.4	6.3	6.23	6.482215	5.977785	0.5
7	6	6.2	6.2	6.2	[illegible]	[illegible]	6.2	6.23	6.482215	5.977785	0
8	7	5.8	5.7	5.9	[illegible]	[illegible]	5.8	6.23	6.482215	5.977785	2
9	8	6.3	5.9	6.2	6.4	[illegible]	6.3	6.23	6.482215	5.977785	0.5
10	9	6.3	5.9	6.2	6.4	6.3	6.3	6.23	6.482215	5.977785	0.5
11	10	7.4	7.4	[illegible]	[illegible]	7.1	7.3	6.23	6.482215	5.977785	0.3
12	11	6.2	6.3	[illegible]	[illegible]	6.2	6.2	6.23	6.482215	5.977785	0.1
13	12	6.4	6.3	[illegible]	[illegible]	6.1	6.2	6.23	6.482215	5.977785	0.3
14	13	6.3	6.4	6.2	6.3	6.1	6.3	6.23	6.482215	5.977785	0.3
15	14	6.1	6.1	6.1	6.1	6.1	6.1	6.23	6.482215	5.977785	0
16	15	6.3	6.4	6.1	6.3	6.1	6.3	6.23	6.482215	5.977785	0.3
17	16	6.4	6.2	6.4	6.2	6.2	6.2	6.23	6.482215	5.977785	0.2
18	17	6.2	6.4	6.3	6.4	6.2	[illegible]	6.23	6.482215	5.977785	[illegible]
19	18	6.1	6.2	6.3	6.4	6.5	[illegible]	6.23	6.482215	5.977785	0.4
20	19	6.2	6.1	6.1	6.1	6.1	6.1	6.23	6.482215	5.977785	0.1
21	20	6.4	6.3	6.2	6.5	6.3	6.3	6.23	[illegible]	5.977785	0.3
22						Mean	6.23				0.365
23	A~2:		0.691								

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Figure 11-15

$\overline{x}$ and *s* charts

- **When you are particularly concerned about the dispersion of the process, it might be the *R* chart is not sufficiently precise so an $\overline{x}$ and *s* chart is used.**

- **Standard deviation (*s*) charts are used where variation in a process is small.**

$\bar{x}$ and *s* charts

- **$\bar{x}$ and *s* charts limits**
  - Center line: $\bar{s} = \Sigma s_i / k$
  - Control limits: $\text{UCL}_s = B_4 \times \bar{s}$

    $\text{LCL}_s = B_3 \times \bar{s}$

Where:

s_i is the standard deviation for sample i

k is the number of samples.

$\overline{x}$ and *s* charts

- **$\overline{x}$ and *s* charts limits if you need to estimate the process standard deviation**

$$\sigma_{\text{est}} = \bar{s} \times \sqrt{(1 - C_4^2)/C_4}$$

  - Control limits:

$$\text{UCL}_{\overline{\overline{x}}} = \overline{x} + A_3(\bar{s})$$

$$\text{LCL}_{\overline{\overline{x}}} = \overline{x} - A_3(\bar{s})$$

$\overline{x}$ and *s* charts

- Table for values for $\overline{x}$ and *s* charts

n	B_3	B_4	C_4	A_3
2	0	3.267	0.7979	2.659
3	0	2.568	0.8862	1.954
4	0	2.266	0.9213	1.628
5	0	2.089	0.9400	1.427
6	0.030	1.970	0.9515	1.287
7	0.118	1.882	0.9594	1.182
8	0.185	1.815	0.9650	1.099
9	0.239	1.761	0.9693	1.032

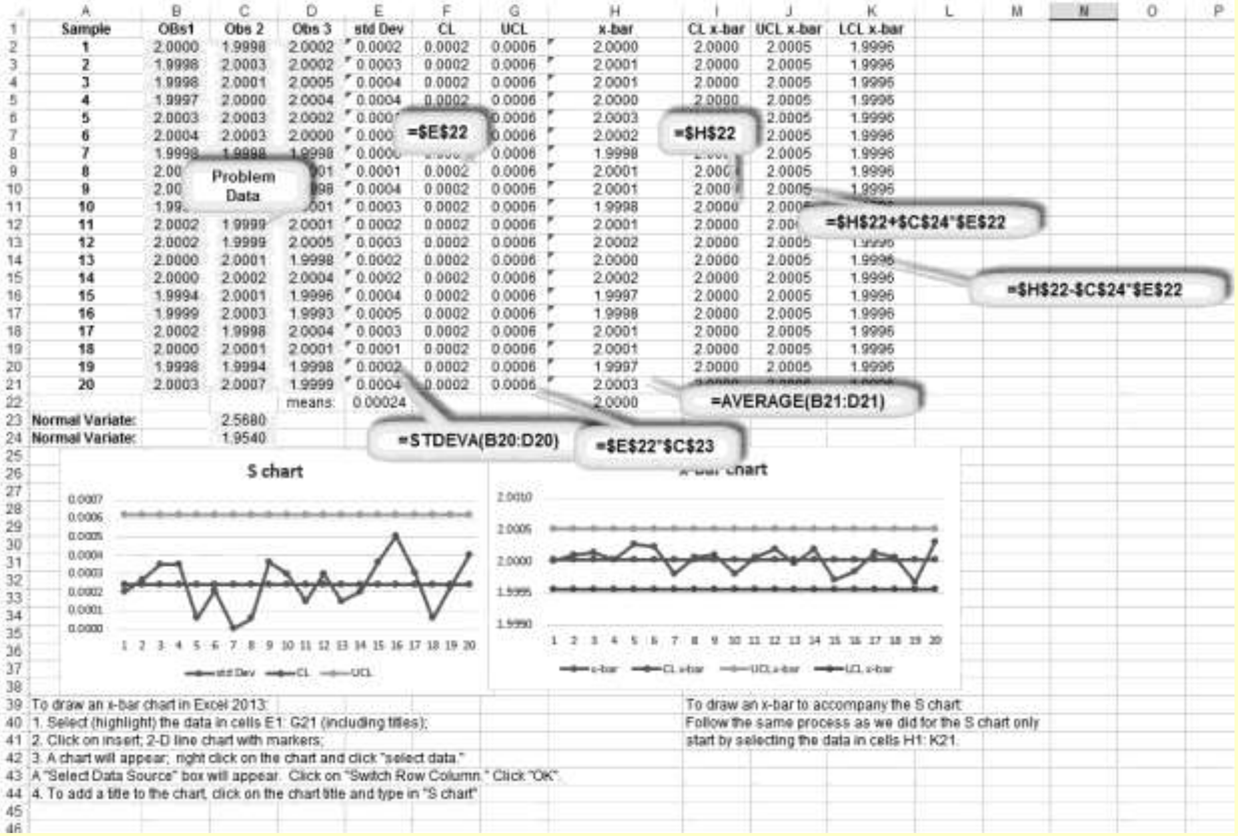
Table 11-3

Example 11-4

- **Problem:** Twenty samples were taken for a milled rod. The diameters are needed to determine whether the process is in control. Because these milled rods must be measured within 1/10,000 of an inch, it is determined that the process dispersion is important.
- Therefore, you need to use an *s* and *x* chart to monitor the process. The data are found in Figure 11-16. We have 20 samples with $n = 3$.

Example 11-4

- **Solution:** The control charts in Figure 11-16 show that the process is in control. There is no need for corrective action. The solution method is demonstrated in the next section.



Sample	OBs1	Obs 2	Obs 3	std Dev	CL	UCL	x-bar	CL x-bar	UCL x-bar	LCL x-bar
1	2.0000	1.9998	2.0002	0.0002	0.0002	0.0006	2.0000	2.0000	2.0005	1.9996
2	1.9998	2.0003	2.0002	0.0003	0.0002	0.0006	2.0001	2.0000	2.0005	1.9996
3	1.9998	2.0001	2.0005	0.0004	0.0002	0.0006	2.0001	2.0000	2.0005	1.9996
4	1.9997	2.0000	2.0004	0.0004	0.0002	0.0006	2.0000	2.0000	2.0005	1.9996
5	2.0003	2.0003	2.0002	[illegible]	[illegible]	0.0006	2.0003	[illegible]	2.0005	1.9996
6	2.0004	2.0003	2.0000	[illegible]	[illegible]	0.0006	2.0002	[illegible]	2.0005	1.9996
7	1.9998	1.9998	1.9998	[illegible]	[illegible]	0.0006	1.9998	[illegible]	2.0005	1.9996
8	[illegible]	[illegible]	[illegible]	0.0001	0.0002	0.0006	2.0001	[illegible]	2.0005	1.9996
9	[illegible]	[illegible]	[illegible]	0.0004	0.0002	0.0006	2.0001	[illegible]	2.0005	1.9996
10	[illegible]	[illegible]	[illegible]	0.0003	0.0002	0.0006	1.9998	2.0000	[illegible]	[illegible]
11	2.0002	1.9999	2.0001	0.0002	0.0002	0.0006	2.0001	2.0000	[illegible]	[illegible]
12	2.0002	1.9999	2.0005	0.0003	0.0002	0.0006	2.0002	2.0000	2.0005	[illegible]
13	2.0000	2.0001	1.9998	0.0002	0.0002	0.0006	2.0000	2.0000	2.0005	1.9996
14	2.0000	2.0002	2.0004	0.0002	0.0002	0.0006	2.0002	2.0000	2.0005	1.9996
15	1.9994	2.0001	1.9996	0.0004	0.0002	0.0006	1.9997	2.0000	2.0005	1.9996
16	1.9999	2.0003	1.9993	0.0005	0.0002	0.0006	1.9998	2.0000	2.0005	1.9996
17	2.0002	1.9998	2.0004	0.0003	0.0002	0.0006	2.0001	2.0000	2.0005	1.9996
18	2.0000	2.0001	2.0001	0.0001	0.0002	0.0006	2.0001	2.0000	2.0005	1.9996
19	1.9998	1.9994	1.9998	0.0002	0.0002	0.0006	1.9997	2.0000	2.0005	1.9996
20	2.0003	2.0007	1.9999	0.0004	0.0002	0.0006	2.0003	[illegible]	[illegible]	[illegible]
			means:	0.00024			2.0000			
Normal Variate:		2.5680								
Normal Variate:		1.9540								

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Figure 11-16

Other Control Charts

Summary of Variables Chart Formulas

Chart	LCL	CL	UCL	Constant Values
$\bar{x}$	$\bar{\bar{x}} - A_2\bar{R}$	$\bar{\bar{x}}$	$\bar{\bar{x}} + A_2\bar{R}$	(Appendix Table A-1)
R	$D_3\bar{R}$	$\bar{R}$	$D_4\bar{R}$	(Appendix Table A-1)
X	$\bar{\bar{x}} - E_2(\overline{MR})$	$\bar{\bar{x}}$	$\bar{\bar{x}} + E_2(\overline{MR})$	(Appendix Table A-1)
Median	$\bar{\tilde{x}} - \tilde{A}_2\bar{R}$	$\bar{\tilde{x}}$	$\bar{\tilde{x}} + \tilde{A}_2\bar{R}$	(Appendix Table A-4)
$\bar{x}$ (with s)	$\bar{\bar{x}} - A_3\bar{s}$	$\bar{\bar{x}}$	$\bar{\bar{x}} + A_3\bar{s}$	(Appendix Table A-3)
s	$B_3\bar{s}$	$\bar{s}$	$B_4\bar{s}$	(Appendix Table A-3)

Table 11-4

Moving Average Chart

A chart for monitoring variables and measurement on a continuous scale by using past information to predict what the next process outcome will be

Cusum Chart

A chart used to identify slight but sustained shifts in a universe in which there is no independence between observations

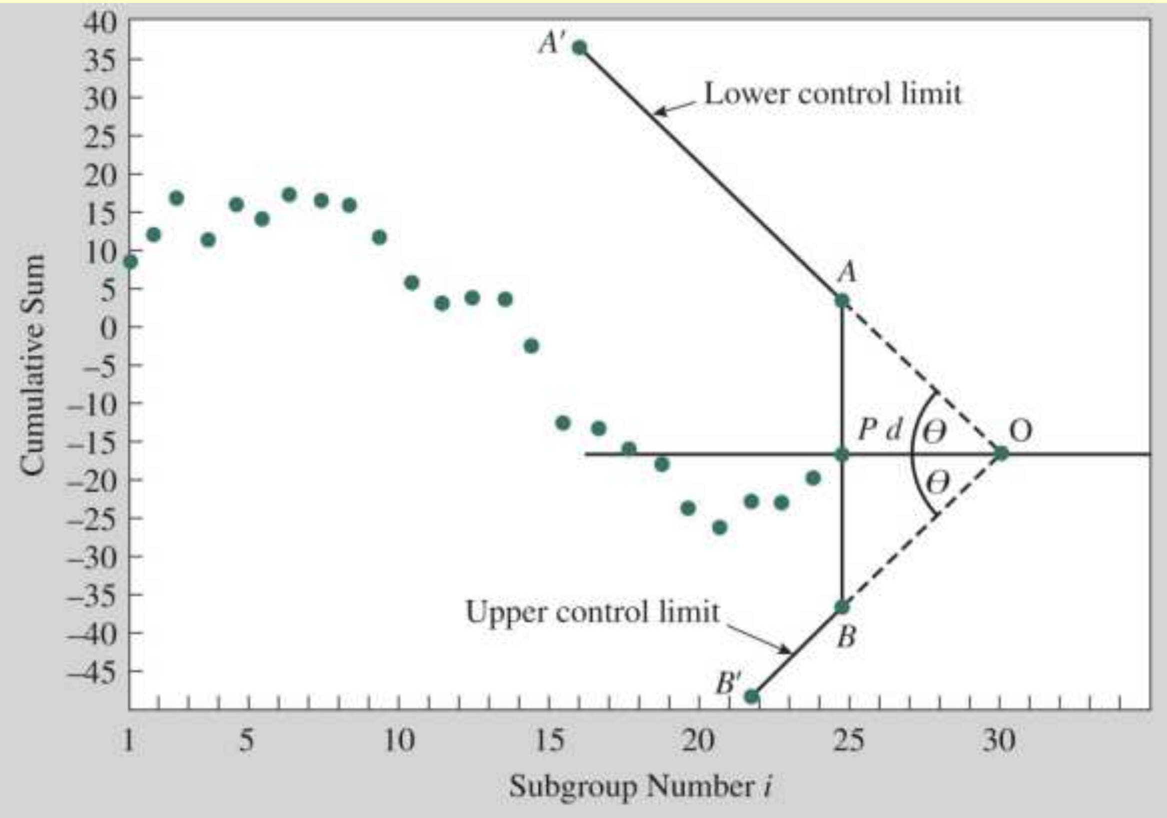

Figure 11-17

Choosing the Correct Variables Control Chart

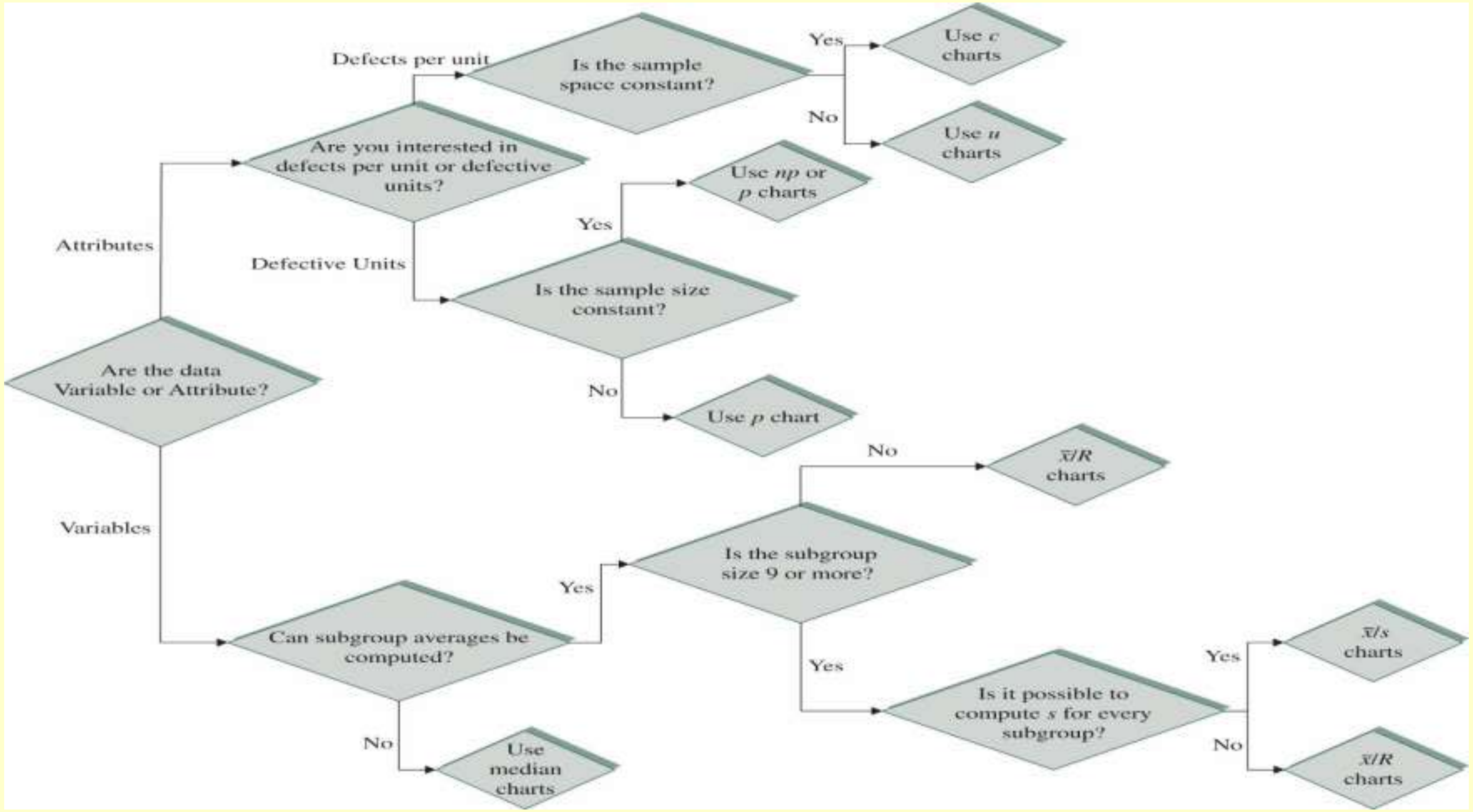

Figure 11-18

Corrective Action

Corrective action steps when a process is out of control:

1. Carefully identify the quality problem.
2. Form the appropriate team to evaluate and solve the problem.
3. Use structured brainstorming along with fishbone diagrams or affinity diagrams to identify causes of problems.
4. Brainstorm to identify potential solutions to problems.
5. Eliminate the cause.
6. Restart the process.
7. Document the problem, root causes, and solutions.
8. Communicate the results of the process to all personnel so this process becomes reinforced and ingrained in the organization.

Using Control Charts to Continuously Improve

Two key concepts:

- The focus of control charts should be on continuous improvement.
- Control chart limits should be updated only when there is a change to the process. Otherwise, any changes are unexpected.

Effects of Tampering with the Process

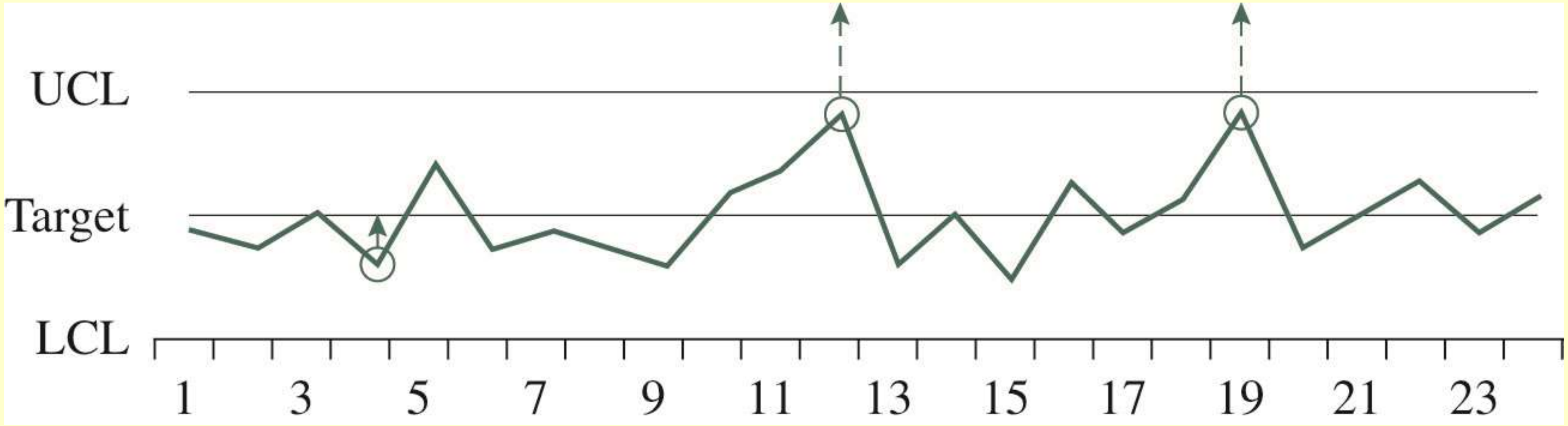

Figure 11-19

Process Capability for Variables

- **The capability of a process to produce a product that meets specification**

- **World-class levels of process capability are measured by parts per million (ppm) defect levels.**

Process Capability for Variables

Six Sigma programs result in highly capable processes and an average of only 3.4 defects per million units produced.

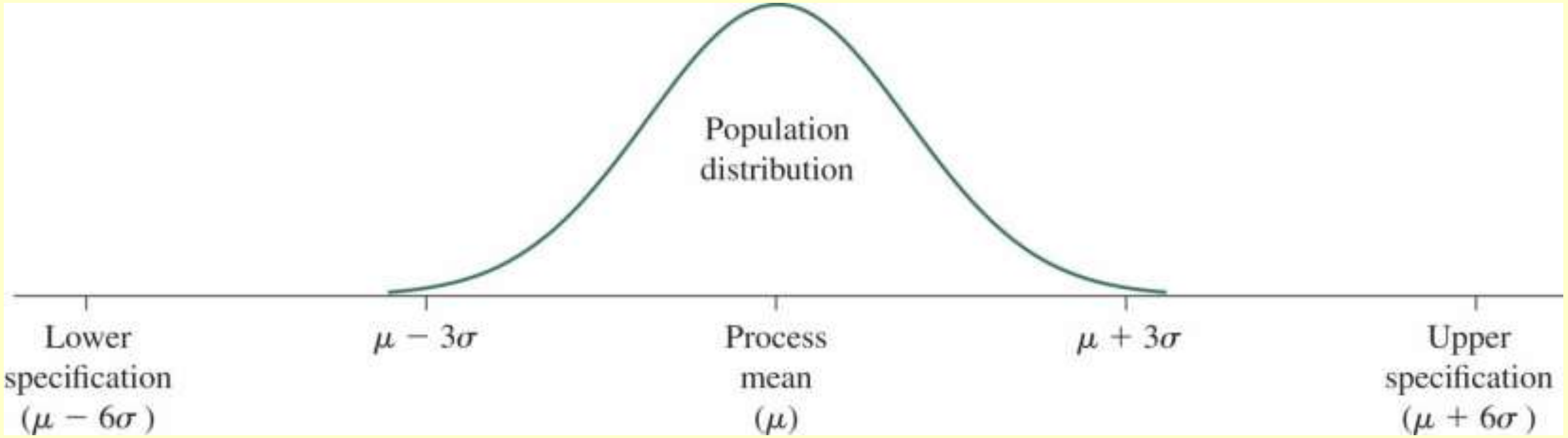

Figure 11-20

Population versus Sampling Distributions

- **Population distributions**
  - Distributions with all individual responses from an entire population
- **Population**
  - A collection of all the items or observations of interest to a decision maker
- **Sample**
  - A subset of the population
- **Sampling distributions**
  - Distributions that reflect the distribution of sample means

Population versus Sampling Distributions

Population and Sampling Distributions for Class Heights

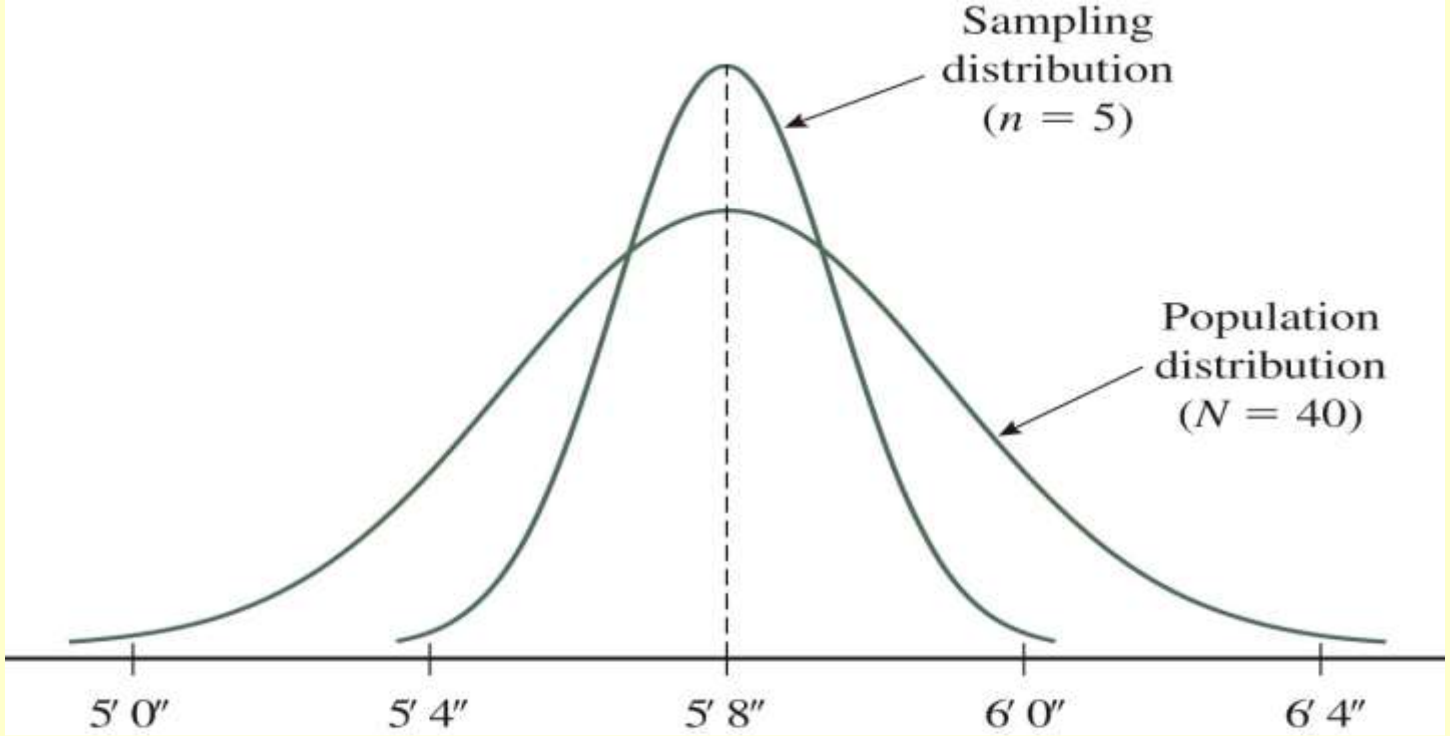

Figure 11-21

Population versus Sampling Distributions

- In the context of quality, specifications and capability are associated with population distributions.
- Sample-based process charts and stability are computed statistically and reflect sampling distributions.
- Quality practitioners should not compare process chart limits with product specifications.

Capability Studies

Reasons to perform a process capability study:

1. To determine whether a process consistently results in products that meet specifications
2. To determine whether a process is in need of monitoring through the use of permanent process charts

Capability Studies

Five steps to perform process capability studies:

1. Select a critical operation. These may be bottlenecks, costly steps of the process, or places in the process in which problems have occurred in the past.
2. Take k samples of size n, where x is an individual observation.
   - Where $19 < k < 26$
   - If x is an attribute, $n > 50$ (as in the case of a binomial)
   - Or if x is a measurement, $1 < n < 11$
3. Use a trial control chart to see whether the process is stable.

Capability Studies

Five steps to perform process capability studies (cont'd):

4. Compare process natural tolerance limits with specification limits. Note that natural tolerance limits are three standard deviation limits for the population distribution. This can be compared with the specification limits.
5. Compute capability indexes. To compute capability indexes, compute an upper capability index (Cpu), a lower capability index (Cpl), and a capability index (Cpk). The formulas are:

$$\text{Cpu} = (\text{USL} - \mu)/3\hat{\sigma}$$
$$\text{Cpl} = (\mu - \text{LSL})/3\hat{\sigma}$$
$$\text{Cpk} = \min\{\text{Cpu}, \text{Cpl}\}$$

Where:

USL = upper specification limit
LSL = lower specification limit
μ = computed population process mean
$\hat{\sigma}$ = Estimated process standard deviation $= \hat{\sigma} = R/d_2$

Capability Studies

- **Although different firms use different benchmarks, the generally accepted benchmarks for process capability are 1.25, 1.33, and 2.0.**

- **We will say that processes that achieve capability indexes (Cpk) of 1.25 are capable, 1.33 are highly capable, and 2.0 are world-class capable (Six Sigma).**

Example 11-5

- ***Problem:*** For an overhead projector, the thickness of a component is specified to be between 30 and 40 millimeters. Thirty samples of components yielded a grand mean ($\bar{\bar{x}}$) of 34 millimeters with a standard deviation ($\boldsymbol{\hat{\sigma}}$) of 3.5. Calculate the process capability index by following the steps previously outlined. If the process is not highly capable, what proportion of product will not conform?

Example 11-5

- **Solution:**

$$Cpu = (40 - 34)/(3)(3.5) = .57$$

$$Cpl = (34 - 30)/(3)(3.5) = .38$$

$$Cpk = .38$$

- **The process capability is poor.**
- **To compute the proportion of nonconforming product being produced:**

$$Z = (x - \mu)/\hat{\sigma}$$

Thus, for the lower end of the distribution:

$$Z = (30 - 34)/3.5 = -1.14$$

For the upper end of the distribution:

$$Z = (40 - 34)/3.5 = 1.71$$

Example 11-5

Proportion of Product Nonconforming

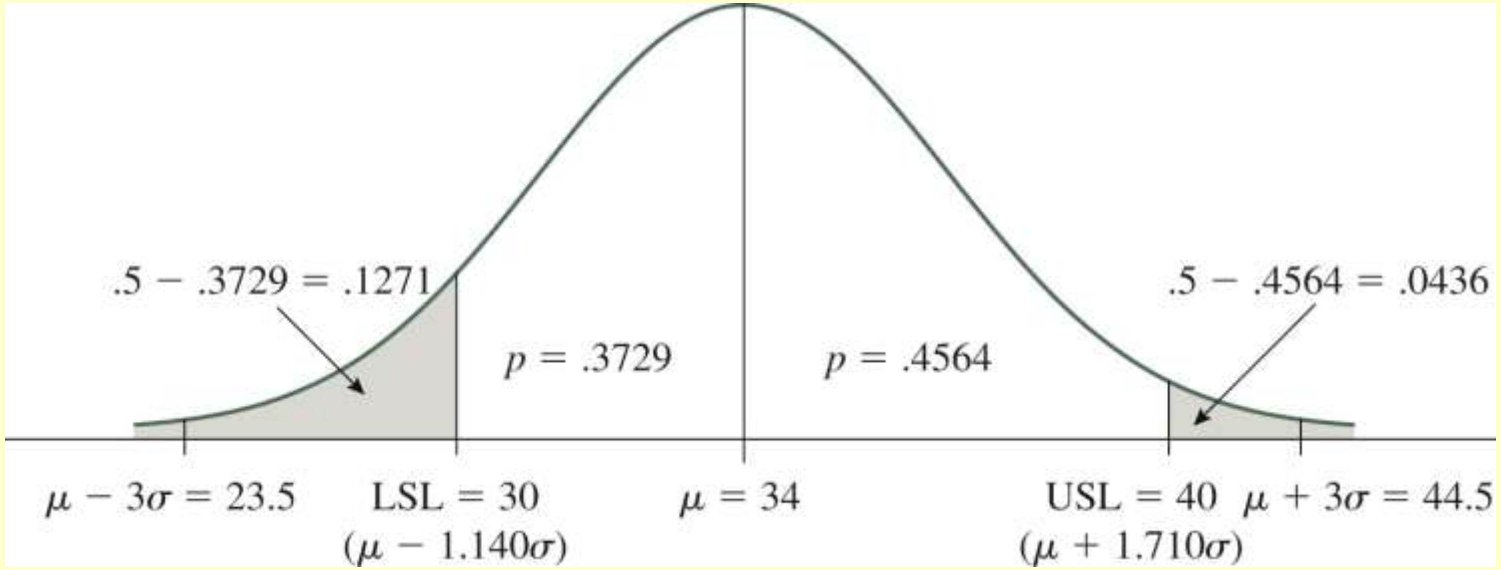

Figure 11-22

Population Capability Index (Ppk)

Rather than using within-groups variation to estimate the sigma that you used in Cpk, use the population standard deviation to compute your capability.

$$\text{Ppk} = \min\{\text{Ppu}, \text{Ppl}\}$$
$$\text{Ppu} = (\text{USL} - \mu)/3\sigma$$
$$\text{Ppl} = (\mu - \text{LSL})/3\sigma$$
$$\sigma = \sqrt{\Sigma(x_i - \overline{x})^2/(n-1)}$$

Where:

USL = upper specification limit
LSL = lower specification limit
μ = population mean
σ = population process standard deviation

Example 11-6

- **Problem:** The upper and lower specification limits (tolerances) for a metal plate are 3 millimeters $\pm$ 0.002 millimeters. A sample of 100 plates yielded a mean *x* of 3.001 millimeters. We know that the population standard deviation is .0002. Compute the Ppk for this product.

- **Solution:**

$$\text{Ppu} = (3.002 - 3.001)/(.0002 \times 3) = 1.67$$

$$\text{Ppl} = (3.001 - 2.998)/(.0002 \times 3) = 5$$

$$\text{Ppk} = 1.67$$

Therefore, the process is highly capable.

Capability versus Stability

- A process is capable if individual products consistently meet specification.
- A process is stable if only common variation is present in the process.

Interlinking

A correlation that is useful in helping to identify causal relationships between variables

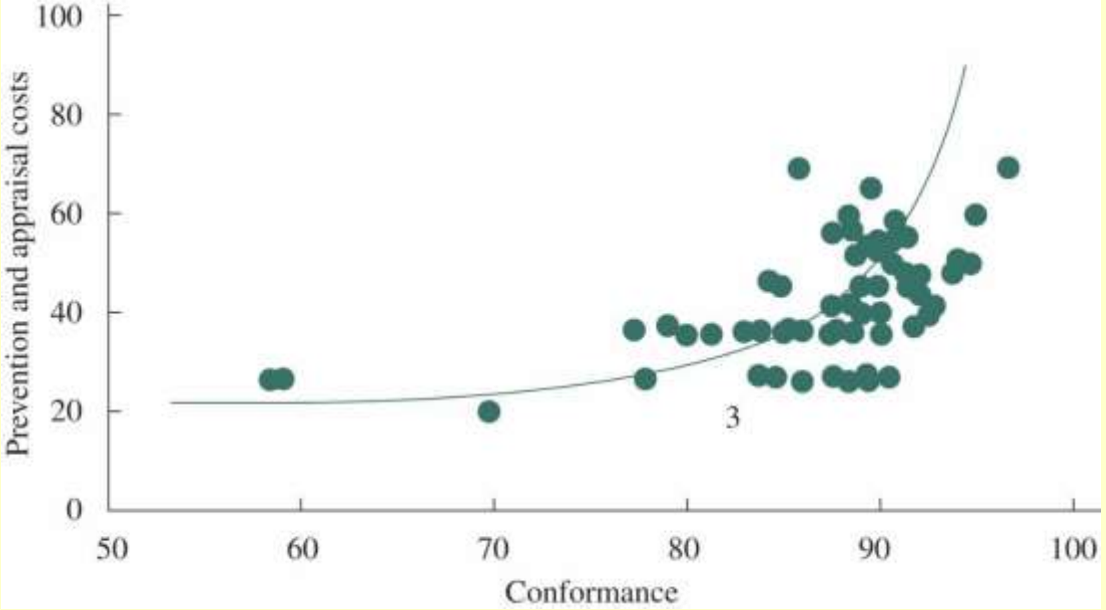

Figure 11-23

Model	R^2	p
First order	0.4002	0.0001
Quadratic	0.4675	0.0001

Table 11-5

Based on S. T. Foster, "Quality Costs Working Paper