Non-Adaptive Distributed Compression in Networks

Mahdy Nabaee

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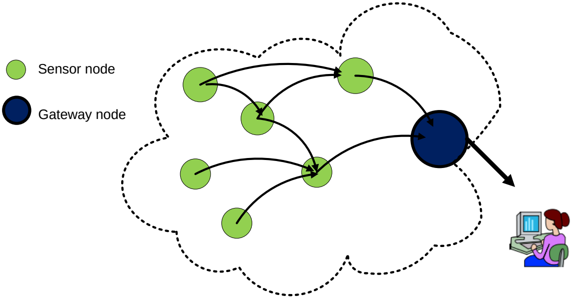

- Nodes: $\mathcal{V} = \{1, \ldots, n\}$, Links (edges): $\mathcal{E} = \{1, \ldots, |\mathcal{E}|\}$
- Each node v has a random source (message) X_v, building up $\underline{X} = [X_v : v \in \mathcal{V}] \in \mathbb{R}^{n \times 1}$,

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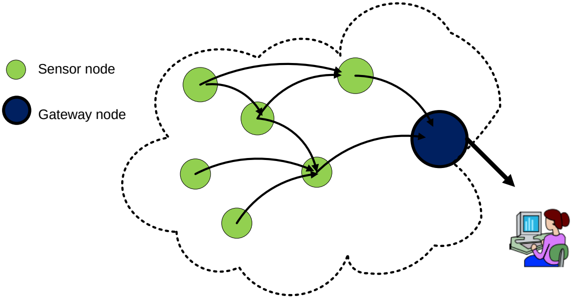

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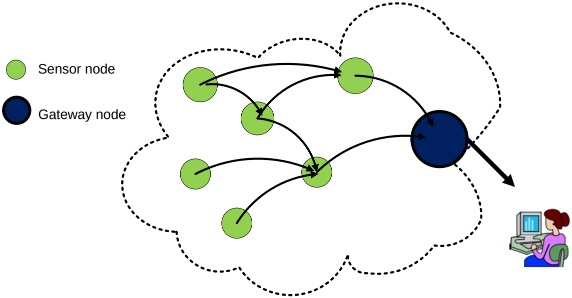

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- Each node v has a random source (message) X_v, building up $\underline{X} = [X_v : v \in \mathcal{V}] \in \mathbb{R}^{n \times 1}$,
- X_v's are *correlated* such that $\exists \phi_{n \times n}$ and $\phi^T \underline{X}$ is k-sparse where $k < n$,
- Knowledge of appropriate marginal rates is not known at the sensor nodes (encoders).

Motivations and Contributions

- Distributed Source Coding needs the appropriate marginal rates and leaves us with Packet Forwarding,
- Motivated by compressed sensing and sparse recovery, Quantized Network Coding[1 2] (QNC) was previously proposed to tackle the need to know the appropriate marginal rates,
- Using computer simulations, QNC was shown to be a better alternative for packet forwarding,[3]
- We aim to provide theoretical performance analysis for QNC, but it is mathematically very difficult,

[1] M. Nabaee and F. Labeau, "Quantized Network Coding for Sparse Messages," in *IEEE SSP12*, Ann Arbor, Michigan, USA, Aug. 2012, pp. 832-835.

[2] M. Nabaee and F. Labeau, "Restricted Isometry Property in Quantized Network Coding of Sparse Messages," in *IEEE Globcom12*, Anaheim, CA, USA, Dec. 2012, pp. 130-135.

[3] M. Nabaee and F. Labeau, "Quantized network coding for Correlated Sources," *submitted for IEEE TSP*, arXiv:1212.5288, Dec. 2012.

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- One-step QNC was proposed as a simplification of QNC,
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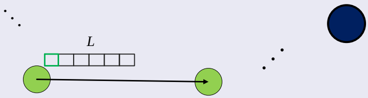
Definition

We define **total network load** to be the product of
- number of packets which need to be delivered to the gateway node, and,
- their length.

♣ It indicates how many symbols have to be delivered to the decoder node.



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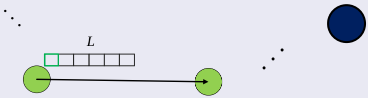

We analyze *network load-distortion* for packet forwarding and one-step QNC.

Quantization and Packet Forwarding (QPF)

As a traditional transmission method, messages are quantized,

$$X_v \to \mathbf{Q}(X_v),$$

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Uniform Messages

For uniform messages, with $|X_v| < q_{\max}, \ \forall v$, we have:

$$\mathbf{E}[|X_v - \mathbf{Q}(X_v)|] = \frac{\Delta_Q}{2},$$

where $\Delta_Q = 2q_{\max}2^{-LC_0}$.
C_0: capacity of the links (bit per channel use)
L: packet length

QPF: Distortion versus Network Load

Corollary

For the QPF scenario with uniform messages, the distortion level of D_0:

$$\boldsymbol{E}[|X_v - \hat{X}_v|] \leq D_0, \ \forall v \in \mathcal{V},$$

is achieved iff the total network load, λ_{QPF}, is such that:

$$\lambda_{\mathrm{QPF}} = (n-1) \cdot L = \frac{n-1}{C_0} \log_2\left(\frac{q_{\max}}{D_0}\right).$$

One-Step Quantized Network Coding[4]

Instead of forwarding the original quantized messages, $\mathbf{Q}(X_v)$'s, we forward random linear combinations of them:

[4] M. Nabaee and F. Labeau, "One-Step Quantized Network Coding for Near Sparse Messages," *submitted for ICASSP13*, arXiv:1210.7399, Oct. 2012.

One-Step Quantized Network Coding[4]

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- $\mathbf{Q}(X_v)$'s are exchanged between the nodes at the first time instant,
- At each node, v, P_v is calculated:

$$P_v = \mathbf{Q}\Big(\sum_{e' \in In(v)} \beta_{v,e'} \mathbf{Q}(X_{tail(e')}) + \alpha_v X_v \Big),$$

where $\beta_{v,e'}$ and α_v are randomly and uniformly chosen from $\{-\kappa, +\kappa\}$.

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  where $\beta_{v,e'}$ and α_v are randomly and uniformly chosen from $\{-\kappa, +\kappa\}$.
- With equal probability, m of these $(n-1)$ linear combinations are selected to be delivered to the decoder node via packet forwarding.

[4] M. Nabaee and F. Labeau, "One-Step Quantized Network Coding for Near Sparse Messages," *submitted for ICASSP13*, arXiv:1210.7399, Oct. 2012.

Theoretical Guarantees[5]

Theorem

For the one-step QNC scenario in random uniform networks, if the total network load, $\lambda_{\mathrm{QNC}} = m \cdot L$, satisfies

$$\lambda_{\mathrm{QNC}} > \min_{\epsilon,\gamma,L'} 48(1+\gamma)\frac{(\kappa^2-1)\, kq'^2_{\max} + q^2_{\max} 2^{2-2L' C_0}}{\epsilon^2} L' \cdot \log(n),$$
$$\textit{subject to: } \epsilon(1-n^{-\gamma}) + 2q_{\max} n^{-\gamma} \leq D_0$$

then the distortion level of D_0 is satisfied:

$$\boldsymbol{E}[|X_v - \hat{X}_v|] \leq D_0, \ \forall v \in \mathcal{V}.$$

- $q'_{\max} = \max_{\underline{X}} ||\phi^T \cdot \underline{X}||_{\ell_\infty}$

[5] M. Nabaee and F. Labeau, "Non-Adaptive Distributed Compression in Networks," *submitted for ISIT13*, arXiv:1301.5973, Jan. 2013.

Theoretical Guarantees (cont.)

By simplifying the result, we can obtain:

Corollary

For the one-step QNC scenario in random uniform networks, if the total network load, $\lambda_{\mathrm{QNC}} = m \cdot L$*, satisfies*

$$\lambda_{\mathrm{QNC}} > 96 \frac{(\kappa^2 - 1)kq_{\max}'^2 + q_{\max}^2 2^{2-2C_0}}{(D_0 - 2q_{\max}/n)^2} \cdot \log(n),$$

then the desired distortion level of D_0 *can be ensured:*

$$\boldsymbol{E}[|X_v - \hat{X}_v|] \leq D_0, \ \forall v \in \mathcal{V}.$$

- $q_{\max} = \max_{\underline{X}} ||\underline{X}||_{\ell_\infty}$
- $q'_{\max} = \max_{\underline{X}} ||\phi^T \cdot \underline{X}||_{\ell_\infty}$
- $k = \max_{\underline{X}} ||\phi^T \cdot \underline{X}||_{\ell_0}$ *(sparsity)*

Comparison of QPF and One-Step QNC

In the random uniform networks, for a given distortion level, D_0, where:

$$\mathbf{E}[|X_v - \hat{X}_v|] \leq D_0, \ \forall v;$$

- QPF requires a network load of

$$\lambda_{\mathrm{QPF}} = \frac{n-1}{C_0} \log_2\left(\frac{q_{\max}}{D_0}\right)$$

- One-Step QNC requires a network load of

$$\lambda_{\mathrm{QNC}} > 96 \frac{(\kappa^2 - 1)kq'^2_{\max} + q^2_{\max} 2^{2-2C_0}}{(D_0 - 2q_{\max}/n)^2} \cdot \log(n)$$

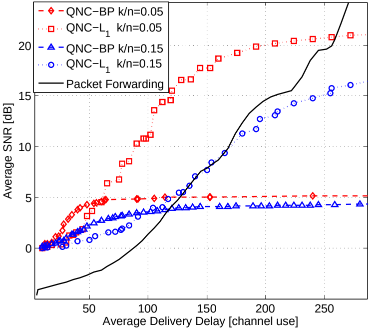
Summary and Future Works

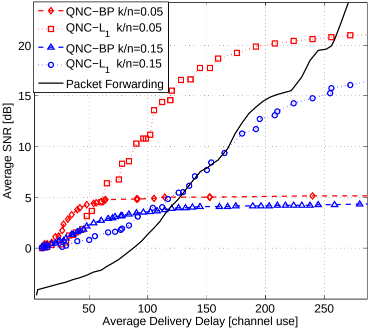
- We have discussed distributed compression and transmission of inter-node correlated messages, without the need to the knowledge of appropriate marginal rates at the sensor nodes. Our theoretical analysis show that the proposed one-step QNC require a smaller order of network load to guarantee the same level of distortion, compared to QPF scenario.
- The reduction of network load can intuitively be interpreted as the reduction of delivery delay.

Summary and Future Works

- We have discussed distributed compression and transmission of inter-node correlated messages, without the need to the knowledge of appropriate marginal rates at the sensor nodes. Our theoretical analysis show that the proposed one-step QNC require a smaller order of network load to guarantee the same level of distortion, compared to QPF scenario.
- The reduction of network load can intuitively be interpreted as the reduction of delivery delay.
- In the future, more sophisticated network models (*e.g.* transmission power decay) have to be considered.
- Full QNC scenario results in better improvements compared to one-step QNC. We need to provide analytic guarantees for full QNC scenario as well.

(bkp) Simulation Results: One-Step QNC vs QPF[6]

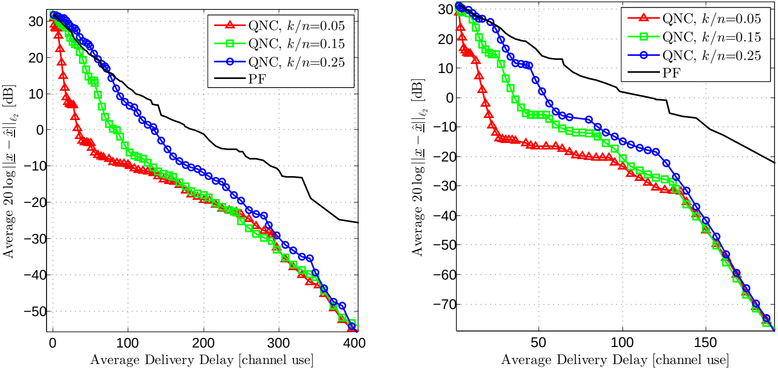




$n = 100$
$|\mathcal{E}| = 400, 800$

[6]M. Nabaee and F. Labeau, "One-Step Quantized Network Coding for Near Sparse Messages," *submitted for ICASSP13*, arXiv:1210.7399, Oct. 2012.

(bkp) Simulation Results: QNC vs QPF[7]



$n = 100$
$|\mathcal{E}| = 1100, 1800$

[7] M. Nabaee and F. Labeau, "Quantized network coding for Correlated Sources," *submitted for IEEE TSP*, arXiv:1212.5288, Dec. 2012.

(bkp) Complimentary Discussion

- Bayesian Compressed Sensing via Belief Propagation[8]
- Message Passing vs Approximate Message Passing for Dense and Low Density QNC scenarios [9] [10]
- Feedback?

[8] D. Baron, S. Sarvotham, R. G. Baraniuk. "Bayesian compressive sensing via belief propagation," *IEEE Transactions on Signal Processing*, 58.1 (2010): 269-280.

[9] M. Bayati, A. Montanari. "The dynamics of message passing on dense graphs, with applications to compressed sensing," *IEEE Transactions on Information Theory*, 57.2 (2011): 764-785.

[10] Rangan, Sundeep. "S. Rangan, "Estimation with Random Linear Mixing, Belief Propagation and Compressed Sensing," arXiv:1001.2228, May 2010.