IO Problem Set 1

Manu Navjeevan

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1 Logit

1. The results from the first OLS Logit are given here. These results are obtained by regressingthe log ratio of $\frac{share_{\text{product}}}{share_{\text{outside option}}}$[1] against product characteristics, price and a dummy for whether a promotion was being run at the time. For this first pass, we assume strict exogeneity of our errors, so that instrumentation is not needed:

	Dependent variable:
	(1)
price	-1.515***
	(0.003)
prom	-2.163***
	(0.053)
Observations	38545.0
R2	0.858
Adjusted R2	0.858
Residual Std. Error	3.02(df = 38542.0)
F Statistic	116599.339*** (df = 2.0; 38542.0)
Note:	*p<0.1; **p<0.05; ***p<0.01

2. The results from the second OLS which includes product dummies are given in the Supplementary Tables Section. Other than including new variables, methodology is the same as in part 1 and we still assume strict exogeneity of our errors. To avoid colinearity, I included all brand dummies but supressed a constant in this specification. Qualitatively, adding brand dummies reduced the magnitude of the coeffecient on price and made the promotion coeffecient positive.

[1] encoded shareDiff

3. The full regression table will not be included here because of the number of regressors. However, I will report the coeffecients and standard deviations on the price and promotion characteristics. The coeffecient on price in this regression is -0.322 with a standard deviation of 0.01. The ccoeffecient on promotion is 0.3290 with a standard deviation of 0.011. This is a similar result to the previous question with just brand dummies. All results are significant at the alpha = 0.01 level.

4. Now I account for endogeneity of the errors using cost as a instrument. The results are given below, though to save space only the price and promotion coeffecients are reported:

Table 1: First three logit specifications using wholesale cost to instrument

	shareDiff I	shareDiff II	shareDiff III
priceHat1	-1.4705	-0.0053	-0.0225
	(0.0042)	(0.0130)	(0.0119)
prom	-2.2420	0.4333	0.4298
	(0.0646)	(0.0124)	(0.0113)

5. Results from using the Hausman Instrument are also given below:

Table 2: First Three logit specifications using the Hausman Instrument

	shareDiff I	shareDiff II	shareDiff III
priceHat2	-1.5194	-0.5128	-0.5310
	(0.0034)	(0.0128)	(0.0116)
prom	-2.0971	0.2948	0.2861
	(0.0528)	(0.0126)	(0.0114)

The two instruments give somewhat different specifications but this could be consistent with them affecting supply in different ways.

6. Now we move to calculate brand elasticities. To do this, we take the coeffecient alpha from the first regression and aggregate price and share by brand. We then use the analytical formulas for elasticities implied by the logit model to get the table of elasticities given below:

Elasticity	Brand 1	Brand 2	Brand 3	Brand 4	Brand 5	Brand 6	Brand 7	Brand 8	Brand 9	Brand 10	Brand 11
Brand 1	-1.814866	-0.001717	-0.001136	-0.001161	-0.000768	-0.000353	-0.000397	-0.000330	-0.000781	-0.000921	-0.000708
Brand 2	-0.002024	-2.621733	-0.001641	-0.001678	-0.001110	-0.000510	-0.000574	-0.000476	-0.001128	-0.001330	-0.001023
Brand 3	-0.002873	-0.003523	-3.723202	-0.002382	-0.001576	-0.000724	-0.000814	-0.000676	-0.001602	-0.001888	-0.001452
Brand 4	-0.001214	-0.001488	-0.000984	-1.572691	-0.000666	-0.000306	-0.000344	-0.000286	-0.000677	-0.000798	-0.000613
Brand 5	-0.002107	-0.002583	-0.001709	-0.001746	-2.730851	-0.000531	-0.000597	-0.000496	-0.001175	-0.001385	-0.001065
Brand 6	-0.003342	-0.004097	-0.002710	-0.002770	-0.001833	-4.331982	-0.000947	-0.000786	-0.001863	-0.002196	-0.001688
Brand 7	-0.001095	-0.001342	-0.000888	-0.000907	-0.000600	-0.000276	-1.419081	-0.000258	-0.000610	-0.000719	-0.000553
Brand 8	-0.001477	-0.001811	-0.001198	-0.001224	-0.000810	-0.000372	-0.000419	-1.915077	-0.000824	-0.000971	-0.000746
Brand 9	-0.001624	-0.001992	-0.001317	-0.001346	-0.000891	-0.000409	-0.000460	-0.000382	-2.105382	-0.001068	-0.000821
Brand 10	-0.000790	-0.000968	-0.000640	-0.000655	-0.000433	-0.000199	-0.000224	-0.000186	-0.000440	-1.023502	-0.000399
Brand 11	-0.001821	-0.002233	-0.001477	-0.001510	-0.000999	-0.000459	-0.000516	-0.000429	-0.001015	-0.001197	-2.360528

In general, we can see that these are not realistic predictions from our model. While the own price elasticities may make sense, the cross price elasticities are far too low. This is a function of the shares being very low for all of our products and because elasticites do not depend on product characteristics. Further, the cross price elasticities are all negative, which suggests that these goods are all complements. This may not be a sensible prediction for the headache medicine market.

2 Random-Coeffecients Logit

1. Python code to do this is involved and is attached. To find the values of σ_B and σ_I I first ran a rough grid search over the area $[-1, 3] \times [-1, 3]$. Doing this I realized that the criterion function was obtaining minimum around values of σ_B and σ_I close to 0. With this in mind, I then ran a gradient descent minimization with a starting point of $(0.1, 0.1)$. The gradient descent returned a minimum at $(\sigma_B, \sigma_I) = (0.1394, 0.0743)$. I then used these to recover the mean utilities and other coeffecients, given below:

Coeffecient	Estimate
σ_b	0.1384
σ_I	0.0743
price	-0.45217
promotion	-1.7133
Brand 1	-6.1956
Brand 2	-12.7887
Brand 3	-11.9699
Brand 4	-6.0623
Brand 5	-4.8449
Brand 6	-13.7221
Brand 7	-6.6258
Brand 8	-7.2018
Brand 9	-5.9018
Brand 10	-16.4058
Brand 11	-8.6397

Table 3: Estimated Random-Coeffecients Logit Model (BLP) Parameters

2. Estimated cross price elasticiteis were given by taking derivatives of the share function with respect to price. Dominated convergence allows us to bring the derivatives into the integration. The integrals are then numerically solved. The results are given below:

Elasticity	Brand 1	Brand 2	Brand 3	Brand 4	Brand 5	Brand 6	Brand 7	Brand 8	Brand 9	Brand 10	Brand 11
Brand 1	-0.102610	53.287470	50.205916	54.212901	52.644678	27.714953	54.228711	54.076155	53.797248	54.355579	53.359324
Brand 2	144.360424	-1.003716	134.026023	144.621892	140.488468	74.092455	144.663388	144.263279	143.530326	144.995183	142.377609
Brand 3	1090.414504	1074.182485	-31.830924	1092.357962	1061.499594	560.808588	1092.666478	1089.693739	1084.237476	1095.124921	1075.642841
Brand 4	40.167548	39.552353	37.259985	-0.052691	39.073744	20.560048	40.253664	40.139866	39.931938	40.348377	39.605615
Brand 5	701.836153	691.295844	651.709492	703.101682	-7.814711	360.420087	703.302540	701.366199	697.817145	704.907058	692.233098
Brand 6	120811.876382	119042.871613	112331.493259	121022.577041	117657.194020	-74760.321087	121056.036767	120733.922444	120141.159128	121321.443238	119205.457062
Brand 7	47.632390	46.902176	44.182254	47.720638	46.334167	24.377168	-0.058094	47.599518	47.352670	47.847119	46.965313
Brand 8	122.448077	120.580235	113.608910	122.673453	119.126205	62.717840	122.709211	-0.258979	121.733373	122.996109	120.742852
Brand 9	432.319919	425.758167	401.218512	433.110410	420.646250	221.618836	433.235842	432.025909	-1.636004	434.240818	426.333371
Brand 10	6.363946	6.265662	5.900743	6.375854	6.189303	3.254419	6.377742	6.359505	6.326229	-0.003136	6.271761
Brand 11	2485.235226	2447.692638	2307.054858	2489.751964	2418.426565	1275.563088	2490.468731	2483.556346	2470.899713	2496.137445	-16.421917

We can see that while the direction of these elasticities are all in the right direction (it it reasonable to believe headache medicines are all substitutes), the magnitudes of these elasticites are far from believable.

3. Marginal Costs are backed out from the matrix equation and are given in the table below. Derivatives were used to calculate elasticites as well, so these were not so difficult to back out.

$$mc = p + \left(\Omega \frac{\partial s}{\partial p}\right)^{-1} s$$

	Predicted Cost	Real Cost
Brand 1	3.286473	2.10
Brand 2	4.893866	3.29
Brand 3	6.433948	5.66
Brand 4	2.816811	2.10
Brand 5	5.340699	3.46
Brand 6	8.440821	5.76
Brand 7	2.710318	1.79
Brand 8	3.369950	2.08
Brand 9	4.018240	3.71
Brand 10	1.482611	0.94
Brand 11	4.5459910	1.92

Overall the predicted costs do a reasonable job of estimating the true marginal costs. Brand 4's marginal cost is predicted quite well, and brand 6 has both the highest predicted costs and real maringal costs.

3 Merger Analysis

1. We rearrange the same matrix equation as before to get

$$p = mc - \left(\Omega \frac{\partial s}{\partial p}\right)^{-1} s$$

To predict prices under the merger, we change the ownership matrix Ω. However, the new Ω is not invertible. We then look for prices to solve:

$$\arg\min_{p} s + \Omega \frac{\partial s}{\partial p}(p - mc)$$

We use a numerical minization routine in python to do this. The resulting predicted prices are given in Table 4. These are in line with what we may expect, prices for the merged brands generally go up, whereas prices for the unmerged brands remain the same.

Brand	Predicted Price	Current Mean Price
Brand 1	4.5128	3.420465
Brand 2	6.28072475	4.942023
Brand 3	7.90219562	7.016067
Brand 4	3.86929228	2.963647
Brand 5	5.74455273	5.145023
Brand 6	8.4353255	8.159743
Brand 7	2.98294236	2.673054
Brand 8	3.86456454	3.607203
Brand 9	4.57602604	3.966644
Brand 10	1.93163615	1.928476
Brand 11	4.4471724	4.447172

Table 4: Predicted Prices Post Merger from Logit Model

2. Finding elasticities with BLP would follow a similar procedure as in the last step. However, solving for the derivative would be somewhat more involved. Fortunately, we already have the derivatives from solving for elasticities.

3. We use the derivatives from the elasticity segment and the matrix equation above to solve for predicted prices under the merger. Once again, we note that the matrix Ω is not invertible, so we look for a numerical solution. Results are given in Table 5.

	Predicted Price	Current Price
Brand 1	3.391746	3.29
Brand 2	4.953639	4.87
Brand 3	6.395938	6.38
Brand 4	2.933929	2.83
Brand 5	5.359535	5.29
Brand 6	7.885246	8.39
Brand 7	2.814275	2.71
Brand 8	3.440935	3.34
Brand 9	4.064824	3.97
Brand 10	1.277707	1.69
Brand 11	4.070670	4.49

Table 5: Predicted Prices Post Merger from BLP Model

From this we see that the model predicts that the merged firms will generally raise their prices. However it also predicts that the non merged firms will lower their prices, which may not be a reasonable prediction from the model. I think there could be a problem here since the matrix equation could have multiple solutions. That is, there could be multiple equilibria to this model.

	Dependent variable:
	(1)
$brand_1$	-6.073***
	(0.036)
$brand_{10}$	-7.278***
	(0.023)
$brand_{11}$	-6.862***
	(0.047)
$brand_2$	-5.384***
	(0.051)
$brand_3$	-5.144***
	(0.072)
$brand_4$	-6.503***
	(0.032)
$brand_5$	-6.266***
	(0.053)
$brand_6$	-6.071***
	(0.083)
$brand_7$	-7.725***
	(0.03)
$brand_8$	-7.665***
	(0.039)
$brand_9$	-6.62***
	(0.042)
price	-0.341***
	(0.01)
prom	0.329***
	(0.013)
Observations	38544.0
R2	0.46
Adjusted R2	0.46
Residual Std. Error	0.668(df = 38531.0)
F Statistic	2733.882*** (df = 12.0; 38531.0)