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Optimal Procurement with Spot Purchases

Business 36106
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1 Problem Description

The Navy Supply System Command is headquartered in Mechanicsberg, Pennsylvania, and is a procurement organization with an annual budget in the billions of dollars. The purpose of the Navy Supply System Command is to resupply the fleet's needs.

The resupply is handled by COG (cognizant ordering group) managers who make buys in their separate areas, e.g., electronics, ordinance, etc. Formally a COG is "A two character alphanumeric cognizant code used to identify and designate the Inventory Control Point office or agency, which exercises supply management. The code identifies the combined technical bureau/command and inventory manager having jurisdiction over the item."[1] Each COG manager is then responsible for the worldwide resupply of his/her unique set of items.

Because of the rapid obsolescence of military inventory, a typical manager might hold back part of his/her budget until late in the year when demand becomes better known. At that time the COG manager can release part of the remaining budget for "spot" purchases. For example, in 1976 the COG managers held back, on average, 11% of their stock replenishment funds for spot buys. (Working Memorandum USN 1976) Much to the COG managers' chagrin and regret these spot buys were at a median increase of 31% over the initial purchase cost.

Our objective is to build an optimization model to determine the optimal

[1] see NAVSO P-3683B Navy and Marine Corps Product Data Reporting and Evaluation Program Manual (http://www.everyspec.com)

order quantity at the start of each year in order to minimize cost.[2] Unfortunately demand is not known when an order is placed. At the time the order is placed, demand must be estimated. In our lingo, demand is a random variable. Obviously, if we knew demand exactly when placing an order, the order quantity would equal demand.

2 Model With No Budget Constraint

First consider the case where there is no budget restriction. In this case the COG manager can make spot purchases to satisfy any demand in excess of the initial purchase quantity. Stock outs are not allowed. Let,

p – purchase price (initial)

v – marginal salvage value (possibly 0 or negative) at year end

s – purchase price in the spot market

x – demand random variable

$g(x)$ – the p.d.f. (probability density function) of demand

Q – the optimal initial purchase quantity

Assume the following relationship among costs.

$$v < p \leq s$$

If $p = s$, what is the optimal initial purchase quantity, i.e. $Q = ?$

If Q is the order quantity then the purchase cost is $p * Q$. What are the other costs? There are two cases to consider. First, realized demand may exceed the order quantity, i.e $x > Q$. Since we do not allow stock outs, we purchase $x - Q$ units at a marginal spot market cost s. However, if demand is less than, or equal to, the order quantity, $Q \geq x$ then we salvage $Q - x$ units for v dollars per unit. Therefore the cost function $c(Q)$ is

[2] see Richard Morey and Dennis Sweeney, "A Budget Holdback Policy for Multi-Item Procurement", *Management Science* 30 (5) 1984 pp. 604-617.

$$c(Q) = pQ - \int_0^Q v * (Q - x)g(x)dx + \int_Q^\infty s * (x - Q)g(x)dx \tag{1}$$

Construct and run a Risk Optimizer model to find the Q that minimizes the total expected expected cost $c(Q)$. Assume that demand for the product under consideration follows a normal distribution with mean 140 and standard deviation 20. Further assume that $p = \$10$, $s = \$12$ and $v = \$7$.

The problem just described is often referred to as the news vendor problem. You may have studied it an Operations Management class. We now add to it a realistic complicating constraint.

3 Model With Budget Constraint

In the previous model stock outs were not allowed. When demand was in excess of the order quantity, additional units were purchased on the spot market. In the presence of a budget, meeting the zero stock out requirement may not be possible. In this case we charge a penalty fee of f dollars per unit that is stocked out.

Assume a budget of B dollars, then the maximum number of units that can be purchased in the spot market is

$$y = (B - p * Q)/s.$$

There are three cases to consider.

1. $x \leq Q$ – In this case there is a return of $v * (Q - x)$.

2. $Q < x \leq Q + y$ – In this case, there is nothing to salvage and we spend $s * (x - Q)$ in the spot market. In this case we are explicitly considering the budget constraint. Since $x \leq Q + y$ total expenditures are

$$\begin{aligned} p * Q + s * (x - Q) \leq p * Q + s * (Q + y - Q) &= p * Q + s * y \\ &= p * Q + (B - p * Q) \\ &= B \end{aligned}$$

3. $Q+y < x$ – This one is a bit more complicated. In this case we exhaust our entire budget and spend

$$s * y = s * (B - p * Q)/s = (B - p * Q)$$

in the spot market. In addition we pay a penalty fee of

$$f * (x - Q - y).$$

Then the cost function is

$$\begin{aligned} c(Q) &= pQ - \int_0^Q v * (Q - x) g(x) dx + \int_Q^{y+Q} s * (x - Q) g(x) dx \qquad (2) \\ &+ \int_{y+Q}^{\infty} (s * y + f * (x - Q - y))\, g(x) dx. \qquad (3) \end{aligned}$$

Assume $f = 15$ and $B = \$1500$. All other parameters remain as before. Use Risk Optimizer to find the order quantity Q that minimizes $c(Q)$.