Problem Set 9 (Astrophysics)

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May 2022

1. Dust grains made of graphite will sublime (turn directly from solid to gas) at a temperature of about 1500 K. We can calculate the temperature of a dust grain by going back to our discussion of the subsolar temperature of planets: $T_{grain} = \left(\frac{R_{star}}{r}\right)^{1/2}(1-A)^{1/4}T_{star}$ where A is the albedo which is about 0.04 for graphite.
   a. How close to an O5 V star (T = 42,000 K and R = 12 R_{sun}) can graphite grains remain solid?
   b. How close to an M2 III star (T = 3540 K and R = 0.5 R_{sun}) can graphite grains remain solid?

To begin, note that we will want to use the given equation for subsolar temperature to find the distance where a grain of graphite can remain solid. That is, we must solve the above equation for r. Doing so, we find

$$r = \left(\frac{T_{Star}}{T_{Grain}}\right)^2 (1-A)^{1/2} R_{Star} \tag{1}$$

In our case, since we wish to find the minimum distance to a star such that graphite remains solid, we can use the sublimation temperature for graphite, 1500 K.

a) We are given $T_{Star} = 42000\,K$ and $R_{Star} = 12\,R_\odot$. Plugging these values into (1), we find

$$r_O = \left(\frac{42000}{1500}\right)^2 0.96^{1/2}(12\,R_\odot)$$

$$\approx 9218\,R_\odot$$

$$\approx 42.87\,AU$$

So graphite can get as close as approximately $42.87AU$ to an O5 V star with radius $12R_\odot$ before it sublimates.

b) We are given $T_{Star} = 3540\,K$, and $R_{Star} = 0.5\,R_\odot$. Plugging these values into (1), we find

$$r_M = \left(\frac{3540}{1500}\right)^2 0.96^{1/2}(0.5\,R_\odot)$$

$$\approx 2.729\,R_\odot$$

$$\approx 0.01269\,AU$$

So graphite can get as close as approximately 2.729 solar radii to an M2III star with radius 0.5 $R_\odot$ before it sublimates.

2. A Cepheid variable star in the Large Magellanic Cloud is observed to vary in luminosity with a period P = 95 days. It has an average apparent magnitude, m_V = 11.80. Calculate the distance to the Large Magellanic Cloud. You may ignore any effects due to dust in the interstellar medium.

Recall the formula that relates absolute magnitude to the period of a pulsating variable star:

$$M_v = -2.76 \log_{10}\left(\frac{P}{10 \text{ days}}\right) - 4.16 \tag{2}$$

We are given that the period (P) is 95 days, so plugging this value into (2), we find

$$\begin{aligned} M_v &= -2.76 \log_{10}\left(\frac{95 \text{ days}}{10 \text{ days}}\right) - 4.16 \\ &\approx -6.86 \end{aligned}$$

So we have that the absolute magnitude of this star is approximately Mag -6.86. Now, we can use the following formula that relates apparent magnitude and absolute magnitude to distance:

$$M_v = m_v + 5 - 5\log_{10}(d) \tag{3}$$

where d is in parsecs. Plugging in the given value for m_v and the value we found for M_v into (2) and solving for d, we find

$$\begin{aligned} d &= 10^{23.66/5} \\ &\approx 54000\, pc \\ &\approx 17600\, ly \end{aligned}$$

So the Large Magellanic Cloud is approximately 54000 pc or 17600 ly away, based on the measurements of this Cepheid variable.

3. What would the rotational period of the Sun be if it collapsed to a radius of 6000 km without losing any angular momentum? (assume I = (2/5) MR²)

To begin, we first wish to calculate the current moment of inertia of the Sun. That is, its moment of inertia before it suddenly collapses.

First, we note that the mass of the sun is approximately $1.988 \times 10^{30}\ kg$ and the current radius of the Sun is $6.957 \times 10^{8}\ m$. Plugging these values into the expression for moment of inertia, we find

$$\begin{aligned} I_\odot &= \frac{2}{5}(1.988 \times 10^{30}\ kg)(6.957 \times 10^{8}\ m) \\ &\approx 3.849 \times 10^{47}\ kg\, m^2 \end{aligned}$$

Now, recall that angular momentum is given by

$$L_\odot = I_\odot \omega \tag{4}$$

where ω is the angular velocity of the Sun's rotation. Now we must find ω for Sun in its current state.

$$\begin{aligned} \omega &= \frac{v}{r} \\ &= \frac{1.990 \times 10^{3}\ m\, s^{-1}}{6.957 \times 10^{8}\ m} \\ &\approx 2.860 \times 10^{-6} \text{ rad } s^{-1} \end{aligned}$$

And plugging this into (4), we find

$$L_\odot \approx 1.100 \times 10^{42}\ kg\, m^2 s^{-1}$$

Now let's calculate the moment of inertia of the Sun if it had a radius of 6000 km (assuming the same mass):

$$I'_\odot = \frac{2}{5}(1.988 \times 10^{30}\ kg)(6000 \times 10^{3}\ m)^2$$

$$\approx 2.863 \times 10^{43}\, kg\, m^2$$

Now, we wish to find ω'. Since angular momentum is assumed to be conserved, we have that

$$L_\odot = I'_\odot \omega' = 1.100 \times 10^{42}\, kg\, m^2 s^{-1}$$

Plugging in the value we found for $I'_\odot$ into the above equation and solving for ω', we find

$$\omega' \approx 0.0384\, \text{rad}\, s^{-1}$$

And to find how long it takes for the Sun to complete one period of rotation, simply divide 2π by ω'.

$$\begin{aligned} P &= \frac{2\pi}{0.0384\, s^{-1}} \\ &\approx 163.6\, s \end{aligned}$$

That is, if the Sun suddenly collapsed down to a radius of 6000 km, it would rotate about once every 163 seconds. Fewer than three minutes!

4. Ca II has two particular spectral lines with the following rest wavelengths: H line: λ_0 = 3968.5 Å and K line: λ_0 = 3933.6 Å. We are observing the spectrum of a distant galaxy in the Corona Borealis Cluster and observe that these lines are shifted to wavelengths λ_H =. 4255.0 Å and λ_K = 4217.6 Å.
    a. What is the redshift (z) of the galaxy?
    b. What is the distance to the galaxy?

a) Recall that the redshift (z) is defined by

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} \tag{5}$$

From the given values, We have that $\lambda_{\text{emitted,H}} = 3968.5$ Å, and $\lambda_{\text{emitted,K}} = 3933.6$ Å. And for the observed values, we have $\lambda_{\text{observed,H}} = 4255.0$ Å and $\lambda_{\text{observed,K}} = 4217.6$ Å. Using these values and plugging them into (5), we find

$$\begin{aligned} z &= \frac{\lambda_{\text{observed,H}} - \lambda_{\text{emitted,H}}}{\lambda_{\text{emitted,H}}} \\ &= \frac{4255.0\text{ Å} - 3968.5\text{ Å}}{3968.5\text{ Å}} \\ &\approx 7.2198 \times 10^{-2} \end{aligned}$$

and we find the same thing for the K line:

$$\begin{aligned} z &= \frac{\lambda_{\text{observed,K}} - \lambda_{\text{emitted,K}}}{\lambda_{\text{emitted,K}}} \\ &= \frac{4217.6\text{ Å} - 3933.6\text{ Å}}{3933.6\text{ Å}} \\ &\approx 7.2198 \times 10^{-2} \end{aligned}$$

So the redshift (z) of this galaxy is approximately 7.2198×10^{-2} or 0.072198.

b) Recall the relationship between d and z:

$$d = \frac{cz}{H_0}$$

where H_0 is the Hubble constant. Using a value of $70 \, \frac{km}{s \, MPc}$ for H_0, we find

$$d = \frac{(3 \times 10^5 \; km/s)(0.072198)}{70 \; km/sMPc}$$

$$\approx 309.42 \; MPc$$

So the distance to this galaxy is approximately $309.42 \; MPc$.