Scientific Computation II HW4

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Exercise 1: Application of RBF Interpolation - 2D reconstruction from a point cloud

Consider an implicit (closed) curve in 2D

$$f(x, y) = 0$$

given by a parameterization $x = x(t)$, $y = y(t)$, $t \in [0, L]$. For a choice of parameter values $\{t_j\} = \{t_1, \dots, t_N\}$, consider the "point cloud" $\{\mathbf{x}_j = (x_j, y_j) = (x(t_j), y(t_j))\}$. The goal of this exercise is to reconstruct (approximately) the implicity function from the given point cloud.

Step 1: Find an (outer) normal direction $\mathbf{n}_j$ to the curve at each point $\mathbf{x}_j$.

Step 2: Fix $\alpha > 0$ small and consider the inner and outer points

$$\mathbf{x_j}^- = \mathbf{x}_j - a\mathbf{n}_j, \quad \mathbf{x}_j^+ = \mathbf{x}_j + \alpha\mathbf{n}_j.$$

Step 3: Interpolate the $3N$ data $(\mathbf{x}_j^-, -\alpha)$, $(\mathbf{x}_j, 0)$, $(\mathbf{x}_j^+, \alpha)$ using RBF interpolants, that is, find a function $F : \mathbb{R}^2 \to \mathbb{R}$,

$$F(\mathbf{x}) = \sum_{j=1}^{N} c_j \phi(\|\mathbf{x} - \mathbf{x}_j\|)$$

such that $F(\mathbf{x}_j) = 0$ and $F(\mathbf{x}_j^-) = -\alpha, F(\mathbf{x}_j^+) = \alpha$.

Step 4: Restrict F to the level set $F(x, y) = 0$ to obtain the RBF interpolation for the implicit curve $f(x, y) = 0$.

Implementing the pointcloud RBF interpolation in `MATLAB`, (see attached `myPointCloudRBF.m` script) we find the following level curves and surfaces for a few select point clouds:

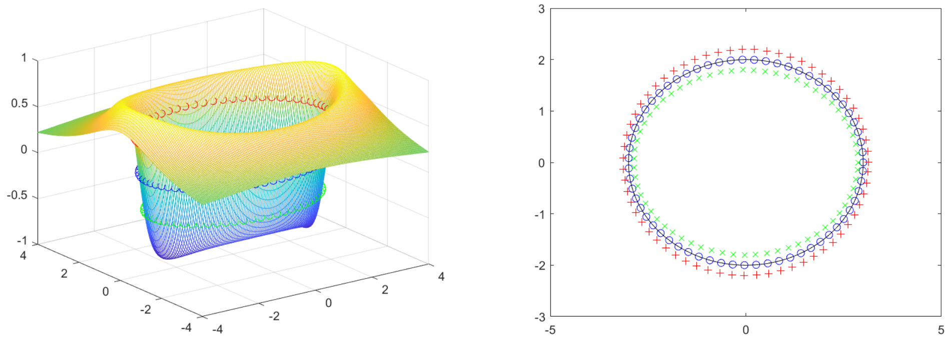

Figure 1: Ellipse Point Cloud with $\alpha = 0.4$, $r_x = 3$, $r_y = 2$ and IMQ RBF interpolant

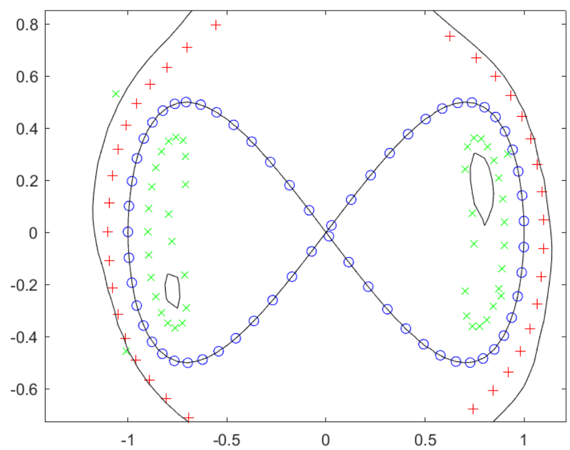

Figure 2: Infinity Lissajous Curve with $\alpha = 0.1$ and IMQ

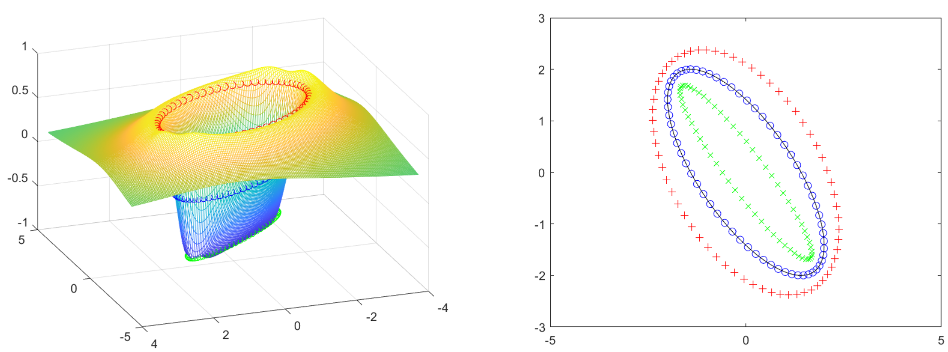

Figure 3: "Skewed" ellipse with $\alpha = 0.7$ and IMQ