Public transport and urban structure⋆

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Abstract

Public transport is central to commuting in most cities. This paper studies the role of public transportation in shaping the urban structure. Its main contribution is to propose a tractable model as a tool to study urban regulations and transport policies in the long-run. Using the classic monocentric city framework, we model public transport as a mode that can only be accessed by walking to a set of stops. By incorporating a discrete transport mode choice and income heterogeneity, the model remains simple yet can reproduce non-monotonous urban gradients observed in cities with public transport, and well-observed spatial patterns of sorting by income and use of public transport. For example, it can reproduce an inverted U-shape of transit usage along the city. To highlight the relevance of the model, we study the effects of pricing pollution externalities together with extending the public transportation network on the urban structure.

Keywords: Monocentric city model; public transport; mode choice; income groups sorting.

1. Introduction

Transportation and commuting are central to the theory of cities and urban structure. Early works from Alonso (1964), Mills (1967), and Muth (1969) developed the starting point of the modern urban economics literature in what is called the monocentric city model. In that model, production takes place at the Central Business District (CBD), where all jobs

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are located. The core of the model is that residents consume a numeraire good, housing, and must commute. As commuting costs increase with distance to the CBD, the differences in these costs along the city must be balanced by differences in the price of living space and consumption of housing (Brueckner, 1987). For this reason, land and rental prices, as well as population density, should decrease with distance to the CBD, and dwelling size should increase with distance to the CBD. This model has been extended in many directions, for example, to study amenities (Brueckner et al., 1999), local public goods (de Bartolome and Ross, 2003), landscape preferences (Turner, 2005), and the institutional frictions in land markets related to slums (Henderson et al., 2020), among others. See Duranton and Puga (2015) for an extensive review.

Public transport is essential to commuting in many cities around the world. For example, the share of public transport trips is 28% in 25 of the European largest cities, while the share of private car trips was 33% in 2015 (EMTA, 2015).[1] The use of public transport is arguably more important in developing countries. The average share of trips made by public transport in 15 of the largest cities in Latin America was 43% in 2009, significantly higher than the share made by car, which was 28% (CAF, 2010). The public transport system's features and technology affects commuting costs and, thus, following the argument of its central importance, it should change the urban form.

This paper studies the role of public transport in shaping cities, through a novel use of the monocentric city model. We focus on three crucial aspects of the urban structure: (i) use of public and private transport according to location; (ii) spatial sorting of different income groups, and (iii) how housing price, land price, dwelling sizes, population density, and structural density (floor-to-area ratio) change with distance to the central business district. We show that the model provides significant predictory power regarding several (ir)regularities that are usually observed in cities with high use of public transport. When cities differ in commuting costs components, their structure and use of public transport will be different. For example, the discrete nature of stations in space allows us to obtain the inverted U-shape of public transport usage along with the city that has been identified in some metropolis. The main contribution of the paper is, therefore, to propose a tractable model that is a useful tool to address the efficiency and impact of transport policies in the long-run.

The literature about the role of transportation on the urban structure of cities is extensive. As this paper, some contributions have developed modifications of the monocentric city model to study population density and urban sprawl. For instance, Baum-Snow (2007) introduces radial highways into the model, retaining the primary standard results, but suggesting that this transportation infrastructure causes suburbanization. Kim (2012)

[1]These metropolitan areas include Amsterdam, Barcelona, Berlin, London, Madrid, and Paris, among others.

incorporates vehicle size choice, arguing that as commuting cost per mile increases the city tends to expand, as opposed to the predictions of the standard model. In a later study, Kim (2016) includes negative externalities into his model of vehicle size choice, showing a positive relationship between population density and fuel-efficiency (closely related to vehicle size) when externalities are priced.

Unlike these contributions, our paper allows residents to choose among commuting by car, public transport, or foot. Certainly, there are studies that incorporate other modes of transportation into the standard monocentric city model. For example, Baum-Snow et al. (2005) study the effect of the incorporation of a rail transit line on ridership and modal shifts through making assumptions about the relative speeds of the transportation modes. Other papers have studied the role of public transit on explaining the sorting of residents by income in a city (LeRoy and Sonstelie, 1983; Glaeser et al., 2008; Su and DeSalvo, 2008). In particular, Glaeser et al. (2008) argue that "the primary reason for central city poverty is public transportation". However, in all these studies, the modeling of public transport is quite simple; indeed, public transport is simply modeled as a car that is slower and less expensive. The consequences of the simplification are that the patterns of car and public transport usage are segregated zones where either one or the other mode is dominant. Our modeling of public transportation allows for more complex patterns that are relevant when studying transport taxes, emissions, vehicle-kilometer traveled, among other key outcomes.

More recently, a growing body of studies have developed quantitative urban models to study more comprehensively the implications of public transport improvements (see, e.g., Tsivanidis, 2019; Severen, 2018). For instance, Tsivanidis (2019) develops a model where multiple skill groups of workers have non-homothetic preferences over transit modes and residential locations. By taking into account time savings but also reallocation and general equilibrium effects, this study shows that a large change in the public transit infrastructure (Transmilenio) in Bogota, Colombia benefited high-skilled workers more because time savings did not compensate for price adjustments which hurt low-skilled workers more. As opposed to these quantitative urban models, our approach takes advantage of its micro-modeling feature to allow for more detailed analyses such as studying the effect on housing price changes in smaller areas than a census-tract, or boarding externalities at public transport stations.

On the other hand, the transport economics literature has modeled public transportation in a significantly more detailed way to study its optimal level of service, the efficient pricing scheme, or agglomeration externalities such as crowding and congestion (see, e.g., Hörcher et al., 2020). Furthermore, many studies have investigated the efficiency of policies such as subsidization, bus lanes, car congestion pricing, and combinations when there is interaction with other transport modes (for recent studies, see, e.g., Proost and Van Dender, 2008; Parry and Small, 2009; Kutzbach, 2009; David and Foucart, 2014; and Basso and Silva, 2014). Nevertheless, this strand of the literature has adopted a short-run view by assuming that the housing market and location of households is exogenously fixed.[2] Our paper contributes to this literature by providing a framework to assess such transportation policies when public transportation and patterns of mode usage and income along with the city are interrelated.

Our paper also contributes to the literature on the benefits of better access to transportation to consumers. There is ample empirical evidence that households value improved accessibility, and several studies have estimated the effect of closer proximity to a rail station on prices (e.g., McMillen and McDonald, 2004; Gibbons and Machin, 2005; Billings, 2011; Diao et al., 2017). Other papers have studied the effect of transportation accessibility improvements on employment and population density. For example, using the opening of a rail transit line in France (the Regional Express Rail, henceforth RER) as a natural experiment, Garcia-López et al. (2017a) show an increase in employment and population density in municipalities located close to the network. Mayer and Trevien (2017) also study the RER opening and show a similar effect on employment, no effect on population density, and a rise in the likelihood of high-skilled residents living near a RER station. Motivated by this evidence, Garcia-López et al. (2017b) delve into the effects of rail transit improvements in Paris between 1968 and 2010, and show an increase in employment subcenters formation. Yet, the theory suggests that the average impact may mask significant heterogeneity concerning overall accessibility (or proximity to the CBD). Our theoretical model delivers price elasticities with respect to the distance to the station that change with distance to the CBD depending on structural parameters. Thus, it can shed light on how the effect of closer proximity changes along a transport corridor such as a rail line.

The discrete nature of the stops and the fact that people may walk downstream or upstream to access public transport drives the action and predictory power while remaining a rather simple model.[3] The non-monotonic commuting costs induce non-monotonic gradients with peaks in prices (as in Figure 1), population density, and structural density at stations, where dwelling sizes are smaller. These predictions have been suggested in the literature, for example, by Duranton and Puga (2015), and modeled in the context of complementary public transport modes, where people take buses to train stations but car is not available as an alternative mode (Kilani et al., 2010).

However, it is the combination of detailed modelling of public transport, a simple discrete choice model between car and transit, and income heterogeneity that makes the model stand out. Our model predicts that the use of cars can appear all along with the city and not only in long stretches of the city, where that mode dominates without any use of public transport, as currently available models predict. We also show that the presence

[2] An exception is Brueckner (2005), who studies the effect of transport subsidies on the spatial expansion of cities, but also using a simplified transit system and without interaction with a different mode.

[3] There is ample evidence showing that the walking time to access public transportation plays a vital role on its demand (see, e.g., Yáñez et al., 2010).

of public transport can break the ordered sorting from the models without the need to have multiple modes of transportation. Our model with only public transportation and two income-groups has a large amount of mixing at the level of the distance between stops because, as the price gradients are non-monotonic due to the access cost to the stations, price bids can cross multiple times.[4]

Finally, even though it is not the main purpose of the paper, we study the policy of pricing pollution externalities and extending the public transportation network. We compare the untolled city with the resulting equilibrium when the marginal external cost per km is charged, the revenues are distributed among residents and public transport service is extended as long as it has demand. We find that the optimally tolled city is 8% more compact, the modal share of car trips is 15 percentage points lower, and 35% fewer kilometers are driven. The demand for public transportation is larger and 4.5 additional km of transit are provided, which had no demand in the untolled city. We also obtain that the utility achieved by both income groups is lower.

Nevertheless, the difference in rental prices and population density is the prediction where our model significantly differs from previous ones. As our model can have more mixing along with the city than traditional models, the changes in prices and density that follow the implementation of a policy have a more significant dispersion. Besides the macro-changes that would be observed in a simpler monocentric model with two zones, it predicts changes due to the presence of stations and changes in the income group. For example, within five kilometers the policy induces changes in household density that are very different over space and can range from an increase of 17% to a decrease of 13%. These results highlight the relevance of the model and the potential it has for future policy analysis.

The rest of the paper is organized as follows. Section 2 introduces the monocentric city model with public transport and characterizes the urban structure equilibrium. Section 3 extends the analysis by including the interaction between public and private transport on the urban structure and explores the implications of our model in the sorting of residents by income. Section 5 concludes.

2. Urban structure in the public transport city

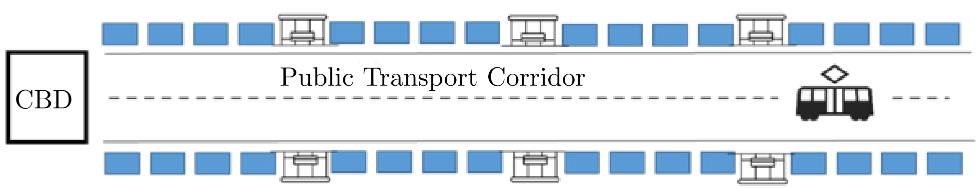
We first describe the model in which public transport is the main transportation mode, to highlight the detailed modeling of stations and its implications. We keep the discussion of the traditional model's features as brief as possible and refer to the reader to Brueckner

[4]Other rationales for social mixing are two-dimensional heterogeneity of individuals such as income and commuting costs (Behrens, Duranton and Robert-Nicoud, 2014) and locations with two-dimensional heterogeneity, distance to the CBD and other exogenous feature such as amenities (Gaigné, Koster, Moizeau and Thisse, 2017).

(1987) and Duranton and Puga (2015). In the following section, we introduce modal choice and income heterogeneity.

Residents commute to their jobs, which are all located at the CBD, along a corridor, where walking and public transport are the only available commuting options. The public transport mode could be a bus rapid transit system, a tramway, or a subway line. In this sense, we consider a linear rather than a circular city as represented in Figure 1. At both sides of the corridor, buildings vary on height, the number of apartments, and apartment size with distance to the CBD.

Figure 1: Linear city with public transport.



Residents' preferences are represented by a standard strictly quasi-concave utility function $U(c, q)$, where c is the consumption of a composite non-housing good, and q is the consumption of housing (measured by floor space). We assume that the price of the composite good is the same everywhere in the city, and is normalized to unity. In equilibrium, q and the rental price per unit of housing floor space, p, can vary with distance to the CBD, x.

Residents' income comes from two sources: a fixed amount E, which is unrelated to the hours of work, and $w \cdot H$, that comes from working H hours per day at a hourly wage of w. Thus, the budget constraint is $E + w \cdot H = c + pq + e$, where e is the public transport fare. The time constraint is $T = H + L + t$, where T is total available time in a day (discounting time for sleeping, eating, and so on), L is leisure time, and t is commuting time. Combining both equations and re-organizing, we obtain,

$$\underbrace{y}_{E+w(T-L)} = c + pq + \underbrace{\rho}_{e+wt} \tag{1}$$

Equation (1) highlights that commuting costs are the sum of the public transport trip fare and travel time costs. This sum is the generalized commuting cost and is represented by ρ.

Firms use inputs of land l and capital K to produce housing, according to the concave constant returns function $H(K, l)$. These firms rent land and capital at prices r and i, respectively. Following Brueckner (1987), we define S as the capital-land ratio (K/l). Using the constant returns property, $H(K, l)$ can be rewritten as $H(S, 1) \equiv h(S)$, which gives floor space per unit of land.[5] It is standard to interpret S as an index of the height of buildings, usually called *structural density* (for details, see Brueckner, 1987). With this notation, the firms' profit function can be defined as $l[ph(S) - iS - r]$. If we assume, without loss of generality, that each household contains one person, then the population density is given by $D = h(S)/q$.

2.1. Public transport modelling and commuting costs

Up to this point, the model and assumptions are conventional, with the exception that the usual constant per-distance cost of commuting is replaced by the generalized cost of commuting, ρ. Indeed, one distinctive feature of this paper is the modeling of public transport, reflected in this commuting cost, which, besides the in-vehicle travel time, includes commuters' walking time to the station.

We consider that the public transport corridor has equally spaced stations at a distance d. We assume that public transport is uncongested and its free-flow speed is denoted v. We further assume that residents do not experience waiting time, because public transport operates based on timetables.

We focus on the case where the indifferent resident between walking to the CBD and commuting by public transport is located between the CBD and the first station.[6] Denote $\bar{x}_0$ the location for which residents are indifferent between walking to the CBD or walking *in the opposite direction* to the first station (see Figure 2). Then, $\bar{x}_0$ is obtained from equating commuting costs:

$$w \cdot \frac{\bar{x}_0}{\mu} = e_1 + w \cdot \frac{d}{v} + w \cdot \frac{d - \bar{x}_0}{\mu} \tag{2}$$

The left side of equation (2) is the generalized cost of walking from $\bar{x}_0$ to the CBD at speed μ, where the wage rate w is the value of time implied by our model. The right side of equation (2) represents the generalized cost of commuting from the first station located at $x = d$. The first term, e_1, is the public transport fare charged at station 1. The second term is the in-vehicle travel time between the first station and the CBD. The third term is the walking time from $\bar{x}_0$ to the first station located at d. From equation (2), we obtain:

$$\bar{x}_0 = \frac{d}{2} + \frac{\mu e_1}{2w} + \frac{d}{2}\frac{\mu}{v} \tag{3}$$

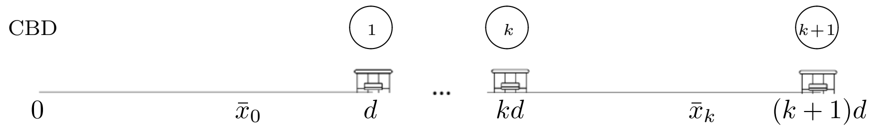
Note that $\bar{x}_0$ is always upstream (to the right) of $d/2$. The necessary condition on parameters needed for this to be interior (i.e. downstream of the station) is: $\frac{\mu e_1}{w(1-\mu/v)} < d$, where $0 < \frac{\mu}{v} < 1$. In other words, if the public transport system is not "good enough",

[5] It follows that the marginal productivity of capital is positive $h'(S) > 0$ and also $h''(S) < 0$ because, for a fixed l, the use of additional capital will be less productive (e.g., thicker walls, deeper foundations).

[6] People would always choose walking if either the public transport fare is prohibitively high or if the walking speed is equal to or higher than public transport speed.

both in terms of fare and speed, then people would prefer walking. Figure 2 shows the public transport line with the stops and the locations at which individuals are indifferent between which one to walk to.

Figure 2: Public transport stations and indifference locations.



The equation that defines the location $\bar{x}_k$ of the indifferent commuter between station k and station $k+1$ is, again, obtained by equating generalized commuting costs:

$$e_k + w\left[\frac{kd}{v} + \frac{\bar{x}_k - kd}{\mu}\right] = e_{k+1} + w\left[\frac{(k+1)d}{v} + \frac{(k+1)d - \bar{x}_k}{\mu}\right] \tag{4}$$

The generalized commuting cost from $\bar{x}_k$, for $k \in \{1, ..., n-1\}$, by walking to station k (located at kd) and to station $k+1$ (located at $(k+1)\cdot d$) are on the left-hand and the right-hand side, respectively, of equation (4). Note that, in principle, e_k is different than e_{k+1}, meaning that the fare may be different at different stations. Of course, people to the right of the last station have no choice but to walk downstream to that station. From (4) we obtain:

$$\bar{x}_k = \left(dk + \frac{d}{2}\right) + \frac{\mu}{2w}(e_{k+1} - e_k) + \frac{d}{2}\frac{\mu}{v} \tag{5}$$

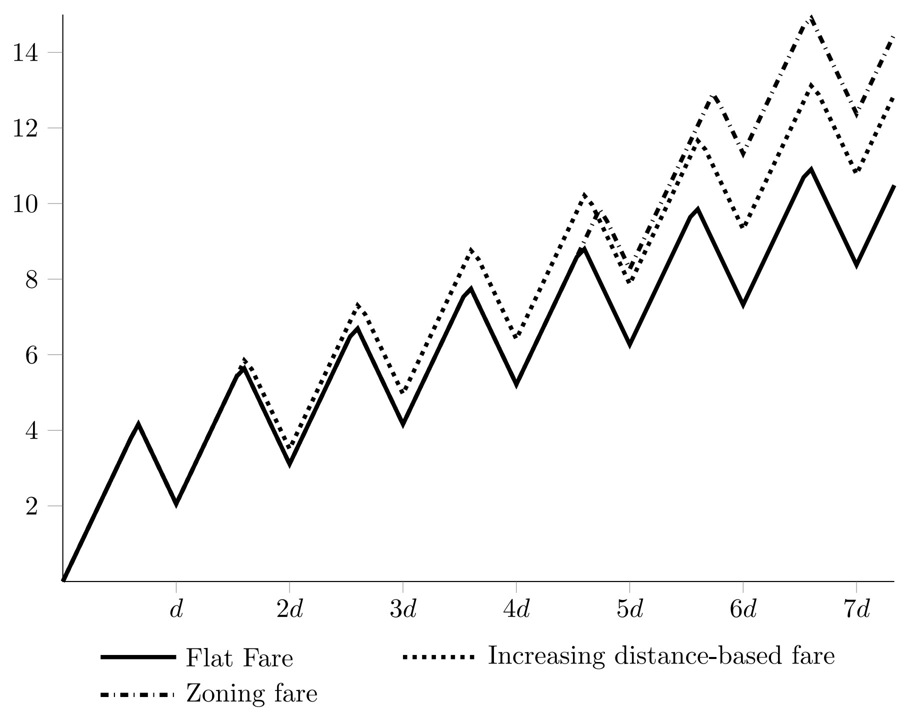
Having characterized the location of the indifferent commuters, we can study how the commuting costs ρ change along the city. This is important since commuting costs are pivotal in the choice of location, which, in turn, is the instrument that enables individuals to trade housing and consumption good. Consider people boarding at station $k > 1$. Everyone living in $[kd, \bar{x}_k]$, with $k > 1$, incur the same in-vehicle time to the CBD since they all board at station k. However, the walking time is increasing in that interval, indicating that total commuting cost is increasing with distance to the CBD in that interval. On the other hand, everyone living in $[\bar{x}_{k-1}, kd]$ also incur the same in-vehicle time since they all also board at station k, but the walking time is *decreasing* as people live closer to the station, indicating that total commuting cost is *decreasing* with x in this interval. For the interval between the CBD and the first station, people that are located downstream of $\bar{x}_0$ walk to the CBD.

Note from equation (5) that, unless transit fares are heavily decreasing with distance, the commuting costs for people located exactly at public transport stations are increasing the farther away from the CBD they are. It is straightforward to conclude that commuting costs –including both in-vehicle and walking time– follow a sawtooth pattern, with an overall increasing trend.

To further characterize commuting costs, consider first the case where the public transport system has a flat fare, i.e., $e_k = e \quad \forall \quad k$. In this case, it follows from equation (5) that the distance between the indifferent commuter $\bar{x}_k$ and the downstream station is given by $\frac{d}{2} + \frac{d}{2}\frac{\mu}{v}$ for all $k \geq 1$. That is, all the indifferent commuters, except for $\bar{x_0}$ are located at the same location relative to their closest stations. Now, consider that the fare is not flat, but increasing with distance: $e_k \equiv e + \Delta \cdot k \quad \forall k > 0$. It follows that the distance from $\bar{x}_k$, for all $k \geq 1$, to the downstream station is now larger by an amount given by $\frac{\mu\Delta}{2w}$. Thus, in each interval the $\bar{x}_k$ when fares increase with distance will be to the left of the indifferent commuter under flat fares, in relative terms, if $\Delta < e$. Note that what changes are the positions of the indifferent commuters but the slope of both parts of a tooth, is always given by the walking speed μ.

With a distance-based fare, then, the sawtooth pattern has more asymmetric teeth, with a shorter decreasing section, overall resembling more the classical τx commuting cost. It is also simple to picture how a zone fare system affects commuting costs by changing the location of the indifferent commuter for sets of stations. Finally, note that changing the distance between stations would change the amplitude of the sawtooth pattern. With very close stations, the amplitude diminishes, moving towards the classic model. All these are shown in Figure 3.

Figure 3: Commuting costs compared to flat fare for (b) increasing distance-based fare (c) zone-based fare.



2.2. Urban structure

In this section, we study how the presence of public transport shapes the internal structure of a closed city. That is, how housing and land prices, dwelling sizes, building heights, and population density change with distance to the CBD in equilibrium. The analysis shows that the model can capture some irregularities that are observed in cities with intensive use of public transport, and that a car-monocentric city model cannot.

As in the standard monocentric (closed) city model, the urban equilibrium is achieved when all individuals maximize and obtain the same utility level $\bar{U}$. That is, the individual optimality condition $\frac{\partial V}{\partial q} / \frac{\partial V}{\partial c} = p$, and the spatial equilibrium condition $V(y - pq - \rho, q) = \bar{U}$ hold, where V is the indirect utility function. Totally differentiating the spatial equilibrium condition with respect to x and using the individual optimality condition, we obtain that $\frac{\partial p}{\partial x} = -\frac{1}{q} \frac{\partial \rho}{\partial x}$. That is, p follows the opposite trend than the commuting cost. From the analysis of commuting costs ρ above, we find that:

$$\operatorname{sign}\left(\frac{\partial p}{\partial x}\right) = \begin{cases} < 0 & \text{when} \quad kd < \; x \; < \bar{x}_k \qquad \forall k \geq 0 \\ > 0 & \text{when} \quad \bar{x}_k < \; x \; < (k+1)d \quad \forall k \geq 0 \end{cases} \tag{6}$$

The housing price decreases with distance to the CBD when the individual walks downstream to a station, as in $(kd, \bar{x}_k)$, and increases with distance to the CBD when she walks upstream, as in $(\bar{x}_k, (k+1)d)$. Thus, the housing price shows peaks at public transport stations, and as opposed to the commuting cost, it follows a sawtooth pattern with an overall decreasing trend. In essence, in each interval between $\bar{x}_k$ and a station, there is a small classic monocentric city. Each station acts as a CBD, and the intuition remains: the change in the housing consumption cost with distance to the CBD has to be exactly offset by the change in commuting costs. Since in each of these small monocentric cities commuting costs are linear –given by the product of walking speed, distance, and the value of time–, it follows that $p(x)$ will be convex in each interval. This pattern is represented in Figure 4. Furthermore, each station is a symmetry point, in that the monocentric city to the right and left of a station are identical, but with different directions. They develop until a $\bar{x}_k$ is reached; and since the $\bar{x}_k$ are to the right of the middle point between two stations, one obtains the overall decreasing pattern.

Dwelling size is also non-monotonic along the city, though its overall pattern is increasing, and it shows local peaks between stations at $\bar{x}_k$, i.e. the commuter's location that is indifferent between the upstream and downstream station (see Figure 4).[7]

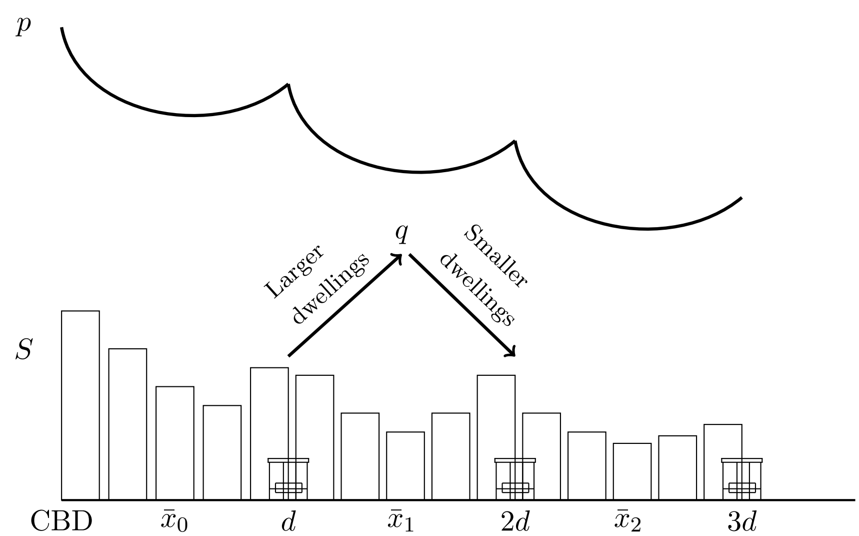
The housing supply side equilibrium is attained when firms maximize their profit, and –because of the competitive setting– the profit is zero in the long-run equilibrium. Just as in the standard model, the land price gradient $\frac{\partial r}{\partial x} = h(S)\frac{\partial p}{\partial x}$, which shows that land price changes follow housing price changes. Therefore, land price is overall decreasing, but with

[7] Differentiating the individual optimality condition with respect to x and reordering we get $\frac{\partial q}{\partial x} = \frac{\partial p}{\partial x} \left[\frac{\partial \left(\frac{\partial V / \partial q}{\partial V / \partial c} \right)}{\partial q} \right]^{-1} = \frac{\partial p}{\partial x} \eta$ where $\eta \equiv \left[\frac{\partial \left(\frac{\partial V / \partial q}{\partial V / \partial c} \right)}{\partial q} \right]^{-1}$ is the slope of the indifference (constant utility) curve. We have already shown that the sign of $\frac{\partial p}{\partial x}$ is positive or negative, depending on the location. η is negative because indifference curves are convex as we assume that the utility function is strictly quasi-concave. Using (6) and the equation above, we obtain that housing consumption changes with distance to the CBD as follows: $\operatorname{sign}\left(\frac{\partial q}{\partial x}\right) > 0 \quad \text{when} \quad kd < x < \bar{x}_k \forall k \geq 0$ and $\operatorname{sign}\left(\frac{\partial q}{\partial x}\right) < 0 \quad \text{when} \quad \bar{x}_k < x < (k+1)d \forall k \geq 0$.

local peaks at stations.

Next, differentiating the firm profit maximization condition with respect to x, yields $\frac{\partial S}{\partial x} = -\frac{\partial p}{\partial x}\frac{h'(S)}{p \cdot h''(S)}$. That is, the height of buildings is overall decreasing but with local peaks at stations. Finally, another important element of the urban structure is the population density. As $D = h(S)/q$, it is straightforward to show that $\frac{\partial D}{\partial x}$ has the same sign as $\frac{\partial S}{\partial x}$. Therefore, the population density follows the same pattern as the structural density: it is overall decreasing with local peaks at stations.[8] All the relevant gradients are displayed in Figure 4, except for the land price r which follows the same pattern as the housing price p.

Figure 4: Housing price (p), housing consumption (q) and structural density (S) along the city.



The two conditions that characterize the equilibrium of the urban area are that the urban land rent r equals the agricultural rent r_A at the city boundary $\bar{X}$, and the total urban population N has to fit inside the city. Both conditions are expressed in equations (7) and (8), respectively.

$$r(\bar{X}, y, \rho, u) = r_A \tag{7}$$

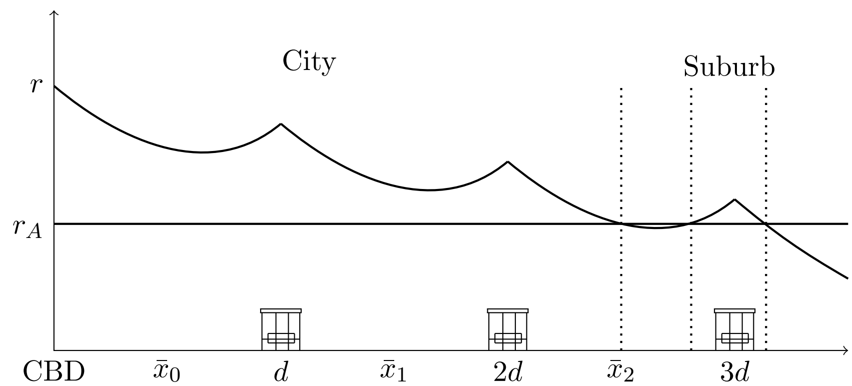
$$\int_0^{\bar{X}} D(x)dx = N \tag{8}$$

Before moving to the full model, it is worth noting that the presence of public transport may also give rise to suburbs that are disconnected from the city. That is, inhabitants that cluster around a transit station, but that has agricultural land on both sides. This is sometimes also known as leapfrog developments (see Duranton and Puga, 2015, for

[8]Concavity of all these functions follow directly from the fact that each interval between the location of the indifferent commuter and the station is a classic monocentric city.

references). The intuition is displayed in Figure 5: as land price is not a monotonically decreasing function, it can be equal to the agricultural rent more than once as long as there is a public transport stop further away from the CBD from the first crossing point.[9]

Figure 5: City boundary and the disconnected suburb.



We have studied how the presence of public transport affects the gradients predicted by the classic monocentric model. The overall trends are the same when moving away from the CBD: the housing price, land price, structural density, and population density decrease, while dwelling size increases. The critical difference is that these gradients are non-monotonic: our model predicts prices, population, and structural density peaks at stations, where dwelling sizes are smaller. In other words, around public transport stations, rental prices are higher, buildings are taller, and apartments are smaller. These predictions seem intuitive and representative of cities with intensive use of public transport. Another example is Bowes and Ihlanfeldt (2001) who use data from Atlanta and find that "[...] properties that are between one and three miles from a station have a significantly higher value compared to those farther away". Other references that sustain the empirical finding that rental prices decrease as properties are farther away from subway stations, and which may lead to the pattern that our model delivers, are Landis et al. (1995); Knaap et al. (2001); Bae et al. (2003); Gibbons and Machin (2005); Armstrong and Rodriguez (2006); Hess and Almeida (2007); Liang and Cao (2007); Gu and Zheng (2010); Feng et al. (2011); Efthymiou and Antoniou (2013). These studies cover many different cities and countries. In terms of specific numbers, Knight and Sirmans (1996) reports that every 0.1 miles further from the metro station contributed to a 2.5% decrease in housing rent.

[9] Indeed, leapfrog development only occurs if that station is built. In work currently being developed, we show that it may well be the case that building the station, and thus inducing a disconnected suburb, is welfare-maximizing.

3. Mode choice and heterogeneous income

3.1. Private transport

We now add the car as a possible transport mode. We continue to abstract away from congestion externalities, and, for the time being, we keep the assumption that individuals are homogeneous. We further assume that everyone has access to commute by car.[10]

If a resident commutes by car, the income constraint becomes $E+wH = c+pq+\gamma+\tau' x$, where γ represents a fixed (distance independent) cost, such as capital, insurance, or parking costs at the CBD. The time constraint is $T = H + L + t_c$, where t_c is the car travel time. As before, we obtain H from the time constraint and replace it on the income constraint. Using that travel time t_c and location x are related through car speed, v_c, we obtain:

$$\underbrace{y}_{E+w(T-L)} = c + pq + \underbrace{\rho_c}_{\gamma+\left(\tau'+\frac{w}{v_c}\right)x}$$

The classic monocentric model would have $\gamma = 0$ and $\tau = \tau' + \frac{w}{v_c}$, such that commuting costs are τx, with τ capturing both operational and time costs.

We have now a discrete-continuous choice model where consumers have to choose where to locate, the dwelling size, consumption of the numeraire, and one of the three transport modes available: car, public transport or walking. At each location x, individuals choose the mode of transport with lower commuting costs, as this maximizes utility; the decision is individual and does not require any belief on what the rest of the people do since there are no externalities. Letting V be the utility associated to the transport mode with lowest cost at each location x, individual optimality and spatial equilibrium conditions and above hold.

Identifying which mode of transport is used at different locations boils down to identifying where each mode has a lower commuting cost. In previous models (e.g. LeRoy and Sonstelie, 1983; Glaeser et al., 2008), as modes differ in both fixed and variable costs, this is reduced to find the intersection between two linear commuting costs. In our model, as public transport commuting costs are not monotonically increasing, there could be multiple crossing points. Mathematically, finding intersection points is simple to do, yet it is not very informative, so we do not linger on this. Instead, we rely on graphical analyses to illustrate the ensuing numerical simulations. Recall that public transport costs have the sawtooth shape discussed in Section 2.2 and displayed in Figure 3. What becomes relevant for our analyses here are the upper and lower *boundaries*, that is, the straight line connecting the lower points of the commuting cost function, which occur at stations, and the straight line

[10] Residents can be car owners, but also can rent a car or take a taxi. Notice that income groups with low enough earnings with respect to the car's fixed and variable costs will actually not be able to commute by car.

connecting the peaks, which are located at $\bar{x}_k$ ($k \in \{1, ..., n\}$). It is simple to see that, in the case of a flat fare, the lower boundary line has an intercept equal to e and a slope given by $\frac{w}{v}$. The upper boundary line has an intercept given by $z = \bar{x}_0 \left(\frac{1}{\mu} - \frac{w}{v} \right) > e$ and the same slope $\frac{w}{v}$.

As discussed, car commuting costs are linear, with intercept γ and slope $\tau' + \frac{w}{v_c}$. There are several possible spatial arrangements of mode use, and the equilibrium pattern depends on the relative magnitude of both the intercepts (γ, e and z) and the slopes ($\tau' + \frac{w}{v_c}$, and $\frac{w}{v}$). It is reasonable to assume that cars go faster than public transport so that $v_c > v$; this pushes for the slope of the car commuting cost to be smaller. However, the cost per kilometer τ' has the opposite effect, as per-kilometer user expenses are higher for car travel than for transit. Consider a first case where $e < \gamma < z$ and suppose that the two opposite effects in the slope of the car commuting cost cancel out, so that $\tau' + \frac{w}{v_c} = \frac{w}{v}$. In this case, as shown in panel (a) of Figure 6, the prediction is that between consecutive stations, people closer to the stations would walk to the public transport system, while people in the center would use the car. This pattern does not change as x increases, so it is present in the entire city. Note that this result implies a decrease of the commuting costs compared with the case of only public transport, around what we denoted $\bar{x}_k$, implying that the previously predicted drop of rental and land prices, building height, and population density between stations would be softened. Consistently, dwelling sizes there would not increase as much between stations.

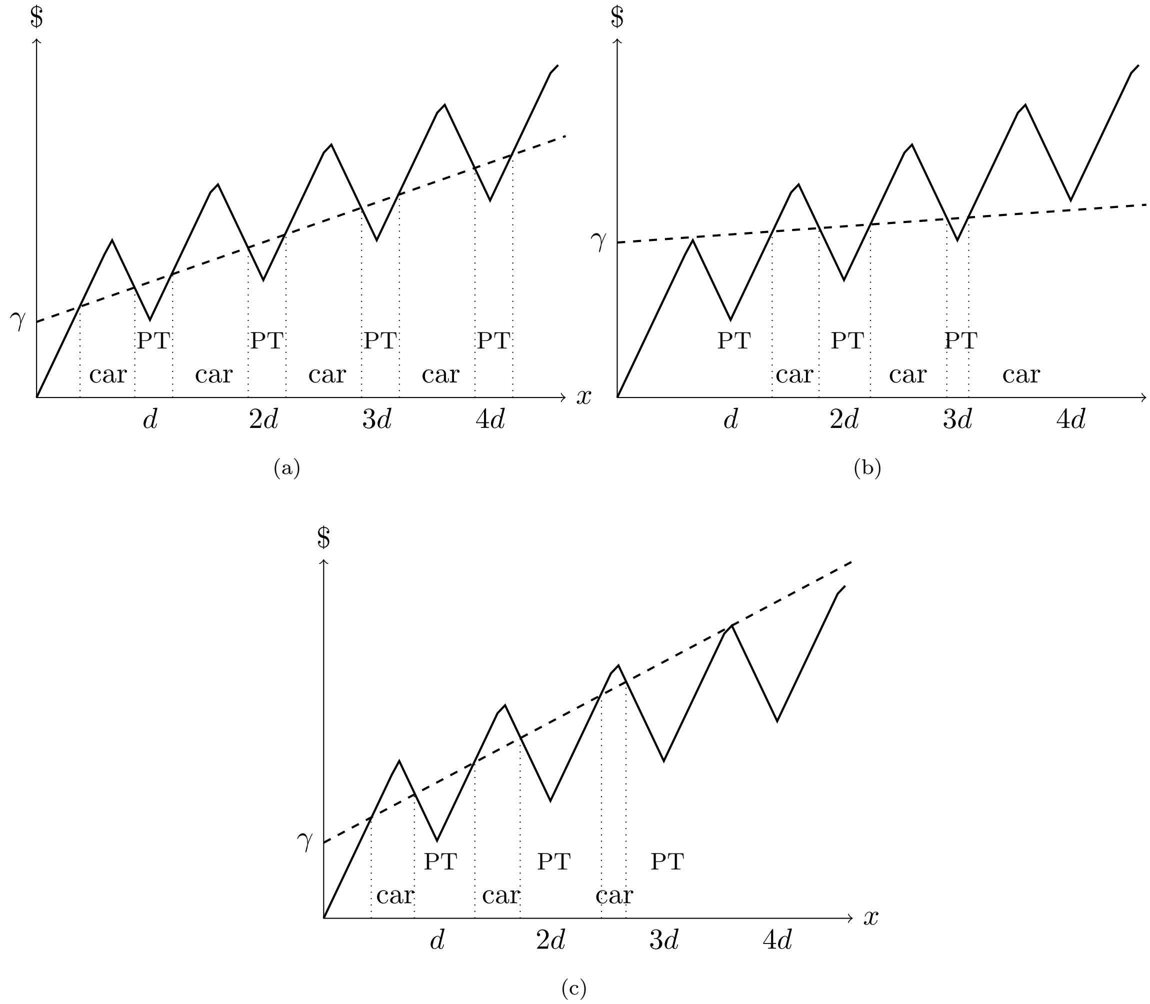
We now turn to two additional cases, one where the slope of the car commuting cost is smaller than that of the lower boundary line, and one where it is larger. If speeds are fixed, which of the two cases arise depends on τ', which in turn is arguably mainly driven by gasoline prices. So, to fix ideas we could think of a North American case, where gasoline is relatively inexpensive, implying a small τ', and a European case, where gasoline is more expensive, meaning a larger τ'.[11]

For a small τ' –the North American case– it may then happen that $\tau' + \frac{w}{v_c} > \frac{w}{v}$. In this case, shown in panel (b) of Figure 6, there can exist a first zone where people either walk to the CBD or use only public transport, followed by a zone where people close to stations take the public transit and people in-between stations commute by car. The share of car use would be increasing as neighborhoods are farther away from the CBD, until a point in which people only commute by car. Note that in this case, there would be no disconnected suburbs. Panel (c) shows what happens when fuel is more expensive. In this case, the mix between car and public transport occurs close to the CBD, but people living farther away, in equilibrium, take public transport.

One of the most compelling results is that the simple public transport/car model with

[11] According to Statista (2018), as of the 2nd quarter of 2017, US gasoline prices were between half and a third of those in the UK, Germany, France, Sweden, Italy, Netherlands, and Norway.

Figure 6: (a) spatial modal split with equal slopes, (b) low-gasoline price city, and (c) high-gasoline price city.PT refers to public transport.



stations and walking can provide a rationale for mixing in transport modes along the city, without needing to resort to different income groups. LeRoy and Sonstelie (1983), when considering two income groups and two modes, obtain up to four different zones but, if only one income group is considered, the city would be divided into two, one zone dominated by the car, and one zone by public transport. In reality, we observe smoother modal split changes as people live away from the CBD. We discuss this issue further in the following Section.

A final point worth mentioning is that not only the price of fuel may lead to these different configurations. Indeed, for fixed fuel prices, various public transport technologies may involve different speeds (variable v), which would affect the trend of the sawtooth pattern, as opposed to that of the straight car line. Several transport policies may affect the slopes of the relevant curves that define the transport mode used at each location, and, therefore, may determine how much, and where, public transport is used. For instance, the pricing of parking, car ownership taxes, or transit subsidies will affect intercepts, while transit policies such as congestion pricing or dedicated bus lanes will affect speeds. We believe that this opens the door to analyzing the effects of these and other measures in terms of the spatial distribution of public transport usage and their welfare implications in a broader urban spatial context.

3.2. Adding heterogeneity of income

We now extend the analysis to include income heterogeneity. We adopt the simplest approach, namely considering two income groups. Suppose first that only public transport (and walking) is available. The income groups, denoted by superscripts h for high-income and l for low-income differ in the wage rate and in the utility function. As nothing else changes, the individual optimality condition and the spatial equilibrium condition apply to each group:

$$\frac{\frac{\partial V^i(y^i - pq - \rho^i, q)}{\partial q}}{\frac{\partial V^i(y^i - pq - \rho^i, q)}{\partial c}} = p \qquad V^i(y^i - pq - \rho^i, q) = \bar{U}^i$$

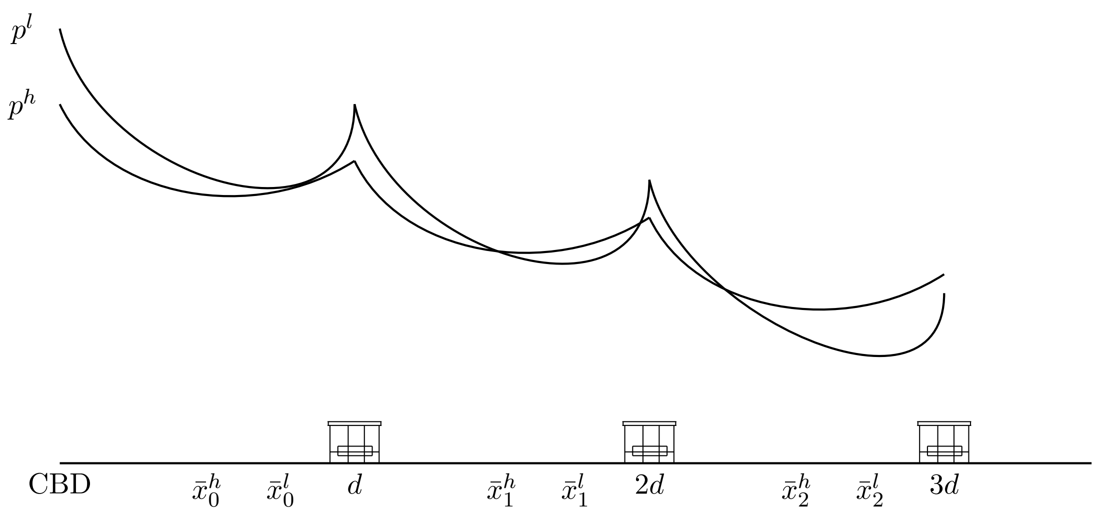
This system of equations, just as derived in Section 2, allows for obtaining the solutions for rental prices and housing consumption for each group as a function of the parameters of the model. As the income and utility function are different, the price of housing and the housing consumption will, in general, be different at each location. To distinguish them we use superscripts so that $p^i(x)$ represents the willingness to pay for housing by the group $i \in \{l, h\}$ at location x. As previous authors have assumed (see, e.g., Wheaton, 1976) land is allocated to the consumers willing to pay more, i.e. those in the group with higher $p^i(x)$. In other words, $p^i(x)$ are the bids for land by consumers.

As housing is essential, both groups consume housing, and therefore, there must be at least one location in which low-income residents outbid the high-income residents, and the reverse must also hold. Suppose there is a point $\hat{x}$ in which both bids are the same, therefore $p^l(\hat{x}) = p^h(\hat{x})$ holds. Using the Alonso-Muth condition (see equation (6)) the slope of the bid rent for each group is given by:

$$\frac{\partial p^i}{\partial x} = -\frac{1}{q^i}\frac{\partial \rho^i}{\partial x}$$

As housing is a normal good, $q^h(\hat{x}) > q^l(\hat{x})$ holds because prices are the same. Therefore, if ρ were the same for both income groups, as is the case in the more classical version of the monocentric city, the price gradient would be steeper for the low-income group, and they would be located nearer to the CBD with the high-income residents in the suburbs. This has been pointed out as early as Alonso (1964). In our model, however, ρ is different for the two income groups for two reasons. First, the wage rate is different so that the opportunity cost of time is different. Second, the location in which residents are indifferent between walking upstream or downstream, $\bar{x}_k$, are generally different.[12] This implies that at some locations the commuting cost and thus the bid rent gradient for low- and high-income groups have different signs. Therefore, due to the sawtooth pattern and the difference in the critical points, the bid rent curves can cross multiple times. This implies that mixing of income groups may happen several times along the city, inducing mixed neighborhoods. In contrast, close to the CBD and the city edge, one income group will eventually prevail. We illustrate the possible multiple crossing in Figure 7.

Figure 7: Example of the possible multiple crossing of the price of housing for low-income individuals (p^l) and high-income individuals (p^h).



Mixing of two income groups, as opposed to having just two regions, one for the poor and one for the rich has been obtained before, but always at somewhat macro scales. For instance, if amenities or open spaces are present, then the mixing of income groups may also occur (Gaigné et al., 2017). LeRoy and Sonstelie (1983) find that mixing is possible with up to four regions along the city if the two income groups have two transport modes available. But the level of mixing we obtain, at a smaller scale, is novel, it does not require a second transport mode, and it provides a much smoother transition of the average income of neighborhoods as distance increases from the CBD.

4. Full model and policy analysis

When substitution between car and public transport is added to income heterogeneity, one obtains a model that remains simple, yet flexible enough to keep track of the non-monotonous nature of urban gradients in cities with public transport. At the same time, the model allows for reproducing well-observed spatial patterns of sorting by income and use of public transport.

[12]From equation (3) it is straightforward to see that $\bar{x}_k$ is always different for the two groups $\forall k > 0$.

To illustrate this better, we combine the extensions discussed in Section 3 and we simulate the urban equilibrium structure of a (closed) city with the full model with two income groups. Previous analytical models, such as those in LeRoy and Sontelie (1983) and Glaeser et al. (2008), can have up to four zones in the city given by the combination of income (high or low) and mode (private or public). With the same number of modes and income groups, our model can have four types of zones, but alternating continuously over space.

4.1. Calibration and equilibrium

The functional forms and parameters of the simulations come mainly from Wu and Plantinga (2003) and Bertaud and Brueckner (2005), which are based on US values. We discuss them briefly. The utility function chosen is a Cobb-Douglas $U(c, q) = c^{1-\alpha^i} q^{\alpha^i}$, which provides closed-form solutions for all the key variables. The taste parameter α^i, where the superscript i denotes the income group, represents the share of the income that is spent on housing in equilibrium. Each income group is of equal size, and we use $\alpha^h = 0.25$ and $\alpha^l = 0.35$, so that the average share of income spent on housing is 30%, which, in turn, is the average of the value used by Wu and Plantinga (2003), 0.5, and Bertaud and Brueckner (2005), 0.1 for an homogeneous population.

We set the average hourly wage as US$ 16.86, which is the value used by Bertaud and Brueckner (2005). It is estimated using the income per household of the 2000 US census (US$ 42,151) and 2,000 work hours/year, which implies 5.48 work hours per day. Wu and Plantinga (2003) use an income of US$ 40,000 which is broadly consistent. We finally assume that the high-income group earns 1.5 times as much as the low-income group.

We also follow Bertaud and Brueckner (2005) in the production side and use a Cobb-Douglas production function normalizing the price of capital to unity. This gives a functional form for the housing output per unit of land $h(S) = g \cdot S^{\beta}$. We set g and β together with the agricultural land value r_A and the city population N to obtain a reasonable back-of-the-envelope estimate for the equilibrium number of dwellings per building in the city center as well as a reasonable length of the city.

The distance between stops, d, is set at 1.5 km, and the public transport fare e is US$ 1. The fixed cost for a car trip is US$ 3.5, and the variable cost per km. is US$ 0.15. Walking speed is 4 km/hr, and the free-flow speed is 24 km/hr and 50 km/hr for public transport and cars, respectively. The main parameters' value are summarized in Table 1.

The market equilibrium under these assumptions is reasonable in that back-of-the-envelope calculations inspired in the linear city of Figure 1 and assuming three buildings per 100m on either side of the line lead to sensible outcomes. In the first 200m of the city, there are 1,950 dwellings in 12 buildings, endogenously giving 163 dwellings per building. The same calculation for the first kilometer gives an average of 128 dwellings per building. Using a reference of 12 dwellings per floor, we obtain buildings of an average height of 14 floors in the first 200m and 11 floors in the first km. The same calculation for the

Table 1: Parameters values of the base case.

Group	Parameter	Value	Explanation
High-income	α^h	0.25	Percentage of the income spent on housing
	y^h	US$48,000	Annual income per household
	w^h	US$20.23	Hourly wage
Low-income	α^l	0.35	Percentage of the income spent on housing
	y^l	US$32,000	Annual income per household
	w^l	US$13.49	Hourly wage
Housing production	g	1	Production function multiplier
	β	0.75	Power of the production function
General	r_A	US$4,000	Agricultural land price
	N	80,000 hab.	Population
Transportation	d	1.5 km	Distance between public transport stops
	e	US$ 1	Public transport (flat) fare
	τ	US$ 3.5	Car trip's fixed cost
	γ	US$ 0.15	Car trip's per-km cost

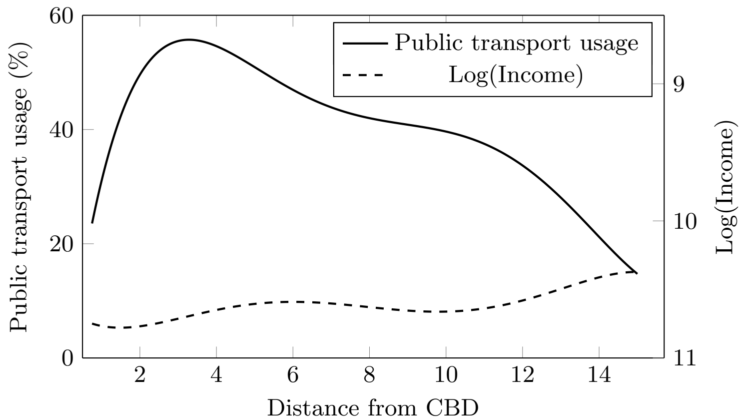
kilometer furthest away from the CBD yields buildings of just one floor. Furthermore, the percentage of individuals that end up walking to the CBD instead of taking public transport or traveling by car is 9.2%. We believe that these are reasonable numbers for a metro or rapid transit line.

The calibration also aims at reproducing observed patterns of public transport usage and income distribution. Our reference is what Glaeser et al. (2008) report as *subway cities* in the US (Boston, Chicago, New York City, and Philadelphia). They show that public transport usage increases from around 30 to 40-50% in the first three miles, and then decreases to about 20% at the 10th mile. The inverted U-shape of the use of public transport mode is attributed to macro changes in the location of income groups, according to the model of LeRoy and Sonstelie (1983). First, close to the CBD, rich people would use public transport; then, poor people would locate and use public transportation, and, finally, rich people would live in the outer part of the city and would commute by car.

The resulting equilibrium income mixing in our simulation is strong, especially in the 12 kilometers closest to the CBD. Transport usage and income sorting along the city are shown in Figure 8. We compute the average income and transit ridership every 1.5km starting from the CBD and plot a smooth interpolation. In this case, both the U-shape of transport usage and the U-and-then-inverse-U-shape for income that Glaeser et al. (2008) report as representative of some American cities are reproduced.

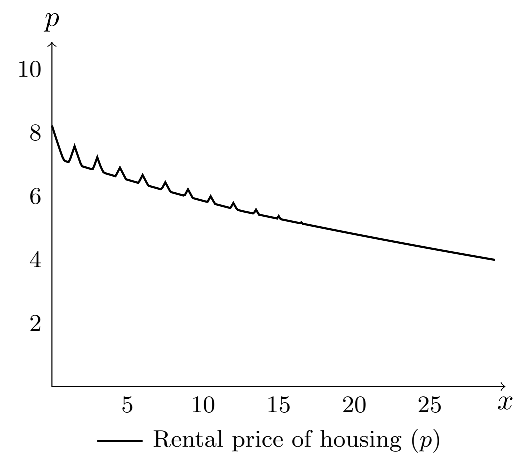
We illustrate the results of the simulation further in Figures 9 and 10, which show the rental price (per unit of floor space) and population (dwellings) density gradient (notice that, respectively, housing price and building heights show a similar but vertically shifted pattern). As discussed in the previous section, the gradients match what a large body of empirical literature has found: that rental prices decrease as units are farther away from

Figure 8: Public transit usage, income, and distance to CBD.



the public transport station. However, one can observe that close to the CBD, the peaking effect of stations is strong, but it weakens as the car becomes more prevalent among people living in the range of 5 to 15 km from the CBD. Eventually, the use of cars by lower-income people takes over completely (which also means that no disconnected suburb occurs for this simulation).

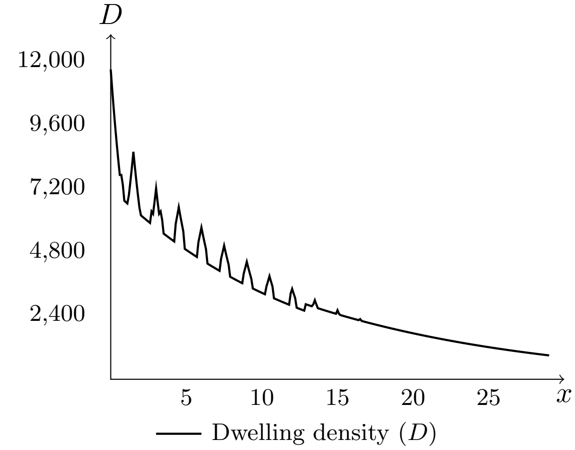
Figure 9: Numerical simulation of a city. Base case. Rental price per unit of floor space.



The city is 27.5km long but half of the population lives within 7km of the CBD. There is demand for public transportation from the first stop located 1.5km away from the CBD to the 11th stop located at 16.5km from the CBD. In between stops, there are always individuals who commute by car.

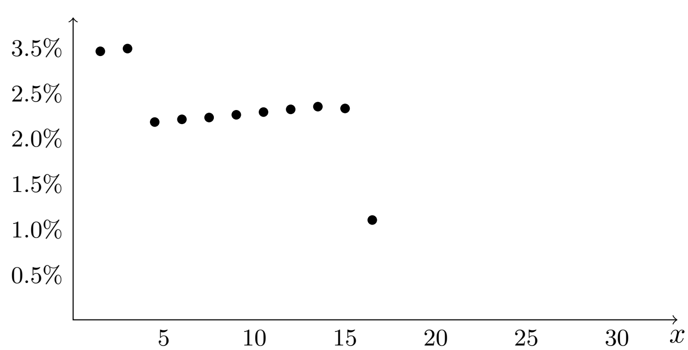
In Figure 11, we show the distance elasticity of rental price, calculated as the change in price 0.1 miles away from a station. In contrast with the traditional monocentric city model, the drop in price around a transit station is driven by the change in walking costs. This feature allows for predicting more significant price changes, which, as we show, approximately match the empirical results obtained in previous studies. Our resulting

Figure 10: Numerical simulation of a city. Base case. Population (dwelling) density pattern.



values range from 1% to 3%, with a non-monotonic spatial pattern, although decreasing as the station is closer to the CDB between kilometers 4 and 15. These figures are roughly consistent with the 2.5% found for Washington DC by Knight and Sirmans (1996). Moreover, Gu and Guo (2008) and Gu and Zheng (2010) found that the distance elasticity is smaller closer to the CBD. To highlight the difference with a traditional model, the drop in price around the station can be compared with the price change in between stations. For example, in the stop located 9km from the CBD, the distance elasticity is 2.3%, at 10km is 0.35% and around the station located at 10.5km is again 2.3%.

Figure 11: Distance elasticity of rental price around transit stations.



4.2. Policy analysis: pollution externalities

In this section, we consider the issue of pricing negative externalities from car use. To keep the model tractable, we consider pollution externalities, so that we can assume that the marginal external cost per km is constant, and that it does not affect the trip's generalized cost.

The market equilibrium in the presence of an externality does not change, as it does not affect commuting costs and the mode choice. Thus, we focus in this section on the resulting equilibrium when the marginal external cost per km is charged. The main difference is that there is a toll of US$5 cents per km driven, which is set based on Parry and Small (2009). The revenues from tolling are distributed in a lump-sum fashion equally among residents, so, in equilibrium:

$$E = \frac{\tau}{N} \int_0^{\overline{x}} \mathbb{1}_{car}(x) \cdot D(x) \cdot x \, dx \tag{9}$$

where $\mathbb{1}_{car}(x)$ indicates if the car dominates public transportation at x.

In this analysis we also consider that public transportation is supplied until there is demand for it; therefore, bus stops are added, keeping the same 1.5 km. distance between them, as long as the commuting cost for someone that is located right at the stop is lower than the car's cost.

With the pollution toll, car trips become more expensive, especially the long ones; therefore, we expect to observe a more compact city, with a larger share of public transport trips, and lower pollution costs. What is not straightforward to predict, besides the magnitude of the effects, is whether some individuals achieve higher utility and how rental prices change.

Table 2 summarizes the result of the policy analysis. The optimally tolled city is 8% more compact, and the modal share of car trips is 15 percentage points lower than the untolled city. These results imply that the tolled city has 35% fewer kilometers driven which directly translate into a the same reduction in pollution costs. The demand for public transportation is significantly larger and 9 additional km of transit are provided that had no demand at all in the untolled city. This is a 55% increase in the network's length. Finally, the utility achieved by both income groups is lower.

Table 2: Main results and comparison of the calibration city with the tolled city.

Variable	**Calibration**	**Tolled city**	**Difference**
City length [km]	29.1	26.8	−2.3
Public transport share [%]	30.9	45.8	+14.9
Car share [%]	59.9	44.3	−15.6
Veh-km driven [km]	614,382	401,444	−212,938
Pollution cost [US$/day]	30,719	20,072	−10,647
Public transport line extension [km]	16.5	25.5	+9
Low-income Utility	12.11	12.08	−0.03
High-income Utility	27.31	27.25	−0.06

The last comparisons we make are of rental price and density. These are especially important as transit stops play a key role in shaping these two variables. The effects we observe cannot be predicted by a model that does not consider them. Figures 12 and 13 show the percentage change between the rental price and the density, respectively, of the tolled and the untolled city.

Figure 12: Differences in rental price between tolled city and untolled city.

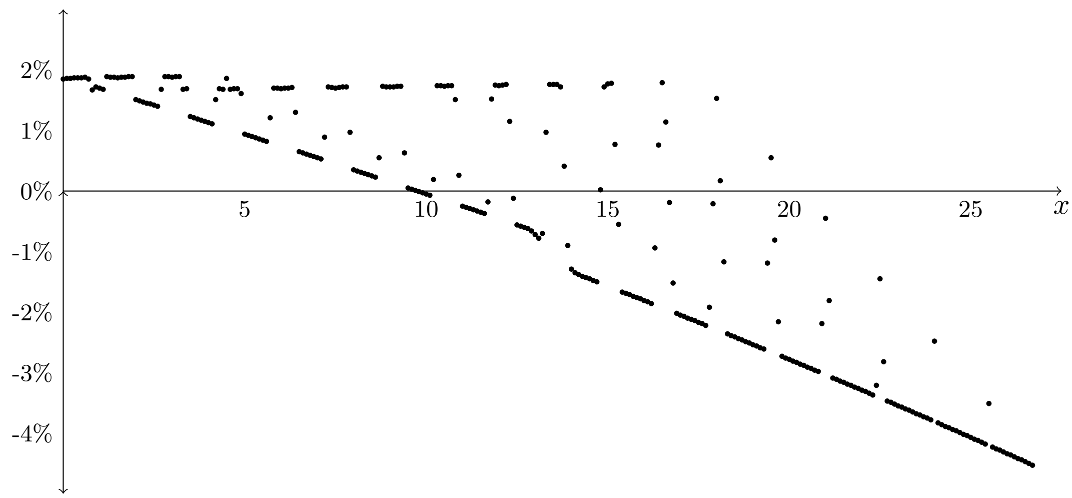
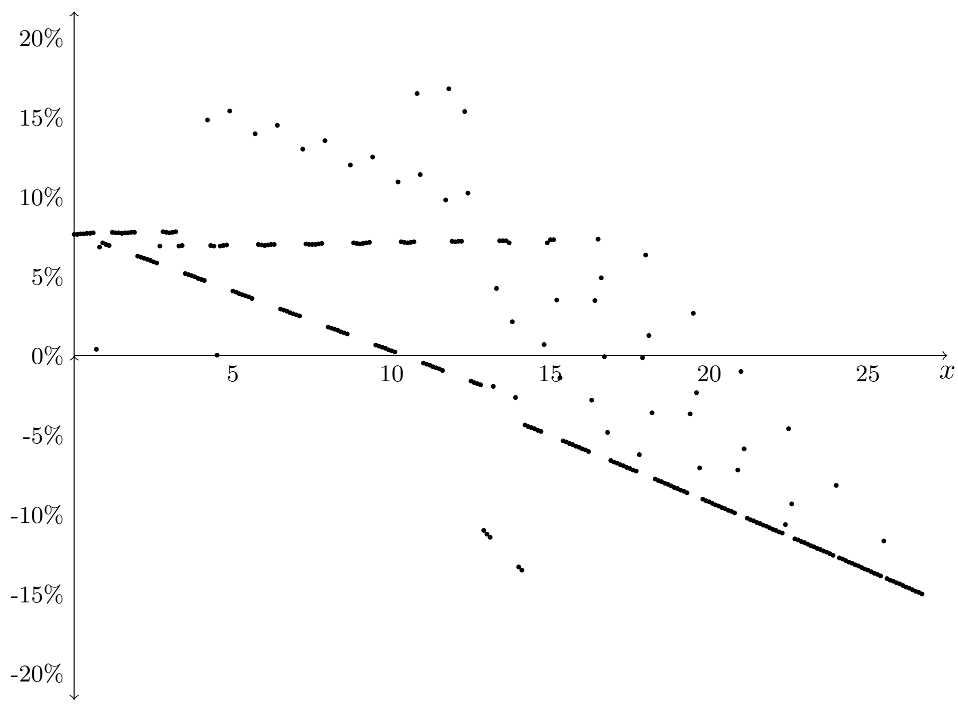


Figure 13: Differences in household density between tolled city and untolled city.



As our model can have more mixing along with the city than traditional models, the changes in prices and density that follow the implementation of a policy have a more significant dispersion. The dots that form the downwards sloping lines are the macro-changes that would be observed in a simpler monocentric model with two zones. The rest of the dots represent changes due to the presence of stations and changes in the income group. With the shift in car travel costs, transit becomes relatively more attractive, especially near stations. As a consequence, price and density increases are larger. The most significant changes, which stand out more in Figure 13, occur around stations due to a change in the income group and the predominant transport mode. For example, one of the largest density percentage increase happens around the station located 10.5 km from the CBD. In kilometer 10.8, without tolls, high-income residents use the car, while, with the pollution tax, low-income residents travel by public transport, which leads to a 17% increase in density.

5. Conclusions

In this paper, we have studied the role of public transport in shaping urban structure. We extend the analysis of public transportation in the monocentric city model by explicitly modeling that it can be accessed through a limited set of stations. This gives rise to non-monotonic gradients for all the essential variables. In particular, our theoretical model shows that around public transport stations rental prices are higher, buildings are taller, and apartments are smaller, as it is observed in reality. This simple model can also explain the presence of disconnected suburbs (leapfrog development) and to reproduce observed patterns of modal and income mixing along the city.

We argue that this model is useful to address the efficiency and impact of public transport policies as it captures many of the urban (ir)regularities. In particular, it helps evaluate the implications of changes in transportation systems or technology, pricing schemes, and taxes, among others, on relevant variables such as vehicle-kilometers traveled emissions and congestion. The main reason why this model is appropriate and improves upon others is that it can predict and capture the critical essential features in a simple way. For example, there is evidence that car ownership and fuel consumption decreases in the catchment area of a new subway station and that the effect is heterogeneous to the distance to the city center (Zhang et al., 2017). Such long-run effects and their heterogeneity are captured in our model.

The analysis can be extended in various directions. Arguably the most natural extension is to analyze different distance-based public transport pricing schemes. There is a current debate about whether distance-based fares should be abandoned or embraced. Yet, we are not aware of a study that addresses this question with the endogenous location of households. Another avenue is incorporating private and public transport externalities and study the efficiency of urban transport pricing policies as well as investment in infrastructure. Car congestion and public transport boarding delays are logical candidates. A different avenue for future research is to consider the dynamics of the cities by considering durable housing and redevelopment. This would make the study closer to a medium-run intra-city analysis.

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