CS 229: Machine Learning

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UC Riverside

Lecture 8

Slides from Lecture 8

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  - CS 229: Machine Learning
  - Professor Christian Shelton
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    - Elements of Statistical Learning (Hastie, et al.)
    - Pattern Recognition and Machine Learning (Bishop)
    - Machine Learning: A Probabilistic Perspective (Murphy)
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Non-linear methods

In

$$f(x) = x^\top w \qquad\qquad f(x) = \sigma(x^\top w)$$

replace x with $\phi(x)$:

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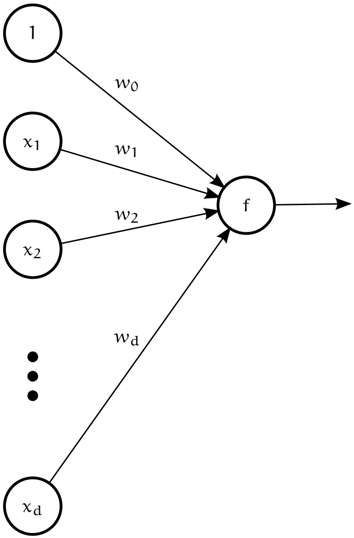
If $\phi(x)$ selected by hand, we are done!

Learning Non-linearity



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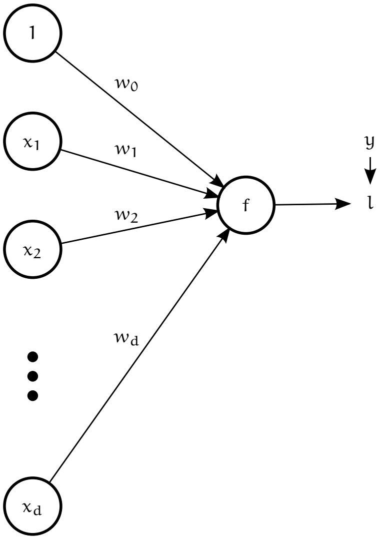

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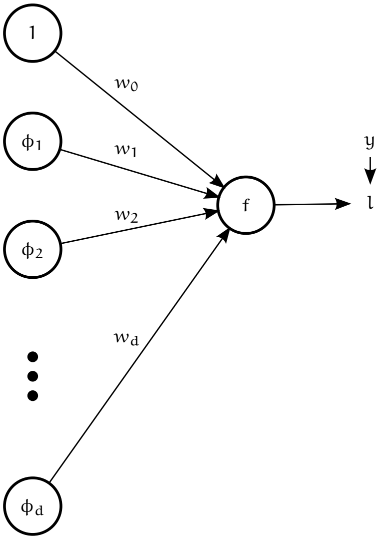

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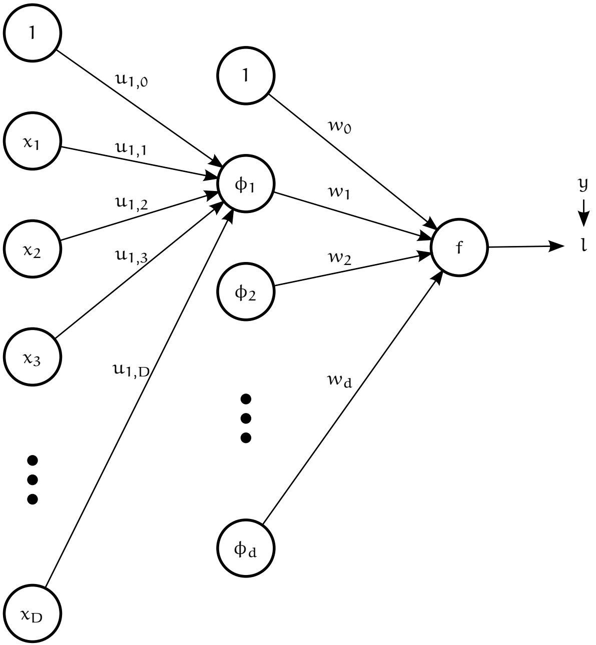

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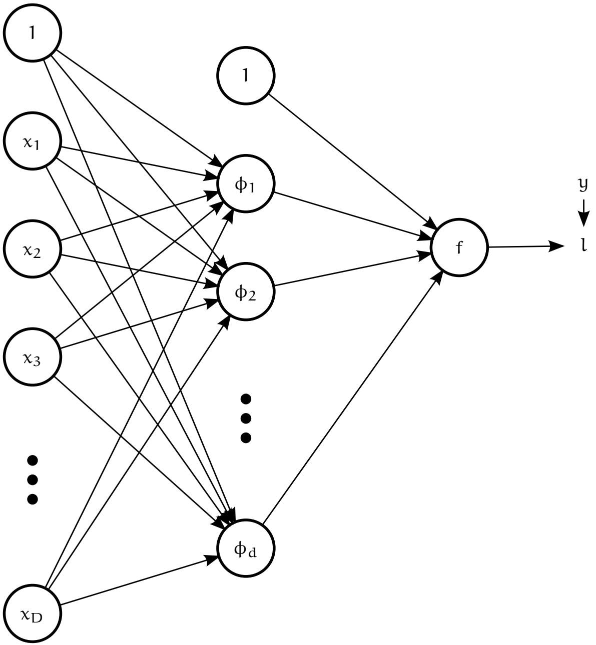

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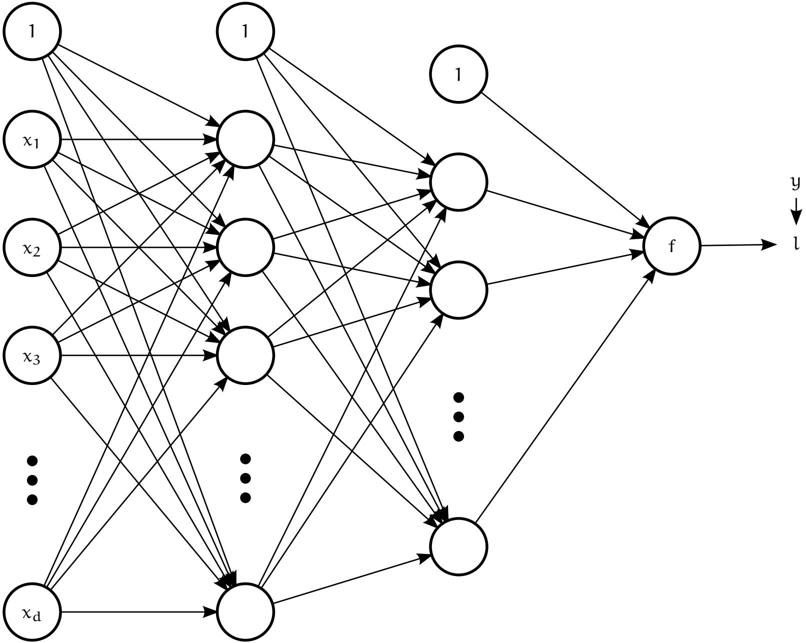

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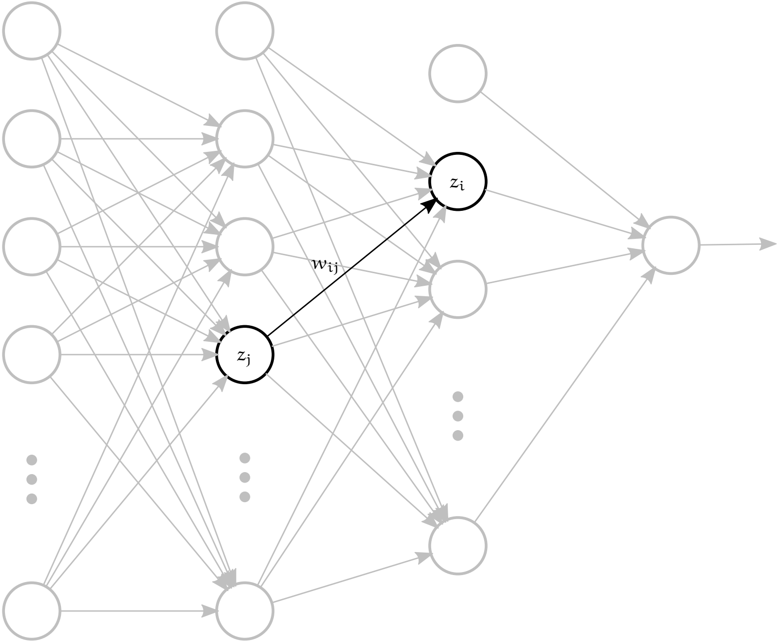
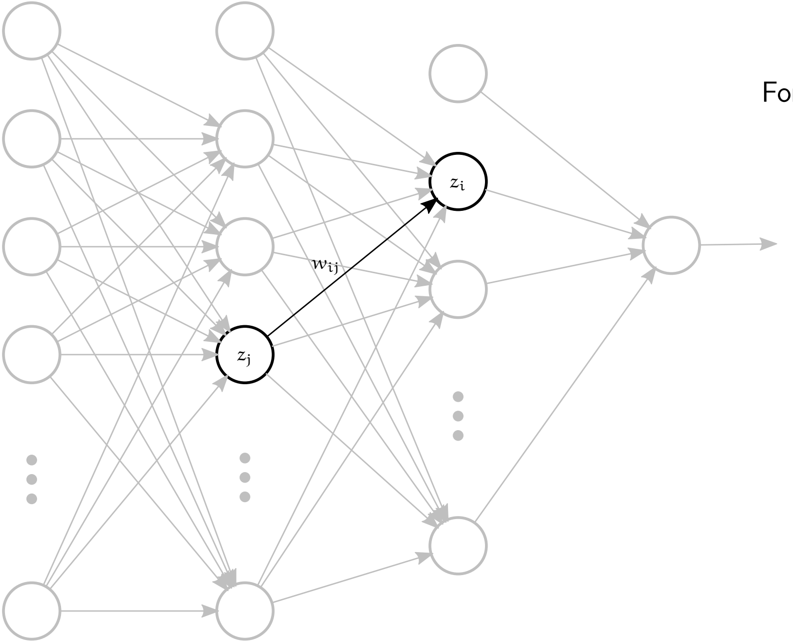
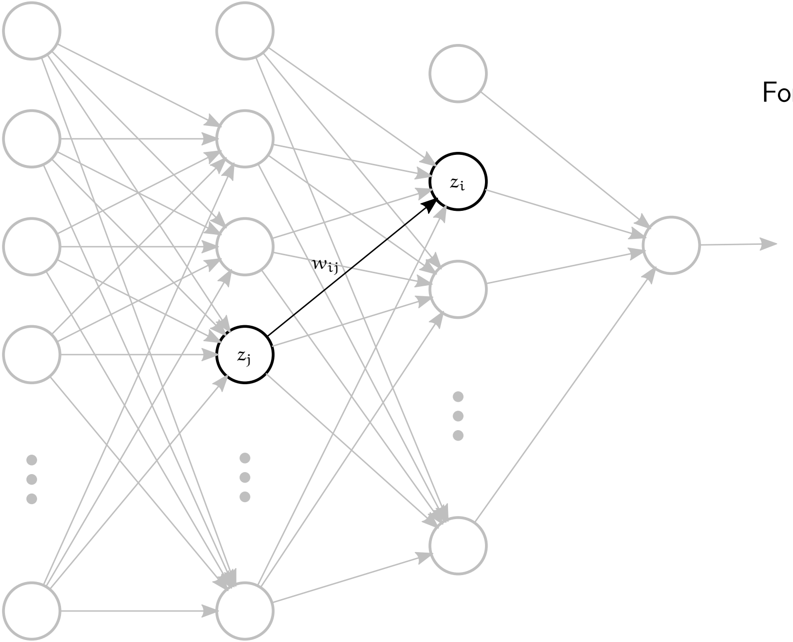




For a given x, y

$$z_i = g_i(a_i)$$
$$a_i = \sum_j w_{ij} z_j$$

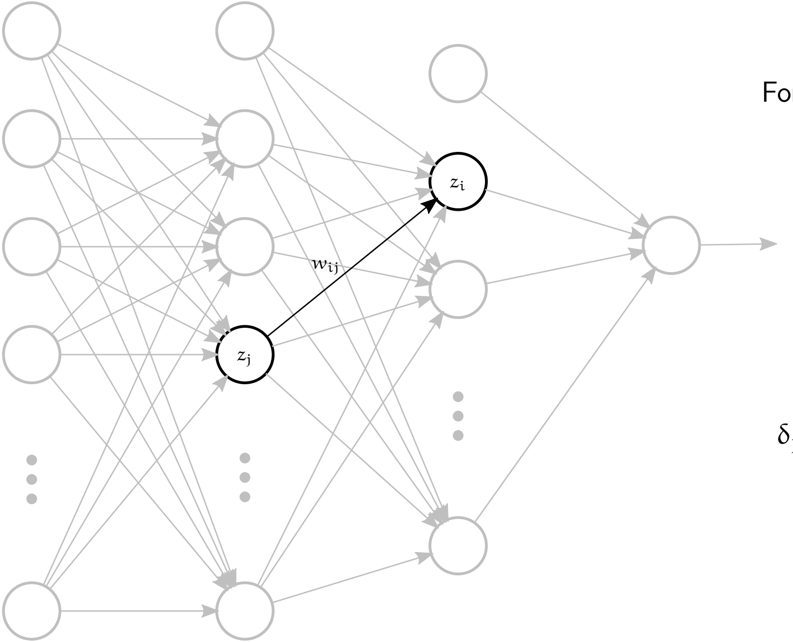
Learning Non-linearity



For a given x, y

$$
\begin{aligned}
z_i &= g_i(a_i) \\
a_i &= \sum_j w_{ij} z_j \\
\frac{\partial l}{\partial w_{ij}} &= \frac{\partial l}{\partial a_i} \frac{\partial a_i}{\partial w_{ij}} \\
&= \delta_i z_j
\end{aligned}
$$

Learning Non-linearity



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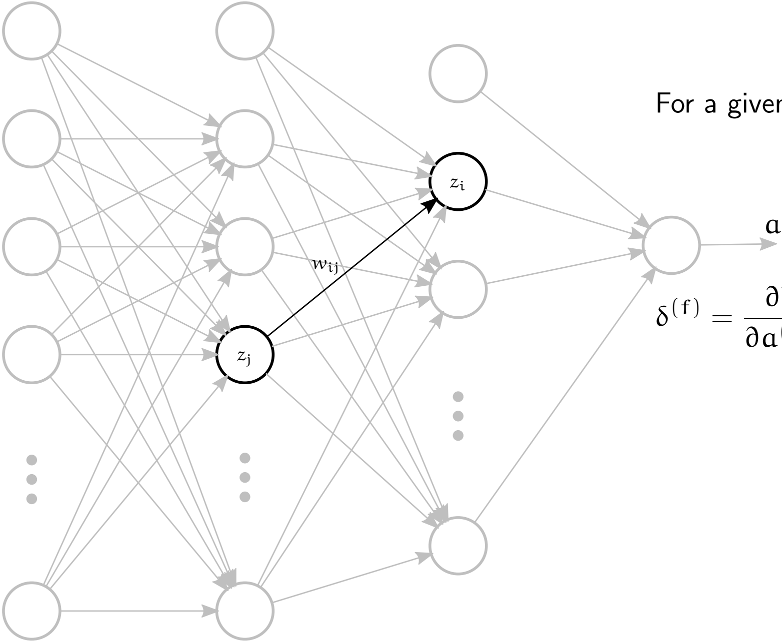
&= \sum_i \frac{\partial l}{\partial a_i} \frac{\partial a_i}{\partial z_j} \frac{\partial z_j}{\partial a_j} \\
&= \sum_i \delta_i w_{ij} g'(a_j)
\end{aligned}
$$

Learning Non-linearity



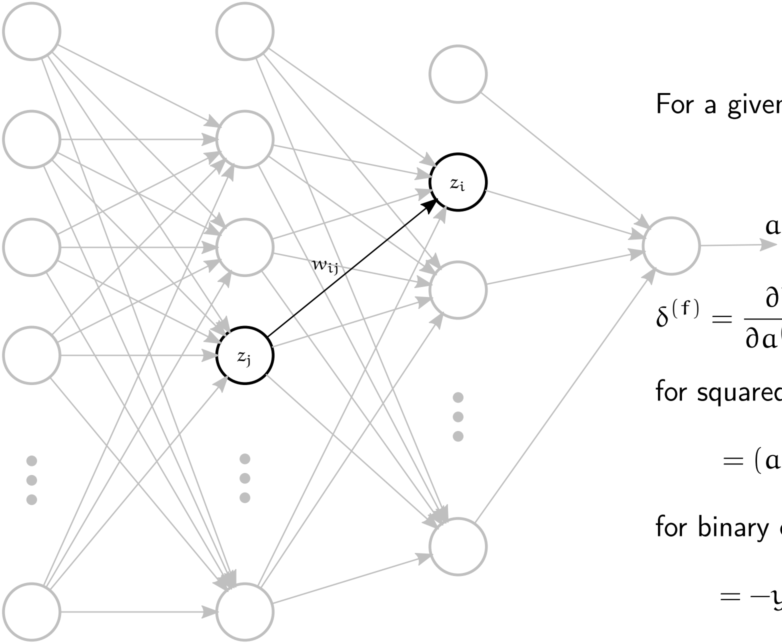
For a given x, y

$$f = g_f(a^{(f)})$$

$$a^{(f)} = \sum_j w_{fj} z_j$$

$$\delta^{(f)} = \frac{\partial l}{\partial a^{(f)}} = \frac{\partial l}{\partial f}\frac{\partial f}{\partial a^{(f)}} = \frac{\partial l}{\partial f} g_f'(a^{(f)})$$

Learning Non-linearity



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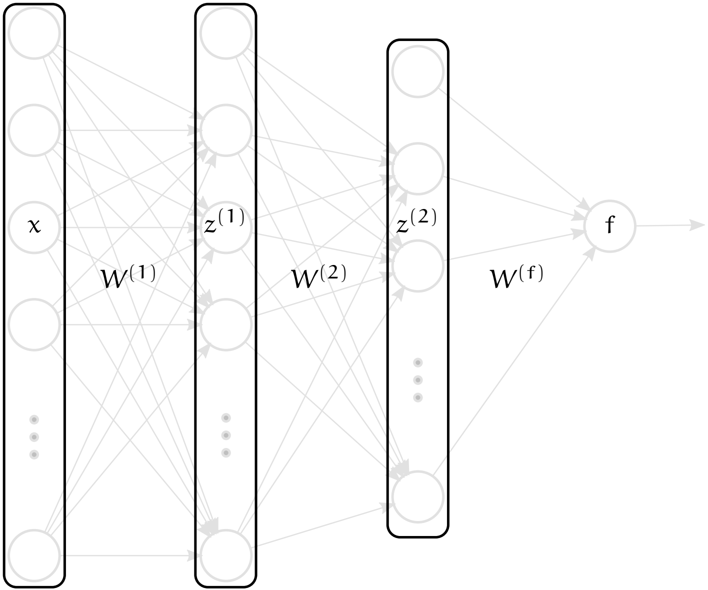
for squared error

$$= (a^{(f)} - y) = (f - y)$$

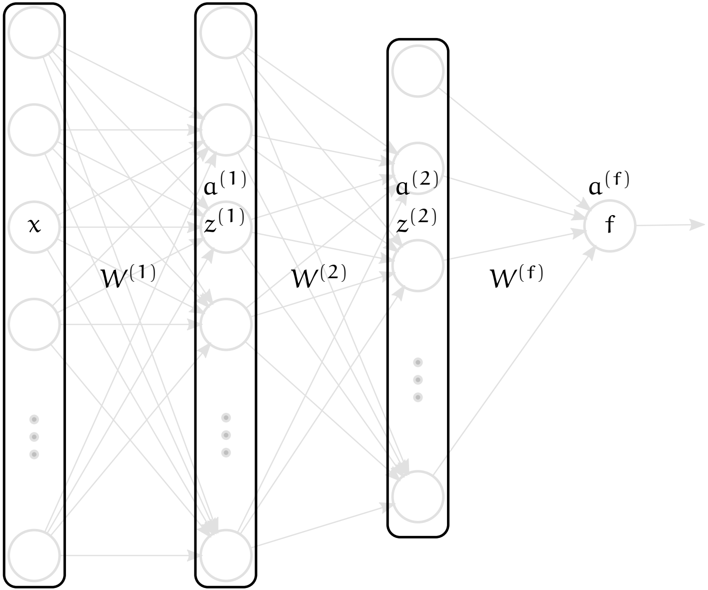
for binary classification

$$= -y(1 - \sigma(ya^{(f)})) = (f - y_{01})$$

Layered Neural Network



Layered Neural Network



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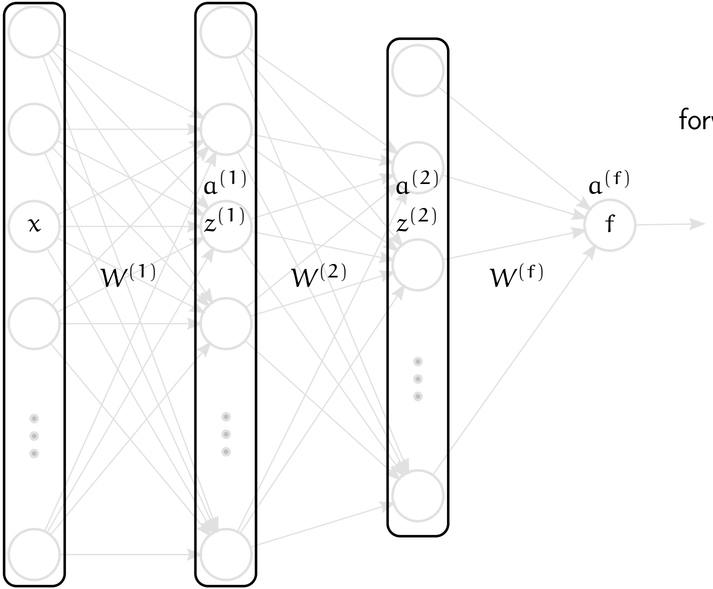

forward:

$$a^{(i)} = W^{(i)} z^{(i-1)}$$
$$z^{(i)} = g(a^{(i)})$$

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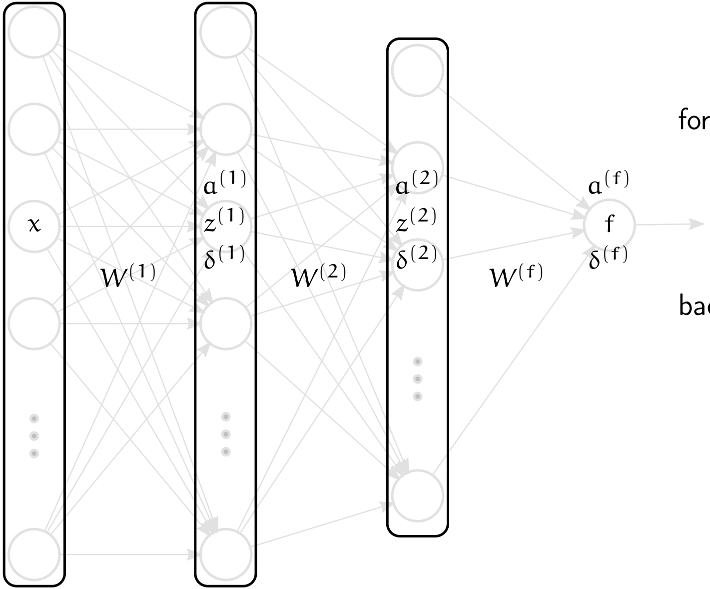

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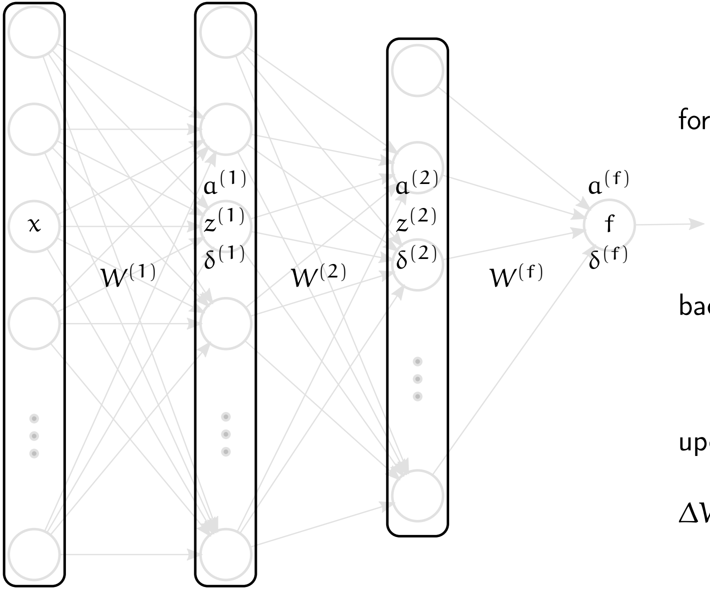
$$\delta^{(i)} = g'(a^{(i)}) \odot {W^{(i+1)}}^\top \delta^{(i+1)}$$

Layered Neural Network



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backward:

$$\delta^{(i)} = g'(a^{(i)}) \odot {W^{(i+1)}}^\top \delta^{(i+1)}$$

update:

$$\Delta W^{(i)} = -\eta \delta^{(i)} {z^{(i-1)}}^\top$$

Output Layer

For regression:

$$g_f(a) = a$$

$$\delta^{(f)} = (f - y)$$

$$l(f, y) = \frac{1}{2}(f - y)^2$$

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For binary classification, $y \in \{-1, +1\}$:

$$g_f(a) = \sigma(a) \qquad l(f, y) = \log(1 + e^{-yf})$$

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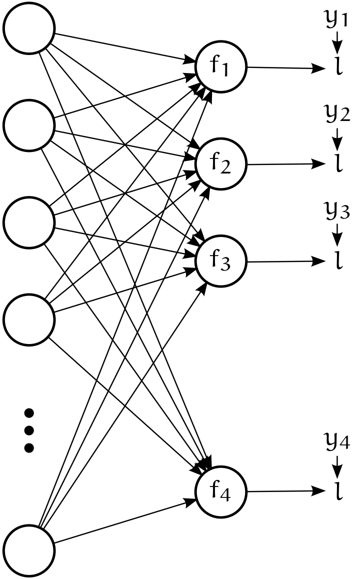
For binary classification, $y \in \{0,1\}$:

$$g_f(a) = \sigma(a) \qquad\qquad l(f,y) = -(y\log(f) + (1-y)\log(1-f))$$

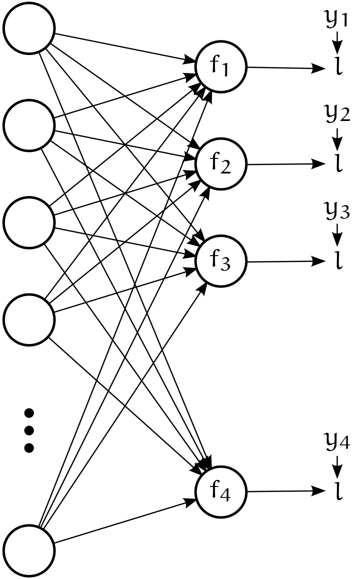
$$\delta^{(f)} = (f-y)$$

Output Layer

For multiclass classification, $y_c \in \{0, 1\}$:



Output Layer



For multiclass classification, $y_c \in \{0, 1\}$:

$$g_{f,c}(a^{(f)}) = \frac{e^{a_c^{(f)}}}{\sum_{c'} e^{a_{c'}^{(f)}}} \qquad l(f, y) = -\sum_c y_c \log(f_c)$$

$$\delta^{(f_c)} = (f_c - y_c)$$

Non-linearities

Most common:

$$g(a) = \sigma(a) = \tanh(2a)/2 + 1$$

$$g(a) = \tanh(a)$$

$$g(a) = \max(0, a)$$

$$g(a) = \max(0, 0.99a) + 0.01a$$

Non-linearities

Most common:

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\begin{aligned}
g(a) &= \sigma(a) = \tanh(2a)/2 + 1 \\
g(a) &= \tanh(a) \\
g(a) &= \max(0, a) \\
g(a) &= \max(0, 0.99a) + 0.01a
\end{aligned}
$$

Also used:

$$z_i = e^{-\sum_j (z_j - w_{ij})^2}$$

(radial basis function network)

Hidden Units

How many layers?

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- ...see next slide

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- 2 can approximate any function (with enough hidden units)
- More cause problems with minimization
- Now common to have many (deep learning)

How many units?

- Too many can cause overfitting...
- ...see next slide
- Usually err on side of too many

Overfitting

Start (stochastic) gradient descent:

- With weights near 0
- But, random!

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$$L = \sum_i l(f(x_i), y_i) + \frac{\lambda}{2} \sum_{ij} w_{ij}^2$$

Overfitting

Start (stochastic) gradient descent:

- With weights near 0
- But, random!

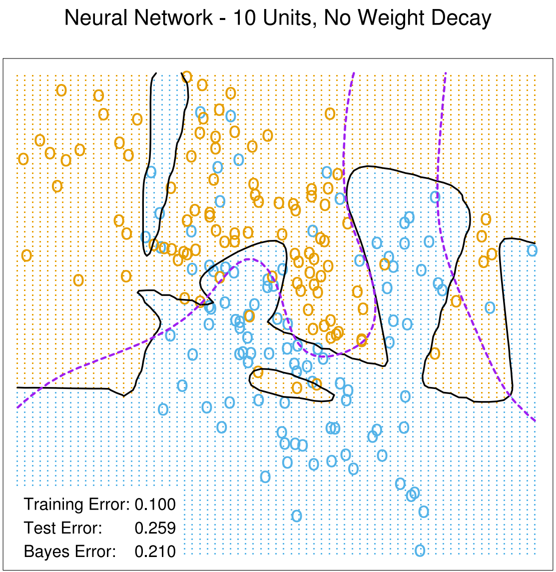
Add regularization to loss:

$$L = \sum_i l(f(x_i), y_i) + \frac{\lambda}{2} \sum_{ij} w_{ij}^2$$

Same as adding $-\eta\lambda w_{ij}$ to batch update of w_{ij}
(or $-\frac{1}{n}\eta\lambda w_{ij}$ to online update)

Called "weight decay"

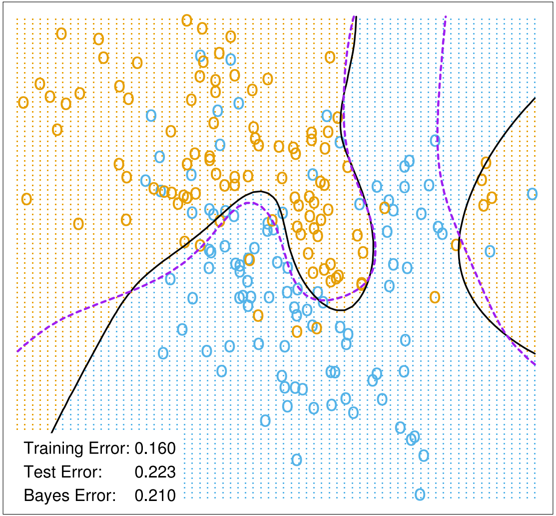
Example



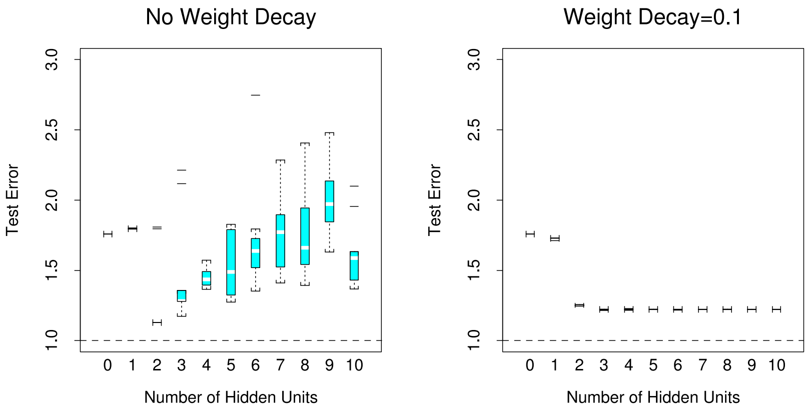
Neural Network - 10 Units, No Weight Decay
Training Error: 0.100
Test Error: 0.259
Bayes Error: 0.210

Example

Neural Network - 10 Units, Weight Decay=0.02



Sum of 2 Sigmoids



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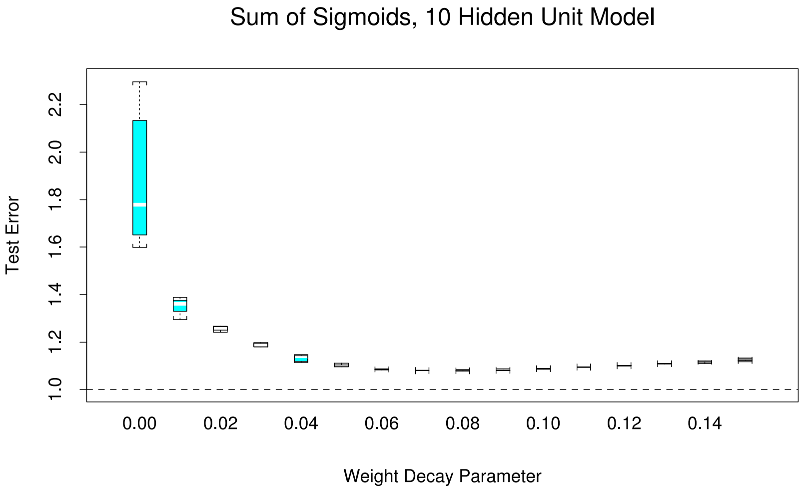

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Either batch (gradient descent) or online (stochastic gradient descent) used.

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Normalize data

Weight Tieing

If we want two weights to be the same...

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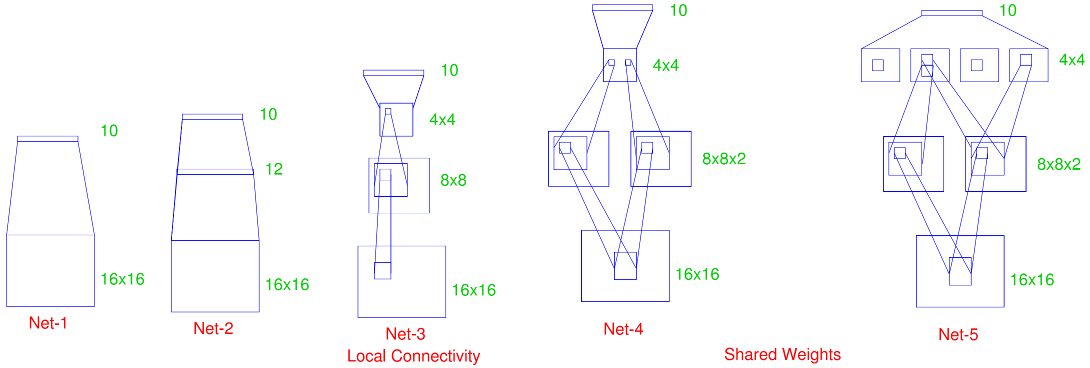
If we want two weights to be the same...

- Start them the same
- Sum their updates and apply to both:

$$\begin{aligned} g(a,b) &= \ldots \\ f(x) &= g(x,x) \\ \frac{\partial f}{\partial x} &= \frac{\partial g}{\partial a}\frac{\partial a}{\partial x} + \frac{\partial g}{\partial b}\frac{\partial b}{\partial x} \\ \frac{\partial f}{\partial x} &= \frac{\partial g}{\partial a} + \frac{\partial g}{\partial b} \end{aligned}$$

Digit Recognizer Networks



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