National Institute of Technology Karnataka, Surathkal
E& E Department, Even semester 2017-2018
EE 243 Mathematics for Electrical Engineers
Assignment 2 (a)

Due Date: 07-03-2018 **Weightage: 2+2=4%,**

Time: 10.00 am **Venue: CR-5**

Instructions:

- **Write the assignment neatly on the standard assignment sheets, utilizing both sides.**
- **Please submit the assignment in person.**
- **Make suitable assumptions wherever necessary. State them explicitly and justify them.**
- For Q1(a), MATLAB/OCTAVE m-file should be uploaded on Moodle link provided. Name the m-file as **Assign_01_EE_xxx.m** where xxx is the last 3 digitsof your roll number. Deadline to upload is till _______. Please make sure that your login credentials are active,well before the submission deadline. No submission through mail will be accepted.]

PART-I (Computer solution/Numerical solution based questions 2%)

1. Consider an nxn matrix A.
   a. Write a MATLAB code to factorise any nxn invertible matrix as $\boldsymbol{A} = \boldsymbol{L} \times \boldsymbol{D} \times \boldsymbol{U}$ where $\boldsymbol{L}$ is a nxn lower triangular matrix with every diagonal entry equal to unity, $\boldsymbol{D}$ is a nxn diagonal matrix, and $\boldsymbol{U}$ is a nxn upper triangular matrix with every diagonal entry equal to unity. [Algorithm need not_consider row exchanges. Do not use the built- in functions for factorisation in the code. Use only basic matrix operations, loops etc.]

      Instruction:

      - Write the algorithm/pseudo code in the assignment sheets. M-file must be uploaded in Moodle link.
      - m-file **must contain** appropriate descriptions (about the logic in the **commented** area) for the code segments.
      - Name the M-file as **assign_02_EExxx_1_a.m**

   b. Write a MATLAB code to factorise any nxn invertible matrix as $\boldsymbol{PA} = \boldsymbol{LU}$ where $\boldsymbol{L}$ is a 5x5 lower triangular matrix with every diagonal entry equal to unity, and $\boldsymbol{U}$ is a 5x5 upper triangular matrix. [Name the M-file as **assign_02_EExxx_1_b.m**]
   c. Obtain the result for the given 5x5 matrix $\mathbf{A_i}$ **using the algorithm in (a) and (b) above in MATLAB.** [Refer to the procedure below, to determine value of i.]
   d. Use the MATLAB built- in command lu($\mathbf{A_i}$) for LU factorisation to obtain L,U and P.
   e. Solve $\boldsymbol{A_i x} = \boldsymbol{b_j}$ for given $\boldsymbol{b_j}$. **[3+2+1+2+1= 9 Marks]**

   [To obtain the $\mathbf{A_i}$ matrix:

   - Identify the last digit of your **roll number**, 16EExxi. Choose $\mathbf{A=A_i}$.**]**

$$A_1 = \begin{bmatrix} 2 & 4 & 2 & 6 & 2 \\ -2 & -1 & -2 & -3 & -2 \\ 4 & 11 & 6 & 7 & 6 \\ 0 & 3 & 2 & -6 & 1 \\ 2 & 4 & 0 & 12 & -4 \end{bmatrix} A_2 = \begin{bmatrix} 3 & 6 & 3 & 9 & 3 \\ -3 & -4 & -3 & -7 & -3 \\ 6 & 14 & 7 & 16 & 7 \\ 0 & 2 & 1 & -3 & 0 \\ 3 & 6 & 2 & 11 & 2 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 2 & 4 & 2 & 6 & 2 \\ -2 & -2 & -2 & -4 & -2 \\ 4 & 10 & 3 & 18 & 3 \\ 0 & 2 & -1 & 8 & 1 \\ 2 & 4 & 3 & 6 & 8 \end{bmatrix} A_4 = \begin{bmatrix} 2 & 4 & 2 & 6 & 2 \\ 4 & 11 & 4 & 15 & 4 \\ 4 & 5 & 6 & 1 & 6 \\ 2 & 10 & 8 & -13 & 7 \\ 2 & 4 & 4 & 0 & 4 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 3 & 6 & 3 & 9 & 3 \\ 6 & 14 & 6 & 20 & 6 \\ 6 & 10 & 7 & 12 & 7 \\ 3 & 10 & 6 & 0 & 5 \\ 3 & 6 & 4 & 7 & 8 \end{bmatrix} A_6 = \begin{bmatrix} 2 & 4 & 2 & 6 & 2 \\ 4 & 10 & 4 & 14 & 4 \\ 4 & 6 & 3 & 14 & 3 \\ 2 & 8 & -1 & 24 & 1 \\ 2 & 4 & 1 & 6 & -2 \end{bmatrix}$$

$$A_7 = \begin{bmatrix} 2 & 4 & 2 & 2 & -4 \\ -2 & -1 & -2 & 1 & 4 \\ 4 & 11 & 6 & 11 & -10 \\ 0 & 3 & 2 & 6 & -3 \\ 2 & 4 & 0 & -4 & -6 \end{bmatrix} A_8 = \begin{bmatrix} 3 & 6 & 3 & 3 & -6 \\ -3 & -4 & -3 & -1 & 6 \\ 6 & 14 & 7 & 10 & -13 \\ 0 & 2 & 1 & 3 & -2 \\ 3 & 6 & 2 & -1 & -5 \end{bmatrix}$$

$$A_9 = \begin{bmatrix} 2 & 4 & 2 & 2 & -4 \\ -2 & -2 & -2 & 0 & 4 \\ 4 & 10 & 3 & 4 & -7 \\ 0 & 2 & -1 & 2 & 3 \\ 2 & 4 & 3 & 8 & 0 \end{bmatrix} A_0 = \begin{bmatrix} 2 & 4 & 2 & 2 & -4 \\ 4 & 11 & 4 & 7 & -8 \\ 4 & 5 & 6 & 5 & -10 \\ 2 & 10 & 8 & 19 & -11 \\ 2 & 4 & 4 & 8 & -6 \end{bmatrix}$$

[To obtain b:

- Consider <u>last two digits</u> of the roll number.
    i. If mod(xx,5)=**j-1**, choose **b**$_j$**.]**

$$b_1 = \begin{bmatrix} 36 \\ -21 \\ 71 \\ -9 \\ 26 \end{bmatrix} \quad b_2 = \begin{bmatrix} 15 \\ -15 \\ 28 \\ -2 \\ 15 \end{bmatrix} \quad b_3 = \begin{bmatrix} 20 \\ -20 \\ 44 \\ 4 \\ 14 \end{bmatrix} \quad b_4 = \begin{bmatrix} 26 \\ 67 \\ 25 \\ 12 \\ 18 \end{bmatrix} \quad b_5 = \begin{bmatrix} 62 \\ 154 \\ 66 \\ 22 \\ 44 \end{bmatrix}$$

2. Using MATLAB bulit-in functions, find a **<u>rational basis</u>** for the null space and column space of B. Write the code segment and also write the result. **[4 Marks]**
[To obtain B:

- Identify the last 3 digits of your **<u>roll number</u>**, 16EExxx.

i. If mod(xxx,3)=0, Choose $B = \begin{bmatrix} 3 & 6 & 9 & 12 & 15 & 5 & 22 \\ 3 & 8 & 9 & 14 & 15 & -8 & 2 \\ 6 & 14 & 22 & 10 & 34 & 69 & 128 \\ 0 & 2 & 20 & -73 & 25 & 337 & 490 \\ 3 & 6 & 13 & 6 & 35 & 63 & 118 \end{bmatrix}$

ii. If mod(xxx,3)=1, Choose $B = \begin{bmatrix} 3 & 6 & 9 & 22 & 15 & 5 & 12 \\ 3 & 8 & 9 & 2 & 15 & -8 & 14 \\ 6 & 14 & 22 & 128 & 34 & 69 & 10 \\ 0 & 2 & 20 & 490 & 25 & 337 & -73 \\ 3 & 6 & 13 & 118 & 35 & 63 & 6 \end{bmatrix}$

iii. If mod(xxx,3)=2, Choose $B = \begin{bmatrix} 3 & 6 & 9 & 5 & 15 & 12 & 22 \\ 3 & 8 & 9 & -8 & 15 & 14 & 2 \\ 6 & 14 & 22 & 69 & 34 & 10 & 128 \\ 0 & 2 & 20 & 337 & 25 & -73 & 490 \\ 3 & 6 & 13 & 63 & 35 & 6 & 118 \end{bmatrix}$

3. A computer program makes use of a strategy called 'partial pivoting' while row reducing the matrices to improve the numerical accuracy. Illustrate the strategy for a 3x3 matrix.

PART-II (Do not use MATLAB/OCTAVE for the solutions, 2%)

I. Show that if the set $\{u, v, w\}$ is linearly independent set, so is the set $\{u, u+v, u+v+w\}$.

II. Consider a transformation, $\boldsymbol{D}: \boldsymbol{P_4} \rightarrow \boldsymbol{P_3}$ where, P_4, P_3 **are real polynomial spaces** s.t $\forall p \in P_4, \boldsymbol{D(p)} = \frac{dp}{dx}$.

a. Show that this is a linear transformation from $\boldsymbol{P_4} \rightarrow \boldsymbol{P_3}$. What is the null space of $\boldsymbol{D(p)} = \frac{dp}{dx}$?

b. Find the range of **D**.

c. Obtain a basis for the Nul D.

d. Find dim (Nul D) and dim (Range D).

e. Find the matrix corresponding to the linear transformation **D**.

III. Consider $\boldsymbol{W} = \{\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} \in \boldsymbol{R^3}, \boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z} = \boldsymbol{0}\}$.

a. Show that $\boldsymbol{W}$ is a subspace of $\boldsymbol{R^3}$.

b. Find a basis for $\boldsymbol{W}$.

IV. An isomorphism between two vector spaces which (i) is a (one-to-one) correspondence and (ii) preserves structure. Show that the map $f : R^1 \rightarrow R^1$ given by $f(x) = x^3$ is one- one and onto. Is it an isomorphism?

V. If $\boldsymbol{T}: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ **and** $\boldsymbol{L}: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ are two linear maps and let $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in \mathbb{R}^n$, such that $\boldsymbol{T(u)} = \boldsymbol{L(u)}, \boldsymbol{T(v)} = \boldsymbol{L(v)},$ ***and*** $\boldsymbol{T(w)} = \boldsymbol{L(w)}$, then show that $\boldsymbol{T(x)} = \boldsymbol{L(x)}, \forall \boldsymbol{x}$ in $W = \boldsymbol{span}\{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}\}$.

VI. Show that $\{x^2+1, 3x-1, -4x+1\}$ is a basis for the vector space P_2 set of all polynomials of degree ≤ 2.

--------- End of Assignment----------

Useful MATLAB Commands/ functions:

- inv
- null
- lu
- eye
- ones
- zeros
- Modify entire ith row with a row vector r : A(i,:)=r
- Modify entire jth coulmn with a coulumn vector c : A(:,j)=v
- Modify (i, j)th entry of A: A(i,j)=c
- k^{th} Power of A= A^k
- rref
- help

Use help to understand the syntax for the functions. E.g:- Type " help rref " to get the syntax for input and output variables for 'rref' function.