36106 Managerial Decision Modeling
Linear Decision Models

Kipp Martin
University of Chicago
Booth School of Business

September 22, 2017

Reading and Excel Files

Reading (Powell and Baker):

- Chapter 9, Sections 1-3

Files used in this lecture:

- `allocation.xlsx`
- `allocation_key.xlsx`
- `click-through.xlsx`
- `click-through-key.xlsx`
- `covering.xlsx`
- `covering-key.xlsx`
- `mpfp.xlsx`
- `mpfp_key.xlsx`
- `mpfpVanilla.xlsx`
- `mpfpVanilla_key.xlsx`
- `mpfpHotFudge.xlsx`

Lecture Outline

Moving From What-If To What's Best

Resource Allocation Problems

- A Manufacturing Example
- Using Solver
- Click-Through Revenue Maximization

Covering Problems

Cash Flow Matching

Style and Spreadsheet Engineering

Learning Objectives

1. Learn how to go from What If to What's Best in Excel.
2. Formulate a problem in terms of variables, objective function, and constraints.
3. Build a Solver model to do the What's Best
4. Gain exposure to the following generic applications:
   - Resource allocation
   - Covering
   - Cash flow matching

Moving From What-If To What's Best

Excel is good for doing simple what-if and scenario analysis.

- Data Table
- Scenario Manager

However, when there are:

- multiple products
- multiple investment opportunities
- time periods
- uncertainty, etc.

Data Table and Scenario Manager are not so good.

When there are a lot of possible options we need a better way to evaluate and pick a good (if not the best) option.

Resource Allocation

The **resource allocation** problem:

- multiple products
- multiple resources
- constraints on the resources
- objective: allocate resources to products in order to maximize profit

Resource Allocation

We use an unrealistically small problem just to illustrate the process. Our example is the Veerman Furniture Company from Section 9.2 of the textbook.

- multiple products – chairs, desks, and tables
- multiple resources – fabrication, assembly, shipping
- constraints on the resources – hours available in the respective departments
- objective: allocate resources to products in order to maximize profit – a profit margin on each product

Resource Allocation

Here are the specific numbers.

3		Chair	Desk	Table		
4	Profit Margin	$ 15.00	$ 24.00	$ 18.00		
5						
6						Available
7						Hours
8	Fabrication(Hours)	4	6	2		1850
9	Assembly(Hours)	3	5	7		2400
10	Shipping(Hours)	3	2	4		1500
11						

Resource Allocation

Let's start easy and assume that the fabrication availability is the only resource. See workbook **allocation.xlsx** and spreadsheet **simple**.

How many chairs, desks, and tables should we make in order to:

- maximize profit
- do not exceed the 1850 hours available in the fabrication department
- we are only worried about fabrication for now, not assembly and shipping

Let's give a formal mathematical statement of the problem.

Resource Allocation

Mathematical Model:

The **decision variables** are

- C = number of chairs produced
- D = number of desks produced
- T = number of tables produced

The **profit function** is $15C + 24D + 18T$.

The **resource usage function** is $4C + 6D + 2T$.

The **model** is

$$\begin{aligned} \max 15C + 24D + 18T & \\ 4C + 6D + 2T &\leq 1850 \end{aligned}$$

Resource Allocation

How do we solve this in Excel? See the spreadsheet `simple`.

	A	B	C	D	E	F	G	H
1		**Powell and Baker Resource Allocation Example -- Section 9.2**						
2								
3		Chair	Desk	Table				
4	Profit Margin	$ 15.00	$ 24.00	$ 18.00				
5								
6						Available		
7						Hours		
8	Fabrication(Hours)	4	6	2		1850		
9								

Resource Allocation

Now what about the full model?

$$\begin{array}{rcll} \max 15C + 24D + 18T & & & \\ 4C + 6D + 2T & \leq & 1850 & \text{(Fabrication)} \\ 3C + 5D + 7T & \leq & 2400 & \text{(Assembly)} \\ 3C + 2D + 4T & \leq & 1500 & \text{(Shipping)} \end{array}$$

Can we apply the same logic from the single constraint case?

Resource Allocation

Now calculate the **bang-for-buck ratios** for each resource.

	A	B	C	D	E	F	G
1		**Powell and Baker Resource Allocation Example -- Section 9.2**					
2							
3		Chair	Desk	Table			
4	Profit Margin	$ 15.00	$ 24.00	$ 18.00			
5							
6						Available	
7						Hours	
8	Fabrication(Hours)	4	6	2		1850	
9	Assembly(Hours)	3	5	7		2400	
10	Shipping(Hours)	3	2	4		1500	
11							
12							
13							
14		3.75	4.00	9.00			
15	Bang For Buck Ratios	5.00	4.80	2.57			
16		5.00	12.00	4.50			

Argle Bargle! The bang-for-buck ratios are different for each resource!

Using Solver

First install Solver. See the handout:

`http://faculty.chicagobooth.edu/kipp.martin/root/htmls/coursework/36106/handouts/InstallSolver.pdf`

In the Excel ribbon, select the **Data** Tab.

Solver is located in the **Analysis** Group.

See the spreadsheet **model-done** in the workbook **allocation.xlsx**.

Using Solver

Trying to figure out the best resource allocation by trial and error is not practical.

Indeed, this is an unrealistically small problem. In practice, we might have thousands of products.

Let's find the **best** solution using the Solver Add-In. It is as simple as learning your ABCs.

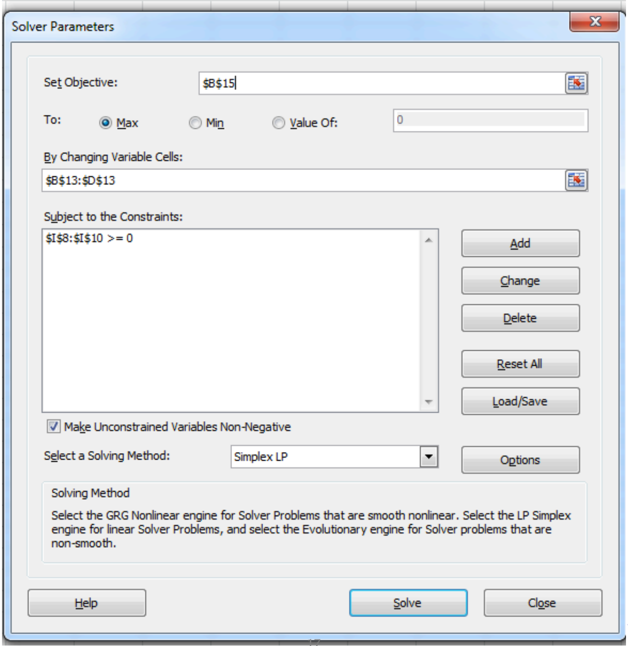
- **A** is for **Adjustable –** determine which cells, you as the decision maker, determine
- **B** is for **Best –** determine your objective
- **C** is for **Constraints –** determine the constraints on your decision

Using Solver

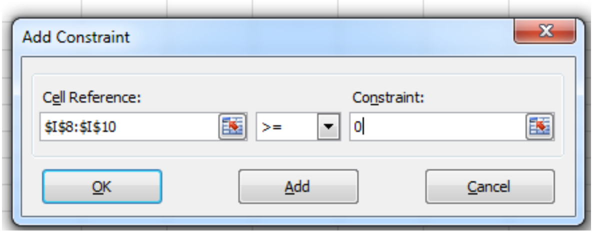
Let's do our ABCs for the resource allocation problem.

- **Adjustable Cells** – in this case how many **chairs**, **desks**, and **tables** you produce (`B13:D13`).
- **Best Cell** (Note there can be only one) – the cell that holds the profit function (`B15`)
- **Constraint Cells** – you cannot exceed the resource limit in fabrication, assembly, and shipping (`I8:I10`)

Using Solver



Using Solver



With **Add Constraint** we force Excel to set values for the adjustable cells such that:

- the constraints are satisfied
- the "best" cell is indeed that – the best possible value while still satisfying the constraints.

Using Solver

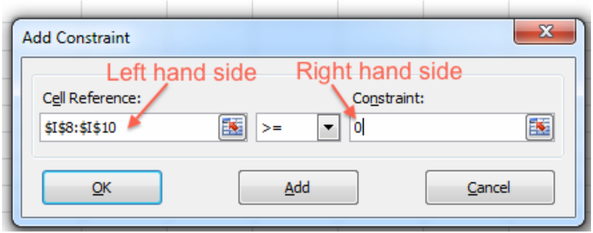
Important Terminology: When talking about constraints we often use the terms **left hand side** and **right hand side**.

Left hand side: Solver refers to this as the Cell Reference. The formula on the left hand side of the inequality.

Right hand side: Solver refers to this as the Constraint. The formula on the right hand side of the inequality.

We are constraining the left hand side (what is in the Cell Reference) by the right hand side. See next slide.

Using Solver



Algebraically the constraints are

$$
\begin{aligned}
1850 - 4C - 6D - 2T &\geq 0 \quad \text{(Fabrication)} \\
2400 - 3C - 5D - 7T &\geq 0 \quad \text{(Assembly)} \\
1500 - 3C - 2D - 4T &\geq 0 \quad \text{(Shipping)}
\end{aligned}
$$

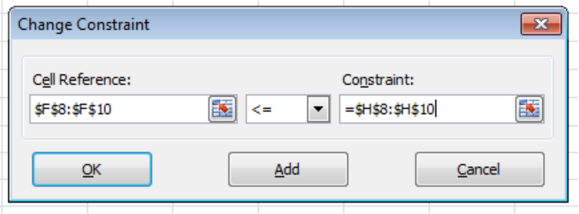
The right hand sides are 0, 0, and 0.

Using Solver

However, an algebraically equivalent statement is

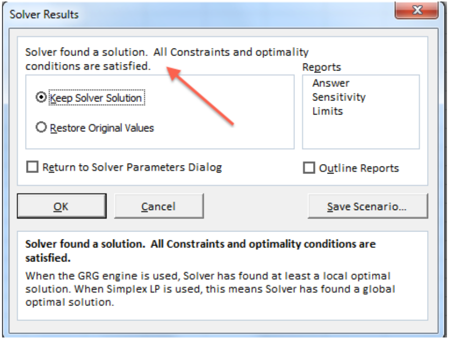
$$
\begin{aligned}
4C + 6D + 2T &\leq 1850 \quad \text{(Fabrication)} \\
3C + 5D + 7T &\leq 2400 \quad \text{(Assembly)} \\
3C + 2D + 4T &\leq 1500 \quad \text{(Shipping)}
\end{aligned}
$$

The right hand sides are now 1850, 2400, and 1500.



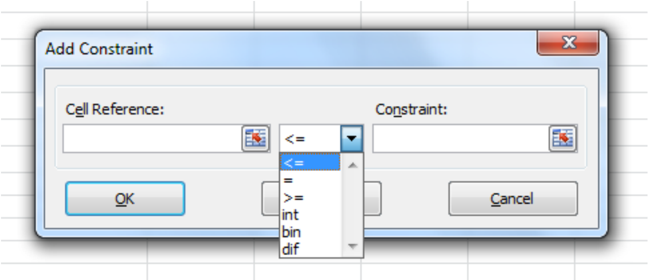
Using Solver

When you hit Solve you should see:



It is **critical** to see: "Solver found a solution. All Constraints and optimality conditions are satisfied."

Using Solver



Constraint Types:

- $\leq$
- $=$
- $\geq$
- `int` – cell must take on an integer value (for example, an integer number of bonds)
- `bin` – cell must take on the value 0 or 1 (used to enforce logical conditions)
- `diff` – cells cannot be equal

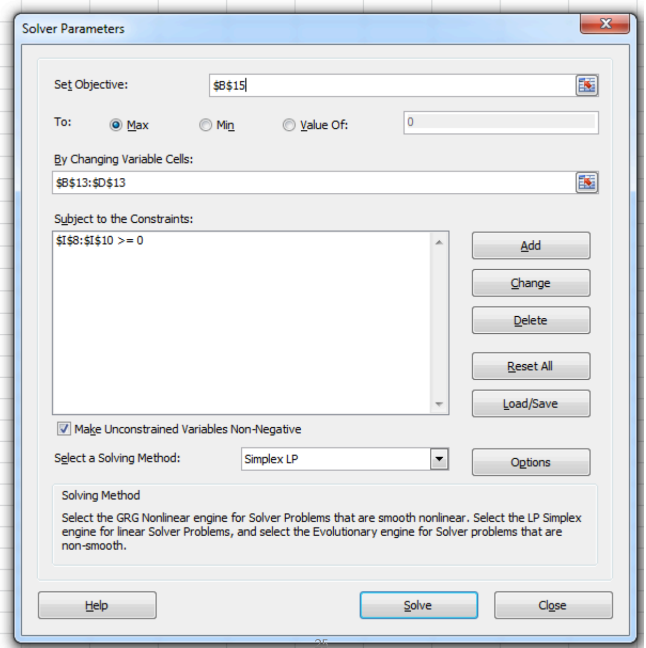
Using Solver

A few other things:

- You can have multiple sets of constraints and they do not need to be contiguous. Just keep adding them!
- In setting the adjustable cells, the rows do not need to be contiguous, separate by commas.
- You can have **only one best** cell.
- My color coding for the font
  1. Red – constraint cells
  2. Blue – adjustable cells
  3. Green – best cell
  4. Black – parameters

Note: We checked the box for nonnegative variables, what happens if we don't?

Using Solver



Using Solver

There are numerous YouTube videos for using Solver.

See, for example,

`https://www.youtube.com/watch?v=K4QkLA3sT1o`

Resource Alloccation

We assumed we could sell every chair, desk, and table produced.

Modify and solve the problem assuming the following demands.

- the chair demand limit is 360
- the desk demand limit is 300
- the table demand limit is 100

Click-Through Revenue Maximization

We are motivated by the following:

https://vimeo.com/193083648

See the handout Maximizing Click-Through Revenue.

Click-Through Revenue Maximization

Players in this market:

- Advertisers (e.g. Bloomingdales)
- Ad exchange (e.g. Double Click)
- Content provider (e.g. *NY Times*)

In this model we assume content provider deals directly with advertisers

Consider the following data for the *NY Times*.

	A	B	C	D	E	F
1			**Advertiser Click-Through Rates**			Daily Page Views
2			KFC	Dot&Bo	Alpha-Wars	
3	**NY Times**	Politics	0.01	0.05	0.01	5
4		Sports	0.04	0.02	0.06	2
5	Required Page Views		2	3	1	
6						

Click-Through Revenue Maximization

Let's see how click-through maximization problem is a resource allocation problem.

- multiple products – the ads from KFC, Dot&Bo, and Alpha-Wars
- multiple resources – sections of the paper, e.g. sports and politics
- constraints on the resources – page views available on each section
- objective: allocate resources to products in order to maximize profit – how many page views to give each advertiser in each section

Click-Through Revenue Maximization

We need to figure out **how many page views to give each advertiser in each paper section.**

Fill in the blue numbers.

	A	B	C	D	E	F
1			**Advertiser Click-Through Rates**			Daily Page Views
2			KFC	Dot&Bo	Alpha-Wars	
3	**NY**	Politics	0.01	0.05	0.01	5
4	**Times**	Sports	0.04	0.02	0.06	2
5	Required Page Views		2	3	1	
6						
7						
8			**Advertiser Click Page Views**			
9			KFC	Dot&Bo	Alpha-Wars	
10	**NY**	Politics	0	0	0	
11	**Times**	Sports	0	0	0	
12						

Click-Through Revenue Maximization

We need to figure out **how many page views to give each advertiser.**

Objective: maximize click-through revenue.

Constraints:

1. the page view contractual agreements
2. cannot give more page views than we have

Click-Through Revenue Maximization

Variables:

- KP – number views given to KFC on the politics page
- DP – number views given to Dot&Bo on the politics page
- AP – number views given to Alpha-Wars on the politics page
- KS – number views given to KFC on the sports page
- DS – number views given to Dot&Bo on the sports page
- AS – number views given to Alpha-Wars on the sports page

Click-Through Revenue Maximization

Objective: maximize click-through revenue

$$\max .1 * (.01 * KP + .04 * KS + .05 * DP + \cdots + .06AS)$$

Constraint 1: page view contractual agreement

$$\begin{aligned} KP + KS &= 2 \\ DP + DS &= 3 \\ AP + AS &= 1 \end{aligned}$$

Constraint 2: cannot give more page views than we have

$$\begin{aligned} KP + DP + AP &\leq 5 \\ KS + DS + AS &\leq 2 \end{aligned}$$

Click-Through Revenue Maximization

Question: What are the implications of

$$
\begin{aligned}
KP + KS &= 2 \\
DP + DS &= 3 \\
AP + AS &= 1
\end{aligned}
$$

versus

$$
\begin{aligned}
KP + KS &\geq 2 \\
DP + DS &\geq 3 \\
AP + AS &\geq 1
\end{aligned}
$$

Click-Through Revenue Maximization

Let's create a cell for the objective in our spreadsheet.

We need to calculate the total number of click-throughs.

A new idea: the **SUMPRODUCT** function!

The SUMPRODUCT function can help reduce errors.

See for example
`https://www.youtube.com/watch?v=rwsX1KVhEnU`

Click-Through Revenue Maximization

The Excel `SUMPRODUCT` function is very useful in model building.

	A	B	C	D	E
1					
2					
3					
4	**range_one**	1	2	-1	
5		3	4	0	
6					
7	**range_two**	3	0	1	
8		2	-1	5	
9					
10	**SUMPRODUCT =**	4			
11					

$$(1 * 3) + (2 * 0) + (-1 * 1) + (3 * 2) + (4 * -1) + (0 * 5) = 4$$

IMPORTANT: the ranges must be of conformable (equal) dimension.

See the `sumproduct` spreadsheet in the `click-through.xlsx` Workbook.

Click-Through Revenue Maximization

Important modeling idea: **Range Names**.

This makes model construction and model debugging much easier!

Rather than

```
=SUMPRODUCT(B1:D5,B7:D8)
```

we write

```
=SUMPRODUCT(range_one,range_two)
```

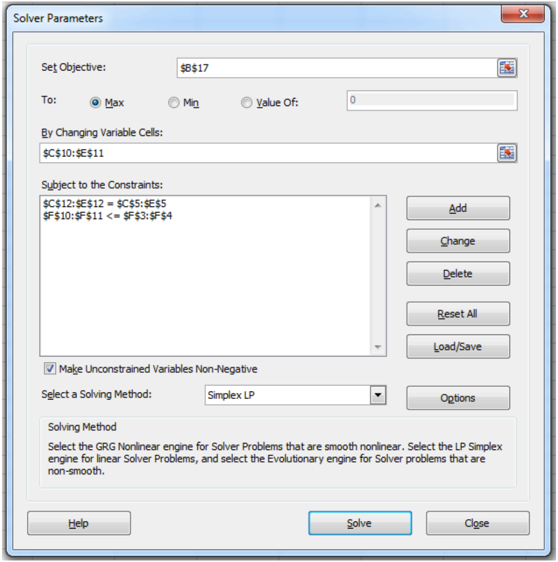
Click-Through Revenue Maximization

Let's do our ABCs for this problem.

- **Adjustable Cells** – in this case how many page views you assign to each section of the paper (sports, politics) for each company (KFC, Dot&Bo, Alpha-Wars) See cells (`C10:E11`).
- **Best Cell** (Note there can be only one) – the cell that holds the profit function which is the total number of click-throughs (`B17`)
- **Constraint Cells** – the contractual page view constraints (`C12:E12`) and upper limits on page views in each section (`F10:F11`)

Click-Through Revenue Maximization

Here is the Solver model.



Using Solver

IMPORTANT: There are different ways to setup a solver model.

Consider the way we write the constraints.

Method 1: Explicitly calculate a set of slack cells and require the slack to be nonnegative.

We did this in the resource allocation model where we calculated the slack in `I8:I10` and then required these cells to be nonnegative.

The primary advantages of this method are:

- You see an explicit calculation of the slack which is useful.
- You may prefer the sensitivity analysis report for this style (more on this later).

Using Solver

IMPORTANT: There are different ways to setup a solver model.

Method 2: This is what we did with the click-through model.

For example, we have constraints that we cannot give more page views than we have.

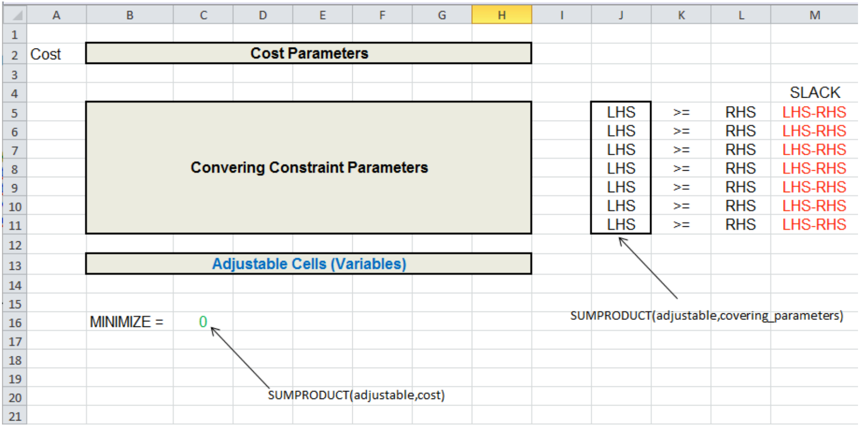
This is expressed as `F10:F11 <= F3:F4`. The advantages are:

- The spreadsheet is less cluttered.
- You may prefer the sensitivity analysis report for this style (more on this later).

My Advice: develop a style that **you** feel most comfortable with.

Covering Problems

Consider a generic covering problem. A possible paradigm is illustrated below.



Historical Notes

Historically, the first constrained optimization problems were **diet problems** solved for the US military in the late 1930s and early 1940s. The diet problem is a covering problem.

Booth (before we were Booth) professor George Stigler and 1982 Nobel Laureate in economics was one of the first to "solve" this problem. See `http://en.wikipedia.org/wiki/Stigler_diet`.

His original problem had 77 potential foods and 9 nutrient requirements. Here is what you should eat.

Food	**Annual Quantities**	**Annual Cost**
Wheat Flour	370 lb.	$13.33
Evaporated Milk	57 cans	$ 3.84
Cabbage	111 lb.	$ 4.11
Spinach	23 lb.	$ 1.85
Dried Navy Beans	285 lb.	$16.80
Total Annual Cost		$ 39.93

Non-vegetarians should consume beef liver.

Covering Problems

This is illustrated for the Powell and Baker Diet problem in Section 9.3.

- S = pounds of seeds in the diet ($4 per pound)
- R = pounds of raisins in the diet ($5 per pound)
- F = pounds of flakes in the diet ($3 per pound)
- P = pounds of pecans in the diet ($7 per pound)
- W = pounds of walnuts in the diet ($6 per pound)

$$
\begin{array}{rcll}
\min 4S + 5R + 3F + 7P + 6W & & & \\
10S + 20R + 10F + 30P + 20W & \geq & 20 & \text{(Vitamin)} \\
5S + 7R + 4F + 9P + 2W & \geq & 10 & \text{(Mineral)} \\
1S + 4R + 10F + 2P + 1W & \geq & 15 & \text{(Protein)} \\
500S + 450R + 160F + 300P + 500W & \geq & 600 & \text{(Calorie)}
\end{array}
$$

Covering Problems

See the `model` spreadsheet in the `covering.xlsx` workbook.

	A	B	C	D	E	F	G	H	I	J	K	L
1		Powell and Baker Diet Example -- Section 9.3										
2												
3		Cost	$ 4.00	$ 5.00	$ 3.00	$ 7.00	$ 6.00					
4												
5												
6												
7			seeds	raisens	flakes	pecans	walnuts		LHS	>=	RHS	SLACK
8		vitamins	10	20	10	30	20		24.642		20	4.642
9		minerals	5	7	4	9	2		10		10	0
10		protein	1	4	10	2	1		15		15	0
11		calories	500	450	160	300	500		600		600	0
12												
13		Adjustable	0.4773	0.3341	1.3186	0	0					
14												
15		Minimize =	$ 7.54									
16												

Using Solver for the Covering Problem

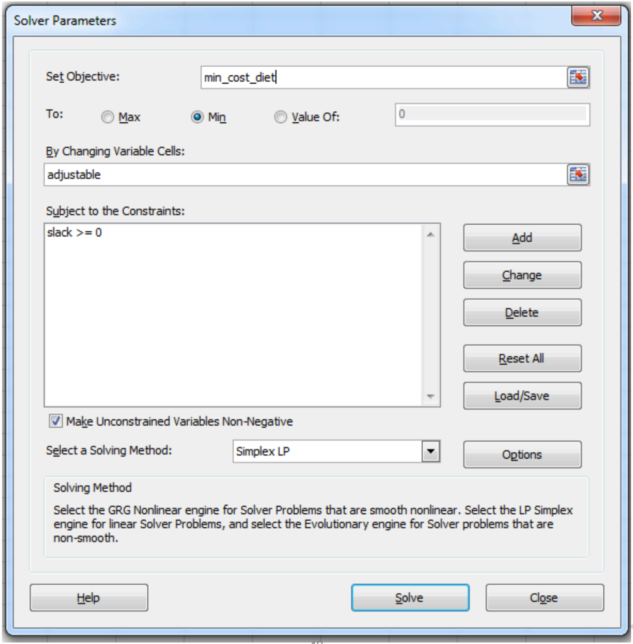
Let's do our ABCs for this covering problem.

- **Adjustable Cells** – how much of each food ingredient goes into the diet (`C13:G13` – range named `adjustable`)
- **Best Cell** (Note there can be only one) – the cell that holds the cost of the diet (`C15` – range named `min_cost_diet`)
- **Constraint Cells** – you must meet the nutritional requirements (`L8:L11` – range named `slack`)

Note the use of **range names**. I highly recommend using range names.

Mention use of **apply names** under the **Define names**

Using Solver for the Covering Problem



Cash Flow Matching

Problem: generate a specified stream of cash flows over a planning horizon.

- pension fund planning
- law suit settlement
- lottery

Assume you win one million dollars in a state lottery. Option 1 is to take the one million up front. Option 2 is to receive an annuity of $50,000 dollars a year for 30 years. Which is better?

If you pick Option 2 the state will discharge the debt to a bank. How much do they pay the bank?

Cash Flow Matching

Let's take some specific numbers. See spreadsheet `NPV` in workbook `mpfp.xlsx`.

Year	0	1	2	3	4	5	6	7	8	$\cdots$	14
Cash ($\times$ \$1000)	10	11	12	14	15	17	19	20	22	$\cdots$	36

What is the equivalent amount of money at time zero?

Cash Flow Matching

Present Value!

Let r be an interest rate and C_t the cash required in period t.

$$C_0 + \sum_{t=1}^{14} \frac{C_t}{(1+r)^t}$$

$$10 + \frac{11}{(1+r)} + \frac{12}{(1+r)^2} + \frac{14}{(1+r)^3} + \cdots + \frac{36}{(1+r)^{14}}$$

Cash Flow Matching

Present Value!

For example, with $r = 0.04$

$$\$230.44 = 10 + \frac{11}{(1+.04)} + \frac{12}{(1+.04)^2} + \frac{14}{(1+.04)^3} + \cdots + \frac{36}{(1+.04)^{14}}$$

See the worksheet `NPV` in the workbook `mpfp.xlsx`.

Cash Flow Matching

Another viewpoint: Assume that we have a savings account that pays 4%.

At time 0 take the money you have to start with and pay the cash requirement (in this case \$10) and put the remaining money in a savings account.

At the *start* of each period $t \geq 1$ we do the following:

1. Withdraw all of the money in our savings account
2. Pay the cash requirement
3. Reinvest the remaining amount of money in the savings account

Cash Flow Matching

At 4% the present value of the cash flows is \$230.44.
Let S_t be the amount of money invested in the savings account at the start of period t. Remember we pay the cash requirement before investing in the savings account.

$$S_0 = \$230.44 - \$10 = \$220.44$$

Then

$$\begin{aligned} S_1 &= (1+.04)S_0 - 11 \\ &= (1+.04)220.44 - 11 \\ &= \$218.25 \end{aligned}$$

In period 2,

$$\begin{aligned} S_2 &= (1+.04)S_1 - 12 \\ &= (1+.04)\$218.25 - 12 \\ &= \$214.98 \end{aligned}$$

Cash Flow Matching

In general, I am writing **sources and uses** constraints. Also called **conservation** or **flow balance** constraints.

In each period

$$\text{Sources of cash} = \text{Uses of cash}$$

Period t **sources of cash** = money from the investing in savings account period $t-1$

Period t **uses of cash** is to pay the cash requirement (C_t) and reinvest in savings

$$\begin{aligned}\text{Sources} &= (1+r)S_{t-1} \\ \text{Uses} &= C_t + S_t\end{aligned}$$

$$\begin{aligned}C_t + S_t &= (1+r)S_{t-1} \\ S_t &= (1+r)S_{t-1} - C_t\end{aligned}$$

Cash Flow Matching

See worksheet `savings` in the `mpfp` workbook.

See for example cell `D10` that has the formula

```
=(1+int_rate)*C9-Cash_Req-Sav_Acct
```

Note that the range `Sav_Acct` is `C6:C20`. In the formula above Excel "knows" that `Sav_Acct` refers to `C10` in this case.

However, I have to refer explicitly to `C9` by the cell index rather than with a range name.

Cash Flow Matching

Now make more realistic. We can buy high grade corporate or government bonds. (See Workbook **mpfpVanilla.xlsx**)

Table: Bond Data

Bond	Current Cost	Coupon Rate	Years to Maturity	Repayment At Maturity
One	\$980	.06	5	\$1000
Two	\$965	.065	12	\$1000

Next *develop sources and uses constraints* – not so easy!

Possible sources of cash in a year are:

- A bond maturing,
- Interest payment on a bond,
- Principal from the previous period savings,
- Interest on the previous period savings.

Cash Flow Matching

Possible **uses of cash** in a year are:

- The cash requirement in each period,
- The investment in bonds in period 0 only,
- The investment in the savings account at the end of the period.

Possible **sources of cash** in a year are:

- Principal plus interest on the savings account
- Bond interest
- A bond maturing
- The initial “lump sum”

Cash Flow Matching – Variables

Variable Definition:

S_t = amount invested in savings account at start of period t

Num_Bnd1 = number of bonds of type 1 purchased at time 0

Num_Bnd2 = number of bonds of type 2 purchased at time 0

Parameters:

Bond prices

Savings interest rate

Bond coupon rate

Which parameters are deterministic and which are stochastic?

Cash Flow Matching – Sources and Uses

TIME 0

Uses of Cash:

- Cash requirement (\$10)
- Money spent on Bond One ($.98 * Num\_Bnd1$)
- Money spent on Bond Two ($.965 * Num\_Bnd2$)
- Money put in the savings account (S_0)

Sources of Cash:

- Initial lump sum of cash (L is what we want to minimize)

$L = 10 + .98 * Num\_Bnd1 + .965 * Num\_Bnd2 + S_0$

Cash Flow Matching – Sources and Uses

TIME 2

Uses of Cash:

- Cash requirement (\$12)
- Money put in the savings account (S_2)

Sources of Cash:

- Interest from Bond One: ($.06 * Num\_Bnd1$)
- Interest from Bond Two: ($.065 * Num\_Bnd2$)
- Interest plus principal from money put into savings account in period one: ($1.04 * S_1$)

$$.06 * Num\_Bnd1 + .065 * Num\_Bnd2 + 1.04 * S_1 = 12 + S_2$$

Cash Flow Matching – Sources and Uses

TIME 5

Uses of Cash:

- Cash requirement (\$17)
- Money put in the savings account (S_5)

Sources of Cash:

- Interest from Bond One ($.06 * Num\_Bnd1$)
- Interest from Bond Two ($.065 * Num\_Bnd2$)
- Face Value of Bond One Maturing (Num_Bnd1)
- Interest plus principal from money put into savings account in period four: ($1.04 * S_4$)

$$.06 * Num\_Bnd1 + .065 * Num\_Bnd2 + Num\_Bnd1 + 1.04 * S_4 = 17 + S_5$$

Cash Flow Matching – Sources and Uses

Sample Test Questions

- What are uses of cash in period 11?
- What are the sources of cash in period 12?

Notes:

- The interest from the bond is based on the face value of the bond **not** the price of the bond.
- When a bond matures you get the face value of the bond.
- Work in a consistent set of units – in this case thousands

Cash Flow Matching – More on Source and Uses

Note: Sources and Uses constraints often written as inequality constraints.

Uses of Cash $\leq$ Sources of Cash

Uses of Cash - Source of Cash ≤ 0

Sources of Cash $\geq$ Uses of Cash

Sources of Cash - Uses of Cash ≥ 0

Cash Flow Matching – More on Source and Uses

If the sources and uses constraint is:

Uses of Cash - Source of Cash ≤ 0

1. What is the interpretation of the value of the left-hand-side = -3?

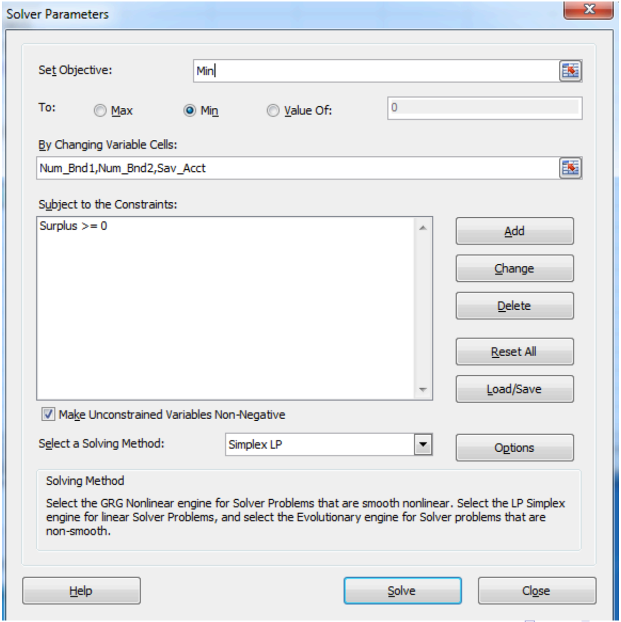
2. If the value of the left-hand-side is -3, what is the slack?

Using Solver

Let's do our ABCs for the cash flow matching problem. Open the workbook **mpfpVanilla.xlsx**.

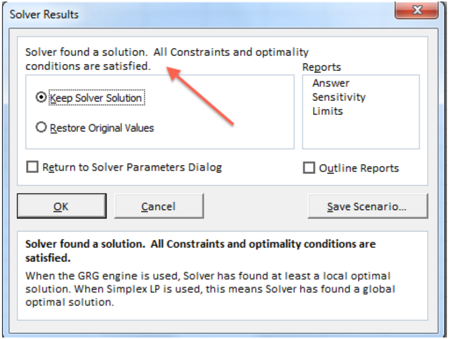
- **Adjustable Cells –** in this case, how many bonds of type 1 you buy, how many bonds of type 2 you buy, and how much you invest in savings in each time period
- **Best Cell** (Note there can be only one) – to minimize the amount of cash at time zero necessary to fund the cash flows
- **Constraint Cells –** you must have enough cash in each time period

Using Solver



Using Solver

When you hit Solve you should see:



Remember: it is critical to see: "Solver found a solution. All Constraints and optimality conditions are satisfied."

Cash Flow Matching

Key Takeaways:

- sources and uses constraints
- how to use Solver to find the BEST value
- present value calculation and why it is restrictive
- **sanity check** – run the model after adding constraints that the number of bonds must be zero – what should the value be?

Style and Spreadsheet Engineering

The Generic Model:

$$
\begin{aligned}
\max(\min) f(x_1, x_2, \ldots, x_n) & & \\
g_1(x_1, x_2, \ldots, x_n) &\leq & b_1 \\
g_2(x_1, x_2, \ldots, x_n) &\leq & b_2 \\
\vdots & \vdots & \vdots \\
g_m(x_1, x_2, \ldots, x_n) &\leq & b_m
\end{aligned}
$$

How to build the model in Solver is a matter of personal style. There is not a single correct way to build the model.

The following are some tips.

Style and Spreadsheet Engineering

1. The BIG ONE – DO NOT *hard code* parameters into the equations! Look at the constraint cells in the range `Surplus`. There are no numbers!

2. Make the formulas as general as possible! Instead of (for period 5)

```
=Num_Bnd1*Int_Bnd1+Num_Bnd2*Int_Bnd2+
C10*(1+Sav_Int)+Num_Bnd1-Sav_Acct-Cash_Req-Sav_Acct
```

use (for period 5)

```
=Int_Bnd1*Num_Bnd1*IF(Mat_Bnd1>=Period,1,0)+
Int_Bnd2*Num_Bnd2*IF(Mat_Bnd2>=Period,1,0)+
$C10*(1+Sav_Int)+Num_Bnd1*IF(Mat_Bnd1=Period,1,0)+
Num_Bnd2*IF(Mat_Bnd2=Period,1,0)-Cash_Req-Sav_Acct
```

See Workbook `mpfpHotFudge.xlsx`.

Style and Spreadsheet Engineering

3. Big errors are good!

If a generic formula is used throughout the spreadsheet, there is a good chance a small mistake will reveal itself in generating obviously wrong results.

For example, if I were to make a small mistake in a single cell formula and type `Num_Bnd1` instead of `Num_Bnd2` I might never notice.

However, if I made this mistake in the generic formula, I would notice that Bond Two is never purchased and would suspect something is wrong.

4. Make use of the `sumproduct` function.

5. Use range names!

Style and Spreadsheet Engineering

6. It is **critical** that you use the IF statement correctly! There is no problem using an IF function when the function arguments are **parameters**.

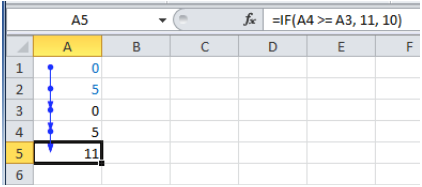
A good example is the cash flow matching model. The augments of the IF function involve bond maturities – not adjustable cells.

DO NOT use an IF function that has an adjustable cell, either directly or recursively, as an argument. Remember Excel is recursive. So in checking for linearity it looks at all precedence relationships. For example, assume:

- cells A1 and A2 are adjustable
- cell A3 has the formula $= \mathrm{A1}$
- cell A4 has the formula $= \mathrm{A2}$
- cell A5 has the formula `=IF(A4 >= A3, 11, 10)`

Style and Spreadsheet Engineering

6. IF statement continued.



The IF statement in A5 does not **directly** involve an adjustable cell, but through Trace Precedents we see that adjustable cells are used.

Bottom Line: test your **IF** statements through Trace Precedents and make sure you do not see any blue cells!

Style and Spreadsheet Engineering

7. Linear is important! **Make the model linear whenever possible.** More on this later.

We focused on the `IF` statement because it is so important.

- Okay to have adjustable cells in a linear function, for example `SUM`
- Bad to have adjustable cells in nonlinear function, for example `LOG` or `AND`.

You need to know if you build a linear or nonlinear model. Life is much harder in the nonlinear lane.

Style and Spreadsheet Engineering

Important: We are working with **linear models!**

$$\begin{array}{ll} 2x_1 + 5x_2 & \text{is linear} \\ 2x_1 + 5x_1x_2 & \text{is nonlinear} \\ 2x_1^2 + x_2 & \text{is nonlinear} \\ x_1/x_2 & \text{is nonlinear} \end{array}$$

Some Excel functions are linear, others nonlinear.

SUM – a linear function

IF, **OR**, **AND**, **MAX**, **MIN** – nonlinear

SUMPRODUCT – could be either

Important: No integer constraints!