# 36106 Managerial Decision Modeling
# Monte Carlo Simulation in Excel: Part II

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# Reading and Excel Files

**Reading:**

- Powell and Baker: Chapter 14.4
- See the handout for Week Eight: **TIAA-CREF Simulation Model**
- See the Documentation and Examples folders in the Palisade RISK7 directory that was installed with your software.

# Learning Objectives

1. Learn to Analyze the Tail of a Distribution
    - Use **RiskTarget**
2. Use Simulation to value an IPO (Netscape)
3. Learn to use compound statistical distributions
4. Learn to run multiple simulations to *optimize*.

# Reading and Excel Files

**Files used in this lecture:**

- riskStatic.xlsx
- retirement_TIAA.xlsx
- retirement_TIAA_key.xlsx
- Ch14-4CorporateValuationBaseModelEmpty.xlsx (Canvas)
- Ch14-4CorporateValuationBaseModel.xlsx (Canvas)
- Ch14-4CorporateValuationSensitivityModel.xlsx (Canvas)
- Ch14-4CorporateValuationRiskModel.xlsx (Canvas)
- compoundDistributions.xlsx
- compoundDistributions_key.xlsx
- retirementRiskTable.xlsx
- retirementRiskTable_key.xlsx
- airlineOverbooking.xlsx
- airlineOverbooking_key.xlsx

# Lecture Outline

Brief Review

Analyze Distribution Tails – TIAA/CREF

Corporate Valuation (Netscape)

Compound Distributions

Multiple Simulations (RiskSimtable)

Airline Overbooking

# Review

**Key Concepts:**

- Using an expected value $E(X)$, instead of a distribution $X$, is bad.
- Nonlinearity can make this even worse (Excel functions like IF, MAX, MIN, are serious offenders)
- Big standard deviations can make this even worse.
- With @RISK we can insert an $X$ into the spreadsheet instead of $E(X)$ and run trials for different values of $X$. @RISK gives us $E(f(X))$ instead of $f(E(X))$.

# Review

We are taking tools from earlier in the quarter and extending them to allow for stochastic parameters.

- Goal Seek $\iff$ @Risk Goal Seek (Last Week)
- Data Table $\iff$ @Risk RiskSimtable (This Week)
- Solver $\iff$ @Risk Optimizer (Week Ten)

# Review

We used **Define Distributions** to put in a distribution. Other options include:

- Use the **Insert Function**
- Just type it in (there is auto completion available)

**IMPORTANT:** Please do not set RiskStatic(). Use default value which is:

- for a continuous distribution (e.g. RiskNormal or RiskUniform) the mean of the distribution
- for a discrete distribution (RiskDiscrete) the distribution value closest to the mean

**Note:** the number you see in a simulation cell is the result of the last trial.

# Review

See the spreadsheet `riskStatic.xlsx` which illustrates the RiskStatic function for the RiskDiscrete() and RiskUniform() distributions.

Here is how to force a value for RiskStatic().

Setting RiskStatic() for the RiskDiscrete distribution

```
=RiskDiscrete(C4:C8,D4:D8, RiskStatic(50))
```

Setting RiskStatic() for the RiskUniform distribution

```
=RiskUniform(C19,D19, RiskStatic(107.36))
```

# Review

**Key @RISK Skills:**

1. Be able to insert a distribution
2. Run (and stop) and set simulation parameters
3. Be able to generate a histogram of results (use **Add Output**)
4. Be able to analyze results
5. Use @RISK simulation functions in your workbook, e.g. RiskMean
6. Use @RISK Goal Seek

# Analyze Distribution Tails – TIAA/CREF

**Key Idea:** we are often interested in the tail of a simulation distribution.

We illustrate with an example from TIAA/CREF Financial Services. A large provider for people working in education, research, and medicine.

In this exercise we show how TIAA/CREF provides analysis of the tail of a simulation distribution for clients.

See also the handout under Week 7.

`http://faculty.chicagobooth.edu/kipp.martin/root/htmls/coursework/36106/handouts/Retirement-Portfolio.pdf`

# Analyze Distribution Tails

Here is part of an actual report from TIAA-CREF.

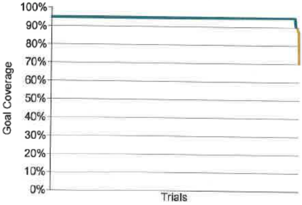
### Current Plan and Life Expectancies to Age 100/Mod. Aggressive

The following graphs illustrate the likelihood of reaching your financial goal. We evaluated the results of 500 hypothetical market simulations ("trials") reflecting various possible market conditions. This assessment helps to determine whether there could be a shortfall in the amount required for you to reach your goal in any given year.

The analysis is performed to simulate a number of possible financial outcomes, rather than assuming an average return year after year. The greater the number of successful trials, the greater the likelihood you will achieve your goal. For this analysis, we define success as being able to cover 90% or more of your goal in at least 450 trials.

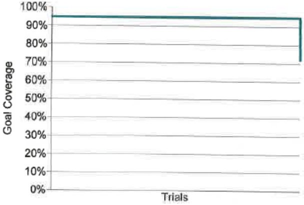
#### Likelihood Of Achieving Your Goal

Current Plan




- In 494 trials, 90% or more of your goal was covered.
- In 6 trials, 65 - 90% of your goal was covered.
- In 0 trials, less than 65% of your goal was covered.

Life Expectancies to Age 100/Mod. Aggressive




- In 499 trials, 90% or more of your goal was covered.
- In 1 trials, 65 - 90% of your goal was covered.
- In 0 trials, less than 65% of your goal was covered.

# Analyze Distribution Tails – TIAA/CREF

**Three Key Points:**

1. "We evaluated the results of 500 hypothetical simulations reflecting various possible market conditions."
2. "The analysis is performed to simulate a number of possible financial outcomes, *rather than assuming an average return year after year*."
3. "For this analysis, we define success as being able to cover 90% or more of your goal in at least 450 trials."

**Note:** results based on life expectancy and how aggressive the portfolio is.

# Analyze Distribution Tails – TIAA/CREF

**Goal:** withdraw $X$ dollars every year for $N$ years and be left with $Y$ dollars

- $X$ – a number selected by the user, for example \$25,000 per year.
- $Y$ – a number selected by the user, for example \$50,000
- $N$ – life expectancy minus age at retirement

**Objective:** Build an @RISK model to find out often we meet 90% or more of our goal.

# Analyze Distribution Tails – TIAA/CREF

Open `retirement_TIAA`. Use the data:

| | |
|---|---|
| 401 K Savings | \$200,000 |
| Years | 20 |
| $\mu$ | 11.55% |
| $\sigma$ | 20.62 % |
| Payment Goal | \$25,000 per year |
| Horizon End Goal | \$50,000 |
| Goal Percentage | 90% |

**Key Question:** what do we mean by meeting 90% or more of the goal?

# Analyze Distribution Tails – TIAA/CREF

**Key Question:** what do we mean by meeting 90% or more of the goal?

I assume we mean that:

1. For each of the 20 years there are sufficient funds to withdraw $25,000.
2. After 20 years I have **at least**

$$0.9 * 50,000 = 45,000$$

dollars in my account.

Now implement in @RISK.

# Analyze Distribution Tails – TIAA/CREF

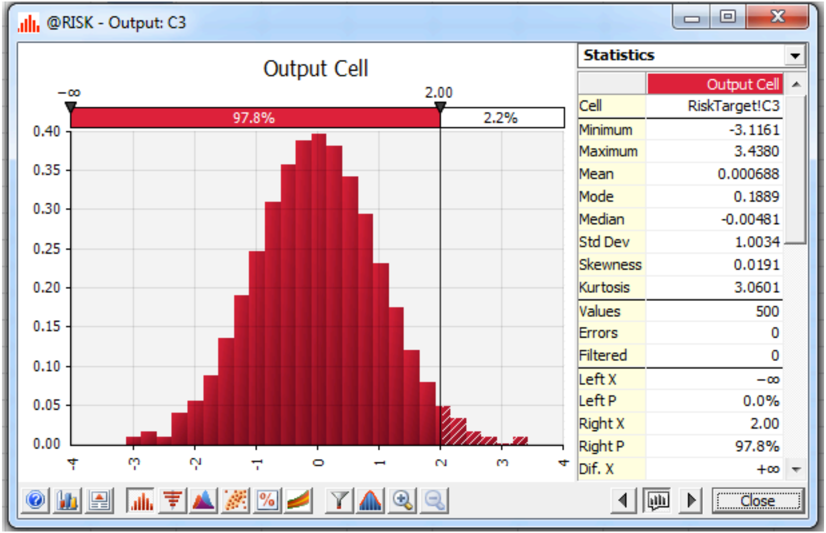
**We do the following:**

1. Run a simulation that records how much money we have after 20 years. **The simulation gives a distribution of returns.**
2. Find the probability associated with a tail value of $45,000.
3. Multiply the probability associated with the tail by the number of trials.

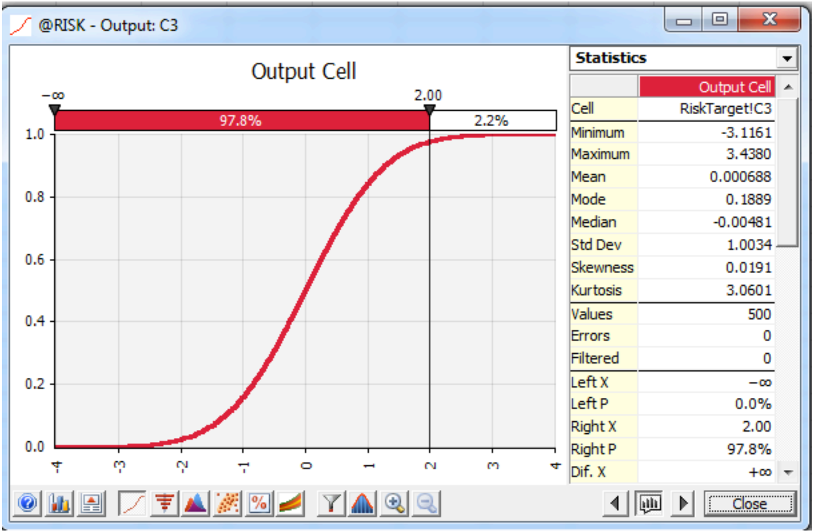
We use `@RiskTargetD` to calculate the tail probability we want.

The use of `@RiskTarget` and `@RiskTargetD` is illustrated spreadsheet `RiskTarget` in the `retirement_TIAA` workbook

# Analyze Distribution Tails – TIAA/CREF



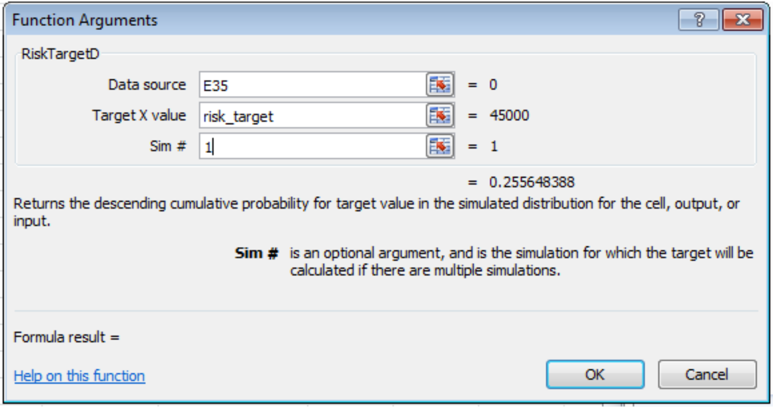
# Analyze Distribution Tails – TIAA/CREF



# Analyze Distribution Tails – TIAA/CREF

We implement in @RISK **two ways**.

**Method 1:** Use the RiskTargetD (descending CDF) function.



# Analyze Distribution Tails – TIAA/CREF

In cell **E38** we put

```
=RiskTargetD(E35,risk_target,1)*num_iterations
```

This is what the TIAA CREF report is doing when they say:

"In 494 trials, 90% or more of your goal was covered."

What is our number?

# Analyze Distribution Tails – TIAA/CREF

We implement in @RISK **two ways**.

**Method 2:** Requires three steps.

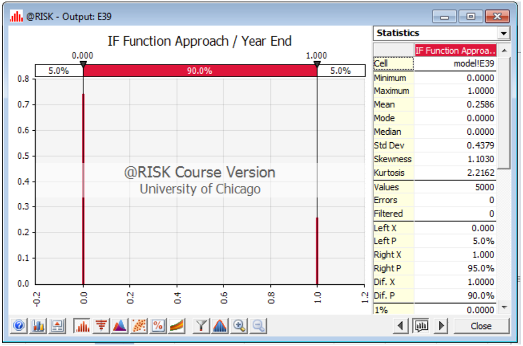
**Step 1:** First, count whether or not we meet our goal using an IF statement.

```
=IF(E35 >= risk_target,1,0)
```

**Step 2:** Build a "distribution" of the simulation results on the IF statement.

- We get a **bimodal** distribution. Either a 0 or 1.
- What does the resulting histogram tell you? (See Figure next page)
- What does the mean tell you?

# Analyze Distribution Tails – TIAA/CREF



# Analyze Distribution Tails – TIAA/CREF

**Step 2 (Continued):** Build a distribution for the IF.

```
=RiskOutput()+IF(E35 >= risk_target,1,0)
```

**Step 3:** Get the mean of this distribution using **RiskMean**.

```
=RiskMean(E39,1)*num_iteration
```

# Analyze Distribution Tails – TIAA/CREF

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 8 | **Horizon End Goal** | $ 50,000.00 | | | | |
| 9 | **Goal Percentage** | 90% | | | | |
| 10 | **Number of Iterations** | 500 | | | | |
| 11 | | | | | | |
| 12 | | | | | | |
| 13 | **Model** | | | | | |
| 14 | | | | | | |
| 15 | **Year** | **Year Start** | **Return** | **Draw Down** | **Year End** | |
| 16 | 1 | $ 200,000.00 | 0.058518474 | $25,000.00 | $ 185,240.73 | |
| 17 | 2 | $ 185,240.73 | 0.221291897 | $25,000.00 | $ 195,700.71 | |
| 18 | 3 | $ 195,700.71 | 0.124677711 | $25,000.00 | $ 191,983.28 | |
| 19 | 4 | $ 191,983.28 | -0.05055506 | $25,000.00 | $ 158,541.43 | |
| 20 | 5 | $ 158,541.43 | 0.058489362 | $25,000.00 | $ 141,352.19 | |
| 21 | 6 | $ 141,352.19 | 0.177155481 | $25,000.00 | $ 136,964.61 | |
| 22 | 7 | $ 136,964.61 | 0.224151198 | $25,000.00 | $ 137,061.62 | |
| 23 | 8 | $ 137,061.62 | -0.018493667 | $25,000.00 | $ 109,989.19 | |
| 24 | 9 | $ 109,989.19 | 0.283824062 | $25,000.00 | $ 109,111.16 | |
| 25 | 10 | $ 109,111.16 | 0.110439522 | $25,000.00 | $ 93,400.36 | |
| 26 | 11 | $ 93,400.36 | 0.154745074 | $25,000.00 | $ 78,984.98 | |
| 27 | 12 | $ 78,984.98 | 0.016909585 | $25,000.00 | $ 54,897.84 | |
| 28 | 13 | $ 54,897.84 | 0.233963112 | $25,000.00 | $ 36,892.83 | |
| 29 | 14 | $ 36,892.83 | 0.329227001 | $25,000.00 | $ 15,808.27 | |
| 30 | 15 | $ 15,808.27 | 0.143422182 | $25,000.00 | $ - | |
| 31 | 16 | $ - | 0.182180891 | $25,000.00 | $ - | |
| 32 | 17 | $ - | 0.140158044 | $25,000.00 | $ - | |
| 33 | 18 | $ - | 0.167116702 | $25,000.00 | $ - | |
| 34 | 19 | $ - | 0.115537345 | $25,000.00 | $ - | |
| 35 | 20 | $ - | 0.064187575 | $25,000.00 | $ - | |
| 36 | | | | | | |
| 37 | | | | Risk Target | $ 45,000.00 | |
| 38 | | | | RiskTarget Function | 128.70 | |
| 39 | | | | IF Function Approach | 0 | |
| 40 | | | | Risk Mean of IF Function | 128.80 | |
| 41 | | | | | | |

# Corporate Valuation

This comes from Powell and Baker, Section 14.4.

- The initial IPO for Netscape was August 9, 1995.
- The underwriters, led by Morgan Stanley & Company, offered five million shares at $28 per share.
- This was the beginning of the Internet boom.
- At $28 per share, Netscape's market value would be more than $1 billion dollars despite a book value of $16 million
- Microsoft copied the Netscape technology and developed Internet Explorer. They gave Internet Explorer away for "free" leading to a major legal battle.

**Our Objective:** Place a value on Netscape given August, 1995 data.

# Corporate Valuation

**The Plan:**

1. Define the problem – put a valuation on the firm (Netscape)
2. Assume all parameters (e.g. revenue growth rate) are deterministic
3. Calculate the NPV of the firm based on free cash flow
4. Determine the sensitivity of the NPV calculation to the inputs
5. Treat the most sensitive inputs as stochastic parameters (i.e. distributions)
6. Run a simulation

# Corporate Valuation

**Spreadsheet Walk Through:**

1. `Ch14-4CorporateValuationBaseModelEmpty.xlsx` (contains parameter assumptions)

2. `Ch14-4CorporateValuationBaseModel.xlsx` (the deterministic valuation model)

3. `Ch14-4CorporateValuationSensitivityModel.xlsx` (used for Step 5 in the previous slide)

4. `Ch14-4CorporateValuationRiskModel.xlsx` (used for Step 6 in the previous slide)

# Corporate Valuation

The basic assumptions for the deterministic model are given below. See Workbook `Ch14-4CorporateValuationBaseModelEmpty.xlsx`.

| | A | B | C |
|---|---|---|---|
| 1 | **Netscape Valuation Model - Deterministic Version** | | |
| 2 | | | |
| 3 | **Assumptions** | | **1996** |
| 4 | Revenue growth rate | 65% | |
| 5 | Terminal value growth rate | 4.00% | |
| 6 | Cost of sales (% revenues) | 10.40% | |
| 7 | R&D (% revenues) | 36.8% | |
| 8 | Tax rate | 34% | |
| 9 | Other operating expenses | | 0.80 |
| 10 | Capital expenditure | | 0.45 |
| 11 | Depreciation (% revenues) | 5.50% | |
| 12 | ΔNWC | 0.00% | |
| 13 | Beta | 1.50 | |
| 14 | Riskless rate | 6.71% | |
| 15 | Market risk premium | 7.50% | |
| 16 | Cost of equity | 17.96% | |
| 17 | Shares outstanding | 38,000 | |

# Corporate Valuation

First, create the deterministic model. You will start with something like the following for homework.

N43

| | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **Netscape Valuation Model - Deterministic Version** | | | | | | | | | | | |
| 2 | | | | | | | | | | | | |
| 3 | **Assumptions** | | **1996** | **1997** | **1998** | **1999** | **2000** | **2001** | **2002** | **2003** | **2004** | **2005** |
| 4 | Revenue growth rate | 65% | | | | | | | | | | |
| 5 | Terminal value growth rate | 4.00% | | | | | | | | | | |
| 6 | Cost of sales (% revenues) | 10.40% | | | | | | | | | | |
| 7 | R&D (% revenues) | 36.8% | | | | | | | | | | |
| 8 | Tax rate | 34% | | | | | | | | | | |
| 9 | Other operating expenses | | 0.80 | 0.65 | 0.55 | 0.45 | 0.35 | 0.25 | 0.20 | 0.20 | 0.20 | 0.20 |
| 10 | Capital expenditure | | 0.45 | 0.40 | 0.30 | 0.20 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 11 | Depreciation (% revenues) | 5.50% | | | | | | | | | | |
| 12 | ΔNWC | 0.00% | | | | | | | | | | |
| 13 | Beta | 1.50 | | | | | | | | | | |
| 14 | Riskless rate | 6.71% | | | | | | | | | | |
| 15 | Market risk premium | 7.50% | | | | | | | | | | |
| 16 | Cost of equity | 17.96% | | | | | | | | | | |
| 17 | Shares outstanding | 38,000 | | | | | | | | | | |
| 18 | | | | | | | | | | | | |
| 19 | **Model** | | | | | | | | | | | |
| 20 | | **1995 actual** | **Forecast** | | | | | | | | | |
| 21 | Revenues | 33,250 | | | | | | | | | | |
| 22 | Cost of goods sold | 3,472 | | | | | | | | | | |
| 23 | R&D expenses | 12,230 | | | | | | | | | | |
| 24 | Depreciation | 1,836 | | | | | | | | | | |
| 25 | Other operating expenses | 26,898 | | | | | | | | | | |
| 26 | Profit before taxes | (11,186) | | | | | | | | | | |
| 27 | Taxes | (3,803) | | | | | | | | | | |
| 28 | Net income | (7,383) | | | | | | | | | | |
| 29 | | | | | | | | | | | | |
| 30 | Capital expenditure | 15,236 | | | | | | | | | | |
| 31 | Change in net working capital | 0 | | | | | | | | | | |
| 32 | | | | | | | | | | | | |
| 33 | Free cash flow | (20,783) | | | | | | | | | | |
| 34 | Terminal value (2006) | 0 | | | | | | | | | | |
| 35 | PV Free cash flow | (20,783) | | | | | | | | | | |
| 36 | PV Terminal value | 0 | | | | | | | | | | |
| 37 | Total PV | **(20,783)** | | | | | | | | | | |
| 38 | | | | | | | | | | | | |

# Corporate Valuation

Then fill in the numbers.

R34

| | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **Netscape Valuation Model - Deterministic Version** | | | | | | | | | | | |
| 2 | | | | | | | | | | | | |
| 3 | **Assumptions** | | **1996** | **1997** | **1998** | **1999** | **2000** | **2001** | **2002** | **2003** | **2004** | **2005** |
| 4 | Revenue growth rate | 65% | | | | | | | | | | |
| 5 | Terminal value growth rate | 4.00% | | | | | | | | | | |
| 6 | Cost of sales (% revenues) | 10.40% | | | | | | | | | | |
| 7 | R&D (% revenues) | 36.8% | | | | | | | | | | |
| 8 | Tax rate | 34% | | | | | | | | | | |
| 9 | Other operating expenses | | 0.80 | 0.65 | 0.55 | 0.45 | 0.35 | 0.25 | 0.20 | 0.20 | 0.20 | 0.20 |
| 10 | Capital expenditure | | 0.45 | 0.40 | 0.30 | 0.20 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 11 | Depreciation (% revenues) | 5.50% | | | | | | | | | | |
| 12 | ΔNWC | 0.00% | | | | | | | | | | |
| 13 | Beta | 1.50 | | | | | | | | | | |
| 14 | Riskless rate | 6.71% | | | | | | | | | | |
| 15 | Market risk premium | 7.50% | | | | | | | | | | |
| 16 | Cost of equity | 17.96% | | | | | | | | | | |
| 17 | Shares outstanding | 38,000 | | | | | | | | | | |
| 18 | | | | | | | | | | | | |
| 19 | **Model** | | | | | | | | | | | |
| 20 | | **1995 actual** | **Forecast** | | | | | | | | | |
| 21 | Revenues | 33,250 | 54,863 | 90,523 | 149,363 | 246,449 | 406,641 | 670,958 | 1,107,081 | 1,826,683 | 3,014,027 | 4,973,145 |
| 22 | Cost of goods sold | 3,472 | 5,706 | 9,414 | 15,534 | 25,631 | 42,291 | 69,780 | 115,136 | 189,975 | 313,459 | 517,207 |
| 23 | R&D expenses | 12,230 | 20,189 | 33,313 | 54,966 | 90,693 | 149,644 | 246,913 | 407,406 | 672,219 | 1,109,162 | 1,830,117 |
| 24 | Depreciation | 1,836 | 3,017 | 4,979 | 8,215 | 13,555 | 22,365 | 36,903 | 60,889 | 100,468 | 165,771 | 273,523 |
| 25 | Other operating expenses | 26,898 | 43,890 | 58,840 | 82,150 | 110,902 | 142,324 | 167,739 | 221,416 | 365,337 | 602,805 | 994,629 |
| 26 | Profit before taxes | (11,186) | (17,940) | (16,023) | (11,501) | 5,668 | 50,017 | 149,624 | 302,233 | 498,684 | 822,829 | 1,357,668 |
| 27 | Taxes | (3,803) | (6,100) | (5,448) | (3,910) | 1,927 | 17,006 | 50,872 | 102,759 | 169,553 | 279,762 | 461,607 |
| 28 | Net income | (7,383) | (11,840) | [illegible] | [illegible] | 3,741 | 33,011 | 98,752 | 199,474 | 329,132 | 543,067 | 896,061 |
| 29 | | | | | | | | | | | | |
| 30 | Capital expenditure | 15,236 | 24,688 | [illegible] | [illegible] | 49,290 | 40,664 | 67,096 | 110,708 | 182,668 | 301,403 | 497,314 |
| 31 | Change in net working capital | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 32 | | | | | | | | | | | | |
| 33 | Free cash flow | (20,783) | [illegible] | [illegible] | (44,185) | (31,994) | 14,712 | 68,558 | 149,655 | 246,931 | 407,436 | 672,270 |
| 34 | Terminal value (2006) | 5,008,313 | | | | | | | | | | |
| 35 | PV Free cash flow | 243,196 | | | | | | | | | | |
| 36 | PV Terminal value | 960,160 | | | | | | | | | | |
| 37 | Total PV | **1,203,357** | | | | | | | | | | |
| 38 | | | | | | | | | | | | |
| 39 | **Performance** | | | | | | | | | | | |

Terminal value is an estimate of the value of the firm after 2005.

PV Free cash flow is the value of the firm from 1995 to 2005.

PV Terminal value is the value as of 1995 of the value after 2005.

The Total PV is the

# Corporate Valuation

**Full disclosure:** *I was a math major. Please do not ask me to justify accounting measures.*

We need to calculate the **free cash flow** in an **income statement.**

The free cash flow is the amount of cash a company can generate after paying out the capital expenditures. See:

http://www.investopedia.com/terms/f/freecashflow.asp

**Free cash flow** is *net income plus depreciation minus capital expenditure and changes in net working capital.*

# Corporate Valuation

**Objective:** Calculate the *present value of free cash flow*. This requires the following key steps.

1. Calculate the present value of the **free cash flow** for the current year 1995 and the forecasted years 1996 - 2005.
2. Calculate the **terminal value** for the firm at the end of year 2005.
3. Calculate the present value of the terminal value.
4. Add Steps 1 and 3 for the net present value.

This number goes in cell `B37` and the value is $1,203,357 (In the book you will see a value of present value calculation of $1,057,167. This is an error and was caught by Professor Che-Lin Su.)

# Corporate Valuation

**Step 1:** Calculate the present value of the free cash flow for the forecasted years (i.e. through the end of 2005.)

Let $C_t$ denote the free cash flow at the end of period $t$. We know $C_{1995}$, this value is in cell `B33` and is equal to (20,738).

We have projected values for the 10-year period 1996 through 2005. These numbers are in the range `C33:L33`.

# Corporate Valuation

**Step 1 (Continued):** Calculate the present value of the free cash flow for the forecasted years (i.e. through the end of 2005.)

Hence the present value of these cash flows is

$$C_{1995} + \sum_{t=1996}^{2005} C_t/(1+i)$$

This number is \$243,196 and appears in cell `B35`. The formula in `B35` is

```
=B33+NPV(B16,C33:L33)
```

The number $i$ is **cost of equity.** In our spreadsheet this number is equal to 17.96% and is in cell `B16`.

# Corporate Valuation

**Step 2:** Calculate the **terminal value.** Use the *Gordon terminal value*. See `http://www.investopedia.com/university/dcf/dcf4.asp#axzz2MS2bjicl`.

The forecasts are through the end of 2005. However, we assume the firm has an "infinite" life.

Let $\alpha$ be the terminal growth rate of free cash flow through infinity. In our case $\alpha = 0.04$ and is given in cell B5. This implies the following values for fresh cash flow starting at the end of 2006.

$$\begin{array}{ccccc} 2006 & 2007 & 2008 & 2009 & \cdots \\ (1+\alpha)C_{2005} & (1+\alpha)^2 C_{2005} & (1+\alpha)^3 C_{2005} & (1+\alpha)^4 C_{2005} & \cdots \end{array}$$

# Corporate Valuation

**Step 2 (Continued):** Calculate the **terminal value.**

Now discount these free cash flows back *to the end of 2005* using the cost of equity (denoted by $i$).

| 2006 | 2007 | 2008 | 2009 | $\cdots$ |
|---|---|---|---|---|
| $\frac{(1+\alpha)}{(1+i)} C_{2005}$ | $\frac{(1+\alpha)^2}{(1+i)^2} C_{2005}$ | $\frac{(1+\alpha)^3}{(1+i)^3} C_{2005}$ | $\frac{(1+\alpha)^4}{(1+i)^4} C_{2005}$ | $\cdots$ |

So we need to find

$$C_{2005} \sum_{t=1}^{\infty} \frac{(1+\alpha)^t}{(1+i)^t}$$

which is all future free cash flow discounted back to the end of 2005.

# Corporate Valuation

**Step 2 (Continued):** Calculate the **terminal value.**
When $0 < r < 1$,

$$\sum_{t=1}^{\infty} r^t = \frac{r}{1-r}$$

As long as $\alpha < i$, $0 < (1+\alpha)/(1+i) < 1$. Doing a little algebra gives

$$C_{2005} \sum_{t=1}^{\infty} \frac{(1+\alpha)^t}{(1+i)^t} = C_{2005} \frac{1+\alpha}{i-\alpha}$$

This is the number calculated in cell B34 with the formula

```
=(L33*(1+B5))/(B16-B5)
```

using range names gives

```
=(c_2005*(1+alpha))/(i-alpha)
```

# Corporate Valuation

**Step 2 (Continued):** Calculate the **terminal value.**

We have now discounted all future cash flows back to the end of 2005.

Discount back to the end of 1995. This is

$$\left( C_{2005} \frac{1+\alpha}{i-\alpha} \right) / (1+i)^{10}$$

This calculation is in cell `B36` and gives \$960,160.

This differs from the value in the text, where the authors use a discount factor of $(1+i)^{11}$. Professor Che-Lin the author of the above derivation found this error.

# Corporate Valuation

Excellent reference: *Financial Modeling* by Simon Benninga, MIT Press, ISBN 0-262-02628-7 (Third Edition).

See Chapter 3, "Financial Statement Modeling."

# Corporate Valuation

You should understand the key part of the spreadsheet highlighted in gray below.

| | | | | | | |
|---|---|---|---|---|---|---|
| 23 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 24 | Depreciation | 1,836 | 3,017 | 4,979 | 8,215 | 13,555 |
| 25 | Other operating expenses | 26,898 | 43,890 | 58,840 | 82,150 | 110,902 |
| 26 | Profit before taxes | (11,186) | (17,940) | (16,023) | (11,501) | 5,668 |
| 27 | Taxes | (3,803) | (6,100) | (5,448) | (3,910) | 1,927 |
| 28 | Net income | (7,383) | (11,840) | [illegible] | [illegible] | 3,741 |
| 29 | | | | | | |
| 30 | Capital expenditure | 15,236 | 24,688 | [illegible] | [illegible] | 49,290 |
| 31 | Change in net working capital | 0 | 0 | 0 | 0 | 0 |
| 32 | | | | | | |
| 33 | Free cash flow | (20,783) | (33,51[illegible] | [illegible] | (44,185) | (31,994) |
| 34 | Terminal value (2006) | 5,008,313 | | | | |
| 35 | PV Free cash flow | 243,196 | | | | |
| 36 | PV Terminal value | 960,160 | | | | |
| 37 | Total PV | **1,203,357** | | | | |
| 38 | | | | | | |
| 39 | **Performance** | | | | | |
| 40 | Total NPV | 1,203,357 | | | | |
| 41 | Ratio TV/Total NPV | 0.80 | | | | |
| 42 | Year FCF >0 | 2002 | | | | |
| 43 | Max loss | (172,278) | | | | |
| 44 | Price/Share | 31.67 | | | | |

**Terminal value is an estimate of the value of the firm after 2005.**

**PV Free cash flow is the value of the firm from 1995 to 2005.**

**PV Terminal value is the value as of 1995 of the value after 2005.**

**The Total PV is the sum of the PV from 1995-2005 and the terminal value after 2005.**

# Corporate Valuation

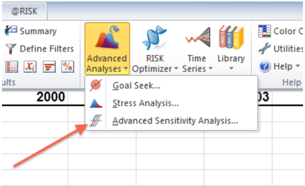
**Deterministic Result:** With the deterministic assumptions in the model, the NPV of Netscape is $1,203,357. See cell `B37`.

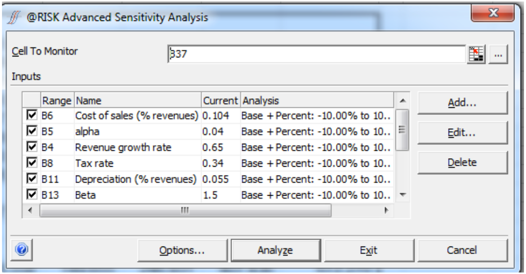
This results in a price per share of $31.67. See cell `B44`.

Let's do a simulation on the NPV.

# Corporate Valuation

The first step is to determine which parameters have the greatest effect on the NPV outcome. We will create a **tornado** chart.



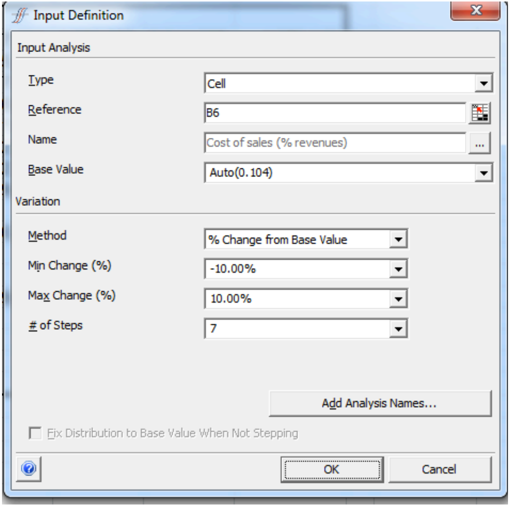
# Corporate Valuation

I did a sensitivity analysis on the following:

- Revenue growth rate
- alpha – terminal value growth rate
- Cost of sales
- R&D (% of revenue)
- Tax Rate
- Depreciation
- Beta
- Riskless rate
- Market Risk Premium

# Corporate Valuation

The settings for cell B6, Cost of sales (% revenues)



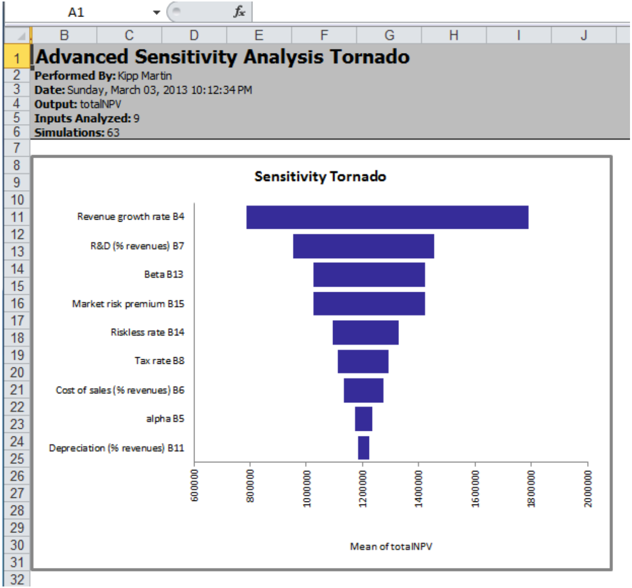
# Corporate Valuation

**Disclaimer:**

- This sensitivity analysis looks at changing only **one** parameter at a time.
- We do not change two or more parameters simultaneously.

# Corporate Valuation

The tornado chart.



# Corporate Valuation

Now **identify key uncertainties.** Based on the tornado chart we examine:

- Revenue Growth Rate – *assume normal with mean of 65 percent and standard deviation of 5 percent*
- R&D (% of Revenue) – *assume triangular, with a minimum of 32 percent, most likely value of 37 percent, and a maximum of 42 percent*
- Market Risk Premium – *assume uniform with a minimum of 5 percent and maximum of 10 percent*

# Corporate Valuation

Next identify model outputs. These are in the range `B40:B44`. See below.

| | | | | |
|---|---|---|---|---|
| 32 | | | | |
| 33 | Free cash flow | (20,783) | (34,014) | (42,58 |
| 34 | Terminal value (2006) | 5,935,710 | | |
| 35 | PV Free cash flow | 289,483 | | |
| 36 | PV Terminal value | 1,508,795 | | |
| 37 | Total PV | **1,798,278** | | |
| 38 | | | | |
| 39 | **Performance** | | | |
| 40 | Total NPV | 1,798,278 | | |
| 41 | Ratio TV/Total NPV | 0.84 | | |
| 42 | Year FCF >0 | 2002 | | Model |
| 43 | Max loss | (177,136) | | Outputs |
| 44 | Price/Share | 47.32 | | |
| 45 | | | | |
| 46 | | | | |

# Corporate Valuation

Run the simulation for 1000 trials. I get the following results:

| Output | Mean | Min | Max | Std. Dev. |
|---|---|---|---|---|
| Total NPV | 1,314,979 | 260,728 | 4,398,352 | 612,383 |
| Ratio TV/Total NPV | .80 | .73 | .91 | .03 |
| Year FCF $> 0$ | 2002 | 2002 | 2003 | .32 |
| Max Loss | -173,339 | -215,962 | -135,408 | 13,723 |
| Price/Share | 34.6 | 6.86 | 115.75 | 16.12 |

# Corporate Valuation

At Canvas please go to Homework 7.

Get the file **FigureOddsInAcquisitionResults.pdf**

You will use this file and a corresponding Excel data file `MergersandAcquisitionsData.xlsx` to build a corporate valuation model.

You will use **FigureOddsInAcquisitionResults.pdf** and the Excel file to get the time 0 data and assumptions about growth rates.

You will then run a simulation based on stochastic parameter assumptions.

# Corporate Valuation

**Important:** Please note the following for your homework on *M&A*.

- See Point 6 in the M&A clarifications homework. The Capital Expenditures in the Ch14-4CorprateValuationBaseModel.xlsx example are calculated differently than in the M&A homework case.
- See Point 7 in the M&A clarifications homework. Calculate taxes and net income as in the Ch14-4CorprateValuationBaseModel.xlsx workbook.

# Compound Distributions

Consider the following scenario: (sales, general, and administrative expenses (SG&A) from your next homework)

1. nature determines an outcome
2. for each possible outcome there is a (potentially different) probability distribution

**Example Data:**

| Scenario Index | Probability | Uniform Distribution Parameters: Min | Uniform Distribution Parameters: Max |
|---|---|---|---|
| 1 | 0.3 | -2% | 1% |
| 2 | 0.5 | 1% | 4% |
| 3 | 0.2 | 4% | 7% |

# Compound Distributions

**Example 1:** Nature determines outcome state 1. In this case the relevant distribution is a uniform distribution with a min of -2% and a max of 1%.

**Example 2:** Nature determines outcome state 3. In this case the relevant distribution is a uniform distribution with a min of 4% and a max of 7%.

The probability of outcome (scenario) 1 is 0.3, the probability of outcome 2 is 0.5, and the probability of outcome 3 is 0.2.

**We want to model this process for seven years.**

You have to do this is in Homework 7, involving Mergers and Acquisitions. It is based on a document about *probability-based scenario planning* prepared by Crédit Lyonnais (now Crédit Agricole) of America.

# Compound Distributions

There are two methods in `compoundDistributions.xlsx` to do this.

**Method 1:**

| Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 | Year 7 |
|---|---|---|---|---|---|---|
| -0.019373378 | -0.018448466 | -0.003034135 | -0.0037663 | 0.007747254 | -0.0002269 | -0.002732214 |
| 0.021636047 | 0.017343423 | 0.012106574 | 0.0362221 | 0.021814185 | 0.01275471 | 0.013879671 |
| 0.049992129 | 0.061583884 | 0.064888847 | 0.0521943 | 0.055888519 | 0.042851273 | 0.060304191 |
| 0.021636047 | 0.017343423 | -0.003034135 | -0.0037663 | 0.055888519 | 0.01275471 | -0.002732214 |

**Step 1:** For each year, generate a realization of each of the three random variables. For example,

For example, the second number generated in Year Three, 0.012106574, is a realization from the uniform distribution with minimum 1% and maximum 4%.

# Compound Distributions

**Important:**

1. In the first part of this example, I am generating a *different uniform distribution* for each of the seven years, but we do not do that in the M&A case. In the M&A it is stipulated which years use the same draw from the uniform distribution.

2. In the M&A case we are using different draws from the RiskDiscrete for each of the different accounting measures, e.g. revenue growth, we may wish to use the SAME scenario probabilities for each accounting measure.

# Compound Distributions

**Method 1 (continued):** The first number generated in Year Five, 0.007747254, is a realization from the uniform distribution with minimum -2% and maximum 1%.

| Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 | Year 7 |
|---|---|---|---|---|---|---|
| -0.019373378 | -0.018448466 | -0.003034135 | -0.0037663 | 0.007747254 | -0.0002269 | -0.002732214 |
| 0.021636047 | 0.017343423 | 0.012106574 | 0.0362221 | 0.021814185 | 0.01275471 | 0.013879671 |
| 0.049992129 | 0.061583884 | 0.064888847 | 0.0521943 | 0.055888519 | 0.042851273 | 0.060304191 |
| 0.021636047 | 0.017343423 | -0.003034135 | -0.0037663 | 0.055888519 | 0.01275471 | -0.002732214 |

**Step 2:** For each year, generate a realization from the **RiskDiscrete** random variable. For example, in the fourth row for Year One we have:

```
=RiskDiscrete(C20:C22,probRange)
```

The range **probRange** contains the probabilities .3, .5, and .2. Note that `C20:C22` is the range with the realizations from the uniform distributions for year one.

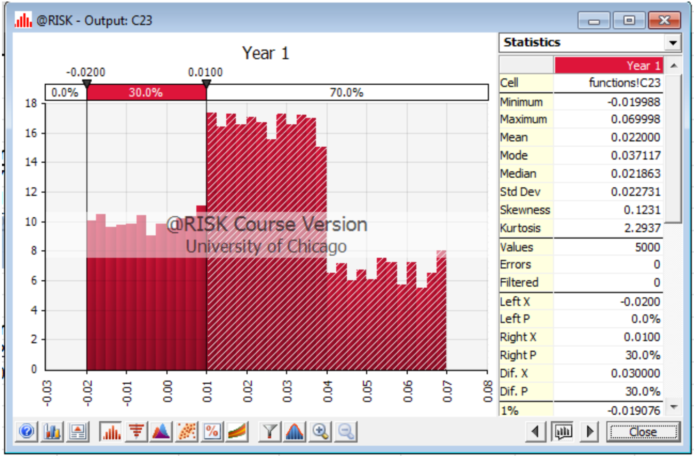
# Compound Distributions

Now do the experiment where we fix in `F12:F14` the same uniform distribution to be used in each year. Then generate the output for years 1-7 in row 27. This is the way to do it for the M&A case.

| Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 | Year 7 |
|---|---|---|---|---|---|---|
| 0.007736761 | 0.060179286 | 0.037699322 | 0.03769932 | 0.060179286 | 0.060179286 | 0.007736761 |

# Compound Distributions

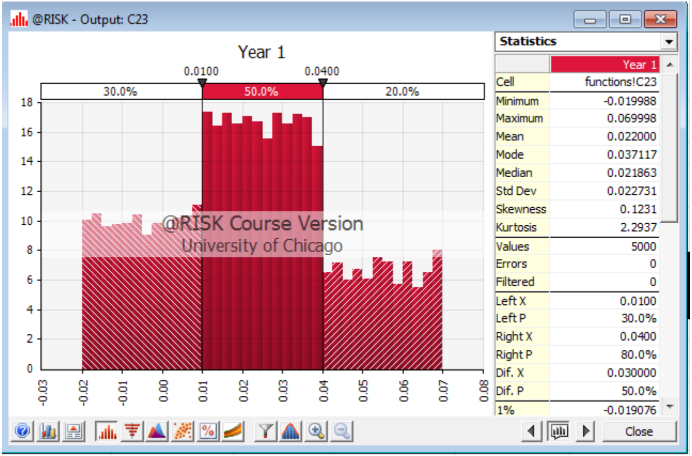
The histogram below illustrates that 30% of returns are between -2% and 1%.




Note: we put an RiskOutput in for the RiskDiscrete in Period 1

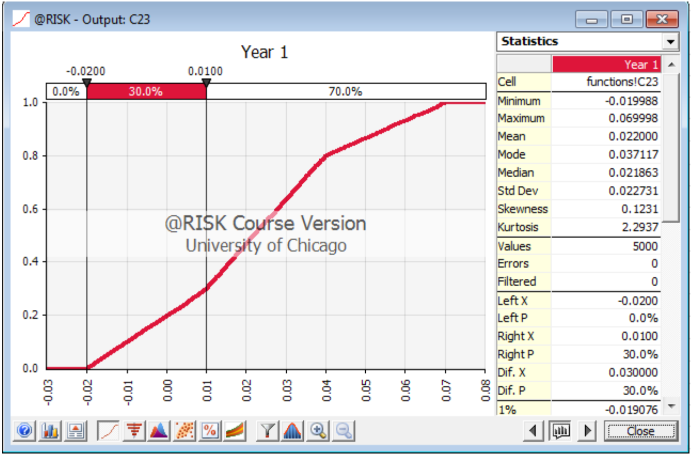
# Compound Distributions

The histogram below illustrates that 50% of returns are between 1% and 4%.



# Compound Distributions

The cumulative distribution motivates the second approach.



# Compound Distributions

**Method 2:** Use the @Risk function **Riskcumul.** For this function to be valid:

1. the probability distributions for each possible outcome must be uniform
2. the intervals defining the uniform distributions cannot overlap

See the explanation and definition in the section Distribution Functions of the User's Manual.

# Compound Distributions

**Method 2: (Continued) Riskcumul** requires the following arguments:

1. a minimum value – in our case -0.02 (this is the minimum value of any realization over the set of uniform distributions)

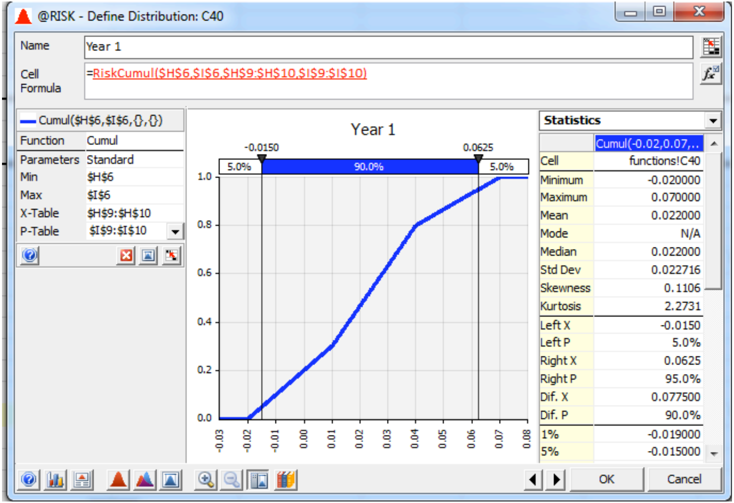
2. a maximum value – in our case 0.07 (this is the maximum value of any realization over the set of uniform distributions)

3. a range of $X$ values – in our case 0.01 and 0.04 (the upper limits of the uniform distributions, not including the uniform distribution that gave the maximum value)

4. a range of $p$ values – in our case 0.3 and 0.8 (the cumulative probabilities associated with the X values)

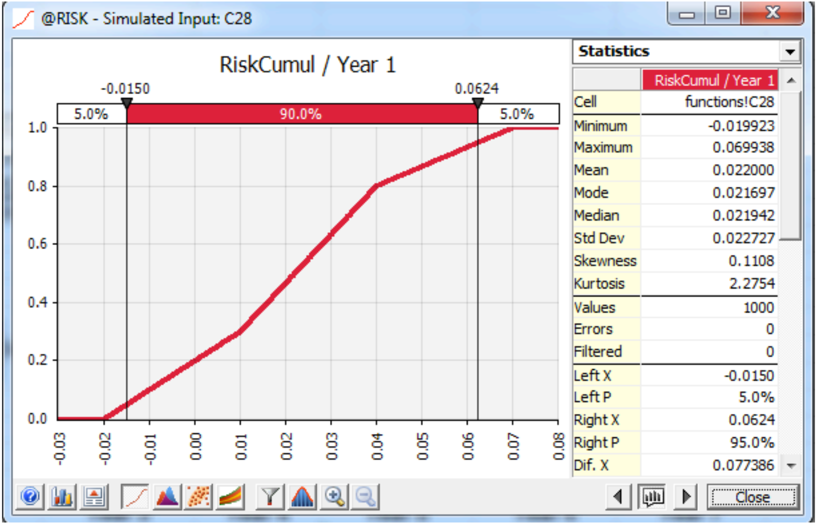
# Compound Distributions

## Method 2: (Continued) Defining **Riskcumul**



# Compound Distributions

**Method 2: (Continued) Riskcumul** simulation results.



# Compound Distributions

**Practice Problem:** Assume there are two outcome distributions.

**Outcome 1:**

- 25 percent probability that this outcome occurs
- this outcome results in a uniform distribution between -12 and -7

**Outcome 2:**

- 75 percent probability that this outcome occurs
- this outcome results in a uniform distribution between 0 and 8

Implement in **Riskcumul.** See

`http://kb.palisade.com/index.php?pg=kb.page&id=51`

# Compound Distributions

**Summary:**

**Method 1:**

**The Good:** this is the most general method. No assumptions are made about the distributions associated with each outcome. They do not even need to be uniform.

**The Bad:** The most cumbersome of the three methods to implement.

**Method 2:**

**The Good:** You can use a built-in @Risk function, **Riskcumul**, nothing else needed.

**The Bad:** The most restrictive of the three methods. The outcome distributions must be non-overlapping uniform distributions.

# Multiple Simulations (RiskSimtable)

**New Idea:** Return to the retirement example and treat the payment value as a **variable**. Run a simulation for different values of this variable.

The current annual payment is $23,327.72. We currently take this as a given.

Let's examine what happens as we vary this from $10,000 to $28,000. *We are now treating the payment as adjustable or a variable.*

Remember the Data Table in the **What If Analysis**?

We do a similar thing for simulation using `RiskSimtable()`.

# Multiple Simulations (RiskSimtable)

The end result of this will be `retirementRiskTable_key.xlsx`. For now, modify `retirementRiskTable.xlsx`.

We had in `B6` (the range **payment**) the value that results from evaluating the Excel **PMT** function.

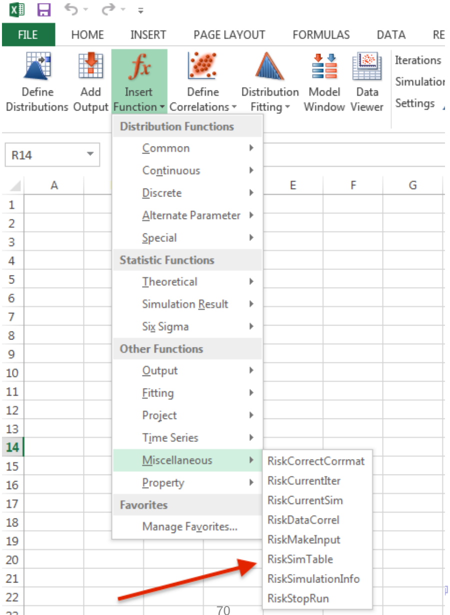
Treat this as a variable. Insert the `RiskSimtable()` function here.

Under **Insert Function** select **Special** and then **RiskSimtable()**.

See next slide for picture.

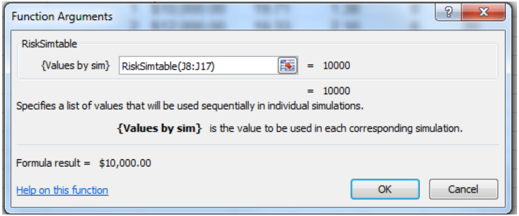
# Multiple Simulations (RiskSimtable)

Here is the location of **RiskSimTable.**



# Multiple Simulations (RiskSimtable)

Now select the range where the test values are. In our case this is `J8:J17`.



# Multiple Simulations (RiskSimtable)

In columns J8:J17 we put the numbers 10,000 to 28,000 in increments of 2,000.

| I | J | K | L | M | N |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| **Simulation** | | | | | |
| **Run** | **Payment** | **Mean** | **Std Dev** | **Min** | **Max** |
| 1 | $10,000.00 | | | | |
| 2 | $12,000.00 | | | | |
| 3 | $14,000.00 | | | | |
| 4 | $16,000.00 | | | | |
| 5 | $18,000.00 | | | | |
| 6 | $20,000.00 | | | | |
| 7 | $22,000.00 | | | | |
| 8 | $24,000.00 | | | | |
| 9 | $26,000.00 | | | | |
| 10 | $28,000.00 | | | | |

# Multiple Simulations (RiskSimtable)

Next in columns K, L, M, N and Row 8 we insert the four functions:

- `=RiskMean(objectiveCell,I8)`
- `=RiskStdDev(objectiveCell,I8)`
- `=RiskMin(objectiveCell,I8)`
- `=RiskMax(objectiveCell,I8)`

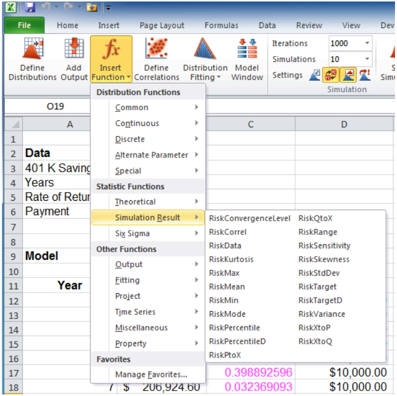
Recall that **objectiveCell** is the green cell `E32`, it is the Risk Output cell

```
=RiskOutput()+COUNTIF(E12:E31,">0")
```

Copy and paste the four functions through row 17.

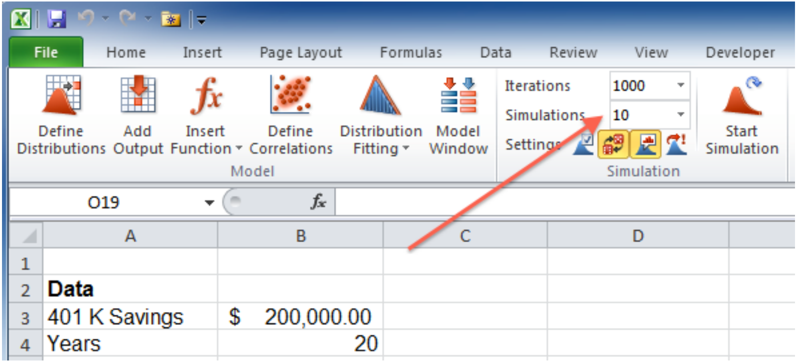
# Multiple Simulations (RiskSimtable)

The functions that we inserted came from **Insert Function**, **Statistic Functions**, then **Simulation Result**.



# Multiple Simulations (RiskSimtable)

Now run 10 simulations since there are 10 possible values for our decision variables.



# Multiple Simulations (RiskSimtable)

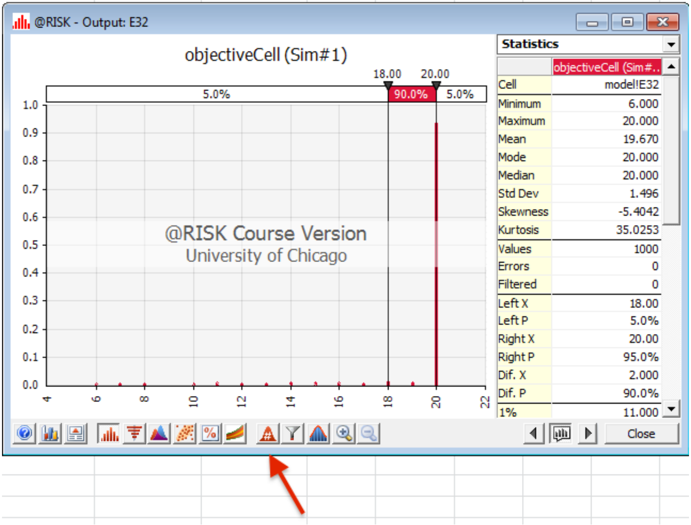
The simulation result is:

| Simulation Run | Payment | Mean | Std Dev | Min | Max |
|---|---|---|---|---|---|
| 1 | $10,000.00 | 19.71 | 1.38 | 8 | 20 |
| 2 | $12,000.00 | 19.33 | 2.16 | 8 | 20 |
| 3 | $14,000.00 | 18.95 | 2.66 | 6 | 20 |
| 4 | $16,000.00 | 18.21 | 3.47 | 5 | 20 |
| 5 | $18,000.00 | 17.31 | 4.13 | 4 | 20 |
| 6 | $20,000.00 | 16.33 | 4.53 | 4 | 20 |
| 7 | $22,000.00 | 15.19 | 4.95 | 5 | 20 |
| 8 | $24,000.00 | 13.85 | 5.16 | 4 | 20 |
| 9 | $26,000.00 | 12.73 | 5.17 | 3 | 20 |
| 10 | $28,000.00 | 11.55 | 4.97 | 3 | 20 |

Are the results logical?

# Multiple Simulations (RiskSimtable)

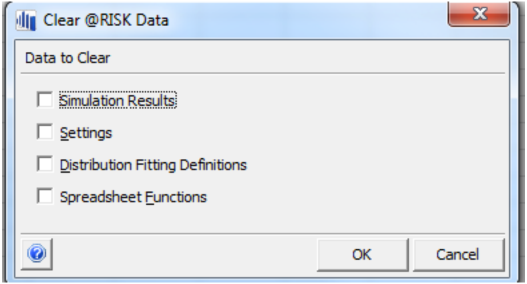
You get a histogram for each simulation run. See the cute little icon.



# Multiple Simulations (RiskSimtable)

A useful trick for getting rid of your simulation results.

Under **Utilities** select **Clear @RISK Data ....**



# Airline Overbooking

See Excel files `airlineOverbooking.xlsx` and `airlineOverbooking_key.xlsx`.

This example is due to Sudhakar D. Deshmukh at Kellogg School of Management, Northwestern University.

- Consider the Chicago to Tokyo leg for North East Airlines (NEA).
- Chicago to Tokyo leg serviced by an NEA Boeing 007 jumbo jet with 300 passenger capacity.
- Each one-way ticket generates $500.
- The flight fixed cost is $80,000.
- Based on past data, there is a 10% chance a ticket holder does show.
- NEA is allowed to overbook, but must give $250 in compensation, plus provide alternative transportation which also costs $500.
- NEA wants to determine how many tickets to sell.

# Airline Overbooking

**Step 1:** The deterministic paramaters in Excel files `airlineOverbooking.xlsx`.

A32

| | A | B | C |
|---|---|---|---|
| 1 | **Airline Overbooking: Source Sudhakar D. Deshmukh** | | |
| 2 | | | |
| 3 | **Economic Data** | | |
| 4 | Fixed Cost Per Flight | $ 80,000.00 | |
| 5 | Revenue Per Ticket | $ 500.00 | |
| 6 | Bumping Cost per Passenger | $ 250.00 | |
| 7 | | | |
| 8 | **Operational Data** | | |
| 9 | Plane Capacity | 300 | |
| 10 | | | |
| 11 | **Probabilistic Data** | | |
| 12 | Probability of No Show | 0.1 | |
| 13 | | | |

# Airline Overbooking

**Step 1:** Continuing with parameters, the *stochastic parameter* is the number of people who show up.

We need a distribution!

We select the **binomial distribution**.

See the You Tube video:
`http://www.youtube.com/watch?v=O12yTz_8EOw`
@Risk provides the **RiskBinomial** distribution.

The **RiskBinomial** has two parameters: 1) number of tickets sold, and 2) the probability of *success*.

# Airline Overbooking

To better understand the **binomial distribution**, think of tossing a fair coin 500 times.

For a given toss consider a **success** to be a Head and a failure a Tail.

The probability of a success is 0.5.

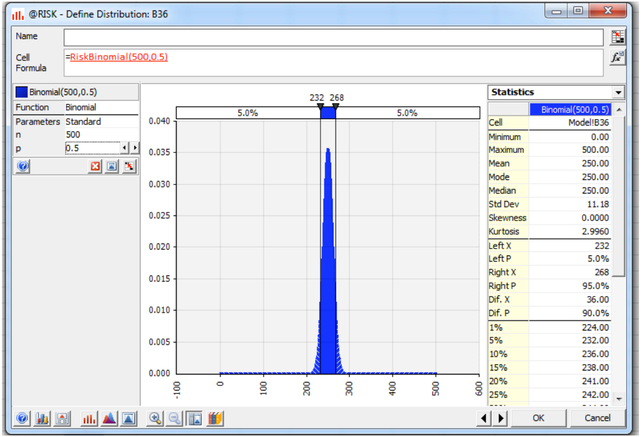
Toss the coin 500 times so there are 500 trials.

The random variable $X$ is the number of successes. This is the binomial random variable.

# Airline Overbooking

Here is the binomial distribution with probability of success .5 and 500 trials.

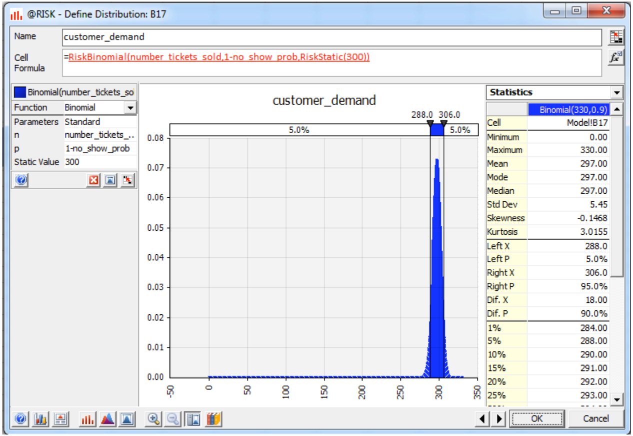
Note that the mean is 250 and 90 percent of the outcomes are between 232 and 268.



# Airline Overbooking

Here is the binomial distribution with probability of success (showing up) .9 and 330 trials.

Note that the mean is 297 and 90 percent of the outcomes are between 288 and 306.



# Airline Overbooking

**Step 2:** Identify the decision variables. In this case there is one, *the number of tickets sold*.

**Step 3:** Determine the model output. This is the profit. Assume no-shows are not refunded. Develop an expression for the profit.

Let $X$ be the number of overbooked customers. This is a random variable.

Let $Y$ denote the number of tickets sold. This is a decision variable.

The profit random variable conditioned on the value of $Y$ is:

$$p(X|Y) = 500 * Y - X * (250 + 500) - 80000$$

# Airline Overbooking

The number of tickets sold (value of $Y$) is contained in cell **B16**. We fix this.

The number of overbooked customers (the value of $X$) is in cell **B18**. It is calculated by

```
=MAX(customer_demand - plane_capacity, 0)
```

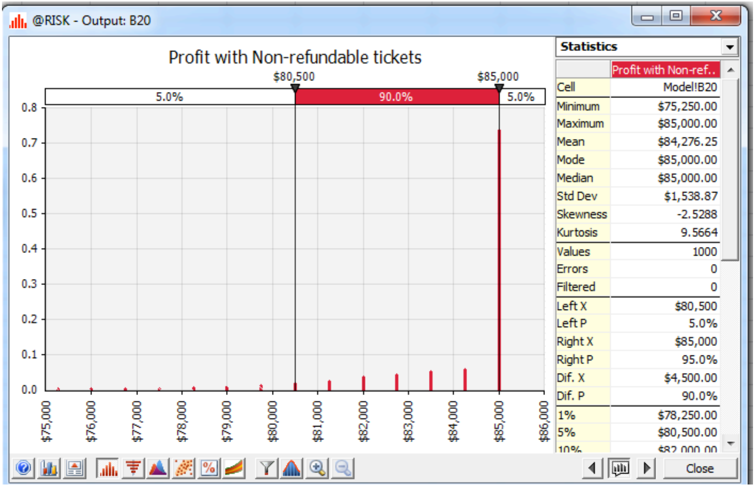
# Airline Overbooking

| | | |
|---|---|---|
| 14 | **The Model** | |
| 15 | | |
| 16 | Number of Tickets Sold | 330 |
| 17 | Number of Customers Arriving | 300 |
| 18 | Number of Overbooked Customers | 0 |
| 19 | Passengers on the flight | 300 |
| 20 | Profit with Non-refundable tickets | $ 85,000.00 |
| 21 | | |

| | | |
|---|---|---|
| 14 | **The Model** | |
| 15 | | |
| 16 | Number of Tickets Sold | 330 |
| 17 | Number of Customers Arriving | =RiskBinomial(number_tickets_sold,1-no_show_prob,RiskStatic(300)) |
| 18 | Number of Overbooked Customers | =MAX(customer_demand-plane_capacity,0) |
| 19 | Passengers on the flight | =MIN(customer_demand,plane_capacity) |
| 20 | Profit with Non-refundable tickets | =RiskOutput()+ticket_revenue*number_tickets_sold-flight_fixed_cost-number_overbooked*(ticket_revenue+bumping_cost) |
| 21 | | |

# Airline Overbooking

**Step 4:** Run the simulation.



# Airline Overbooking

**Step 5:** Analyze the outputs. Let $X$ be the random variable denoting the number of overbooked customers. Let $f(X)$ denote the profit function.

**Sample Exam Question:** In this example, what is f(E(X))?

**Sample Exam Question:** In this example, what is E(f(X))?

You must understand the difference between $E(f(X))$ and $f(E(X))$. *I just pounded very loudly on the blackboard!*

**KEY CONCEPT:** Even when $E(f(X)) = f(E(X))$ simulation is very useful. Why?

# Airline Overbooking

**Modification:** Assume that tickets are refundable. How does this affect the profit calculation?

**Step 3:** Determine the model output. This is the profit.

Again, let $X$ be the number overbooked customers. This is a random variable.

Let $Y$ denote the number of tickets sold.

The profit random variable conditioned on the value of $Y$ without refunds was

$$p(X|Y) = 500 * Y - X * (250 + 500) - 80000.$$

With refunds the revenue is $500 * \min(Y, 300)$ where 300 is the plane capacity. The new profit function is

$$p(X|Y) = 500 * \min(Y, 300) - X * (250 + 500) - 80000.$$

# Airline Overbooking

**Run Simulation and Analyze Results:**

| | **Non-refundable** | **Refundable** |
|---|---|---|
| Mean Profit | \$84,276 | \$67,294 |
| Minimum Profit | \$73,750 | \$57,000 |
| Maximum Profit | \$85,000 | \$70,000 |
| Std. Dev. | \$1,541 | \$1,990 |

# Airline Overbooking

**Optimization:** Treat the number of tickets sold as a variable / adjustable cell and find the *optimal number of tickets to sell*.

Consider values between 300 and 440 in increments of 10 tickets.

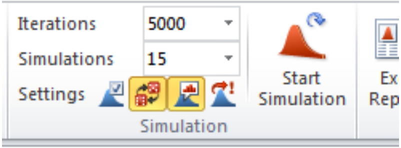
Run simulations for both the non-refundable and refundable profit functions.

Record the Mean, Min, Max, and Standard Deviation statistics for each run.

Put the `RiskSimTable` in cell `B16`. This is the cell with the range name **number_tickets_sold**.

# Airline Overbooking

We are running **15 simulations** on **two** @Risk Output Cells.



# Simulation Basics

What is the optimal number of tickets to sell for the non-refundable profit function? What is the optimal number of tickets to sell for the refundable profit function?

| Simulation Run | Tickets Sold | Profit with Non-Refundable Tickets | | | | Profit with Refundable Tickets | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Mean | Min | Max | Std. Dev. | Mean | Min | Max | Std. Dev. |
| 1 | 300 | $ 70,000.00 | $ 70,000.00 | $ 70,000.00 | $ - | $ 55,000.10 | $ 44,500.00 | $ 63,500.00 | $ 2,597.46 |
| 2 | 310 | $ 74,999.85 | $ 74,250.00 | $ 75,000.00 | $ 10.61 | $ 59,500.15 | $ 49,000.00 | $ 69,250.00 | $ 2,642.80 |
| 3 | 320 | $ 79,989.65 | $ 74,750.00 | $ 80,000.00 | $ 150.03 | $ 63,982.35 | $ 53,000.00 | $ 70,000.00 | $ 2,647.71 |
| 4 | 330 | $ 84,276.25 | $ 71,500.00 | $ 85,000.00 | $ 1,544.24 | $ 67,293.95 | $ 56,500.00 | $ 70,000.00 | $ 1,992.67 |
| 5 | 340 | $ 85,185.75 | $ 72,000.00 | $ 90,000.00 | $ 3,608.74 | $ 64,976.45 | $ 52,000.00 | $ 70,000.00 | $ 3,394.04 |
| 6 | 350 | $ 83,741.45 | $ 69,500.00 | $ 95,000.00 | $ 4,186.01 | $ 58,736.25 | $ 44,500.00 | $ 70,000.00 | $ 4,173.11 |
| 7 | 360 | $ 82,000.60 | $ 67,750.00 | $ 100,000.00 | $ 4,270.08 | $ 52,000.50 | $ 37,750.00 | $ 69,500.00 | $ 4,269.67 |
| 8 | 370 | $ 80,249.85 | $ 63,000.00 | $ 98,250.00 | $ 4,330.04 | $ 45,249.85 | $ 28,000.00 | $ 63,250.00 | $ 4,330.04 |
| 9 | 380 | $ 78,500.00 | $ 62,750.00 | $ 95,000.00 | $ 4,384.42 | $ 38,500.00 | $ 22,750.00 | $ 55,000.00 | $ 4,384.42 |
| 10 | 390 | $ 76,750.30 | $ 62,500.00 | $ 97,000.00 | $ 4,445.31 | $ 31,750.30 | $ 17,500.00 | $ 52,000.00 | $ 4,445.31 |
| 11 | 400 | $ 75,000.15 | $ 60,000.00 | $ 92,250.00 | $ 4,498.51 | $ 25,000.15 | $ 10,000.00 | $ 42,250.00 | $ 4,498.51 |
| 12 | 410 | $ 73,250.30 | $ 57,500.00 | $ 90,500.00 | $ 4,556.53 | $ 18,250.30 | $ 2,500.00 | $ 35,500.00 | $ 4,556.53 |
| 13 | 420 | $ 71,500.30 | $ 56,500.00 | $ 89,500.00 | $ 4,610.37 | $ 11,500.30 | $ (3,500.00) | $ 29,500.00 | $ 4,610.37 |
| 14 | 430 | $ 69,750.15 | $ 53,250.00 | $ 89,250.00 | $ 4,665.58 | $ 4,750.15 | $(11,750.00) | $ 24,250.00 | $ 4,665.58 |
| 15 | 440 | $ 68,000.15 | $ 50,750.00 | $ 87,500.00 | $ 4,721.70 | $ (1,999.85) | $(19,250.00) | $ 17,500.00 | $ 4,721.70 |