Quantitative Analysis of Physical data: Assignment 4

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The neutrino oscillation probability is a quantum mechanical probability for an electron anti-neutrino, $\bar{\nu}_e$, to remain a $\bar{\nu}_e$. This paper discusses the non-linear analysis of the data extracted from the KamLAND experiment. The global minimum for χ^2 is calculated over the parameters Δm^2 and $sin^2 2\theta$. The procedure is extended to three degrees of freedom by taking the length of the detector as a nuisance parameter.

INTRODUCTION

The Kamioka Liquid scintillator Anti-Neutrino Detector (KamLAND) investigates neutrino oscillation parameters by observing electron anti-neutrinos ($\bar{\nu}_e$) emitted from distant nuclear reactors [1]. Previously, by observing distortion of the reactor $\bar{\nu}_e$ energy spectrum, KamLAND has provided direct evidence of neutrino oscillation. The experiment described in [1] has determined a precise value for the neutrino oscillation parameter Δm_{21} and stringent constraints on θ_{12} based on the data collected from March 9, 2002 to May 12, 2007 along with the data that was already available. The enlargement of the fiducial volume radius, reduction of the systematic uncertainties from the background and the number of target protons has and longer lifetime has increased the exposure to the nuclear reactors $\bar{\nu}_e$ almost fourfold over the previous studies. This has enabled the observation of almost two complete oscillation cycles and more precise values of the oscillation parameters [1].

Electron anti-neutrinos are detected via inverse β-decay, $\bar{\nu}_e + p \rightarrow e^+ + n$, with a 1.8 MeV threshold. A number of improvements are made to the system to obtain more precise measurements. The "off-axis" calibration system enables the positioning radioactive sources away from the central vertical axis of the detector [1]. This provides the information about the radius and angle dependence, which in turn determines the fiducial volume to 1.6% uncertainty up to 5.5 m. This off-axis calibration measurements and numerous central-axis deployments of radioactive sources established the event reconstruction performance. Further, the nonlinear effects from quenching and Cherenkov light production are corrected while recording the scintillator response. This results in a 1.9% uncertainty in Δm^2_{21} due to distortion of the energy scale, which makes the total uncertainty on Δm^2_{21} be 2%. However, θ_{12} is affected by 4.1% by these uncertainties.

This experiment also puts constraints on the events to check for background noise. The accidental coincidence events were taken into account by constructing a probability density function to pair events in a 10- to 20-s delayed-coincidence window [1]. The source of dominant background is caused by the reactions from α- decay and hence, the decay-rate is included in the analysis with proper uncertainty values for both the ground and the excited states. The background arising from the cosmogenic beta delayed-neutron emitters is diminished by applying a 2s veto within a 3-m-radius cylinder around well-identified muon tracks [1]. This experiment shows that considering errors from background and the systematic errors in the analysis provides a precise result and also allows the consistency with theoretical interpretations.

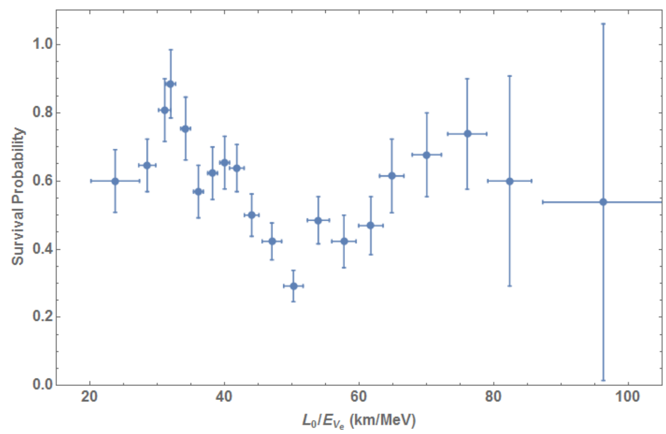

FIG. 1: Ratio of the background and geoneutrino-subtracted $\bar{\nu}_e$ spectrum to the expectation for no-oscillation as a function of L_0/E. L_0 is the effective baseline taken as a flux-weighted average ($L_0 = 180km$). The energy bins are equal probability bins of the best-fit including all backgrounds. The histogram and curve show the expectation accounting for the distances to the individual reactors, time-dependent flux variations and efficiencies. The error bars are statistical only and do not include, for example, correlated systematic uncertainties in the energy scale. [1].

The basic goal of the experiment described here is to extract the parameters of the neutrino oscillations from the data of the KamLAND experiment [1], i.e., Δm^2 and the angle θ. The following equation 1 gives the neutrino oscillation probability, which is a quantum mechanical probability for an electron anti-neutrino, $\bar{\nu}_e$, to remain a $\bar{\nu}_e$.

$$P_{\bar{\nu}_e\bar{\nu}_e} = 1 - sin^2 2\theta sin^2\left(\frac{1270\Delta m^2 L}{E}\right) \tag{1}$$

Here, Δm^2 is called the squared mass splitting and is one of the measured parameters. This probability depends on the mixing angle θ and is linear in $sin^2 2\theta$. The distance between the source and the detector is given by L and E represents the neutrino energy. The analysis is repeated for three degrees of freedom by taking L as a nuisance parameter. However, both the fitting models are not a good representation of the data points. Therefore, it is necessary to either change the fitting models or to introduce other parameters into the one that is already provided.

METHODS

The minimum χ^2 analysis using a non-linear fitting method is used to analyze the data points and the error bars extracted from the fig.1. Since the bins are not of equal widths, the following definition is used for χ^2.

$$\chi^2 = 2\sum_{i=1}^{b} \frac{\left(\langle x_i\rangle - x_i\right)^2}{\sigma_i^2} \tag{2}$$

The σ_i is the standard deviation of data x_i and the summation is done over the number of bins, b. The $\langle x_i\rangle$ is the corresponding expectation value computed using equation 3 where the variable bin width is denoted by ΔE_i. For the analysis, L_0 is taken to be 180km.

$$\langle x_i\rangle = \int_{\Delta E_i} P\left(E, \Delta m^2, sin^2 2\theta\right) \tag{3}$$

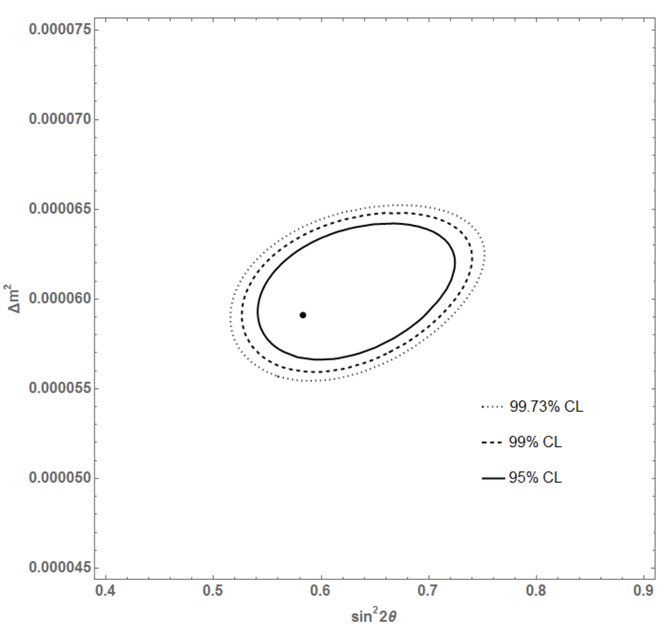

FIG. 2: Contour Plot showing the values of $\Delta\chi^2$ for 2 degrees of freedom, Δm^2 and $\sin^2 2\theta$ for χ^2 values 95%, 99% and 99.73%.

The global minimum of the χ^2 function is calculated using Wilks theorem for 2 degrees of freedom. The minimization is done with respect to two parameters, Δm^2 and $sin^2 2\theta$. The contour plots in fig. 2 shows the allowed region for neutrino oscillation parameters. The profile likelihoods for these parameters are plotted in fig. 3 and 4. These plots show how $\Delta\chi^2$ varies with one parameter when the other one is kept constant.

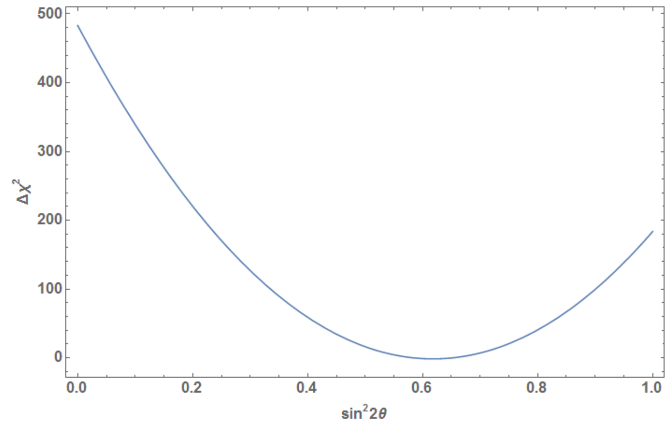

FIG. 3: Variation of $\Delta\chi^2$ with $sin^2 2\theta$ at a constant value of $\Delta m^2 = 5.9 \times 10^{-5}$ for 2 degree of freedom.

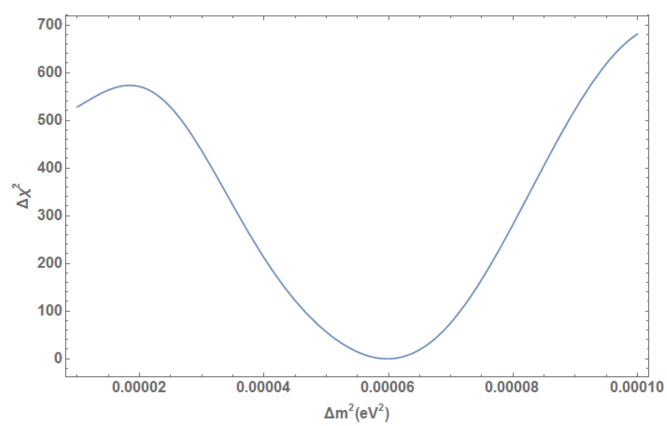

FIG. 4: Variation of $\Delta\chi^2$ with Δm^2 at a constant value of $sin^2 2\theta = 0.58$ for 2 degrees of freedom.

Further, this analysis is extended to 3 degrees of freedom. In this part of the problem, the minimization is also performed over L_0. It is treated as a nuisance parameter and its 1 standard deviation uncertainty is taken as 20km. This changes the definition of χ^2 to the following.

$$\chi^2 = 2\sum_{i=1}^{b} \frac{\left(\langle x_i\rangle - x_i\right)^2}{\sigma_i^2} + \frac{(L_0 - L)^2}{20^2} \tag{4}$$

The contour plots in fig. 5 shows the new allowed regions of the parameters Δm^2 and $sin^2 2\theta$ when L_0 is considered uncertain. The fig. 6 and 7 show how the profile likelihood of these parameters change from the case when L_0 is taken to be the value that is obtained from minimization of χ^2. These plots behave very differently when compared to those obtained by taking a constant $L_0 = 180$km.

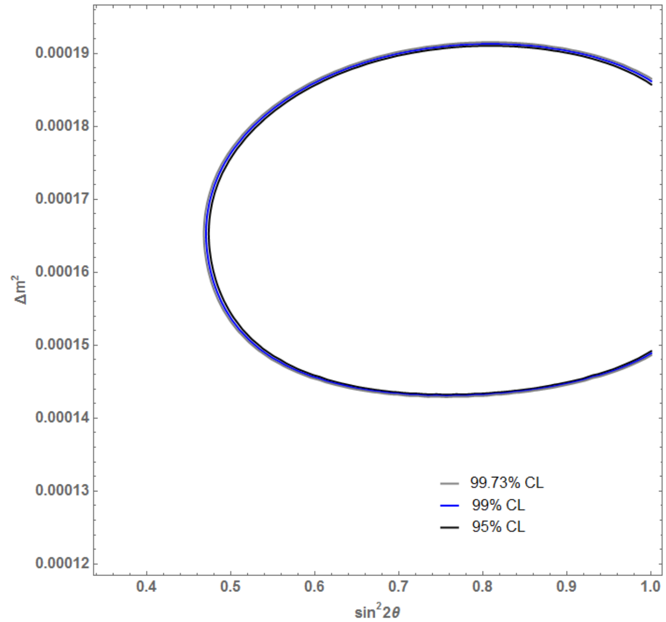

FIG. 5: Contour Plot showing the values of $\Delta\chi^2$ for 3 degrees of freedom, Δm^2 and $\sin^2 2\theta$ for χ^2 values 95%, 99% and 99.73%.

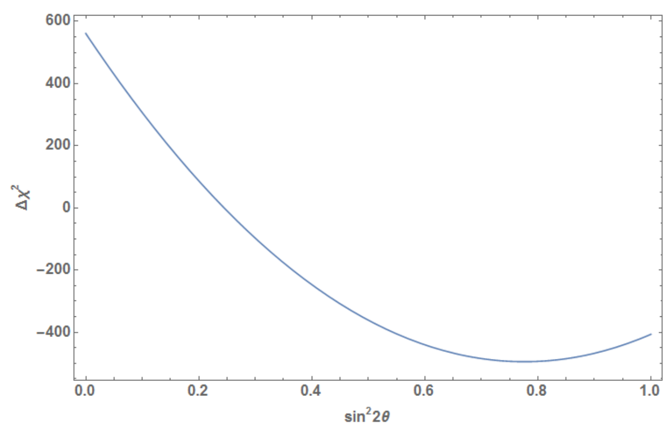

FIG. 6: Variation of $\Delta\chi^2$ with $sin^2 2\theta$ at a constant value of $\Delta m^2 = 1.63 \times 10^{-4}$ for the case of 3 degrees of freedom.

DISCUSSIONS

As mentioned in the introduction, the KamLAND experiment confirms the neutrino oscillations and determines the most precise values of the parameters, Δm^2 and θ. This experiment takes into account technical improvements and carefully analyzes the events for background events. The spectrum showing the ratio of the background-subtracted $\bar{\nu}_e$ candidate events, including the subtraction of geoneutrinos, to no-oscillation expectation as a function of L_0/E indicates almost two cycles of the periodic feature expected from neutrino oscillation [1]. The data extracted from this observation is used to perform minimum χ^2 analysis with non-linear fits.

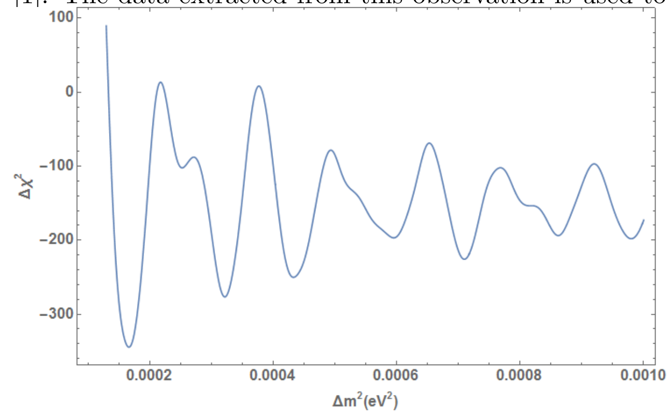

FIG. 7: Variation of $\Delta\chi^2$ with Δm^2 at a constant value of $sin^2 2\theta = 0.58$ for the case of 3 degrees of freedom.

# of Parameters	χ^2	Δm^2	$sin^2 2\theta$
Two Degrees of Freedom	964.86	5.91×10^{-5}	0.58
Three Degrees of Freedom	621.53	1.63×10^{-4}	0.48

TABLE I: Statistics obtained from the analyses.

The minimization over two degrees of freedom, Δm^2 and $sin^2 2\theta$ gives the χ^2 value from which the contour plots are obtained. These show the allowed regions for these parameters for three different χ^2 values, 95%, 99% and 99.73%. Table I shows the values of the parameters and the respective χ^2 values obtained from both the analyses. The value of L_0 that minimizes χ^2 is 163km when it is taken as a nuisance parameter. The model with three degrees of freedom seems to give a better fit compared to the one with L_0 constant. From the values of the χ^2, it can concluded that both the methods are not a good fit to the data points extracted from the KamLaND experiment shown in fig. 1. Using a different fitting model or introduction of some other parameters that affects the oscillation probability in the analysis might provide better results.

HONOR CODE

I have neither given nor received unauthorized assistance on this assignment.

[1] S. Abe, *et al.* Precision Measurement of Neutrino Oscillation Parameters with KamLAND. Phys. Rev. Lett., 100:221803, 2008.