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NON-ADAPTIVE DISTRIBUTED COMPRESSION IN NETWORKS

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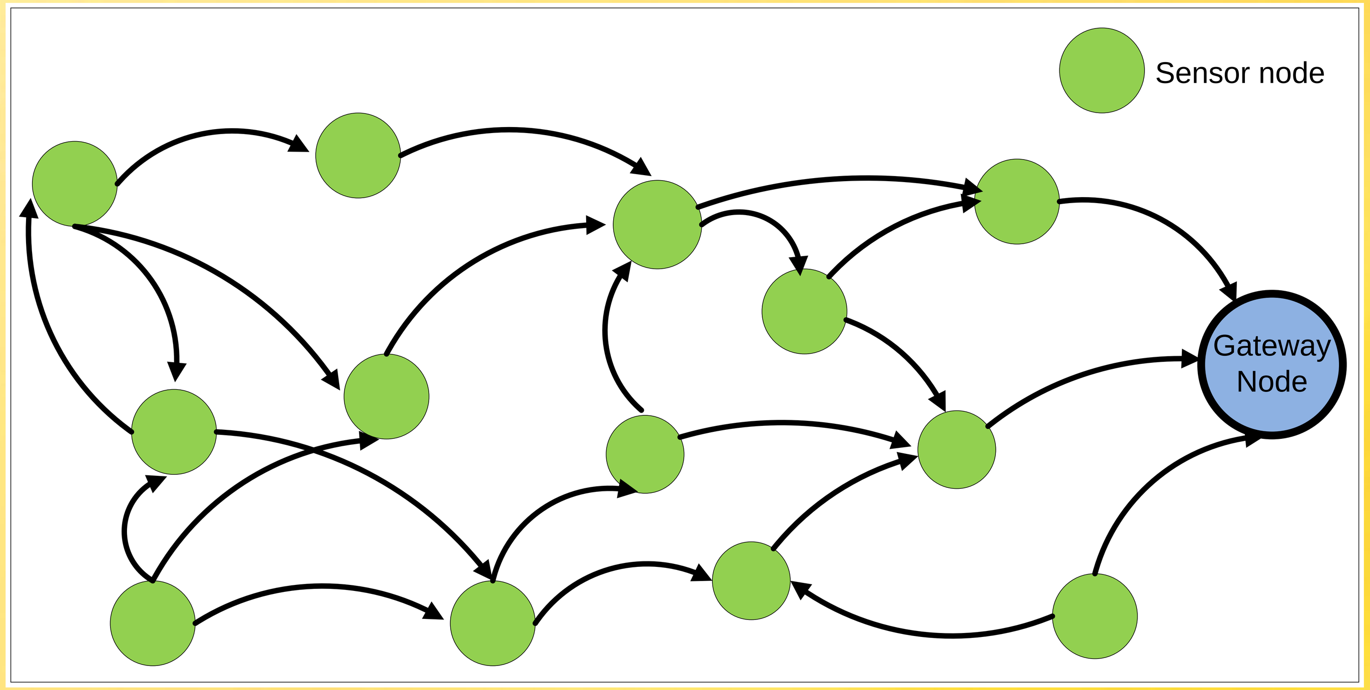
Introduction

We study data gathering scenario in sensor networks, in which the *correlated* readings (messages) of all sensor nodes are transmitted to a single gateway (decoder) node.



Goal: We try to find the possibility of efficient distributed compression without having the knowledge of inter-node correlation of messages (or the appropriate rates for marginal encoding) at the encoders' side.

Problem Description and Modeling

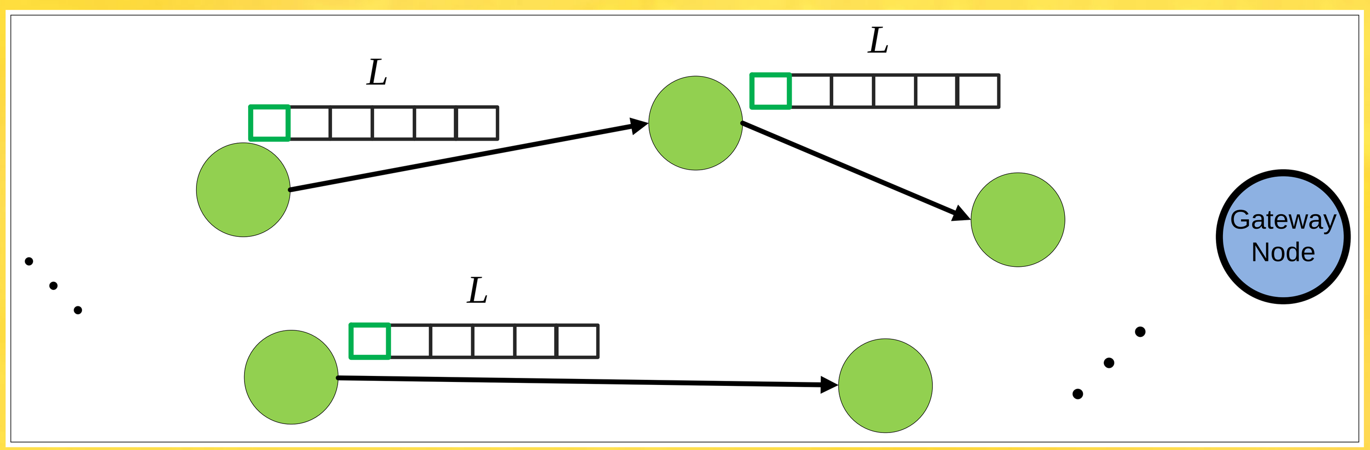
Messages: $\underline{X} = [X_v : v = 1, \dots, n]$
- are uniformly distributed between $-q_{max}$ and $+q_{max}$,
- are k-sparse in $\phi_{n\times n}$ transform domain: $\|\phi^T \underline{X}\|_{\ell_0} = k$.

Network Deployment:
- has uniformly distributed links,
- with lossless communication of limited capacity, C_0.

Distortion Level, D_0**:** The maximum average distance of decoded messages, $\hat{X}_v$'s, from their original value:

$$D_0 \geq \mathbf{E}[|X_v - \hat{X}_v|],\ \forall v.$$

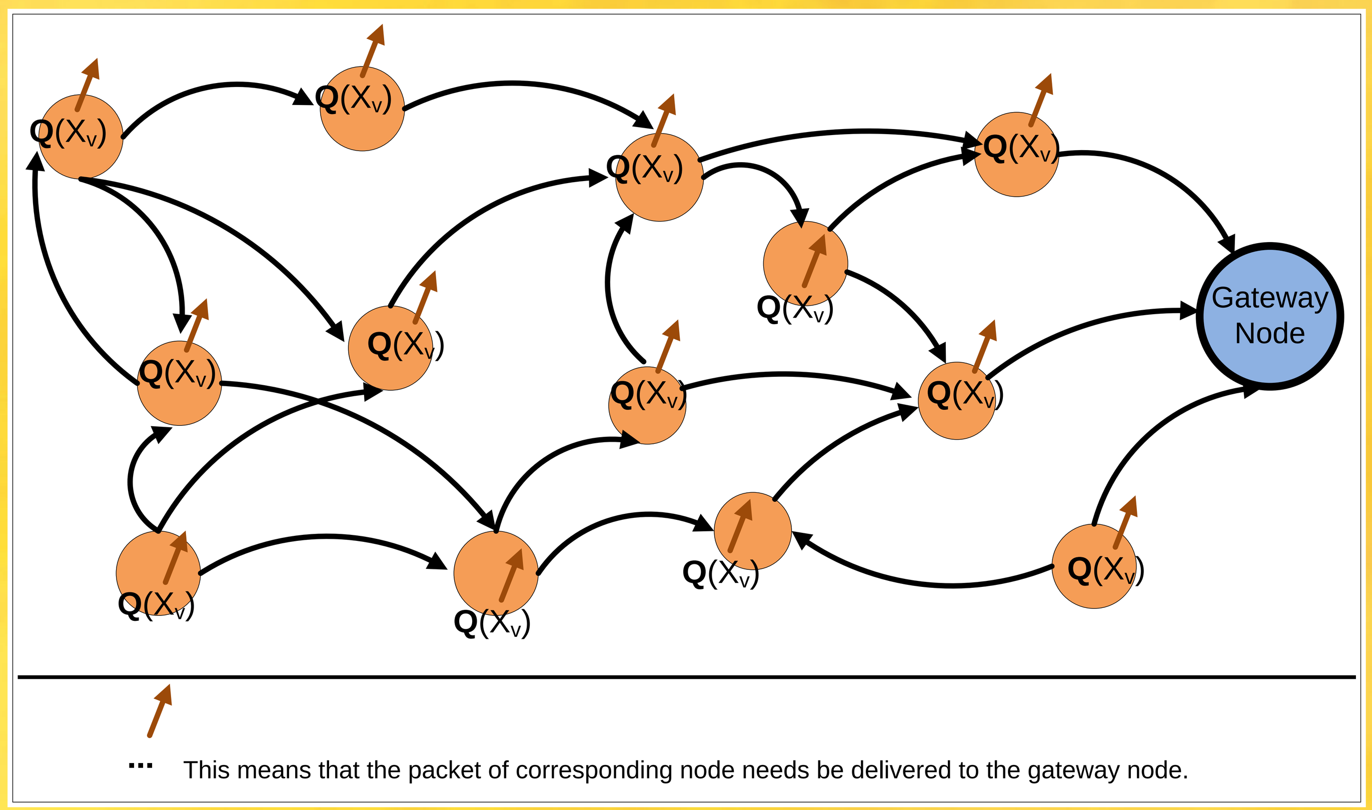
Network Load: Number of bits to be delivered to the decoder node from all the nodes, to ensure a distortion level.



This only includes the packets that have to travel through multiple hops to arrive at the decoder node from their originating nodes.

Quantization and Packet Forwarding (QandPF)

Limited capacity of links require us to quantize the messages at their source node before being transmitted (via packet forwarding) to the decoder node.



Corollary: For the QandPF scenario with real-valued uniform messages, the distortion level of D_0,

$$\mathbf{E}[|X_v - \hat{X}_v|] \leq D_0,\ \forall v \in \mathcal{V},$$

is achieved if and only if the adopted packet length, L, to transmit $(n-1)$ quantized messages is such that:

$$L \simeq \frac{1}{C_0}\log_2(\frac{q_{\max}}{D_0}),$$

resulting a *total network load* of

$$\lambda_{\mathrm{QPF}} = L\cdot(n-1) = \frac{n-1}{C_0}\log_2(\frac{q_{\max}}{D_0}).\ \blacksquare \qquad (1)$$

One-Step Quantized Network Coding (QNC)

In our proposed one-step QNC scheme, we forward a few number (less than the number of messages) of random linear combinations of messages.

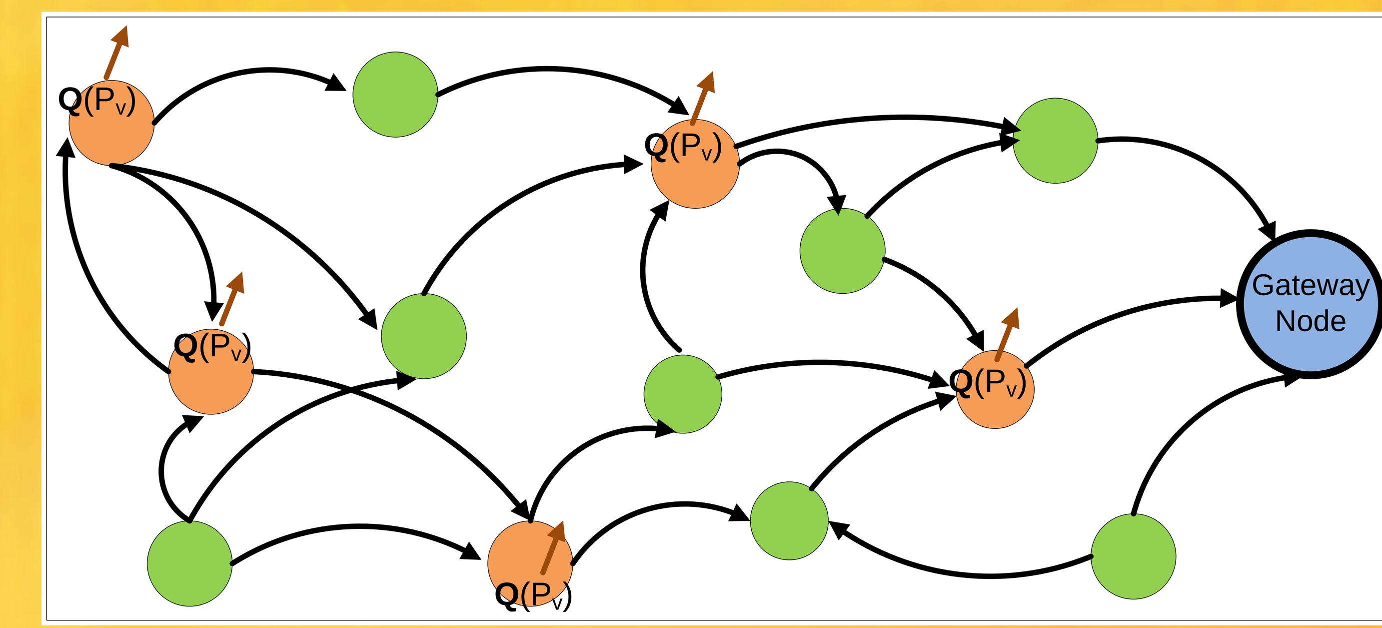
The steps of one-step QNC are as follows:
- $\mathbf{Q}(X_v)$'s are exchanged between the nodes at the first time instant.
- Then, at each node, v, the random linear combination of incoming quantized messages to that node, called P_v, is calculated:

$$P_v = \sum_{e' \in In(v)} \beta_{v,e'}\mathbf{Q}(X_{tail(e')}) + \alpha_v X_v,$$

where $\beta_{v,e'}$ and α_v are randomly and uniformly chosen from $\{-\kappa, +\kappa\}$. The constant κ is: $\kappa = \sqrt{2n^2/(n+|\mathcal{E}|)}$, where $|\mathcal{E}|$ is the number of network links (edges).

One-Step Quantized Network Coding (cont.)

- With equal probability, m of these $(n-1)$ linear combinations are selected to be delivered to the decoder node via packet forwarding.



Motivated by theory of compressed sensing and sparse recovery, we show that these fewer number of packets can be used to recover all the messages, with respect to a distortion level.

Theorem: For the described one-step QNC scenario, if the total network load, $\lambda_{\mathrm{QNC}} = m\cdot L$, satisfies

$$\lambda_{\mathrm{QNC}} = m\cdot L > 96\frac{(\kappa^2-1)kq'^2_{\max} + q^2_{\max}2^{2-2C_0}}{(D_0 - 2q_{\max}/n)^2}\cdot\log(n),\quad (2)$$

then we have:

$$\mathbf{E}[|X_v - \hat{X}_v|] \leq D_0,\ \forall v \in \mathcal{V}.\ \blacksquare$$

Conclusion

Our mathematical derivations show that the required network load in conventional QandPF is reduced in one-step QNC scenario (from order n to order $\log(n)$). Explicitly, for a given distortion level, D_0, where:

$$\mathbf{E}[|X_v - \hat{X}_v|] \leq D_0,\ \forall v;$$

- QandPF requires a network load of

$$\lambda_{\mathrm{QPF}} = \frac{n-1}{C_0}\log_2(\frac{q_{\max}}{D_0})$$

- One-Step QNC requires a network load of

$$\lambda_{\mathrm{QNC}} > 96\frac{(\kappa^2-1)kq'^2_{\max} + q^2_{\max}2^{2-2C_0}}{(D_0 - 2q_{\max}/n)^2}\cdot\log(n)$$

This resulting decrease in the total network load shows a potential decrease in the overall transmission power in the network. One may also weakly interpret this decrease as a decrease on the delivery delay, required to achieve the desired distortion level.

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