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Quantized Network Coding of Correlated Sources in Wireless Sensor Networks

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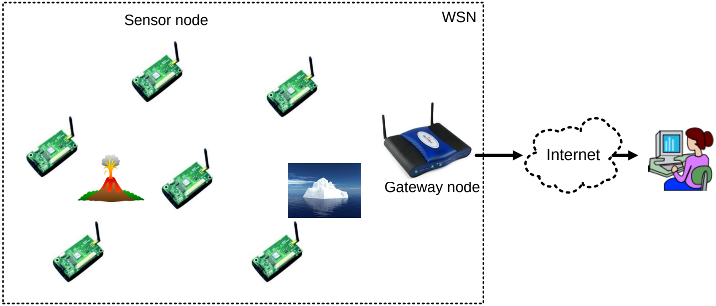
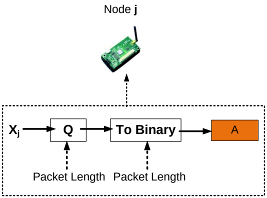
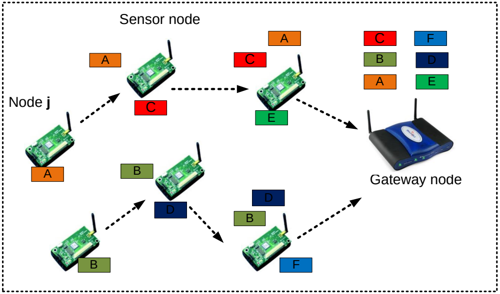
Scenario: Data Gathering in a WSN



- Sensor Nodes measure **scalar-valued** data
- WSN needs to **gather** all the sensed data at the **gateway**
- **Sensed Data** usually have **inter-node** correlation

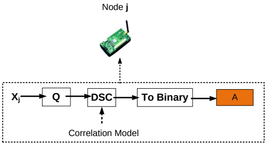
Conventional Approach: Quantization and Packet Forwarding

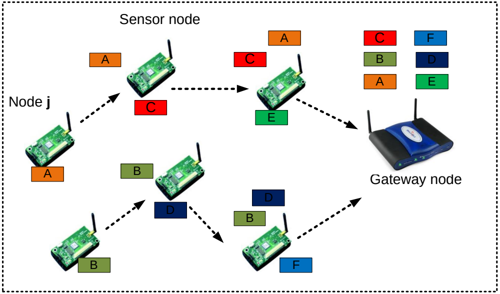




- Sensed Data are quantized and put into **packets**
- Packets are **forwarded** to gateway via multiple hops

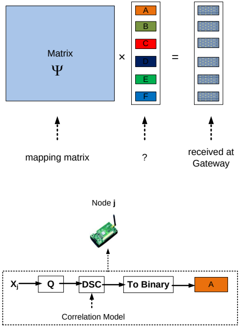
Distributed Source Coding

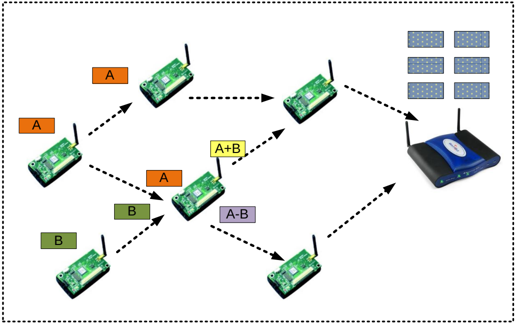




- DSC is necessary since data are correlated
- DSC needs **correlation model** at sensor nodes

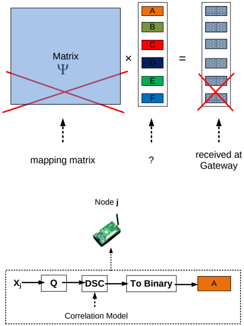
Linear Network Coding

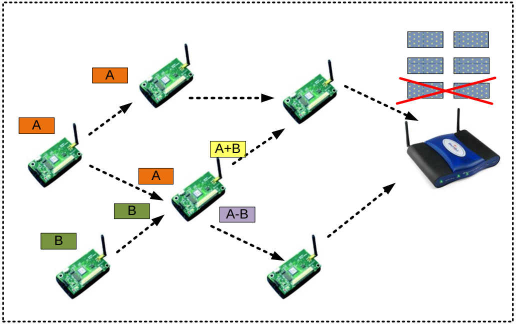




- Network Coding sends linear **combinations** of packets
- **All-Or-Nothing** decoding is done using **matrix inversion**

Linear Network Coding





- Network Coding sends linear **combinations** of packets
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What if we have fewer number of received packets than number of data?

Compressed Sensing and Sparse Recovery

Consider an **under-determined** set of linear equations and measurements:

$$\underline{z}_{m\times 1} = \Psi_{m\times n} \cdot \underline{x}_{n\times 1}, \qquad m < n$$

Can we find $\underline{x}$ from $\underline{z}$? **Yes!** If some conditions are met.

Compressed Sensing (CS) and Sparse Recovery

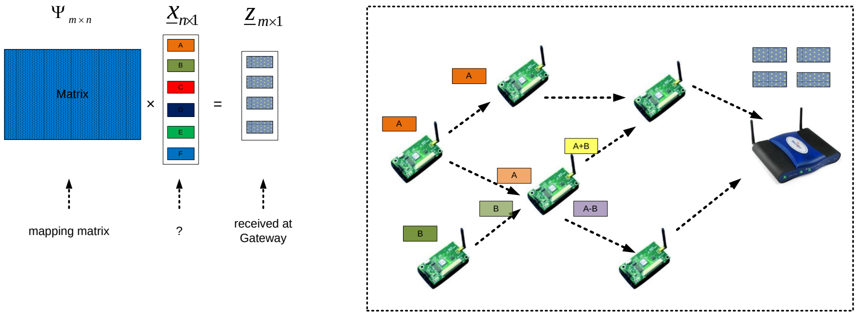
Compressed sensing literature claims that if (Donoho, 06):

1. $\underline{x}$ has many zero elements; *i.e.* **sparse**
2. Ψ satisfies **Restricted Isometry Property (RIP)**; *e.g.* **i.i.d Gaussian random matrices**

then one can find $\underline{x}$ with no error using ℓ_1-minimization (Candes and Tao, 05):

$$\hat{\underline{x}} = \arg\min_{\underline{x}'} ||\underline{x}'||_{\ell_1}, \ \ s.t.\ \Psi\underline{x}' = \underline{z}$$

Linear Network Coding + Sparse Recovery



- CS framework fits into random linear network coding
- correlated sources are near-sparse in some domain
- embedded **distributed compression** with **random** encoding
- Real vs Finite field

Compressed Sensing in Sensor Networks

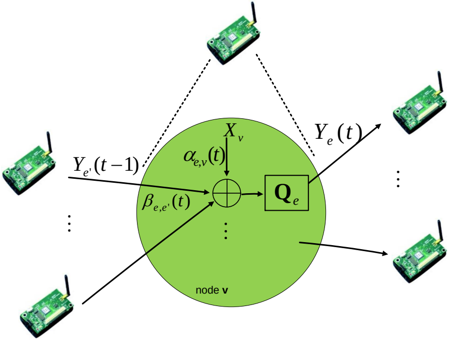
Previous Works

- Source Coding to reduce temporal redundancy
  - Compressive wireless sensing by Bajwa *et al.*, 2006
  - Compressed sensing for networked data by Haupt *et al.*, 2008
- Network Coding and Compressed Sensing
  - Feizi *et al.* on Information Theory side, 2011
  - Bassi *et al.* and Iwaza *et al.* on Finite Field operations, 2012

Current Challenges and Issues

- Provide theoretical guarantee
- Local network coding coefficients and Satisfaction of RIP

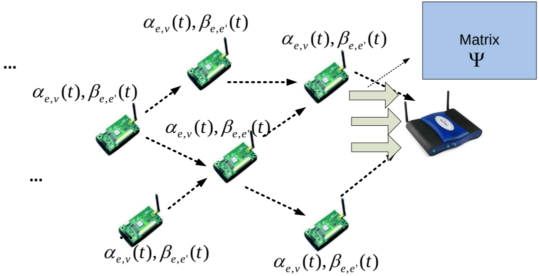
Quantized Network Coding (QNC)



$$Y_e(t) = \mathbf{Q}_e\Big(\sum_{e' \in In(v)} \beta_{e,e'}(t) Y_{e'}(t-1) + \alpha_{e,v}(t) X_v\Big)$$

- operations on the field of **real** numbers
- real-vaued linear combinations are **quantized**

QNC Design for RIP



We proposed an appropriate design for **local** network coding coefficients to have **global** Ψ which satisfies RIP:

- $\alpha_{e,v}(t) = 0,\ \forall t > 2$ and $\alpha_{e,v}(2)$'s are i.i.d zero-mean Gaussian RVs
- $\beta_{e,e'}(t)$'s are locally orthogonal

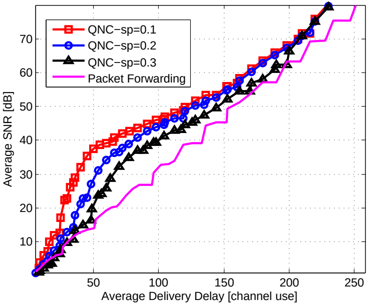
Then, our analysis showed that such design results in a Ψ which has a **similar characteristic to an i.i.d Gaussian matrix** (appropriate for sparse recovery).

QNC and ℓ_1-min (CS-based) Decoding

$$\underline{z} = \Psi \cdot \underline{x} + \underline{n}$$

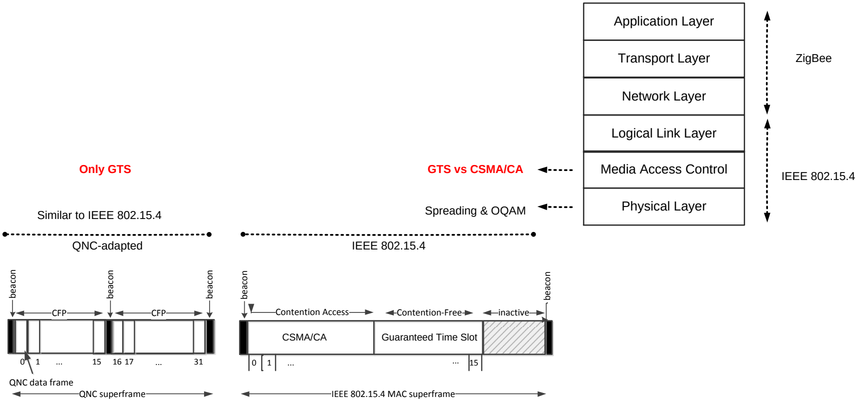
- Ψ total (global) measurement matrix
- $\underline{n}$ due to quantization noises with $||\underline{n}||_{\ell_2} \leq \epsilon$
- $\underline{x}$'s are sparse in transform domain ϕ: $\underline{s} = \phi^T \underline{x}$

$$\hat{\underline{x}} = \phi \cdot \arg\min_{\underline{s}'} ||\underline{s}'||_{\ell_1}, \quad , s.t. \; ||\Psi \phi \underline{s}' - \underline{z}||_{\ell_2} \leq \epsilon$$

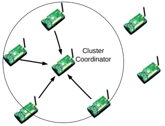


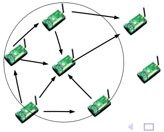
- n = 100 nodes, 1400 links
- k non-zero elements in the sparse domain
- lossless links with 1 bit per use capacity
- lower k/n means higher correlation of data

Lossy Networks: IEEE 802.15.4



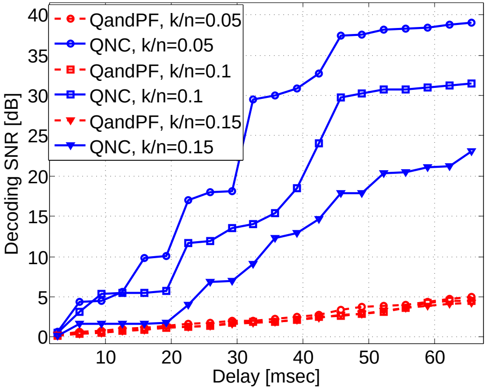
TX to coordinator
(Packet Forwarding)





TX to all neighbours
(Network Coding)

Simulation Results for Lossy Networks



- 50 nodes = 50 messages
- exactly sparse in ϕ domain
- packet drop rate 3×10^{-3}

Summary and Conclusion

Thesis Summary

- Network Coding + Compressed Sensing → Quantized Network Coding
- Quantized Network Coding → (non-adaptive) distributed compression + **no need for correlation model at sensor nodes** + Only at decoder
- Theoretical Guarantees → RIP Analysis + Bayesian CS
- Decoding Algorithm → ℓ_1-minimization + message-passing (for Bayesian framework)
- Practicality → IEEE 802.15.4 + noisy physical channel model

Conclusions

- QNC can gain **higher decoding SNR** for **low delay regions**
- Compressed Sensing offers **random** encoding (compression)
- Not as well as Transform Coding but worth the **Flexibility**