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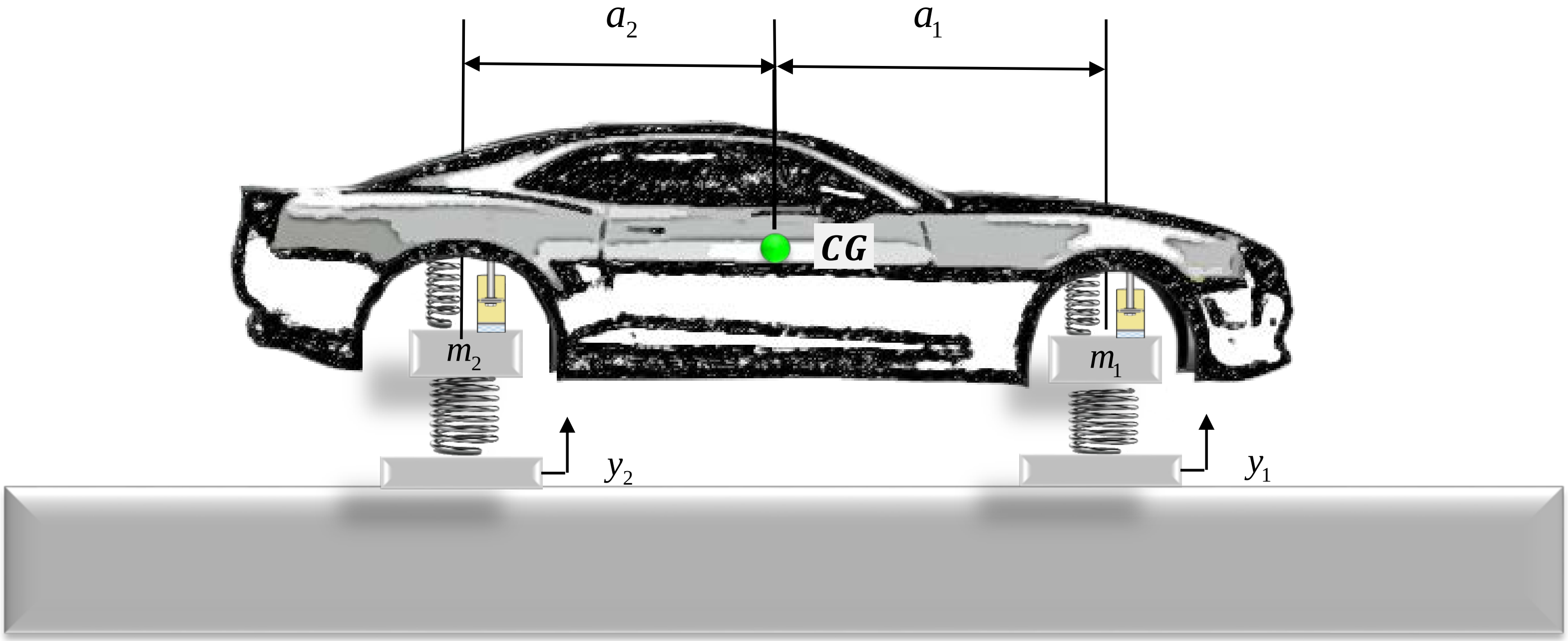

$$\boldsymbol{M\ddot{x}} + \boldsymbol{C\dot{x}} + \boldsymbol{Kx} = \boldsymbol{f}$$

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f}$$

$$x = \begin{bmatrix} x \\ \theta \\ x_1 \\ x_2 \end{bmatrix} \qquad M = \begin{bmatrix} m & & & \\ & I_z & & \\ & & m_1 & \\ & & & m_2 \end{bmatrix}$$

$$C=\begin{bmatrix} c_1+c_2 & a_2c_2-a_1c_1 & -c_1 & -c_2 \\ a_2c_2-a_1c_1 & c_1a_1^2+c_2a_2^2 & a_1c_1 & -a_2c_2 \\ -c_1 & a_1c_1 & c_1 & 0 \\ -c_2 & -a_2c_2 & 0 & c_2 \end{bmatrix}$$

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$$K = \begin{bmatrix} k_1 + k_2 & a_2k_2 - a_1k_1 & -k_1 & -k_2 \\ a_2k_2 - a_1k_1 & k_1a_1^2 + k_2a_2^2 & a_1k_1 & -a_2k_2 \\ -k_1 & a_1k_1 & k_1 + k_{p1} & 0 \\ -k_2 & -a_2k_2 & 0 & k_2 + k_{p2} \end{bmatrix}$$

$$f = \begin{bmatrix} 0 \\ 0 \\ y_1k_{p1} \\ y_2k_{p2} \end{bmatrix}$$

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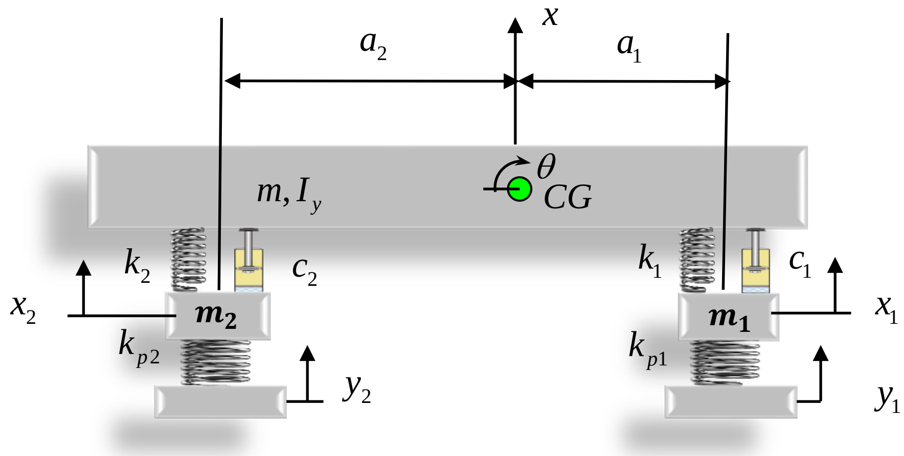
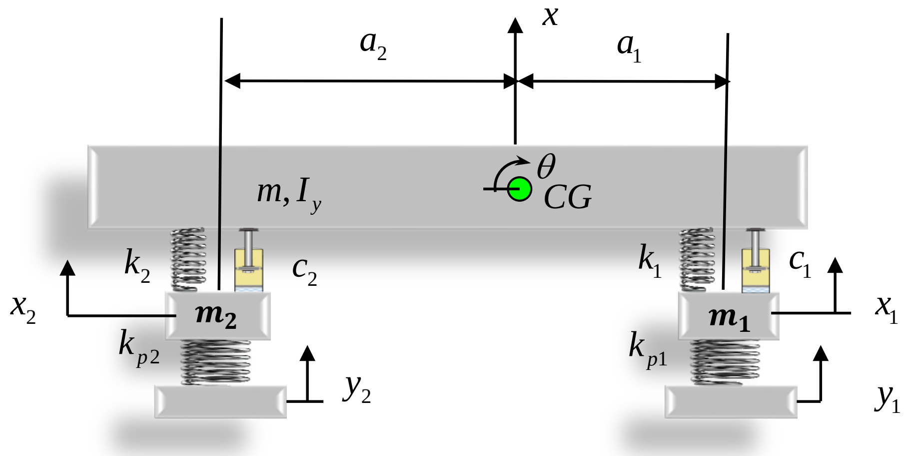


$$m_1\ddot{x}_1 - c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + k_{p1}(x_1 - y_1) - k_1(x - x_1 - a_1\theta) = 0$$

$$m_2\ddot{x}_2 - c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta}) + k_{p2}(x_2 - y_2) - k_2(x - x_2 + a_2\theta) = 0$$



$$m\ddot{x} + c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta})$$
$$+ k_1(x - x_1 - a_1\theta) + k_2(x - x_2 +_2 \theta) = 0$$

$$I_z\ddot{\theta} - a_1c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + a_2c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta})$$
$$- ak_1(x - x_1 - a_1\theta) + a_2k_2(x - x_2 + a_2\theta) = 0$$

$$m\ddot{x} + c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta})$$
$$+ k_1(x - x_1 - a_1\theta) + k_2(x - x_2 +_2 \theta) = 0$$

$$I_z\ddot{\theta} - a_1c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + a_2c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta})$$
$$- ak_1(x - x_1 - a_1\theta) + a_2k_2(x - x_2 + a_2\theta) = 0$$

$$m_1\ddot{x}_1 - c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + k_{p1}(x_1 - y_1) - k_1(x - x_1 - a_1\theta) = 0$$

$$m_2\ddot{x}_2 - c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta}) + k_{p2}(x_2 - y_2) - k_2(x - x_2 + a_2\theta) = 0$$

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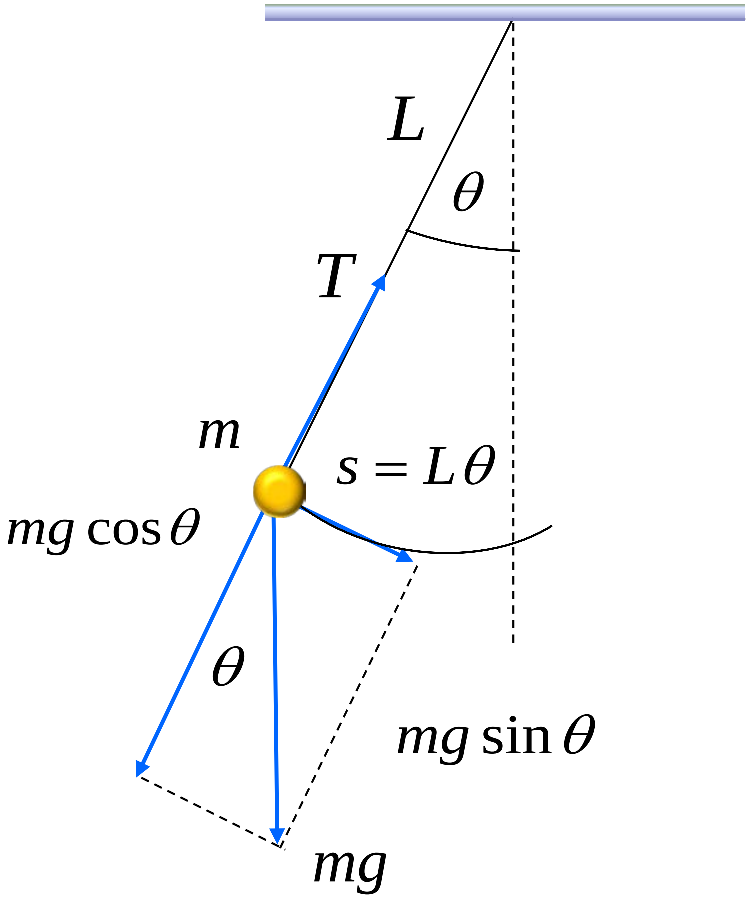
Le pendule



$$F = -mg\sin\theta$$

$$m\frac{d^2s}{dt^2} = -mg\sin\theta$$

$$s = L\theta \rightarrow$$

$$mL\frac{d^2\theta}{dt^2} = -mg\sin\theta$$

Le pendule pour θ petit

$$mL\frac{d^2\theta}{dt^2} = -mg\sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

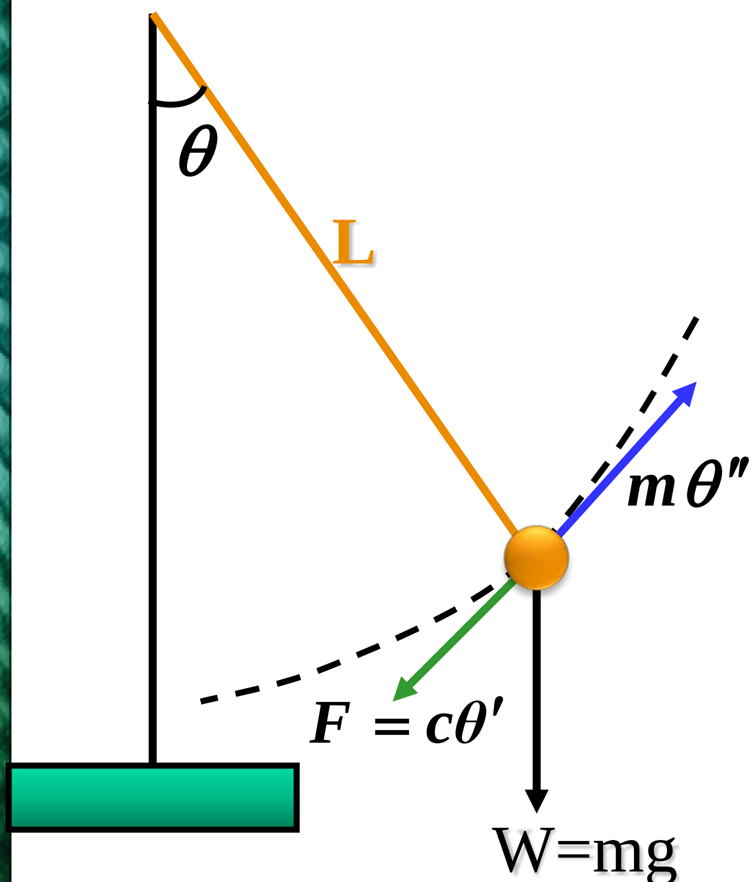
lorsque θ est petit

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \omega\end{bmatrix} = \begin{bmatrix}\omega\\ -\theta\, g/l\end{bmatrix}$$

Le pendule + amortissement



$$\begin{cases} \textit{Poids} & \mathrm{W} = \mathrm{mg} \\ \text{Amortissement} & \mathrm{F} = -c\theta' \end{cases}$$

$$mL\theta'' = -(mg\sin\theta + c\theta')$$

$$\theta'' + \frac{c}{mL}\theta' + \frac{g}{L}\sin\theta = 0$$

Pendule

$$y''+\frac{c}{mL}y'+\frac{g}{L}\sin y=0,\quad y(0)=a,\quad y'(0)=b.$$

avec $g/L=1$, $c/(mL)=0.3$,$a=\pi/2$ et $b=0$

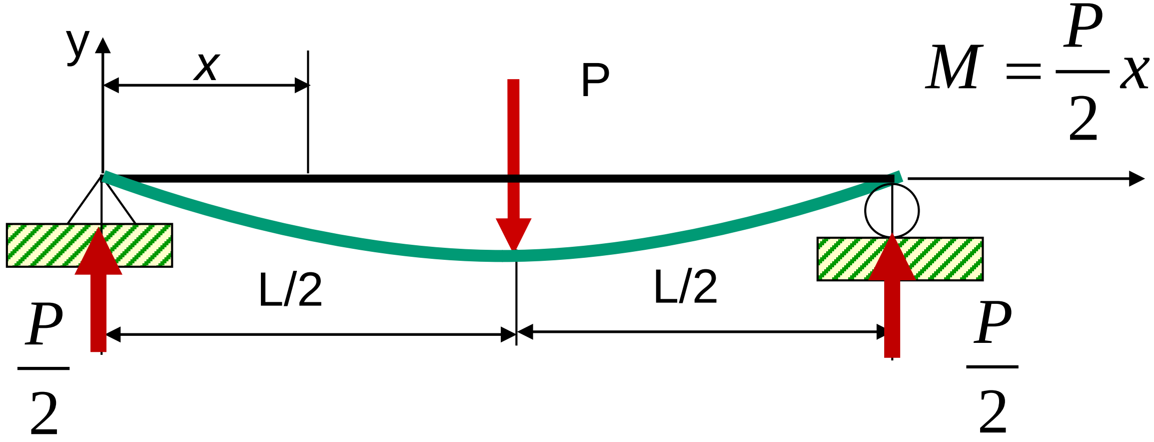
$$y''=-0.3y'-\sin y$$

$$u=y,\quad v=y'$$

$$u'=v=f(x,u,v),$$
$$v'=-0.3v-\sin u=g(x,u,v),$$
$$\text{avec } u(0)=\pi/2, v(0)=0.$$

EDOs :PVF

Déflection d'une poutre



$$M = \frac{P}{2}x \qquad 0 < x < \frac{L}{2}$$

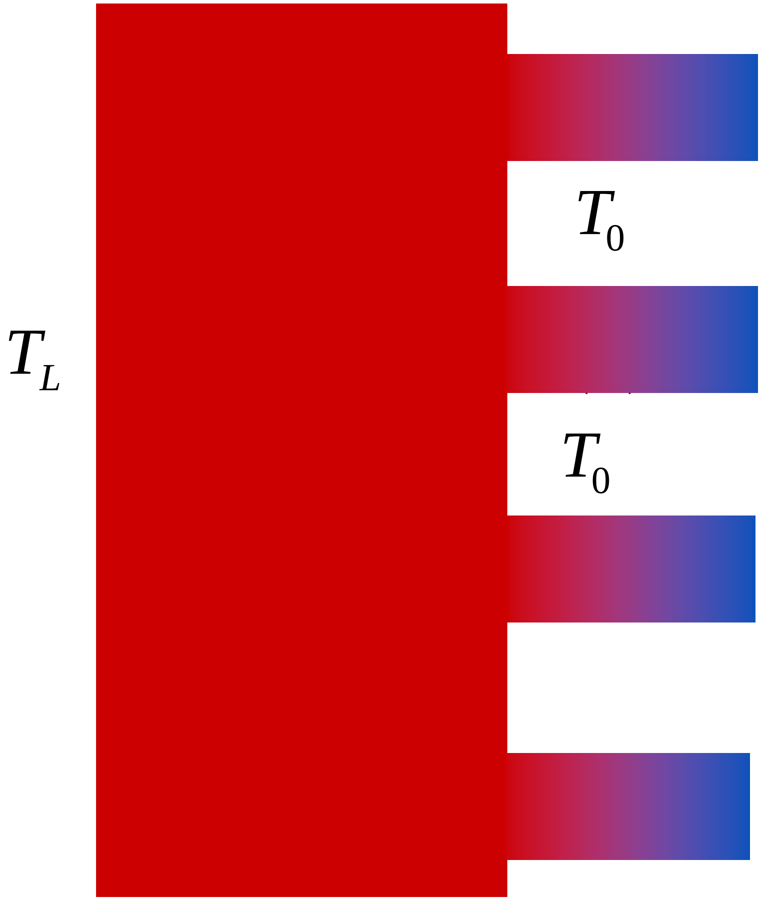
$$EI\frac{d^2y}{dx^2} = \frac{P}{2}x$$

$$y(0) = 0$$

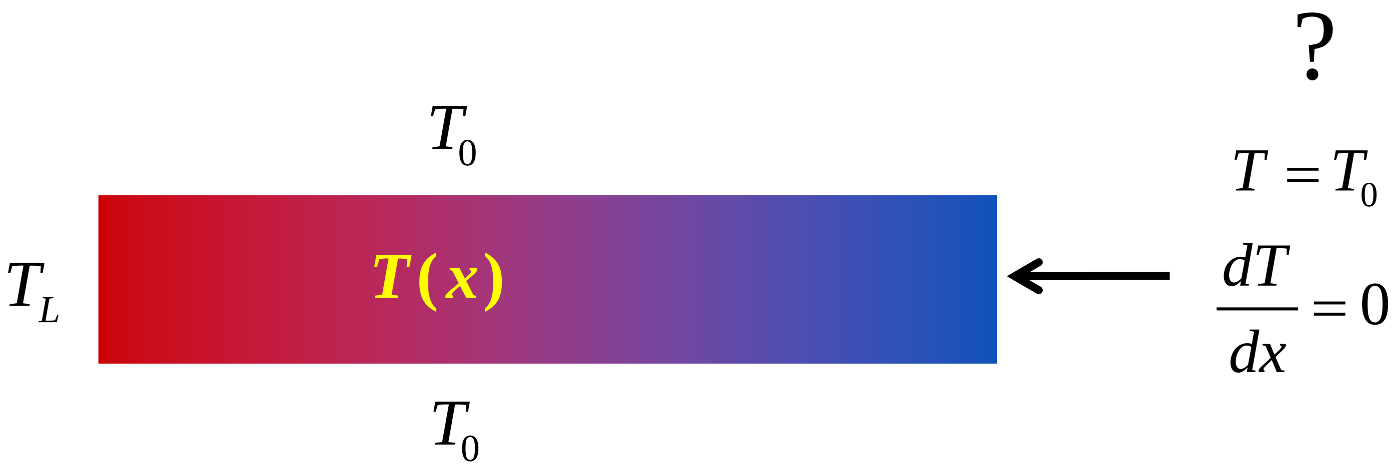
$$y(L) = 0$$

Refroidissement à ailettes

L'ailette



L'ailette



$$k\frac{d^2T}{dx^2} = h\left(T(x) - T_0\right)$$

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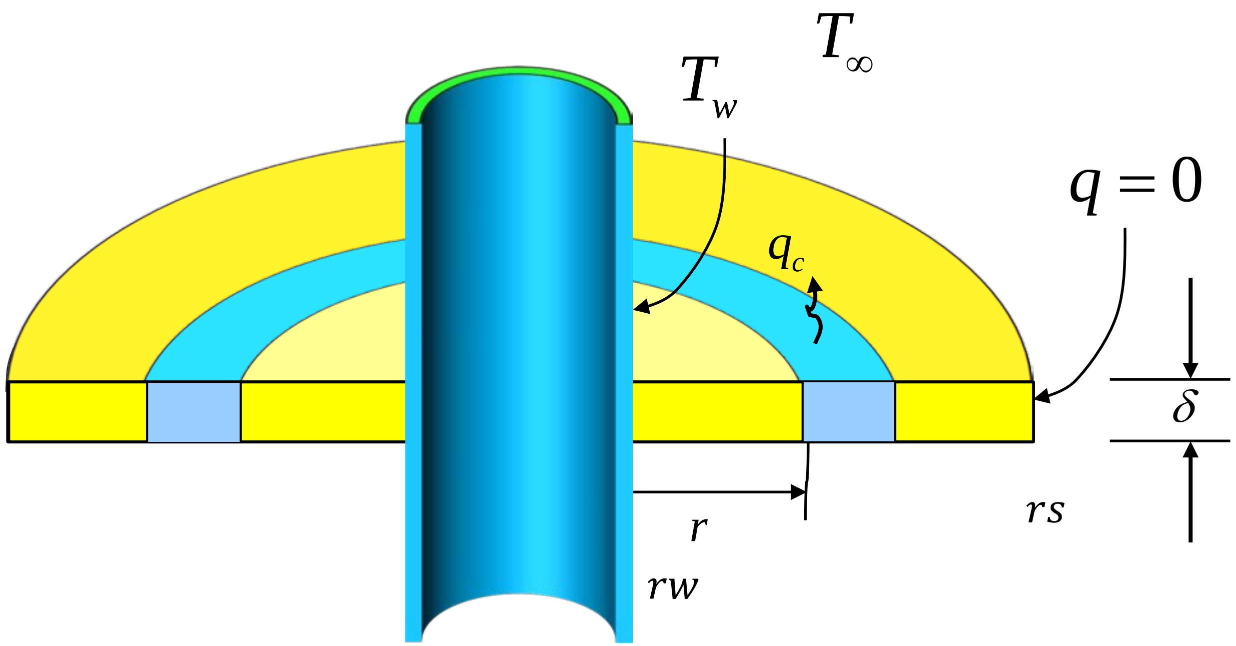
Ailette circulaire



$$r^2 \frac{d^2T}{dr^2} + r\frac{dT}{dr} - \frac{2h}{k\delta} r^2 (T - T_\infty) = 0$$

$$T(r_w) = T_w, \quad \left.\frac{dT}{dr}\right|_{rs} = 0$$

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Frome adimensionnelle

$$u = \frac{T - T_\infty}{T_w - T_\infty}, \qquad x = \frac{r}{r_s}$$

$$r^2 \frac{d^2T}{dr^2} + r\frac{dT}{dr} - \frac{2h}{k\delta} r^2 (T - T_\infty) = 0$$

$$T(r_w) = T_w, \qquad \left.\frac{dT}{dr}\right|_{rs} = 0$$

$$x^2 \frac{d^2u}{dx^2} + x\frac{du}{dx} - \alpha^2 x^2 u = 0$$

$$\begin{bmatrix} u(a) \\ u'(b) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \alpha^2 = \frac{2hr_s^2}{k\delta}$$

$$\frac{r_w}{r_s} = a \qquad u(a) = 1$$

$$x = \frac{r_s}{r_s} = 1 = b, \qquad u'(b) = 0$$

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} u \\ u' \end{bmatrix}$$

$$z_1' = z_2$$

$$z_2' = -\frac{1}{x} z_2 + \alpha^2 z_1$$

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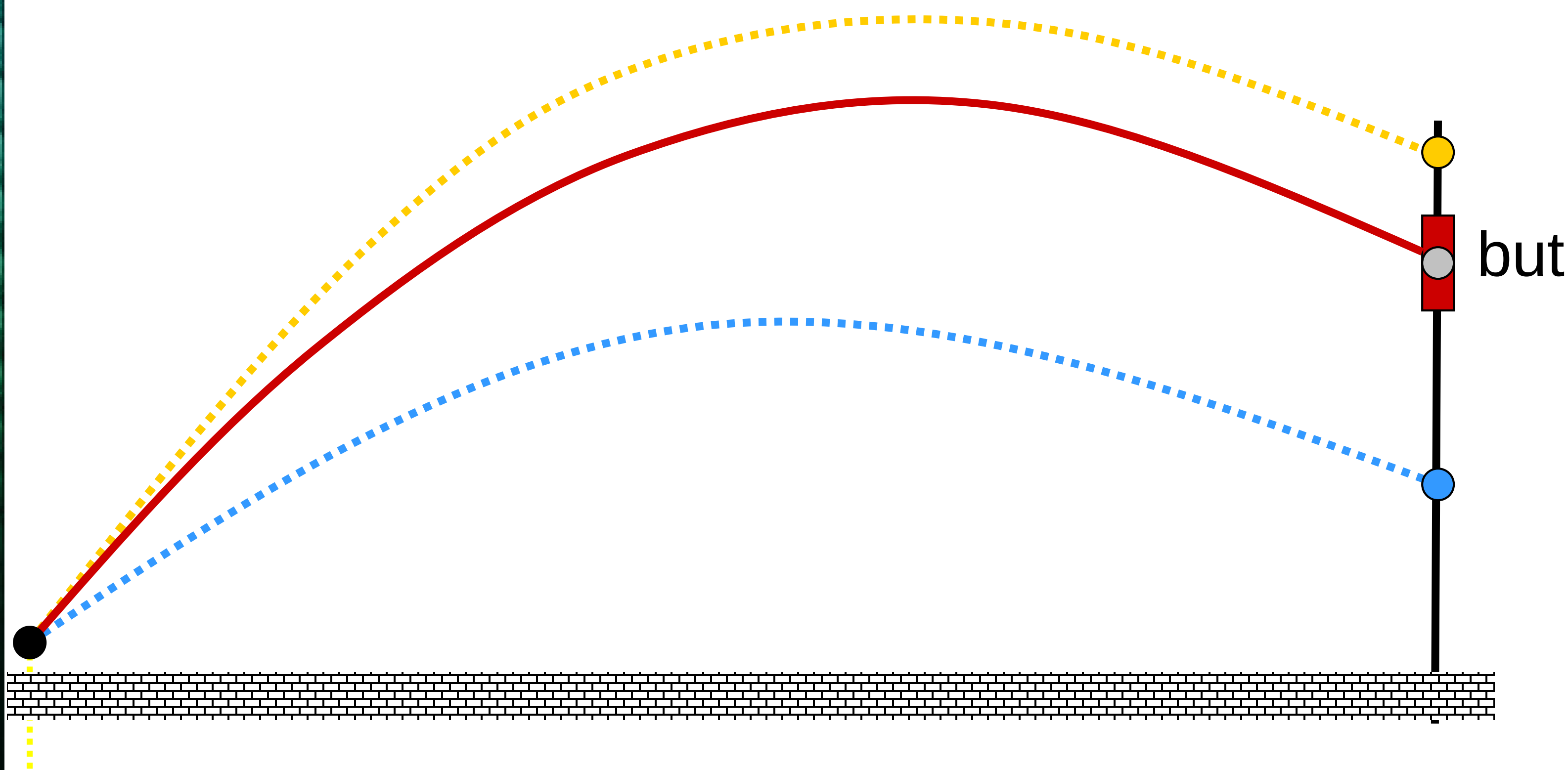
L'idée du Tir

- *On doit atteindre un but à une distance donnée*
- *On connaît les valeurs aux extémités*

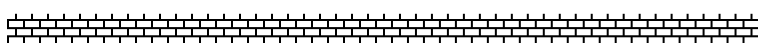


La méthode de Tir

Problème linéaire

$$\frac{d^2y}{dx^2} = a_2(x)\frac{dy}{dx} + a_1(x)y + a_0(x)$$

$$y(a) = y_a \qquad\qquad y(b) = y_b$$

a  ***b***

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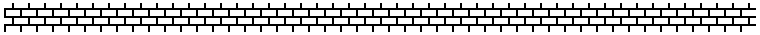
Deux tirs

$$\frac{d^2 y}{d x^2} = a_2(x)\frac{dy}{dx} + a_1(x)y + a_0(x)$$

$$y(a) = y_a \qquad \boxed{y(b) = y_b}$$

a  ***b***

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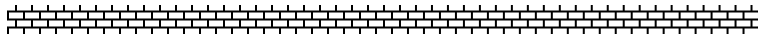
Tir #2

$$\frac{d^2 y}{d x^2} = a_2(x)\frac{dy}{dx} + a_1(x)y + a_0(x)$$

$$y(a) = y_a \qquad\qquad y(b) = y_b$$

a  ***b***

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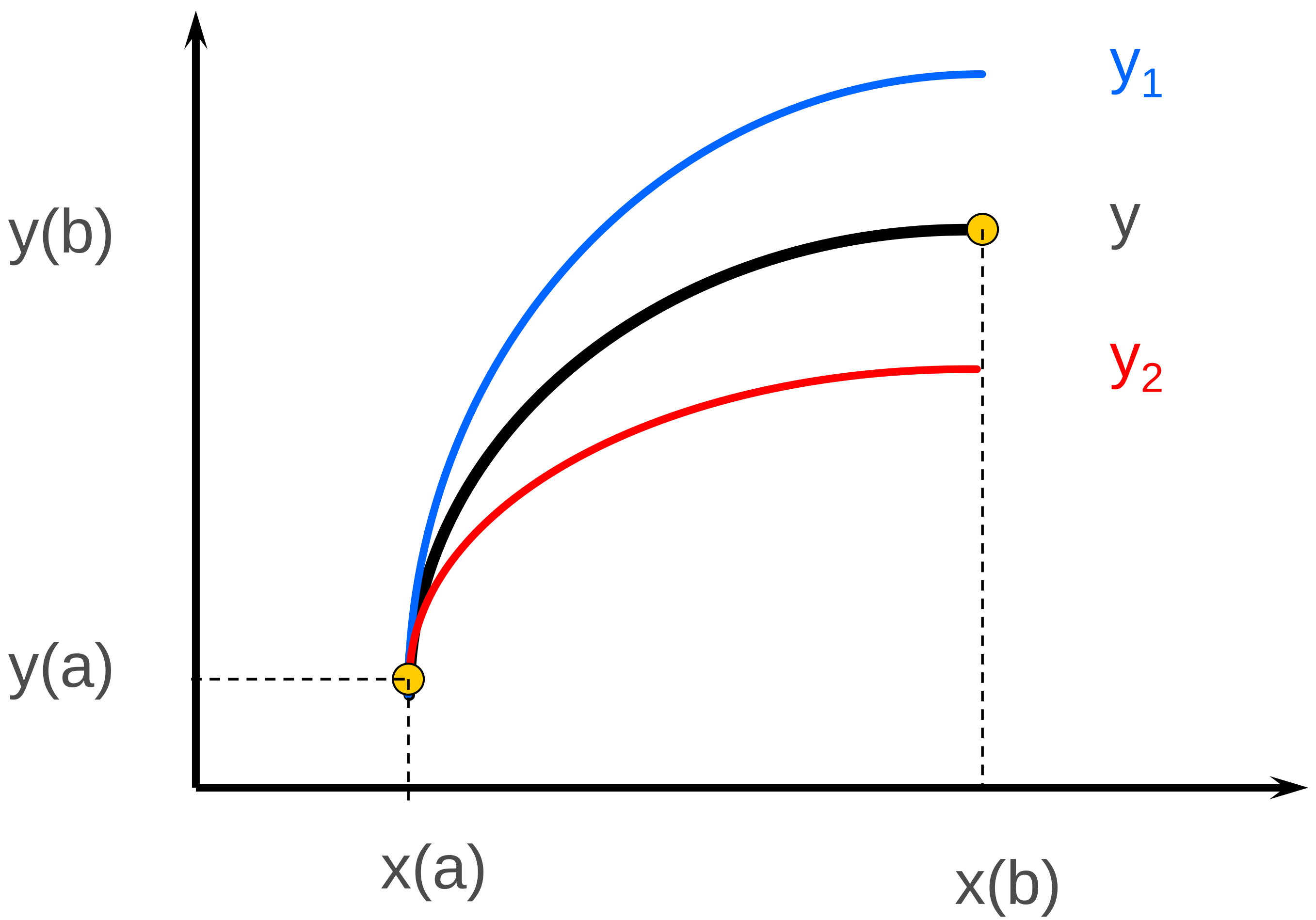
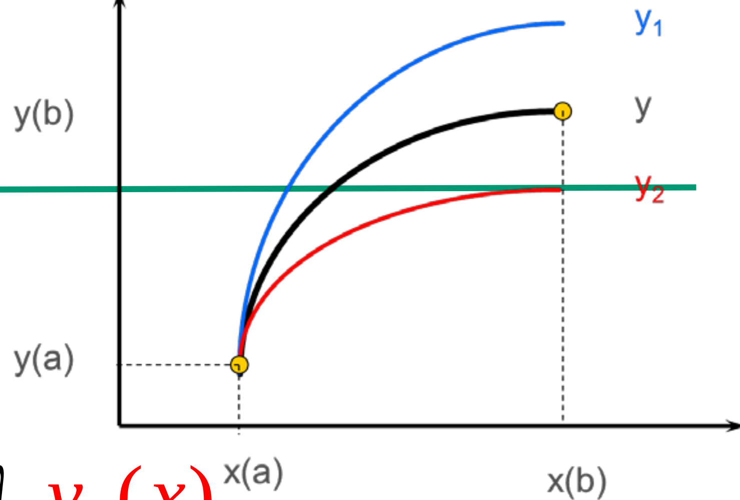
Combinaison



Cas linéaire



$$y(x) = \lambda_1 y_1(x) + \lambda_2 y_2(x)$$

Pour $y=y_a$ $$y(a) = \cancel{y_a} = \lambda_1 \cancel{y_a} + \lambda_2 \cancel{y_a} \qquad 1 = \lambda_1 + \lambda_2$$

Pour $y=y_b$ $$y_b = \lambda_1 y_1(b) + (1-\lambda_1) y_2(b)$$

$$y_b = \lambda_1 y_1(b) + (1-\lambda_1)\, y_2(b)$$

$$\boxed{\lambda_1 = \frac{y(b) - y_2(b)}{y_1(b) - y_2(b)}}$$

EDO-PVF
$$\begin{cases} y'' = \dfrac{2x}{x^2+1}y' - \dfrac{2}{x^2+1}y + x^2 + 1 \\ y(0) = 2,\ \ y(1) = \dfrac{5}{3} \end{cases}$$

$$p(x) = \frac{2x}{x^2+1},\quad q(x) = -\frac{2}{x^2+1},\quad r(x) = x^2+1$$

est tranformée en deux ODE-PVI

$$u'' = \frac{2x}{x^2+1}u' - \frac{2x}{x^2+1}u + x^2 + 1, \quad u(0) = 2, \quad u'(0) = 0$$

$$v'' = \frac{2x}{x^2+1}v' - \frac{2x}{x^2+1}v + x^2 + 1, \quad v(0) = 2, \quad v'(0) = 1$$

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$$z_1 = u, \quad z_2 = u', \quad z_3 = v, \quad z_4 = v'$$

$$u'' = \frac{2x}{x^2+1}u' - \frac{2x}{x^2+1}u + x^2 + 1, \quad u(0) = 2, \quad u'(0) = 0$$

$$v'' = \frac{2x}{x^2+1}v' - \frac{2x}{x^2+1}v + x^2 + 1, \quad v(0) = 2, \quad v'(0) = 1$$

$$\begin{cases} z_1' = z_2 \\ z_2' = \dfrac{2x}{x^2+1}z_2 - \dfrac{2}{x^2+1}z_1 + x^2 + 1 \\ z_3' = z_4 \\ z_4' = \dfrac{2x}{x^2+1}z_4 - \dfrac{2}{x^2+1}z_3 + x^2 + 1 \end{cases} \qquad \begin{cases} z_1(0) = 2 \\ z_2(0) = 0 \\ z_3(0) = 2 \\ z_4(0) = 1 \end{cases}$$

$$z_1 = u, \quad z_2 = u' \quad z_3 = v, \quad z_4 = v'$$

$$\begin{cases} z_1' = z_2 \\ z_2' = \dfrac{2x}{x^2+1} z_2 - \dfrac{2}{x^2+1} z_1 + x^2 + 1 \\ z_3' = z_4 \\ z_4' = \dfrac{2x}{x^2+1} z_4 - \dfrac{2}{x^2+1} z_3 + x^2 + 1 \end{cases} \qquad \begin{cases} z_1(0) = 2 \\ z_2(0) = 0 \\ z_3(0) = 2 \\ z_4(0) = 1 \end{cases}$$

$$y(i) = \lambda_1 z_1(i) + (1 - \lambda_1) z_3(i)$$

$$\lambda_1 = \frac{y_b - z_3(n)}{z_1(n) - z_3(n)}$$

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Cas non linéaire

$$\begin{cases} y'' = f(x, y, y'), & a \le x \le b \\ y(a) = y_a, \quad y(b) = y_b \end{cases}$$

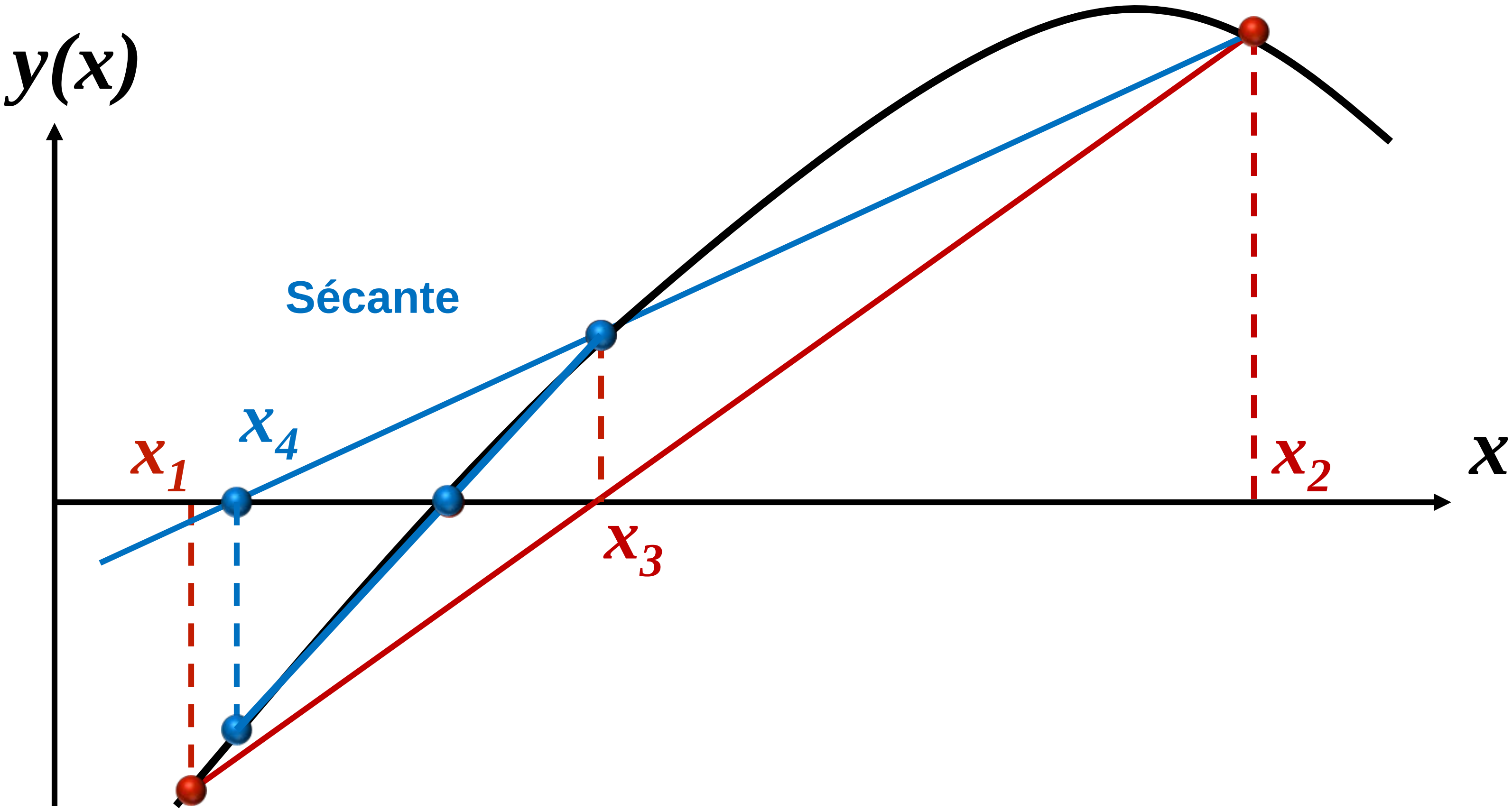
Soit $\begin{array}{c} u'' = f(x, u, u') \\ u(a) = y_a, \boldsymbol{u'(a) = \theta} \end{array}$ avec une pente estimée $\boldsymbol{\theta}$

La différence entre **la valeur calculée** $\boldsymbol{u(b, \theta)}$ et la **condition frontière** $\boldsymbol{y_b}$ est utilisée pour ajuster $u'(a) = \theta$

L'erreur $\boldsymbol{m(\theta) = u(b, \theta) - y_b}$ est une fonction de $\boldsymbol{\theta}$

On applique la méthode de la sécante (ou la_méthode de Newton) pour trouver une valeur de $\boldsymbol{\theta}$ telle que $\boldsymbol{m(\theta) = 0}$

Rappel: La méthode de la sécante



La sécante cherche la solution à gauche et à droite de la racine

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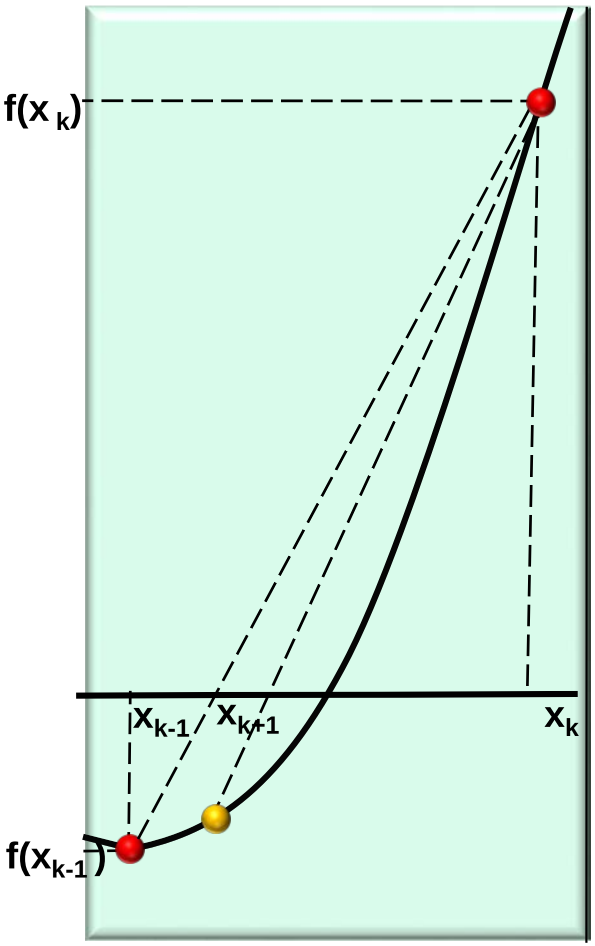
Itérations avec la sécante

$$x_{k+1} = x_k - f(x_k)\frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$$



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Tir avec la Sécante

EDO

Non linéaire

$$\begin{cases} y'' = f(x, y, y'), \quad a \le x \le b \\ y(a) = y_a, \quad h(y(b), y'(b)) = 0 \end{cases}$$

1) Utiliser $u(a) = y_a$, $u'(a) = \theta(1) \rightarrow Erreur = m(1)$

2) Utiliser $u(a) = y_a$, $u'(a) = \theta(2) \rightarrow Erreur = m(2)$

3) Corriger l'angle θ avec la formule pour la sécante

$$\boxed{\theta(i) = \frac{\theta(i-1) - \theta(i-2)}{m(i-1) - m(i-2)} m(i-1)}$$

4) Itérer jusqu'à satisfaire $|\theta(i) - \theta(i-1)| \le tol$

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Tir avec la Sécante

- Tir avec la méthode de la sécante

$$\begin{cases} y'' = -2yy', & 0 \le x \le 1 \\ y(0) = 1, & h(y(1), y'(1)) = y(1) + y'(1) - 0.25 = 0 \end{cases}$$

$$\text{solution exacte} \quad \mathrm{y} = 1/(x+1)$$

- Convertir en deux EDOS aux VI de premier ordre

$$soit \;\; \mathrm{z}_1 = y, \;\; \mathrm{z}_2 = y'$$

$$\begin{cases} z_1' = z_2, & \mathrm{z}_1(0) = 1 \\ z_2' = -2z_1z_2, & \mathrm{z}_2(0) = t \end{cases}$$

- Mise à jour de $z(t)$ avec la sécante

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Méthode aux différences

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Méthode aux différences

- On divise le domaine en plusieurs sous-intervalles

$$x_0 = a,\ x_n = b, \qquad h = \frac{b-a}{n} = x_{i+1} - x_i$$

- On remplace les dérivées par des approximations
- On résout un système algébrique d'équations
- Si nécessaire, on applique des méthodes pour la résolution de problèmes non linéaires

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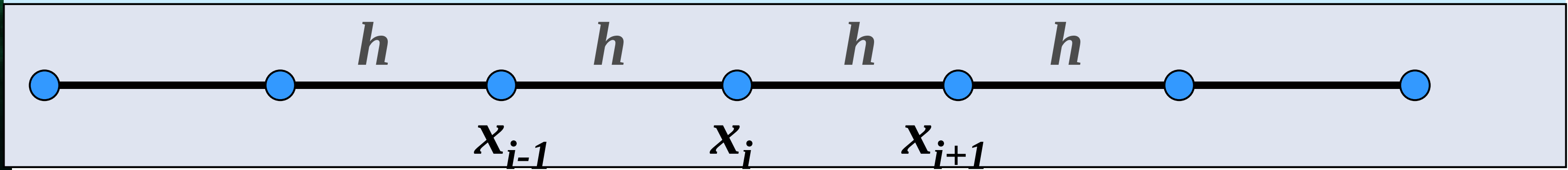
Méthode aux différences

- Formulation générale PVF

$$\begin{cases} y'' = p(x)y' + q(x)y + r(x), \quad a \le x \le b \\ y(a) = \alpha, \quad y(b) = \beta \end{cases}$$

- On remplace les derivées par des approximations aux differénces finies

$$y''(x_i) = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}, \quad y'(x_i) = \frac{y_{i+1} - y_{i-1}}{2h}$$



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Méthode aux différences

$$y'' = p(x)y' + q(x)y + r(x), \qquad a \le x \le b$$

- Différences centrées

$$y'(x_i) = \frac{y_{i+1} - y_{i-1}}{2h}$$

$$y''(x_i) = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = p_i \frac{y_{i+1} - y_{i-1}}{2h} + q_i y_i + r_i$$

- Système tridiagonal

$$\left(1 + \frac{h}{2} p_i\right) y_{i-1} - \left(2 + h^2 q_i\right) y_i + \left(1 - \frac{h}{2} p_i\right) y_{i+1} = h^2 r_i$$

$$\left(1+\frac{h}{2}p_i\right)y_{i-1}-\left(2+h^2q_i\right)\,y_i+\left(1-\frac{h}{2}p_i\right)y_{i+1}=h^2r_i$$

$$\left(1+\frac{h}{2}p_{\mathbf{2}}\right)y_1-(2+h^2q_{\mathbf{2}})\boldsymbol{y_2}+\left(1-\frac{h}{2}p_{\mathbf{2}}\right)y_{\mathbf{3}}=h^2r_{\mathbf{2}}$$ 2

$$\left(1+\frac{h}{2}p_i\right)y_{i-1}-(2+h^2q_i)\boldsymbol{y_i}+\left(1-\frac{h}{2}p_i\right)y_{i+1}=h^2r_i$$ i

$$\left(1+\frac{h}{2}p_{\boldsymbol{n-2}}\right)y_{\boldsymbol{n-3}}-(2+h^2q_{\boldsymbol{n-2}})\boldsymbol{y_{n-2}}+\left(1-\frac{h}{2}p_{\boldsymbol{n-2}}\right)y_{\boldsymbol{n-1}}=h^2r_{\boldsymbol{n-2}}$$ n-2

$$\left(1+\frac{h}{2}p_i\right)y_{i-1}-\left(2+h^2q_i\right)y_i+\left(1-\frac{h}{2}p_i\right)y_{i+1}=h^2r_i$$

❶ ❷ ...

$$\left(1+\frac{h}{2}p_1\right)\boldsymbol{y_0}-(2+h^2q_1)\boldsymbol{y_1}+\left(1-\frac{h}{2}p_1\right)y_2=h^2r_1$$ 1

$$\left(1+\frac{h}{2}p_{n-1}\right)y_{n-2}-(2+h^2q_{n-1})\boldsymbol{y_{n-1}}+\left(1-\frac{h}{2}p_{n-1}\right)y_n=h^2r_{n-1}$$ n-1

$$\left(1+\frac{h}{2}p_i\right)y_{i-1}-\left(2+h^2q_i\right)y_i+\left(1-\frac{h}{2}p_i\right)y_{i+1}=h^2r_i$$

❶ ❷

$$-(2+h^2q_1)y_1+\left(1-\frac{h}{2}p_1\right)y_2=h^2r_1-\left(1+\frac{h}{2}p_1\right)y_0$$

$$\left(1+\frac{h}{2}p_{n-1}\right)y_{n-2}-(2+h^2q_{n-1})y_{n-1}=h^2r_{n-1}-\left(1-\frac{h}{2}p_{n-1}\right)y_n$$

$$\left(1+\frac{h}{2}p_i\right)y_{i-1}-\left(2+h^2q_i\right)y_i+\left(1-\frac{h}{2}p_i\right)y_{i+1}=h^2r_i$$

$$-(2+h^2q_1)y_1+\left(1-p_1\frac{h}{2}\right)y_2 = h^2r_1-\left(1+p_1\frac{h}{2}\right)\alpha,$$

$$\left(1+p_2\frac{h}{2}\right)y_1-(2+h^2q_2)y_2+\left(1-p_2\frac{h}{2}\right)y_3 = h^2r_2,$$

$$\vdots \qquad\qquad \vdots$$

$$\left(1+p_i\frac{h}{2}\right)y_{i-1}-(2+h^2q_i)y_i+\left(1-p_i\frac{h}{2}\right)y_{i+1} = h^2r_i,$$

$$\vdots \qquad\qquad \vdots$$

$$\left(1+p_{1n-2}\frac{h}{2}\right)y_{n-3}-(2+h^2q_{n-2})y_{n-2}+\left(1-p_{n-2}\frac{h}{2}\right)y_{n-1}=h^2r_{n-2},$$

$$\left(1+p_{n-1}\frac{h}{2}\right)y_{n-2}\ -(2+h^2q_{n-1})y_{n-1}=h^2r_{n-1}-\left(1-p_{n-1}\frac{h}{2}\right)\beta,$$

Méthode aux différences

$$\begin{bmatrix} -(2+h^2q_1) & 1-(\mathrm{h}/2)p_1 & 0 & \cdots & 0 \\ 1+(\mathrm{h}/2)p_2 & -(2+h^2q_2) & 1-(\mathrm{h}/2)p_2 & \cdots & 0 \\ 0 & 1+(\mathrm{h}/2)p_3 & -(2+h^2q_3) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -(2+h^2q_{n-1}) \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{n-1} \end{Bmatrix}$$

$$= \begin{Bmatrix} h^2r_1 - {\color{red}(1+h\mathrm{p}_1/2)y_0} \\ h^2r_2 \\ h^2r_3 \\ \vdots \\ h^2r_{n-1} - {\color{red}(1-h\mathrm{p}_{\mathrm{n-1}}/2)y_n} \end{Bmatrix} = \begin{Bmatrix} h^2r_1 - {\color{red}(1+h\mathrm{p}_1/2)\ \alpha} \\ h^2r_2 \\ h^2r_3 \\ \vdots \\ h^2r_{n-1} - {\color{red}(1-h\mathrm{p}_{\mathrm{n-1}}/2)\beta} \end{Bmatrix}$$

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Méthode aux différences

Méthode aux différences

$$
\begin{bmatrix}
-(2+h^2 q_1) & 1-(\mathrm{h}/2)p_1 & 0 & \cdots & 0 \\
1+(\mathrm{h}/2)p_2 & -(2+h^2 q_2) & 1-(\mathrm{h}/2)p_2 & \cdots & 0 \\
0 & 1+(\mathrm{h}/2)p_3 & -(2+h^2 q_3) & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & -(2+h^2 q_{n-1})
\end{bmatrix}
\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{n-1} \end{Bmatrix}
$$

$$
= \begin{Bmatrix}
h^2 r_1 - \color{red}{(\mathbf{1} + \boldsymbol{h}\,\boldsymbol{p_1}/\mathbf{2}))\boldsymbol{y_0}} \\
h^2 r_2 \\
h^2 r_3 \\
\vdots \\
h^2 r_{n-1} - \color{red}{(\mathbf{1} - \boldsymbol{h}\,\boldsymbol{p_{n-1}}/\mathbf{2})\boldsymbol{y_n}}
\end{Bmatrix}
\qquad
= \begin{Bmatrix}
h^2 r_1 - \color{red}{(\mathbf{1} + \boldsymbol{h}\,\boldsymbol{p_1}/\mathbf{2})\,\boldsymbol{\alpha}} \\
h^2 r_2 \\
h^2 r_3 \\
\vdots \\
h^2 r_{n-1} - \color{red}{(\mathbf{1} - \boldsymbol{h}\,\boldsymbol{p_{n-1}}/\mathbf{2})\boldsymbol{\beta}}
\end{Bmatrix}
$$

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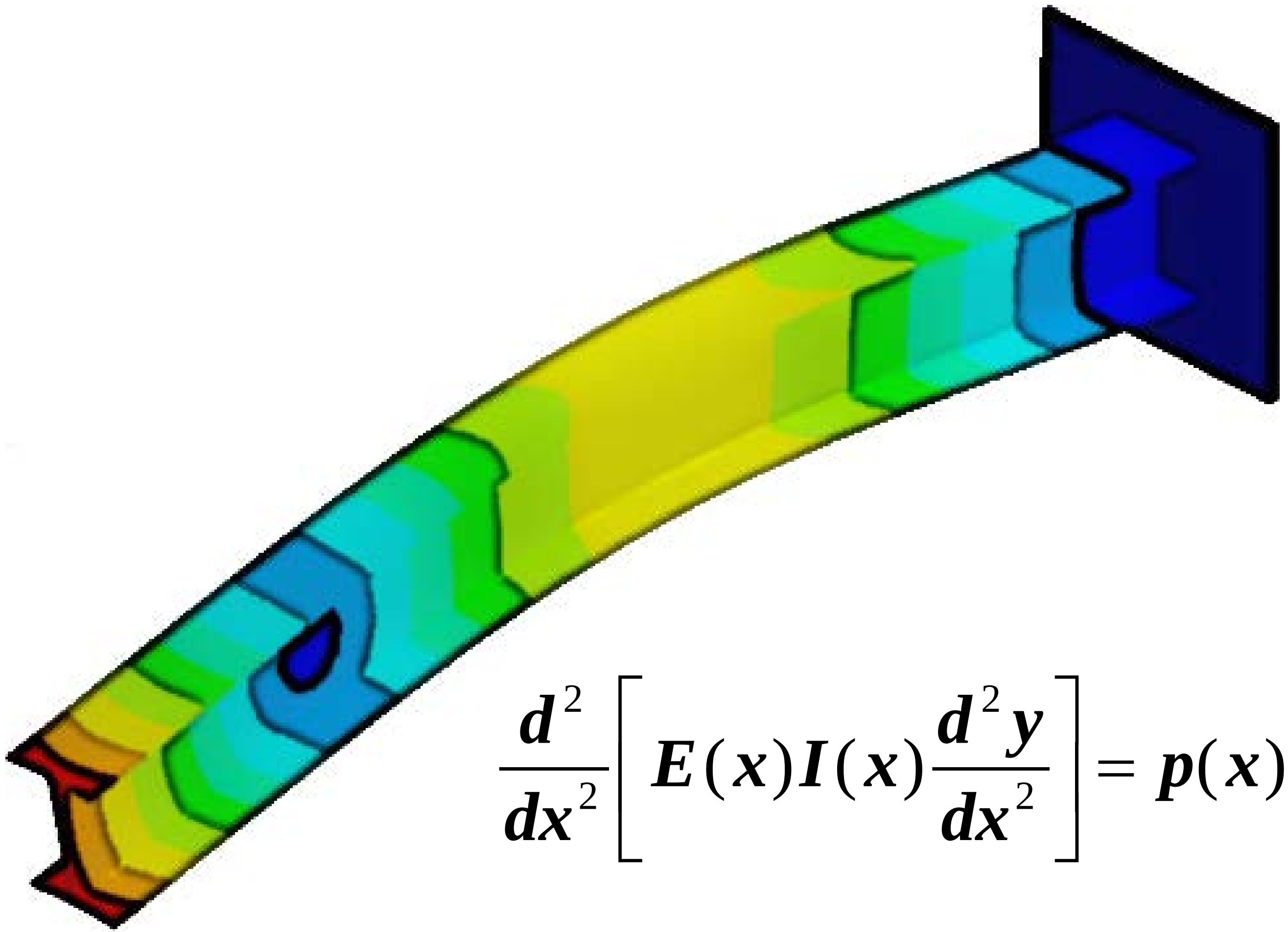
Poutre fléchie en porte-à-faux



$$\frac{d^2}{dx^2}\left[E(x)I(x)\frac{d^2y}{dx^2}\right] = p(x)$$

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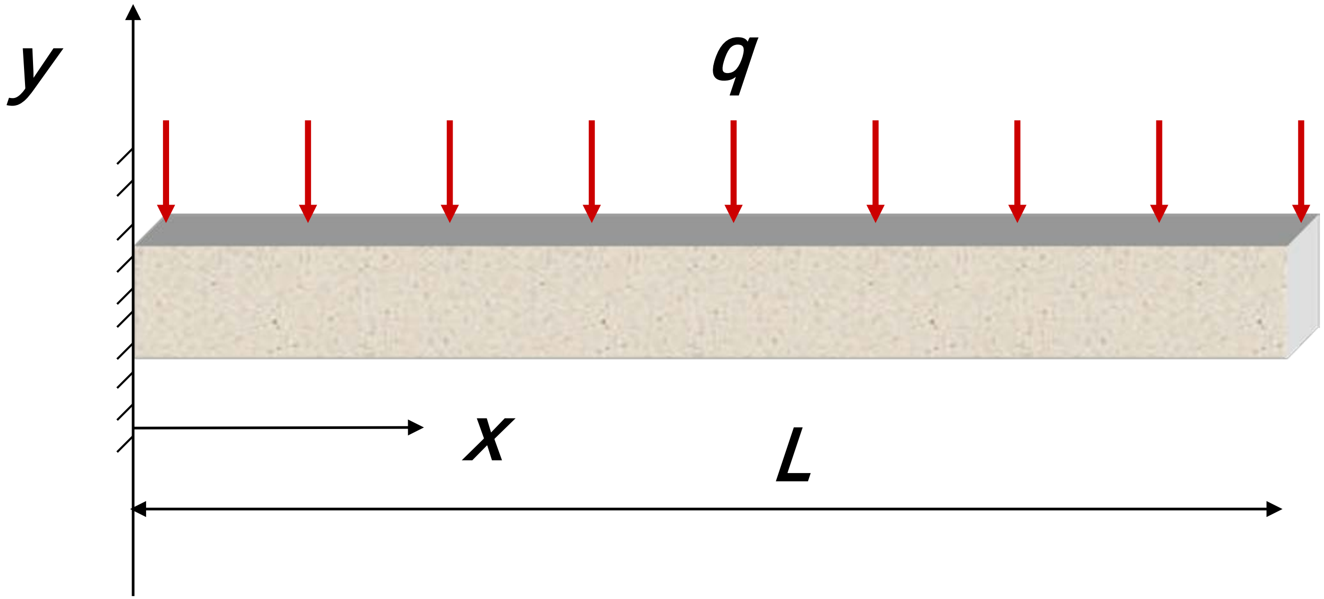
Poutre fléchie en porte-à-faux



$$\frac{d^2}{dx^2}\left[E(x)I(x)\frac{d^2y}{dx^2}\right] = p(x)$$

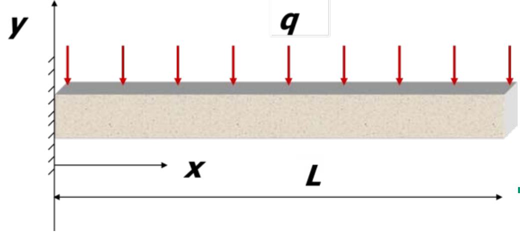
Ordre supérieur

$$\frac{d^2}{dx^2}\left[E(x)I(x)\frac{d^2y(x)}{dx^2}\right] = p(x)$$

$$E(x)I(x)\frac{d^4}{dx^4}\Big[y(x)\Big] + 2\Big[E'(x)I(x) + E(x)I'(x)\Big]\frac{d^3}{dx^3}\Big[y(x)\Big]$$
$$+\Big[E''(x)I(x) + 2E'(x)I'(x) + E(x)I''(x)\Big]\frac{d^2}{dx^2}\Big[y(x)\Big] = p(x)$$

$$E\cdot I(x)\frac{d^4}{dx^4}\Big[y(x)\Big] + 2E\cdot I'(x)\frac{d^3}{dx^3}\Big[y(x)\Big]$$
$$+E\cdot I''(x)\frac{d^2}{dx^2}\Big[y(x)\Big] = p(x).$$

E=cnste.



$$E \cdot I(x)\frac{d^4}{dx^4}\Big[y(x)\Big] + 2E \cdot I'(x)\frac{d^3}{dx^3}\Big[y(x)\Big] + E \cdot I''(x)\frac{d^2}{dx^2}\Big[y(x)\Big] = p(x).$$

y(0)=0 Aucun déplacement (flèche imposée) (CF1)

y'(0)=0 Pente nulle (rotation imposée) (CF2)

y''(L)=0 Moment fléchissant nul (CF3)

y'''(L)=0 Effort tranchant imposé (CF4)

Pour un appui simple, y(L)=0: (CF3) et y''(L)=0: (CF4)

$$\begin{aligned} y_n'' &\approx [y_{n+1} - 2y_n + y_{n-1}]/h^2 \\ y_n''' &\approx [y_{n+2} - 2y_{n+1} + 2y_{n-1} - y_{n-2}]/2h^3 \\ y_n'''' &\approx [y_{n+2} - 4y_{n+1} + 6y_n - 4y_{n-1} + y_{n-2}]/h^4 \end{aligned}$$

n=1, 2,........,N avec x_n=n(L-0)/N

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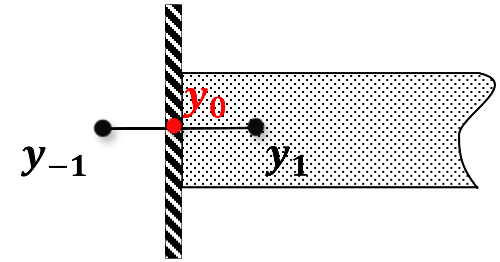
$$y_0 = 0$$

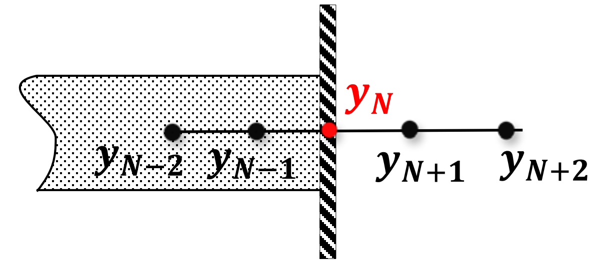


$$y'(0) = y'_0 = \frac{y_1 - y_{-1}}{2h} = 0 \quad \Rightarrow \quad y_{-1} = y_1$$

$$y''_N = \frac{y_{N-1} - 2y_N + y_{N+1}}{h^2} = 0 \quad \Rightarrow \quad y_{N+1} = 2y_N - y_{N-1}$$

$$y'''_N = \frac{-y_{N-2} + 2y_{N-1} - 2y_{N+1} + y_{N+2}}{2h^3} = 0 \quad \Rightarrow$$

$$y_{N+2} = y_{N-2} - 4y_{N-1} + 4y_N$$



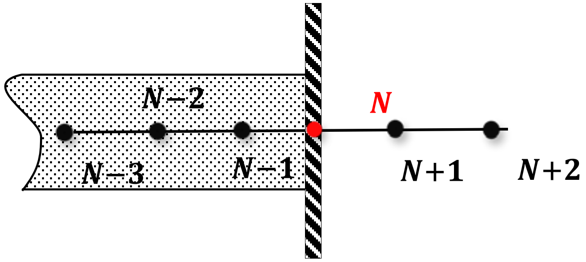
$$I'_n = \frac{I_{n+1} - I_{n-1}}{2h}$$

$$I'_N = \frac{3I_N - 4I_{N-1} + I_{N-2}}{2h}$$

$$I''_n = \frac{I_{n+1} - 2I_n + I_{n-1}}{h^2}$$

$$I''_N = \frac{2I_N - 5I_{N-1} + 4I_{N-2} - I_{N-3}}{h^2}$$

$$E \cdot I(x) \frac{d^4}{dx^4}\Big[y(x)\Big] + 2E \cdot I'(x) \frac{d^3}{dx^3}\Big[y(x)\Big] + E \cdot I''(x) \frac{d^2}{dx^2}\Big[y(x)\Big] = p(x).$$



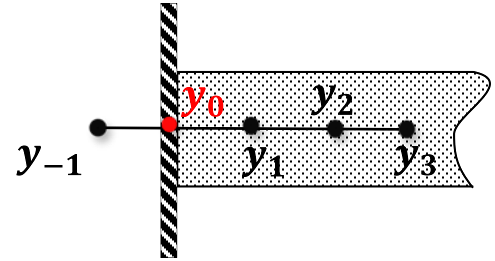
$$E \cdot I_n \left[\frac{y_{n+2} - 4y_{n+1} + 6y_n - 4y_{n-1} + y_{n-2}}{h^4} \right] + \\ +2E \cdot I'_n \left[\frac{y_{n+2} - 2y_{n+1} + 2y_{n-1} - y_{n-2}}{2h^3} \right] + \\ +E \cdot I''_n \left[\frac{y_{n+1} - 2y_n + y_{n-1}}{h^2} \right] = p_n$$

$$y_n'' \approx [y_{n+1} - 2y_n + y_{n-1}]/h^2$$
$$y_n''' \approx [y_{n+2} - 2y_{n+1} + 2y_{n-1} - y_{n-2}]/2h^3$$
$$y_n'''' \approx [y_{n+2} - 4y_{n+1} + 6y_n - 4y_{n-1} + y_{n-2}]/h^4$$

$$E \cdot I(x)\frac{d^4}{dx^4}\left[y(x)\right] + 2E \cdot I'(x)\frac{d^3}{dx^3}\left[y(x)\right] + E \cdot I''(x)\frac{d^2}{dx^2}\left[y(x)\right] = p(x).$$

$n = 1$

$$E \cdot I_1\left[\frac{y_3 - 4y_2 + 6y_1 - 4\mathbf{y_0} + \mathbf{y_{-1}}}{h^4}\right] + 2E \cdot I_1'\left[\frac{y_3 - 2y_2 + 2\mathbf{y_0} - \mathbf{y_{-1}}}{2h^3}\right] + E \cdot I_1''\left[\frac{y_2 - 2y_1 + \mathbf{y_0}}{h^2}\right] = p_1$$



$$E \cdot I_1\left[\frac{y_3 - 4y_2 + 7y_1}{h^4}\right] + 2E \cdot I_1'\left[\frac{y_3 - 2y_2 - y_1}{2h^3}\right] + E \cdot I_1''\left[\frac{y_2 - 2y_1}{h^2}\right] = p_1$$

$\mathbf{y_0 = 0}$
$\mathbf{y_{-1} = y_1}$

$$E \cdot I_n\left[\frac{y_{n+2} - 4y_{n+1} + 6y_n - 4y_{n-1} + y_{n-2}}{h^4}\right] + 2E \cdot I_n'\left[\frac{y_{n+2} - 2y_{n+1} + 2y_{n-1} - y_{n-2}}{2h^3}\right] + E \cdot I_n''\left[\frac{y_{n+1} - 2y_n + y_{n-1}}{h^2}\right] = p_n$$

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$n = N$

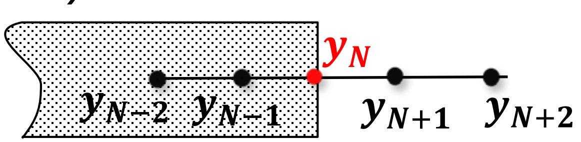
$$E \cdot I_N \left[\frac{y_{N+2} - 4y_{N+1} + 6y_N - 4y_{N-1} + y_{N-2}}{h^4} \right] +$$

$$+2E \cdot I'_N \left[\frac{y_{N+2} - 2y_{N+1} + 2y_{N-1} - y_{N-2}}{2h^3} \right] +$$

$$+E \cdot I''_N \left[\frac{y_{N+1} - 2y_N + y_{N-1}}{h^2} \right] = p_N$$

y''(0)=0 Moment fléchissant nul (CF3)

y'''(0)=0 Cisaillement nul (CF4)



(CF3) $y_{N+1} = 2y_N - y_{N-1}$

(CF4) $y_{N+2} = y_{N-2} - 4y_{N-1} + 4y_N$

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$n = N$

$$E \cdot I_N \left[\frac{\mathbf{y_{N-2}} - 4\mathbf{y_{N-1}} + 4\mathbf{y_N} - 4(2\mathbf{y_N} - \mathbf{y_{N-1}}) + 6y_N - 4y_{N-1} + y_{N-2}}{h^4} \right] +$$

$$+2E \cdot I'_N \left[\frac{\mathbf{y_{N-2}} - 4\mathbf{y_{N-1}} + 4\mathbf{y_N} - 2(2\mathbf{y_N} - \mathbf{y_{N-1}}) + 2y_{N-1} - y_{N-2}}{2h^3} \right] +$$

$$+E \cdot I''_N \left[\frac{(2\mathbf{y_N} - \mathbf{y_{N-1}}) - 2y_N + y_{N-1}}{h^2} \right] = p_N$$

$$E \cdot I_N \left[\frac{2y_{N-2} - 4y_{N-1} + 2y_N}{h^4} \right] + 2E \cdot I'_N \left[\frac{\mathbf{0}}{2h^3} \right] + E \cdot I''_N \left[\frac{\mathbf{0}}{h^2} \right] = p_N$$

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$n = (N-1)$

$$E \cdot I_{N-1}\left[\frac{\mathbf{y_{N+1}} - 4y_N + 6y_{N-1} - 4y_{N-2} + y_{N-3}}{h^4}\right] +$$

$$+2E \cdot I'_{N-1}\left[\frac{\mathbf{y_{N+1}} - 2y_N + 2y_{N-2} - y_{N-3}}{2h^3}\right] +$$

$$+E \cdot I''_{N-1}\left[\frac{y_N - 2y_{N-1} + y_{N-2}}{h^2}\right] = p_{N-1}$$

$$y_{N+1} = 2y_N - y_{N-1}$$

$$E \cdot I_{N-1}\left[\frac{\mathbf{2y_N} - \mathbf{y_{N-1}} - 4y_N + 6y_{N-1} - 4y_{N-2} + y_{N-3}}{h^4}\right] +$$

$$+2E \cdot I'_{N-1}\left[\frac{\mathbf{2y_N} - \mathbf{y_{N-1}} - 2y_N + 2y_{N-2} - y_{N-3}}{2h^3}\right] +$$

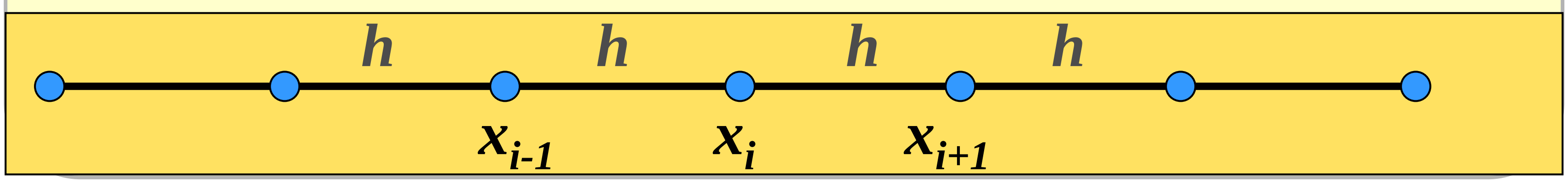
$$+E \cdot I''_{N-1}\left[\frac{y_N - 2y_{N-1} + y_{N-2}}{h^2}\right] = p_{N-1}$$

PVF non linéaire

$$\begin{cases} y'' = f(x, y, y'), & a \le x \le b \\ y(a) = \alpha, \; y(b) = \beta \end{cases}$$

- On évalue par des formules aux différences

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - f_i = 0$$



PVF non linéaire

$$y'' = f(x, y, y')$$

$$y(a) = \beta, \quad y(b) = \beta$$

$$y_i' = \frac{y_{i+1} - y_{i-1}}{2h}$$

$$y_i'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

i

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PVF non linéaire

$$y'' = f(x, y, y')$$

$$\Downarrow$$

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = f\left(x_i, y_i, \frac{y_{i+1} - y_{i-1}}{2h}\right)$$

$$\Downarrow$$

$$\boldsymbol{F_i} = -\left(\frac{y_{i+\mathbf{1}} - 2y_i + y_{i-\mathbf{1}}}{h^2}\right) + f\left(x_i, y_i, \frac{y_{i+1} - y_{i-1}}{2h}\right) = 0$$

$$y_0 = \alpha \qquad y_{N+1} = \beta$$

$$i = 1,2,3, \dots N$$

Rappel:Méthode de Newton

$$F(x) = F(x_k) + \frac{dF(x_k)}{dx}(x - x_k) \Leftarrow F(x) = 0$$

$$F(x_k) + \frac{dF(x_k)}{dx}(x - x_k) = 0$$

$$J(x_k) = \frac{dF(x_k)}{dx}$$

$$J(x_k)(x - x_k) = -F(x_k) \Rightarrow J(x_k)\overbrace{(x_{k+1} - x_k)}^{\delta_k} = -F(x_k)$$

Formule itérative: x_k a remplacée x

Mise à jour

$$\delta_{\boldsymbol{k}} = (\boldsymbol{x}_{\boldsymbol{k+1}} - \boldsymbol{x}_{\boldsymbol{k}})$$

$$J(x_k)\delta_{\boldsymbol{k}} = -\boldsymbol{F}(\boldsymbol{x}_{\boldsymbol{k}})$$

$$\boldsymbol{x}_{\boldsymbol{k+1}} = \boldsymbol{x}_{\boldsymbol{k}} + \delta_{\boldsymbol{k}}$$

$$k = 1, \ldots$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta \qquad \Leftarrow \qquad \frac{g}{L} = 1$$

$$\frac{d^2\theta}{dt^2} = -\sin\theta$$

$$\boxed{\frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{h^2} + \sin(\theta_i) = 0} \quad \Rightarrow \quad \mathrm{F_i}(\theta_{i-1}, \theta_i, \theta_{i+1}) = 0$$

$$\boldsymbol{F}(\boldsymbol{\theta}) = \begin{cases} F_1(\theta_0, \theta_1, \theta_2) \\ F_2(\theta_1, \theta_2, \theta_3) \\ . \\ . \\ F_{n-1}(\theta_{n-2}, \theta_{n-1}, \theta_n) \\ F_n(\theta_{n-1}, \theta_n, \theta_{n+1}) \end{cases} = 0$$

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$$J_{ij} = -\frac{\partial}{\partial \theta_j} F_i(\theta_j) \qquad j = i-1, i, i+1$$

$$\frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{h^2} + \sin(\theta_i) = 0$$

$$J_{ij} = \begin{cases} 1/h^2 & j = i-1, i+1 \\ -2/h^2 + \cos(\theta_i) & j = i \\ 0 \quad \rightarrow & ailleurs \end{cases}$$

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$$J_{ij} = \frac{1}{h^2}\begin{bmatrix} (-2+h^2\cos(\theta_1)) & 1 & & & \\ 1 & (-2+h^2\cos(\theta_2)) & 1 & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & 1 & (-2+h^2\cos(\theta_n)) \end{bmatrix}$$

Problème courant

$$y'' = f(x, y, y')$$

$$\Downarrow$$

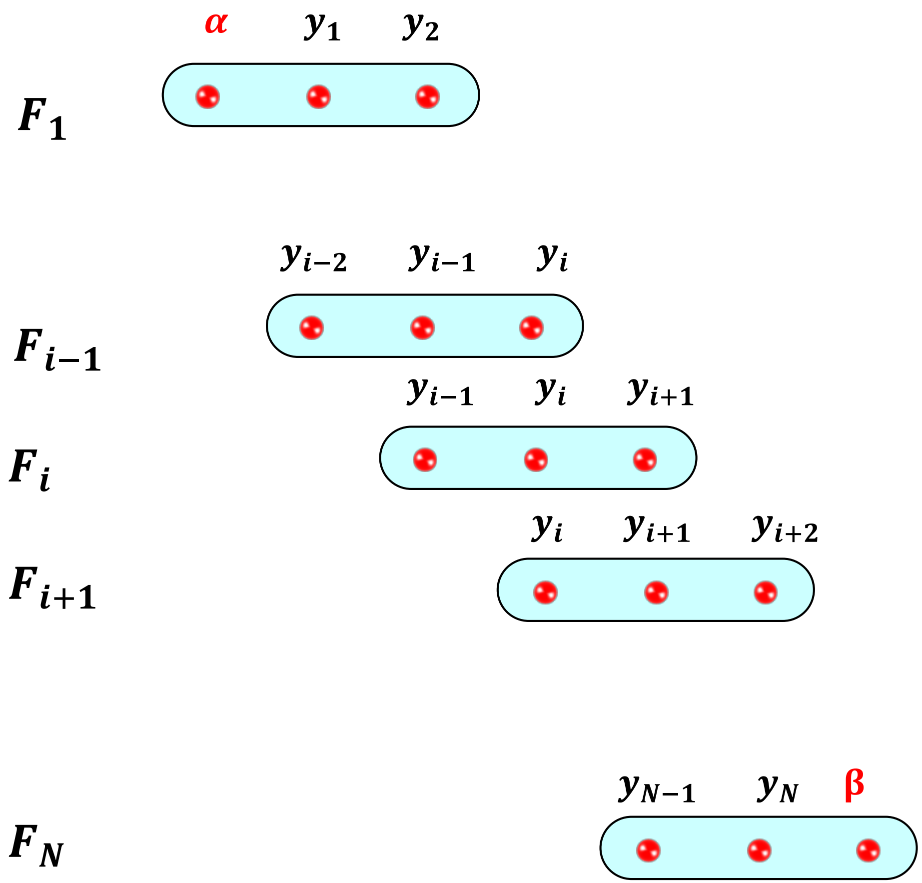
$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = f\left(x_i, y_i, \frac{y_{i+1} - y_{i-1}}{2h}\right)$$

$$\Downarrow$$

$$\boxed{\boldsymbol{F_i} = -\left(\frac{y_{i+\mathbf{1}} - 2y_i + y_{i-\mathbf{1}}}{h^2}\right) + f\left(x_i, y_i, \frac{y_{i+1} - y_{i-1}}{2h}\right) = 0}$$

$$\boxed{y_0 = \alpha \qquad y_{N+1} = \beta}$$

$$\boxed{i = 1,2,3, \ldots N}$$


α y_1 y_2
F_1
y_{i-2} y_{i-1} y_i
F_{i-1}
y_{i-1} y_i y_{i+1}
F_i
y_i y_{i+1} y_{i+2}
F_{i+1}
y_{N-1} y_N β
F_N

Les équations

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = f\left(x_i, y_i, \frac{y_{i+1} - y_{i-1}}{2h}\right)$$

$$-y_2 + 2y_1 - \alpha + h^2 f\left(x_1, y_1, \frac{y_2 - \alpha}{2h}\right) = 0 \qquad 1$$

$$-y_3 + 2y_2 - y_1 + h^2 f\left(x_2, y_2, \frac{y_3 - y_1}{2h}\right) = 0 \qquad 2$$

$$\vdots$$

$$-y_N + 2y_{N\text{-}1} - y_{N\text{-}2} + h^2 f\left(x_{N\text{-}1}, y_{N\text{-}1}, \frac{y_N - y_{N\text{-}2}}{2h}\right) = 0 \qquad \text{n-1}$$

$$-y_{N\text{-}1} + 2y_N - \beta + h^2 f\left(x_N, y_N, \frac{\beta - y_{N-1}}{2h}\right) = 0 \qquad \text{n}$$

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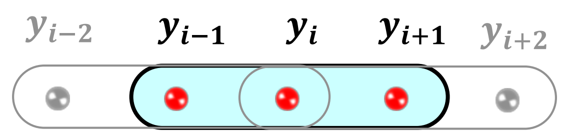
La matrice Jacobienne

$$\boldsymbol{F_i} = -\left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}\right) + f\left(x_i, y_i, \frac{y_{i+1} - y_{i-1}}{2h}\right) = 0$$

$\boldsymbol{F_i}$



$$\ldots\ldots \cancel{\frac{\partial F_i}{\partial y_{i-2}}} \quad \frac{\partial \boldsymbol{F_i}}{\partial \boldsymbol{y_{i-1}}}, \frac{\partial \boldsymbol{F_i}}{\partial \boldsymbol{y_i}}, \frac{\partial \boldsymbol{F_i}}{\partial \boldsymbol{y_{i+1}}}, \cancel{\frac{\partial F_i}{\partial y_{i+2}}} = 0 \ldots\ldots$$

La matrice Jacobienne

$$\boldsymbol{F_i} = -\left(\frac{y_{i+\mathbf{1}} - 2y_i + y_{i-\mathbf{1}}}{h^2}\right) + f\left(x_i, y_i, \frac{y_{i+1} - y_{i-1}}{2h}\right) = 0$$

Tridiagonal

$$J\left(y_1, y_2, \ldots, y_N\right) = \begin{cases} -1 + \dfrac{h}{2}\dfrac{\partial f}{\partial y'}\left(x_i, y_i, \dfrac{y_{i+1} - y_{i-1}}{2h}\right) & i = j-1,\ j = 2, \ldots, N \\ 2 + h^2 \dfrac{\partial f}{\partial y}\left(x_i, y_i, \dfrac{y_{i+1} - y_{i-1}}{2h}\right) & i = j,\ j = 1, \ldots, N \\ -1 - \dfrac{h}{2}\dfrac{\partial f}{\partial y'}\left(x_i, y_i, \dfrac{y_{i+1} - y_{i-1}}{2h}\right) & i = j+1,\ j = 1, \ldots, N-1 \end{cases}$$

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Calcul des δ (variations)

$$J\left(y_1, y_2, \ldots, y_N\right)\left(\delta_1, \delta_2, \ldots, \delta_N\right)^T =$$

$$\begin{pmatrix}
-2y_1 + y_2 - \alpha + h^2 f\left(x_1, y_1, \dfrac{y_2 - \alpha}{2h}\right) \\
-y_1 + 2y_2 - y_3 + h^2 f\left(x_2, y_2, \dfrac{y_3 - y_1}{2h}\right) \\
\vdots \\
-y_{N\text{-}2} + 2y_{N\text{-}1} - y_N + h^2 f\left(x_{N\text{-}1}, y_{N\text{-}1}, \dfrac{y_N - y_{N\text{-}2}}{2h}\right) \\
-y_{N\text{-}1} + 2y_N - \beta + h^2 f\left(x_N, y_N, \dfrac{\beta - y_{N-1}}{2h}\right)
\end{pmatrix}$$

Mise à jour des y

$$J(\boldsymbol{y})\boldsymbol{\delta} = -\boldsymbol{F}(\boldsymbol{y}) \Rightarrow \boldsymbol{\delta}$$

$$\left(y_1^{k+1}, y_2^{k+1}, \dots, y_N^{k+1}\right)^T =$$

$$\left(y_1^{k}, y_2^{k}, \dots, y_N^{k}\right)^T + (\delta_1, \delta_2, \dots, \delta_N)^T$$

Méthode de Newton

$$F(x) = F(x_k) + \frac{dF(x_k)}{dx}(x - x_k) \Leftarrow F(x) = 0$$

$$F(x_k) + \frac{dF(x_k)}{dx}(x - x_k) = 0 \qquad J(x_k) = \frac{dF(x_k)}{dx}$$

$$J(x_k)(x - x_k) = -F(x_k) \Rightarrow J(x_k)(x_{k+1} - x_k) = -F(x_k)$$

Formule itérative: x_k a remplacée x

Mais, le calcul du Jacobien $J(x_k)$ n'est pas toujours facile en pratique

Méthode de Broyden

La méthode de Broyden approxime la matrice Jacobienne de manière récurrente.

$$\boldsymbol{F}(\boldsymbol{x}_{k+1`}) = \boldsymbol{F}(\boldsymbol{x}_k) + \boldsymbol{J}(\boldsymbol{x}_k)(\boldsymbol{x}_{k+1} - \boldsymbol{x}_k) \qquad \text{Newton}$$

$$\boldsymbol{F}(\boldsymbol{x}_{k+1`}) = \boldsymbol{F}(\boldsymbol{x}_k) + \boldsymbol{B}(\boldsymbol{x}_k)(\boldsymbol{x}_{k+1} - \boldsymbol{x}_k) \qquad \text{Broyden}$$

$$\boldsymbol{F}(\boldsymbol{x}_{k+1`}) - \boldsymbol{F}(\boldsymbol{x}_k) = \boldsymbol{B}(\boldsymbol{x}_k)(\boldsymbol{x}_{k+1} - \boldsymbol{x}_k)$$

Formule itérative de la sécante à n dimensions

En une dimension $\boldsymbol{B(x_k)}$ correspond à une droite passant par $\boldsymbol{F(x_k)}$ et $\boldsymbol{F(x_{k+1})}$. En $\boldsymbol{n}$ dimensions il y a une infinité de possibilités pour $B(x_k)$ La méthode de Broyden correspond à un choix particulier de $B(x_k)$

Algorithme de Broyden :I

$F(x_k)$: *fonction vectorielle*

$B_1 = J(x_1)$: *la matrice de Broyden = Jacobienne*

$k = 1, \ldots$

$B_k \delta_k = -F(x_k)$: *calcul de* δ_k

$x_{k+1} = x_k + \delta_k$: *mise à jour*

$y_k = F(x_{k+1}) - F(x_k)$: *changement de F*

$$B_{k+1} = B_k + (y_k - B_k \delta_k) \frac{\delta_k^T}{\delta_k^T \delta_k} : \textit{mise à jour}$$

Algorithme de Broyden :II

$$F(x_k): \text{ fonction vectorielle}$$

$$B_1^{-1} = J^{-1}(x_1): \text{ la matrice de Broyden} = \text{Jacobienne}$$

$$k = 1, \ldots$$

$$\delta_k = -B_k^{-1} F(x_k): \text{ calcul de } \delta_k$$

$$x_{k+1} = x_k + \delta_k : \text{ mise à jour}$$

$$y_k = F(x_{k+1}) - F(x_k): \text{ changement de } F$$

$$B_{k+1}^{-1} = B_{-1}^{k} + (\delta_k - B_k^{-1} y_k) \frac{\delta_k^T B_k^{-1}}{\delta_k^T B_k^{-1} \delta_k} : \text{ mise à jour}$$

Marcelo Reggio

$$\mathbf{g}(x,y,z) = \begin{bmatrix} x^2+y^2+z^2-3 \\ x^2+y^2-z-1 \\ x+y+z-3 \end{bmatrix}$$

$$\mathbf{x}^{(0)} = (x^{(0)}, y^{(0)}, z^{(0)}) = (1,0,1).$$

$$J_{\mathbf{g}}(x,y,z) = \begin{bmatrix} 2x & 2y & 2z \\ 2x & 2y & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} \mathbf{2} & \mathbf{0} & \mathbf{2} \\ \mathbf{2} & \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix}$$

$$J_{\mathbf{g}}(\mathbf{x}^{(0)})(\mathbf{x}^{(1)} - \mathbf{x}^{(0)}) = -\mathbf{g}(\mathbf{x}^{(0)})$$

$$J_{\mathbf{g}}(x,y,z) = \begin{bmatrix} 2x & 2y & 2z \\ 2x & 2y & -1 \\ 1 & 1 & 1 \end{bmatrix} .$$

$$\mathbf{g}(x,y,z) = \begin{bmatrix} x^2+y^2+z^2-3 \\ x^2+y^2-z-1 \\ x+y+z-3 \end{bmatrix}$$

$$J_{\mathbf{g}}(\mathbf{x}^{(0)})(\mathbf{x}^{(1)}-\mathbf{x}^{(0)}) = -\mathbf{g}(\mathbf{x}^{(0)})$$

$$B_0\, \mathbf{d}^{(0)} = -\mathbf{g}(\mathbf{x}^{(0)})$$

$$B_0 = J_{\mathbf{g}}(\mathbf{x}^{(0)}) = \begin{bmatrix} 2x^{(0)} & 2y^{(0)} & 2z^{(0)} \\ 2x^{(0)} & 2y^{(0)} & -1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \mathbf{d}^{(0)} = \begin{bmatrix} x^{(1)}-x^{(0)} \\ y^{(1)}-y^{(0)} \\ z^{(1)}-z^{(0)} \end{bmatrix}$$

$$\mathbf{g}(\mathbf{x}^{(0)}) = \begin{bmatrix} (x^{(0)})^2+(y^{(0)})^2+(z^{(0)})^2-3 \\ (x^{(0)})^2+(y^{(0)})^2-z^{(0)}-1 \\ x^{(0)}+y^{(0)}+z^{(0)}-3 \end{bmatrix}$$

$$\boldsymbol{g}(\boldsymbol{x}^{\mathbf{0}}) = \begin{bmatrix} \mathbf{-1} \\ \mathbf{-1} \\ \mathbf{-1} \end{bmatrix}$$

$$(x^{(0)}, y^{(0)}, z^{(0)}) = (1,0,1)$$

$$J_{\mathbf{g}}(x,y,z) = \begin{bmatrix} 2x & 2y & 2z \\ 2x & 2y & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

$$B_{k+1} = B_k + \frac{(yk - B_k s_k) s_k^T}{s_k^T s_k} = B_k + \frac{F(x_{k+1}) s_k^T}{s_k^T s_k}$$

$$\begin{bmatrix} 2 & 0 & 2 \\ 2 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x^{(1)} - 1 \\ y^{(1)} \\ z^{(1)} - 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$(x^{(0)}, y^{(0)}, z^{(0)}) = (1, 0, 1)$$

$$\mathbf{d}^{(0)} = (\tfrac{1}{2}, \tfrac{1}{2}, 0)$$

$\boldsymbol{B_0}$ $\boldsymbol{d^{(0)}}$ $\boldsymbol{-g(x^0)}$

$$\mathbf{x}^{(1)} = (\tfrac{3}{2}, \tfrac{1}{2}, 1)$$

$$J_{\mathbf{g}}(x,y,z) = \begin{bmatrix} 2x & 2y & 2z \\ 2x & 2y & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

$$B_{k+1} = B_k + \frac{(yk - B_k s_k) s_k^T}{s_k^T s_k} = B_k + \frac{F(x_{k+1}) s_k^T}{s_k^T s_k}$$

$$\mathbf{g}(x,y,z) = \begin{bmatrix} x^2+y^2+z^2-3 \\ x^2+y^2-z-1 \\ x+y+z-3 \end{bmatrix}$$

$\Leftarrow$ $\mathbf{x}^{(1)} = (\frac{3}{2}, \frac{1}{2}, 1)$

$$g(x^1) = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}$$

$$d(x^0)^T = [1/2 \quad 1/2 \quad 0]$$

$$d(x^0)^T \cdot d(x^0) \quad = [1/2 \quad 1/2 \quad 0] \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix} \quad = 1/2$$

$$J_{\mathbf{g}}(x,y,z) = \begin{bmatrix} 2x & 2y & 2z \\ 2x & 2y & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

$$B_{k+1} = B_k + \frac{(yk - B_k s_k)s_k^T}{s_k^T s_k} = B_k + \frac{F(x_{k+1})s_k^T}{s_k^T s_k}$$

$$\boldsymbol{g}(\boldsymbol{x}^1)\boldsymbol{d}(\boldsymbol{x}^0)^T = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 1/4 & 1/4 & 0 \\ 1/4 & 1/4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = B_0 + \frac{1}{\mathbf{d}^{(0)} \cdot \mathbf{d}^{(0)}} \mathbf{g}(\mathbf{x}^{(1)}) \otimes \mathbf{d}^{(0)}$$

$$\begin{bmatrix} 2 & 0 & 2 \\ 2 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{1}{1/2} \begin{bmatrix} 1/4 & 1/4 & 0 \\ 1/4 & 1/4 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 5/2 & 1/2 & 2 \\ 5/2 & 1/2 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{x}^{(1)} = \left(\tfrac{3}{2}, \tfrac{1}{2}, 1\right) \Rightarrow \mathbf{g}(x,y,z) = \begin{bmatrix} x^2+y^2+z^2-3 \\ x^2+y^2-z-1 \\ x+y+z-3 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 5/2 & 1/2 & 2 \\ 5/2 & 1/2 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\boldsymbol{g}(\boldsymbol{x}^1) = \begin{bmatrix} \mathbf{1/2} \\ \mathbf{1/2} \\ \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} 5/2 & 1/2 & 2 \\ 5/2 & 1/2 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x^{(2)} - \frac{3}{2} \\ y^{(2)} - \frac{1}{2} \\ z^{(2)} - 1 \end{bmatrix} = \left[\begin{bmatrix} \mathbf{1/2} \\ \mathbf{1/2} \\ \mathbf{0} \end{bmatrix}\right]$$

$$B_1 d^1 = -g(x^1)$$

$$\mathbf{x}^{(2)} = \left(\tfrac{5}{4}, \tfrac{3}{4}, 1\right)$$

FIN