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Numerical Simulation of Complex Free Surface Flows Using a Stable Mesh-Free Lagrangian Method

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Motivation

Numerical methods have been used extensively in free surface hydraulics. The methods can be classified into two categories; grid methods and particles methods. In recent years, particles methods are gaining further attentions among numerical model developers for hydraulic engineering. Particularly, topological deformation of the fluid can be analyzed by particles, while it is impossible to fit and move a grid continuously in such domains. Also, convection is directly calculated by the motion of particles without numerical diffusion. Thus, interfaces are kept clear. In addition, grid generation which recently seems to be used to analyze complex domains is not necessary. Problems with sever and sharp changes of free water surface can be successfully simulated with numerical methods based on the Lagrangian approach.

Introduction

A mesh-Free numerical approach, called the moving particle semi implicit method (MPS), is presented to solve inviscid Navier-Stokes equations in a fully Lagrangian form using a fractional step method. This method consists of splitting each time step in two steps. The fluid is represented with particles and the motion of each particle is calculated through interactions with neighboring particles by means of a kernel function.

Ataie-Ashtiani and Farhadi (2006) applied various kernel functions to improve the stability of MPS method. Based on their studies, a kernel function is introduced which improves the stability of dam break with impact problem. In this research, the MPS method with the most stable form of kernel function is used to simulate a dynamic system consisting of a heavy box sinking vertically into a water tank, known as Scott Russell's wave generator problem. This problem is an example of a falling rock avalanche into natural or artificial reservoirs. By simulating this problem, the applicability of MPS method to model complex free surface flows is shown. Moreover, we test the stability of Scott Russell's wave generator problem with various kernel functions to confirm the accuracy of the most stable form of this kernel function.

MPS Formulation

Governing Equations: $\frac{1}{\rho}\frac{D\rho}{Dt} = -\nabla . u$; $\frac{Du}{Dt} = -\frac{1}{\rho}\nabla P + f$

Conservation of mass — Conservation of momentum

Time Splitting:
$$\frac{D\rho}{Dt} = \frac{\rho^{n+1}-\rho^{n}}{\Delta t} = \frac{\rho^{n+1}-\rho^{*}+\rho^{*}-\rho^{n}}{\Delta t} = \frac{\Delta\rho' + \Delta\rho^{*}}{\Delta t}$$
$$\frac{Du}{Dt} = \frac{u^{n+1}-u^{n}}{\Delta t} = \frac{u^{n+1}-u^{*}+u^{*}-u^{n}}{\Delta t} = \frac{\Delta u' + \Delta u^{*}}{\Delta t}$$

Discretization of governing equations:

Conservation of momentum
$$\frac{\Delta u^{*}}{\Delta t} = f \; ; \frac{\Delta u'}{\Delta t} = -\frac{1}{\rho}\nabla P$$

Conservation of mass
$$\frac{1}{\rho}\frac{\Delta\rho^{*}}{\Delta t} = -\nabla . u^{*} \; ; \frac{1}{\rho}\frac{\Delta\rho'}{\Delta t} = -\nabla . \Delta u'$$

Fictitious time step: $u^{*} = f\,\Delta t + r^{n}$; $r^{*} = u^{*}\Delta t + r^{n}$

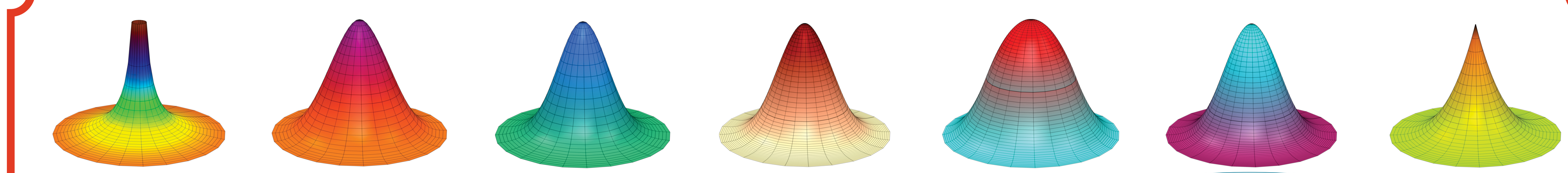
MPS interpolation:
$$\langle n\rangle_i = \sum w(|r_j - r_i|) \quad ; \quad w(r) = \frac{r_e}{r} - 1$$
$$\langle\rho\rangle_i = m\langle N\rangle_i = \frac{m\langle n\rangle_i}{\int_V w(r)dv}$$

Modeling the incompressibility:
$$n^{*} + \Delta n' = n^{0} \; ; \; \frac{1}{n^{0}}\frac{\Delta n'}{\Delta t} = -\nabla.\Delta u' \; ; \left\langle \nabla^2 P^{n+1}\right\rangle_i = -\frac{\rho}{dt^2}\frac{\left\langle n^{*}\right\rangle_i - n^{0}}{n^{0}}$$

MPS Gradient and Laplacian:
$$\left\langle \nabla^2\varphi\right\rangle_i = \frac{2d}{\lambda n^{0}}\sum_{i\neq j}[(\varphi_j - \varphi_i)\; w(|r_j - r_i|)]$$
$$\left\langle \nabla\varphi\right\rangle_i = \frac{d}{n^{0}}\sum_{i\neq j}\frac{\varphi_j - \hat{\varphi}_i}{|r_j - r_i|^2}(r_j - r_i)\; w(|r_j - r_i|)$$

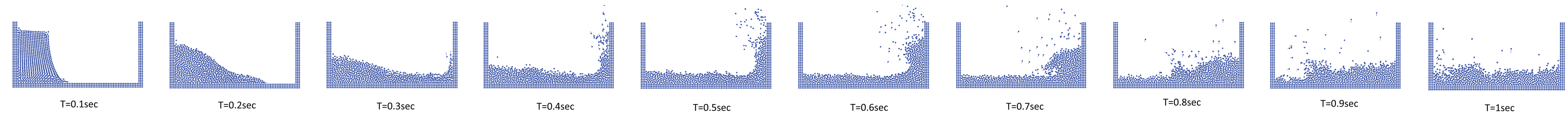
Real time step: $u^{n+1} = u^{*} + \Delta u'$; $r^{n+1} = r^{n} + u^{n+1}\Delta t$

First Simulation: Dam Break with Impact Problem



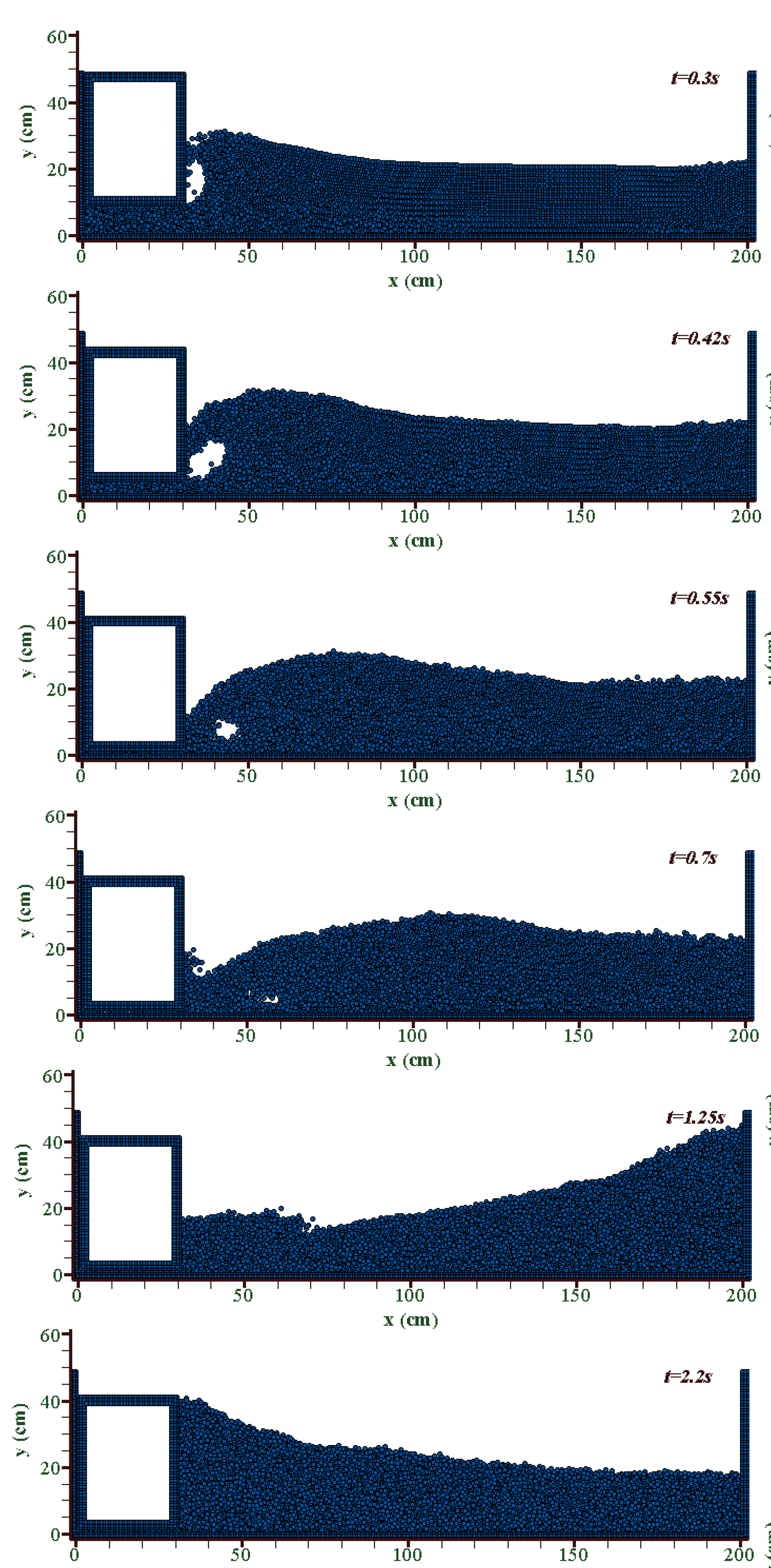
Kf1: A simple rational function | Kf2: An exponential function | Kf3: A quartic spline function | Kf4: A cubic spline function | Kf5: A polynomial function | Kf6: A cubic spline function (Provides the best stability of MPS method) | Kf7: A polynomial function

Based on the studies on 7 different kernel functions (Ataie-Ashtiani and Farhadi (2006)), a kernel function is introduced (KF6) that improves the stability of dam break with impact simulation by MPS method. Using this kernel function, dam break with impact problem is successfully simulated until the water loses all its momentum and becomes completely horizontal. At 0.4s, the water goes up to the right vertical wall and at t= 0.5s, it begins to come down. A breaking wave is formed at 0.7s. The reflected wave moves toward the left side of water tank and at t=0.9s reaches the left vertical wall. At t=4s, the water surface is approximately horizontal.



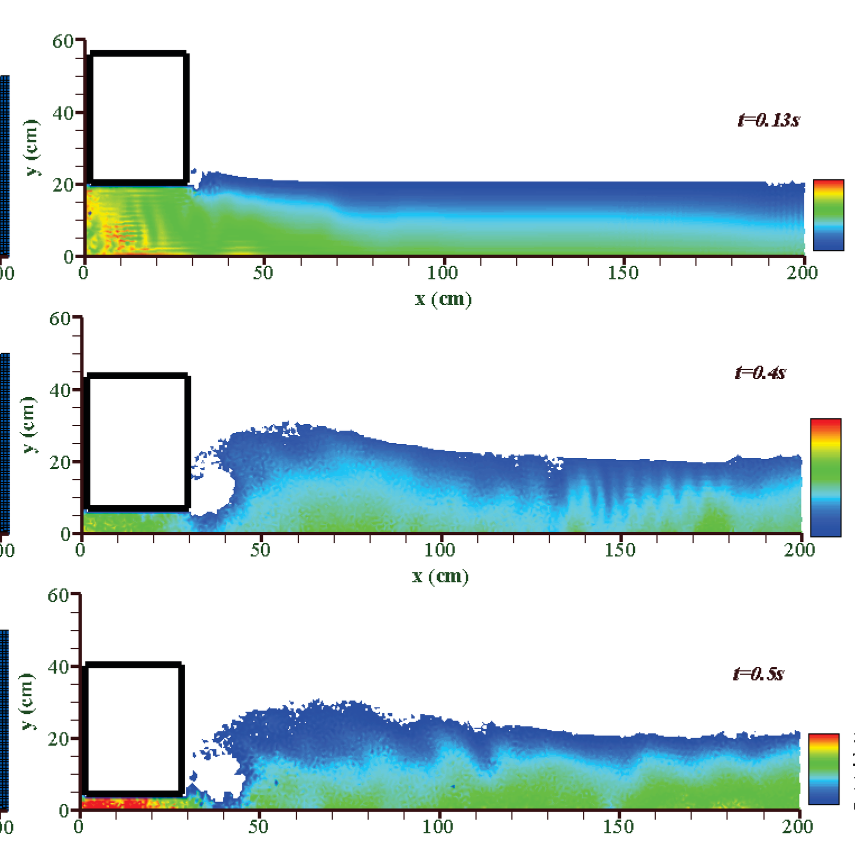
Second Simulation: Scott Russell's Wave Generator

Water Surface Profile

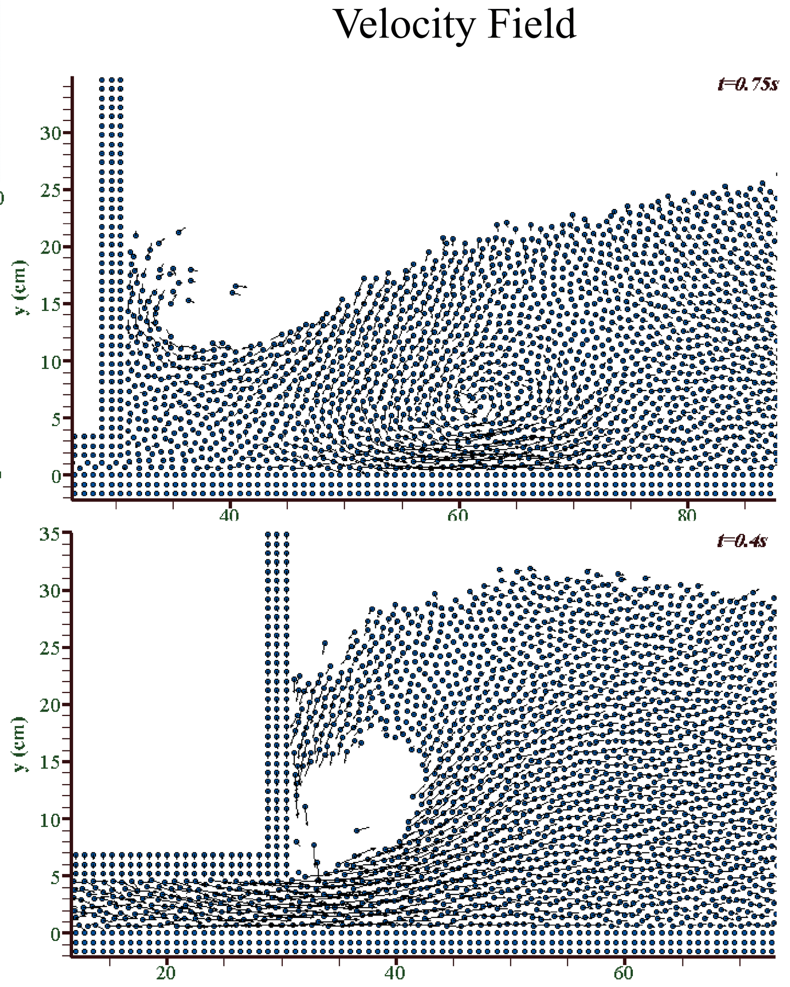


As the box sinks into water tank, the water is heaved up to form a solitary wave and a reverse plunging wave which forms a vortex. This vortex follows the solitary wave down the water tank.

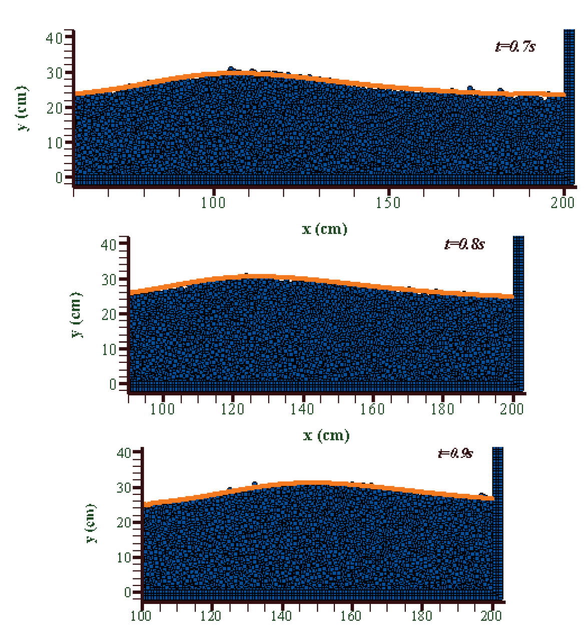
Pressure Field



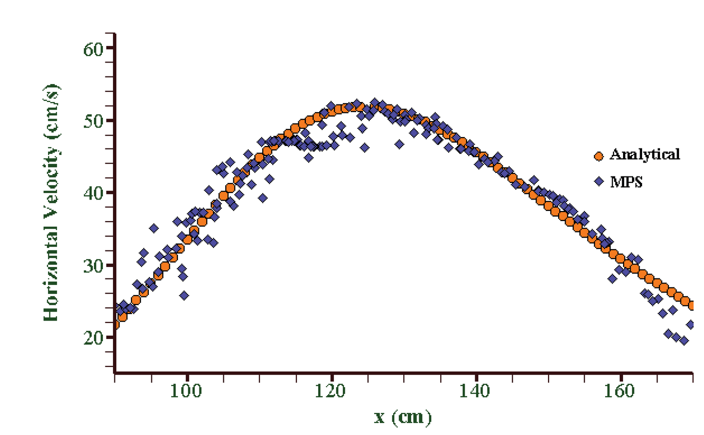
Velocity Field



Solitary Wave Profile Compared with Analytical Solution



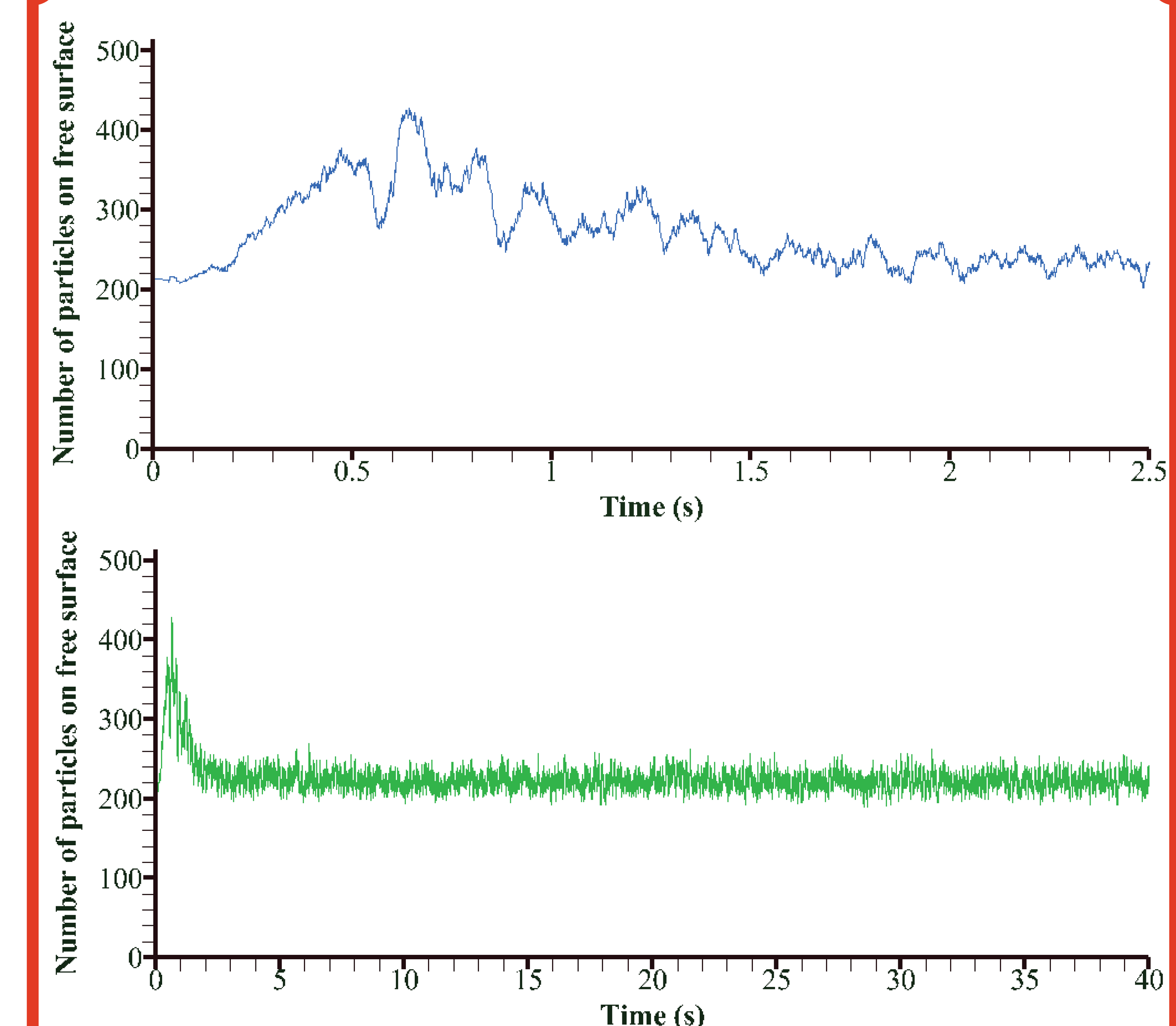
Horizontal Velocity of Particles on Free Surface Compared with Analytical Solutions



The wave amplitude is 11 cm, measured by the present model. Around t=1.25s, t=3.4s, t=5.6s, t=7.9s and t=10.5s, the solitary wave strikes the right vertical wall and accordingly, the direction of wave propagation is changed. Around t=2.2s, t=4.4s, t=6.7s, t=9s and t=11.5s, the solitary wave impinges on the right side of box and its direction is changed. At t=17s, the water surface profile is approximately horizontal.

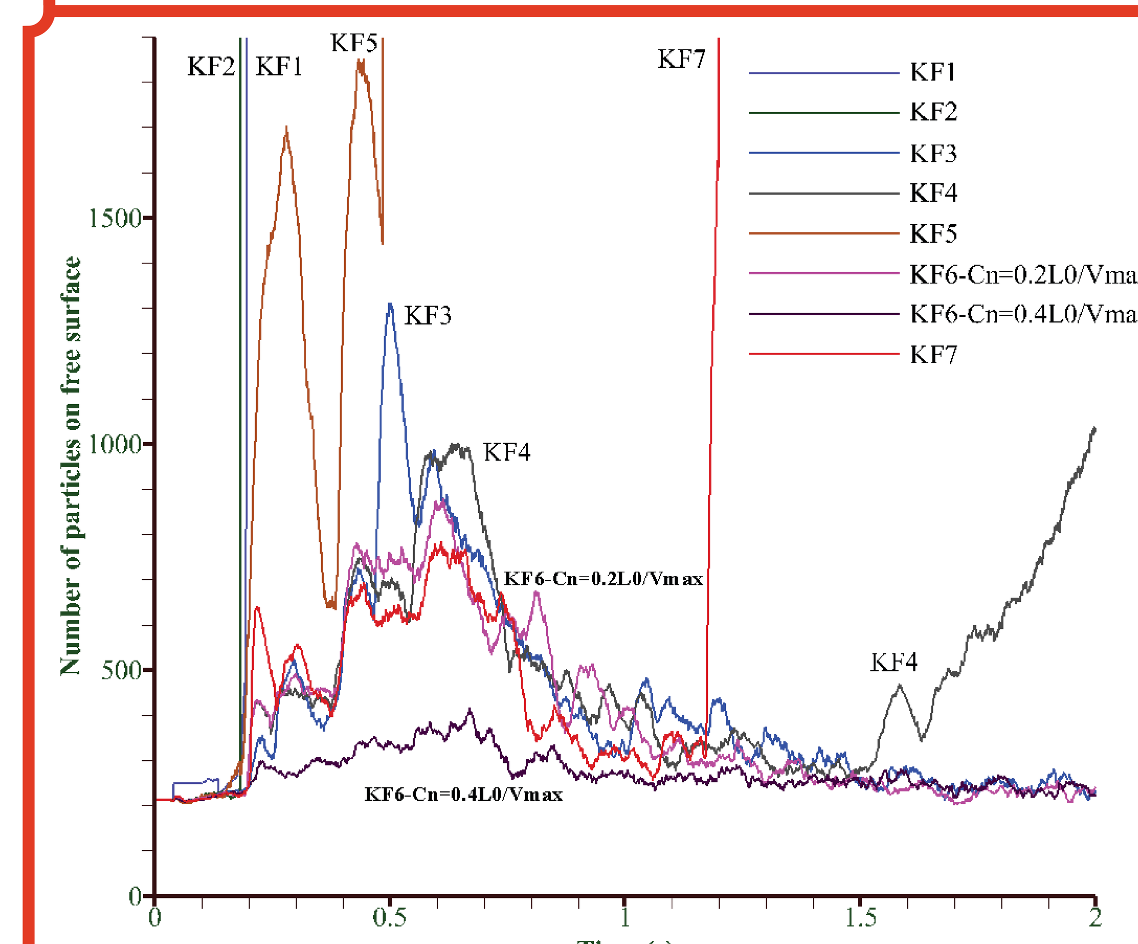
Results are in good agreement with the analytical solutions, showing the applicability and validity of present model in simulating complex free surface flow problems.

Stability of Second Simulation



There is no unphysical variation or fluctuation in the number of particles on free surface. Simulation is performed without instability occurrence for a long period. Accordingly, the present model is stable.

Kernel Function and Stability



The figure shows that once again, KF6 results in the best stability of MPS simulations.(The figure corresponds to the second simulation.)

Conclusion

This research presents application of MPS method in simulating complex free surface flows. The dam break with impact problem and Scott Russell's wave generator problem were successfully simulated without any instability occurence. This is a remarkable ability amongst other particle methods. Results are compared with analytical solutions, showing good agreement. Additionally, it is shown that the kernel function introduced by Ataie-Ashtiani and Farhadi (2006) will significantly improve the stability of Scott Russell wave generator simulation by MPS method. Stability and accuracy of the two simulations proves that the present method is a very useful utility for solving problems with irregular and complex free surface in hydraulic and coastal engineering when an accurate prediction of free water surface is required.