MINIA UNIVERSITY
FACULTY OF COMPUTERS & INFORMATION

Discrete Structure
Mid Term Exam
2^{nd} Year Students

19 November,2023
Code: CS201
Time Allowed: 1 hour

Answer the following questions:

Question 1: **[5 Marks]**

A. Determine whether $\neg p \vee p$ is a *tautology* or not. **(1 Mark)**

p	$\neg p$	$\neg p \vee p$
T	F	T
F	T	T

is a *tautology*

B. For each of these lists of integers, **provide a simple formula or rule** that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, **determine the type and the next three terms of the sequence**. **(2 Marks)**

a) 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 1, . . .

alternates 1's and 0's, increasing the number of 1's and 0's each time. The sequence starts with 1 (one, and zero), then 2 (ones, and zeros), then 3 (ones, and zeros), then 4 (ones, and zeros), then 5 (ones, and zeros). Assuming the pattern continues, we then expect 6 (ones, and zeros).

1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, …

b) 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, . . .

Yes, the given sequence numbers are represented in binary number format. In the binary number system to represent number, we use only 0's and 1's. The positive integers go like this: 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110……..

Here explicit formula is the digits of a binary number represent powers of 2.

Examples:

1. $(1100)_2=(1\times2^3)+(1\times2^2)+(0\times2^1)+(0\times2^0)=8+4+0+0=(12)_{10}=12$
2. $(1101)_2=(1\times2^3)+(1\times2^2)+(0\times2^1)+(1\times2^0)=8+4+0+1=(13)_{10}=13$

and so on....

Therefore the sequence is:

1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110,1111……..

C. State True or False with reasons **(1 Marks)**

i. If $2^S = \{\emptyset\}$, then $S = \{\{\emptyset\}\}$ **Reason:** $S=\emptyset$, or $2^S = \{\emptyset,\{\{\emptyset\}\}\}$	**(F)**
ii. The composition $f \circ g$ cannot be defined unless the range of f is a subset of the domain of g. **Reason:** The range of g is a subset of the domain of f.	**(F)**

D. Express the statement "**Every student in MFCI has an email**" in logical expression. Where $P(x)$ is "x in MFCI", $F(x)$ is "x has an email". **(1 Mark)**
The domain of x is the set of all students in Minia University.

$$\forall x(P(x) \rightarrow F(x))$$

Question 2: **[5 Marks]**

A. Use set builder notation and logical equivalences to prove the second distributive law, which states that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ for all sets A, B, and C. (2 Marks)

$$A \cap (B \cup C) = \{x \mid x \in A \ \wedge \ x \in (B \cup C)\}$$
$$A \cap (B \cup C) = \{x \mid x \in A \ \wedge (x \in B \vee x \in C)\}$$
$$A \cap (B \cup C) = \{x \mid (x \in A \ \wedge x \in B) \vee (x \in A \wedge x \in C)\}$$
$$A \cap (B \cup C) = \{x \mid (x \in (A \cap B)) \vee (x \in (A \cap C))\}$$
$$A \cap (B \cup C) = \{x \mid x \in ((A \cap B) \ \cup (A \cap C))\}$$

B. Define composite function and find fog(3) and gof(3) where, $f(x) = 3x^2 - 1$ and $g(x) = 5+2x$ **(1 Marks)**

$fog(3) = f(g(x)) = 3(5+2x)^2 - 1 = 3(25+4x^2+20x)-1 = 3(25+(3)^2+20(3))-1 = 281$

$gof(3) = g(f(x)) = 5+2(3x^2 - 1) = 5+6x^2-2 = 5+6(3)^2 - 2 = 57$

C. Let P(x) be the predicate "x > 1/x." **[2 Marks]**

a) Write P(2), P(1/2), P(−1), P(−1/2), and P(−8), and indicate which of these statements are true and which are false.
b) Find the truth set of P(x) if the domain of x is R, the set of all real numbers.
c) If the domain is the set R+ of all positive real numbers, what is the truth set of P(x)?

P(2)→T 2>1/2	**P(1/2)→F ½ ≯2**	**P(-1/2→T -1/2 >-2**	**P(-8)→F-8≯ -1/8**

b) The truth set x>1 or -1<x<0
c) The truth set x>1