How universal are ice fracture patterns?

Applying a Crumpling Framework to Icy Moons' Surfaces

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Project Motivation and Goals

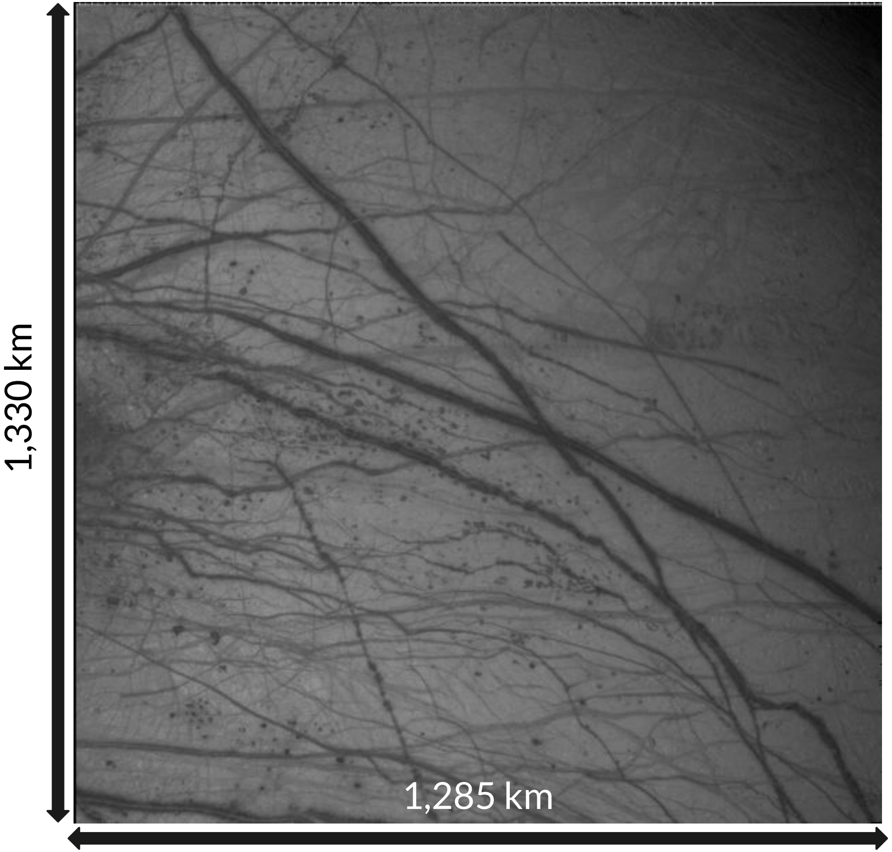
- Investigating mechanisms driving surface fracture patterns on icy moons is made difficult by long timescales on which they occur.
- This summer we are studying images of Europa and Enceladus, drawing an analogy to crumpled thin sheets that consist of:
  - creases (↔ fractures on icy moons), and
  - facets (↔ areas enclosed by fractures).
- The goal is to harness available satellite images to study how the ice crust fractures by obtaining scaling relationships between
  - perimeter,
  - facet area, and
  - topographic variations,

  that can be used to probe fracture mechanisms.

Image Analysis Procedure

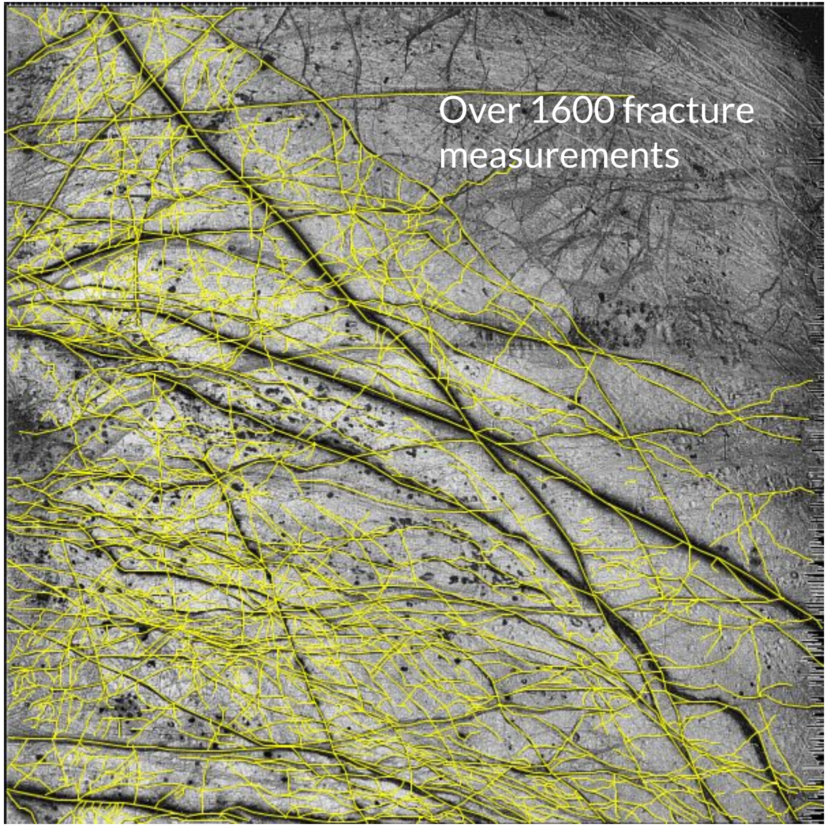
5178r

Raw Image



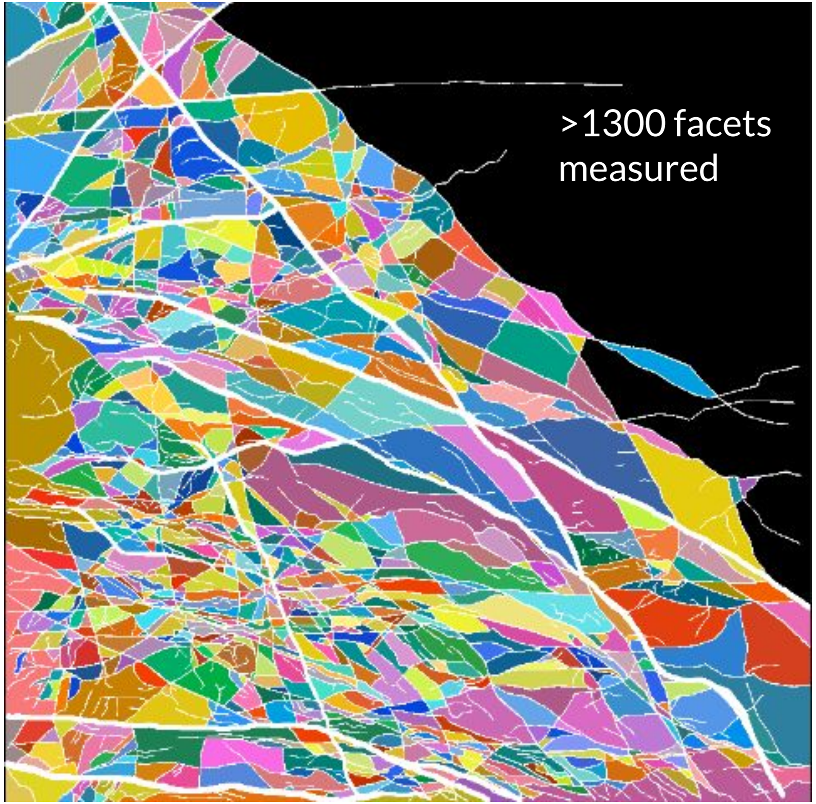
Sharpen

Trace Fractures



Extract Overlay + Thicken

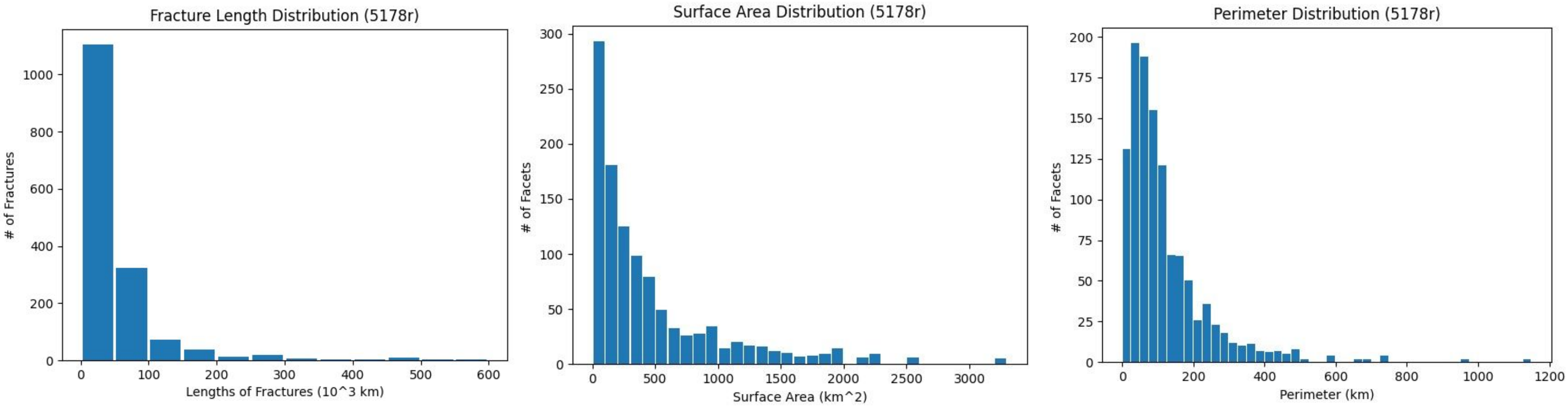
Paint the Facets



Surface area vs. perimeter scalings

5178r

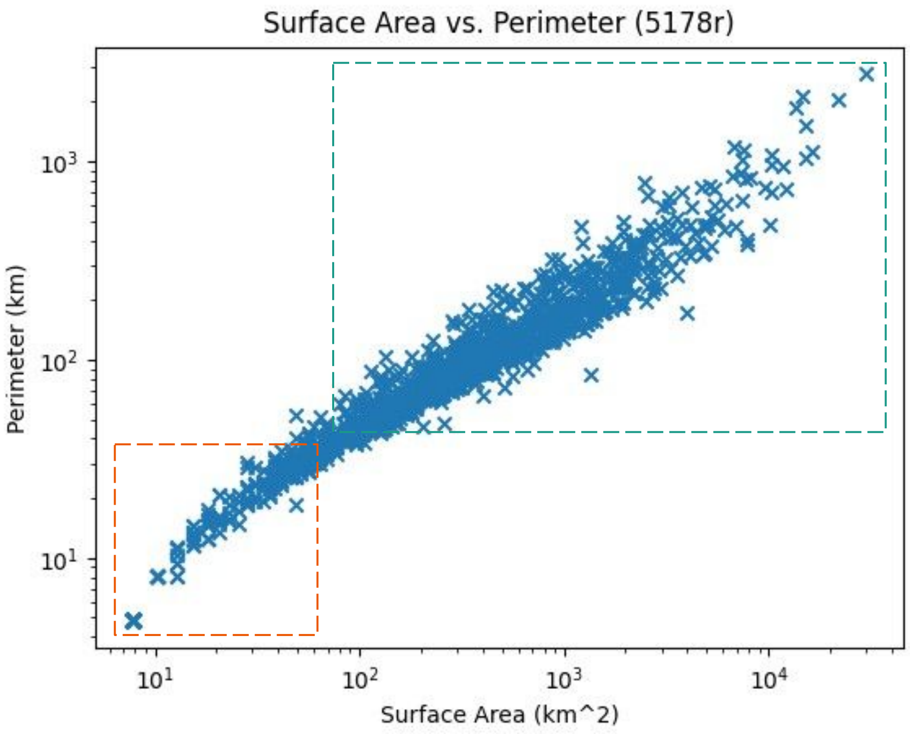
Distributions: fracture length, surface area, perimeter



Key takeaways:

- Distributions skewed towards small scales, but have long tails
- Both small highly fractured regions <u>and</u> large unfractured regions exist

Surface Area vs. Perimeter



Key takeaways:

- Power law relationship between perimeter and surface area
- Possibly different relationship at small versus large scales

Other aspects to explore:

- Surface topography of Europa
- Coloration v fracture in images
- Correction for surface curvature (for large images)

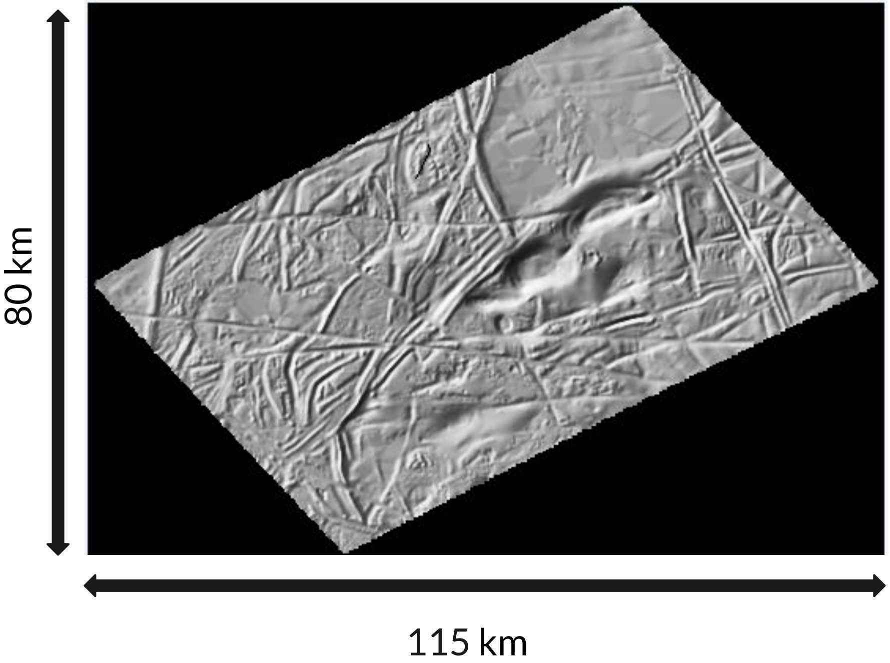
We proceed by study the topographic variation on Europa

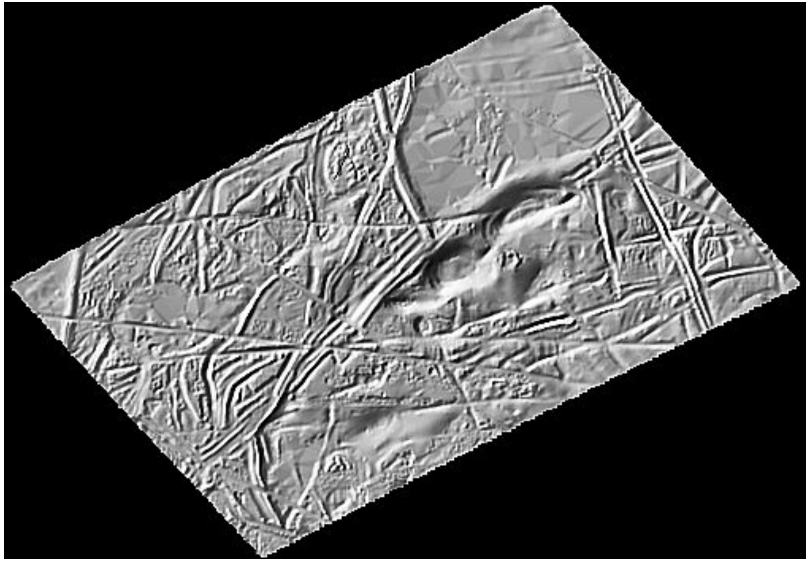
Surface Topography

Rhadamanthys

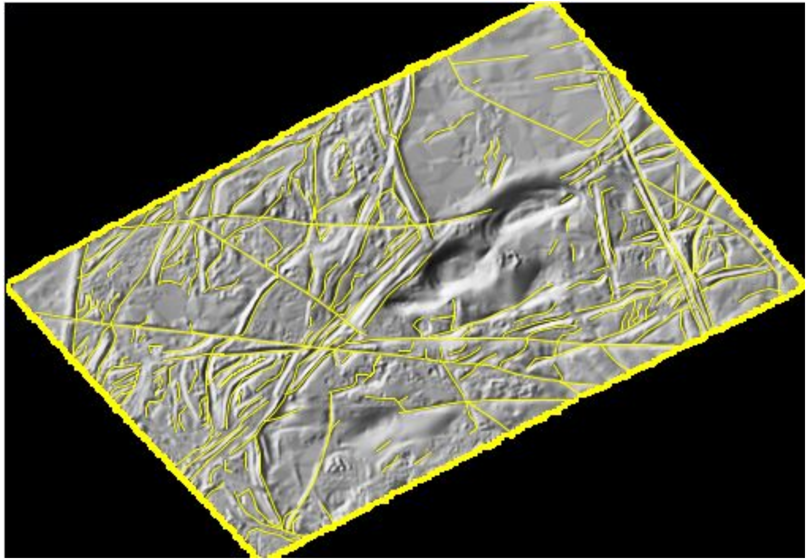
Raw Image (DTM)

Sharpen

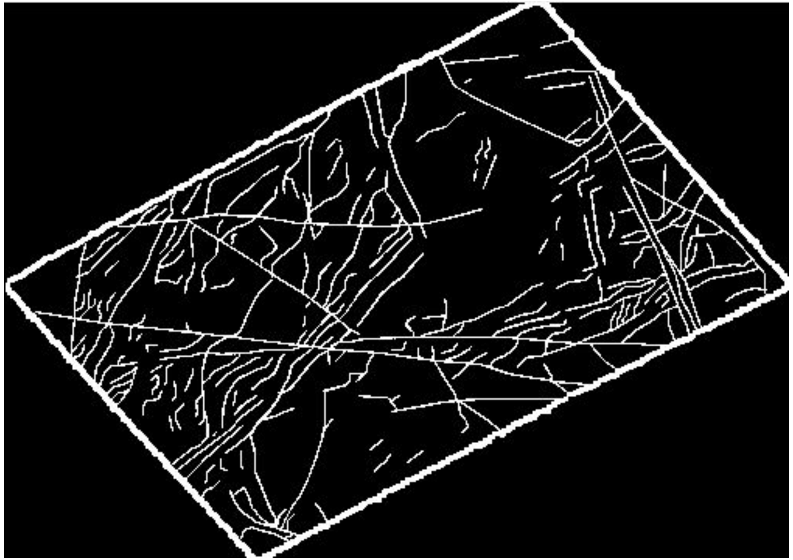




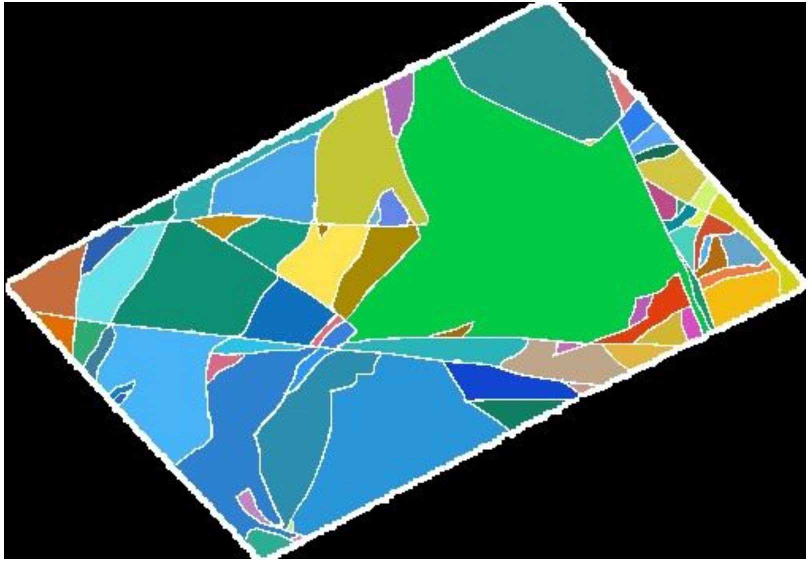
Trace Fractures



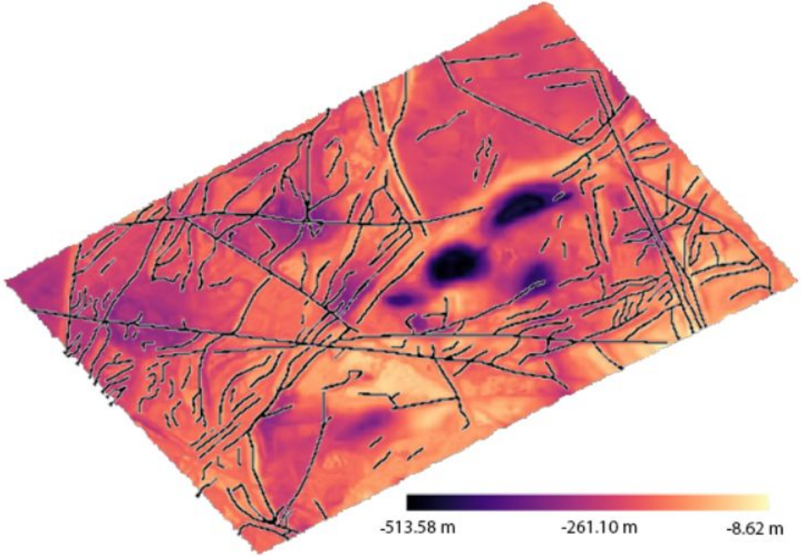
Extract Overlay + Thicken



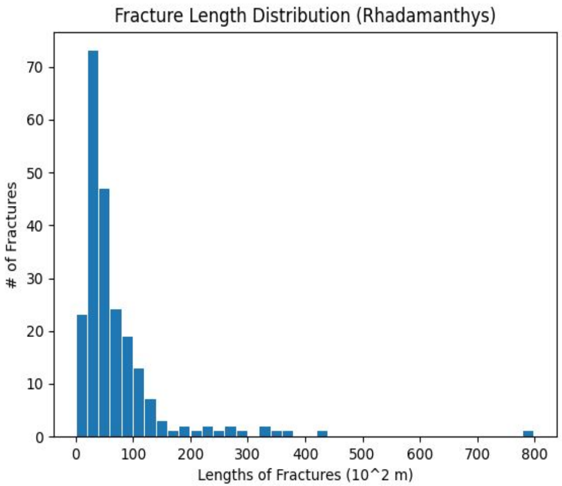
Paint the Facets

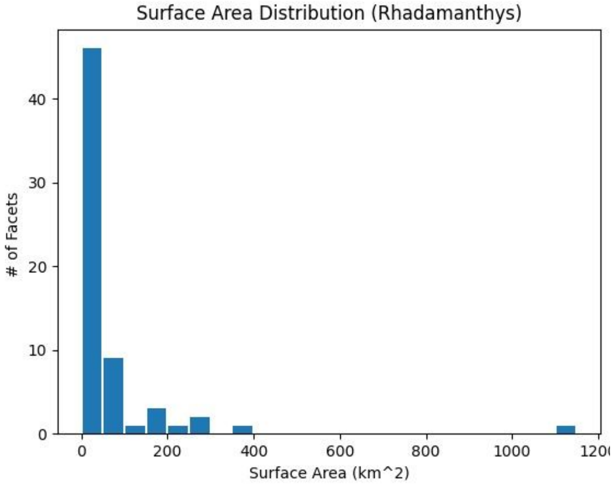


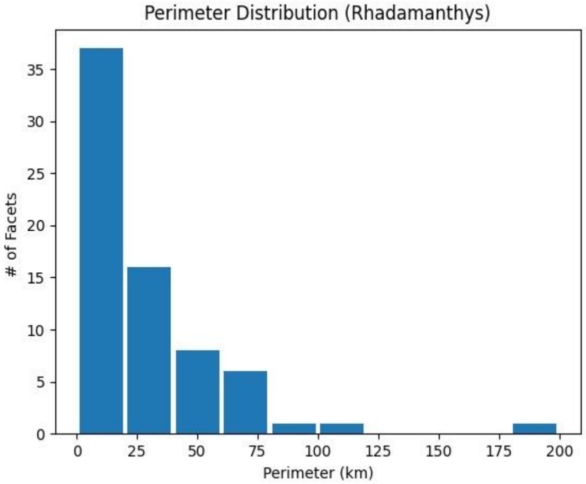
Extract Overlay + Thicken



Distributions: fracture length, surface area, perimeter



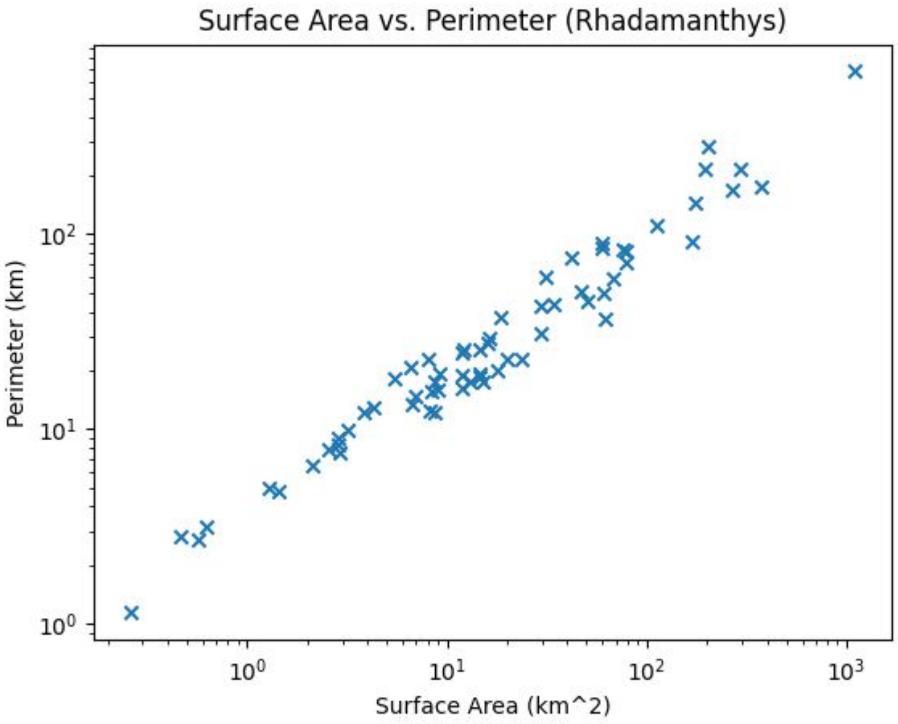




Key takeaways: (similar to 5178r)

- Distributions skewed towards small scales, but have long tails
- Both small highly fractured regions <u>and</u> large unfractured regions exist

Surface Area vs. Perimeter



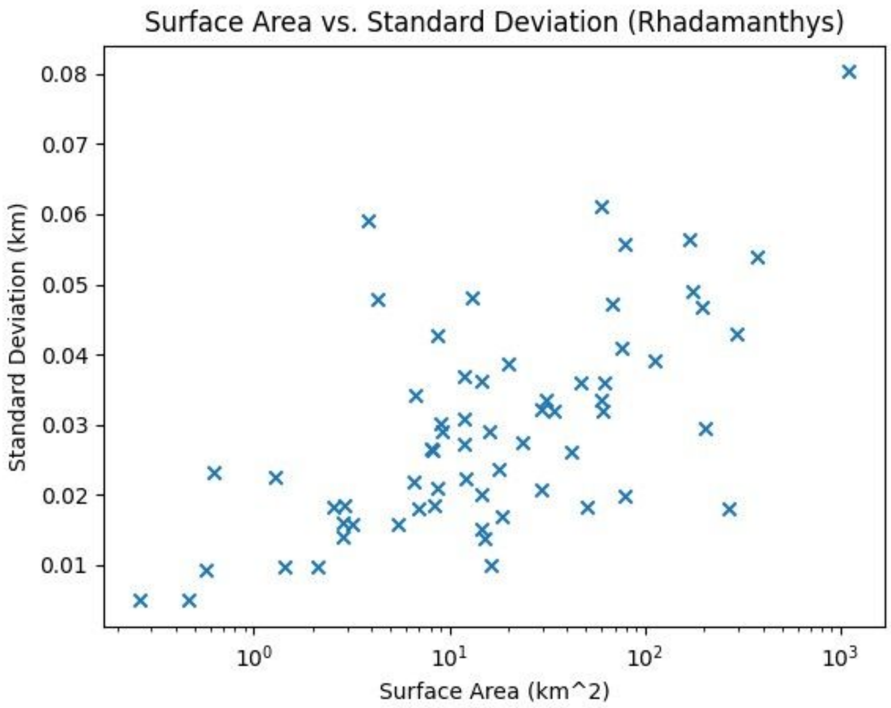
Key takeaways:

- Power law relationship between perimeter and surface area
- No evidence for scale dependent relationship

Caveats:

- Fewer points, smaller image

Elevation Variation within Facets



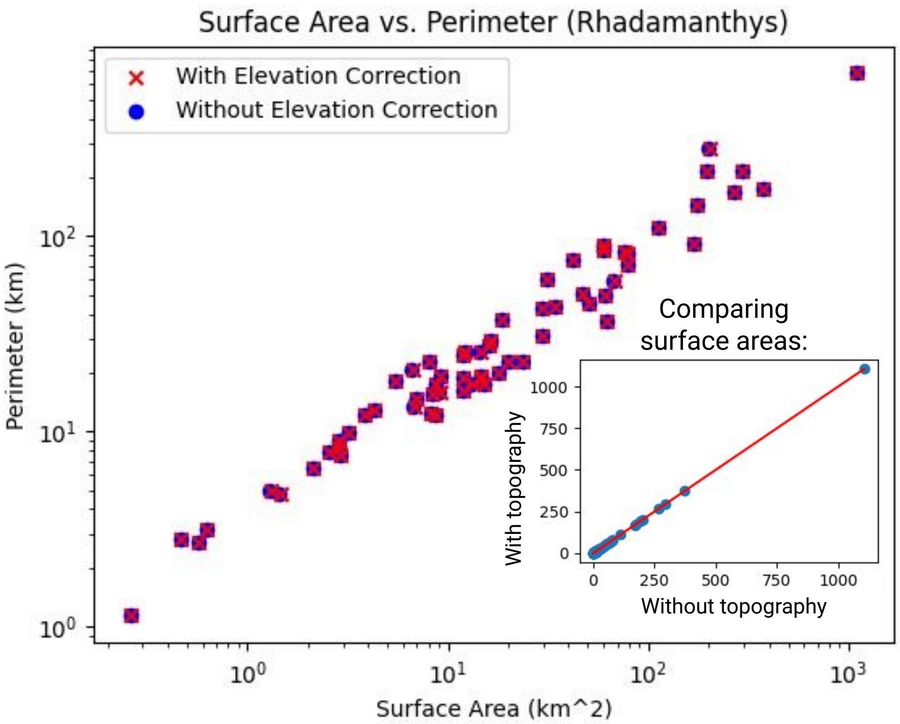
Key takeaways:

- Smaller facets have less topographic variation (as less area for topography to vary over)
- Medium-sized facets have more topographic variations than others

Caveats:

- Region-specific, cannot generalise

Surface Area vs. Perimeter (scaled by topography)



Key takeaways:

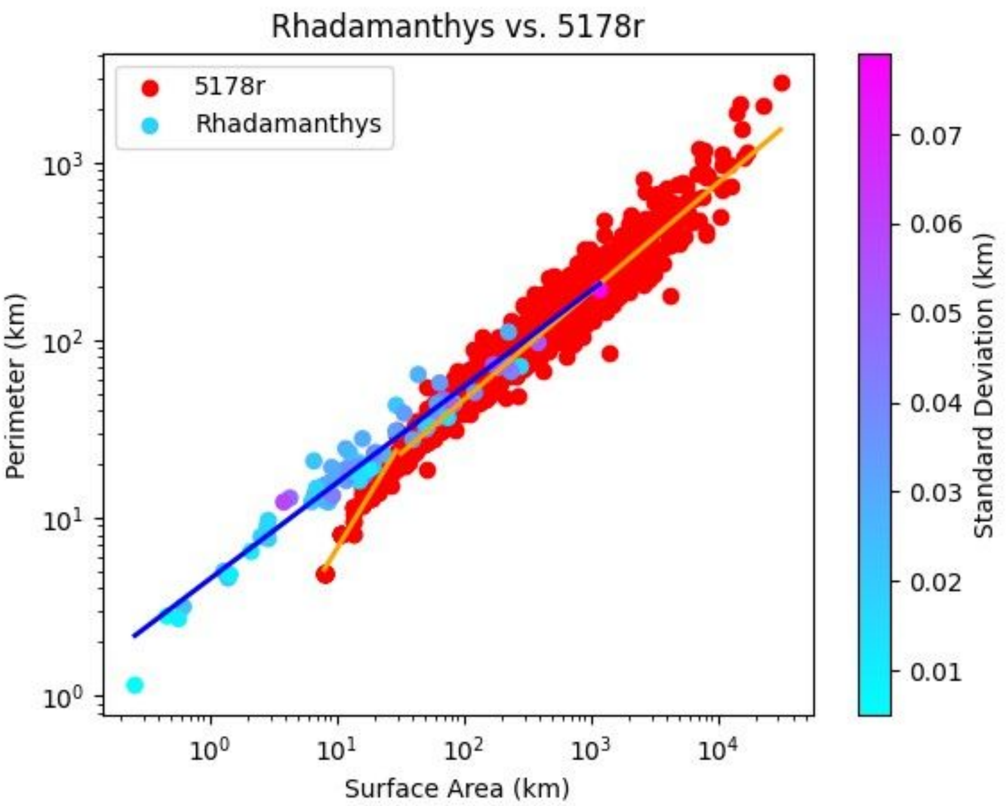
- Correction makes minimal difference
- Variation of the elevation at a given pixel is much smaller than the horizontal and vertical pixel scales, so facets are effectively flat

Caveats:

- Small image at coarse resolution

$$\iint_{Facet} d\mathbf{S} \rightarrow \sum_{pixel \in Facet} \underbrace{\sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + 1}}_{\text{scaling function}} \times PixelArea$$

5178r v Rhadamanthys: Steeper Slope



$$P \sim S^{\lambda}$$

	λ	r-value	p-value
Rhadamanthys	0.54 ± 0.014	0.98	10^{-47}
5178r	1.18 ± 0.024 / 0.610 ± 0.005	0.98 / 0.96	10^{-75} / 0.0

Key takeaways:

- At large scales, only 5178r has data
- At medium scales: obey **similar** scaling. Same physical processes possibly at work here.
- At small scales, different scalings:
  - For a given perimeter, surface area is larger in 5178r.
  - For a given surface area, perimeter is larger in Rhadamanthys.

Caveats:

- Smaller pixel scales present in Rhadamanthys than 5178r.
- At large scales, no data for Rhadamanthys.

Two scalings at small scales

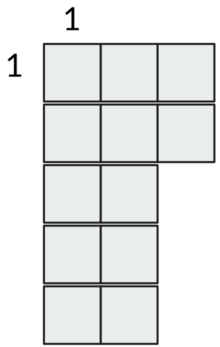
A geometric interpretation

5178r v Rhadamanthys: physical explanation

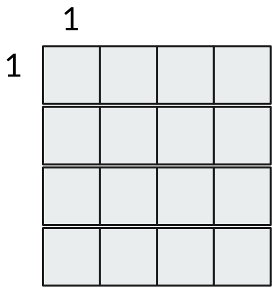
How can a fixed **perimeter** enclose different surface areas?

5178r v Rhadamanthys: physical explanation

How can a fixed **perimeter** enclose different surface areas?



Perimeter = 16
Area = 12
i.e., Rhadamanthys

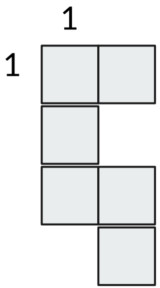


Perimeter = 16
Area = 16
i.e., 5178r

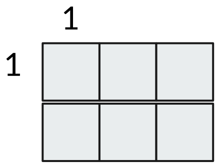
Slide inspired by some high school maths teachers!

5178r v Rhadamanthys: physical explanation

How can a fixed **area** have a different surface perimeter?



Perimeter = 14
Area = 6
i.e., Rhadamanthys



Perimeter = 10
Area = 6
i.e., 5178r

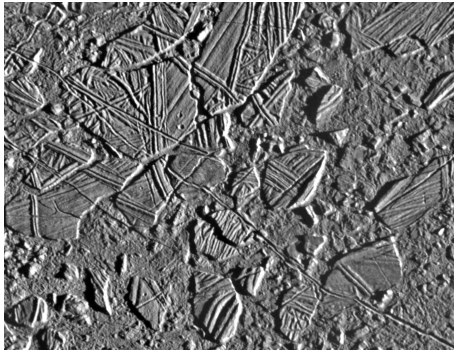
Slide inspired by some high school maths teachers!

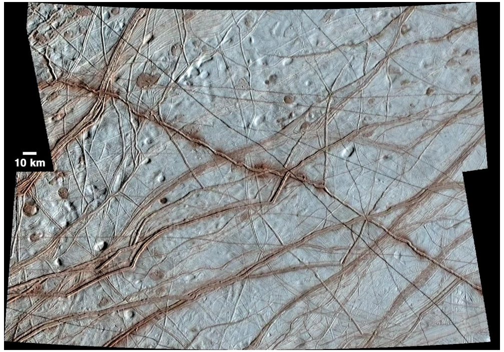
Looking ahead

Future work

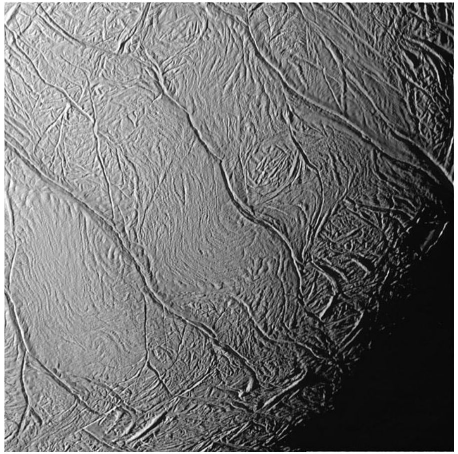
Apply our methodology to different surface features on Europa & Enceladus to obtain empirically-derived relationships between surface area and perimeter (and, where relevant, topographic variations) **at small, medium, large scales**

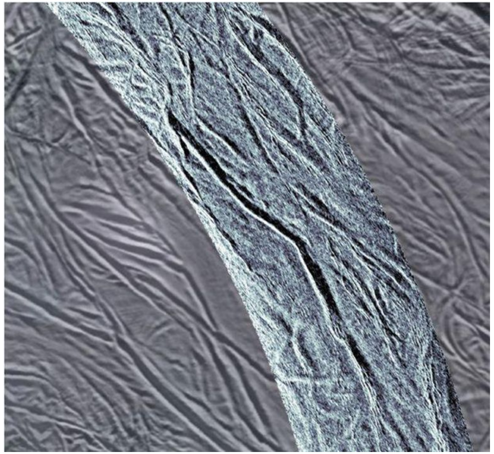
[long-term goal] Physical explanation for the relationships


Conamara Chaos, Europa



Rhadamanthys Linea, Europa


Tiger Stripes, Enceladus

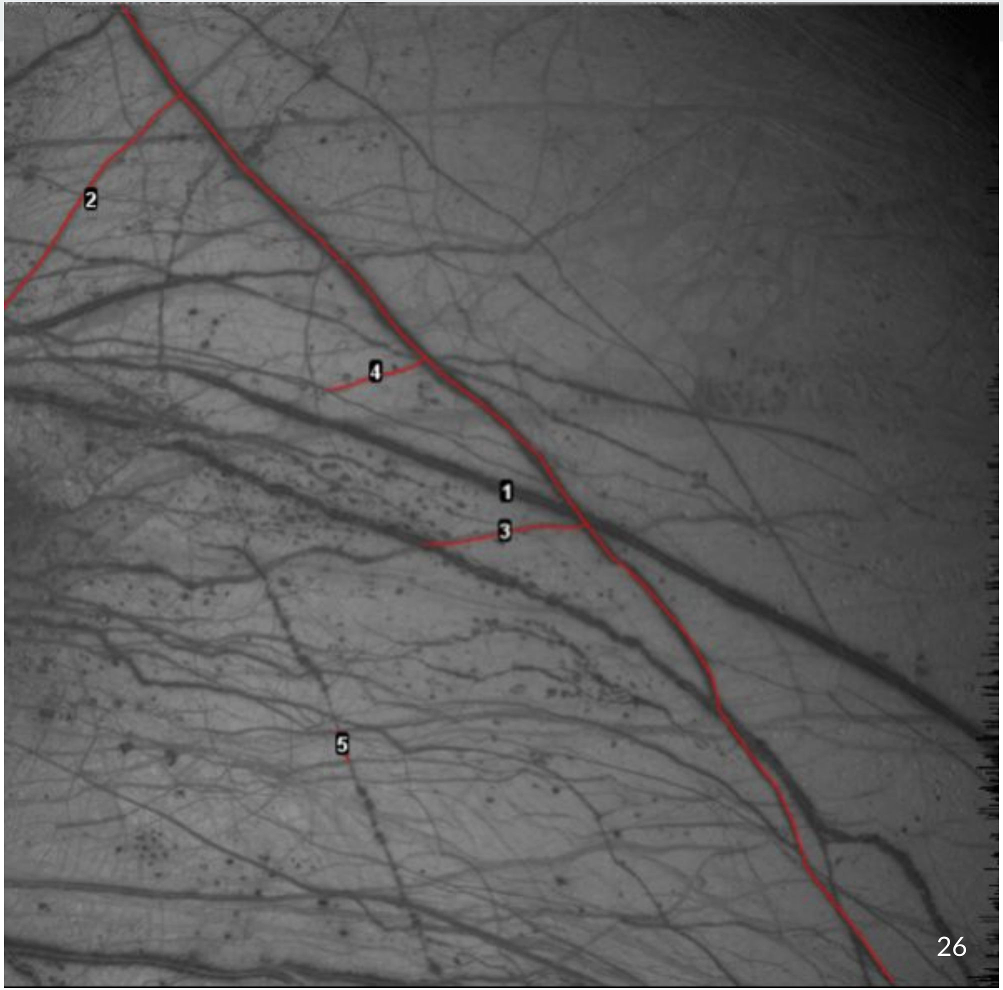

Grooves, Enceladus

Additional Checks

Variability in Measurements

Fracture Measurements in ImageJ vs. Photoshop

Fracture #	ImageJ	Photoshop
1	1022.147	1015.589
2	226.635	229.908
3	130.08	125.392
4	82.57	83.05
5	27.406	27.513



Repeated Fracture Measurements in ImageJ

Trial	Small Fracture (0-50px)	Medium Fracture (50-500 px)	Large Fracture (500 -)
1	38.626	225.297	817.134
2	38.052	225.927	824.96
3	39.351	225.401	820.601
4	39.587	224.685	822.793
5	38.103	224.475	820.667
6	39.695	226.515	824.855
7	37.831	226.052	829.308
8	39.574	226.105	828.285
9	38.061	226.317	824.093
10	38.367	223.242	828.009
Standard Deviation	0.183140656	1.453104435	7.689786245

Additional Checks

Automated Segmentation Techniques

Automated segmentation techniques

- ImageJ oversegments the image
- The extent of oversegmentation is consistent regardless of chosen threshold to convert the image to black and white
- 13,000 facets in this image, versus 1300 found in manual segmentation

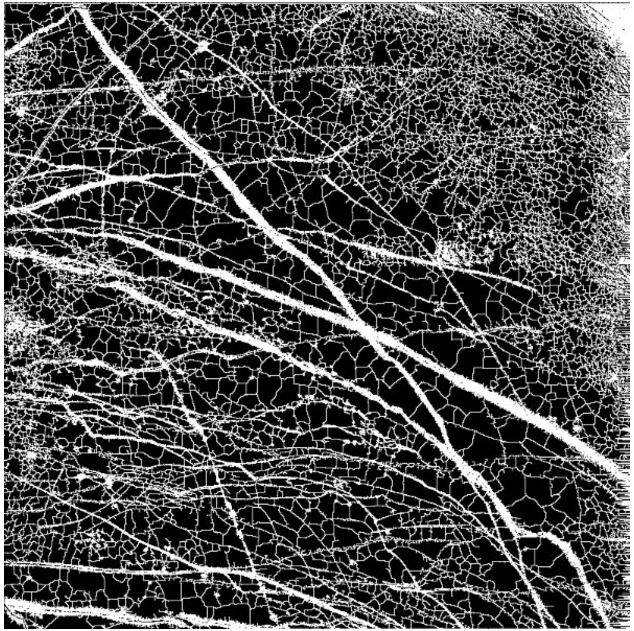
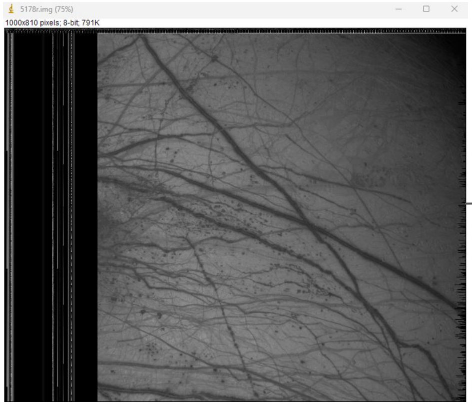
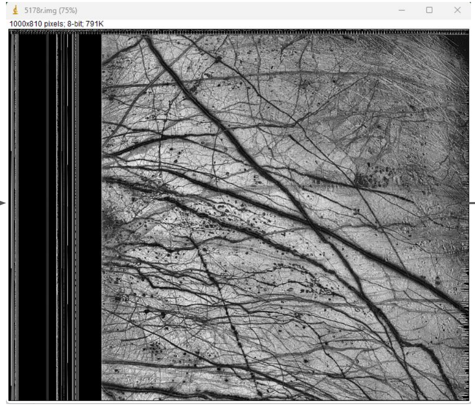


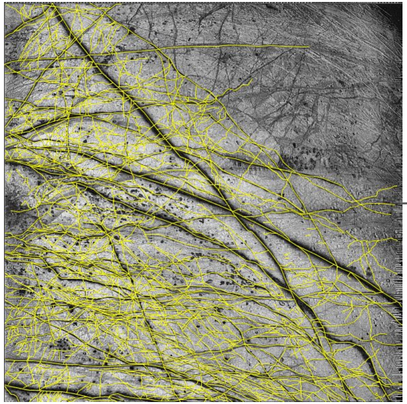
Image Analysis Procedure [rough slide]



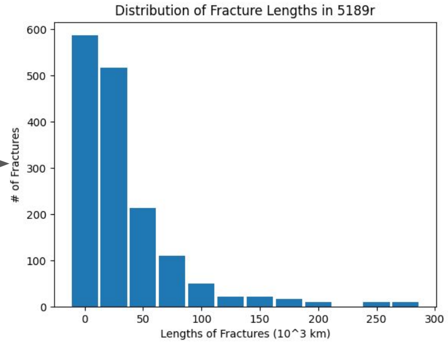
Raw image



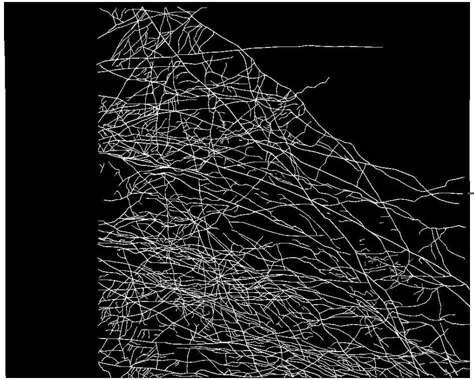
Sharpened



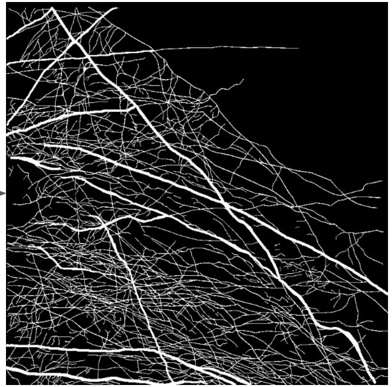
Tracing fractures



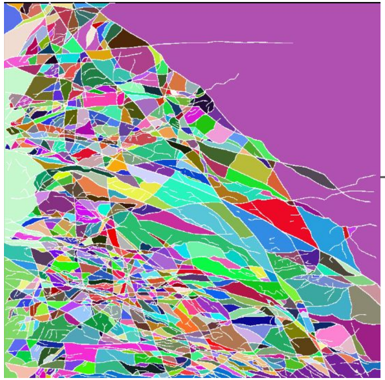
Distribution



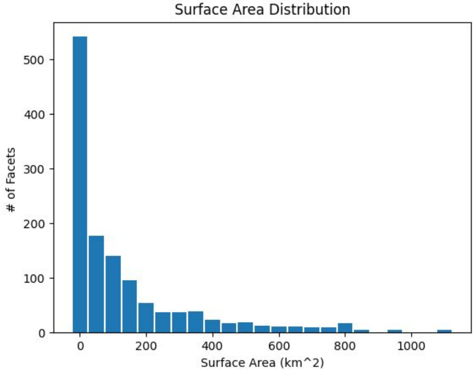
Overlay



Thicken

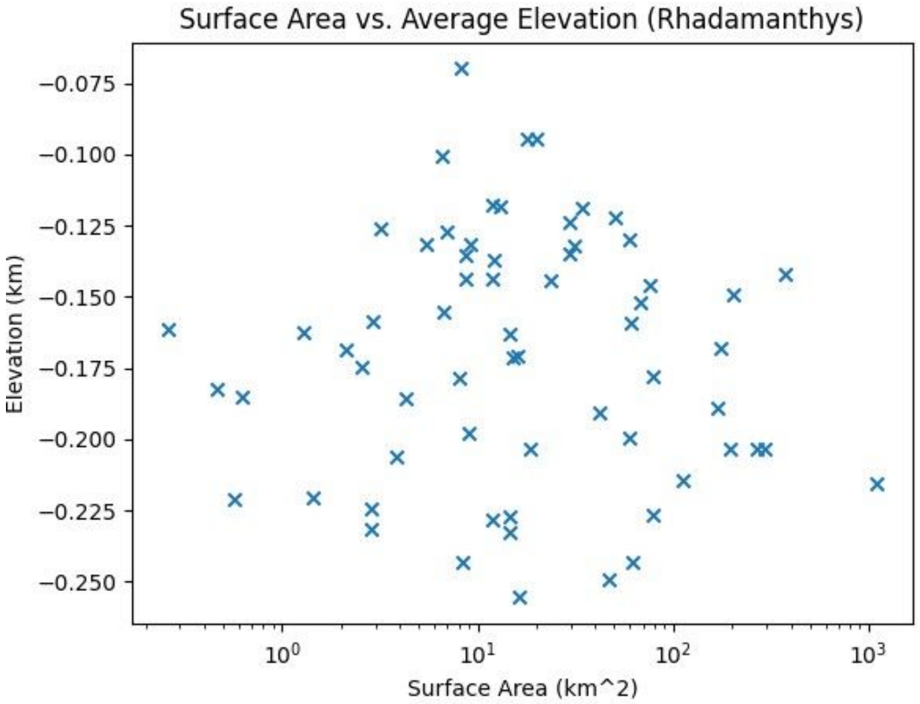


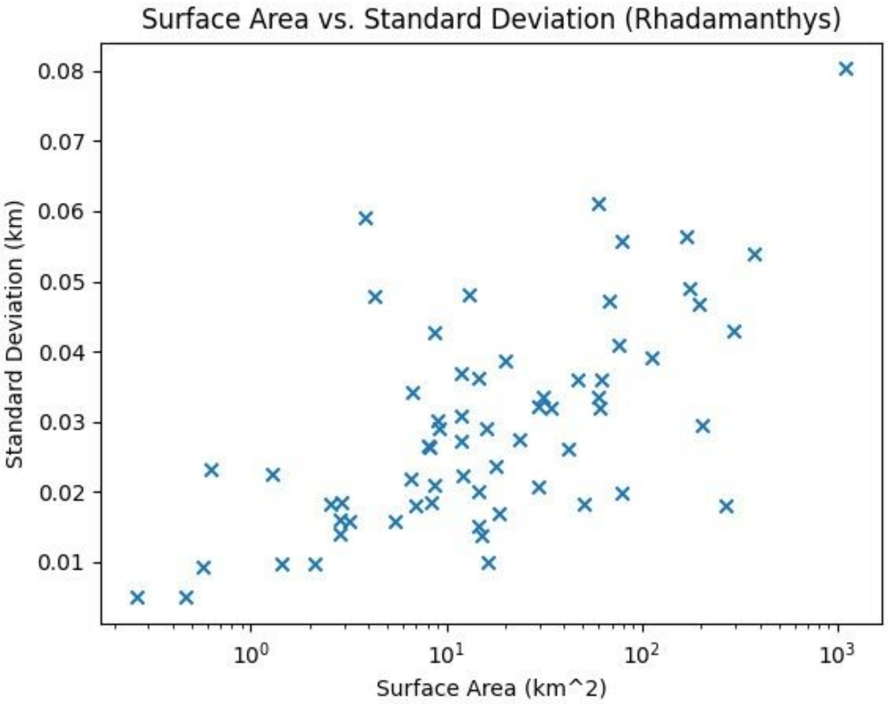
Coloring facets



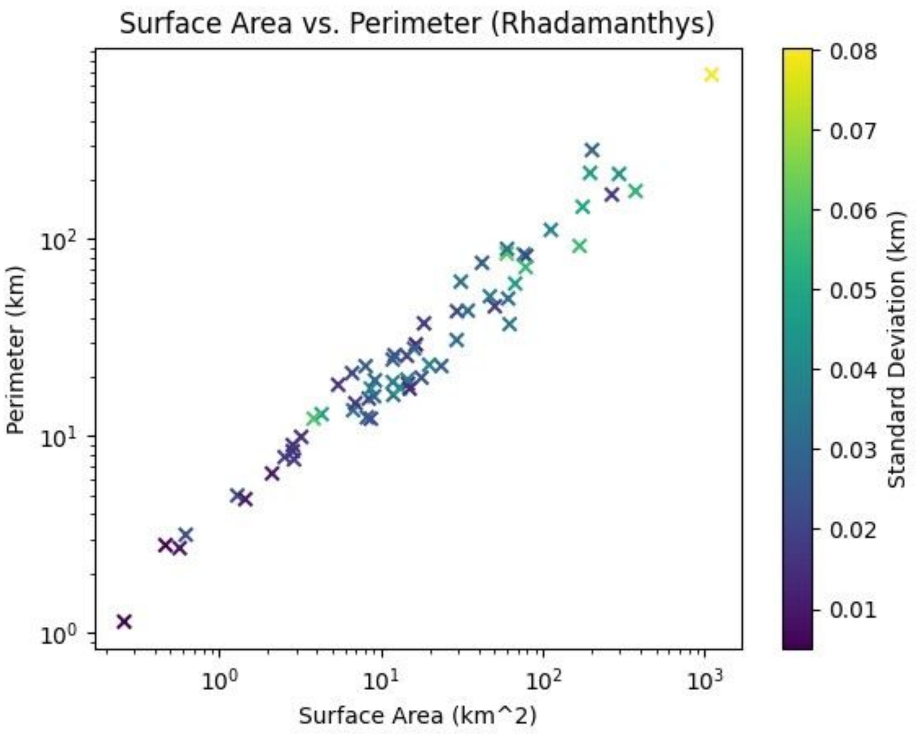
Distribution

Mean & std of elevation of each facet





Elevation Variation within Facets



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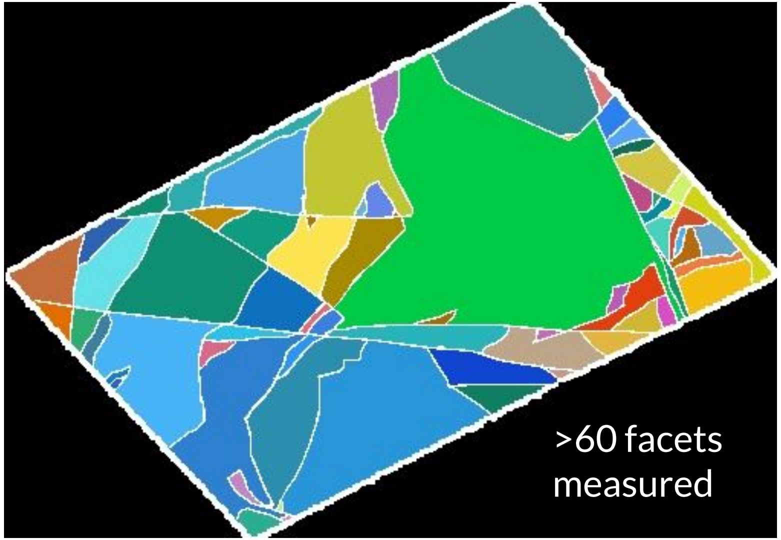
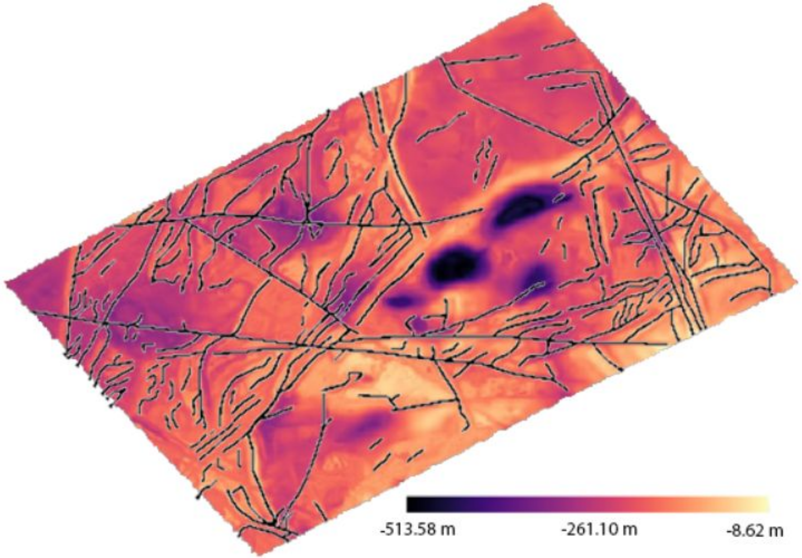
Paint the Facets



Surface Elevation + Overlay



Key takeaways:

- Entire region depressed below reference elevation (0 m)
- Smaller image than 5178r ⇒ fewer enclosed facets
- Large facets have topographic variations (craters and high points)
- Small facets have little topographic variation