Noise-Induced Stochastic Resonance and Coherence Resonance in the Duffing Oscillator

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ABSTRACT

This report gives a brief introduction to the non-linear Duffing oscillator, its chaotic behaviour and effects of noise addition into the system. The Duffing equation has been modelled using MATLAB. Also the Duffing oscillator circuit is simulated using PSPICE and implemented using an electronic circuit. The results of chaotic behaviour of the Duffing system are reported. The resonance characteristics resulting due to noise addition are also indicated in this report.

INTRODUCTION

The Duffing system is based on a linear second order non homogeneous differential equation which describes the motion of a classical particle in a double well potential, Duffing oscillator can be regarded as a model of a periodically forced steel beam which is deflected toward the two magnets as shown in Figure 1. It is an example of a periodically forced oscillator with a nonlinear elasticity[1].

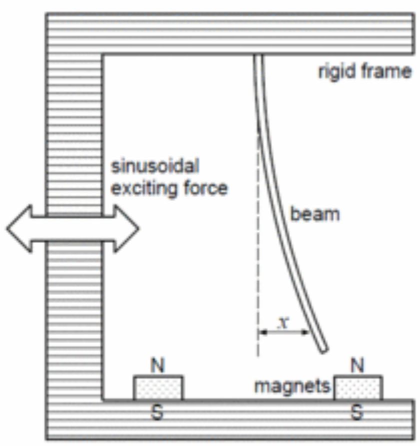


Figure 1

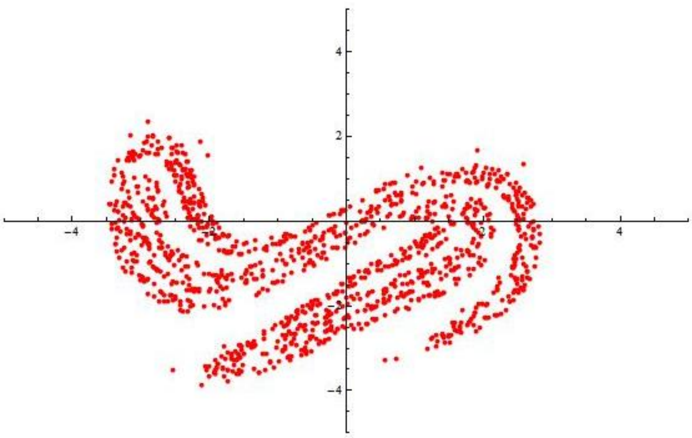


Figure 2

The Duffing system has a strange attracter to which the trajectory tends every period of the driving force, It is shown in Figure 2.

SYSTEM EQUATION

The system equation is represented as

$\ddot{x}+\delta\dot{x}+\beta x+\alpha x^3=\gamma\cos\omega t$[3]

Here, the (unknown) function $x=x(t)$ is the displacement at time t, $\dot{x}$ is the first derivative of x with respect to time, i.e. velocity, and $\ddot{x}$ is the second time-derivative of x, i.e. acceleration. The numbers δ, α, β, γ and ω are given constants.

Parameters

1. δ controls the size of the damping.
2. α controls the size of the stiffness.
3. β controls the amount of non-linearity in the restoring force. If $\beta = 0$, the Duffing equation describes a damped and driven simple harmonic oscillator.
4. γ controls the amplitude of the periodic driving force. If $\gamma = 0$ we have a system without driving force.
5. ω controls the frequency of the periodic driving force.

The Duffing oscillator can have either of three possible states

1. **The forced system**

The most general forced form of the Duffing equation is

$$\ddot{x}+\delta\dot{x}+\beta x+\alpha x^3=\gamma\cos(\omega t+\phi)$$

2. **The unforced system**

The dynamics of the unforced system ($\gamma=0$) is examined. When there is no damping ($\delta=0$), the Duffing equation can be integrated as

$$E(t) \equiv 1/2\,(\dot{x})^2 + 1/2\,\beta(x)^2 + 1/4\,\alpha(x)^4 = const\ .$$

3. **The weakly forced system : case of nonlinear resonance[2]**

Here, the response of the Duffing oscillator to a weak periodic forcing is considered and the equation can be reduced to,

$$\ddot{x}+\omega_0^2 x = \epsilon\,(\,-\delta\dot{x}-\,\alpha x^{\,3}+\,\gamma\cos\omega t\,)$$

The equation is an example of a dynamical system that exhibits chaotic behaviour.

MATLAB Code for solving Duffing System [4]

Duffing equation is modelled in MATLAB using ODE45 function which is capable of solving ordinary differential equations of up to 4th order.

1. We first declare our function @duffing

```
function xdot=duffing(t,x)

global gamma omega epsilon GAM OMEG

xdot(1)=-gamma*x(1)+omega^2*x(2)-epsilon*x(2)^3+GAM*cos(OMEG*t);
xdot(2)=x(1);

xdot=xdot';

end
```

2. Now we will call our function @duffing in main program

```
close all
clear
clc

global gamma omega epsilon GAM OMEG

gamma=0.1;
omega=1;
epsilon=0.25;
OMEG=2;

% For gamma 0.5

GAM=0.5;

[t x]=ode45(@duffing,0:2*pi/OMEG/100:4000,[0 1]);

figure(1)
plot(t(2000:6000),x(2000:6000,1),'r')
axis tight
title('time series')
figure(2)
plot(x(5000:10000,2),x(5000:10000,1),'b')
axis tight
title('phase space')

% For gamma 1.5

GAM=1.5;

[t x]=ode45(@duffing,0:2*pi/OMEG/100:4000,[0 1]);
```

```
figure(3)
plot(t(2000:6000),x(2000:6000,1),'r')
axis tight
title('time series')
figure(4)
plot(x(2000:10000,2),x(2000:10000,1),'b')
axis tight
title('phase space')
figure(5)
for i=5000:100:127300
    n=(i-4900)/100;
    x1(n)=x(i,2);
    x2(n)=x(i,1);
end
plot(x1(:),x2(:),'g.')
axis tight
title('Poincaré section')
```

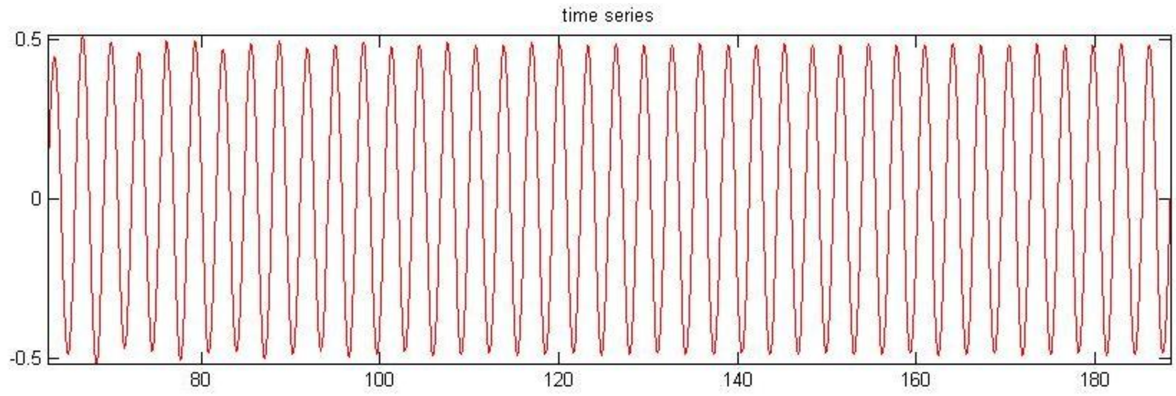
MATLAB MODELLING

We analysed the forced Duffing oscillator system using mathematical modelling in matlab The equation was analysed for following set of values, Only the driving force was varied.

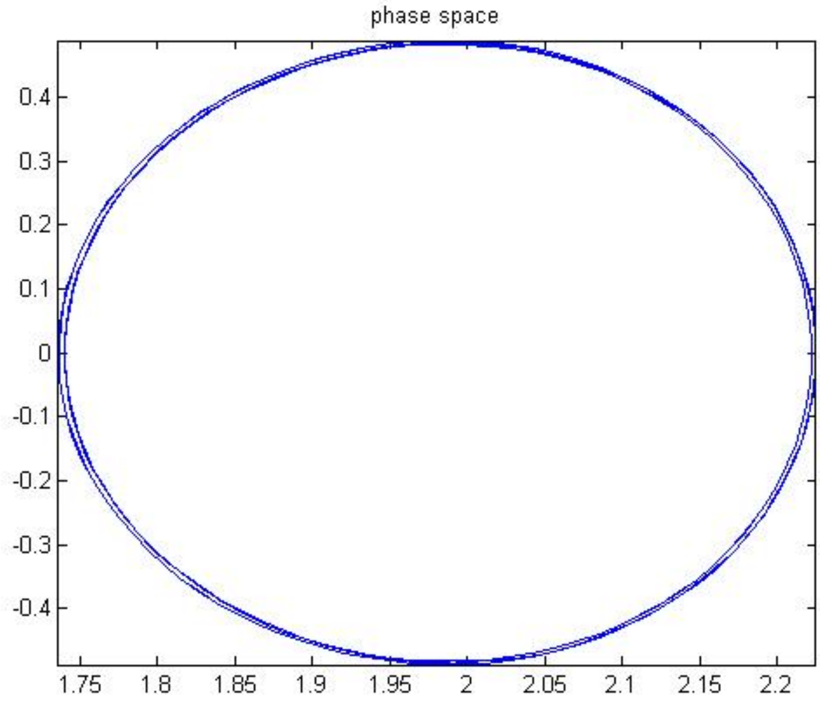
```
gamma=0.1;
omega=1;
epsilon=0.25;
OMEG=2;
```

When the driving force GAM was set to 0.5, we got a limit cycle.

1. Time series plot for GAM=0.5

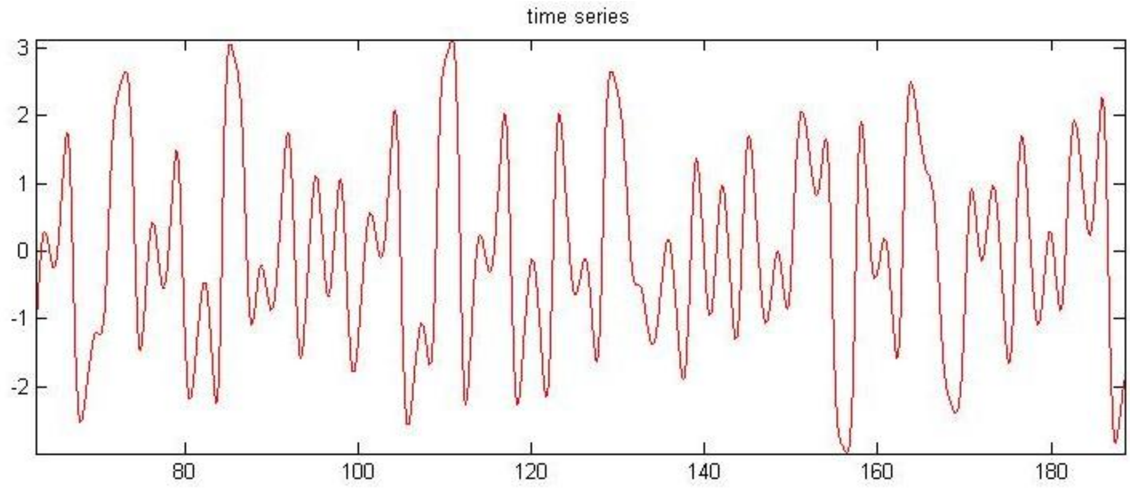


2. Phase space plot for GAM=0.5

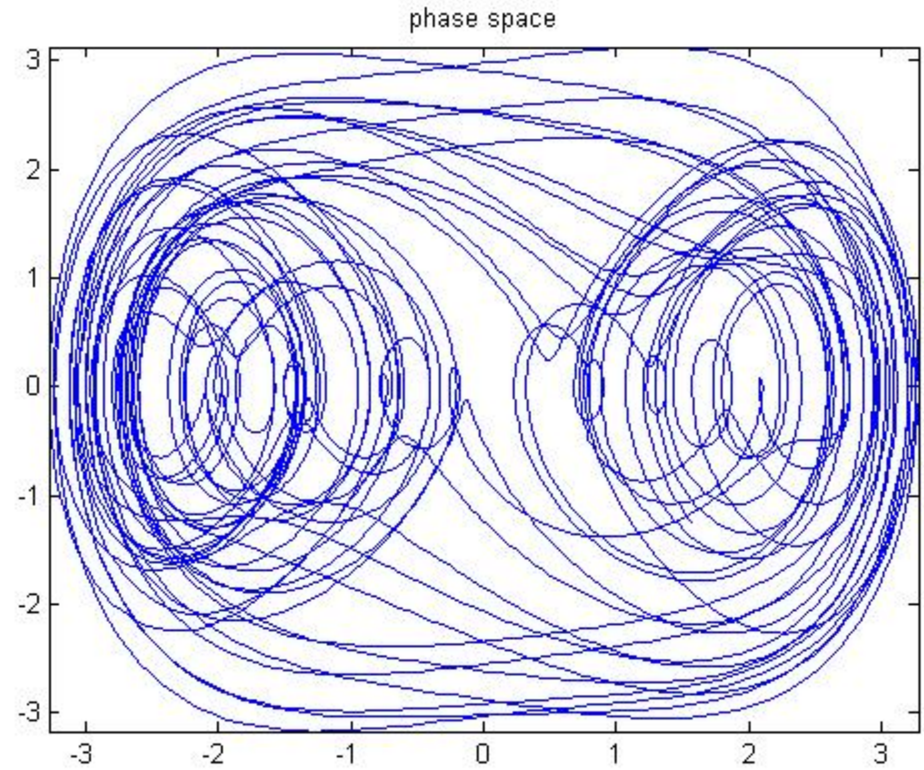


When the GAM was set to 1.5, we got a periodic time series, strange chaotic attractor, and a plot of Poincaré section

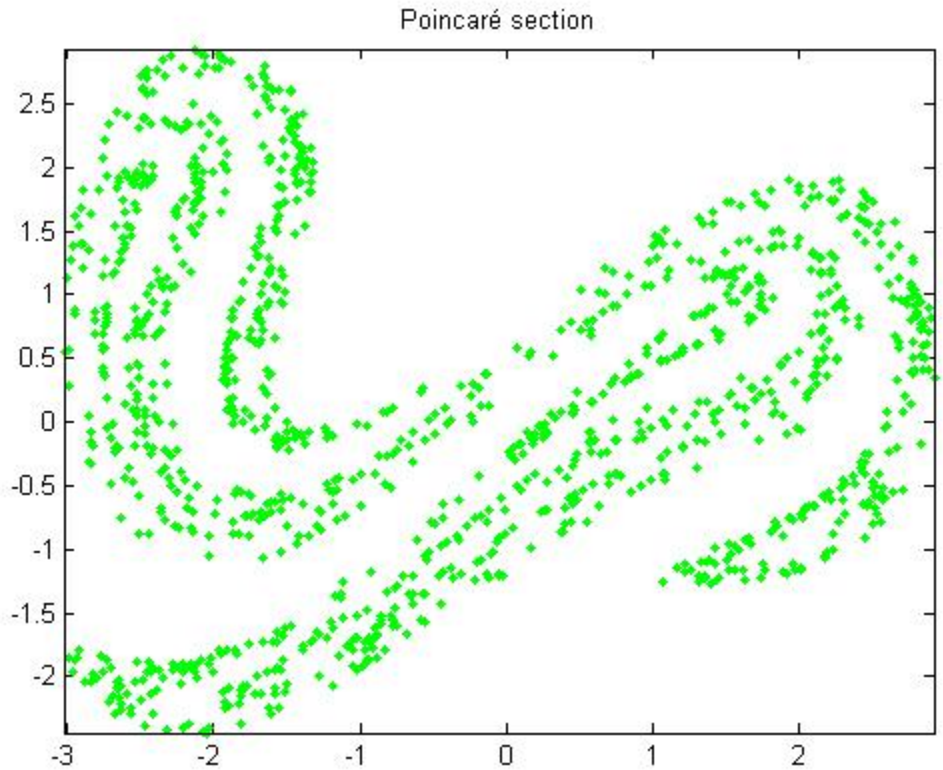
1. Time series plot for GAM=1.5



2. Phase space plot for GAM=1.5



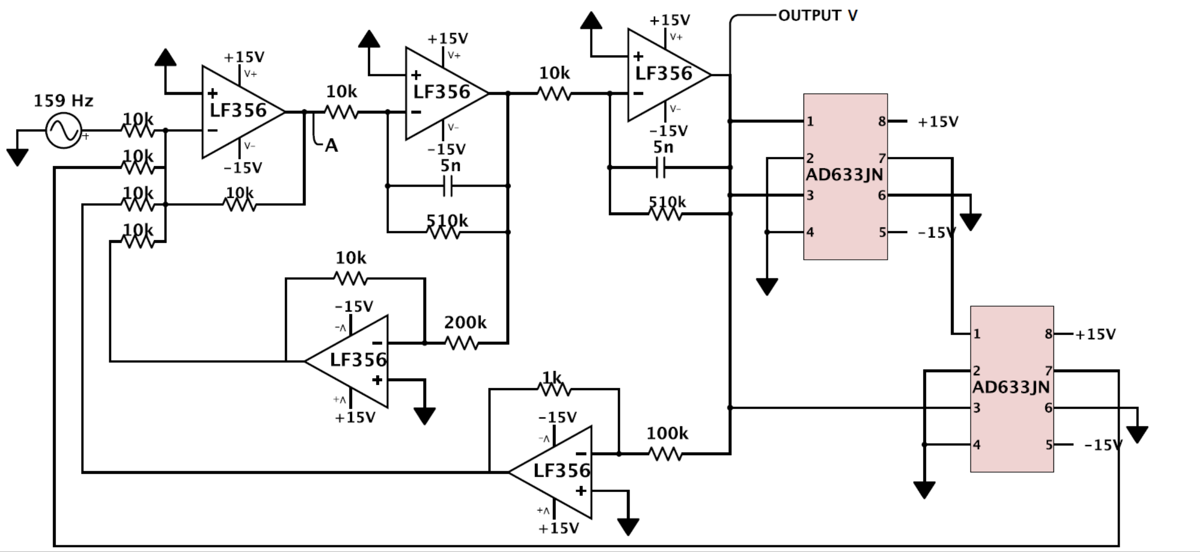
3. Poincaré Section at GAM=1.5



Poincaré section is a complicated curve namely a fractal

DESIGNING OF HARDWARE CIRCUIT AND EXPERIMENTAL RESULT

The Schematic diagram of Duffing oscillator is shown below



The hardware electronic circuit of the Duffing oscillator has been designed using Op-Amp IC LF356, 4-bit analogue multiplier AD633JN, carbon resistors and ceramic capacitors. The circuit runs on a 20V function generator which is capable of producing noise too.

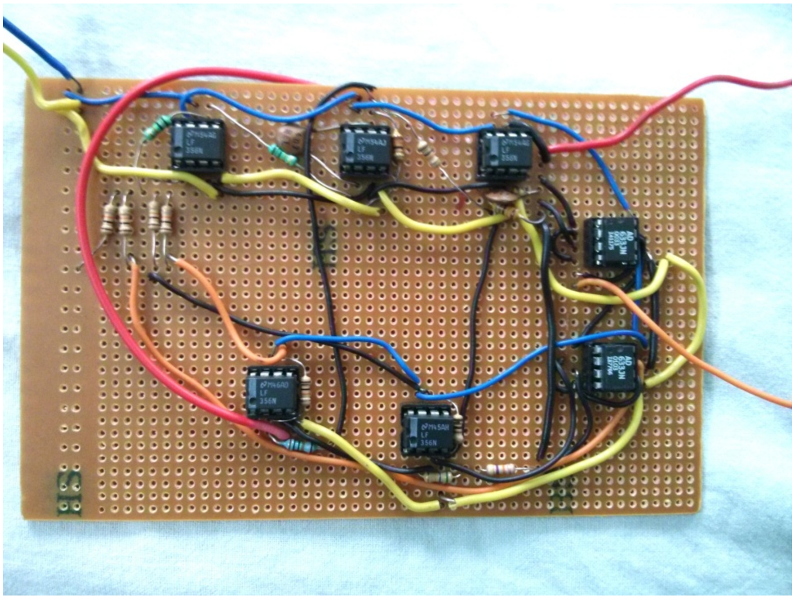


Fig. Electronic circuit implementation of Duffing oscillator

- First, we observed the output signal at F=159Hz and varied amplitude of function generator

  Following waveforms were observed on the digital storage oscilloscope.

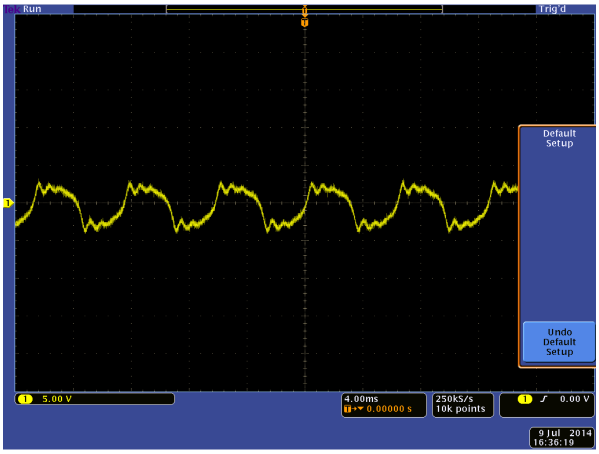


Fig. Output at 300mv input

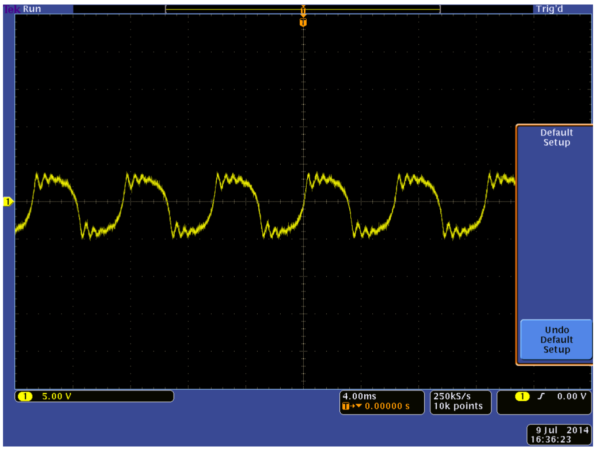


Fig. Output at 2v input

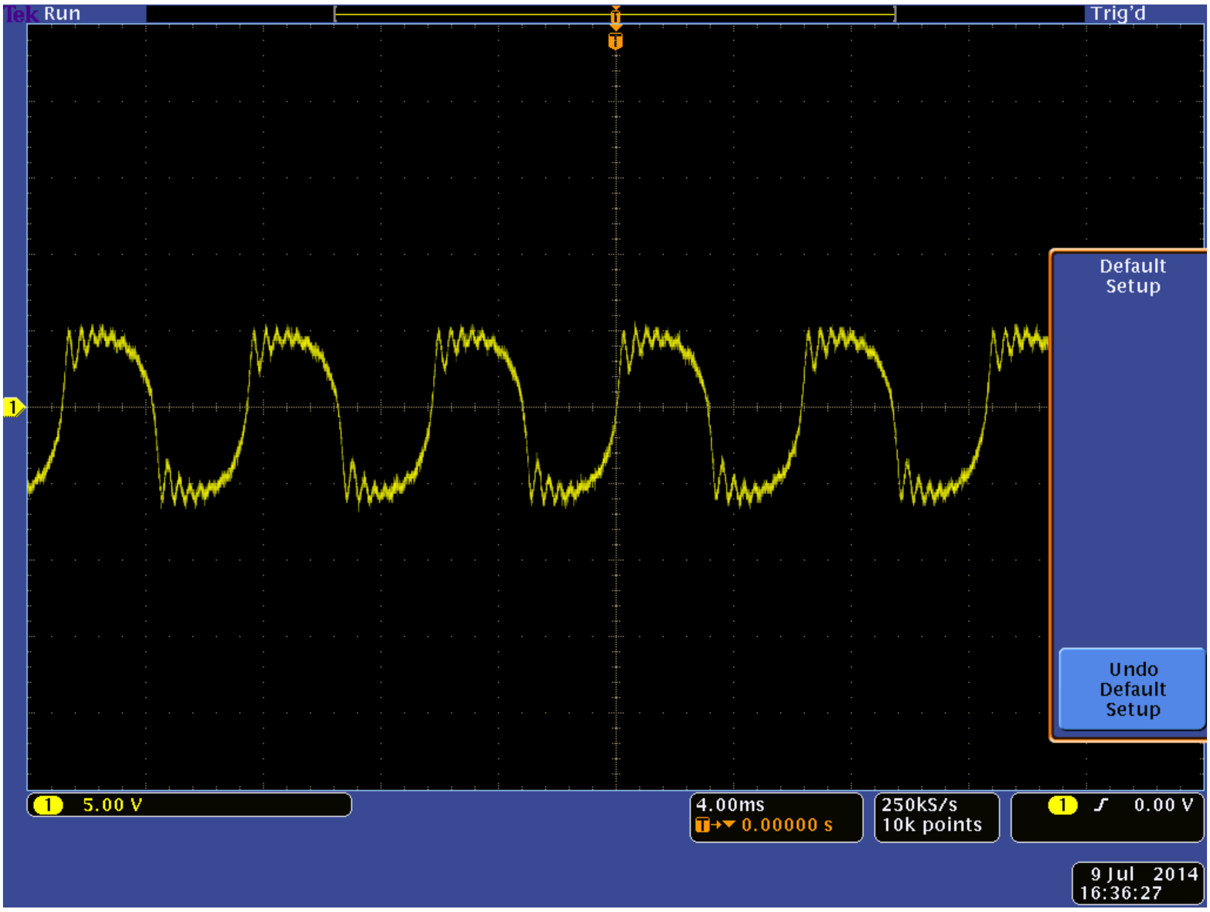
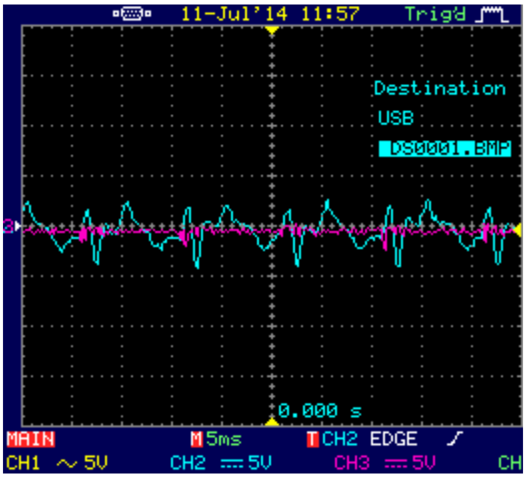
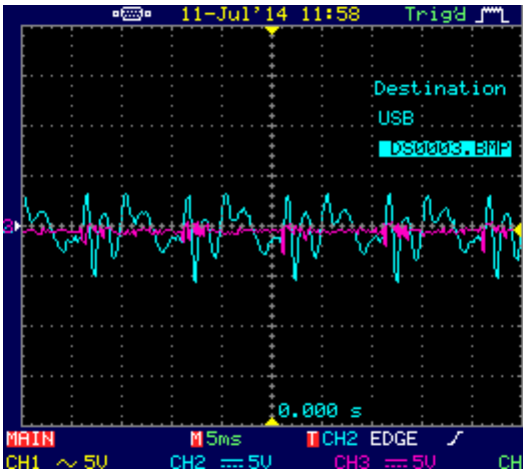


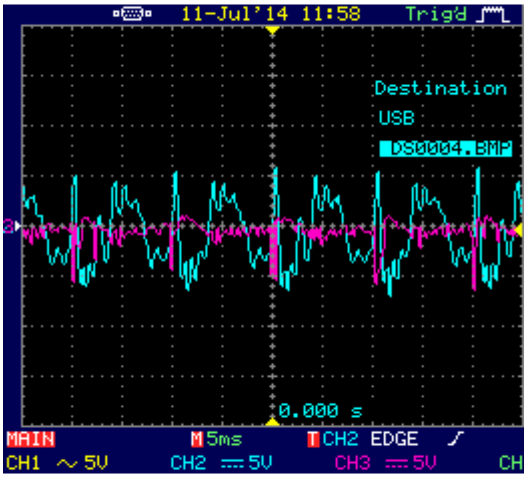
Fig. Output at 8v input

- Now the noise was introduced in the system.





The coherence resonance was established up to a certain noise level.



CONCLUSION

The main objective of this project was to perform experiments on a chaotic signals generated by Duffing oscillator circuit and obtain output characteristics by varying the amplitude, frequency and type of forcing input applied to the Duffing system. I conclude the report by pointing out that, Up to a certain level the addition of noise in the input signal amplifies the output and makes the output coherent, thus coherence resonance occurs; But if we further increase the amplitude the output becomes stochastic. Thus, by addition noise to the system, we saw coherence resonance and stochastic resonance being induced in the system.

REFERENCES

[1] E. Ott, *Chaos in Dynamical Systems (2nd edition)*, Cambridge University Press, 2002.

[2] P.J. Holmes, A nonlinear oscillator with a strange attractor, *Philosophical Transactions of the Royal Society A*, **292**, 419-448, 1979.

[3] Bender & Orszag (1999, p. 546)

[4] FDO, Housam Binous 16 Oct 2007 (Poincaré section of the forced Duffing oscillator)