36106 Managerial Decision Modeling
Monte Carlo Simulation in Excel: Part III

Kipp Martin
University of Chicago
Booth School of Business

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Reading and Excel Files

Reading:

- Powell and Baker: Chapter 14.6-14.8
- Simulation Elections Using @RISK – Handout link for Week Nine.

Files used in this lecture:

- `distributions.xlsx`
- `confidence_interval.xlsx`
- `markowitzCorrelation.xlsx`
- `markowitzCorrelation_key.xlsx`

Learning Objectives

1. Learn how to select among various probability distributions

2. Learn how to select a distribution based on data

3. Learn how to build a confidence interval on simulation outputs

4. Learn how to generate correlated random variables

Lecture Outline

Errors

We would like to know $E(f(X))$ but there are pitfalls:

1. We must estimate $f(X)$ – the function is just an estimate for reality
2. We must estimate X (the random variable)
3. We must run enough trials so that we can have confidence in $f(X)$

Probability Distributions

THE KEY CONCEPT: Don't use averages, use distributions!

A PROBLEM: if you use a woefully miscast distribution you may get bad results.

A bad distribution may be worse than no distribution at all.

- maybe reality is very "skewed" and you use a symmetric distribution
- maybe outcomes are very correlated and you pick independent distributions

First, address the problem of selecting a distribution. Then address the problem of correlated random variables.

Probability Distributions

Some Options:

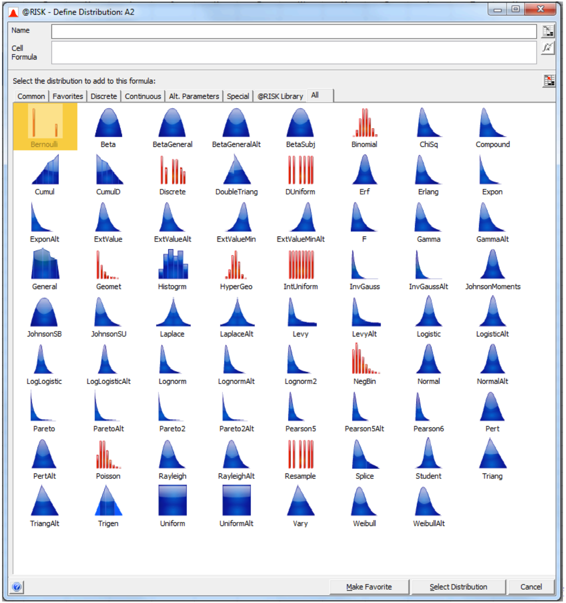
If you have some historical data you could:

1. build a histogram and use **RiskDiscrete** (more on this later)
2. fit a distribution to your data (more on this later) using @RISK
3. use your data to estimate certain parameters such as a mean and standard deviation and use these as inputs to a distribution (e.g. normal)

If you do not have any data you could *subjectively pick a distribution*. In this case it important to understand which distribution might be most appropriate.

Probability Distributions

@RISK Distributions (there are 71)



Probability Distributions

Characteristics:

- Discrete versus Continuous
- Symmetric versus skewed (measure of asymmetry – if you must know it is the third moment about the mean)
- Bounded versus unbounded
- Nonnegative versus positive and negative values

Probability Distributions

Key Distributions:

- Discrete
  1. RiskDiscrete $(\{x_1, x_2, \ldots, x_n\}, \{p_1, p_2, \ldots, p_n\})$
  2. RiskBinomial (N, p)
  3. RiskBernoulli(p)
  4. Poisson(Mean)
- Continuous
  1. RiskUniform (Min, Max)
  2. RiskTriang(Min, Most Likely, Max)
  3. RiskPert(Min, Most Likely, Max)
  4. RiskNormal(Mean, Std Dev)
  5. RiskLognorm(Mean, Std Dev)
  6. Exponential(Mean)

Probability Distributions

RiskDiscrete $(\{x_1, x_2, \ldots, x_n\}, \{p_1, p_2, \ldots, p_n\})$**:**

This is a **discrete** distribution that may be **skewed**. It is **bounded** and may have **negative** values.

If you have historical data and make a histogram, you can use the histogram to produce a **RiskDiscrete** distribution.

See `distributions.xlsx`

Probability Distributions

RiskBinomial (N, p)**:** where there are N "trials" and the probability of "success" is p.

This is a **discrete** distribution that may be **skewed**. It is **bounded** and it is nonnegative.

The mean of this distribution is pN.

This applies when the trials are independent events, for example flipping a fair coin.

You might use this as follows: you have sold 300 tickets for a flight. In the past the probability of a no-show is .1. The random variable for the number of people showing up is binomial with $N = 300$ and $p = .9$.

See `distributions.xlsx`

Probability Distributions

RiskBernoullil (p): The values of the random variable are either $x = 1$ with probability p, or 0 with probability $1 - p$.

This is like the binomial when $N = 1$.

This is a **discrete** distribution that may be **skewed**. It is **bounded** and it is nonnegative.

We use this distribution is our simulation of election outcomes.

See `distributions.xlsx`

Probability Distributions

RiskUniform (Min, Max): "Equally likely" events in an interval.

- A continuous distribution
- A bounded distribution
- May assume positive or negative values

See `distributions.xlsx`

Probability Distributions

RiskPert (Min, Most Likely, Max) and RiskTriang (Min, Most Likely, Max): where Min is the minimum possible value, Max is the maximum possible value, and Most Likely, is well, most likely.

Both are **continuous** distributions and may be **asymmetric**. They are **bounded** and may take on negative values.

The Pert distribution put less emphasis on extreme values.

See `distributions.xlsx` where the triangle and pert are superimposed.

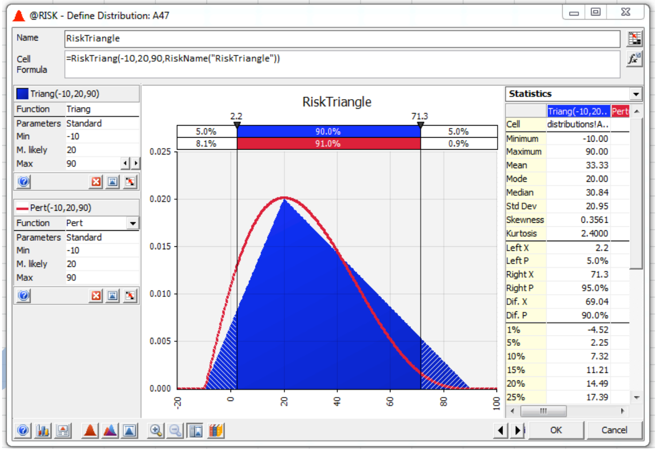
These are good distributions to use when:

- You want an asymmetric distribution (although Triangle and Pert can be symmetric).
- You do not have historical data, but have subjective guesses as to minimum, maximum, and most likely values.

Probability Distributions

The mean of the triangle distribution is $(a + b + c)/3$. All values weighted equally.

The mean of the Pert distribution is $(a + 4b + c)/6$.



Probability Distributions

RiskNormal(Mean, Std. Dev.):

A **continuous** distribution that is **symmetric**. It is **unbounded** and may take on negative values.

Good to use when Central Limit Theorem applies.

Central Limit Theorem applies when you are summing independent random variables with well defined mean and variance.

Probability Distributions

RiskLognorm(Mean, Std. Dev.):

A **continuous** distribution that is **asymmetric**. It is **unbounded** but does not take on negative or zero values. The lognormal random variable is given by

$$X = e^{(\mu+\sigma Z)}$$

where Z is a standard normal variable. For this random variable

$$E(X) = e^{(\mu+\sigma^2/2)}$$

$$Var(X) = (e^{\sigma^2} - 1)e^{(2\mu+\sigma^2)}$$

Probability Distributions

The lognormal is often used in finance to model stock prices.

If you use continuous compounding and then take the log of the ratio of stock prices you get normally distributed returns.

$$P_t = P_0 e^{\left((r-0.5\sigma^2)t+\sigma t^{0.5} Z\right)}$$

- P_0 is the stock price at time 0
- P_t is the estimated stock price at time t
- r is the mean continuous compounding growth rate (estimate from data)
- σ is the continuous compounding growth rate standard deviation (estimate from data)
- Z is the Normal(0,1)

Probability Distributions

Using the expression for Expected value of the lognormal, one can show:

$$E(P_t) = P_0 e^{rt}$$

See spreadsheet **lognormal** in the **distributions.xlsx** workbook. We estimate a stock price six periods out, two ways.

1. First way: generate values of N(0,1) and plug into the exponential formula

$$P_t = P_0 e^{\left((r-0.5\sigma^2)t+\sigma t^{0.5} Z\right)}$$

2. Second way: generate lognormal random values based on $E(x)$ and $Var(X)$ where

$$\begin{aligned} X &= e^{\left((r-0.5\sigma^2)t+\sigma t^{0.5} Z\right)} \\ P_t &= P_0 X \end{aligned}$$

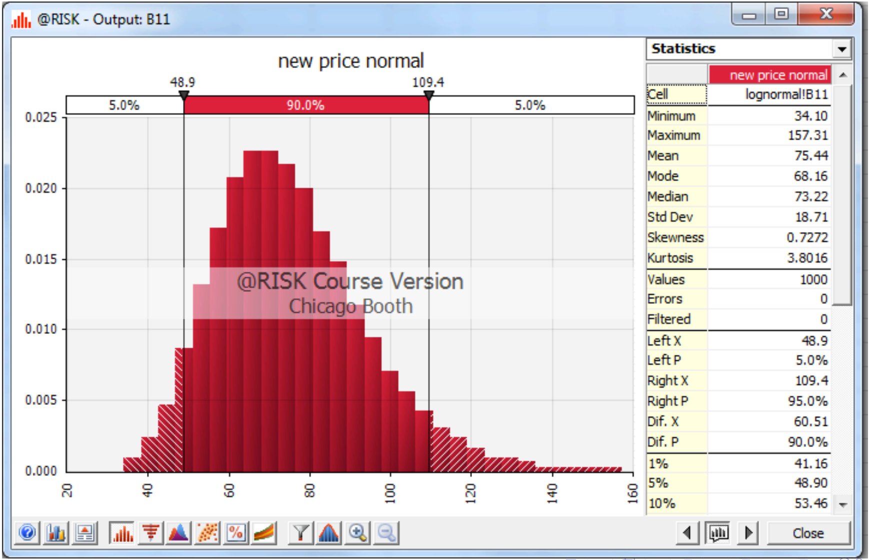
Probability Distributions

Simulate the stock price two ways.

	A	B
1		
2		
3	rate--r	1.25%
4	sigma	10.00%
5	t	6
6	Initial Price	70
7		
8	**First Way**	
9	Normal (0, 1)	-0.50899719
10	exp argument	-0.07967834
11	new price normal	64.63893269
12		
13		
14	**Second Way**	
15	lognormal mu	0.045
16	lognormal sigma	0.244948974
17	lognormal mean	1.077884151
18	lognormal std dev	0.26803697
19	new price lognormal	93.72709334

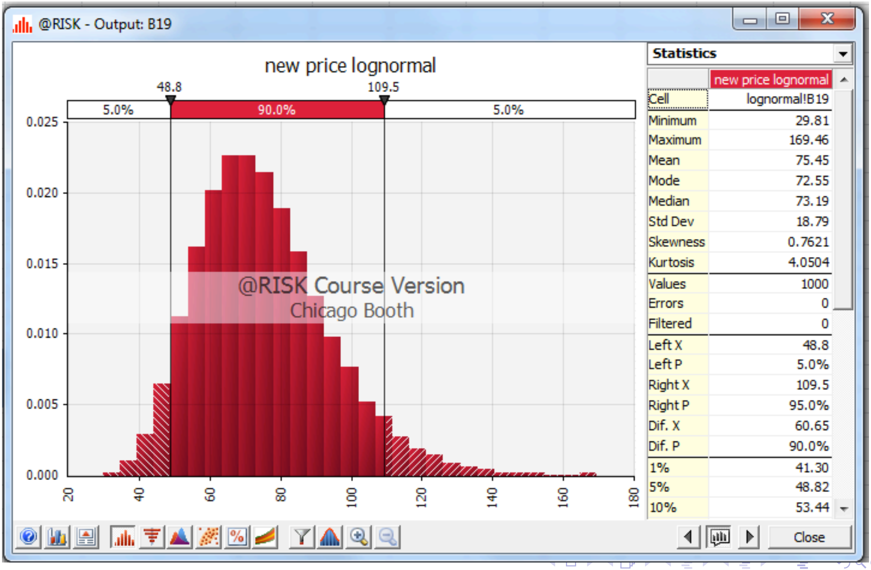
Probability Distributions

Result from using N(0,1) in exponential formula.



Probability Distributions

Result from using lognormal distribution.



Probability Distributions

Two important distributions often used in **queuing** (call centers, banks teller lines, etc.) are

1. Poisson – model arrival rates (may lead to fishy results (ugh!))

2. Exponential – model service times

Probability Distributions

You can even create your own.

We did this for the Mergers and Acquisitions case.

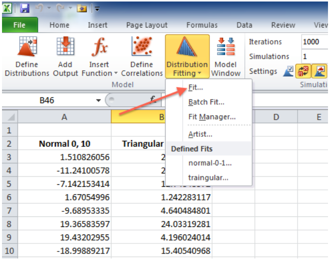
You can put them on a corporate SQL Server.

Selecting a Distribution From Data

You can use @RISK to pick a distribution for you based on your data.

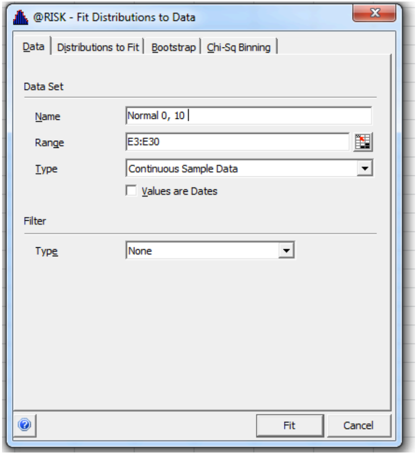
It does a *goodness of fit test* based on the long laundry list of distributions supported by @RISK.

Best of all, it is an easy process. We illustrate with the **fitting** spreadsheet in the **distributions** Workbook.



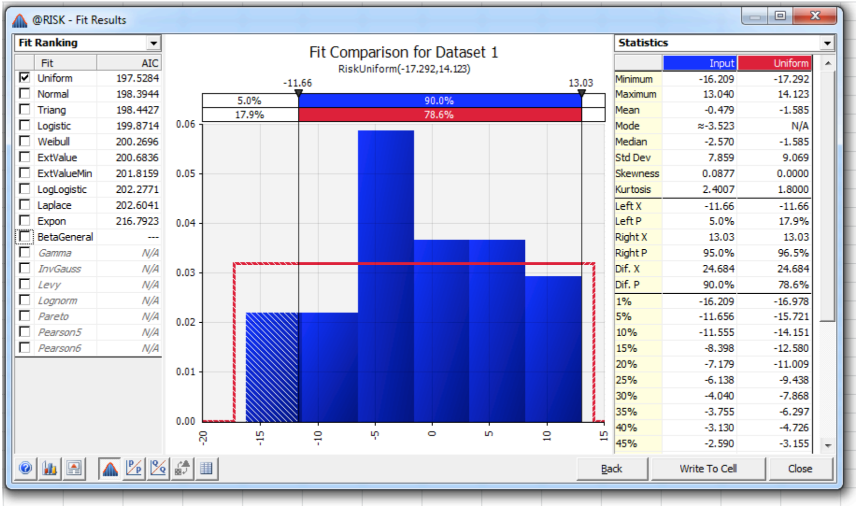
Selecting a Distribution From Data

The numbers in `E3:E30` were generated by a normal distribution with mean 0 and standard deviation 10. This is what we are trying to fit below.



Selecting a Distribution From Data

@RISK fits the normal distribution with a uniform with max 14.123 and min -17.292.



Selecting a Distribution From Data

This example illustrates the difficulty of using a small sample size for fitting a distribution.

Some sample sizes may not pick up the low probability events.

In this sample, there were no realizations more than two standard deviations below the mean, and two standard deviations above the mean.

@RISK will often fit a non-symmetric triangular distribution to the normal sample data – why do you think this might happen?

How Many Trials?

See the workbook **confidence_intervall.xlsx**.

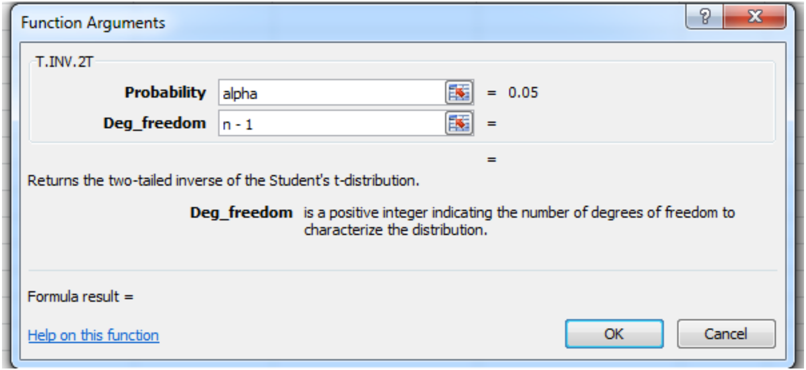
Interval estimate of a population mean.

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

- $\bar{X}$ is the sample mean
- s is the sample standard deviation
- n is the sample size
- $t_{\alpha/2}$ is the t value for a $(1 - \alpha)$ confidence interval with $n - 1$ degrees of freedom (see next slide for appropriate Excel function)

How Many Trials?

Use Excel function **T.INV.2T** to calculate $t_{\alpha/2}$.



See cell **F32**. It has the formula

`=T.INV.2T(alpha,$C32-1)`

How Many Trials?

Consider the new product introduction model from Homework 6.

The expected profit $E(p(X, Y, Z))$ is 7,100,000.

We estimate this number through simulation.

Simulation is nothing more than sampling.

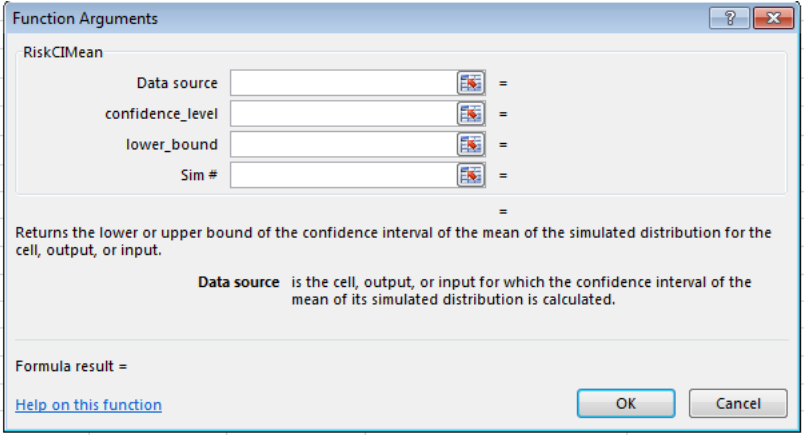
n	Mean	Std. Dev	95% Confidence Interval
10	7,084,302	2,473,681	$7,084,302 \pm 2.26 * 2,473,681/\sqrt{10}$
100	7,135,080	2,617,859	$7,135,080 \pm 1.98 * 2,617,859/\sqrt{100}$
1000	7,102,757	2,468,202	$7,102,757 \pm 1.96 * 2,468,202/\sqrt{1000}$
10000	7,100,207	2,466,928	$7,100,207 \pm 1.96 * 2,466,928/\sqrt{10000}$

How Many Trials?

I	J	K	L	M	N
n	mean	std. dev	Student t	Lower Confidence Interval	Upper Confidence Interval
10	7,084,302	2,473,681	2.26215716	5,314,737.21	8,853,866.79
100	7,135,080	2,617,859	1.98421695	6,615,639.98	7,654,520.02
1000	7,102,757	2,468,202	1.96234146	6,949,593.50	7,255,920.50
10000	7,100,207	2,466,928	1.96020126	7,051,850.25	7,148,563.75

How Many Trials?

You can create a confidence interval using RiskCIMean().




```
=RiskCIMean(profit,0.9,FALSE,1)
```

Correlation

Consider the Workbook `distributions.xlsx` pictured below.

D41

	A	B	C	D
1	**Correlation Examples**			
2				
3				
4				
5				
6				
7	**Uncorrelated**			
8	**Normal Distribution (0, 10)**	9.6392147		
9	**Risk Output -- Risk Uniform (-10, 10)**	3.8487698		
10				
11				
12			**Correlation Matrix**	
13			1	-0.8
14			-0.8	1
15				
16	**Correlated**			
17	**Normal Distribution (0, 10)**	-3.650734		
18	**Risk Output -- Risk Uniform (-10, 10)**	1.3523502		
19				
20				

Correlation

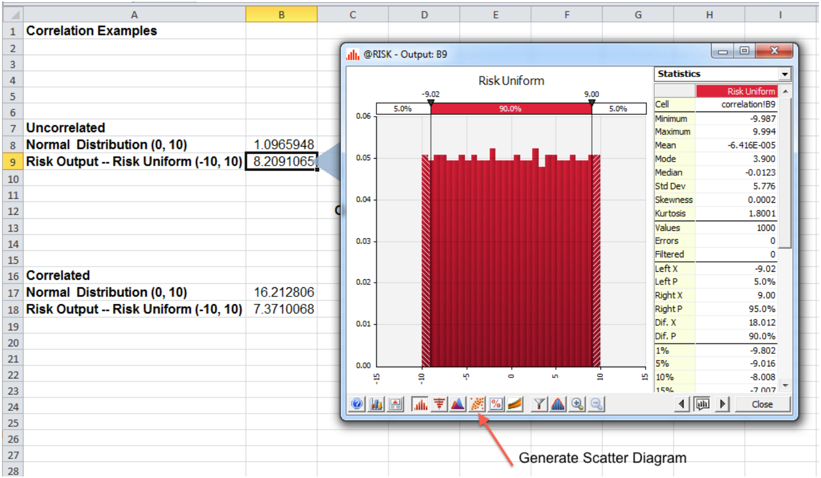
Objective: generate a **scatter diagram.** Here are the steps:

Step 1: Insert a Risk Normal(0,10) in cell B8 and a Risk Uniform(-10,10) in cell B9.

Step 2: Generate a histogram for Cell B9 which is the Risk Output for the Risk Uniform distribution.

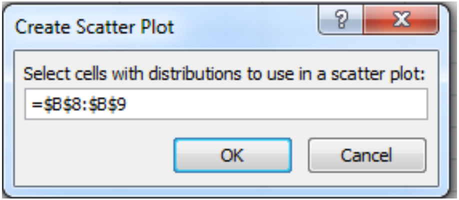
Step 3: Select a scatter plot. See next slide.

Correlation



Correlation

Step 4: Select Cell B8 with the Risk Normal as the second distribution for the scatter plot.

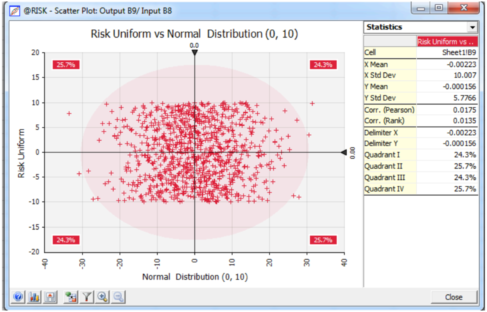


Correlation

Cell B8 is the output from a normal random variable with mean 0 and standard deviation 10.

Cell B9 is the output from a uniform random variable with minimum value -10 and maximum value 10.

These two random variables **have zero correlation**. A run of 1000 simulations gives the scatter plot below.



Correlation

Important Takeaway: If you insert @RISK distributions in an Excel workbook, by default, they will have **zero correlation**.

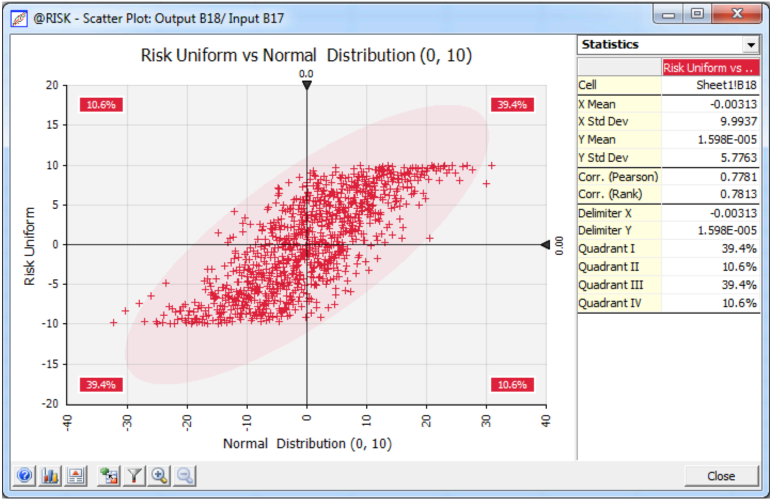
They will have a scatter diagram like on the previous slide.

Zero correlation looks like a *shotgun blast of bird shot.*

On the next slide we illustrate scatter diagrams for positive and zero correlation. For now, don't worry about how we generated these graphs.

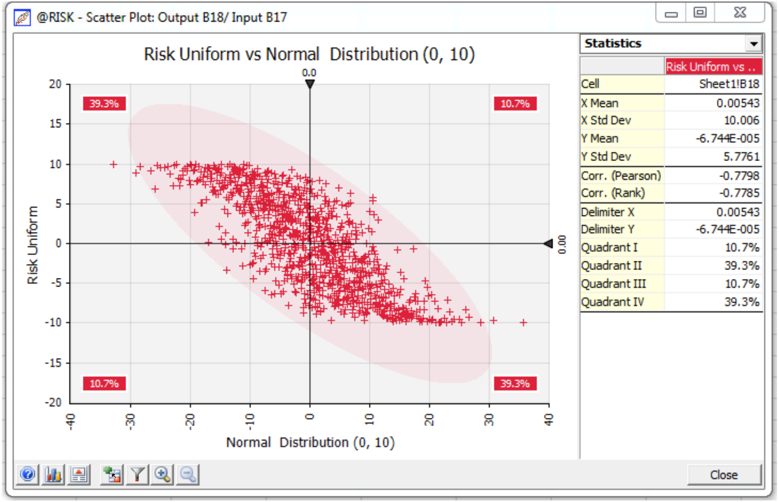
Correlation

A correlation of 0.8 between a normal and uniform random variable. Note the **positive** slope.



Correlation

A correlation of -0.8 between a normal and uniform random variable. Note the **negative** slope.



Correlation

Let X and Y be two random variables. The **covariance** between X and Y is

$$
\begin{aligned}
cov(X, Y) &= E\left((X - \mu_X) * (Y - \mu_Y)\right) \\
&= E\left((X * Y - \mu_X * Y - X\mu_Y + \mu_X\mu_Y\right) \\
&= E(X * Y) - \mu_X * E(Y) - E(X) * \mu_Y + E(\mu_X * \mu_Y) \\
&= E(X * Y) - \mu_X * \mu_Y - \mu_X * \mu_Y + \mu_X * \mu_Y \\
&= E(X * Y) - \mu_X * \mu_Y
\end{aligned}
$$

Note: if X and Y have zero correlation, then

$$E(X * Y) - \mu_X * \mu_Y = 0$$

and this implies

$$E(X * Y) = \mu_X * \mu_Y$$

Correlation

Let X and Y be two random variables. The **correlation** between X and Y "scales" the covariance to be between -1 and 1.

$$\begin{aligned}\rho_{XY} &= cor(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y} \\ &= \frac{E\left((X-\mu_X)*(Y-\mu_Y)\right)}{\sqrt{E\left((X-\mu_X)^2\right)*E\left((Y-\mu_Y)^2\right)}}\end{aligned}$$

Given sample means $\overline{X}$ and $\overline{Y}$, that estimate μ_X and μ_Y, respectively, the sample correlation coefficient for X and Y is

$$\frac{\sum_{i=1}^{n}(X_i-\overline{X})(Y_i-\overline{Y})}{\sqrt{\sum_{i=1}^{n}(X_i-\overline{X})^2\sum_{i=1}^{n}(Y_i-\overline{Y})^2}}$$

Correlation

Okay math nerds, how does one generate correlated random variables?

Here is the basic idea.

1. generate uncorrelated random variables
2. generate new random variables that are linear combinations of the uncorrelated random variables
3. the new random variables will be correlated and we find the right coefficients in the linear combination to give the desired correlations

Here are some links with details:

http://www.numericalexpert.com/blog/correlated_random_variables/

http://math.stackexchange.com/questions/446093/generate-correlated-normal-random-variables

Correlation

In @RISK you can generate correlated random variables.

The user inputs the correlations using the **RiskCorrmat** function.

There are two methods to specify correlations using **RiskCorrmat**.

Regardless of which method you choose, you must first use **Define Distributions** to insert the random variables of interest.

Correlation

Method One:

Step 1: Decide which variables you think should be correlated. In our example, cells B17 and B18 contain the result of Define Distributions

```
=RiskNormal(0,10)
=RiskUniform(-10,10)
```

Step 2: Generate a correlation matrix for these variables. The correlations typically come from historical data. I created a correlation matrix in the range **correlationMatrix**.

Step 3: Manually edit cells B17 and B18 and insert the RiskCorrmat argument. Link the distributions back to the correlation matrix. See cells B17 and B18 with the edited formulas.

```
=RiskNormal(0,10,RiskCorrmat(correlationMatrix,1))
=RiskUniform(-10,10,RiskCorrmat(correlationMatrix,2))
```

Correlation

Important: In the formulas below

```
=RiskNormal(0,10,RiskCorrmat(correlationMatrix,1))

=RiskUniform(-10,10,RiskCorrmat(correlationMatrix,2))
```

it **is important to “link” the random variable to the correlation matrix.**

This is what the 1 and 2 do, respectively.

Correlation

Step 4: Run the simulation.

Step 5: Look at the simulation output and create a scatter diagram.

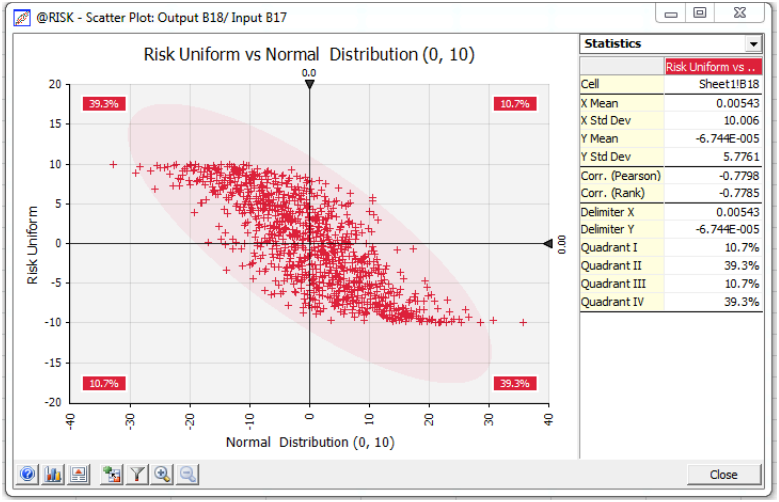
In the following slide I put -.8 into the correlation matrix.

I added an @RISK output to cell `B18`.

I then treated cell `B17` as the input. The scatter diagram plots the output versus the input.

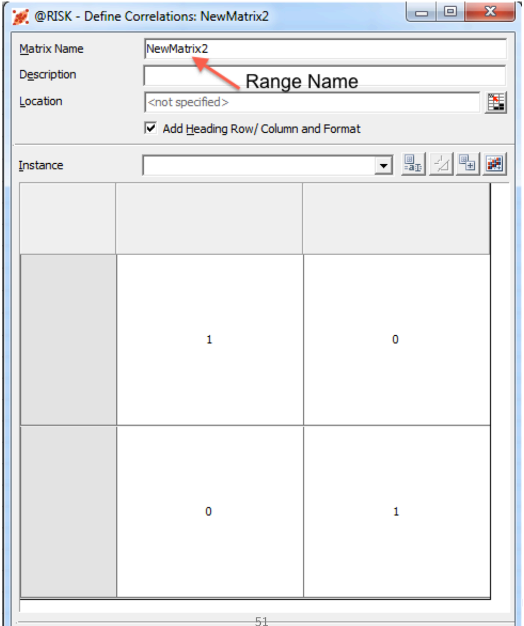
Correlation

Note the sample correlation is -.7798.



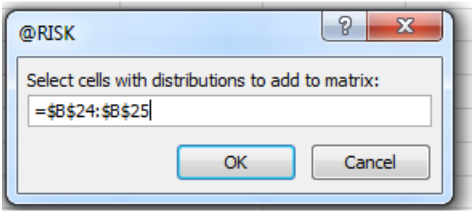
Correlation

Method 2: Use @RISK **Define Correlations**.



Correlation

Select the random variables to correlate.



Then type in the correlations.

Stock Correlations

Creating correlated random variables is incredibly useful!

Where we are headed: take the Markowitz model, and for a given portfolio, simulate returns based on the stock correlations.

Stock Correlations

Open Workbook `markowitzCorrelation.xlsx`. In our the Markowitz optimization model there are three **random variables**:

- the monthly return on Apple stock – denote by r_X
- the monthly return on AMD stock – denote by r_Y
- the monthly return on Oracle stock – denote by r_Z

There is a constraint on expected return. Let X denote the percentage of the portfolio invested in Apple, Y denote the percentage of the portfolio invested in AMD, and Z denote the percentage of the portfolio invested in Oracle.

The unity constraint is

$$X + Y + Z = 1.$$

Stock Correlations

The unity constraint is straightforward and does not involve random variables.

However, the **required return constraint** does involve stochastic parameters (random variables). The required return constraint is

$$E(r_X X + r_Y Y + r_Z Z) \geq .12.$$

We replaced this constraint with

$$E(r_X)X + E(r_Y)Y + E(r_Z)Z \geq .12.$$

Why is this a legitimate thing to do? What is $f(r_X, r_Y, r_Z)$?

Stock Correlations

Here is the Markowitz optimal solution: $X \approx .12$, $Y = 0$, and $Z \approx .88$.

H23

	A	B	C	D	E
2					
3		**Variance Covariance Matrix**			
4		AAPL	AMD	ORCL	
5	AAPL	0.0047	-0.0111	0.0058	
6	AMD	-0.0111	0.4341	0.1120	
7	ORCL	0.0058	0.1120	0.2134	
8	Mean Returns	-0.0501	0.0359	0.1425	
9	Investment Level	0.116921314	0	0.88307969	
10					
11	Expected Return	0.1200			
12	Required Return	0.1200			
13	Expected Return Slack	0.0000			
14					
15	Portfolio Variance	0.1677			
16					
17	Investment Level Sum	1.00			
18	Unity Constraint	-1E-06			
19					

Stock Correlations

The expected return of the portfolio is indeed, 12%.

But what returns can we expect?

Let's run a **simulation** of the portfolio based on the values of $X = .12$, $Y = 0$, and $Z = .88$.

In order to do this, we must generate realizations of stock returns that are correlated!

Let's examine the simulation model starting in row 21 of the spreadsheet `markowitzCorrelation.xlsx`.

Stock Correlations

Objective: Simulate portfolio returns.

20					
21	**Simulation Model**				
22					
23					
24	**Correlation Matrix**		AAPL	AMD	ORCL
25		AAPL	1	-0.247733082	0.18268999
26		AMD	-0.247733082	1	0.36787431
27		ORCL	0.182689985	0.367874313	1
28					
29	**Standard Deviations**		AAPL	AMD	ORCL
30			0.068287379	0.658838102	0.46196004
31					
32					
33	**Random Variables**		AAPL	AMD	ORCL
34	Fitted		-0.004948048	1.233849332	-0.20334602
35	Normal		-0.106549838	-0.339803046	-0.27627742
36					
37					
38	**Portfolio Return Fitted**		-0.180149272		
39	**Portfolio Return Normal**		-0.256432928		
40					

Stock Correlations

Step 1: Calculate the correlation matrix based on the sample returns using the formula below for each pair of random variables.

$$\frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \overline{X})^2 \sum_{i=1}^{n}(Y_i - \overline{Y})^2}}$$

Use the Excel function **CORREL(Range1, Range2)**.

Stock Correlations

The returns for the three stocks are in the **returns** spreadsheet.

For example, Cell **D25** contains the correlation between Apple and Oracle. The formula is:

```
=CORREL(apple_returns,orcl_returns)
```

For example, Cell **C27** contains the correlation between Oracle and AMD. The formula is:

```
=CORREL(orcl_returns,amd_returns)
```

Stock Correlations

Step 2: Now decide on the distribution for each of the random variables r_X, r_Y, and r_Z. We took two approaches:

- Use @RISK **Distribution Fitting** to fit a distribution to the return data in the spreadsheet **returns.** This gives

    1. For Apple the fitted distribution of returns is `RiskUniform(-0.19809,0.082945)`
    2. For AMD the fitted distribution of returns is `RiskUniform(-1.3596,1.2825)`
    3. For Oracle the fitted distribution of returns is `RiskExtvalue(-0.074098,0.37482)`

    See Row 34

- Assume a normal distribution with mean and standard deviation set to the sample mean and standard deviation.

    See Row 35

Stock Correlations

Step 3: Now add the `RiskCorrmat` function to each distribution. For example for the Apple fitted distribution we have

```
=RiskUniform(-0.19809,0.082945,RiskName("AAPL"),
  RiskCorrmat(correlationMatrix,1,"fitted"))
```

For the Oracle normal distribution we have

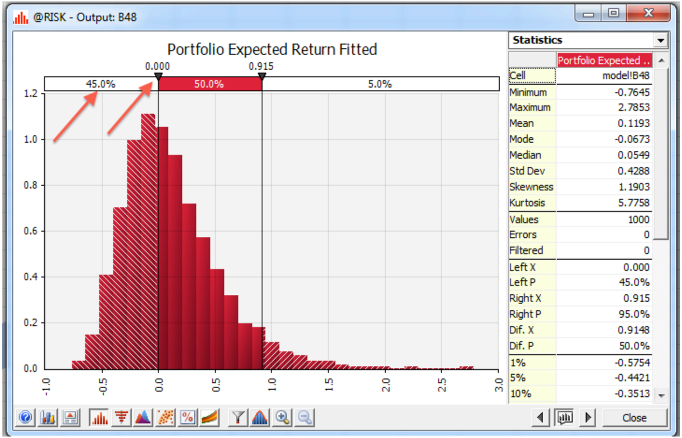
```
=RiskNormal(orcl_mean,orcl_std_dev,RiskName("ORCL"),
   RiskCorrmat(correlationMatrix,3,"normal"))
```

Important: we are using the same correlation matrix to generate the realizations for two distinct sets of random variables.

Step 4: Run the simulation and look at the distribution of returns that are calculated in cells B38 and B39.

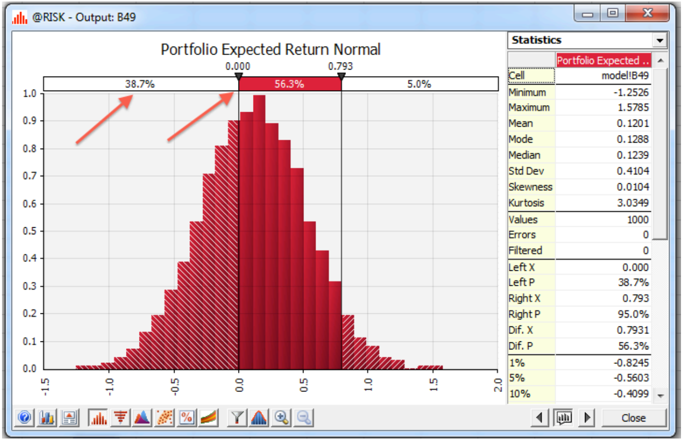
Stock Correlations

Argle Bargle! 45% of the time we experience a negative return for the fitted random variables! What percentage of the returns are below .12?



Stock Correlations

Argle Bargle! 38.7% of the time we experience a negative return for the normal random variables! What percentage of the returns are below .12?



Stock Correlations

Next Week:

1. Use Risk Optimizer

2. Put a constraint on the fraction of time we can have negative returns.

Correlation in Senate Races

Larry Robinson correlation example. See:

1. http://faculty.chicagobooth.edu/kipp.martin/root/htmls/coursework/36106/handouts/politics-correlation.pdf

2. http://blog.palisade.com/2014/10/08/risk-helps-zero-in-on-u-s-senate-race-outcomes/

St. Bernard Case

Please see corresponding Excel file `stbernardData.xlsx`.

This a capstone case combining many concepts used throughout the quarter.

The theme of this case is that a municipal bond underwriter is submitting bids to a municipality.

St. Bernard Case

Step 1: The municipal bond underwriter makes a **bid** to the municipality of St. Bernard. This bid includes the following cash flows to St. Bernard.

- The **face value** of all the bonds that are going to be sold. In this case the total is $5 million.
- A **premium** which is the difference between the total coupon interest payments and the underwriter's profit (or spread). More on this later.

Step 2: St. Bernard accepts or rejects the bid. Assume for now that they accept the bid of the underwriter in our case.

St. Bernard Case

Step 3: Two weeks elapse and the underwriter sells the bonds in the marketplace. Assume all bonds are sold. For purposes of this case, the sole function of the underwriter is acting as a bond salesman for St. Bernard. The revenue to the underwriter is price of the bonds times the number sold. Hence the underwriter's profit is:

bond sales revenue - 5,000,000 - premium

Step 4: from 2009-2014 the municipality pays the coupon rates on the bonds and the total face value of bonds (which is $5 million). The total cost to St. Bernard is then the face value of the bonds plus the coupon interest payments minus the premium they are paid. Hence they want total cost to be as small as possible and accept the bid that minimizes total cost.

St. Bernard Case

The sales price for the coupon rate - bond maturity

St. Bernard Municipal Bond

		Year					
		2009	**2010**	**2011**	**2012**	**2013**	**2014**
Coupon Rate %	3.00%	$ 1,009.64	$ 1,000.00	$ 981.63	$ 966.38	$ 947.58	$ 925.66
	3.25%	$ 1,014.46	$ 1,007.07	$ 990.82	$ 977.58	$ 960.68	$ 940.53
	3.50%	$ 1,019.27	$ 1,014.14	$ 1,000.00	$ 988.79	$ 973.79	$ 955.40
	3.75%	$ 1,024.09	$ 1,021.21	$ 1,009.18	$ 1,000.00	$ 986.89	$ 970.27
	4.00%	$ 1,028.91	$ 1,028.29	$ 1,018.37	$ 1,011.21	$ 1,000.00	$ 985.13
	4.25%	$ 1,033.73	$ 1,035.36	$ 1,027.55	$ 1,022.42	$ 1,013.11	$ 1,000.00
	4.50%	$ 1,038.55	$ 1,042.43	$ 1,036.73	$ 1,033.62	$ 1,026.21	$ 1,014.87
	4.75%	$ 1,043.37	$ 1,049.50	$ 1,045.91	$ 1,044.83	$ 1,039.32	$ 1,029.73
	5.00%	$ 1,048.19	$ 1,056.57	$ 1,055.10	$ 1,056.04	$ 1,052.42	$ 1,044.60
	Number of Bonds	250	425	1025	1050	1100	1150
	Years to Maturity	2	3	4	5	6	7
	Yield to Maturity	0.025	0.03	0.035	0.0375	0.04	0.0425

bond face value = 1000

St. Bernard Case

Understand:

1. yield curve
2. yield to maturity
3. coupon rate
4. face value
5. sales price

St. Bernard Case

An illustration of coupon rates assigned to maturities. Note the total revenue collected by the underwriter.

Maturity	Coupon Rate (%)	Number Of Bonds	Face Value ($)	Maturity Payment ($)	Sales Price ($)	Revenue ($)
2009	3	250	1,000	250,000	1,009.64	252,409.28
2010	4 1/2	425	1,000	425,000	1,042.43	443,032.40
2011	4 3/4	1025	1,000	1,025,000	1,045.91	1,072,061.33
2012	4 1/2	1050	1,000	1,050,000	1,033.62	1,085,305.69
2013	4 1/2	1100	1,000	1,100,000	1,026.21	1,128,831.75
2014	4 1/2	1150	1,000	1,150,000	1,014.87	1,167,097.60
TOTAL		5000		5,000,000		5,148,738.05

St. Bernard Case

For the given assignment, here are the interest payments by St. Bernard.

Maturity	Coupon Rate (%)	Number Of Bonds	Face Value	Years To Maturity	Interest Payment ($)
2009	3	250	1000	2	15,000
2010	4 1/2	425	1000	3	57,375.00
2011	4 3/4	1025	1000	4	194,750.00
2012	4 1/2	1050	1000	5	236,250.00
2013	4 1/2	1100	1000	6	297,000.00
2014	4 1/2	1150	1000	7	362,250.00
TOTAL					1,162,625.00

St. Bernard Case

Here are the cash flows:

1. From underwriter to St. Bernard:
   - \$5,000,000 (face value of the bonds)
   - premium = \$5,148,738.05 - \$5,000,000 - \$40,000 = \$108,738.05
2. From investors to underwriter
   - \$5,148,738.05 (bond sales)
3. From St. Bernard to Investors
   - \$5,000,000 (bond face value at maturity)
   - \$1,162,625 (bond face interest payments)

St. Bernard Case

Objective: Minimize NIC (net interest charge) for St. Bernard in order to win the bid.

For this example, the NIC is

$$\texttt{NIC} = \$1,162,625 - \$108,738.05 = \$1,053,886.95$$

Final Offer Arbitration

1. We know $f(E(X))$ is basically meaningless.
2. We can estimate $E(f(X))$ using simulation. However:
   - $E(f(X))$ may be totally irrelevant,
   - Basing a decision on $E(f(X))$ may lead to terrible results.