# Solid State Physics HW 9

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**6.** ***Frequency Dependence of the electrical conductivity.*** Use the equation $m(dv/dt+v/\tau) = -eE$ for the electron drift velocity $v$ to show that the conductivity at frequency $\omega$ is

$$\sigma(\omega) = \sigma(0)\left(\frac{1+i\omega\tau}{1+(\omega\tau)^2}\right)$$

where $\sigma(0) = ne^2\tau/m$.

To begin, suppose $v = v_0e^{-i\omega t}$. then $E = E_0e^{-i\omega t}$ since the drift velocity is directly related to the electric field. Now, from the differential equation above, we have

$$\begin{aligned}
m\left(\frac{dv}{dt}+\frac{v}{\tau}\right) &= -eE\\
m\left(\frac{d}{dt}(v_0e^{-i\omega t})+\frac{v_0e^{-i\omega t}}{\tau}\right) &= -eE\\
m\left(-i\omega v_0e^{-i\omega t}+\frac{v_0e^{-i\omega t}}{\tau}\right) &= -eE_0e^{-i\omega t}\\
m\left(-i\omega v_0+\frac{v_0}{\tau}\right) &= -eE_0
\end{aligned}$$

Recall that $J = \sigma E$ and $J = nq\mathbf{v}$ (here, $q = -e$) and so

$$\mathbf{v} = \frac{\sigma E}{-ne}$$

Using this, our above equation becomes

$$\begin{aligned}
\frac{-im\omega E_0}{ne}\sigma + \frac{mE_0}{ne\tau}\sigma &= -eE_0\\
\frac{-im\omega}{-ne}\sigma + \frac{m}{-ne\tau}\sigma &= -e\\
-i\omega\tau\sigma + \sigma &= \frac{ne^2\tau}{m}\\
\sigma(1-i\omega\tau) &= \sigma(0)\\
\sigma(\omega) &= \sigma(0)\left(\frac{1}{1-i\omega\tau}\right)\\
\sigma(\omega) &= \sigma(0)\left(\frac{1+i\omega\tau}{1+(\omega\tau)^2}\right)
\end{aligned}$$

**9.** ***Static magnetoconductivity tensor.*** For the drift velocity theory of (51), show that the static current density can be written in matrix form as

$$\begin{pmatrix} j_x\\ j_y\\ j_z\end{pmatrix} = \frac{\sigma_0}{1+(\omega_c\tau)^2}\begin{pmatrix} 1 & -\omega_c\tau & 0\\ \omega_c\tau & 1 & 0\\ 0 & 0 & 1+(\omega_c\tau)^2\end{pmatrix}\begin{pmatrix} E_x\\ E_y\\ E_z\end{pmatrix}$$

From the drift velocity theory of (51), we have

$$m\left(\frac{d}{dt}+\frac{1}{\tau}\right)v_x=-e\left(E_x+\frac{B}{c}v_y\right)$$
$$m\left(\frac{d}{dt}+\frac{1}{\tau}\right)v_y=-e\left(E_y-\frac{B}{c}v_x\right)$$
$$m\left(\frac{d}{dt}+\frac{1}{\tau}\right)v_z=-eE_z$$

For static current density, we have $d\mathbf{v}/dt=0$; using this and rearranging the above equations to solve for $v_x, v_y$, and $v_z$, we find

$$v_x=\frac{1}{1+(\omega_c\tau)^2}\left(\frac{-e\tau}{m}(E_x+\omega_c\tau E_y)\right)$$
$$v_y=\frac{1}{1+(\omega_c\tau)^2}\left(\frac{-e\tau}{m}(E_y+\omega_c\tau E_x)\right)$$
$$v_z=-\frac{-e\tau E_z}{m}$$

Further recall that current density is defined by

$$j=nq\mathbf{v}$$

here, $q=-e$. Using this and the equations we found above for $v_x, v_y$, and $v_z$, and writing the current density in matrix notation, we find

$$\begin{pmatrix}j_x\\j_y\\j_z\end{pmatrix}=\frac{\sigma_0}{1+(\omega_c\tau)^2}\begin{pmatrix}1&-\omega_c\tau&0\\\omega_c\tau&1&0\\0&0&1+(\omega_c\tau)^2\end{pmatrix}\begin{pmatrix}E_x\\E_y\\E_z\end{pmatrix}$$

Where $\sigma_0=ne^2\tau/m$, which is what we sought to show.

Now, we wish to show in the limit $\omega_c\tau\gg 1$, we have that

$$\sigma_{yx}=\frac{nec}{B}=-\sigma_{xy}$$

To begin, rewriting the matrix system above in tensor notation, we have

$$j_i=\sigma_{ij}E_j$$

Thus,

$$j_y=\sigma_{yx}E_x+\sigma_{yy}E_y+\sigma_{yz}E_z$$

Then

$$\begin{aligned}\sigma_{yx}&=\frac{\sigma_0\omega_c\tau}{1+(\omega_c\tau)^2}\\&=\frac{ne^2\tau(\omega_c\tau)}{m(1+(\omega_c\tau))^2}\\&=\frac{ne^2\tau}{m(1/(\omega_c\tau)+\omega_c\tau)}\\&=\frac{ne^2\tau}{m(1/(\omega_c\tau)+eB/(mc)\tau)}\\&=\frac{ne^2\tau}{m(eB/(mc))\tau}\quad(\omega_c\tau\gg 1)\\&=\frac{nec}{B}\end{aligned}$$

Now, notice that

$$\sigma_{xy} = \frac{-\sigma_0 \omega_c \tau}{1 + (\omega_c \tau)^2}$$

so by the same process as above, we have that

$$\sigma_{yx} = -\sigma_{xy}$$

which is what we wanted to show.