Quantized Network Coding for Sensor Networks with Applications in Smart Power Grid

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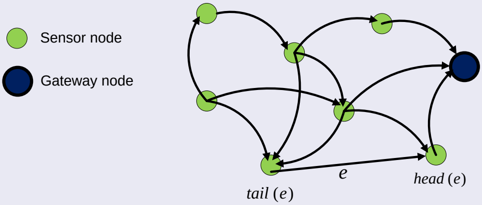
Sensor Networks in Substations

-Photos are taken from the following paper:
A. Nasipuri, R. Cox, J. Conrad, L. Van der Zel, B. Rodriguez, and R. McKosky, Design considerations for a large-scale wireless sensor network for substation monitoring," in 2010 IEEE 35th Conference on Local Computer Networks, pp. 866-873, oct. 2010.

Data Gathering in Sensor Networks



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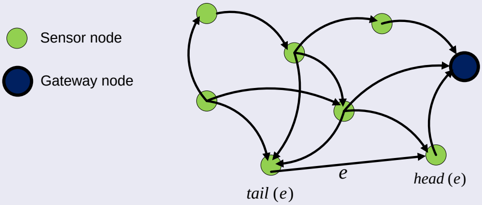
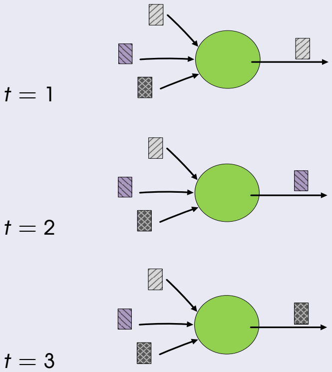


Application based assumptions:

- Correlation of sensed data in substations
  because of the nature of sensed data.
- We model sensed data as *k-sparse* or *k-compressible* data.

Transmission Methods in Network

Packet Forwarding (PF)



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Packet Forwarding (PF)

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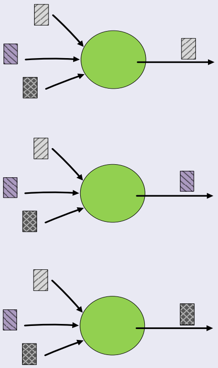
$t = 3$



Network Coding (NC)

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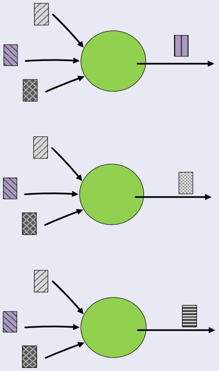

NC vs PF for Data Gathering

For data gathering scenario, in *lossless* networks:

- Packet Forwarding is **sufficient** for transmission of independent messages.
  - Knowledge of *network connectivity* is required for optimal flow of information.
  - Achievable rate region is the same as *min max upper bound*.

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    - Knowledge of *network connectivity* is required for optimal flow of information.
    - Achievable rate region is the same as *min max upper bound*.

- Packet Forwarding + Distributed Source Coding is **sufficient** for transmission of arbitrarily dependent messages.
    - Knowledge of *inter-node dependency* is required for optimal SW coding.
    - DSC and PF can be done separately, as long as work point in rate tube is determined.

Why NC over PF?

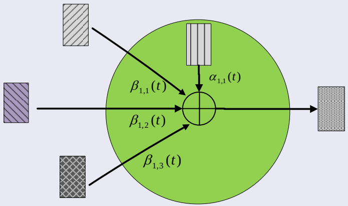
- **Lossy Networks:** achievable rate region using NC is better than that of PF; *e.g. erasure networks*.

- **Error Probability and Diversity:** embedded path diversity in NC increases robustness to link failures.

- **Flexibility:** robustness to network changes makes it more flexible to add/remove nodes.

- **Distributed Source Coding:** random linear network coding can perform embedded inter-node compression.

Linear Network Coding in lossless networks



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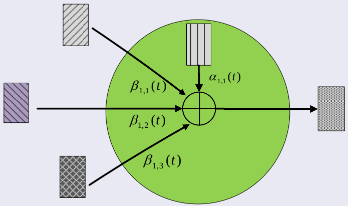

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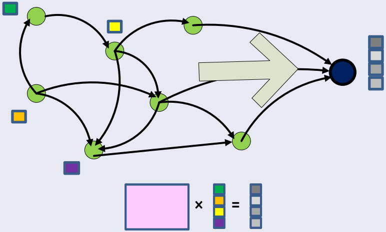

- Decoding is possible, by using matrix inversion in the field, if measurement matrix is full rank.

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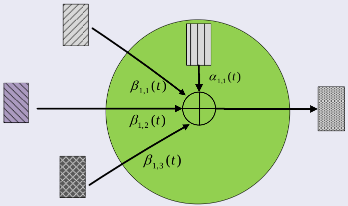

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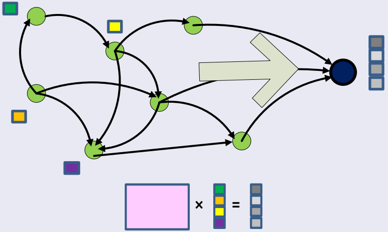

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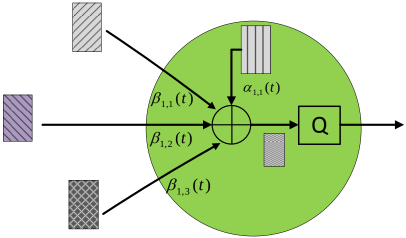
Question

What happens if not enough measurements are collected at decoder?

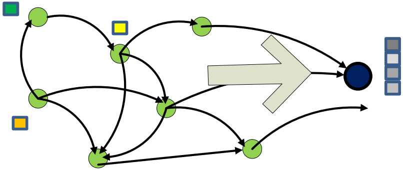
Quantized Network Coding

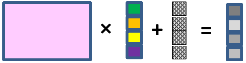


Network Coding + Quantization $\rightarrow$ QNC

- Linear network coding in real field with semi-random coefficients,
- *Quantization* to couple NC result with finite capacity of links.

QNC meets Compressed Sensing





$\underline{n}_{eff,tot}$: effect of quantization noise at decoder

Decoding for: $[\Psi_{tot}]_{m\times n} \cdot [\underline{x}]_{n\times 1} + [\underline{n}_{eff,tot}]_{m\times 1} = [\underline{z}_{tot}]_{m\times 1}$

- If Ψ_{tot} is full rank, a matrix inversion can recover $\underline{x}$, with respect to an error, caused by $\underline{n}_{eff,tot}$.
- If not, we have an under-determined set of equations, which for **Compressed Sensing** decoding (ℓ_1-minimization) *may* help.

ℓ_1-min decoding for QNC

CS claims recovery may be made for $m < n$:

$$[\Psi_{tot}]_{m\times n} \cdot [\underline{x}]_{n\times 1} + [\underline{n}_{eff,tot}]_{m\times 1} = [\underline{z}_{tot}]_{m\times 1}, \quad (1)$$

or,

$$\Psi_{tot} \cdot \phi \cdot \underline{s} + \underline{n}_{eff,tot} = \underline{z}_{tot}, \quad (2)$$

where $\underline{s}$ is k-sparse, and $\left|\left|\underline{n}_{eff,tot}\right|\right|_2 \leq \epsilon$, for a finite ϵ.

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ℓ_1-min recovery

$$\hat{\underline{x}} = \phi \cdot \arg\min_{\underline{s}'} \left|\left|\underline{s}'\right|\right|_1, \quad s.t. \left|\left|\underline{z}_{tot} - \Psi_{tot} \cdot \phi \cdot \underline{s}'\right|\right|_2 \leq \max \left|\left|\underline{n}_{eff,tot}\right|\right|_2 \tag{3}$$

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Advantage:

If we have robust recovery, when $m < n$, we have a *saving* in the required number of channel uses $\longrightarrow$ **inter-node compression**.

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How to ensure robust recovery?

Theorem: If $\Theta_{tot} = \Psi_{tot} \cdot \phi$ satisfies Restricted Isometry Property (RIP) of order $2k$ with constant $\delta_{2k} < \sqrt{2} - 1$, then:

$$||\underline{x} - \hat{\underline{x}}||_2 = ||\underline{s} - \hat{\underline{s}}||_2 \leq c_1(\delta_{2k}) \cdot \max \left|\left|\underline{n}_{eff,tot}\right|\right|_2. \quad (4)$$

Problem I: Satisfaction of RIP

Restricted Isometry Property

As a *norm conservation* property, $\Theta_{m\times n}$ is said to satisfy RIP of order k, with constant δ_k, if for all k-sparse vector $\underline{s}$, we have:

$$1-\delta_k \leq \frac{||\Theta \cdot \underline{s}||_2^2}{||\underline{s}||_2^2} \leq 1+\delta_k. \tag{5}$$

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Our results:

We could find simple conditions on NC coefficients which ensures the entries of resulting Θ_{tot} are random independent Gaussian.

Problem II: Effective Measurement Noise in QNC

$$\Psi_{tot} \cdot \underline{x} + \underline{n}_{eff,tot} = \underline{z}_{tot}$$

Quantization Noise at Nodes

- We use *uniform* quantizer in a constant range for all nodes and vary the step size, depending on the capacity of edges.

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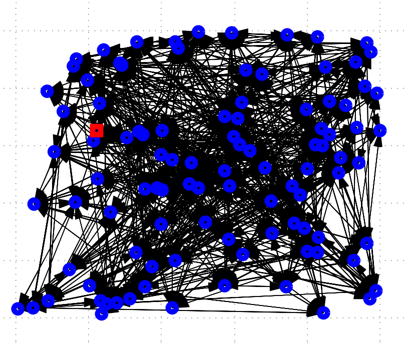
Quantization Noise Propagation

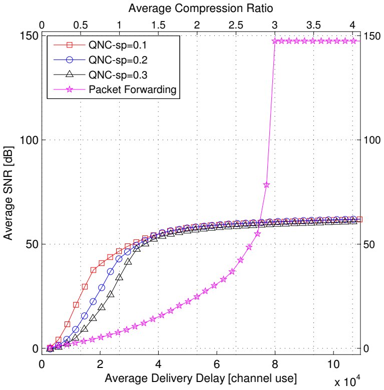
- Quantization noises at each outgoing edge is considered as a random source which propagates noise in the network and has a transfer function to decoder ports.
- We have calculated an *upper bound* on the ℓ_2-norm of effective noise, at the decoder ports, $\left|\left|\underline{n}_{eff,tot}\right|\right|_2$.

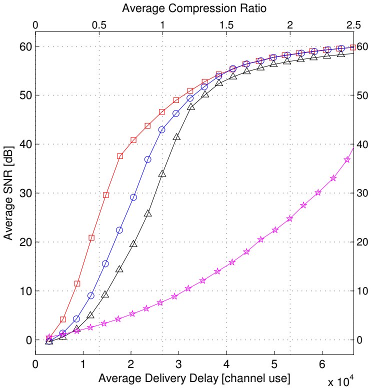
Simulation Setup

- Random deployment with 100 nodes and 1100 edges,
- *Uniformly* distributed sparse vector, $\underline{s}$,
- Random sparsity transform, ϕ,
- $\frac{\text{non zero elements}}{\text{all elements}} = 0.1, 0.2, 0.3,$
- Unit capacity for edges,
- Block length of 25,
- QNC with ℓ_1-min decoding,
- Packet forwarding using shortest distance routing,



Simulation Results: Compression Ratio vs SNR





Conclusion

- Although network coding has no advantage over packet forwarding, in terms of achievable rate for both independent and correlated messages, it is a good alternative because of its *flexibility*, and *embedded route diversity*.
- Moreover, experiments show that QNC with ℓ_1-min decoding has compression advantageous over packet forwarding and conventional network coding, even when the knowledge of inter-node dependency is not available at encoders.

Current Work and Future Plans

Currently working on:

- theoretical RIP based results for robust ℓ_1-min decoding.

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Plans for the future:

- power efficient quantized network coding,
- estimation of sparsity transform at decoder,
- quantized network coding in lossy networks and error probability analysis,
- message generation synchronization,
- numerical stability of decoding, for ill-conditioned measurement matrices.