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$$f_G = 4\pi \int_0^{\infty} r^2 \frac{\sin(Gr)}{Gr} n(r)\, dr$$

$$n(r) = (\pi a_0^3)^{-1} \cancel{\exp}\, e^{-\frac{2r}{a_0}}$$

$$f_G = \frac{4\cancel{\pi}}{G\cancel{\pi} a_0^3} \int r\sin(Gr) e^{-\frac{2r}{a_0}} dr$$

let $\cancel{x} = Gr$ $\quad r = \frac{\cancel{x}}{G}$

$d\cancel{x} = G\,dr$

$$= \frac{4}{G^3 a_0^3} \int_0^{\infty} \cancel{x}\sin(\cancel{x}) e^{-\frac{2\cancel{x}}{Ga_0}} d\cancel{x}$$

$\cancel{u}$ $u = x\sin(x)$ $\quad dv = e^{-\frac{2x}{Ga_0}} dx$

$du = \sin(x) + x\cos(x)$ $\quad V = \frac{-Ga_0}{2} e^{-\frac{2x}{Ga_0}}$

$$= \frac{4}{G^3 a_0^3}\left[\frac{Ga_0}{2} x\sin(x) e^{-\frac{2x}{Ga_0}} + \frac{Ga_0}{2}\int \sin(x) e^{-\frac{2x}{Ga_0}} dx + \frac{Ga_0}{2}\int x\cos(x) e^{-\frac{2x}{Ga_0}} dx\right]$$

$$\Rightarrow \frac{Ga_0}{2}\int \sin(x) e^{-\frac{2x}{Ga_0}} dx$$

let $u = \sin(x)$ $\quad dv = e^{-\frac{2x}{Ga_0}} dx$

$du = \cos(x)\,dx$ $\quad V = \frac{-Ga_0}{2} e^{-\frac{2x}{Ga_0}}$

$$= \frac{Ga_0}{2}\left[\frac{-Ga_0}{2}\sin(x) e^{-\frac{2x}{Ga_0}} + \frac{Ga_0}{2}\int \cos(x) e^{-\frac{2x}{Ga_0}} dx\right]$$

let $u = \cos(x)$ $\quad dv = e^{-\frac{2x}{Ga_0}} dx$

$du = -\sin(x)\,dx$ $\quad V = \frac{-Ga_0}{2} e^{-\frac{2x}{Ga_0}}$

$$= \frac{Ga_0}{2}\left[\frac{-Ga_0}{2}\sin(x) e^{-\frac{2x}{Ga_0}} + \frac{Ga_0}{2}\left[\frac{-Ga_0}{2}\cos(x) e^{-\frac{2x}{Ga_0}} + \frac{-Ga_0}{2}\int \sin(x) e^{-\frac{2x}{Ga_0}} dx\right]\right]$$

$$= \frac{Ga_0}{2}\left[\frac{-Ga_0}{2}\sin(x) e^{-\frac{2x}{Ga_0}} - \frac{G^2a_0^2}{4}\cos(x) e^{-\frac{2x}{Ga_0}} + \frac{-G^2a_0^2}{4}\int \sin(x) e^{-\frac{2x}{Ga_0}} dy\right]$$

$$= -\frac{G^2a_0^2}{4}\sin(x) e^{-\frac{2x}{Ga_0}} - \frac{G^3a_0^3}{8}\cos(x) e^{-\frac{2x}{Ga_0}} + \frac{-G^3a_0^3}{8}\int \sin(x) e^{-\frac{2x}{Ga_0}} dx$$

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$$\left(\frac{Ga_0}{2}+\frac{G^3a_0^3}{8}\right)\int \sin(x)e^{\frac{-2x}{Ga_0}}dx = \frac{-G^2a_0^2}{4}\sin(x)e^{\frac{-2x}{Ga_0}} - \frac{G^3a_0^3}{8}\cos(x)e^{\frac{-2x}{Ga_0}}$$

$$\frac{Ga_0}{2}\int \sin(x)e^{\frac{-2x}{Ga_0}}dx = -e^{\frac{-2x}{Ga_0}}\left[\frac{\frac{G^2a_0^2}{4}\sin(x)+\frac{G^3a_0^3}{8}\cos(x)}{1+\frac{G^2a_0^2}{4}}\right]$$

$$= -e^{\frac{-2x}{Ga_0}}\left[\frac{G^2a_0^2\sin(x)+\frac{G^3a_0^3}{2}\cos(x)}{4+G^2a_0^2}\right]$$

$$\Rightarrow = \frac{2}{G^2a_0^2}x\sin(x)e^{\frac{-2x}{Ga_0}} - e^{\frac{-2x}{Ga_0}}\left[\frac{4\frac{\sin(x)}{Ga_0}+2\cos(x)}{4+G^2a_0^2}\right] + \frac{2}{G^2a_0^2}\int x\cos(x)e^{\frac{-2x}{Ga_0}}dx$$

$$u = x\cos(x) \qquad dv = e^{\frac{-2x}{Ga_0}}dx$$

$$du = (\cos(x) - x\sin(x))dx \qquad v = \frac{-Ga_0}{2}e^{\frac{-2x}{Ga_0}}$$

$$\Rightarrow \int x\cos(x)e^{\frac{-2x}{Ga_0}}dx = \frac{-Ga_0}{2}x\cos(x)e^{\frac{-2x}{Ga_0}} + \frac{Ga_0}{2}\int \cos(x)e^{\frac{-2x}{Ga_0}} - \frac{Ga_0}{2}\int x\sin(x)e^{\frac{-2x}{Ga_0}}dx$$

$$\Rightarrow \int \cos(x)e^{\frac{-2x}{Ga_0}}dx \qquad \text{let } u = \cos(x) \qquad dv = e^{\frac{-2x}{Ga_0}}dx$$

$$du = -\sin(x)dx \qquad v = \frac{-Ga_0}{2}e^{\frac{-2x}{Ga_0}}$$

$$= \frac{-Ga_0}{2}\cos(x)e^{\frac{-2x}{Ga_0}} - \frac{Ga_0}{2}\int \sin(x)e^{\frac{-2x}{Ga_0}}dx$$

$$\text{let } u = \sin(x) \qquad dv = e^{\frac{-2x}{Ga_0}}dx$$

$$du = \cos(x)dx \qquad v = \frac{-Ga_0}{2}e^{\frac{-2x}{Ga_0}}$$

$$= -\frac{Ga_0}{2}\cos(x)e^{\frac{-2x}{Ga_0}} + \frac{G^2a_0^2}{4}\sin(x)e^{\frac{-2x}{Ga_0}} - \frac{G^2a_0^2}{4}\int \cos(x)e^{\frac{-2x}{Ga_0}}dx$$

$$\Rightarrow \int \cos(x)e^{\frac{-2x}{Ga_0}}dx = +e^{\frac{-2x}{Ga_0}}\left[\frac{-\frac{Ga_0}{2}\cos(x)+\frac{G^2a_0^2}{4}\sin(x)}{1+\frac{G^2a_0^2}{4}}\right]$$

$$= e^{\frac{-2x}{Ga_0}}\left[\frac{-2Ga_0\cos(x)+G^2a_0^2\sin(x)}{4+G^2a_0^2}\right]$$

$$\Rightarrow \frac{Ga_0}{2}\int \cos(x)e^{\frac{-2x}{Ga_0}}dx = e^{\frac{-2x}{Ga_0}}\left[\frac{-G^2a_0^2\cos(x)+\frac{1}{2}\sin(x)}{4+G^2a_0^2}\right]$$

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$$\Rightarrow \int x\cos(x) e^{\frac{-2x}{Ga_0}} dx = \frac{-Ga_0}{2} x\cos(x) e^{\frac{-2x}{Ga_0}} + e^{\frac{-2x}{Ga_0}} \left[\frac{G^2 a_0^2 \left(\cos(x) + \frac{1}{2}\sin(x)\right)}{4+G^2 a_0^2} \right]$$

$$- \frac{Ga_0}{2} \int x\sin(x) e^{\frac{-2x}{Ga_0}} dx$$

$$= \frac{2}{G^2 a_0^2} x\sin(x) e^{\frac{-2x}{Ga_0}} - e^{\frac{-2x}{Ga_0}} \left[\frac{\frac{4\sin(x)}{Ga_0} + 2\cos(x)}{4+G^2 a_0^2} \right] + \frac{2}{G^2 a_0^2} \left[\frac{-Ga_0}{2} e^{\frac{-2x}{Ga_0}} x\cos(x) \right.$$

$$+ e^{\frac{-2x}{Ga_0}} \left[\frac{G^2 a_0^2 \left(\cos(x) + \frac{1}{2}\sin(x)\right)}{4+G^2 a_0^2} \right]$$

$$\left. - \frac{Ga_0}{2} \int x\sin(x) e^{\frac{-2x}{Ga_0}} dx \right]$$

$$= \frac{2}{G^2 a_0^2} x\sin(x) e^{\frac{-2x}{Ga_0}} - e^{\frac{-2x}{Ga_0}} \left[\frac{\frac{4\sin(x)}{Ga_0} + 2\cos(x)}{4+G^2 a_0^2} \right] + \frac{1}{Ga_0} e^{\frac{-2x}{Ga_0}} x\cos(x) + e^{\frac{-2x}{Ga_0}} \left[\frac{2\cos(x) + \frac{1}{Ga_0}\sin(x)}{4+G^2 a_0^2} \right]$$

$$- \frac{1}{Ga_0} \int x\sin(x) e^{\frac{-2x}{Ga_0}} dx$$

$$= \frac{Ga_0}{1+Ga_0} \left[\frac{2}{G^2 a_0^2} x\sin(x) e^{\frac{-2x}{Ga_0}} - e^{\frac{-2x}{Ga_0}} \right.$$

$$\frac{4}{G^3 a_0^3} + \frac{1}{Ga_0}$$

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$$e^{\frac{-2x}{Ga_0}} \left[\frac{Ga_0}{2} x\sin(x) - \frac{G^2 a_0^2 \sin(x) + \frac{1}{2} G^3 a_0^3 \cos(x)}{4+G^2 a_0^2} - \frac{G^2 a_0^2}{4} x\cos(x) + \frac{\frac{G^3 a_0^3}{2}\left(\cos(x) + \frac{Ga_0}{4}\sin(x)\right)}{4+G^2 a_0^2} \right]$$

$$- \frac{G^2 a_0^2}{4} \int x\sin(x) e^{\frac{-2x}{Ga_0}} dx$$

$$1 + \frac{G^2 a_0^2}{4}$$

$$= \frac{e^{\frac{-2x}{Ga_0}} \left[\frac{Ga_0}{2} x\sin(x) - \frac{G^2 a_0^2 \sin(x) + \frac{1}{2} G^3 a_0^3 \cos(x)}{4+G^2 a_0^2} - \frac{G^2 a_0^2}{4} x\cos(x) + \frac{\frac{G^3 a_0^3}{2}\left(\cos(x) + \frac{Ga_0}{4}\sin(x)\right)}{4+G^2 a_0^2} \right]}{1 + \frac{G^2 a_0^2}{4}}$$

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$$\int_0^\infty x\sin(x)e^{\frac{-2x}{\epsilon a_0}}dx = \left[e^{\frac{-2x}{\epsilon a_0}}\left[\frac{2\epsilon a_0 x\sin(x)}{4+\epsilon^2 a_0^2} - \frac{4\epsilon^2 a_0^2\sin(x) + \frac{2}{\epsilon^3 a_0^3}\cos(x)}{(4+\epsilon^2 a_0^2)^2} - \frac{\epsilon^2 a_0^2 x\cos(x)}{(4+\epsilon^2 a_0^2)^2} + \frac{\frac{2}{\epsilon^3 a_0^3}\cos(x) - \frac{4}{\epsilon a_0}\sin(x)}{(4+\epsilon^2 a_0^2)^2}\right]\right]\Bigg|_0^\infty$$

$$= \frac{\frac{2}{\epsilon^3 a_0^3}}{(4+\epsilon^2 a_0^2)^2} + \frac{\frac{2}{\epsilon^3 a_0^3}}{(4+\epsilon^2 a_0^2)^2}$$

$$= \frac{\frac{4}{\epsilon^3 a_0^3}}{(4+\epsilon^2 a_0^2)^2}$$

$$\frac{4}{\epsilon^3 a_0^3}\int_0^\infty x\sin(x)e^{\frac{-2x}{\epsilon a_0}}dx = \boxed{\frac{16}{(4+\epsilon^2 a_0^2)^2}}$$