36106 Managerial Decision Modeling
Monte Carlo Simulation in Excel: Part I

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November 1, 2017

Reading and Excel Files

Reading:

- Powell and Baker: Chapter 14, 14.1-14.3
- See the Documentation and Examples folders in the Palisade RISK6 directory that was installed with your software.

Files used in this lecture:

- `retirement.xlsx`
- `retirementRisk.xlsx`
- `riskStatic.xlsx`
- `retirementRiskGoal.xlsx`

Learning Objectives

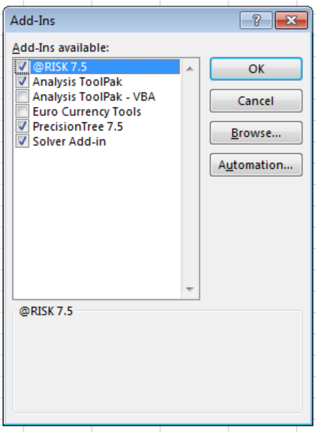
1. Understand why stochastic models are important
2. Understand the **Flaw of Averages**
   - a truly critical concept for anyone working with spreadsheets
   - perhaps the biggest takeaway of the quarter
3. Learn to run a basic **Monte Carlo simulation** in @Risk
   - learn how to replace a parameter with a distribution
   - learn how to use simulation result statistical functions
4. Learn to use @Risk Goal Seek

Lecture Outline

Software Install – DecisionTools Suite

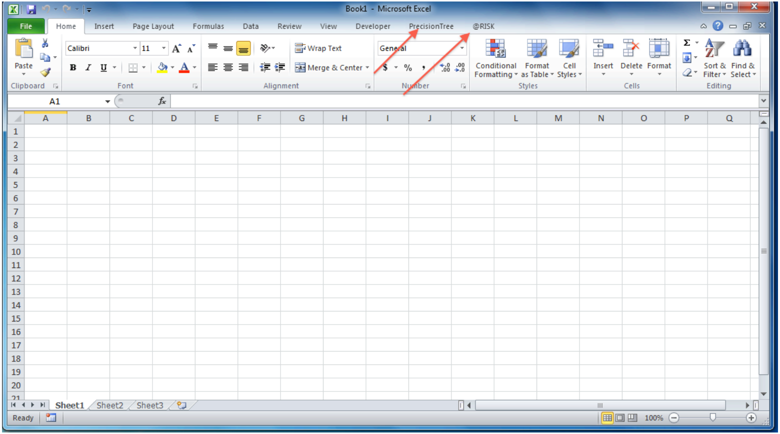
After installing the Precision Tools Suite:

1. Click on the @RISK icon on your desktop
2. Use AddIns to put @RISK on the Ribbon for future use.



Software Install – DecisionTools Suite

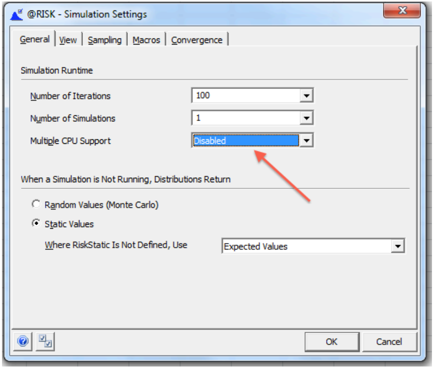
From now on, you can open Excel and do not need to use the desktop icons. You should see Tab items for both PrecisionTree and @Risk.



Software Install – DecisionTools Suite

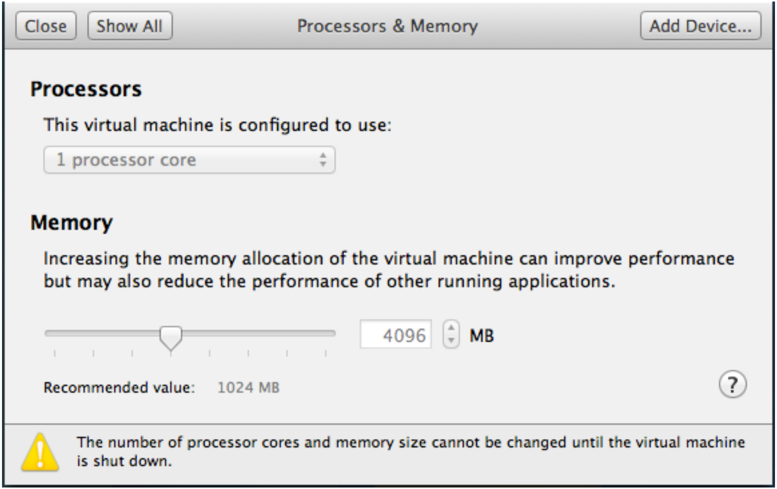
Tips:

- turn off extra CPU's
- close other apps
- minimize number of open workbooks



Software Install – DecisionTools Suite

Tips: Mac User – Setting for VM Ware



Where We Are Headed

The natural progression of modeling:

1. Build a purely deterministic model – all parameters known with certainty
2. Build a model (like Markowitz) where certain parameters are means and variances of a statistical distribution
3. Build a model where parameters are actually a random variable (a statistical distribution)

Where We Are Headed

We are taking tools from earlier in the quarter and extending them to allow for stochastic parameters.

Deterministic Tool	**Stochastic Tool**
Goal Seek	@Risk Goal Seek (Week 7)
Data Table	@Risk RiskSimtable (Week 8)
Solver	@Risk Optimizer (Week 10)

Important: Understand the difference between a *deterministic model* and *stochastic model.*

Motivation – The Flaw of Averages

Here is the **key concept** or **big take away:**

If there are numbers in your spreadsheet that are not known with certainty, putting in **estimated average values** *can lead to terrible results!*

With @RISK we will enter "distributions" rather than numbers into cells.

Consider the following retirement example:

Motivation – The Flaw of Averages

Assume a retirement 401 K account of $200,000 in the S&P.

You want to withdraw equal amounts for the next 20 years until the fund is depleted.

How much can you withdraw each year before hitting 0 at the end of year 20?

Please open Workbook **retirement.xlsx**.

Motivation – The Flaw of Averages

In Workbook **retirement.xlsx** there are two spreadsheets:

- **S&P 500**: this spreadsheet has the S&P returns for 1926-2012. This also has the calculations:
    1. mean $= 0.115486207$
    2. standard deviation $= 0.206161235$
- **model:** this spreadsheet has the retirement calculations

Motivation – The Flaw of Averages

A Few Things:

1. I am using the *arithmetic mean* not the *geometric mean*.
2. I am using annual compounding.
3. You may wish to use continuous compounding.
4. With continuous compounding, the arithmetic and geometric mean the same.
5. Using continuous does not affect the result I am demonstrating.

Motivation – The Flaw of Averages

Take a first cut using the **PMT** function in Excel. See cell B6, it has range name **payment.**

There are five arguments:

- **rate** – the interest rate to use in the calculation
- **nper** – the number of payment periods
- **pv** – the present value of your money, in this example the $200,000 at the beginning of the period
- **fv** – the future value – what you want to end up with after all the payments are made, in our case 0
- **type** – 0 if payments made at the end of the period, 1 if made at the beginning, the value is 0 if this argument is omitted

Motivation – The Flaw of Averages

In this example we use the formula in cell named **payment** which is

```
=PMT(interest_rate,years,-savings,0,1)
```

and this translates to

PMT(.1155, 20, -200000, 0, 1)

The value is $23,327.72

If you take out $23,327.72, while earning 11.55% on your money each year, you will drawn the account down to 0 at the end of 20 years.

Motivation – The Flaw of Averages

See the spreadsheet **retirement.xls**.

P29 f_x

	A	B	C	D	E	F	G	H
1								
2							S&P 500 (1926-2012)	
3	401 K Savings	$ 200,000.00					Mean =	11.55%
4	Years	20					STD =	20.62%
5	Interest Rate	11.55%						
6	Payment	$23,327.72						
7								
8								
9								
10								
11	Year	Year Start	Int Rate	Draw Down	Year End			
12	1	$ 200,000.00	0.115486207	$23,327.72	$ 197,075.50			
13	2	$ 197,075.50	0.115486207	$23,327.72	$ 193,813.25			
14	3	$ 193,813.25	0.115486207	$23,327.72	$ 190,174.26			
15	4	$ 190,174.26	0.115486207	$23,327.72	$ 186,115.02			
16	5	$ 186,115.02	0.115486207	$23,327.72	$ 181,586.99			
17	6	$ 181,586.99	0.115486207	$23,327.72	$ 176,536.04			
18	7	$ 176,536.04	0.115486207	$23,327.72	$ 170,901.77			
19	8	$ 170,901.77	0.115486207	$23,327.72	$ 164,616.82			
20	9	$ 164,616.82	0.115486207	$23,327.72	$ 157,606.04			
21	10	$ 157,606.04	0.115486207	$23,327.72	$ 149,785.62			
22	11	$ 149,785.62	0.115486207	$23,327.72	$ 141,062.05			
23	12	$ 141,062.05	0.115486207	$23,327.72	$ 131,331.02			
24	13	$ 131,331.02	0.115486207	$23,327.72	$ 120,476.20			
25	14	$ 120,476.20	0.115486207	$23,327.72	$ 108,367.79			
26	15	$ 108,367.79	0.115486207	$23,327.72	$ 94,861.03			
27	16	$ 94,861.03	0.115486207	$23,327.72	$ 79,794.43			
28	17	$ 79,794.43	0.115486207	$23,327.72	$ 62,987.84			
29	18	$ 62,987.84	0.115486207	$23,327.72	$ 44,240.32			
30	19	$ 44,240.32	0.115486207	$23,327.72	$ 23,327.72			
31	20	$ 23,327.72	0.115486207	$23,327.72	$ -			
32								

Motivation – The Flaw of Averages

See the spreadsheet **retirement.xls**.

The key formulas are in columns B and E. For example, in cell `B14` we have

```
E13
```

This says that the amount of money at the start of year 3 is equal to the amount of money at the end of year 2.

In cell `E14` we have

```
=IF(D14 > B14, 0, (1 + C14)*(B14-D14))
```

In the formula above we are saying that if the withdrawal amount is less than what we have in the account, the amount in the account at the end of the year is the beginning balance minus the withdrawal times $(1 +$ return$)$.

Motivation – The Flaw of Averages

In this analysis we assume that the S&P return is a **constant** 11.55% per year.

But 11.55% per year is the **average** from 1926 - 2012.

Historically, the **standard deviation** over this period is 20.62%

There is a serious problem with this!

Motivation – The Flaw of Averages

In the spreadsheet **retirement.xls** we make calculations based on the **average** interest rate. Everything is **deterministic**!

In the spreadsheet **retirementRisk.xls** we use **random interest rates** with a mean of 11.55% and standard deviation of 20.62%. We **simulate** reality.

Don't worry about how to generate random interest rates for now.

First, let's look at some potential retirement "scenarios."

Then we will recreate **retirementRisk.xls** from **retirement.xlsx**.

Motivation – The Flaw of Averages

Argle Bargle!!! I am out of money after only five years! Handout anyone?

	A	B	C	D	E	F	G	H
1								
2							**S&P 500 (1926-2012)**	
3	401 K Savings	$ 200,000.00					Mean	11.55%
4	Years	20					STD	20.62%
5	Interest Rate	11.55%						
6	Payment	$23,327.72						
7								
8								
9								
10								
11	**Year**	**Year Start**	**Int Rate**	**Draw Down**	**Year End**			
12	1	$ 200,000.00	-0.017702883	$23,327.72	$ 173,544.67			
13	2	$ 173,544.67	-0.599454979	$23,327.72	$ 60,168.65			
14	3	$ 60,168.65	0.446098661	$23,327.72	$ 53,275.63			
15	4	$ 53,275.63	0.053387546	$23,327.72	$ 31,546.76			
16	5	$ 31,546.76	0.019280501	$23,327.72	$ 8,377.51			
17	6	$ 8,377.51	0.110842395	$23,327.72	$ -			
18	7	$ -	0.064664258	$23,327.72	$ -			
19	8	$ -	-0.210102747	$23,327.72	$ -			
20	9	$ -	0.093862811	$23,327.72	$ -			
21	10	$ -	0.384393041	$23,327.72	$ -			
22	11	$ -	-0.12871117	$23,327.72	$ -			
23	12	$ -	0.367858693	$23,327.72	$ -			
24	13	$ -	-0.007671136	$23,327.72	$ -			
25	14	$ -	0.189868873	$23,327.72	$ -			
26	15	$ -	0.543874087	$23,327.72	$ -			
27	16	$ -	0.011208835	$23,327.72	$ -			
28	17	$ -	-0.165243196	$23,327.72	$ -			
29	18	$ -	0.146846915	$23,327.72	$ -			
30	19	$ -	0.153326718	$23,327.72	$ -			
31	20	$ -	0.01924298	$23,327.72	$ -			
32					5			

Motivation – The Flaw of Averages

What! Now I die with $561,151 left unspent. That is NOT fair!

	A	B	C	D	E	F	G	H
1								
2							**S&P 500 (1926-2012)**	
3	401 K Savings	$ 200,000.00					Mean	11.55%
4	Years	20					STD	20.62%
5	Interest Rate	11.55%						
6	Payment	$23,327.72						
7								
8								
9								
10								
11	**Year**	**Year Start**	**Int Rate**	**Draw Down**	**Year End**			
12	1	$ 200,000.00	0.115736532	$23,327.72	$ 197,119.72			
13	2	$ 197,119.72	0.431873852	$23,327.72	$ 248,848.23			
14	3	$ 248,848.23	0.419598292	$23,327.72	$ 320,148.53			
15	4	$ 320,148.53	0.108268446	$23,327.72	$ 328,957.14			
16	5	$ 328,957.14	0.368861279	$23,327.72	$ 418,364.29			
17	6	$ 418,364.29	0.393558616	$23,327.72	$ 550,506.61			
18	7	$ 550,506.61	0.145572987	$23,327.72	$ 603,921.90			
19	8	$ 603,921.90	0.075788887	$23,327.72	$ 624,596.77			
20	9	$ 624,596.77	0.18281848	$23,327.72	$ 711,192.15			
21	10	$ 711,192.15	-0.281099181	$23,327.72	$ 494,506.31			
22	11	$ 494,506.31	0.164793774	$23,327.72	$ 548,825.89			
23	12	$ 548,825.89	-0.074696464	$23,327.72	$ 486,245.32			
24	13	$ 486,245.32	-0.011460911	$23,327.72	$ 457,612.14			
25	14	$ 457,612.14	0.364424574	$23,327.72	$ 592,548.34			
26	15	$ 592,548.34	0.190884976	$23,327.72	$ 677,876.29			
27	16	$ 677,876.29	-0.137185597	$23,327.72	$ 564,753.94			
28	17	$ 564,753.94	0.153453956	$23,327.72	$ 624,510.21			
29	18	$ 624,510.21	0.261038641	$23,327.72	$ 758,114.36			
30	19	$ 758,114.36	-0.085009822	$23,327.72	$ 672,322.56			
31	20	$ 672,322.56	-0.13535223	$23,327.72	$ 561,151.94			
32					20			

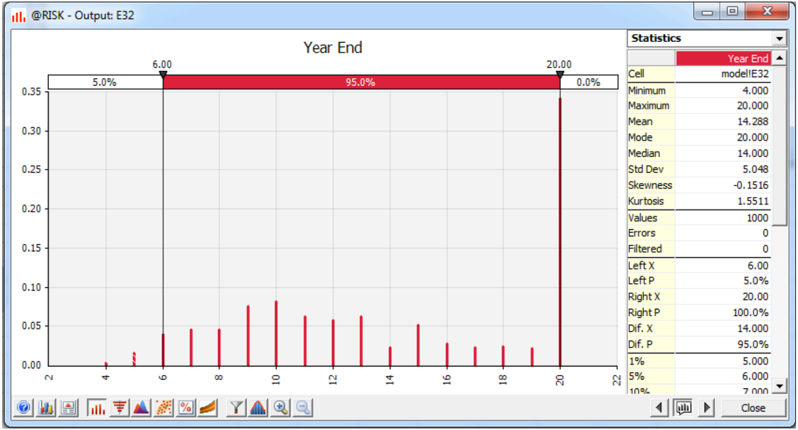
Motivation – The Flaw of Averages

Argle Bargle!!! Out of money yet again.

	A	B	C	D	E	F	G	H
1								
2							S&P 500 (1926-2012)	
3	401 K Savings	$ 200,000.00					Mean	11.55%
4	Years	20					STD	20.62%
5	Interest Rate	11.55%						
6	Payment	$23,327.72						
7								
8								
9								
10								
11	**Year**	**Year Start**	**Int Rate**	**Draw Down**	**Year End**			
12	1	$ 200,000.00	0.267120496	$23,327.72	$ 223,865.07			
13	2	$ 223,865.07	0.194938666	$23,327.72	$ 239,629.84			
14	3	$ 239,629.84	0.245957077	$23,327.72	$ 269,503.16			
15	4	$ 269,503.16	0.038238802	$23,327.72	$ 255,588.90			
16	5	$ 255,588.90	0.083271065	$23,327.72	$ 251,601.82			
17	6	$ 251,601.82	0.124140603	$23,327.72	$ 256,612.18			
18	7	$ 256,612.18	0.146688806	$23,327.72	$ 267,504.69			
19	8	$ 267,504.69	-0.074902766	$23,327.72	$ 225,887.44			
20	9	$ 225,887.44	-0.017346877	$23,327.72	$ 199,045.94			
21	10	$ 199,045.94	-0.217321715	$23,327.72	$ 137,530.84			
22	11	$ 137,530.84	-0.165813257	$23,327.72	$ 95,266.73			
23	12	$ 95,266.73	0.022100596	$23,327.72	$ 73,528.91			
24	13	$ 73,528.91	0.421284207	$23,327.72	$ 71,350.16			
25	14	$ 71,350.16	-0.181733152	$23,327.72	$ 39,295.18			
26	15	$ 39,295.18	0.261936138	$23,327.72	$ 20,149.92			
27	16	$ 20,149.92	0.221146436	$23,327.72	$ -			
28	17	$ -	-0.062091717	$23,327.72	$ -			
29	18	$ -	0.084619609	$23,327.72	$ -			
30	19	$ -	0.363818936	$23,327.72	$ -			
31	20	$ -	0.028921607	$23,327.72	$ -			
32					15			

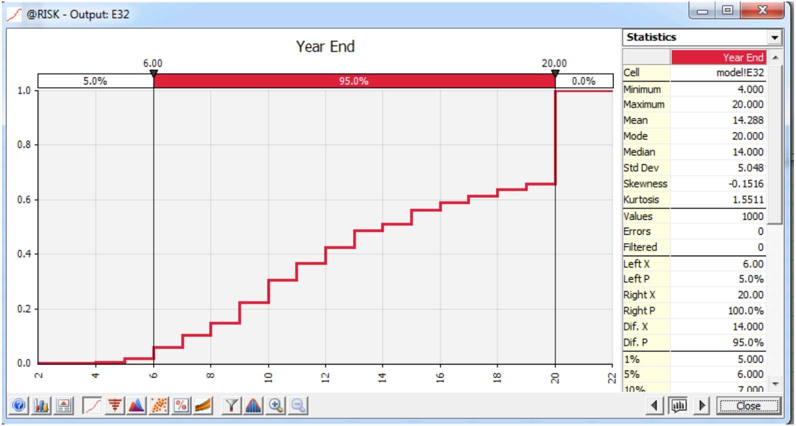
Motivation – The Flaw of Averages

Living a thousand lives. Distribution of years.



Motivation – The Flaw of Averages

Living a thousand lives, cumulative probabilities.



Motivation – The Flaw of Averages

Sam Savage has a good explanation for what is going on.

See his book the *Flaw of Averages*

YouTube link

`http://www.youtube.com/watch?v=0ah03QcRxGY`

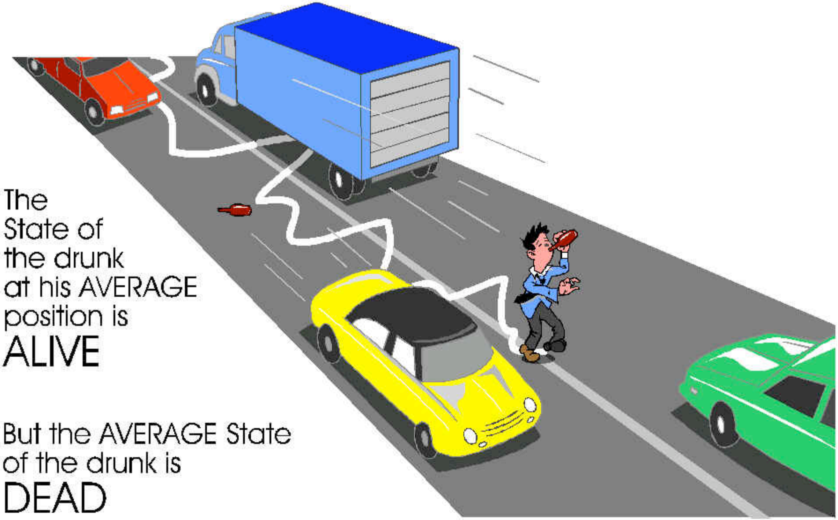
Motivation – The Flaw of Averages

This lecture is best summed up by the pictures on the following slides.

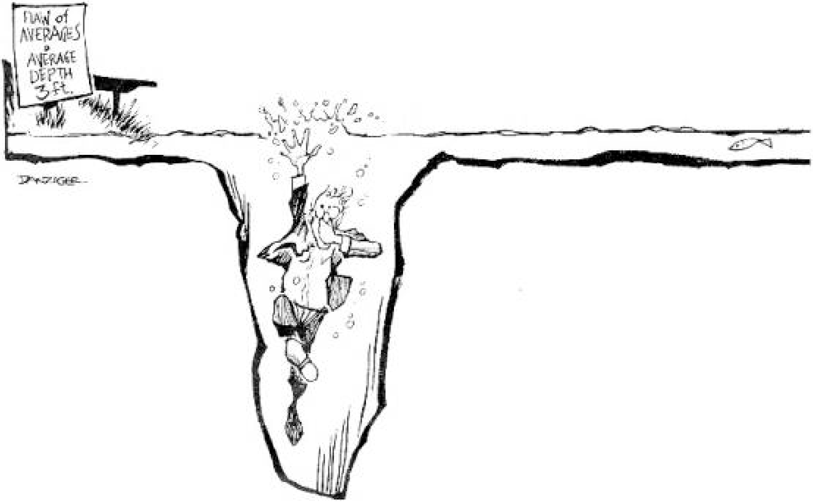
The first picture was shamelessly stolen from the Web site of Sam Savage. But Sam is a nice guy and probably will not sue me.

The second picture was from the *San Jose Mercury Times*. Hopefully they won't sue either.

Motivation – The Flaw of Averages



Motivation – The Flaw of Averages



Motivation – The Flaw of Averages

The bottom line – the average can be pretty useless!

A second rule of thumb:

The more nonlinear the problem, the worse our results will be by using average values. Here is the problem:

$$E(f(X)) \neq f(E(X))$$

When you put average values into the cells (i.e. $E(X)$) you are calculating $f(E(X))$ but you really want $E(f(X))$.

What is X in our retirement example?

What is $f(X)$ in our case?

Warning: $f(X)$ is a real ARGLE BARGLE!

Motivation – The Flaw of Averages

In our case, $f(X)$ is

```
COUNTIF(E12:E31,">0")
```

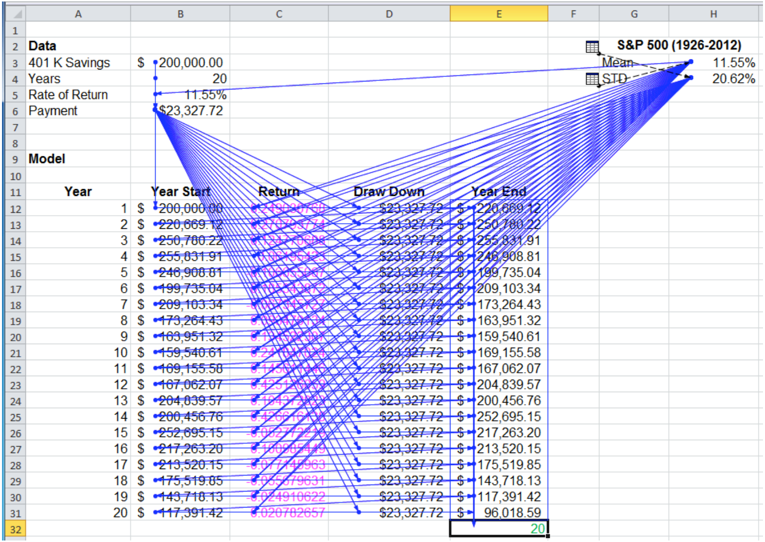
and X corresponds to the values in **C12:31**.

Not only is an IF involved, but we are **counting IF statements**!

Pass me the **defibrillator** please!

Motivation – The Flaw of Averages

The Trace Precedents for $f(X)$!



	A	B	C	D	E	F	G	H
1								
2	**Data**							**S&P 500 (1926-2012)**
3	401 K Savings	$ 200,000.00					Mean	11.55%
4	Years	20					STD	20.62%
5	Rate of Return	11.55%						
6	Payment	$23,327.72						
7								
8								
9	**Model**							
10								
11	**Year**	**Year Start**	**Return**	**Draw Down**	**Year End**			
12	1	$ 200,000.00	0.249030760	$23,327.72	$ 220,669.12			
13	2	$ 220,669.12	0.270793774	$23,327.72	$ 250,780.22			
14	3	$ 250,780.22	0.124770608	$23,327.72	$ 255,831.91			
15	4	$ 255,831.91	0.06195423[illegible]	$23,327.72	$ 246,908.81			
16	5	$ 246,908.81	-0.106655067	$23,327.72	$ 199,735.04			
17	6	$ 199,735.04	0.185343872	$23,327.72	$ 209,103.34			
18	7	$ 209,103.34	-0.067345727	$23,327.72	$ 173,264.43			
19	8	$ 173,264.43	0.093470134	$23,327.72	$ 163,951.32			
20	9	$ 163,951.32	0.134522307	$23,327.72	$ 159,540.61			
21	10	$ 159,540.61	0.241847024	$23,327.72	$ 169,155.58			
22	11	$ 169,155.58	0.145611446	$23,327.72	$ 167,062.07			
23	12	$ 167,062.07	0.425126059	$23,327.72	$ 204,839.57			
24	13	$ 204,839.57	0.104372023	$23,327.72	$ 200,456.76			
25	14	$ 200,456.76	0.426616108	$23,327.72	$ 252,695.15			
26	15	$ 252,695.15	-0.052772216	$23,327.72	$ 217,263.20			
27	16	$ 217,263.20	0.100985449	$23,327.72	$ 213,520.15			
28	17	$ 213,520.15	-0.077145963	$23,327.72	$ 175,519.85			
29	18	$ 175,519.85	-0.055679631	$23,327.72	$ 143,718.13			
30	19	$ 143,718.13	-0.024910622	$23,327.72	$ 117,391.42			
31	20	$ 117,391.42	0.020782657	$23,327.72	$ 96,018.59			
32					20			

Motivation – The Flaw of Averages

Here is a good example to help your intuition on why

$$E(f(X)) \neq f(E(X))$$

Consider a random variable X that can take on two values: the two values are $X = 1$ and $X = -1$ (the market is either up or down).

Assume each event is equally likely and so the probabilities are:

$p(X = 1) = .5$ and $p(X = -1) = .5$.

The function of interest is: $f(X) = \max(0, X)$.

- What is $f(E(X))$?
- What is $E(f(X))$?

Motivation – The Flaw of Averages

What is $f(E(X))$?

$$E(X) = 1 * .5 - 1 * .5 = 0$$

$$f(E(X)) = f(0) = \max(0, 0) = 0$$

What is $E(f(X))$?

$$E(f(X)) = .5 * \max(0, 1) + .5 * \max(0, -1) = .5 * 1 + .5 * 0 = .5$$

$$E(f(X)) \neq f(E(X))$$

Motivation – The Flaw of Averages

Remember: *A linear life is a good life.* Some “bad” functions:

- IF
- MAX, MIN
- COUNTIF

Before you took this course, you probably thought the IF function in Excel was pretty nice. Little did you know of the dangers that lurked below.

Simulation Basics

Our goal is to replace **parameters** with **distributions.**

We then **sample** from the distributions.

Simulation Basics

In the following slides we take the workbook `retirement.xlsx` and create the content in workbook `retirementRisk.xlsx`.

Step 1: use the Excel function `COUNTIF`.

Put the formula

```
=COUNTIF(E12:E31,">0")
```

in cell `E32`.

This will count the number of years where we end the year with a *positive amount of money.*

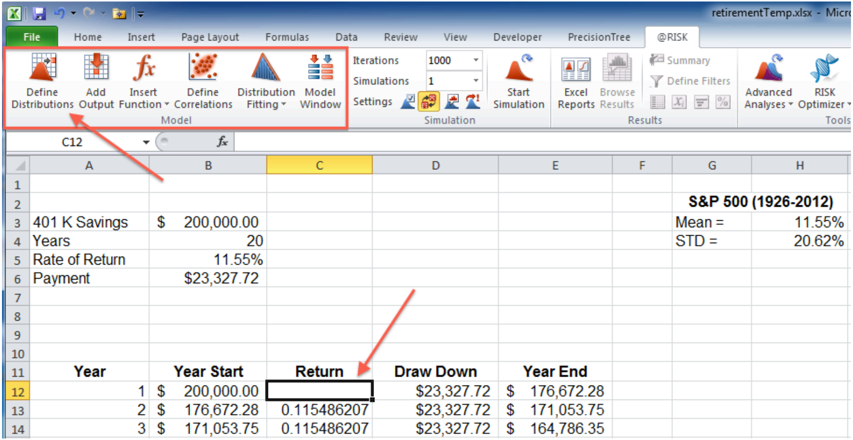
Objective: run a 1000 “trials” and see the “behavior” of this number.

Simulation Basics

Step 2 Incorporate Uncertainty: Here is how we incorporate uncertainty.

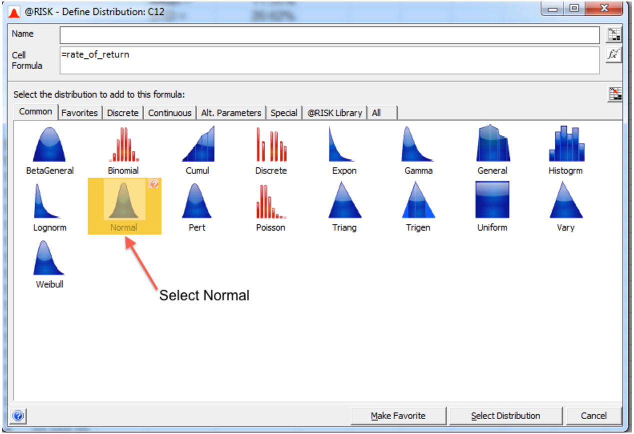
- We were using *average* values for the rates of return. This changes.
- In Column C, replace the "fixed" rates with random draws.
- Select the @Risk Tab in the Ribbon.
- Select and clear the contents of cell `C12`.
- Select Cell `C12` and then **Define Distribution** from the **Model** group. See next slide.

Simulation Basics



Simulation Basics

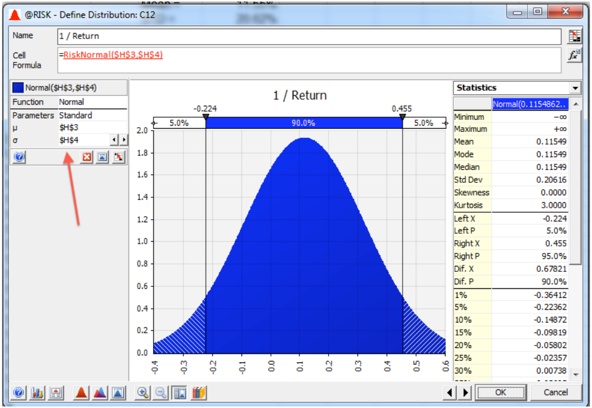
When you click on **Define Distribution** you will see:



Select the Normal Distribution.

Simulation Basics

Put in the mean and standard deviation.



Note the absolute addressing H3 and H4.

We could have used the range names **mean** and **std**.

Simulation Basics

We used **Define Distributions** to put in a distribution. Other options include:

- Use the **Insert Function**
- Just type it in (there is auto completion available)

Simulation Basics

Step 3: Copy Cell `C12` and paste it to Cells `C13` to `C31`.

Every time you recalculate the spreadsheet (F9 or recalculate under Formulas). We make a "draw" of a rate of return from the normal distribution with mean .1155 and standard deviation .2062 for cells `C12` - `C31`.

These draws are all independent of each other. That is, *they are not correlated*. More on this topic later.

Make the font in cells `C12` - `C31` magenta. We are going to make our uncertain parameter cells **magenta**. I am open to other color selections.

Simulation Basics

Step 4: Now, back to Cell E32. This is the cell we care about. It is telling how many years we end with a positive amount of money (a good thing).

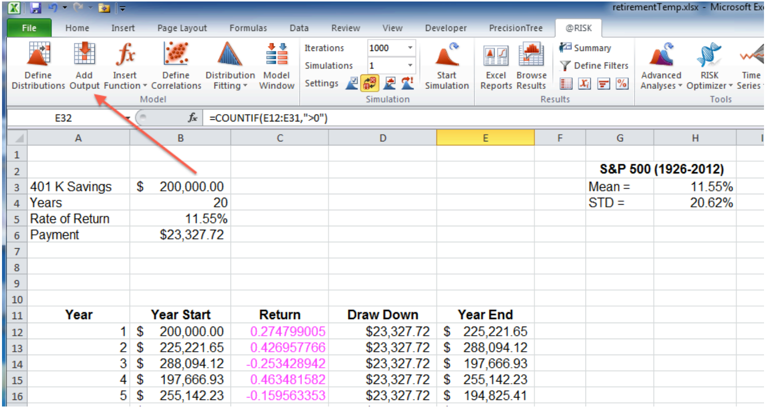
Key Idea: Cell E32 is really a random variable. Its value depends on other random variables.

We care about the probability distribution of Cell E32!

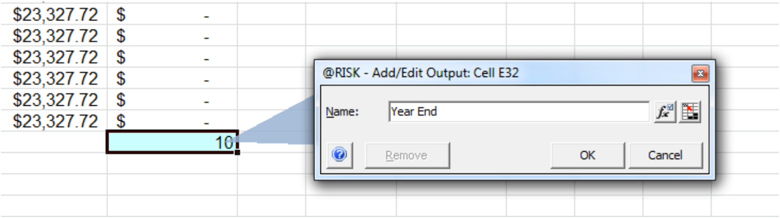
We tell @RISK to consider Cell E32 an output cell. See next slide.

Simulation Basics

Select Cell E32 and then click **Add Output** from the **Model** group.



Simulation Basics



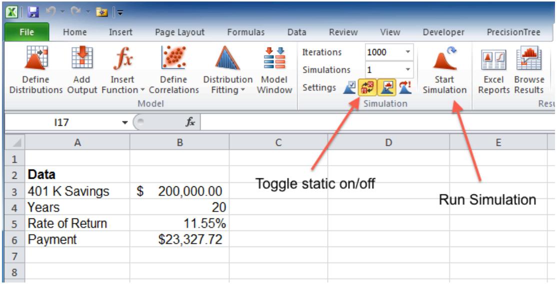
In Cell E32 you should see

`=RiskOutput()+COUNTIF(E12:E31,">0")`

@RISK will now treat Cell E32 as a random variable and generate probability distribution information for the cell output.

Simulation Basics

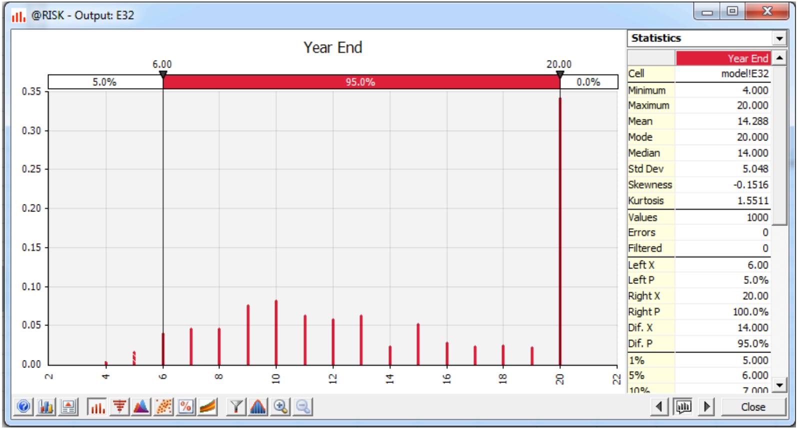
Step 5: Run the simulation!



My default is set for a 1000 trials. Don't worry about how many trials for now.

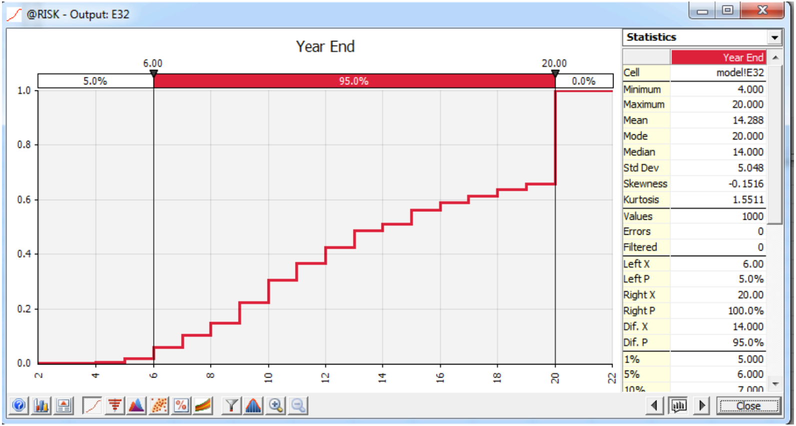
Simulation Basics

Step 5: Examine the output for Cell E32.



Simulation Basics

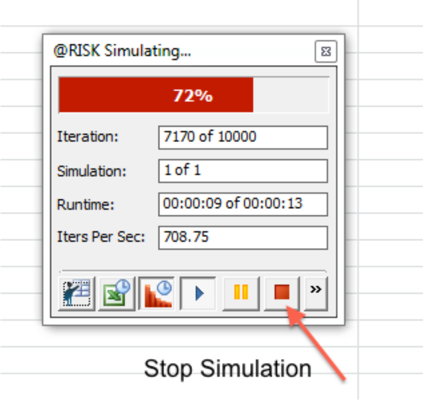
Here is the cumulative histogram.



Simulation Basics

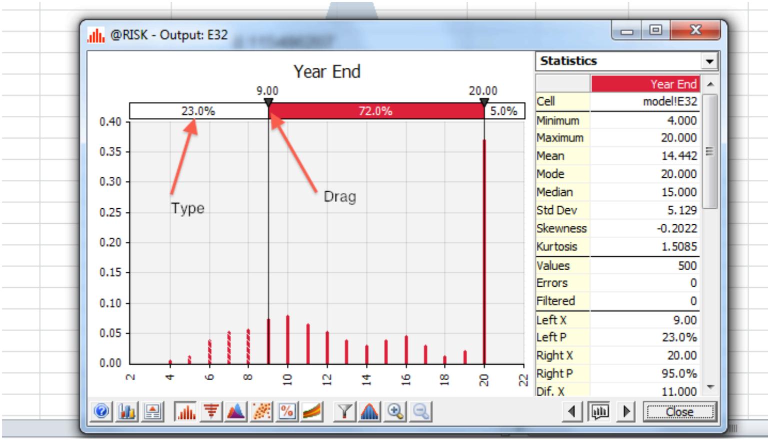
How to stop the simulation.

What to do if your simulation seems to be taking too long.



Simulation Basics

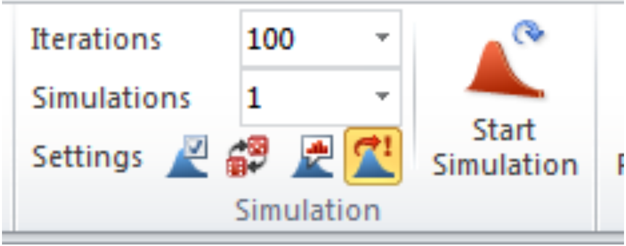
You can adjust the tails of the histogram by dragging or typing in the tail value you care about.



Simulation Basics

Other Hints:

1. The simulation will run **much** faster if you turn **demo** off.
2. I recommend you turn on "Automatically Show Output Graph."



Simulation Basics

Other Distributions: In Week 9 we will discuss more thoroughly selecting distributions for simulation.

For now we consider two other distributions: (see `riskStatic.xlsx`)

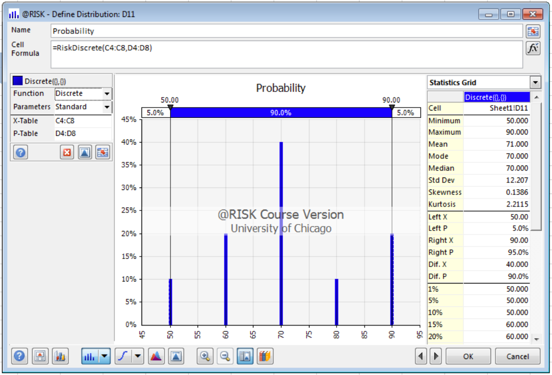
1. **RiskDiscrete**

2. **RiskUniform**

Simulation Basics

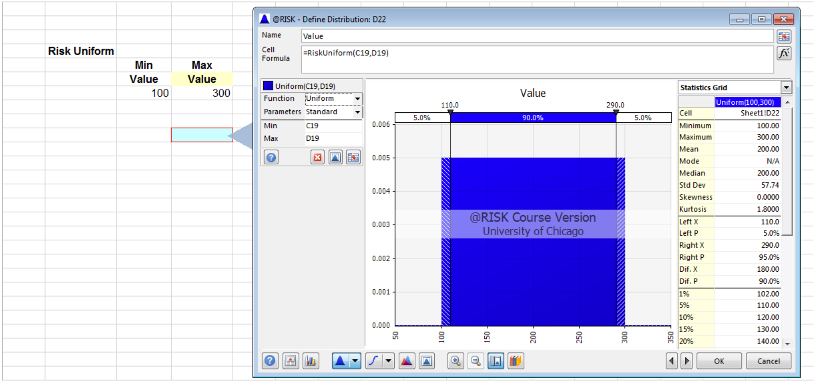
Risk Discrete Distribution

Risk Discrete		
	Value	Probability
	50	0.1
	60	0.2
	70	0.4
	80	0.1
	90	0.2



Simulation Basics

Risk Uniform Distribution



Simulation Basics

Sample Exam Question 1: In the retirement example, what is f(E(X)) when the payment is $23,327.72 and

```
f(payment, returns) = COUNTIF(E12:E31,">0")
```

Sample Exam Question 2: In this example, what is E(f(X)) when the payment is $23,327.72 and

```
f(payment, returns) = COUNTIF(E12:E31,">0")
```

Simulation Basics

The bottom line – using averages can lead to bad results when the standard deviation is large!

A rule of thumb:

The greater the variance, the worse our results will be by using average values.

The average high temperature for Chicago on April 1 is 53 degrees.

Conclusion: If I wear a light jacket walking to the Harper Center I will be comfortable.

The average high temperature for Honolulu on April 1 is 82 degrees.

Conclusion: If I wear a tee shirt and jeans I will be comfortable.

Simulation Basics

The bottom line – using averages can lead to bad results when the standard deviation is large!

	Running Back 1	Running Back 2
Avg. yrds. per carry	4.0	4.5
Std. Dev.	0.3	2.5

Situation 1: Own 20, ahead by 1 point, 5 minutes left in fourth quarter. Which back should carry?

Situation 2: Own 20, down by 1 point, 5 minutes left in fourth quarter. Which back should carry?

Simulation Basics

Summary:

- Using an expected value instead of a distribution is bad.
- Nonlinearity makes this even worse.
- Big standard deviations make this even worse.

Simulation Basics

Color Coding:

- Black – a **parameter cell** known with certainty (or at least treated as such)
- Magenta – an **uncertain parameter cell**, one with an @RISK distribution
- Blue – an adjustable cell, a variable cell, something that we choose using `Goal Seek` or `RiskSimtable()`
- Green – an @RISK output cell (contains `=RiskOutput()` from **Add Output**)

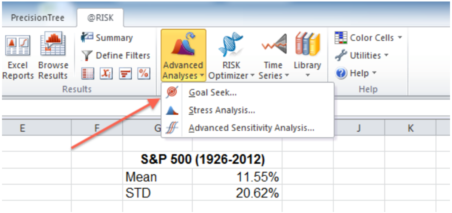
Simulation Goal Seek

If we withdraw \$23,327.72 per year, on average, it will last 14.42 years. Not very satisfying.

Even worse, there was a draw of only 3 years.

Let turn the question around and see how much we can take out each year, and have a minimum draw of 18.

In @RISK there is a Goal Seek very similar to the Excel Goal Seek we used earlier in the quarter.

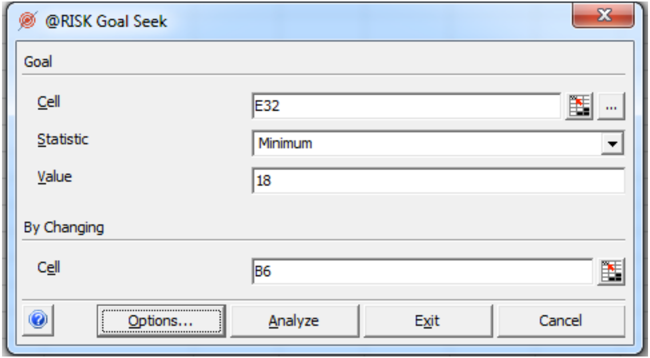


Simulation Goal Seek

The **Goal** cell is E32. This is the cell that counts the number of periods with a positive amount of money.

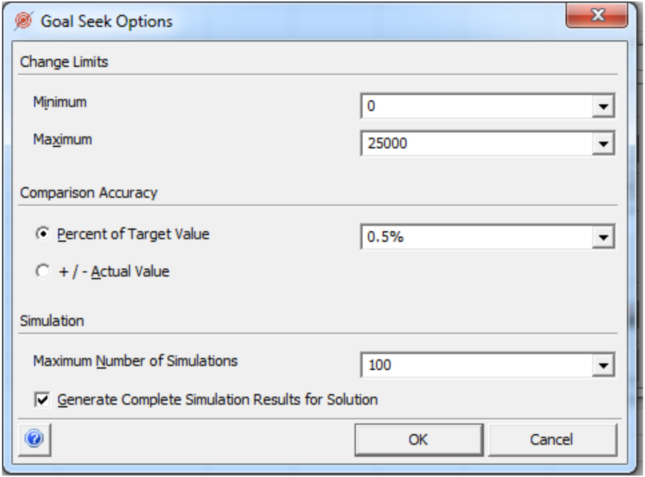
We are interested in recording the **minimum** number of years for this cell.

We change cell B6 which is the amount of money we withdraw each year.



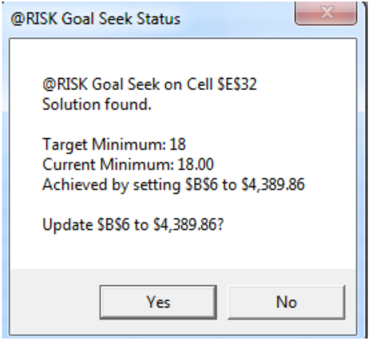
Simulation Goal Seek

Notice we are restricting our search to be between 0 dollars and 25000 dollars.



Simulation Goal Seek

We should withdraw \$4,389.86. Notice we did only 500 iterations per simulation. This should probably be larger.



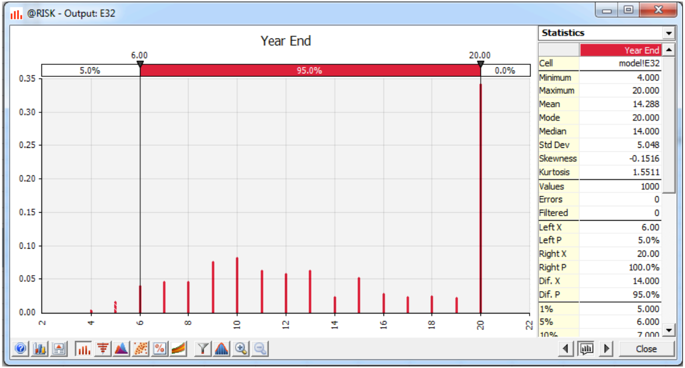
Simulation Goal Seek

Note: If you did not invest in the market, and got a 0 percent rate of return each year, you could withdraw $10,000 a year and die with zero dollars left.

This suggests running a more complicated simulation where you can allocate your money between the stock market and a less risky asset.

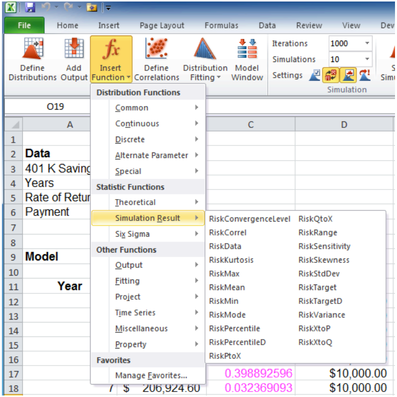
Summary

Running a simulation creates a histogram for each cell in the spreadsheet that contains a `RiskOutput()`. You can have @RISK put the summary statistics from the simulation directly in the spreadsheet.



Summary

You can insert functions with **Insert Function**, **Statistic Functions**, then **Simulation Result**.



Summary

See the workbook `retirementRisk`.

1. In cell **E33** I have inserted `=RiskMean(E32,1)`.

2. In cell **E34** I have inserted `=RiskVariance(E32,1)`.

After running a new simulation, these cells will hold, respectively, the simulation sample mean and variance.

Cell **E32** is the output of the simulation, it is `=RiskOutput()+COUNTIF(E12:E31,">0")`.

Summary

What we are doing is called **Monte Carlo Simulation**. The process was developed by Stanislaw Ulam and Nicholas Metropolis.

Process developed while working on nuclear weapons projects at Los Alamos. Named after Monte Carlo where Ulam's uncle often gambled.

Building a Monte Carlo Simulation Model:

1. Identify model inputs:
   - deterministic inputs (black cells)
   - random inputs (magenta cells) – must also determine the distribution
2. Identify decision variable or variables (blue cells)
3. Identify model outputs (green cells)
4. Run the simulation
5. Analyze the outputs

Summary

The benefit of Monte Carlo simulation – **Booth Students** winning the 2012 SABR (Society for American Baseball Research) Competition.

http://sabr.org/analytics/case/2012

"One of the things that distinguished the Booth (presentation) was their outstanding use of risk analysis, **particularly their Monte Carlo simulations** on playoff probabilities and (looking at) some of the other considerations that weighed into their decision," Gennaro (society president) said.

Summary

Deja Vu all over again.

Sample Exam Question: In this example, what is f(E(X))?

Sample Exam Question: In this example, what is E(f(X))?

You must understand the difference between $E(f(X))$ and $f(E(X))$.

KEY CONCEPT: Even when $E(f(X)) = f(E(X))$ simulation is very useful. Why?