Quantitative Analysis of Physical Data: Assignment 1

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Abstract

Two landmark discoveries in the history of science have been discussed: the structure of the atom and the detection of gravitational waves. These two experiments take place almost a century apart. The development of modern computers and advanced engineering have made it possible to make detectors with far higher accuracy and precision than before. This, however, is accompanied by the demand to provide an exhaustive analysis to support results.

1 Introduction

The experiments conducted between 1908 and 1913 by Geiger and Marsden under the guidance of Ernest Rutherford were of historical importance, as they helped the scientific community discover that a positively charged, massive nucleus is present at the centre of every atom [1]. It was preceded by the well known Rutherford gold foil experiment wherein it was discovered that α-particles are scattered by both air and solid matter (gold foil). The experiments described in [2] were performed in order to verify if the α-particles could be scattered at larger angles. These set of experiments formed the basis for atomic structure that "the atom consisted of mostly empty spaces, with all its positive charges concentrated at the centre in a tiny volume, surrounded by a cloud of electrons"[1].

The scientists have detected gravitational waves, the ripples in the fabric of space-time for the first time on September 14, 2015 by both of the twin Laser Interferometer Gravitational-wave Observatory (LIGO) detectors, located in Livingston, Louisiana, and Hanford, Washington, USA. These are the tiny disturbances the waves make to space and time as they pass through the earth, confirming a major prediction of Albert Einsteins 1915 general theory of relativity. Based on the observed signals, LIGO scientists estimate that these waves were produced during the final fraction of a second of the merger of two black holes to produce a single, more massive spinning black hole. It is estimated that the black holes for this event were about 29 and 36 times the mass of the sun, and the event took place 1.3 billion years ago [3].

The time difference between these two significant papers is almost a century, and during this time period, there has been major advancements in the process of data taking and analysis. The use of advanced devices and engineering allows the experimental setup to be more sensitive. The analysis of large amounts of data is possible with the development of modern computers. This in turn affects the number of collaborators, as well, and has made it possible for the scientists at LIGO to detect signals whose effect is $\sim 10^{-21}$ m. However, with these advancements has come the ability to do in depth statistical analyses on the data gathered, thereby requiring that scientists publish very comprehensive and extensive results and be able to answer crucial questions regarding how well their data fits to their model.

2 The Older Paper

The 1909 paper by Gieger and Marsden [2] discusses the experiments they conducted that provide conclusive evidence for the existence of a diffuse reflection of the α-particles. Such effects were known for β-particles, and it was not expected that similar effects would be observed for the α-particles because of the relatively small scattering they experience when traveling into matter. Their observation of this unexpected phenomenon motivated the authors to further investigate this effect and understand the behaviour of the reflected α-particles. In these experiments, they examined the fraction of the incident particles reflected, the relative amounts of reflection from different materials, and the relative amounts of reflection from a metal of varying thickness.

In the paper, there is a basic description of the experimental setup that was used. Details on the reasons for certain procedures were pointed out and it was stated that care was taken so that the apparatus was accurate to their standards. The limitations of the experiments, such as the failure to use a parallel and very intense source of homogeneous α rays to study the variation of the effect with the angles of incidence and emergence, were pointed out. The first experiment examined the relative amounts of reflection of the α-particles from different materials. The number of scintillations observed per minute were recorded and the presence of impurities in one of the materials was blamed for the anomalous data point. For the next experiment, the paper discusses the effect of the thickness of the gold foil on the number of scintillations. The data was plotted on a graph, and an approximate best fit line was drawn through the data points. The goal of the last experiment was to determine an estimate of the total number of particles reflected. The conclusion of 1 in 8000 reflection of the incident α-particles was made from three different measurements. The last experiment was repeated at low pressure and the results of grazing incidence were reported since tangential scattering of α-particles from a radioactive source along a glass tube favours this condition.

3 The Newer Paper

The 2016 paper written by Abbott et al.[4] characterizes the first direct detection of gravitational waves and estimates parameters of the source system. Attempts to detect gravitational waves have been made since they were first postulated by Einstein, and as a result their detection marks one of the great technological and intellectual achievements of our time. The importance of this discovery is evidenced by the thoroughness of the resulting data analysis. Facing a large burden of proof, it is the task of the LIGO team to convince fellow scientists through proper analysis that the discovery was statistically significant.

First, the graph of the signal is compared both between the two LIGO sites at Hanford and Livingston and with relativistic models for the event. The setup of the interferometer and related limitations are discussed, specifically photon shot noise and other noise sources. The calibration error of the equipment is cross-validated using multiple methods to increase accuracy, and the background noise is quantified. With these measurements in place, they are able to quantify the maximum significance of the discovery to 5.1σ. Once it is determined that the event is significant, it is necessary to characterize the source of the waves. This is done by matching the data to a nearly exhaustive waveform set of 250,000 templates and determining the χ^2 statistic to quantify the fit. Again, cross-validation is done here to increase robustness of results. The estimated mass of each black hole, as well as the mass of the final black hole post-merger, are calculated using a 90% confidence interval and while reporting a systematic error from combining waveform templates. Where source parameters such as the primary black hole's spin cannot be confidently quantified, they are constrained. In addition to the thorough statistical analysis throughout the paper, the researchers were also extensive in their referencing of previous papers. This lends credibility to their results by showing that there is precedent for each step in their analytic methodology.

4 Similarities and differences

Both papers show milestones achieved in their own times. The results of the experiments by Geiger and Marsden were a breakthrough in determining the inner structure of the atom. Even though it was a naive description, it was the first time that concrete evidence was available about the inner details of an atom. The detection of gravitational waves by LIGO was another major leap in understanding the universe. The gravitational radiation detected from the binary black hole merger was the first concrete evidence demonstrating the existence of gravitational waves, predicted by Einstein's general theory of relativity, and it will provide a basis to understanding the universe.

However, a number of differences stand out between the two papers, especially in their data analysis. The techniques applied by Geiger and Marsden appear to be straightforward, and their data analysis is largely qualitative when it comes to reporting how well the results match various models and pointing out interesting patterns. For the first experiment, after reporting the measured data in a table, they pointed out that the ratio of the element's atomic number to the number of scintillations observed decreased when atomic weight decreased, with the exception of lead, which they blamed on impurities without following up to see if that was really the case. For the second experiment, they made a plot of the variation of the number of scintillations with the thickness of the gold foil and included a curve to be the best fit for all the observed data points. The authors did not point out what method they used to draw the best fit curve. Further, this result misses any statistical significance tests, i.e. R^2 values representing the goodness of fit of the drawn line to the data points or any σ or χ^2 values with which to compare the data to models declaring the reflection as proportional to the volume as opposed to the surface area. The report of the third experiment was just one number, the ratio of alpha particles that are reflected given their set-up, and they did not include any approximations of how uncertain that number is.

With the help of computers, LIGO's analysis is much more elaborate and quantitative. They have been meticulous in presenting their data and demonstrating the validity of their results. The interest of this paper was focused on a ~ 100 ms long signal observed at two different sites. Their report included graphs of the snippet raw data, that data with the background subtracted, and a best fit curve as obtained through their comparison to models with a shaded region representing the area included in a 90% confidence level fit to that curve. They also stated to what confidence level that curve fit the data and that the shift in time between the two sites' observations of the signal was within bounds of what is allowed by the difference in positions of the sites. They talked about the methods they used to decrease background noise and to make sure this signal could not possibly have come from the instruments used, showing that they went to great lengths to validate their results before presenting them. At every step, the LIGO paper used statistical confidences and analysis using several different methods in order to further legitimize their results.

In regards to analyses of background noise, Geiger and Marsden just measured an average rate of scintillations without the reflective material in place and subtracted that background from all of their measured rates. For the counting experiments, the paper states that the total number of scintillations counted on the screen is a sum of the number of particles striking the screen and those reflected from the glass walls of the tube. They point out that the entry of the reflected particles to the ionization vessel was prevented by a crude method, i.e., narrow constriction of a stopcock. This does not guarantee that the results were not affected due to this addition of reflected particles. The authors simply state that the correction term did not influence the final result, without giving any substantial proof. LIGO's analysis of backgrounds was much more complex, in part because the nature of the experiment produced a lot of background noise, which required them to put in a lot of effort to reduce it as much as possible. They could not simply measure the background without the signal, as Geiger and Marsden did, since there is no way to shield gravitational waves. For the background noise they could not prevent, they used a comparison of the two sites' data to cut out the parts that were not alike, as well as comparing them with various time shifts that are bigger than the amount of time it would take a gravitational wave to travel between the sites, thereby excluding the possibility of a gravitational wave producing any common signals, and calculating how much of the time the comparison yielded signals of various strengths. They then graphed their result and demonstrated how incredibly unlikely this signal was just background noise. The time-shifting of their background noise data allowed them to compare the anomalous event against what is equivalent to 608,000 years of background data, eliminating nearly all possibility of the event occurring from random chance alone.

In conclusion, LIGO's analysis was much more comprehensive in every aspect than was that of Geiger and Marsden. Using statistics, they quantified the likelihoods of backgrounds producing similar signals and how well the model of a merging binary black hole system fit the signal observed. In defense of the latter, though, this type of statistical analysis of large amounts of data would not be possible without the advent of computers. The data analysis completed by the two teams, however, did follow the same general structure of minimizing background noise, discussing limitations to experimental procedure, and comparing their results both to theory and previous research.

Honor Code

I have neither given nor received unauthorized assistance on this assignment.

References

[1] https://en.wikipedia.org/wiki/Geiger-Marsden_experiment

[2] H. Geiger, and E. Marsden, Proceedings of the Royal Society of London, Series A, Containing Papers of a Mathematical and Physical Character 82.557:495-500, (1909).

[3] https://www.ligo.caltech.edu/news/ligo20160211

[4] B.P. Abbott, et al., Phys. Rev. Lett., 116.6:061102, (2016).