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Quantized Network Coding for Sparse Messages

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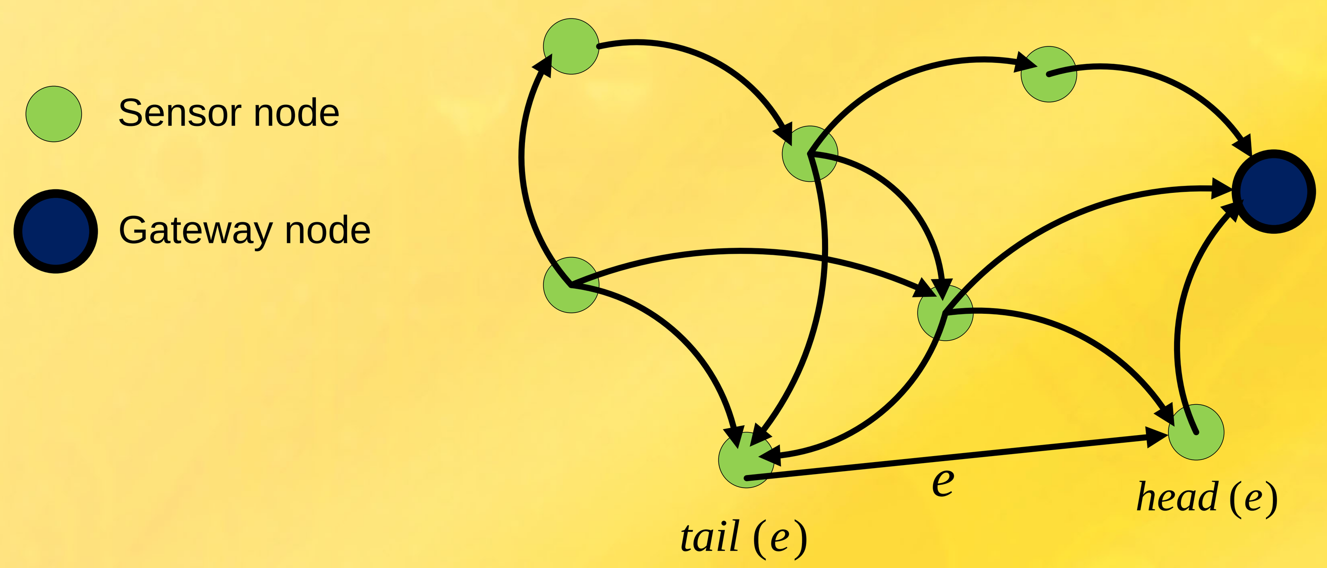
Joint Source-Network Coding

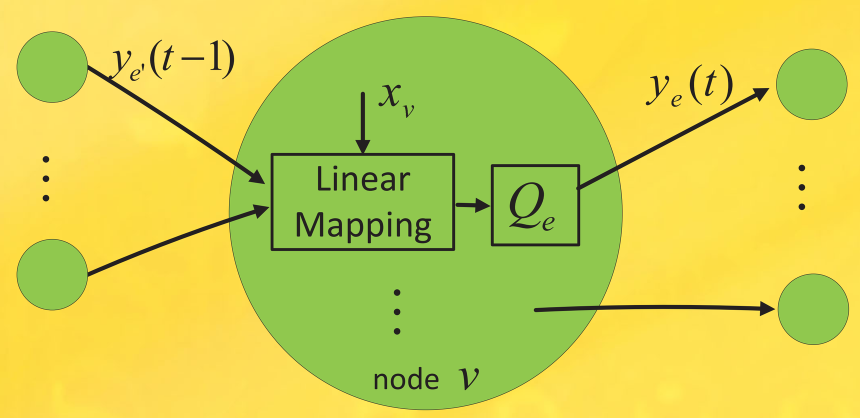
While conventional routing based packet forwarding along distributed source coding is information theoretically sufficient for transmission of inter-node correlated sources in the network, random linear network coding plays a significant role in the next generation communication schemes because of its flexibility and robustness to changes.



Compressed sensing based network coding is a new approach for *non-adaptive* joint source-network coding of sparse (or compressible) messages.

Quantized Network Coding

Consider a directed graph (network), $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, of nodes, $\mathcal{V} = \{1, \ldots, n\}$, and lossless edges, $\mathcal{E} = \{1, \ldots, |\mathcal{E}|\}$. Messages, $\underline{x} = [x_v, v \in \mathcal{V}]$, are k-sparse in transform domain ϕ.



We define *Quantized Network Coding* at each node:

$$y_e(t) = \mathbf{Q}_e\Big[\sum_{e' \in In(v)} \beta_{e,e'}(t)\, y_{e'}(t-1) + \alpha_{e,v}(t)\, x_v\Big], \qquad (1)$$

where $\mathbf{Q}_e$ is the quantizer of the outgoing edge e. Representing quantization noise by $n_e(t)$, we have:

$$y_e(t) = \sum_{e' \in In(v)} \beta_{e,e'}(t)\, y_{e'}(t-1) + \alpha_{e,v}(t)\, x_v + n_e(t), \qquad (2)$$

which implies a linear network (system) with $\underline{x}$ and $n_e(t)$'s, as its inputs.

Received packets at the decoder node, at time t, are:

$$\underline{z}(t) = [y_e(t) : e \in In(v_0)] = B \cdot \underline{y}(t) \qquad (3)$$
$$= \Psi(t) \cdot \underline{x} + \underline{n}_{\mathrm{eff}}(t), \qquad (4)$$

where B is defined such that:

$$\{B\}_{i,e} = \begin{cases} 1\ , & i \text{ corresponds to } e,\ e \in In(v_0) \\ 0\ , & \text{otherwise} \end{cases} . \qquad (5)$$

By storing enough marginal measurements, at the decoder, we build up the *total measurements vector*, $\underline{z}_{tot}(t)$, as follows:

$$\underline{z}_{\mathrm{tot}}(t) = \begin{bmatrix} \underline{z}(2) \\ \vdots \\ \underline{z}(t) \end{bmatrix}_{m \times 1} = \Psi_{\mathrm{tot}}(t) \cdot \underline{x} + \underline{n}_{\mathrm{eff,tot}}(t). \qquad (6)$$

Now, we try to recover $\underline{x}$ from the noisy under-determined measurements, $\underline{z}_{tot}(t)$ ($\Psi_{tot}(t)$ is not full column rank). This can be achieved by adopting compressed sensing in this scenario, and if successful, we have embedded distributed source coding in the network coding.

CS based Decoding

We use an ℓ_1**-min decoder** at the gateway node:

$$\hat{\underline{\mathbf{x}}}(\mathbf{t}) = \phi \cdot \arg\min_{\underline{s}'} ||\underline{s}'||_1, \qquad (7)$$
$$\text{subject to: } ||\underline{z}_{\mathrm{tot}}(t) - \Psi_{\mathrm{tot}}(t)\, \phi\, \underline{s}'||_2^2 \le \epsilon^2(t)$$

with $\epsilon^2(t)$ as an upper bound on ℓ_2 norm of measurement noise, *i.e.* $\underline{n}_{eff,tot}(t)$.

Design of Network Coding Coefficients

Measurement matrices which satisfy Restricted Isometry Property (RIP) are appropriate for compressed sensing of sparse signals. Gaussian ensembles are a choice of matrices with such property.

Theorem: *If the network coding coefficients, $\alpha_{e,v}(t)$ and $\beta_{e,e'}(t)$, are such that:*

- *$\alpha_{e,v}(t) = 0,\ \forall\, t > 2$, and $\alpha_{e,v}(2)$'s are independent zero mean Gaussian random variables,*
- *$\beta_{e,e'}(t)$'s are deterministic,*

then the resulting total measurement matrix, $\Psi_{\mathrm{tot}}(t)$, has zero-mean Gaussian entries, and for every $v, v' \in \mathcal{V}$, where $v \ne v'$, $\{\Psi_{\mathrm{tot}}(t)\}_{iv}$ and $\{\Psi_{\mathrm{tot}}(t)\}_{iv'}$ are independent.

This can be done in a distributed way, where the coefficients are pseudo-randomly generated.

Recovery Error Bound

Theorem: If $\Psi_{\mathrm{tot}}(t)$ and ϕ are such that $\Psi_{\mathrm{tot}}(t) \cdot \phi$ satisfies RIP of order $2k$ with a constant $\delta_{2k} < \sqrt{2} - 1$, and $\hat{\underline{x}}(t)$ is the recovery result of ℓ_1-min decoder of Eq. 7, with $\epsilon^2(t)$ calculated according to Eq. 9, then:

$$||\underline{x} - \hat{\underline{x}}(t)||_2^2 \le c_1\, \epsilon^2(t) \qquad (8)$$

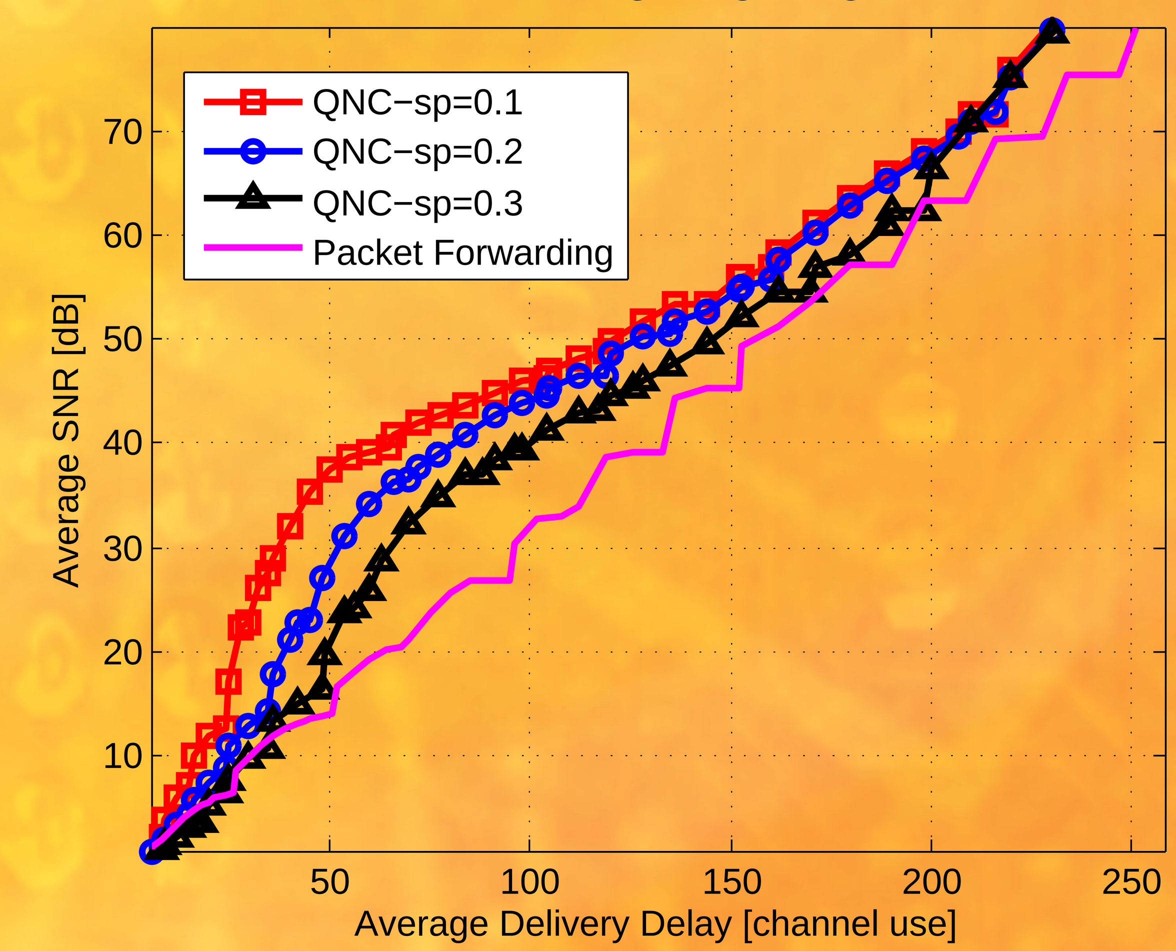
$$\epsilon^2(t) = \frac{1}{4} \sum_{t'=2}^{t} \Big(\sum_{t''=1}^{t'-1} \Delta_Q^T \Big| \prod_{t'''=t''+2}^{t} F(t''') \Big|^T \cdot B^T B \cdot \sum_{t''=1}^{t'-1} \Big| \prod_{t'''=t''+2}^{t'} F(t''') \Big| \Delta_Q \Big) \qquad (9)$$

$$\underline{\Delta}_Q = [\Delta_{Q,e} : e \in \mathcal{E}], \qquad (10)$$

where $\Delta_{Q,e}$ is the step size of uniform quantizer, used for outgoing edge e.

Simulation Results

A random deployment of 100 nodes with uniform distribution of 1400 *unit capacity* links is considered. Messages are k-sparse with $\frac{k}{n} = 0.1, 0.2, 0.3$, and uniformly distributed between -0.5 and $+0.5$. Moreover, a *uniform quantizer* is used at each outgoing edge.



Conclusions

The compressed sensing results helped us develop an efficient network coding scheme for *non-adaptive* practical joint source-network coding of correlated sources.

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