Bayesian Quantized Network Coding via Generalized Approximate Message Passing

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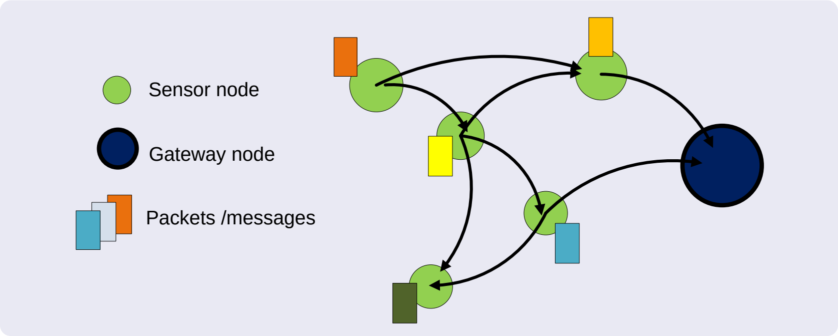
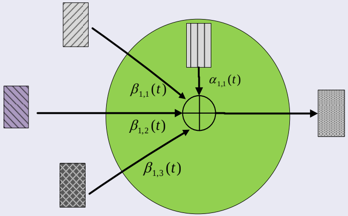
Scenario: Data Gathering in Sensor Networks

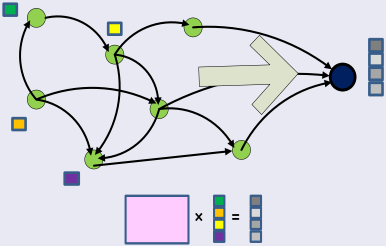


- messages are correlated,
- links are lossless without any interference.

Linear Network Coding in lossless networks



- Calculates linear combinations in a finite field, according to network coding coefficients.

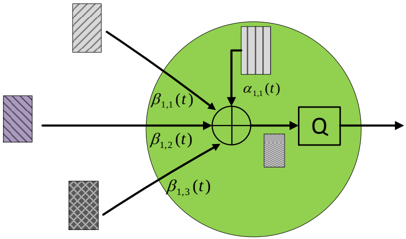


- Perfect decoding is possible, by using matrix inversion in the field, if measurement matrix is full rank.

Using Quantized Network Coding (QNC),

robust recovery is possible even if there are fewer measurements.

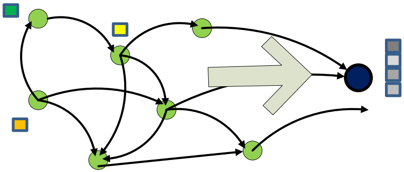
Quantized Network Coding

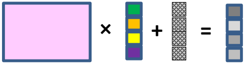


Network Coding + Quantization $\rightarrow$ QNC

- Linear network coding in real field with semi-random coefficients,
- *Quantization* to cope with the finite capacity of links.

QNC meets Compressed Sensing



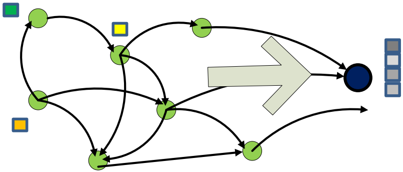


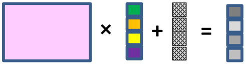
$$\Psi_{tot} \cdot \underline{x} + \underline{n}_{eff,tot} = \underline{z}_{tot}$$

Decoding for: $[\Psi_{tot}]_{m\times n} \cdot [\underline{x}]_{n\times 1} + [\underline{n}_{eff,tot}]_{m\times 1} = [\underline{z}_{tot}]_{m\times 1}$

- If Ψ_{tot} is full rank, a matrix inversion can recover $\underline{x}$, with respect to an error, caused by $\underline{n}_{eff,tot}$.
- If not, we have an under-determined set of equations, which for **Compressed Sensing** decoding *may* help.

Traditional Approach: ℓ_1-min decoding





$$\Psi_{tot} \cdot \underline{x} + \underline{n}_{eff,tot} = \underline{z}_{tot}$$

CS claims recovery is possible for $m < n$, if:

- $\underline{x} = \phi \cdot \underline{s}$, where $\underline{s}$ is k-sparse,
- bounded measurement noise, $\left|\left|\underline{n}_{eff,tot}\right|\right|_{\ell_2} \leq \epsilon_{rec}$.

ℓ_1-min recovery

$$\hat{\underline{x}} = \phi \cdot \arg\min_{\underline{s}'} \left|\left|\underline{s}'\right|\right|_{\ell_1}, \quad s.t. \ \left|\left|\underline{z}_{tot} - \Psi_{tot} \cdot \phi \cdot \underline{s}'\right|\right|_{\ell_2} \leq \epsilon_{rec}$$

ℓ_1-min decoding for QNC

Advantage:

If we have robust recovery, when $m < n$, we have a *saving* in the required number of channel uses $\longrightarrow$ **inter-node compression**.

Nabaee and Labeau, 2012

Theoretical conditions for robust recovery using ℓ_1-min decoding was studied, using Restricted Isometry Property (RIP) of measurement matrix. [a]

[a] M. Nabaee and F. Labeau, "Restricted Isometry Property in Quantized Network Coding of Sparse Messages," in *IEEE Globcom12*, Anaheim, CA, USA, Dec. 2012, pp. 130-135.

Drawbacks:

- Not using signal prior $\mathbf{p}_{\underline{S}}(.)$ beyond its sparsity,
- Complexity of the ℓ_1-min decoder

Bayesian QNC

Bayesian compressed sensing

In compressed sensing, we assume that:

- messages have a few non-zero elements.

In *Bayesian* compressed sensing, we also assume that:

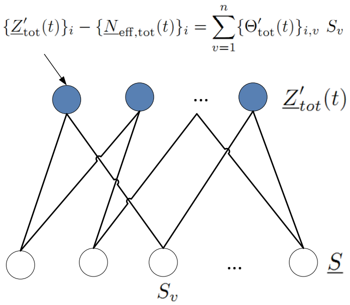
- the prior of those non-zero elements is known at the decoder.

Bayesian QNC

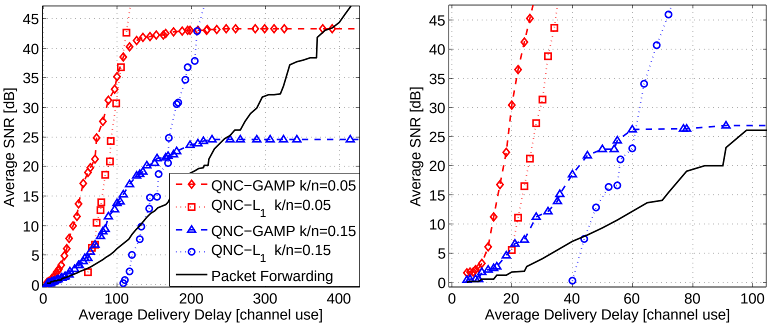
- Similarly, in QNC, the ℓ_1-min can not use the prior information if available.
- Minimum Mean Squared Error (MMSE) decoding is not practical because of its computational complexity.
- A number of iterative near-optimal MMSE decoding algorithms has been developed to tackle this issue.

Generalized Approximate Message Passing (GAMP)

- we model the estimation problem with a bipartite graph,
- passes messages (values) between connected nodes in the bipartite graph,
- at the nodes, the messages are updated according to the priori or measurement information.



Simulation Results



100 nodes - 400,1200 edges - $k/n = 0.05, 0.15$ sparsity factor -
prior: two-state gaussian mixture model
lossless channels of limited capacity, variable block length (uniform quantizer step size)

Summary and Conclusion

Summary

- Quantized Network Coding in sensor networks with near-sparse sources (messages)
- ℓ_1-min decoding for QNC scenario
- Bayesian scenario and GAMP based Decoding
- More details on the algorithm functionality and parameters can be found in the paper.

Conclusion

- Message passing based algorithm offers simpler computational complexity than ℓ_1-min decoding,
- it also offers better decoding performance by considering signal prior in the estimation process.

GAMB based Decoding: Measurement Noise