Problem set 1 (Astrophysics)

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1

Angular and linear distances:

1.6. (a) Consider two points on the Earth's surface that are separated by 1 arcsecond as seen from the center of the (assumed to be transparent) Earth. What is the physical distance between the two points?

To begin, first note that

$$1\, arcsecond = \frac{1}{3600}^{\circ} \tag{1.1}$$

And recall that there are 2π radians per $360°$

Then, to convert from arcseconds to radians, simply multiply (1.1) by $\frac{2\pi}{360°}$

Which results in

$$1\, arcsecond = (\tfrac{1}{3600}^{\circ})(\tfrac{2\pi}{360°})$$

$$\approx 4.848 \times 10^{-6}\, rad^{1} \tag{1.2}$$

Now recall the formula for arc length:

$$s = r\theta \tag{1.3}$$

Where s is the arc length, r is the radius, and theta is the subtended angle in radians.

Using the value $r = 6.371 \times 10^{6}$ meters for the radius of Earth, and plugging (1.2) in for θ and r into equation (1.3), we get the following for the arclength:

[1] For the entirety of this assignment, I will be using four-digit rounding arithmetic.

$$s = (6.371 \times 10^6\, m)(4.848 \times 10^{-6}\, rad)$$
$$\approx 30.89\, m$$

Thus, the physical distance between two points on Earth's surface separated by 1 arcsecond have a physical distance of approximately 30.89 m.

2

Astronomical Distance Units (show your work, don't just look it up.)

2.5. A light-year is defined as the distance traveled by light in a vacuum during one tropical year. How many light-years are in a parsec?

Let's begin by finding how many seconds are in a sidereal year:

$$t = (365.25\, \cancel{days})(24\, \frac{\cancel{h}}{\cancel{day}})(60\, \frac{\cancel{min}}{\cancel{h}})(60\, \frac{s}{\cancel{min}})$$
$$\approx 3.156 \times 10^7\, s \tag{2.1}$$

And recall the speed of light (in a vacuum):

$$c = 3.000 \times 10^8\, \frac{m}{s} \tag{2.2}$$

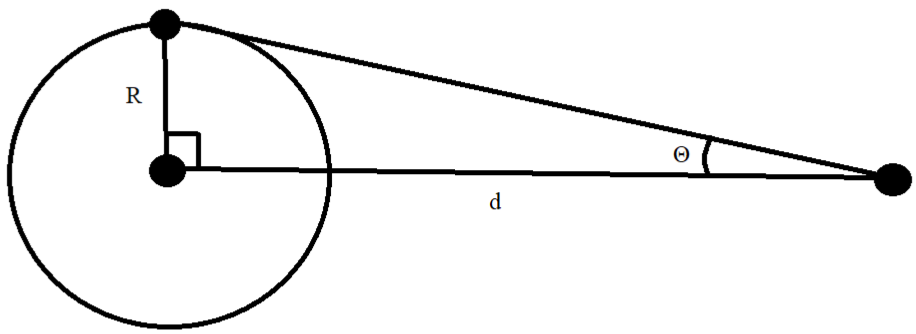
Now, let's find how many meters are in a light year by multiplying (2.1) by (2.2):

$$1\, ly = (3.0000 \times 10^8\, \frac{m}{\cancel{s}})(3.156 \times 10^7\, \cancel{s})$$
$$\approx 9.468 \times 10^{15}\, m \tag{2.3}$$

Now we have the amount of meters in a light year. Let's calculate how many meters are in a parsec and take the ratio between the amount of meters in a light year and parsec to find how many light years are in a parsec.

In the following diagram[2], when $\theta = 1\, arcsecond$, $d = 1\, parsec$ given R is the average distance between the Earth and the Sun.

[2] I made this diagram in MS Paint.



Given the average distance between the Earth and the Sun is

$$R = 1.500 \times 10^{11}\, m \tag{2.4}$$

From the above diagram, it is clear we can use the law of sines and equation (2.4) to find that

$$1\, pc = 1.500 \times 10^{11}\, m\left(\frac{sin(90^\circ - \frac{1}{3600}{}^\circ)}{sin(\frac{1}{3600}{}^\circ)}\right)$$

$$\approx 3.094 \times 10^{16}\, m \tag{2.5}$$

Finally, we can find how many light years are in one parsec by simply finding the ratio between a parsec and a light year. That is, the ratio between equations (2.5) and (2.3):

$$\frac{1\, pc}{1\, ly} = \frac{3.094 \times 10^{16}\, \cancel{m}}{9.468 \times 10^{15}\, \cancel{m}} \approx 3.268 \tag{2.6}$$

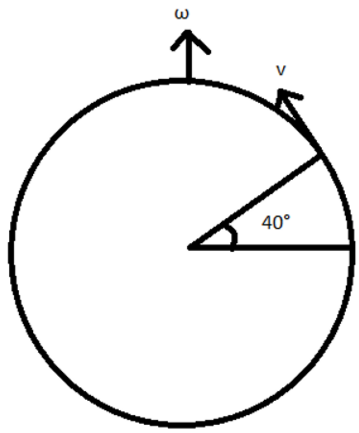
So there are approximately 3.268 light years in a parsec.

3

Coriolis Effect

2.7. Consider a football thrown directly northward at a latitude 40° N. The distance of the quarterback from the receiver is 20 yards (18.5 m), and the speed of the thrown ball is 25 m s^{-1}. Does the Coriolis force deflect the ball to the right or to the left? By what amount (in meters) is the ball deflected? Does the receiver need to worry about correcting for the deflection, or should he be more worried about being nailed by the free safety? [Hint: Remember that the angular velocity $\vec{\omega}$ of the Earth's rotation is parallel to the rotation axis.]

The football is thrown at $40^\circ\ N$ with a velocity of 25 $\frac{m}{s}$ Consider the diagram below.



Using the sidereal day of $23\,h56\,m$ find that

$$\vec{\omega} = \frac{2\pi}{86160} s^{-1} \hat{y} \tag{3.1}$$

And since the ball is being thrown to a target $18.5\,m$ away, we can calculate the amount of time the ball is in the air:

$$t = \frac{18.5\,\cancel{m}}{25\,\cancel{m} * s^{-1}} = 0.7400\,s \tag{3.2}$$

Now, from the equation of Coriolis acceleration,

$$a_{cor} = -2(\vec{\omega} \times \vec{v}) \tag{3.3}$$

We can say that the drift of the football will be to the left of where the quarterback was aiming, from his perspective. Now we just need to find the magnitude of the Coriolis acceleration vector (since we know the direction).
Well, we know that the velocity vector will make an angle of $40°$ with $\vec{\omega}$.

To find the magnitude of the Coriolis acceleration, recall that the magnitude of the acceleration is given by

$$|a_{cor}| = 2|\omega||v|sin(40°) \tag{3.4}$$

Clearly,

$$|v| = 25\,\frac{m}{s} \tag{3.5}$$

And plugging (3.1), and (3.5) into (3.4), we will get the following:

$$|a_{cor}| = \frac{2\pi * 25\,\frac{m}{s}}{86160\,s} sin(40°) \approx 1.172 \times 10^{-3}\,\frac{m}{s^2} \tag{3.6}$$

Now, to find the approximate drift distance, let us use the approximate distance formula as follows:

$$d \approx \frac{1}{2} a_{cor} t^2 \tag{3.7}$$

Plugging in (3.2) and (3.6) into (3.7), we will get the following:

$$d \approx \frac{1}{2} * 1.172 \times 10^{-3} \, \frac{m}{\cancel{s}} * (0.7400 \, \cancel{s})^2$$
$$\approx 3.209 \times 10^{-4} \, m \tag{3.8}$$

So the drift due to the Coriolis effect on the football will be approximately $3.209 \times 10^{-4} \, m$
I would say it's a safe assumption that the receiver does not need to worry about correcting for the deflection.

4

Find the distance from the more massive object to the system center of mass for:

a. The Sun-Jupiter system
b. The Earth-Moon system

Useful data:
$m_{Jupiter}$ = 317.8 x 5.974 x 10^{24} kg = 1.898 x 10^{27} kg
m_{Sun} = 1.989 x 10^{30} kg
m_{Earth} = 5.974 x 10^{24} kg
m_{Moon} = 7.36 x 10^{22} kg
semimajor axis of Jupiter's orbit = 5.203 AU
semimajor axis of Moon's orbit = 384.4 x 10^3 km

To find the center of mass between the Sun and Jupiter, let's use the following equation:

$$r_{com} = \frac{m_1}{m_1 + m_2} r \tag{4.1}$$

Which gives us the distance from m_2 to m_1. In our case, let m_2 be the mass of the Sun and m_1 be the mass of Jupiter. From the table above,

$$m_{Jupiter} = 1.898 \times 10^{27} \, kg \tag{4.2}$$

$$m_{Sun} = 1.989 \times 10^{30} \, kg \tag{4.3}$$

With the distance between the Sun and Jupiter as given:

$$r_{Sun/Jupiter} = 7.784 \times 10^{11} m \tag{4.4}$$

Plugging (4.2), (4.3), and (4.4) into (4.1), we get the following:

$$r_{com} = \left(\frac{1.898 \times 10^{27} \, \cancel{kg}}{1.898 \times 10^{27} \, \cancel{kg} + 1.989 \times 10^{30} \, \cancel{kg}}\right) 7.784 \times 10^{11} \, m$$
$$\approx 7.421 \times 10^{8} \, m \tag{4.5}$$

Thus, from (4.5), the common center of mass between the Sun and Jupiter is approximately $7.421 \times 10^{8} \, m$ from the center of the Sun.

Now, for the center of mass of the Earth-Moon system: From the table above, the mass of the Earth and the Moon are, respectively,

$$m_{Earth} = 5.974 \times 10^{24}\ kg \tag{4.6}$$

$$m_{Moon} = 7.360 \times 10^{22}\ kg \tag{4.7}$$

Where the semi-major axis of the Moon's orbit around the Earth is given by:

$$r_{Earth/Moon} = 3.844 \times 10^{8}\ m \tag{4.8}$$

Now, plugging (4.6), (4.7), and (4.8) into (4.1), we get the following:

$$r_{com} = \left(\frac{7.360\times10^{22}\ \cancel{kg}}{7.360\times10^{22}\ \cancel{kg}+5.974\times10^{24}\ \cancel{kg}}\right)3.844 \times 10^{8}\ m$$

$$\approx 4.678 \times 10^{6}\ m \tag{4.9}$$

Thus, by (4.9), the center of mass of the Earth-Moon system is approximately 4.678×10^{6} meters from the center of the Earth.

5

Compare the mutual gravitational force between:

a. You and another person with mass of 100 kg located 2 m from you

b. You and Mars at opposition (This is when the Sun and Mars are on opposite sides of the Earth which is also when Mars is closest to Earth in their orbits.)

(It is OK to lie about your weight as long as you are reasonable.)

Recall Newton's law of gravitation:

$$F_{grav} = G\frac{m_1 m_2}{r^2} \tag{5.1}$$

Where G is given by:

$$G = 6.670 \times 10^{-11}\ \frac{Nm^2}{kg^2} \tag{5.2}$$

Now let $m_2 = 100\ kg$ and $m_1 = 80.74\ kg$ (my mass)
Now plugging in (5.2), m_1, m_2, and $r = 2\ m$ into (5.1), we will get:

$$F_{m_1 m_2} = 6.670 \times 10^{-11}\ \frac{N\cancel{m^2}}{\cancel{kg^2}}\left(\frac{80.74\ \cancel{kg}*100.0\ \cancel{kg}}{(2.000\ \cancel{m})^2}\right)$$

$$\approx 1.346 \times 10^{-7}\ N \tag{5.3}$$

Thus the force of gravity between me and a person of mass 100 kg standing 2 m away from me is approximately $1.346 \times 10^{-7}\ N$, as given by (5.3).

Now, let's find the force of gravity between me and Mars at opposition. First note that the mass of Mars is:

$$m_{Mars} = 6.390 \times 10^{23}\ kg \tag{5.4}$$

And that the distance between Earth and Mars at opposition is:

$$d_{EarthMars} = 6.207 \times 10^{10}\ m \tag{5.5}$$

Now, let's plug in (5.4), (5.5), (5.2), and my mass into (5.1) to find the mutual force of gravity between me and Mars at opposition:

$$\begin{aligned} F_{Me,Mars} &= 6.67 \times 10^{-11}\ \frac{N\cancel{m^2}}{\cancel{kg^2}}\left(\frac{80.74\ \cancel{kg} * 6.39 \times 10^{23}\ \cancel{kg}}{(6.207 \times 10^{10}\ \cancel{m})^2}\right) \\ &\approx 8.932 \times 10^{-7}\ N \end{aligned} \tag{5.6}$$

Thus, the mutual gravitational force between me and Mars at opposition is approximately $8.932 \times 10^{-7}\ N$, as given by (5.6).

6

At what distance from the center of the Earth will an artificial satellite in an equatorial circular orbit (the orbit lies entirely in the Earth's equatorial plane) have a period equal to 1 day ? (Such a synchronous or geostationary satellite remains fixed above a point on the equator and can be used as a fixed relay station in the worldwide communications network.)

Recall Kepler's third law:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3 \tag{6.1}$$

Where P is the period, a is the semi-major axis, and G is given in equation (5.2).
Now let m_1 be Earth's mass and m_2 be the mass of the satellite. It goes without saying that the mass of the Earth is going to be much, much larger than the mass of the satellite, so we can approximate Kepler's third law as

$$P^2 \approx \frac{4\pi^2}{Gm_1} a^3 \tag{6.2}$$

We wish to solve for a, so rearranging (6.2) into terms of a gives us

$$a \approx \left(\frac{P^2 * G * m_1}{4\pi^2}\right)^{\frac{1}{3}} \tag{6.3}$$

For the satellite to be geosynchronous, we want the period of its orbit to be equal to the period of Earth's rotation, For this calculation, I will use a sidereal day, for which the period is

$$P = 86160\ s \tag{6.4}$$

Given that the mass of the Earth is

$$m_1 = 5.972 \times 10^{24}\ kg$$

and plugging this along with (5.2), (6.4) into (6.3), we get the following:

$$\begin{aligned} a &\approx \left(\frac{(86160\ \cancel{s})^2 * 6.67\times 10^{-11}\ \frac{m^3}{\cancel{kg * s^2}} * 5.972\times 10^{24}\ \cancel{kg}}{4\pi^2}\right)^{\frac{1}{3}} \\ &\approx 4.215 \times 10^7\ m \end{aligned} \tag{6.5}$$

Thus, the radius of a geosynchronous orbit of a satellite for Earth is approximately $4.215 \times 10^7\ m$ from the center of the Earth, as given by (6.5).