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Higher National Diploma in Information Technology

First Year, First Semester Examination – 2015

HNDIT 11072: Mathematics for Computing

Modal Answers

01.

(i) Define the following terms with examples (08 marks)

a) Set

A set is an unordered collection of zero or more distinct well defined objects

Eg: $A = \{1, 2, 3\}$

b) Cardinality of a set

Cardinality refers to number of elements in a set

E.g.: $A = \{a, e, i, o, u\}$ then $|A| = 5$

c) Complement of a set

The complement of a set "A" is the set of elements which belongs to the universal set but which does not belong to "A"

Eg: let $Y = \{x: x \geq 0\}$ then $Y^c = \{x: x < 0\}$

d) Singleton set

A set which contain only one element is called a singleton set

$A = \{i\}$

(ii) If U={1,……,10} A={1,2,3} B={3,4,5} find the following sets (06 m

a) $A \oplus B = \{1,2,4,5\}$

b) $(A \cup B) \cap B^C = \{1,2\}$

c) $A \cap (B \cap A^C) = \{\}$

(iii) Show that $(A \cup B)^C = A^C \cap B^C$ using laws. (06 m

(Total 25 Marks)

02.

(i) Given $U = Z^+$

$A = \{x \mid x \text{ is an integer and } 1 \leq x \leq 5\}$

$B = \{3, 5, 17\}$

$C = \{1, 2, 3, \ldots\}$

Find

a) $A \cup (B \cap C)$ $= \{1,2,3,4,5,17\}$ (02 marks)

b) $A \backslash B$ $= \{1,2,4\}$ (02 marks)

c) $P(B)$ $= \{\{3\},\{5\},\{17\},\{3,5\},\{3,17\},\{5,17\},\{3,5,17\},\{\ \}\}$ (03 marks)

d) $A^{'} \times B = \{(1,3),(1,5),(1,17),(2,3),(2,5),(2,17),(3,3),(3,5),(3,17),$
$(4,3),(4,5),(4,17),(5,3),(5,5),(5,17)\}$ (03 marks)

(ii) If $X = \{\{a,b\}, \{c\}\}$, $Y = \{\{a\}, \{b,c\}\}$ and $Z = \{\{a,b,c\}\}$, then show that X, Y and Z are mutually disjoint. (04 marks)

$X \cap Y = \{\}$, $Z \cap Y = \{\}$, $X \cap Z = \{\}$

Hence X,Y,Z are mutually disjoint

(iii) Write the dual of the followings

a) $(U \cap A) \cup (B \cap A) = A$ (02 marks)

$(\emptyset \cup A) \cap (B \cup A) = A$

b) $(A \cap U) \cap (\emptyset \cup A^C) = \emptyset$ (02 marks)

$(A \cup \emptyset) \cup (U \cap A^C) = U$

then $p \in (A \cup B)'$

$\Rightarrow p \in (A \cup B)$

$\Rightarrow p \notin A$ and $p \notin B$

$\Rightarrow p \in A'$ and $p \in B'$

$\Rightarrow p \in A' \cap B'$

$\therefore (A \cup B)' \subseteq A' \cap B'$ (1)

Let q be an arbitrary element of $A' \cap B'$.

Then $q \in (A' \cap B')$

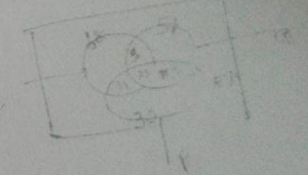
$\Rightarrow q \in A'$ and $q \in B'$

$\Rightarrow q \notin A$ and $q \notin B$

$\Rightarrow q \notin A \cup B$

$\Rightarrow q \in (A \cup B)'$

$\therefore A' \cap B' \subseteq (A \cup B)'$ (2)



From (1) and (2), we get $(A \cup B)' = A' \cap B'$

(iv) In a city three daily newspapers are P, Q, and R are published. 42% of the people in that city read P, 68% read Q, 51% read R; 30% read P and Q; 28% read Q and R; 36% read P and R; 8% do not read any of the three newspapers. Find the percentage of persons who read all the three newspapers. (05 marks)

$(P \cap Q \cap R) = 25$ // hence 25% people read all the three papers

(Total 25 Marks)

02.

(Total 25 Marks)

02.

(i) Given $U=Z^+$

$A = \{x \mid x \text{ is an integer and } 1 \leq x \leq 5\}$

$B = \{3, 5, 17\}$

$C = \{1, 2, 3, \ldots\}$

Find

a) $A \cup (B \cap C) = \{1,2,3,4,5,17\}$ (02 marks)

b) $A \setminus B = \{1,2,4\}$ (02 marks)

c) $P(B) = \{\{3\},\{5\},\{17\},\{3,5\},\{3,17\},\{5,17\},\{3,5,17\},\{\ \}\}$ (03 marks)

d) $A \times B = \{(1,3),(1,5),(1,17), (2,3),(2,5),(2,17), (3,3),(3,5),(3,17), (4,3),(4,5),(4,17), (5,3),(5,5),(5,17)\}$ (03 marks)

(ii) If $X = \{\{a,b\}, \{c\}\}$, $Y = \{\{a\}, \{b,c\}\}$ and $Z = \{\{a,b,c\}\}$, then show that X, Y and Z are mutually disjoint. (04 marks)

$X \cap Y = \{\}, Z \cap Y = \{\}, X \cap Z = \{\}$

Hence X,Y,Z are mutually disjoint

(iii) Write the dual of the followings

a) $(U \cap A) \cup (B \cap A) = A$ (02 marks)

$(\emptyset \cup A) \cap (B \cup A) = A$

b) $(A \cap U) \cap (\emptyset \cup A^C) = \emptyset$ (02 marks)

$(A \cup \emptyset) \cup (U \cap A^C) = U$

(iv) Prove: $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$ (07 marks)

LHS

$(A \cup B) \setminus (A \cap B)$

$= (A \cup B) \cap (A \cap B)^C$

$= (A \cup B) \cap (A^C \cup B^C)$ (De Morgan's law)

$=((A \cup B) \cap A^C) \cup ((A \cup B) \cap B^C)$ (Distributive law)

$= (A \cap A^C) \cup (B \cap A^C) \cup (A \cap B^C) \cup (B \cap B^C)$ (Distributive law)

$= \emptyset \cup (A \cap B^C) \cup (B \cap A^C) \cup \emptyset$ (Complement law)

$= (A \cap B^C) \cup (B \cap A^C)$ (Identity law)

$= (A \setminus B) \cup (B \setminus A)$

(Total 25 Marks)

03.

(i) Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$ and $C = \{x, y, z\}$. Consider the relations R from A to B and S from B to C as follows

$R = \{(1, b), (3, a), (3, b), (4, c)\}$ and $S = \{(a, y), (c, x), (a, z)\}$

Find

a) R^{-1} (02 marks)

$R^{-1} = \{(b,1),(a,3),(b,3),(c,4)\}$

b) domain and range of R (02 marks

$\ldots B^c) \cup (B \cap A^c)$ *(Identity law)*

$= (A \backslash B) \cup (B \backslash A)$

03. **(Total 25 Marks)**

(i) Let A = {1, 2, 3, 4}, B = {a, b, c} and C = {x, y, z}. Consider the relations R from A to B and S from B to C as follows

R = {(1, b), (3, a), (3, b), (4, c)} and S = {(a, y), (c, x), (a, z)}

Find

a) R^{-1} (02 marks)

$R^{-1} = \{(b,1),(a,3),(b,3),(c,4)\}$

b) domain and range of R (02 marks)

Domain = {1,3,4}

Range={a,b,c}

c) matrix representation M_R of R and M_S of S (04 marks)

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$M_S = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

d) the composition relation R∘S (04 mark

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

$R \circ S = \{(3, y),(3, z), (4, x)\}$

(ii) Consider the following relations on set A={1,2,3,4}

R = {(1, 1), (1, 2), (1, 3), (3, 3)}

S = {(1, 1)(1, 2), (2, 1)(2, 2), (3, 3)}

Determine whether the above relations are reflexive, symmetric or anti-symm

R is not reflexive since 2 ∈ A but (2, 2) ∉ R

S is reflexive not

R is not symmetric since (1, 2) ∈ R but (2, 1) ∉ R

S is symmetric

R is anti symmetric

S is not antisymmetric since $1 \neq 2$, and (1, 2) and (2, 1) both belong to S

(06 marks)

(iii) Let X={1,2,3,4}. Determine whether or not each relation below is a function from X into X with giving reasons.

a) f={(2,3),(1,4),(2,1),(3,2),(4,4)} *No. domain is repeated*

b) g={(3,1),(4,2),(1,1)} *No. 2 in, the domain is left unmapped*

c) h={(1, 1),(2, 1),(3, 1),(4, 1)} *function*

(03 marks)

(iv) Consider the functions $f(x)=x^2+3x+1$ and $g(x)=2x-3$. Find a formula defining the composition function:

a) $f \circ g = f(g(x)) = f(2x-3) = (2x-3)^2+3(2x-3)+1$

b) $g \circ f = g(f(x)) = g(x^2+3x+1) = 2(x^2+3x+1)-3$

(04 marks)

(Total 25 Marks)

04.

(i) Define the following matrices with examples.

a) Diagonal matrix

If all non diagonal elements in a matrix are zero then the matrix is a diagonal matrix

(iv) Consider the functions $f(x)=x^2+3x+1$ and $g(x)=2x-3$. Find a formula definin g t composition function:

a) $f\circ g = f(g(x)) = f(2x-3) = (2x-3)^2+3(2x-3)+1$

b) $g\circ f = g(f(x)) = g(x^2+3x+1) = 2(x^2+3x+1)-3$ (04 m

(Total 25 M

04.

(i) Define the following matrices with examples.

a) Diagonal matrix

If all non diagonal elements in a matrix are zero then the n atrix is a diagonal matrix

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

b) Symmetric matrix

Square matrices for which $a_{ij} = a_{ji}$

$$\begin{bmatrix} -3 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

c) Upper triangular matrix

It is a square matrix where all the entries below the main diagonal are zero

$$\begin{bmatrix} 6 & 4 & 2 & 1 \\ 0 & 6 & 4 & 2 \\ 0 & 0 & 6 & 4 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

(06 marks)

(ii) If $A = \begin{bmatrix} 3 & -2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ then compute

a) (A+B)C (04 m

$$A+B = \begin{bmatrix} 4 & 0 \\ 6 & 8 \end{bmatrix}$$

$$(A+B)C = \begin{bmatrix} 8 & 12 \\ 44 & 10 \end{bmatrix}$$

b) $A^T + B^T$ (03 m

$$A^T = \begin{bmatrix} 3 & 3 \\ -2 & 4 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 4 & 6 \\ 0 & 8 \end{bmatrix}$$

c) Verify that (A+B)C = AC + BC (05 marks)

$$AC = = \begin{bmatrix} -2 & 11 \\ 22 & 5 \end{bmatrix}$$

$$BC = \begin{bmatrix} 10 & 1 \\ 22 & 5 \end{bmatrix}$$

$$AC + BC = = \begin{bmatrix} 8 & 12 \\ 44 & 10 \end{bmatrix}$$

(iii) If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$ and I is the identity matrix of order 3, then evaluate A^2-3A+9I

$$\begin{bmatrix} -12 & -5 & 11 \\ 11 & 4 & 1 \\ -7 & 11 & -6 \end{bmatrix} - \begin{bmatrix} 3 & -6 & 9 \\ 6 & 9 & -3 \\ -9 & 3 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 2 \\ 5 & 4 & 4 \\ 2 & 8 & -3 \end{bmatrix}$$

(07 marks)

(Total 25 Marks)

05.

(i) If $A = \begin{bmatrix} 2 & 5 \\ 3 & 9 \end{bmatrix}$ then find determinant of A (03 marks)

18-15 =3

$\begin{bmatrix} 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$ and I is the identity matrix of order

3A+9I

$$\begin{bmatrix} -12 & -5 & 11 \\ 11 & 4 & 1 \\ -7 & 11 & -6 \end{bmatrix} - \begin{bmatrix} 3 & -6 & 9 \\ 6 & 9 & -3 \\ -9 & 3 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 2 \\ 5 & 4 & 4 \\ 2 & 8 & - \end{bmatrix}$$

05.

(i) If $A = \begin{bmatrix} 2 & 5 \\ 3 & 9 \end{bmatrix}$ then find determinant of A

18-15 =3

(ii) If $A = \begin{bmatrix} 2 & 0 & 3 \\ -2 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$

Find

a) adjoint of A

matrix of minors $\begin{bmatrix} -2 & -2 & -2 \\ -3 & 2 & 2 \\ 0 & 10 & 0 \end{bmatrix}$

matrix of cofactors $\begin{bmatrix} -2 & 2 & -2 \\ 3 & 2 & -2 \\ 0 & -10 & 0 \end{bmatrix}$

adjoint of A $\begin{bmatrix} -2 & 3 & 0 \\ 2 & 2 & -10 \\ -2 & -2 & 0 \end{bmatrix}$

b) determinant of A

-10

c) A^{-1} using adjoint matrix of A (04 marks)

$$\frac{1}{-10}\begin{bmatrix} -2 & 3 & 0 \\ 2 & 2 & -10 \\ -2 & -2 & 0 \end{bmatrix}$$

(iii) Solve the following system of linear equations using Cramer's rule (10 marks)

$2X + Y + Z = 3$
$X - Y - Z = 0$
$X + 2Y + Z = 0$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

Det D = 3

$$D_x = \begin{bmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{bmatrix} = 3$$

$$D_y = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix} = -6$$

$$D_z = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{bmatrix} = 9$$

X= 1, y= -2, z= 3

(Total 25 Marks)