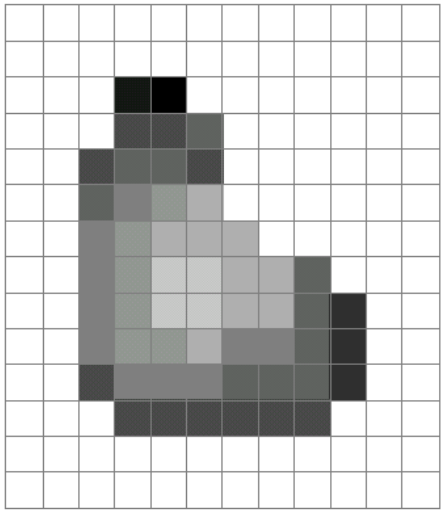
CSE 6367: Computer Vision

Image Filtering and Edge Detection

Slide Courtesy: Richard Szeliski, Computer Vision: Algorithms and Applications, 2nd Edition

What is an image?

- A grid (matrix) of intensity values



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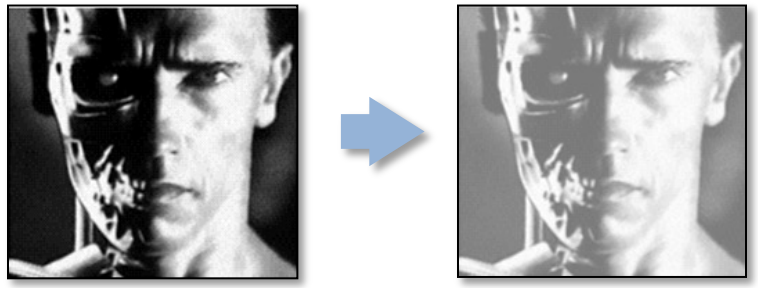
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	20	0	255	255	255	255	255	255	255
255	255	255	75	75	75	255	255	255	255	255	255
255	255	75	95	95	75	255	255	255	255	255	255
255	255	96	127	145	175	255	255	255	255	255	255
255	255	127	145	175	175	175	255	255	255	255	255
255	255	127	145	200	200	175	175	95	255	255	255
255	255	127	145	200	200	175	175	95	47	255	255
255	255	127	145	145	175	127	127	95	47	255	255
255	255	74	127	127	127	95	95	95	47	255	255
255	255	255	74	74	74	74	74	74	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255

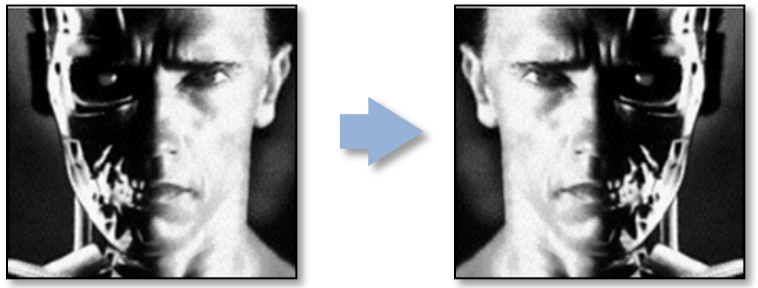
(common to use one byte per value: 0 = black, 255 = white)

Image transformations

- As with any function, we can apply operators to an image



$g(x,y) = f(x,y) + 20$



$g(x,y) = f(-x,y)$

- Today we'll talk about a special kind of operator, *convolution* (linear filtering)

Filters

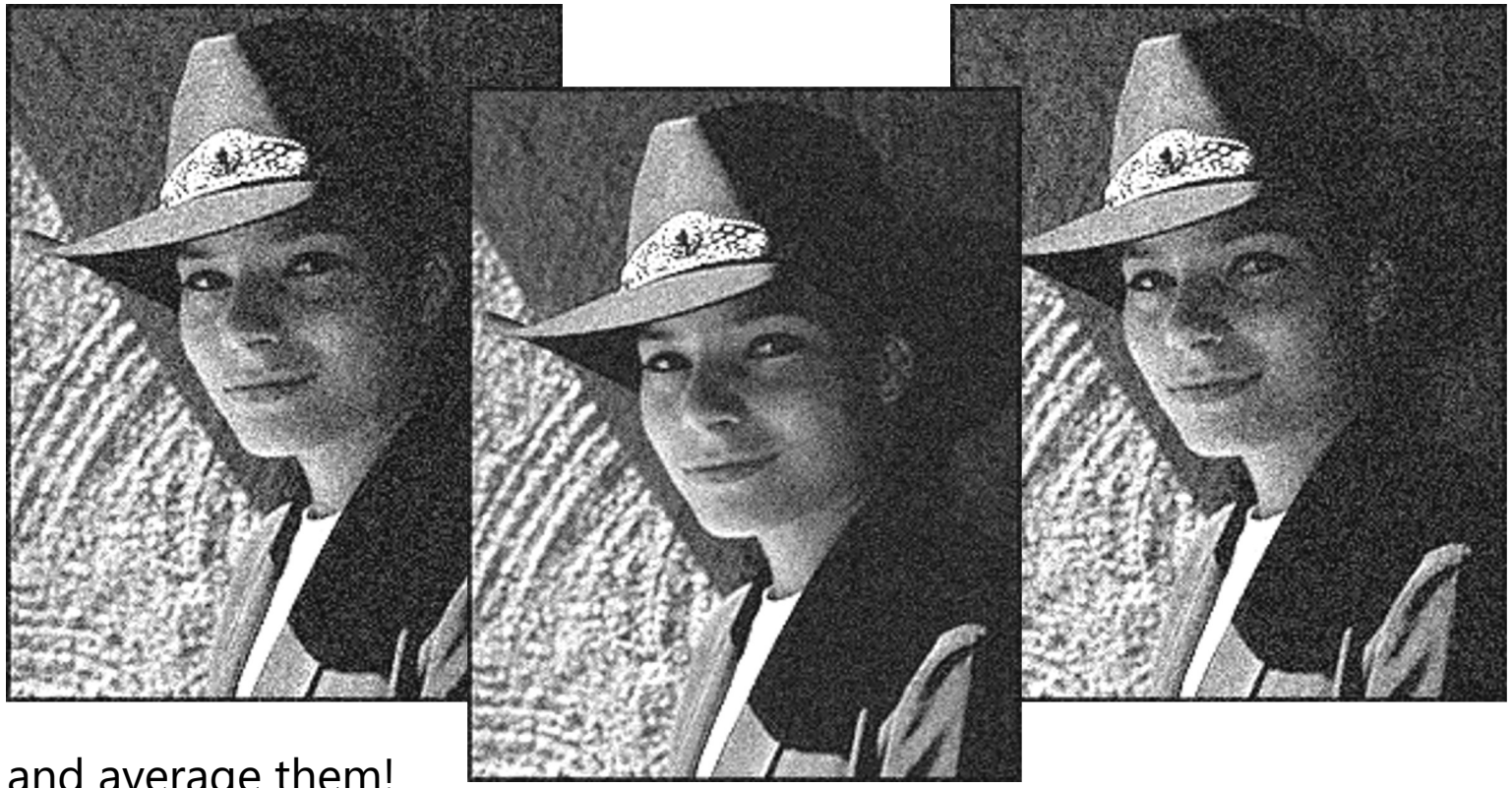
- Filtering
  - Form a new image whose pixel values are a combination of the original pixel values
- Why?
  - To get useful information from images
    - E.g., extract edges or contours (to understand shape)
  - To enhance the image
    - E.g., to remove noise
    - E.g., to sharpen and “enhance image” a la CSI
  - A key operator in Convolutional Neural Networks

Canonical Image Processing problems

- Image Restoration
  - denoising
  - deblurring
- Image Compression
  - JPEG, HEIF, MPEG, …
- Locating Structural Features
  - corners
  - edges

Question: Noise reduction

- Given a camera and a still scene, how can you reduce noise?



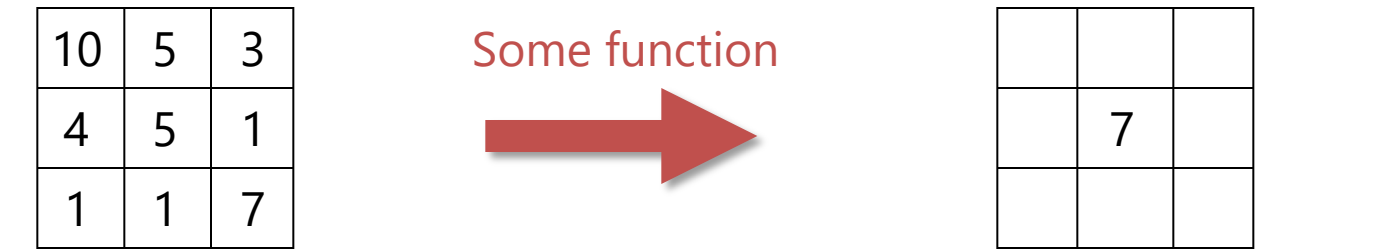
Take lots of images and average them!

What's the next best thing?

Source: S. Seitz

Image filtering

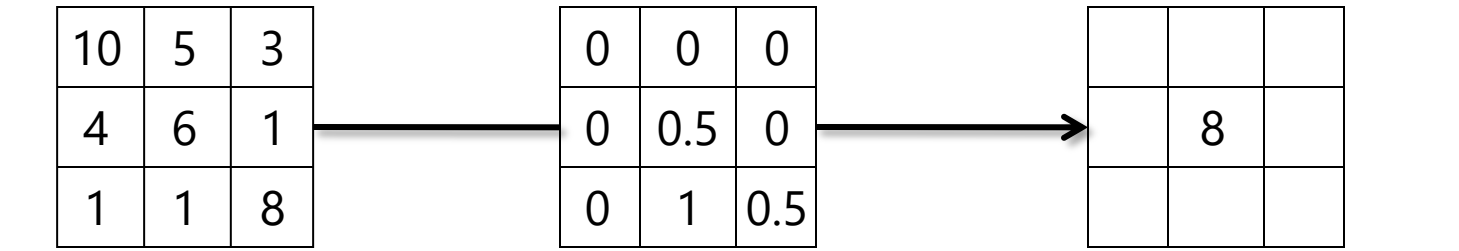
- Modify the pixels in an image based on some function of a local neighborhood of each pixel



Source: L. Zhang

Linear filtering

- One simple version of filtering: linear filtering (cross-correlation, convolution)
  - Replace each pixel by a linear combination (a weighted sum) of its neighbors
- The prescription for the linear combination is called the “kernel” (or “mask”, “filter”)



Source: L. Zhang

Cross-correlation

Let F be the image, H be the kernel (of size $2k+1$ x $2k+1$), and G be the output image

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

This is called a **cross-correlation** operation:

$$G = H \otimes F$$

- Can think of as a "dot product" between local neighborhood and kernel for each pixel

Convolution

- Same as cross-correlation, except that the kernel is "flipped" (horizontally and vertically)

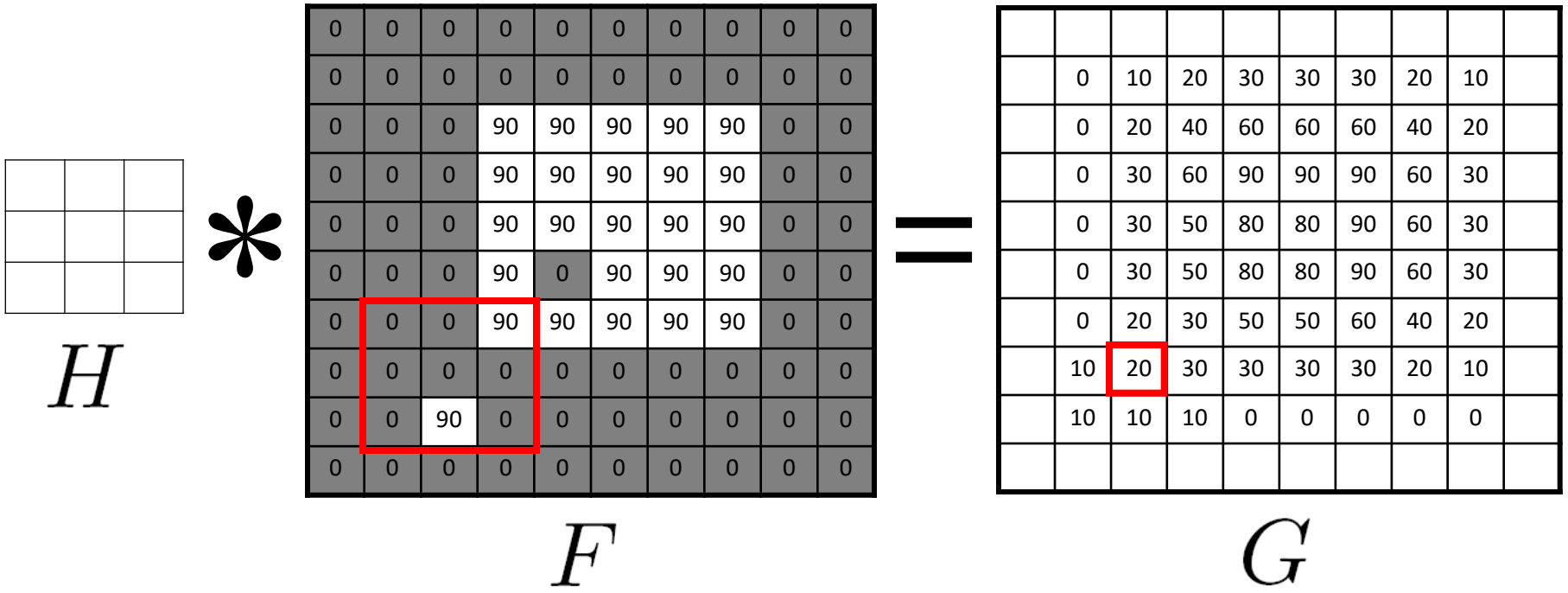
$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

This is called a **convolution** operation:

$$G = H * F$$

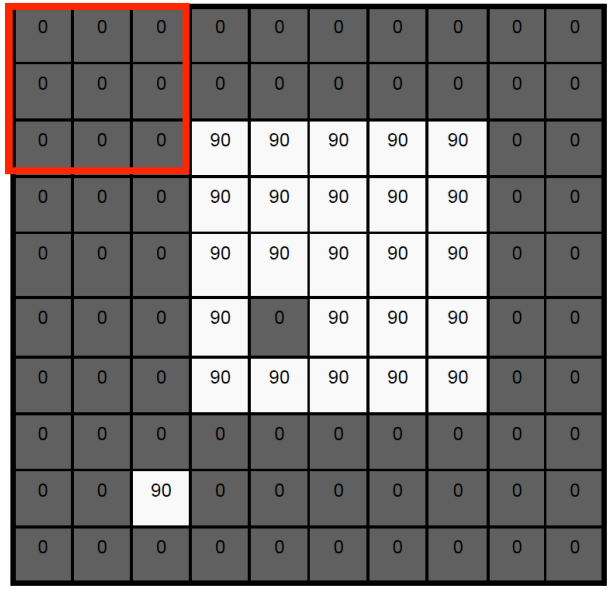
- Convolution is **commutative** and **associative**

Mean filtering

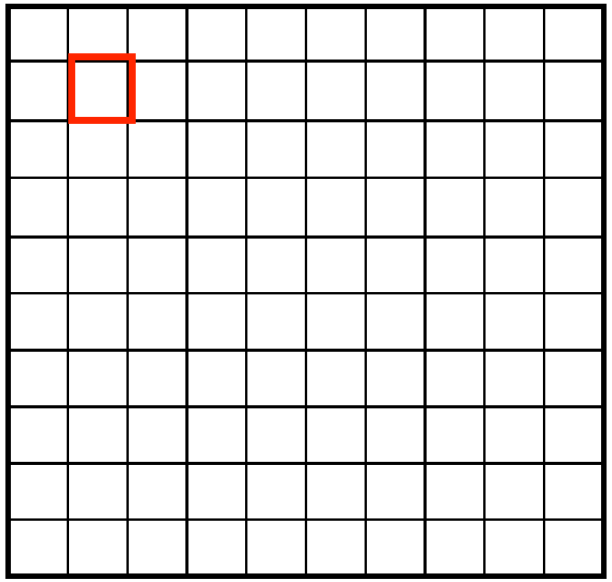


Mean filtering/Moving average

$F[x, y]$

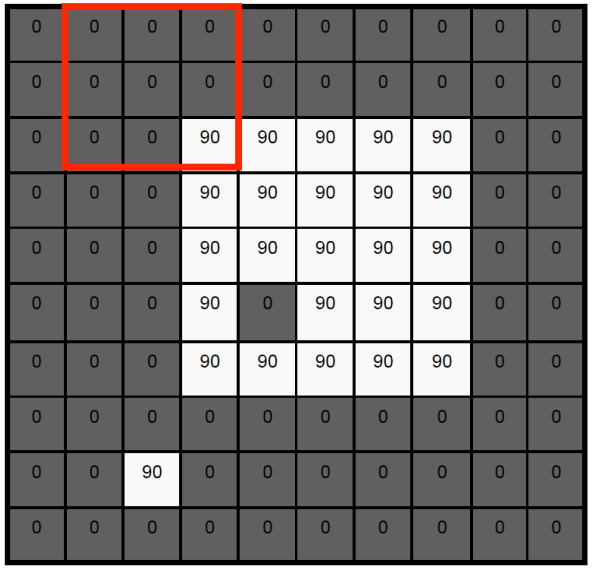


$G[x, y]$

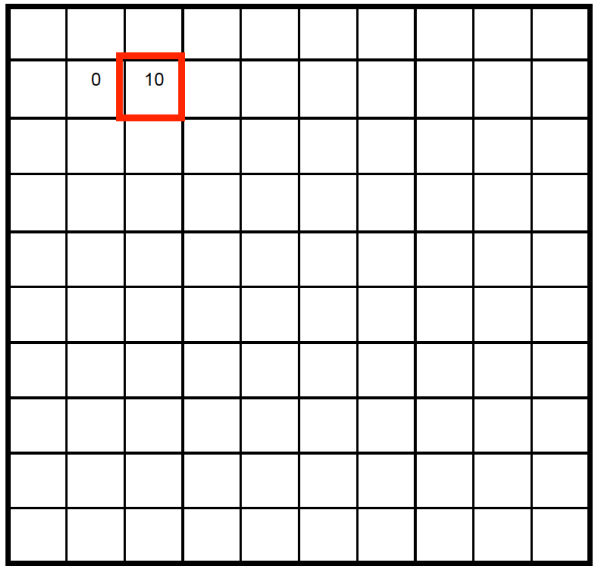


Mean filtering/Moving average

$F[x, y]$

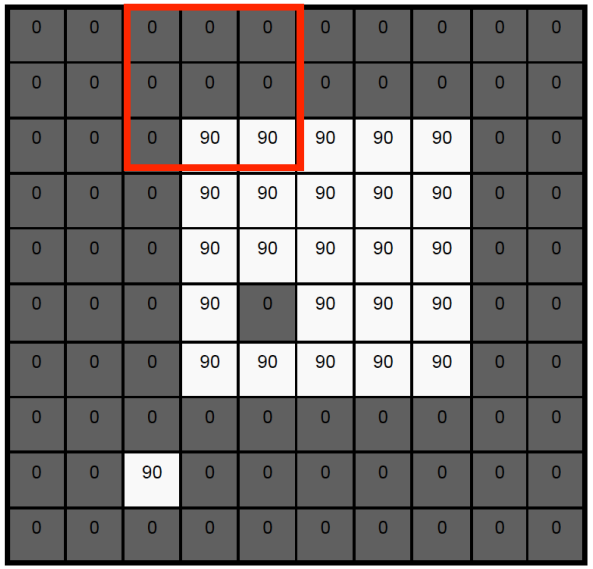


$G[x, y]$

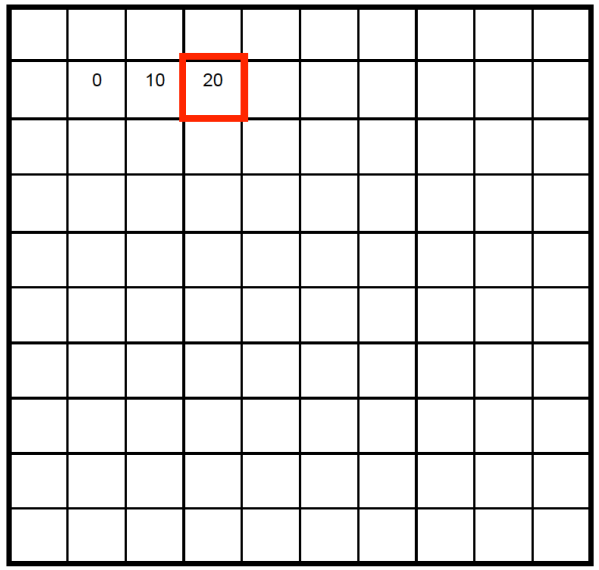


Mean filtering/Moving average

$F[x, y]$

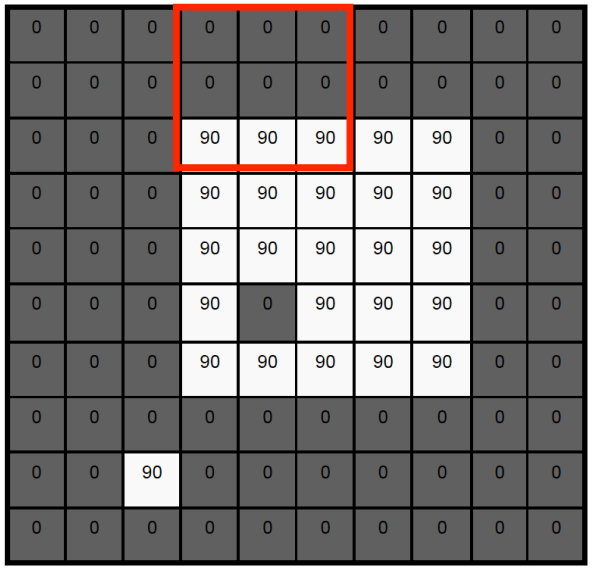


$G[x, y]$

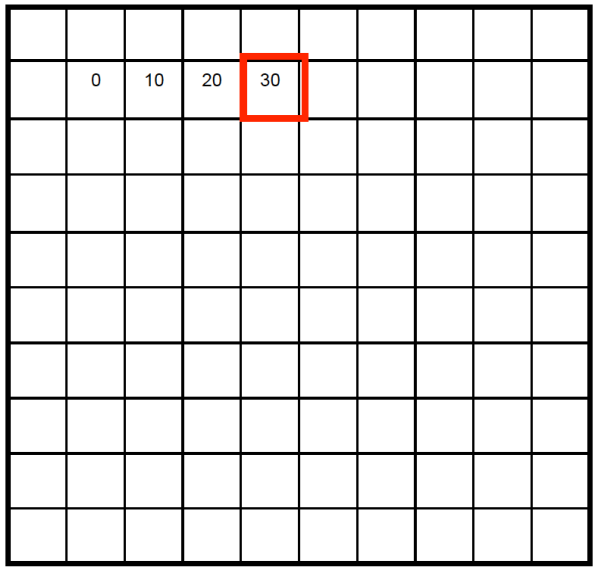


Mean filtering/Moving average

$F[x, y]$



$G[x, y]$



Mean filtering/Moving average

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30				

Mean filtering/Moving average

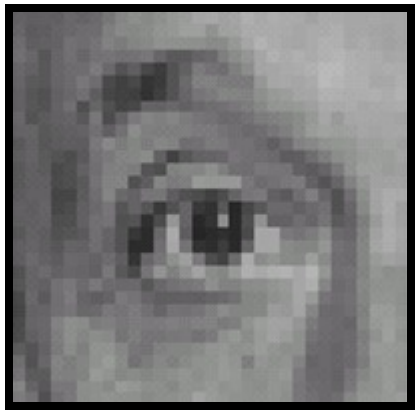
$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Linear filters: examples

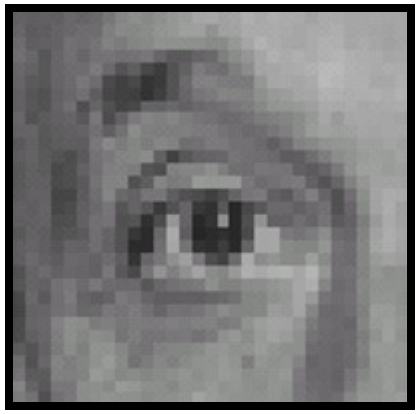


Original

$$*\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Source: D. Lowe

Linear filters: examples



Original

$*$

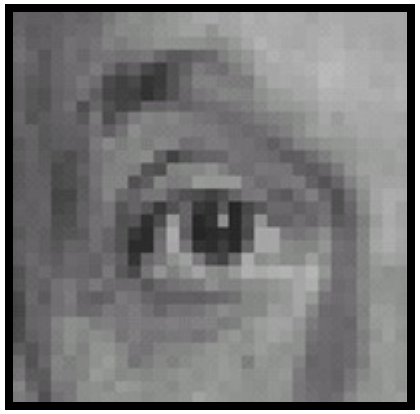
0	0	0
0	1	0
0	0	0

$=$

Identical image

Source: D. Lowe

Linear filters: examples



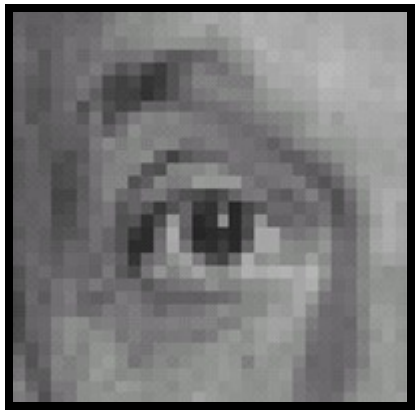
Original

*

0	0	0
1	0	0
0	0	0

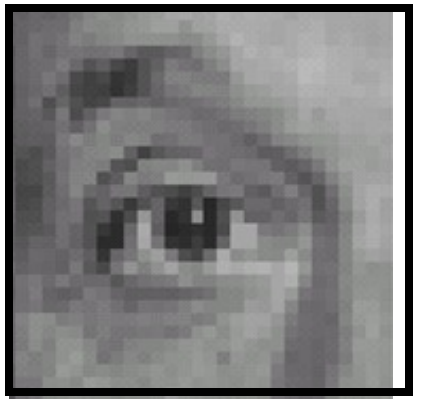
Source: D. Lowe

Linear filters: examples



Original

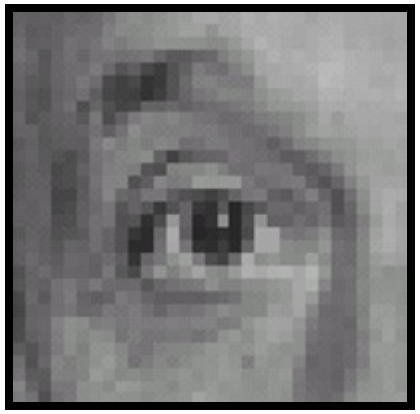
$$*\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$$



Shifted left by 1 pixel

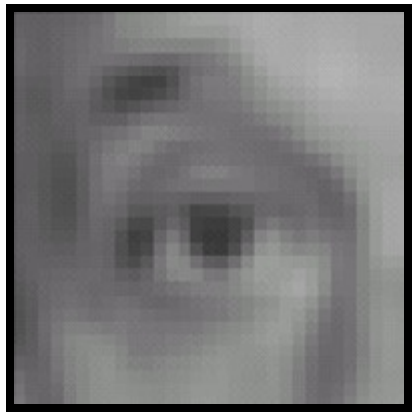
Source: D. Lowe

Linear filters: examples



Original

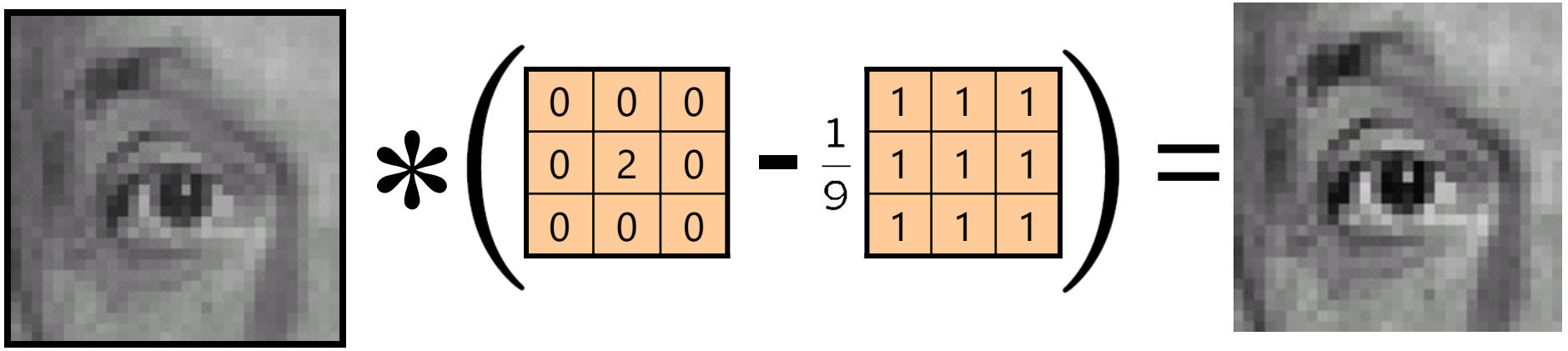
$$* \quad \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad =$$



Blur (with a mean filter)

Source: D. Lowe

Linear filters: examples

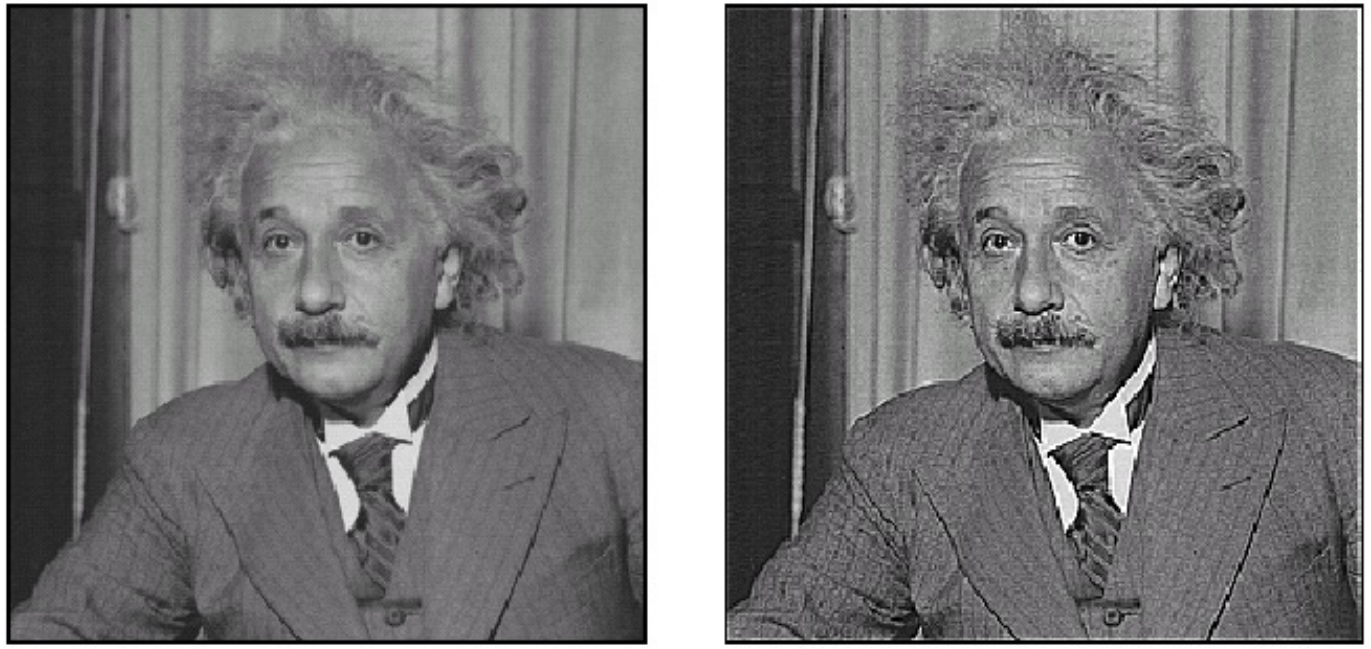


Original

Sharpening filter
(accentuates edges)

Source: D. Lowe

Sharpening

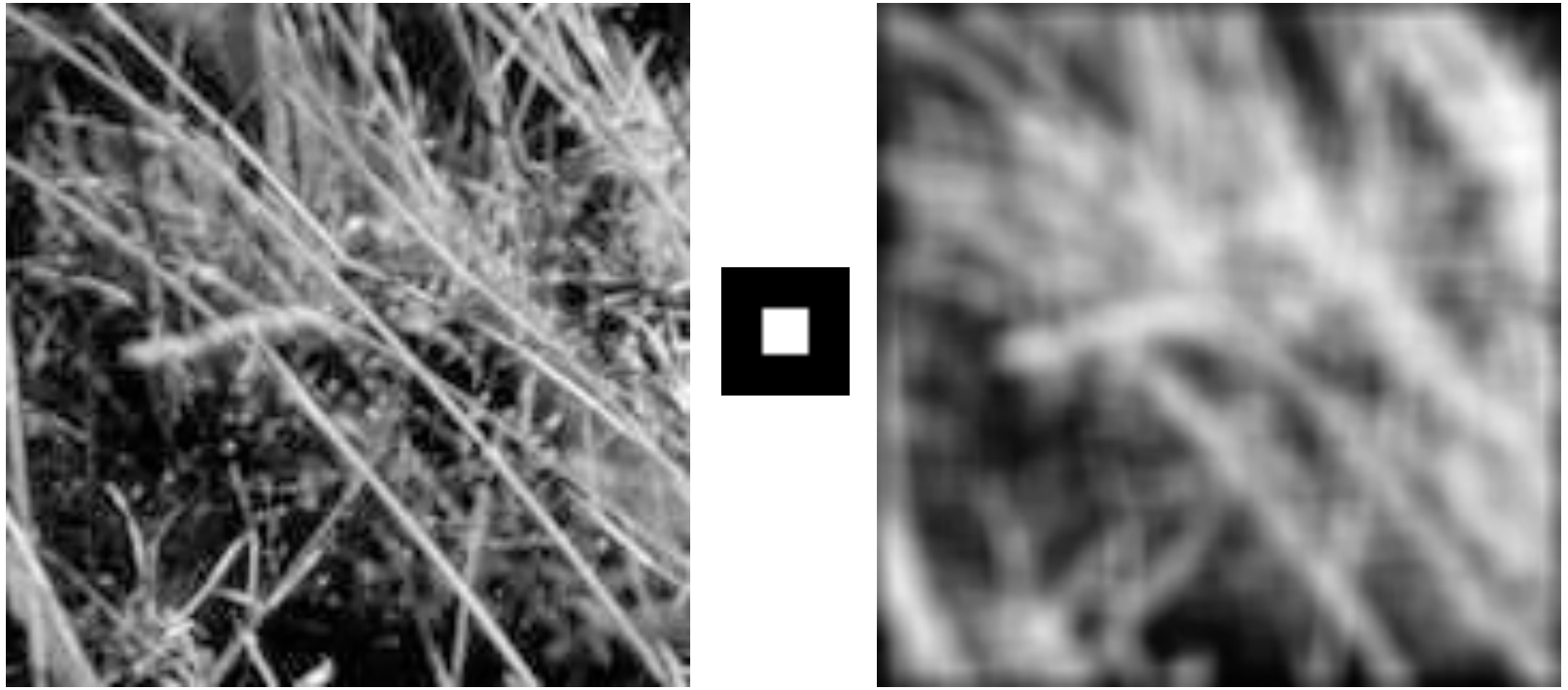


before

after

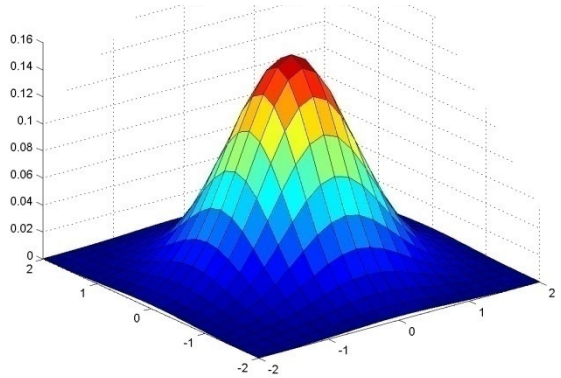
Source: D. Lowe

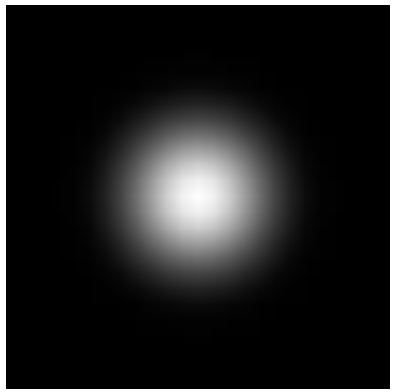
Smoothing with box filter revisited



Source: D. Forsyth

Gaussian kernel

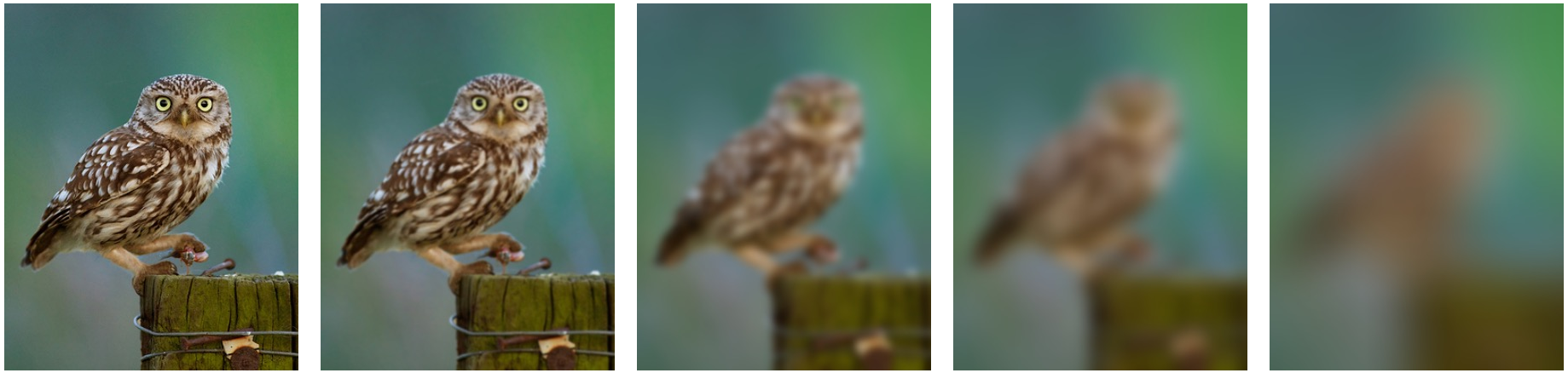




$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

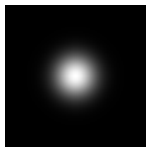
Source: C. Rasmussen

Gaussian filters



σ = 1 pixel

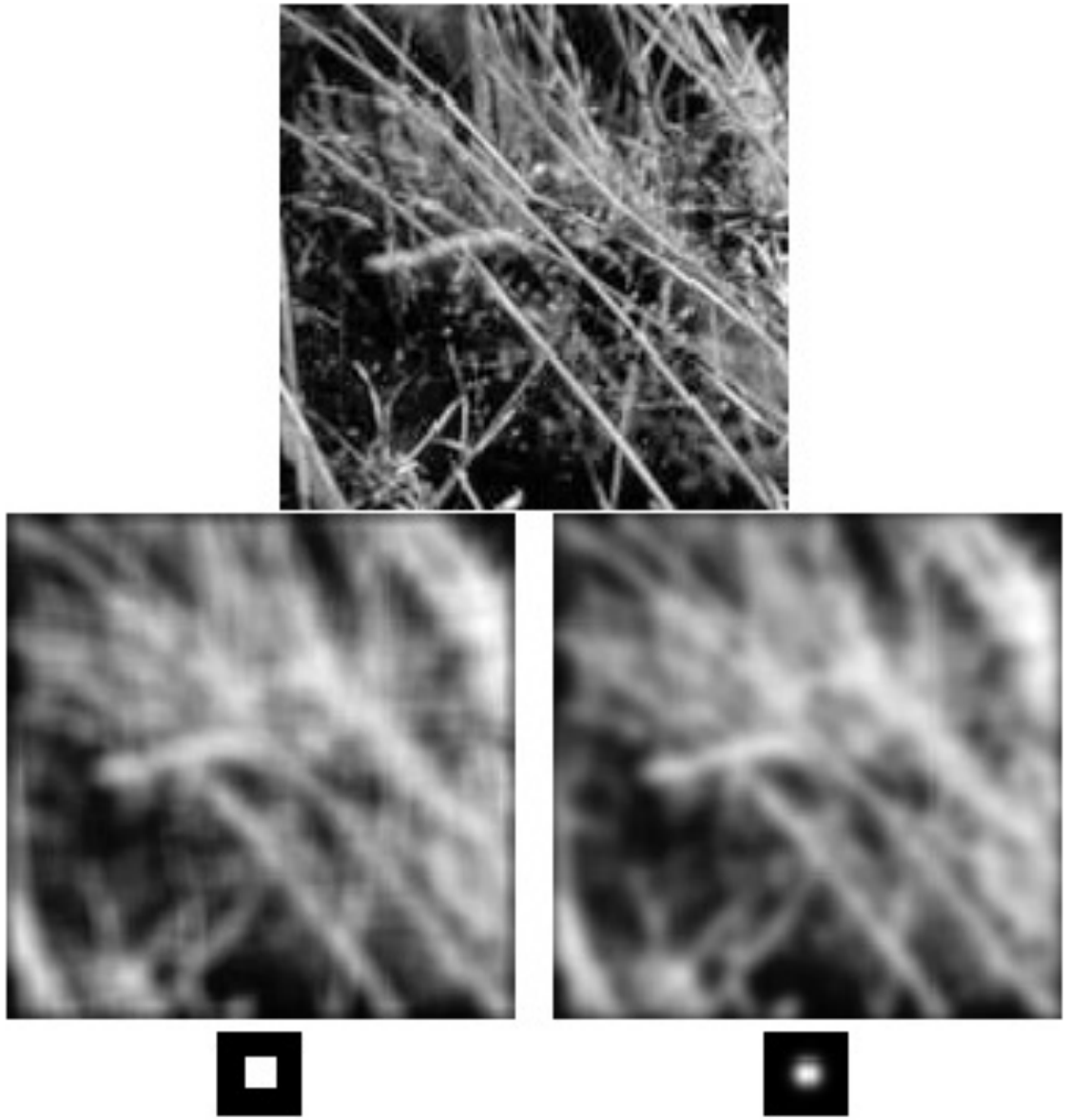
σ = 5 pixels



σ = 10 pixels

σ = 30 pixels

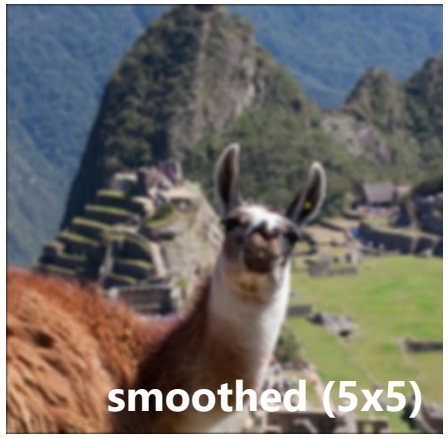
Mean vs. Gaussian filtering



Sharpening revisited

- What does blurring take away?

−



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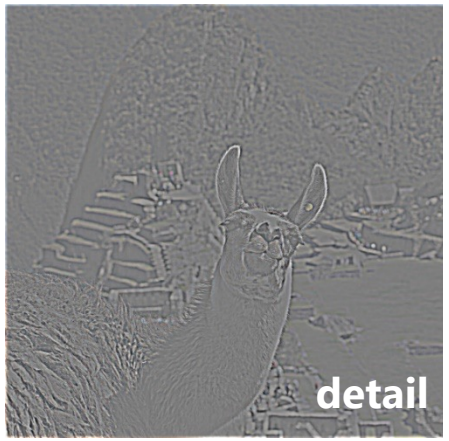

(This “detail extraction” operation is also called a ***high-pass filter***)

Let’s add it back:

+ α

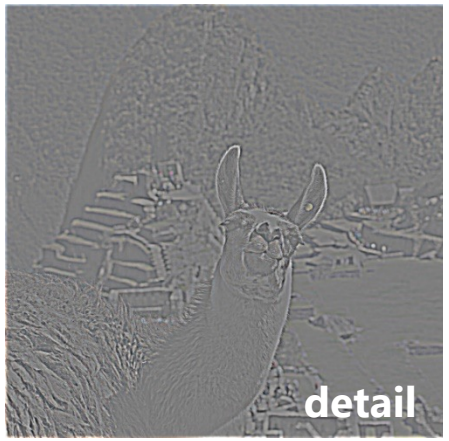

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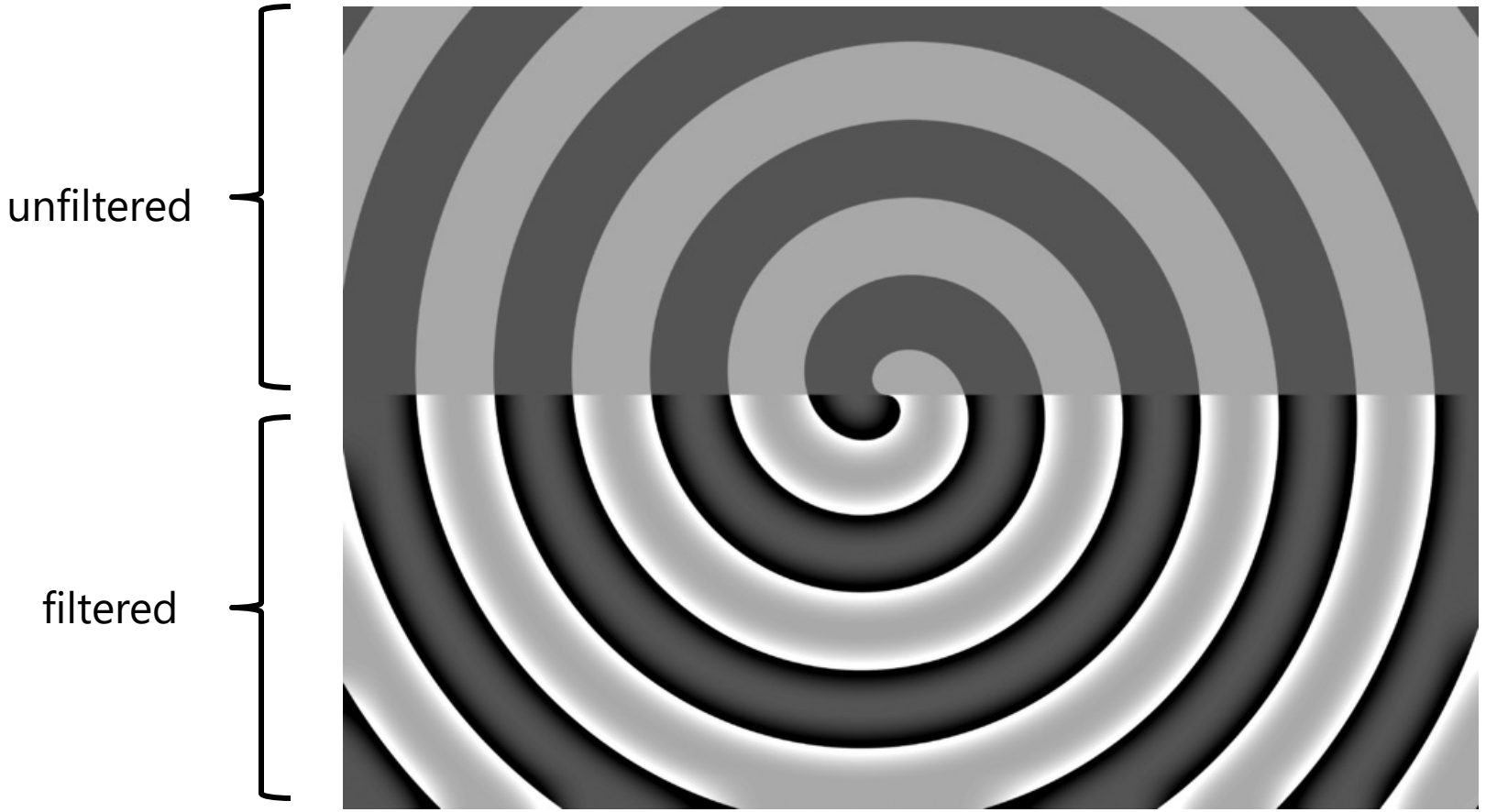
Photo credit: https://www.flickr.com/photos/geezaweezer/16089096376/

Sharpen filter



Filters: Thresholding

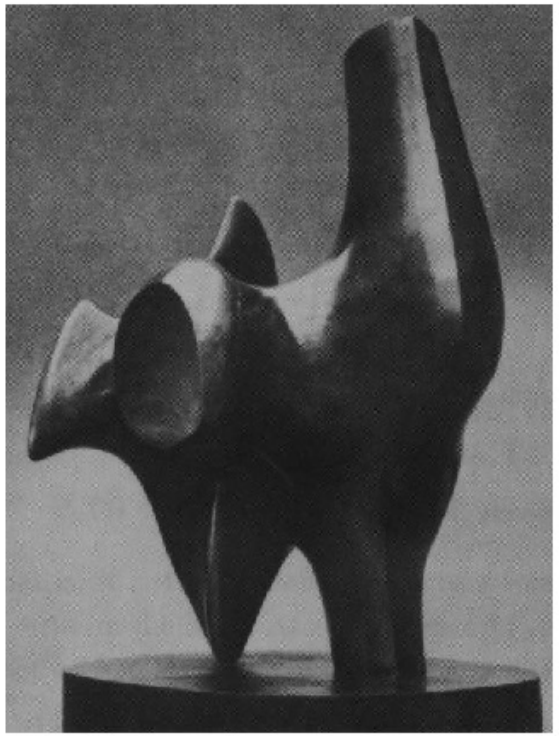
$$g(m,n) = \begin{cases} 255, & f(m,n) > A \\ 0 & otherwise \end{cases}$$

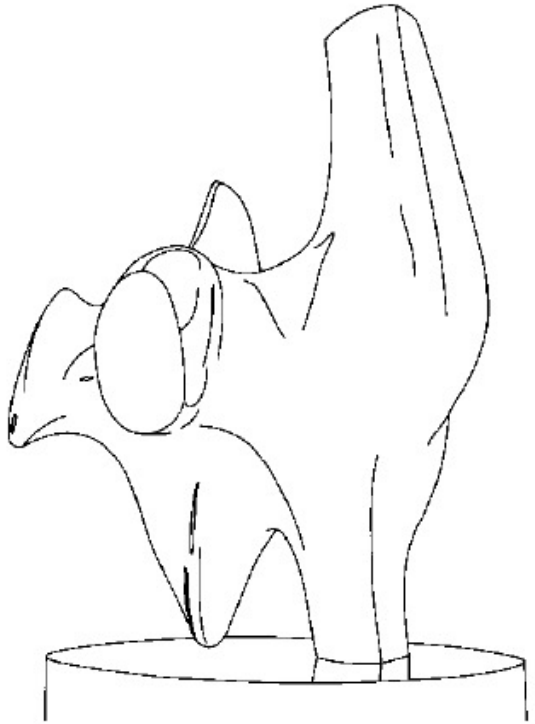
Edge Detection

- Edge detection is an image processing technique for finding the boundaries of objects within images.

- It looks for discontinuities in brightness.

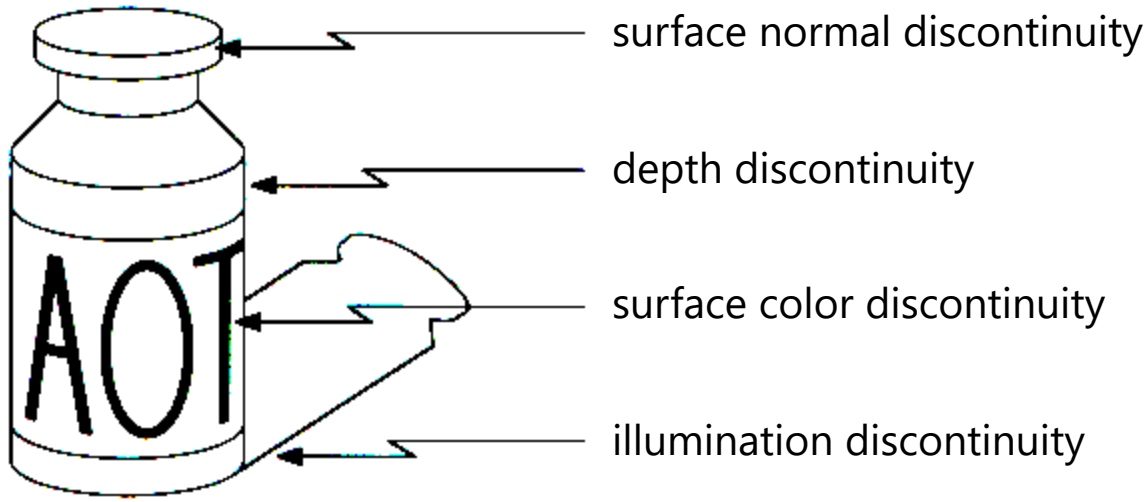
Edge detection





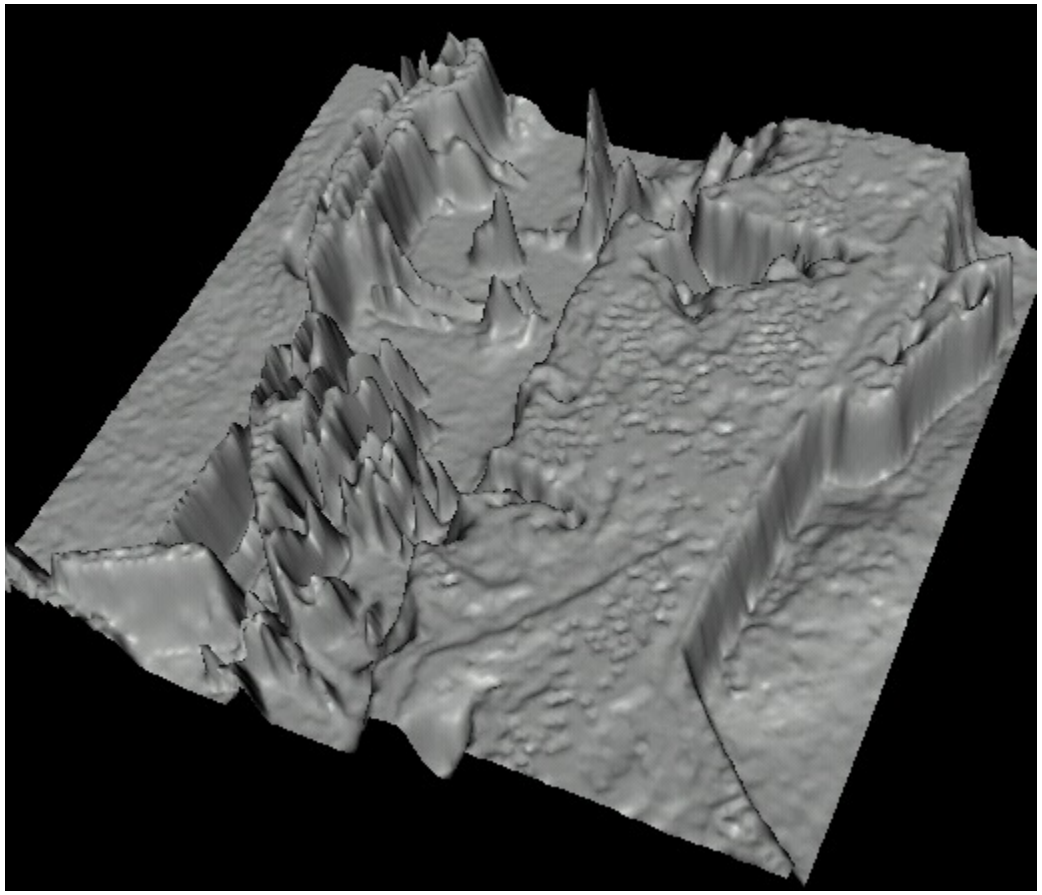
- Convert a 2D image into a set of curves
  - Extracts salient features of the scene
  - More compact than pixels

Origin of edges



- Edges are caused by a variety of factors

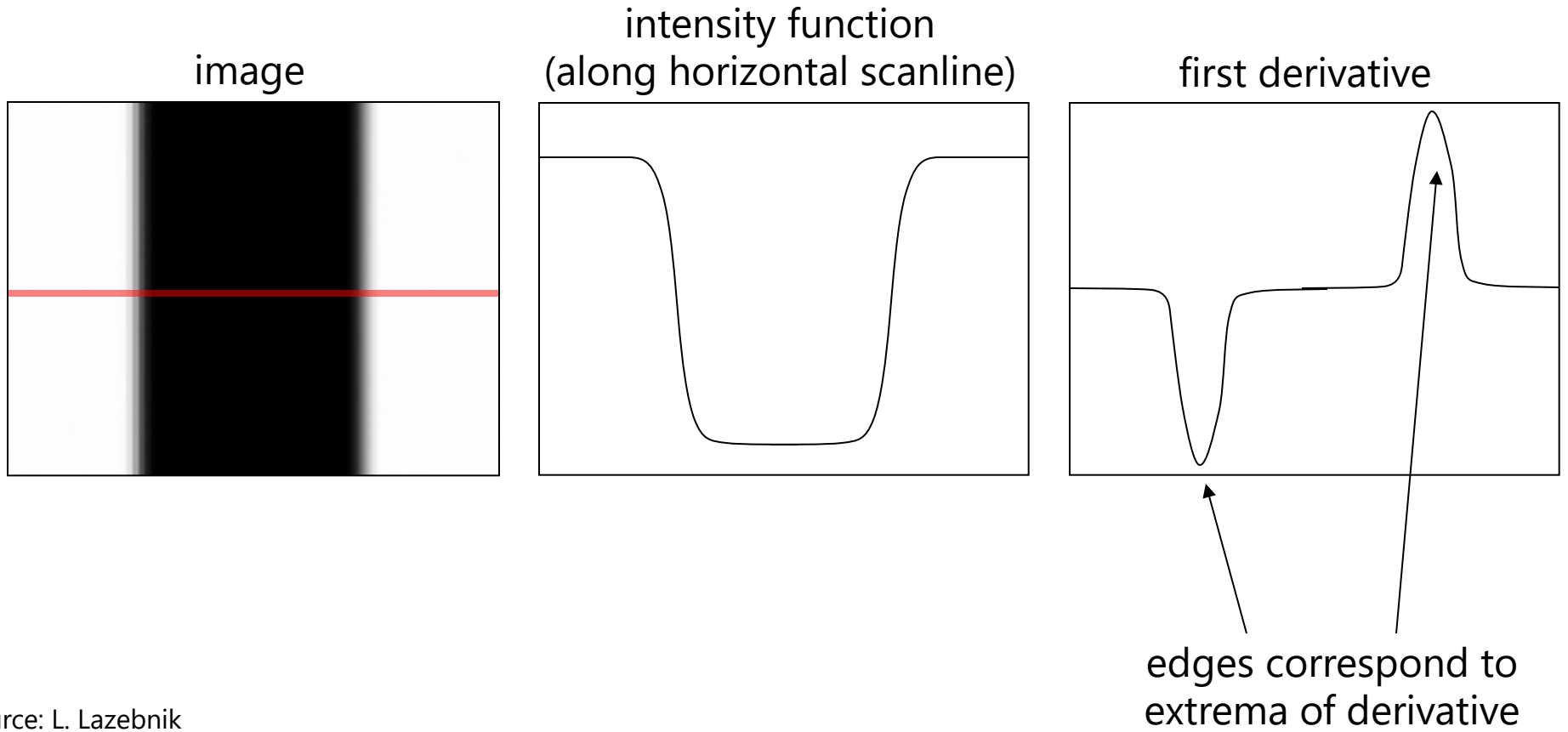
Images as functions…



- Edges look like steep cliffs

Characterizing edges

- An edge is a place of *rapid change* in the image intensity function



Source: L. Lazebnik

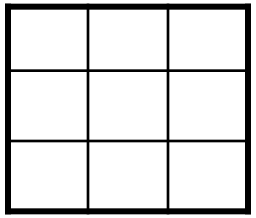
Image derivatives

- How can we differentiate a *digital* image F[x,y]?
  - Option 1: reconstruct a continuous image, *f*, then compute the derivative
  - Option 2: take discrete derivative (finite difference)

$$\frac{\partial f}{\partial x}[x, y] \approx F[x+1, y] - F[x, y]$$

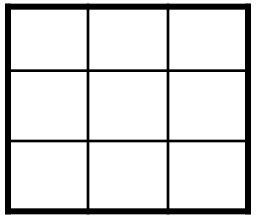
How would you implement this as a linear filter?

$\frac{\partial f}{\partial x}$:



H_x

$\frac{\partial f}{\partial y}$:



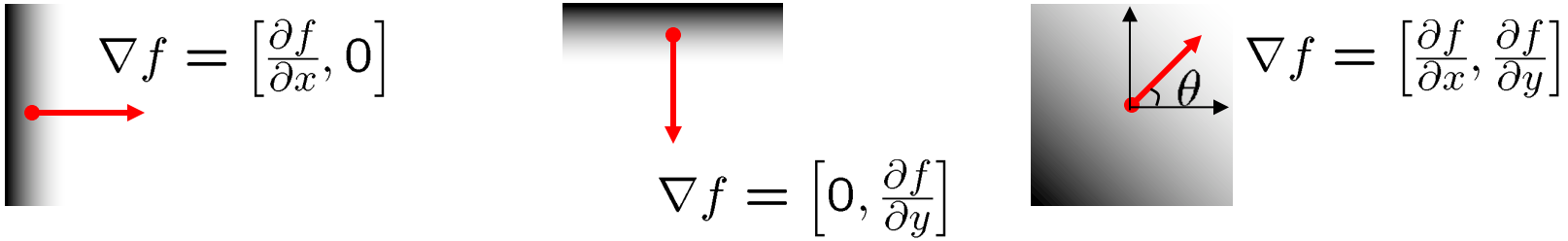
H_y

Source: S. Seitz

Image gradient

- The *gradient* of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

  The gradient points in the direction of most rapid increase in intensity



The *edge strength* is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

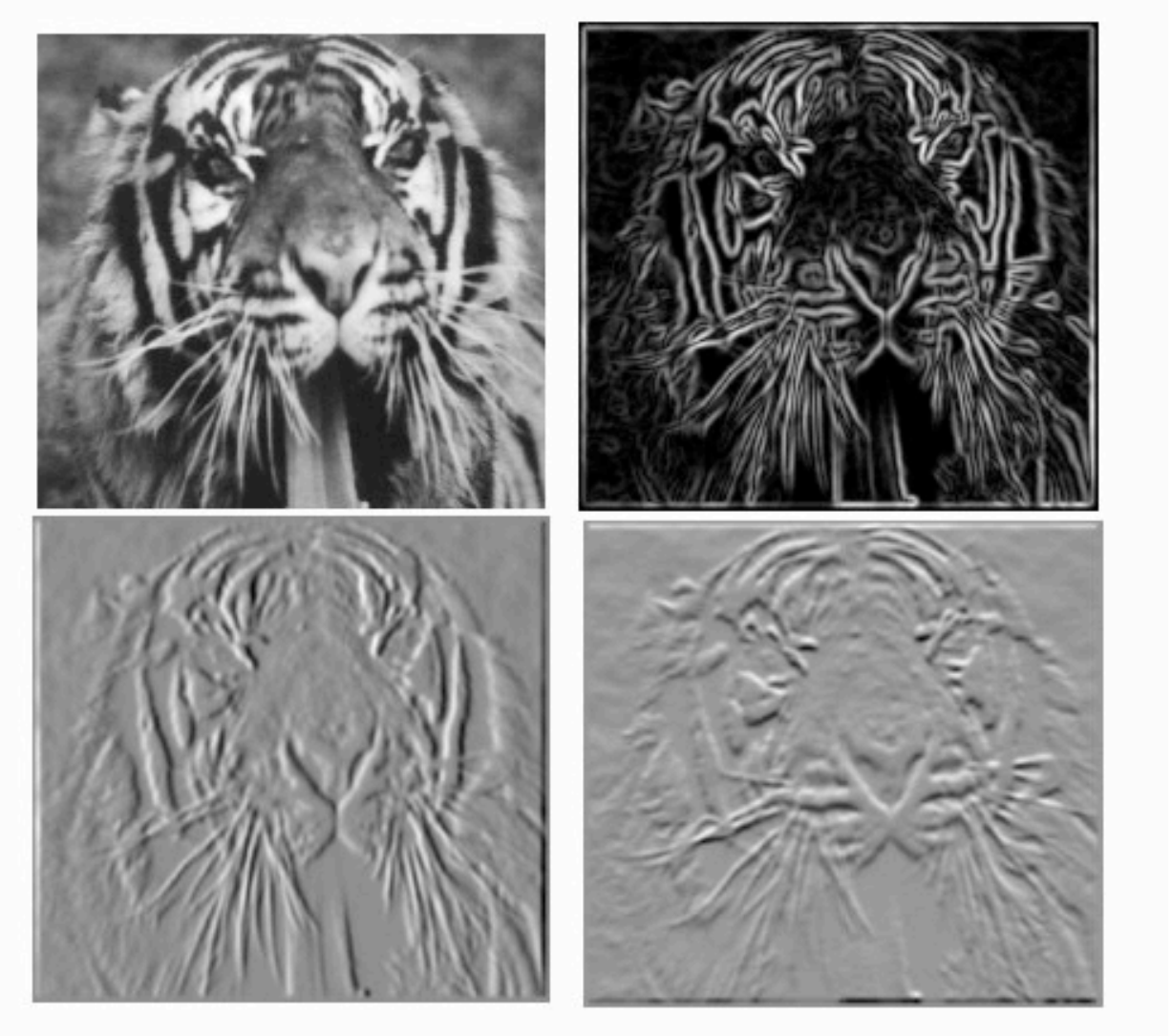
The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

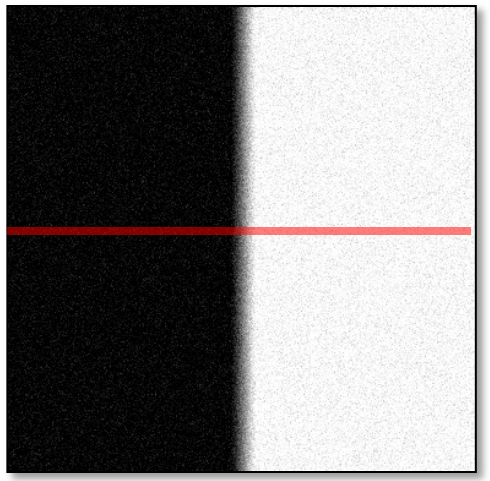
Source: Steve Seitz

Image gradient



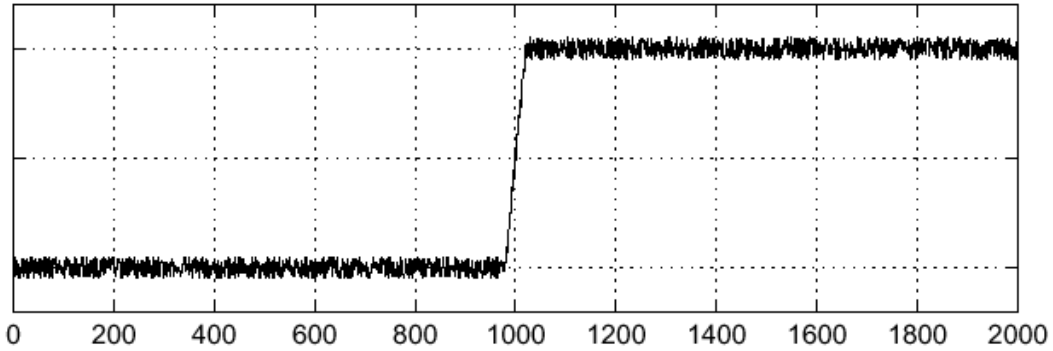
Source: L. Lazebnik

Effects of noise

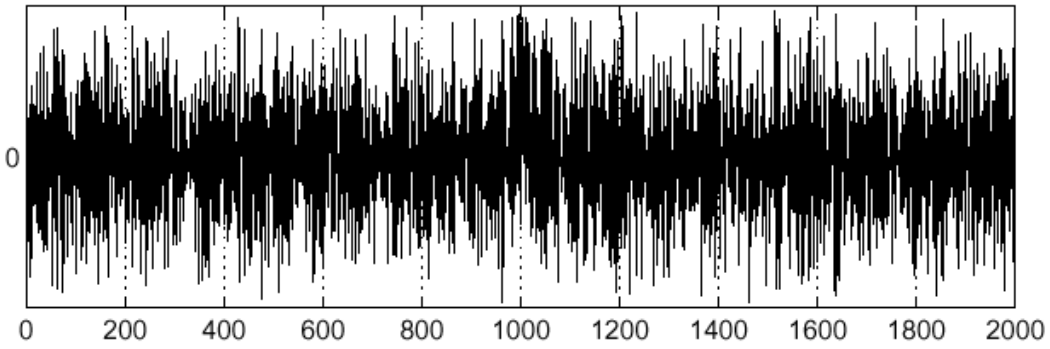


Noisy input image

$f(x)$



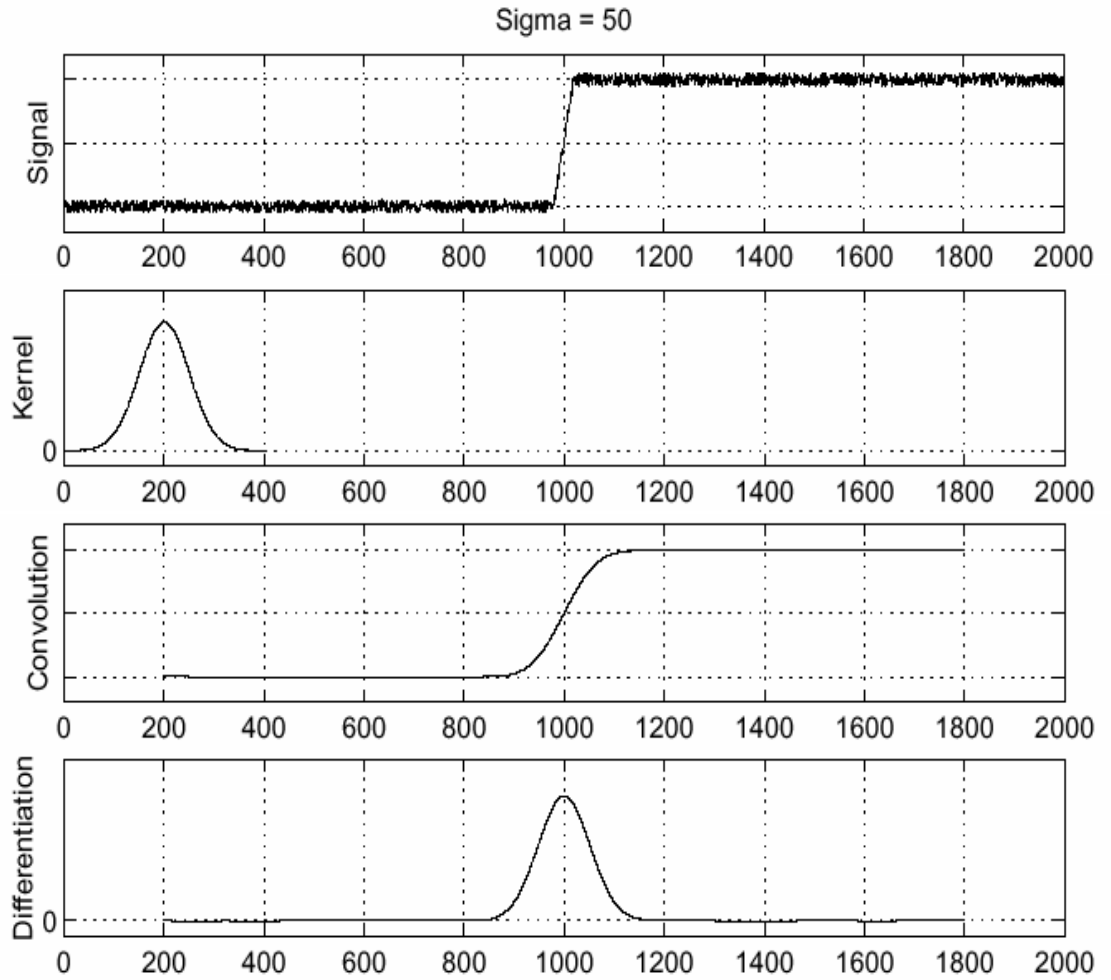
$\frac{d}{dx}f(x)$



Where is the edge?

Source: S. Seitz

Solution: smooth first



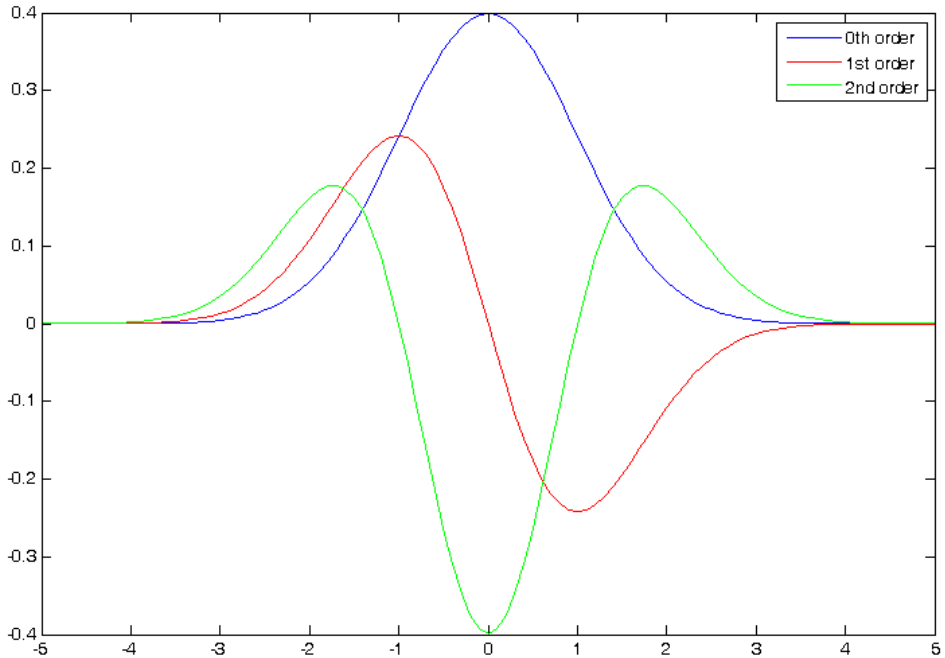
To find edges, look for peaks in $\frac{d}{dx}(f * h)$

Source: S. Seitz

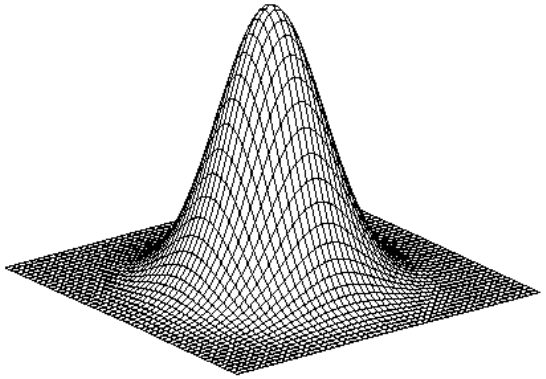
The 1D Gaussian and its derivatives

$$G_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

$$G'_\sigma(x) = \frac{d}{dx} G_\sigma(x) = -\frac{1}{\sigma}\left(\frac{x}{\sigma}\right) G_\sigma(x)$$

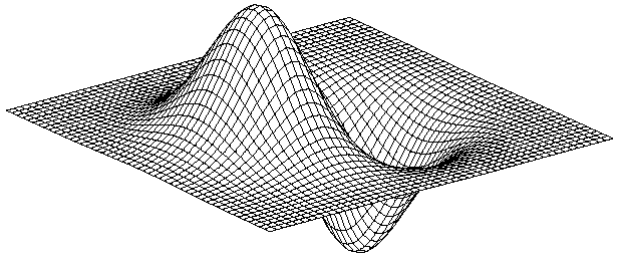


2D edge detection filters



Gaussian

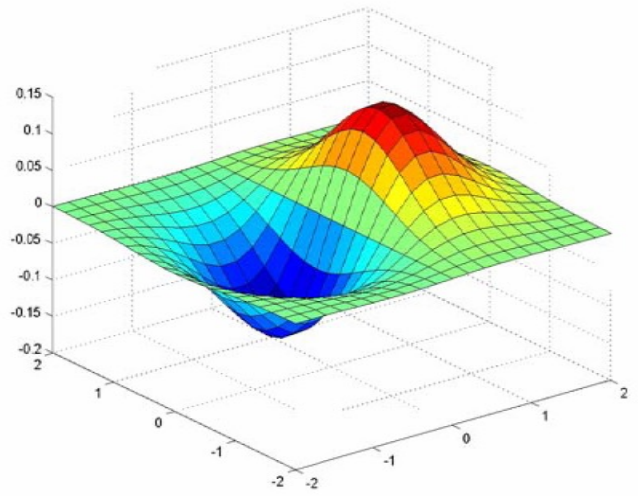
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



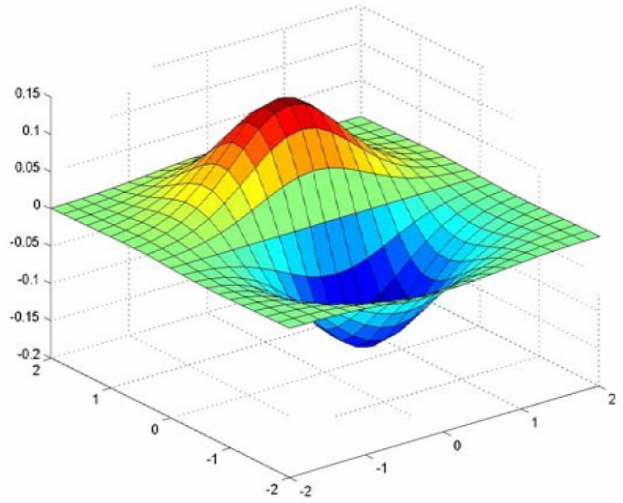
derivative of Gaussian (x)

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

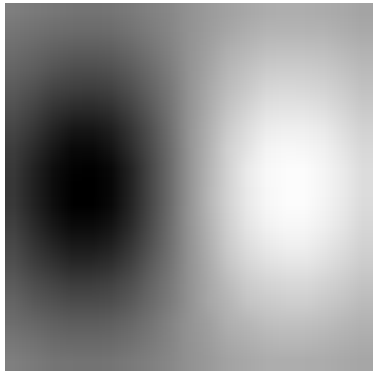
Derivative of Gaussian filter

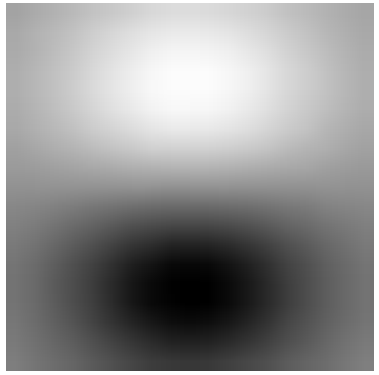


x-direction



y-direction





The Sobel operator

- Common approximation of derivative of Gaussian

$\frac{1}{8}$

-1	0	1
-2	0	2
-1	0	1

s_x

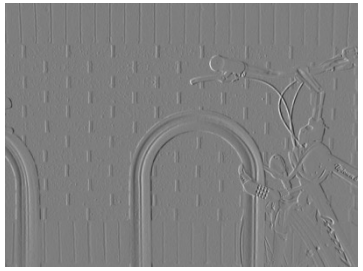
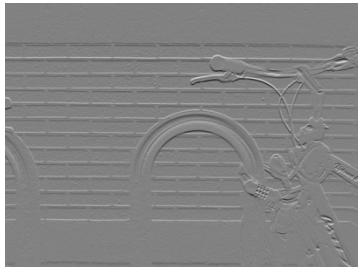
$\frac{1}{8}$

1	2	1
0	0	0
-1	-2	-1

s_y

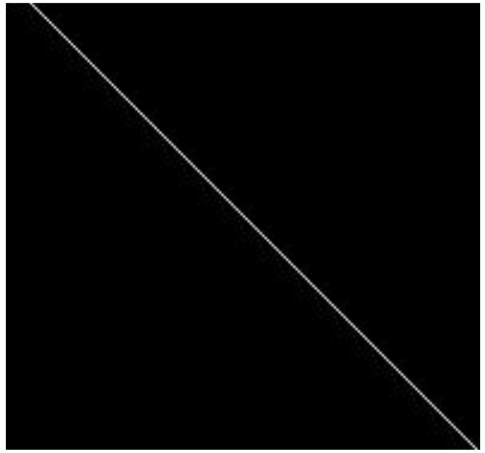
- The standard definition of the Sobel operator omits the 1/8 term
  - doesn't make a difference for edge detection
  - the 1/8 term **is** needed to get the right gradient magnitude

Sobel operator: example


Source: Wikipedia

Image with Edge



Edge Location

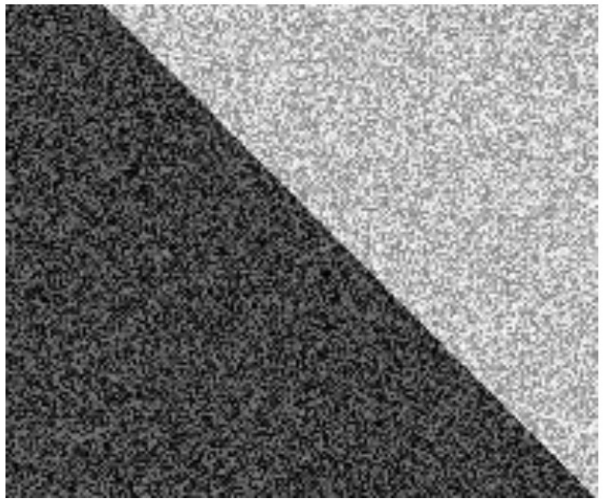
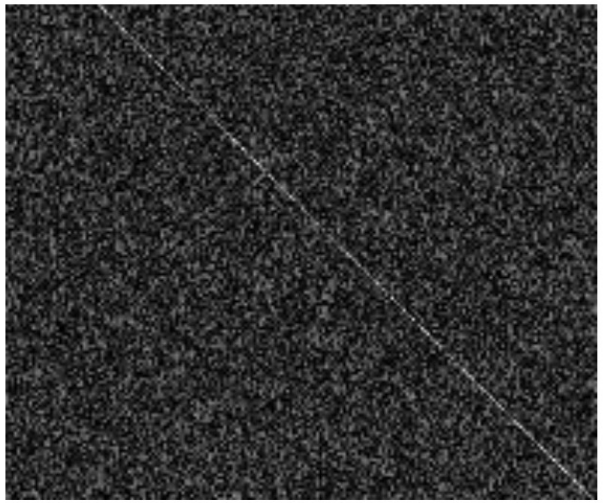
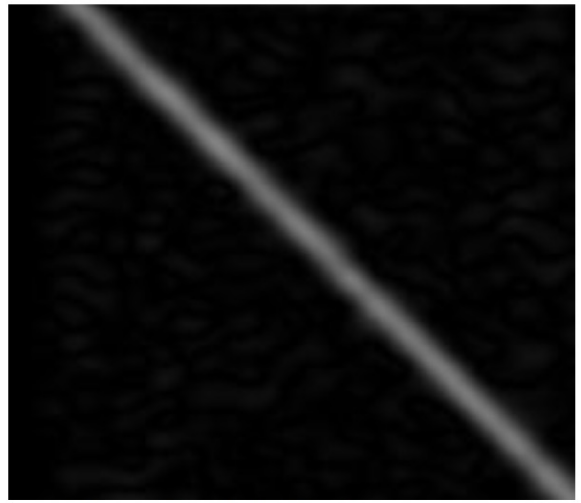


Image + Noise



Derivatives detect
edge *and* noise



Smoothed derivative removes
noise, but blurs edge

Example

original image

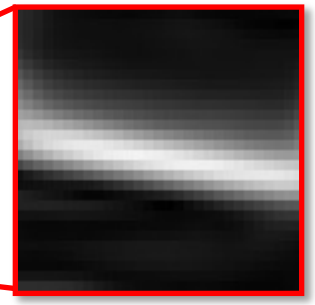
Demo: http://bigwww.epfl.ch/demo/ip/demos/edgeDetector/

Image credit: Joseph Redmon

Finding edges

smoothed gradient magnitude

Finding edges

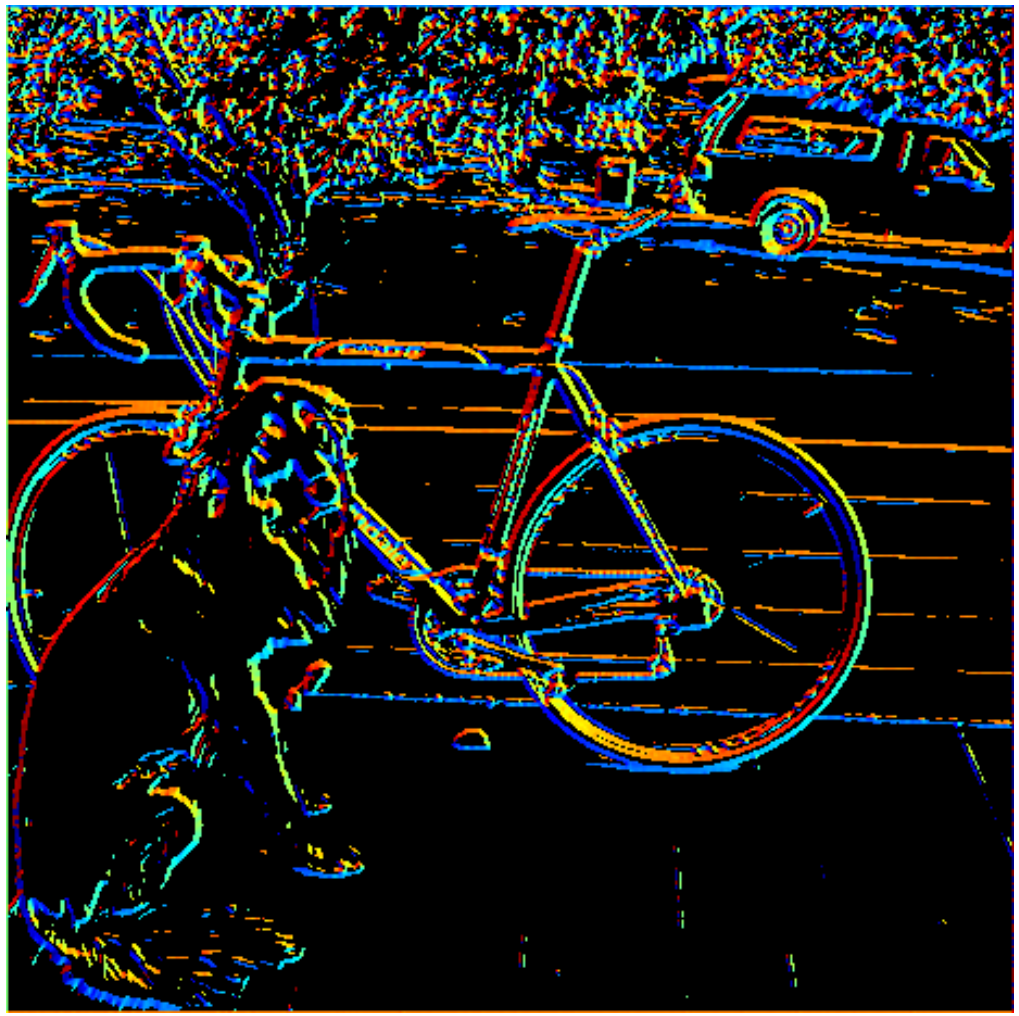


where is the edge?

thresholding

Get Orientation at Each Pixel

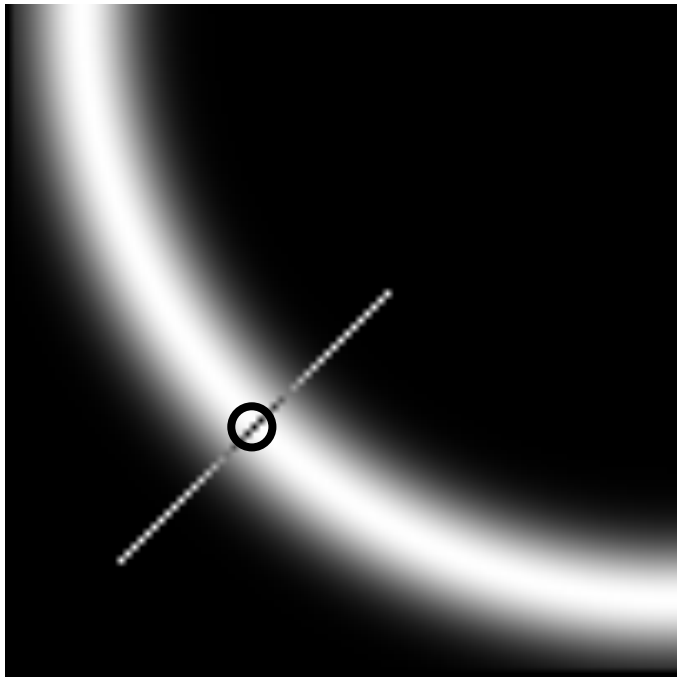
- Get orientation (below, threshold at minimum gradient magnitude)

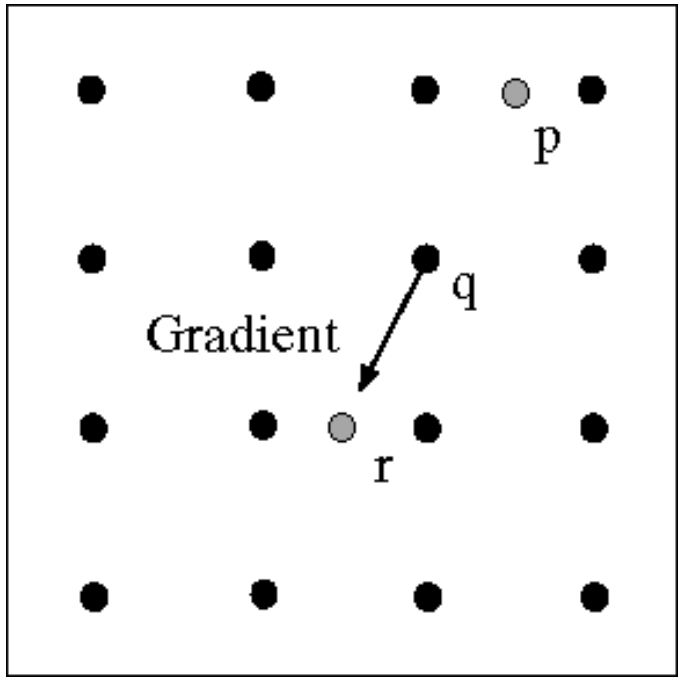


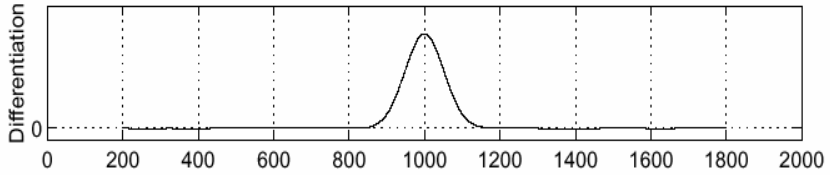
theta = atan2(gy, gx)

Gradient orientation angle

Non-maximum supression







- Check if pixel is local maximum along gradient direction
  - requires *interpolating* pixels p and r

Before Non-max Suppression

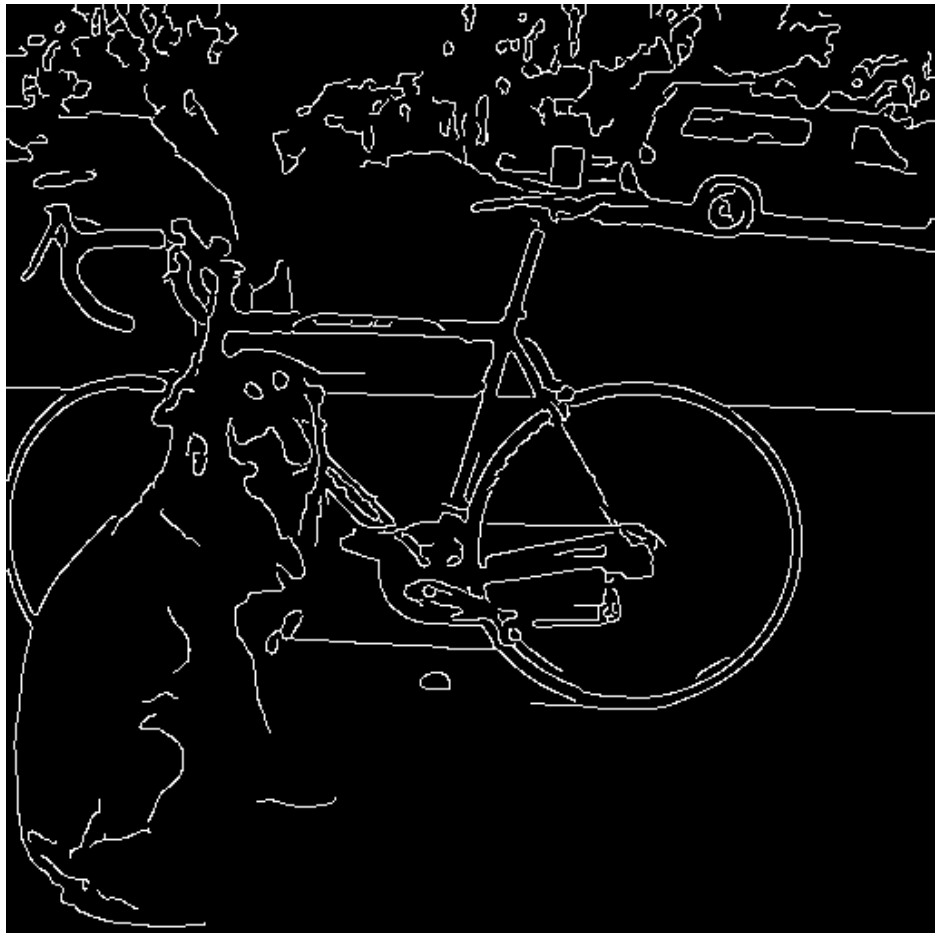
After Non-max Suppression

Thresholding edges

- Still some noise
- Only want strong edges
- 2 thresholds, 3 cases
  - R > T: strong edge
  - R < T but R > t: weak edge
  - R < t: no edge
- Why two thresholds?

Connecting edges

- Strong edges are edges!
- Weak edges are edges
  if they connect to strong
- Look in some neighborhood
  (usually 8 closest)



Canny edge detector

MATLAB: `edge(image,'canny')`

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression
4. Linking and thresholding (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

Source: D. Lowe, L. Fei-Fei, J. Redmon

Canny edge detector

- Our first computer vision pipeline!
- Still a widely used edge detector in computer vision

J. Canny, ***A Computational Approach To Edge Detection***, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

- Depends on several parameters:

high threshold
low threshold

σ : width of the Gaussian blur

Canny edge detector

original

Canny with $\sigma = 1$

Canny with $\sigma = 2$

- The choice of σ depends on desired behavior
  - large σ detects "large-scale" edges
  - small σ detects fine edges

Source: S. Seitz