GCH8108 **Nom:** ______________________

Homework 2 - Lattice Boltzmann Method

Winter 2020

This is the second evaluated homework. It will have to be handed in by February 19th. The report must be submitted in the form of a Powerpoint and you must hand-in your code. This code will be evaluated.

1 Écoulement de Poiseuille

Using the given template, write a software that uses the Lattice Boltzmann method (LBM) to solve a Poiseuille flow in a 2D channel.

Take into account the following considerations

1. Consider the flow in the X axis in a domain of width 1 starting from -0.5 and going to 0.5. Consider the flow oriented in the Y axis. Choose the size of the channel (in the $\boldsymbol{e}_y$ direction) as you see fit.
2. The term driving the Poiseuille flow can be either a pressure differential or a source term. In the present case, I suggest you use a source term (gravity) and periodic boundary conditions in the ($\boldsymbol{e}_y$) direction.
3. Use the $D2Q9$ stencil.
4. Use second order boundary conditions of type *halfway bounceback*.

1. (Slide 1) Demonstrate (briefly) where the analytical solution to the Poiseuille flow comes from. This analytical solution is already programmed in the software.
2. (Slide 2) Show the transient evolution of the Poiseuille flow in an animation.
3. (Slide 3) Compare the analytical solution with the steady-state numerical solution.
4. (Slide 4) Show that your code recovers second order convergence.

Suggestions:

- The first step is how the time step is calculated from the value of $\bar{\tau}$, ρ_0 et μ. Then we can calculate the celerity $\boldsymbol{\xi_i}$ of the lattice. Then the weights of the lattice can be established. The weights are constant for a given lattice, they can therefore be hard-coded.
- Afterwards, we must establish the initial values of $f_i \ \ \forall \ \ [0, 8]$, which are initially at equilibrium. This can be calculated from the initial density and weights if you assume that the initial velocity is zero.
- The LBM algorithm can be divided in a number of elements:
  1. The propagation step

2. The collision step
3. The imposition of the boundary conditions (bounce back and periodicity)
4. The calculation of the macroscopic properties (rho and $\boldsymbol{u}$)
5. Continue to the next time steps