A Mesh-Free Particle Model for Simulation of Free Surface Multiphase Flow

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Introduction

- The MPS method is a mesh-free Lagrangian method, which is originally proposed by Koshizuka and Oka (1996) for simulation of free surface flow.
- The fluid is represented with particles, and the motion of each particle is calculated through the interactions with neighboring particles by means of a kernel (weight) function.
- The MPS method has been successfully applied to a variety of complex hydraulic problems such as dam-break (e.g. Ataie-Ashtiani and Farhadi 2006) and landslide-induced water waves (e.g. Nabian and Farhadi 2014).
- **In the present research, the MPS method is modified to simulate multiphase and granular flows. The accuracy of the proposed formulation is shown through the successful simulation of two test cases: deformable submarine landslide, and mobile-bed dam break.**

Formulation

The governing equations are the mass and momentum equations:

Mass Eq.	Momentum Eq.
$\frac{1}{\rho}\frac{D\rho}{Dt} = -\nabla . u$	$\frac{Du}{Dt} = -\frac{1}{\rho}\nabla P + f$

In the MPS, a fractional step method is used and the time differentiation of density and velocity are expressed as:

$$\frac{Du}{Dt} = \frac{u^{n+1} - u^n}{\Delta t} = \frac{u^{n+1} - u^* + u^* - u^n}{\Delta t} = \frac{\Delta u' + \Delta u^*}{\Delta t}$$

$$\frac{D\rho}{Dt} = \frac{\rho^{n+1} - \rho^n}{\Delta t} = \frac{\rho^{n+1} - \rho^* + \rho^* - \rho^n}{\Delta t} = \frac{\Delta \rho' + \Delta \rho^*}{\Delta t}$$

It can be shown that the density of the fluid is approximated by a fraction of the particle number density, defined as

$$\langle n \rangle_i = \sum_{i \neq j} w(| r_j - r_i |)$$

w(r) is a kernel function. The following kernel function, suggested by Ataie-Ashtiani and Farhadi (2006), is used to improve the stability of the simulations:

$$w(r) = \begin{cases} \frac{40}{7\pi r_e^2}\left(1 - 6\left(\frac{r}{r_e}\right)^2 + 6\left(\frac{r}{r_e}\right)^3\right) \\ \frac{10}{7\pi r_e^2}\left(2 - 2\frac{r}{r_e}\right)^3 \end{cases}$$

In the prediction step, the viscous and gravitational forces are explicitly calculated without enforcing incompressibility and an intermediate velocity is calculated. Then, to impose the incompressibility, the mass and momentum conservation equations are combined, which yields a Poisson equation of pressure in form of:

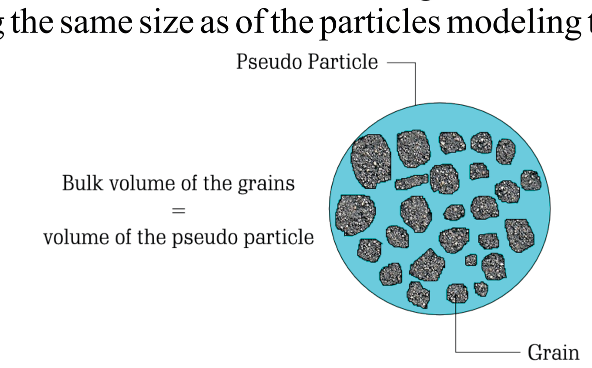
$$\langle \nabla^2 P^{n+1} \rangle_i = -\frac{\rho}{dt^2}\frac{\langle n^* \rangle_i - n^0}{n^0}$$

Once the pressure is calculated, the position of the particles are corrected.
WC-MPS (Weakly Compressible MPS) method proposed by Shakibaeinia and Jin (2010; 2011) is a simplified form of MPS method which uses an equation of state within an explicit scheme while considering the fluid to be slightly compressible.
In this research, the MPS formulation is modified to model multiphase and granular flows:

- A high-order accurate density smoothing scheme is applied to avoid pressure discontinuity at the phase interfaces:

$$\rho_i = \frac{1}{\sum w(|r_i - r_j|)}\sum\left(\rho_j - \frac{\partial \rho_i}{\partial x_{ij}}x_{ij} - \frac{\partial \rho_i}{\partial y_{ij}}y_{ij}\right)w(|r_i - r_j|)$$

- A viscosity arithmetic averaging algorithm is applied to the interface to avoid shear stress discontinuity.
- A developed adaptive density algorithm is applied to the particles modeling fixed boundaries.
- the granular and sediment materials are merged to form a number of pseudo particles, having the same size as of the particles modeling the water phase:



- The granular media is treated as a non-Newtonian fluid, meaning that the viscosity is a function of the shear rate. The Bingham plastic model is used to model the rheology of the flow.

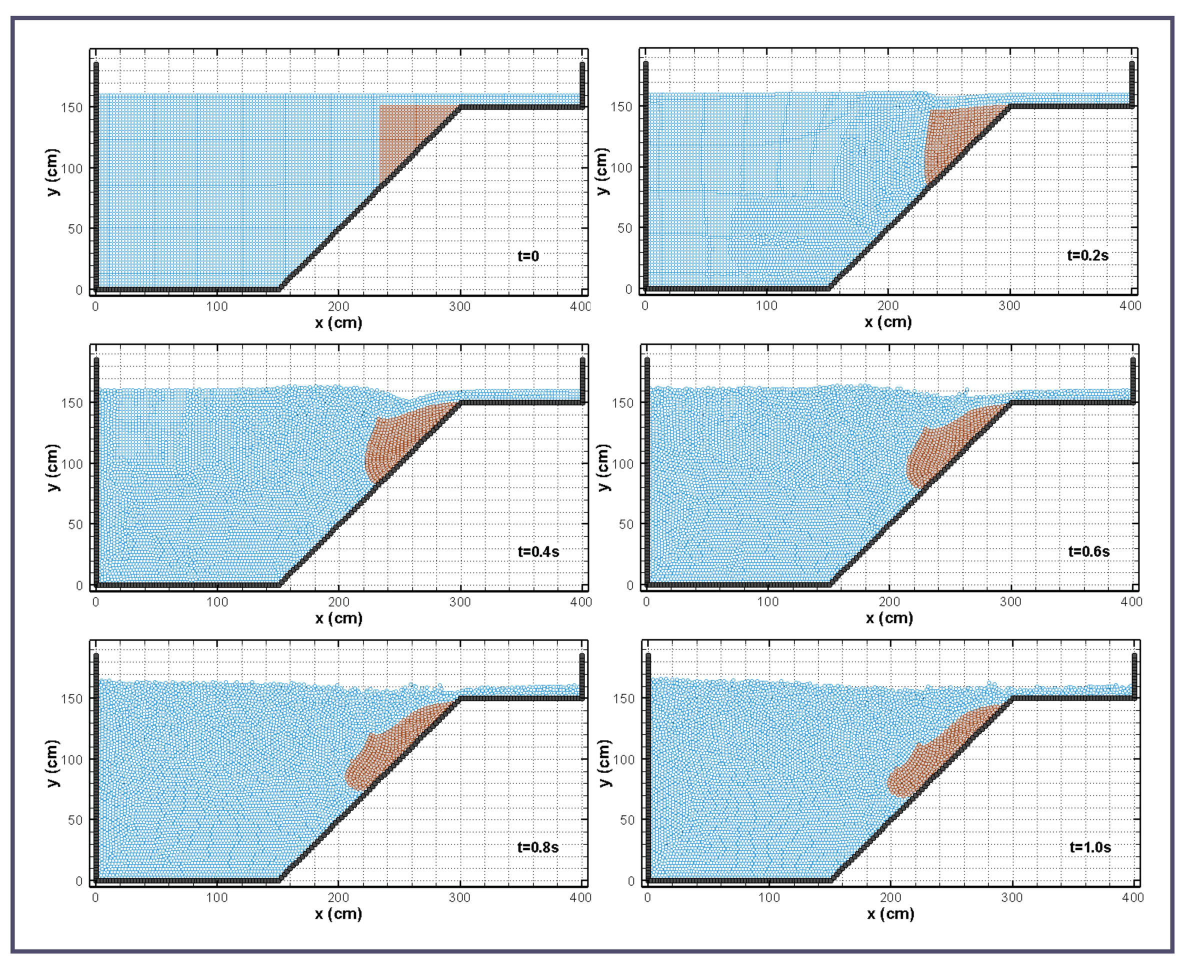
Motivation

- There are a wide range of free surface multiphase flows, in which the interfaces and boundaries, e.g. free surface, will experience high deformation or fragmentation.
- The fluid interfaces and free surface boundaries can be treated simply and efficiently using mesh-free particle methods.
- Moving Particle Semi Implicit (MPS) method is a mesh-free particle method, which is recently gaining a lot of attention due to its efficiency and ease of free surface tracking, straightforward boundaries treatment, ease of coding, and better stability compared to other particle methods.
- **The application of the original MPS method is mostly limited to single-phase flows. The motivation of this research is to extend the MPS formulation to free surface multiphase flow, and to verify the accuracy of the proposed method by comparing the results with the available experimental data.**

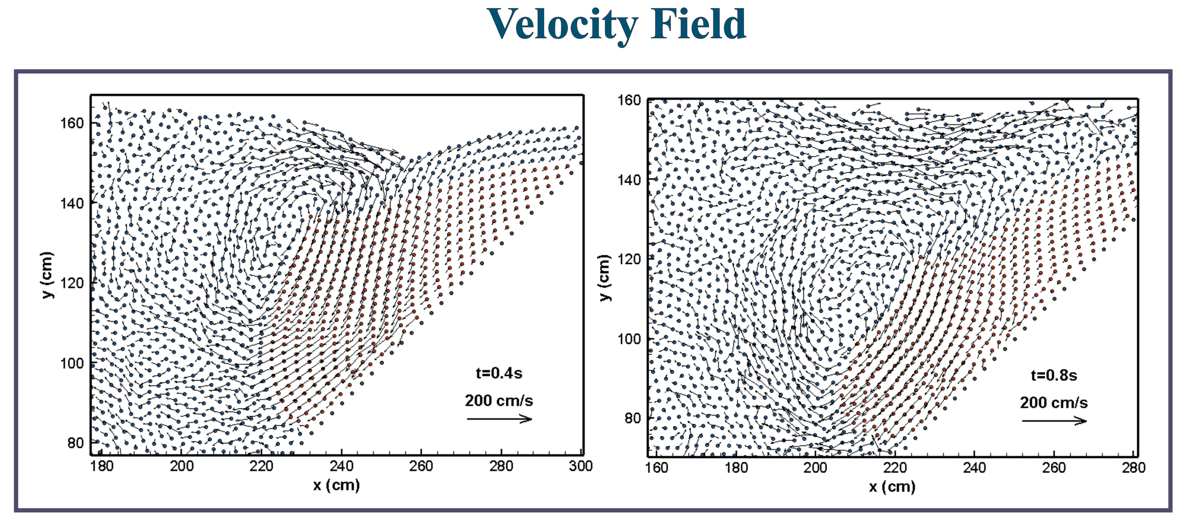
Deformable Submarine Landslide

Landslides, usually caused by slope failures or liquefaction of sediments, can generate water waves and small-scale tsunamis in coastal areas. Once the landslide generated water waves reach the coast or structures, they are able to produce disasters including loss of life and collapse of facilities and infrastructures. Therefore, it is of importance to predict the damage of landslide-generated water waves in flood hazard assessment of coastal zones.

Particle Configuration

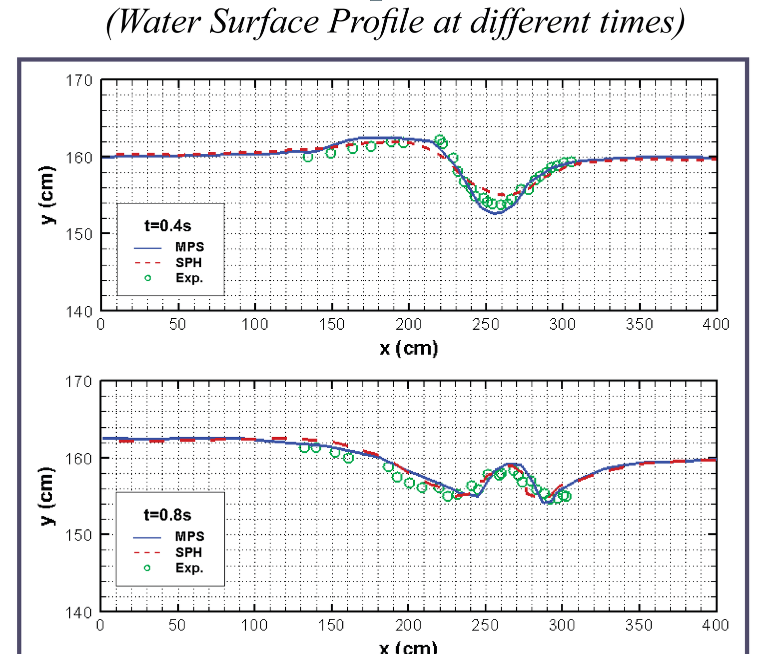


Velocity Field



Comparison

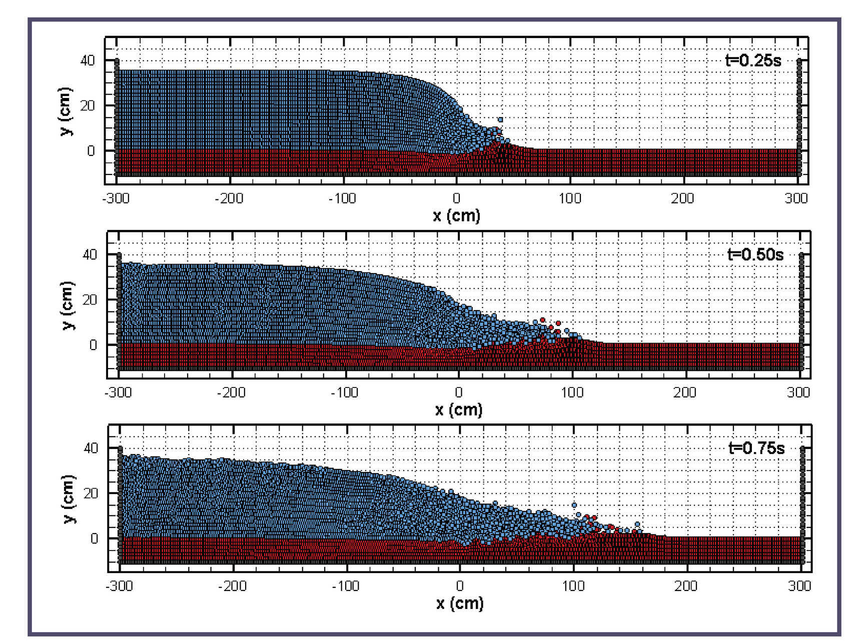
(Water Surface Profile at different times)



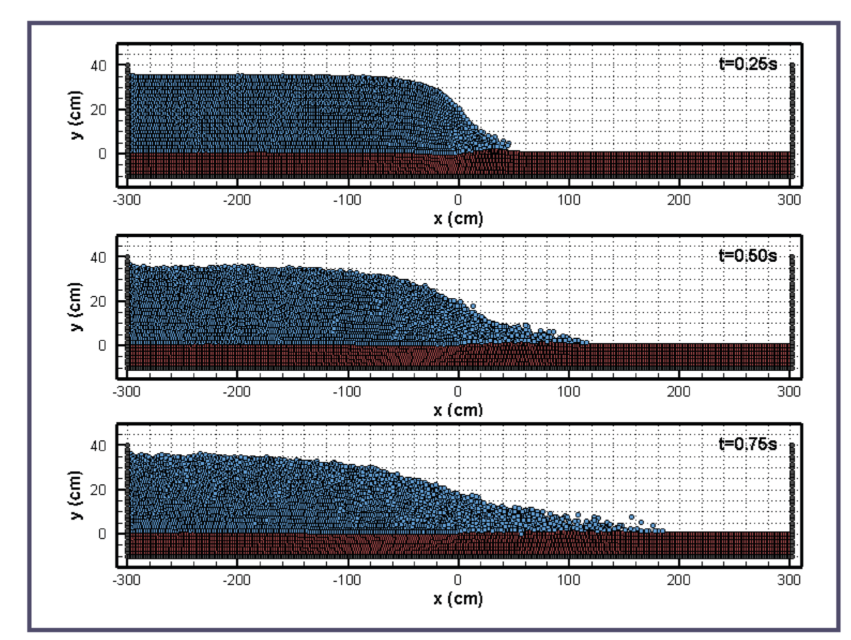
Mobile-Bed Dam-Break

The majority of the existing numerical models for the dam-break problem are applicable only for fixed beds. However, dam-break waves usually propagate along floodplains and rivers, in which the interaction between the dam-break flow and the bed sediments should be taken into account.

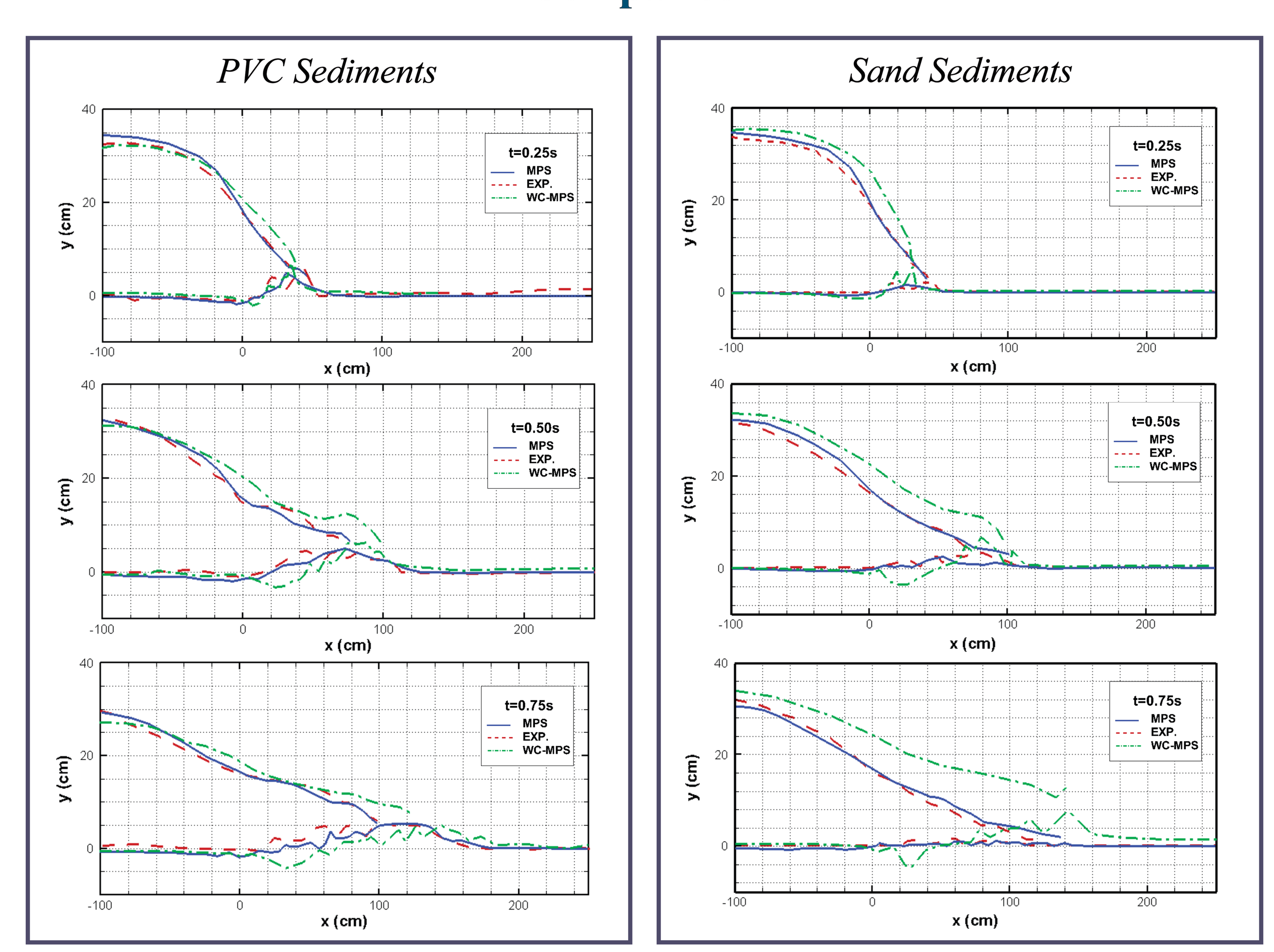
Case I: PVC Sediments



Case II: Sand Sediments



Comparison



Summary of Results

Deformable Submarine Landslide

- As the landslide mass slides along the inclined wall, it generates a reverse plunging wave which creates a vortex.
- The good agreement between the MPS results and experimental data shows the applicability and accuracy of the present model to predict the water surface profile.

Mobile-Bed Dam-Break

- It is observed that the maximum sediment displacement occurs almost under the wave front.
- At late stages, as the water wave energy decreases, the shear stress on the sediments will also decrease, making them to deposit gradually.
- The curvature of the water wave profile is predicted accurately by the MPS results.
- the MPS results are accurately predicting some important features of the sediment motion.
- The peak sediment displacement in experiment and MPS results are almost the same at different times.
- The local peaks in sediment profile are almost accurately captured by the MPS.
- The general fluctuation trend of the sediment profile at different times is predicted accurately by the MPS.
- Having the PVC as the bed sediments, the position of wave front at different times is almost the same in the MPS and experimental results.
- It is evident that the MPS results are predicting the water wave profile to a satisfactory extent.
- Unlike the MPS, the WC-MPS method does not detect the accurate position of the wave front and the wave curvature.
- Results of the WC-MPS method show overestimation of the water wave height at the regions close to the wave front.
- In overall, the WC-MPS results are showing more bed erosion than the MPS and experimental results do.

Conclusion

- The applicability of the proposed model is shown through the simulation of deformable submarine landslide and dam break over erodible beds.
- By comparing the results with available experimental data, the accuracy of the simulations are shown.
- In simulation of dam-break over erodible beds, it is shown that the results of the present MPS model are significantly in closer agreement with the experimental data than the WC-MPS results and this is a remarkable ability of this method.
- This study shows that realistic prediction of the free surface profile can be obtained by the proposed multiphase MPS method, showing its applicability in hydraulics, coastal and ocean engineering when an accurate prediction of water surface profile is necessary.
- This opens the door to more extensive simulations of multiphase flows using the MPS method.

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