Problem Set 7 (Astrophysics)

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1. Absolute magnitude (M), apparent magnitude (m), distance (d), and parallax(π") are all related. For each of the following stars, calculate whichever of these parameters is missing.
   a. m = -1.6 mag, d = 4.3 pc. Find M.
   b. M = 14.3 mag, m = 10.9 mag. Find d.
   c. m = 5.6 mag, d = 88 pc. Find M.
   d. M = -0.9 mag, d = 220 pc. Find m.
   e. m = 0.2 mag, M = -9.0 mag. Find d.
   f. m = 7.4 mag, π" = 0".0043. Find M.

Recall the formula that relates absolute magnitude, apparent magnitude, and distance (in parsecs):

$$M = m + 5 - 5\log_{10}(d) \tag{1}$$

a) Plugging in the given values into equation (1), we find that

$$M = -1.6 + 5 - 5\log_{10}(4.3)$$
$$\approx 0.233$$

So the absolute magnitude of a star with apparent magnitude of -1.6 at a distance of 4.3 pc is approximately mag 0.233.

b) Plugging in the values into equation (1), we find that

$$14.3 = 10.9 + 5 - 5\log_{10}(d)$$

and rearranging and solving for d, we get

$$d = 10^{1.6/5} \approx 2.09\, pc$$

So the distance of a star with an absolute magnitude of 14.3 and apparent magnitude of 10.9 is approximately 2.09 pc.

c) Plugging in the given values into equation (1), we find that

$$M = 5.6 + 5 - 5\log_{10}(88)$$
$$\approx 0.878$$

So the absolute magnitude of a star at a distance of 88 pc and an apparent magnitude of 5.6 is approximately mag 0.878.

d) Plugging in the given values into equation (1) and solving for the apparent magnitude, we find that

$$m = -5.9 + 5\log_{10}(220)$$
$$\approx 5.81$$

So the apparent magnitude of a star at a distance of 220 pc with an absolute magnitude of -0.9 is approximately 5.81.

e) Plugging in the given values into equation (1) and solving for d, we find

$$d = 10^{14.2/5}$$

$$\approx 691.8\,pc$$

So the distance of a star with an apparent magnitude of 0.2 and an absolute magnitude of -9 is approximately 691.8 pc.

f) To begin, we must find the distance in parsecs by using the relationship

$$d = \frac{1}{\pi''}$$

Doing so, we find

$$d \approx 232.6\,pc$$

And plugging this along with the other given values into equation (1), we find that

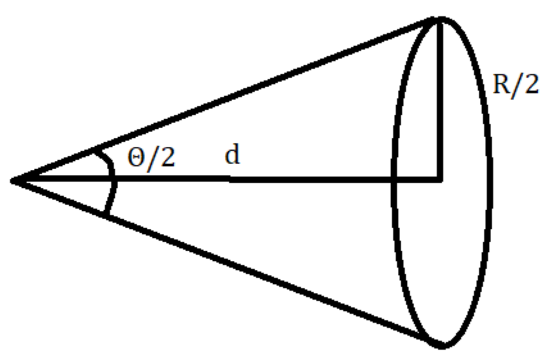
$$M = 7.4 + 5 - 5\log_{10}(232.6)$$

$$\approx 0.567$$

So the absolute magnitude of a star with apparent magnitude of 7.4 at a distance of 232.6 pc is approximately Mag 0.567.

2. What are the angular diameters of the following objects as seen from the Earth? Use a small angle approximation. Remember to convert radians to degrees or arc-seconds. In part b) you will need to first find the distance from the apparent and absolute magnitude.
   a. The Sun. radius $R_{sun} = 7 \times 10^5$ km, distance from Earth = 1.5×10^8 km
   b. Betelgeuse. M_V = -5.5 mag, m_V = 0.8 mag, radius R = 650 R_{sun}
   c. The galaxy M31, with R = 30 kpc at a distance of 0.7 Mpc
   d. The Coma cluster of galaxies with R = 3 Mpc at a distance of 100 Mpc

Notice from the diagram below that the angular size is given by $\tan\left(\frac{\theta}{2}\right) = \frac{R}{2d}$ and using the small angle approximation, we find that $\theta \approx \frac{R}{d}$.



a) Using the approximation we found above, and the data given, we find that

$$\theta_{Sun} \approx \frac{7 \times 10^5\,km}{3 \times 10^8\,km}$$

$$\approx 0.00233\, rad$$
$$= 0.27^\circ$$

So the angular diameter of the Sun as seen from Earth is approximately 0.27°.

b) To begin, we must figure out the distance to Betelgeuse. Using equation (1) we find that

$$d \approx 10^{11.3/5}$$
$$\approx 181.97\, pc$$
$$\approx 5.615 \times 10^{15}\, km$$

then the angular diameter is

$$\theta \approx \frac{650 \times 7 \times 10^5\, km}{5.615 \times 10^{15}\, km}$$
$$\approx 8.10 \times 10^{-8}\, rad$$
$$\approx 0.017 \text{ arcsec}$$

So the angular diameter of Betelgeuse as seen from the Earth is approximately 0.017 arcseconds.

c) Using the given data, we find that

$$\theta \approx \frac{30\, kpc}{700\, kpc}$$
$$\approx 0.043\, rad$$
$$\approx 2.46^\circ$$

So the Andromeda Galaxy has an angular diameter as seen from the Earth of approximately 2.46°.

d) Using the given data, we find that

$$\theta \approx \frac{3\, Mpc}{100\, Mpc}$$
$$\approx 1.72^\circ$$

So the Coma Cluster has an angular diameter as seen from the Earth of approximately 1.72°.

3. The star 9 Sagittarii is a main sequence star with spectral type O5. O5 stars typically have an absolute magnitude M_V = -5.7. Its apparent magnitude is m_v=6.0. What is the distance to 9 Sagittarii (ignoring any extinction by dust)?

Using equation (1), we find that

$$d = 10^{16.7/5}$$
$$\approx 2187.76\, pc$$

So the distance to 9 Sagittarii is approximately 2187.76 pc. (9 Sagittarii is the brightest star seen imposed on M8, the Lagoon nebula, a nice summertime object that is visible to the naked eye in many dark sky locations.)

4. At the center of the Sun, the mass density if $\rho = 1.52 \times 10^5$ kg m^{-3} and the mean opacity is κ = 0.12 m^{2}kg^{-1}. What is the mean free path for a photon at the Sun's center? Mean free path of photons was defined when we discussed radiative transfer in Module 6.

The equation for the mean free path of a photon given density and opacity is given by

$$mfp = \frac{1}{\rho\kappa} \tag{2}$$

Plugging in the given values into equation (2), we find

$$mfp = \frac{1}{(1.52 \times 10^5\, kgm^{-3})(0.12\, m^2kg^{-1})}$$

$$\approx 5.48 \times 10^{-5}\ m$$

So the mean free path for a photon at the center of the Sun is approximately $5.48 \times 10^{-5}\ m$.

5. From the equation of hydrostatic equilibrium and assuming a constant density we derived an approximate expression for the pressure at a center of a star. A somewhat better approximation yields $P_C = \frac{3}{8\pi} G \left(\frac{M}{R^2}\right)^2$. It is often convenient to express the Mass and Radius of a star in solar units by dividing those terms by the solar values and then multiplying the equation by the same values. The result is: $P_C = \frac{3}{8\pi} G \left(\frac{M}{M_{sun}}\right)^2 \left(\frac{R}{R_{sun}}\right)^{-4} \left[\frac{1.99x10^{30}}{(6.96x10^8)^2}\right]^2$. Use this equation to calculate the approximate central pressures for the following stars:
   a. A K0 V star with M = 0.8 M_{sun} and R = 0.85 R_{sun}
   b. A K0 III star with M = 4 M_{sun} and R = 16 R_{sun}.
   c. A K0 I star with M = 13 M_{sun} and R = 200 R_{sun}.

$$P_C = \frac{3}{8\pi} G \left(\frac{M}{M_{Sun}}\right)^2 \left(\frac{R}{R_{Sun}}\right)^{-4} \left[\frac{1.99 \times 10^{30}}{(6.96 \times 10^8)^2}\right]^2 \tag{3}$$

a) Using the given data and plugging it into equation (3), we find

$$P_{C_{K0\ V}} = \frac{3}{8\pi} G \left(\frac{0.8\ M_{Sun}}{M_{Sun}}\right)^2 \left(\frac{0.85\ R_{Sun}}{R_{Sun}}\right)^{-4} \left[\frac{1.99 \times 10^{30}}{(6.96 \times 10^8)^2}\right]^2$$

$$\approx 1.65 \times 10^{14}\ Pa$$

b) Using the given data and plugging it into equation (3), we find

$$P_{C_{K0\ III}} = \frac{3}{8\pi} G \left(\frac{4\ M_{Sun}}{M_{Sun}}\right)^2 \left(\frac{16\ R_{Sun}}{R_{Sun}}\right)^{-4} \left[\frac{1.99 \times 10^{30}}{(6.96 \times 10^8)^2}\right]^2$$

$$\approx 3.280 \times 10^{10}\ Pa$$

c) Using the given data and plugging it into equation (3), we find

$$P_{C_{K0\ I}} = \frac{3}{8\pi} G \left(\frac{13\ M_{Sun}}{M_{Sun}}\right)^2 \left(\frac{200\ R_{Sun}}{R_{Sun}}\right)^{-4} \left[\frac{1.99 \times 10^{30}}{(6.96 \times 10^8)^2}\right]^2$$

$$\approx 1.42 \times 10^{7}\ Pa$$