36106 Managerial Decision Modeling
Revenue Management

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Reading and Excel Files

Reading (Powell and Baker):

- Section 9.5
- Appendix 9.1

Files used in this lecture:

- `revenueMgmt.xlsx`
- `revenueMgmt_key.xlsx`
- `revenueMgmt_macro.xlsx`
- `revenueMgmt_alt.xlsx`

Lecture Outline

Revenue Management

Revenue Management - Sensitivity Analysis

Allowable Increase and Decrease

Objective Function Coefficient Sensitivity

Learning Objectives

1. Learn about revenue management, a very important application of modeling.

2. Apply our dual price methodology to find the value of "overbooking."

3. Learn to build a revenue management model in Solver.

Revenue Management

Revenue Management: managing the short-term demand for **perishable inventory** in order to maximize the revenue potential for an organization.

For example, seat 35C on United Flight 219 from ORD to HNL on October 30, 2017 is an item held in inventory. This item expires on October 30, 2017 and has zero value after this date.

Started with the *American Airlines Sabre* system. Back then it was called **yield management**. It is a form of price **discrimination**.

Was used to determine how many airline flight seats to sell at an early reservation discount fare and how many airline flight seats to sell at full fare.

Used by all major airlines today.

Generates about a billion in incremental revenue for American Airlines.

40108 Revenue Management is offered this Spring.

Revenue Management

Revenue Management has been applied to:

- Airline seat allocation
- Hotel room booking (**my Phoenix experience**)
- Apartment rentals
- Car rentals
- Cruise lines
- Golf courses

The Internet greatly facilitates revenue management.

Revenue Management

Objective:

- Show how to formulate revenue management problems as a linear program in Solver.
- Understand how **dual prices** are used in revenue management.

Revenue Management is **driven by technology**.

1. fast computers
2. the Internet

Revenue Management

This presentation is based on Leisure Air. See *Management Science* by Anderson, Sweeney, Williams, and Camm.

- Two Boeing 757-200 jets, each with 132 seat capacity.
- One jet originates in Pittsburgh, the other in Newark.
- The Pittsburgh jet flies to Orlando with a stop in Charlotte
- The Newark jet flies to Myrtle Beach with a stop in Charlotte

There are two fare classes:

- Class Q – discount fare
- Class Y – full fare

There are three destinations: Orlando, Myrtle Beach, and Charlotte.

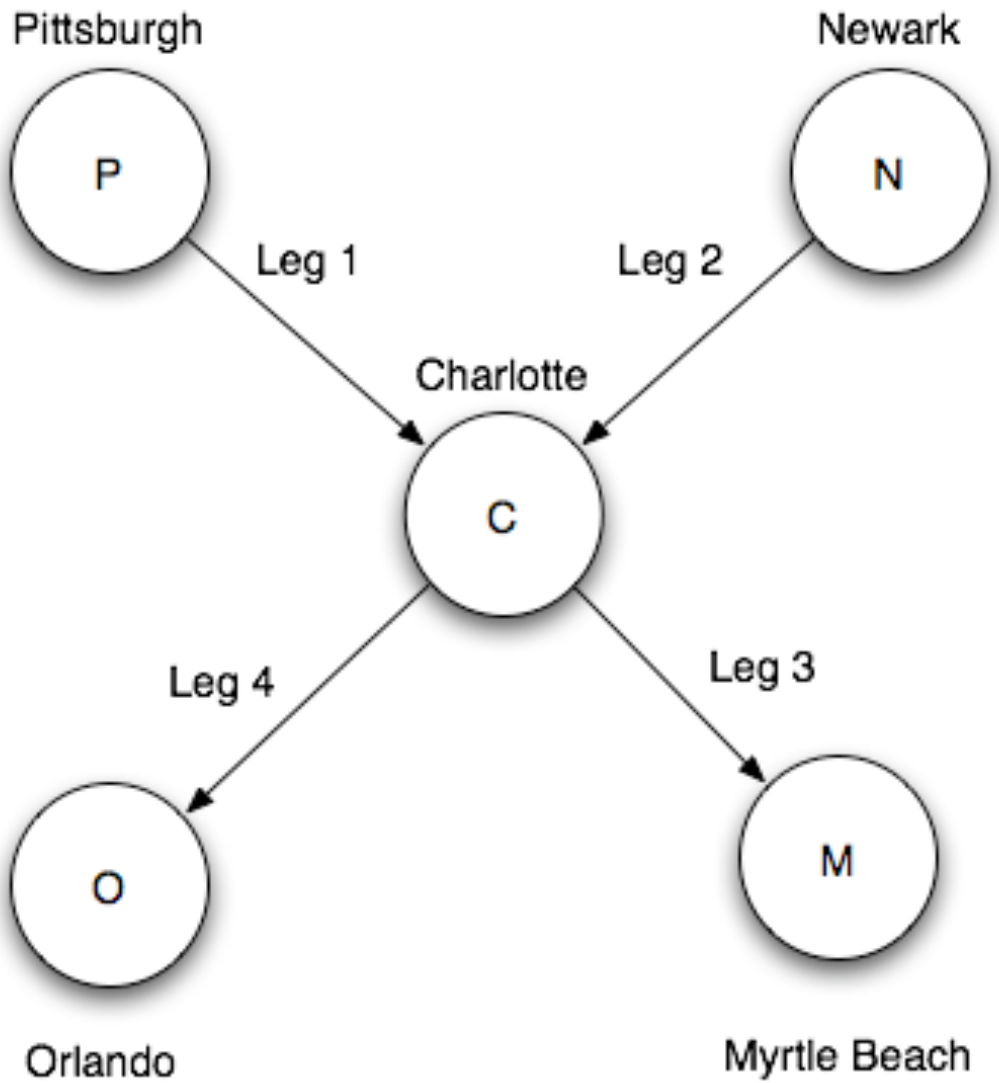
Revenue Management

Possible Itineraries:

- Pittsburgh-Orlando (stop in Charlotte)
- Pittsburgh-Myrtle Beach (stop in Charlotte)
- Pittsburgh-Charlotte
- Newark-Orlando (stop in Charlotte)
- Newark-Myrtle Beach (stop in Charlotte)
- Newark-Charlotte
- Charlotte-Orlando
- Charlotte-Myrtle Beach

Revenue Management

Here is the network topology.



Revenue Management

Key Concept: ODIF origin-destination-itinerary fare

This is a valid combination of legs in a class.

A **leg** is a flight between two cities without a stop.

There are 16 possible ODIF fares.

There are four distinct legs.

Note: In the BlueSky case for homework an itinerary is called a **route** and a leg is called a **flight.**

Revenue Management

	A	B	C	D	E	F	G
1	**Leisure Air**						
2							
3							
4		**ODIF**	**Origin**	**Destination**	**Fare Class**	**Fare**	**Demand**
5	Leg 1	**PCQ**	Pittsburgh	Charlotte	Q	$ 178.00	33
6		**PMQ**	Pittsburgh	Myrtle Beach	Q	$ 268.00	44
7		**POQ**	Pittsburgh	Orlando	Q	$ 228.00	45
8		**PCY**	Pittsburgh	Charlotte	Y	$ 380.00	16
9		**PMY**	Pittsburgh	Myrtle Beach	Y	$ 456.00	6
10		**POY**	Pittsburgh	Orlando	Y	$ 560.00	11
11	Leg 2	**NCQ**	Newark	Charlotte	Q	$ 199.00	26
12		**NMQ**	Newark	Myrtle Beach	Q	$ 249.00	56
13		**NOQ**	Newark	Orlando	Q	$ 349.00	39
14		**NCY**	Newark	Charlotte	Y	$ 385.00	15
15		**NMY**	Newark	Myrtle Beach	Y	$ 444.00	7
16		**NOY**	Newark	Orlando	Y	$ 580.00	9
17	Leg 3	**CMQ**	Charlotte	Myrtle Beach	Q	$ 179.00	64
18		**CMY**	Charlotte	Myrtle Beach	Y	$ 380.00	8
19	Leg 4	**COQ**	Charlotte	Orlando	Q	$ 224.00	46
20		**COY**	Charlotte	Orlando	Y	$ 582.00	10

See **revenueMgmt.xlsx**.

Revenue Management

Objective: *Determine how many seats to make available for each ODIF in order to maximize revenue.* This is called **revenue management.**

How are costs treated in these models?

Important:

1. We treat *price as parameter*!
2. We treat *demand as a deterministic parameter!*

Variable Definition:

PCQ = number of seats allocated to PCQ ODIF (Pittsburg-Charlotte Q class)

POY = number of seats allocated to POY ODIF (Pittsburg-Orlando Y class)

There is one such variable for each class. The optimal value of the variable is how many seats to **allow** in that ODIF.

Revenue Management

Objective: *Determine how many seats to make available for each ODIF in order to maximize revenue.*

Therefore maximize the sum product of the ODIF variables with the corresponding revenue, e.g.

$$\max \quad 178PCQ + 268PMQ + \cdots + 224COQ + 582COY$$

Sample Test Question: Write out the revenue for just the Charlotte-Orlando leg.

Revenue Management

Plane Capacity Constraint: *The number of seats allocated to a leg cannot exceed the plane capacity (in this case 132) of the leg*. There is one such constraint for each leg for a total of four constraints.

Consider Pittsburgh-Charlotte leg capacity constraint

$$PCQ + PMQ + POQ + PCY + PMY + POY \leq 132$$

Sample Test Question: Write out the Charlotte-Orlando leg capacity constraint.

Revenue Management

ODIF Demand Constraints: *The number of seats allocated to an ODIF cannot exceed the demand on the ODIF.*

$$PCQ \leq 33$$

$$PMQ \leq 44$$

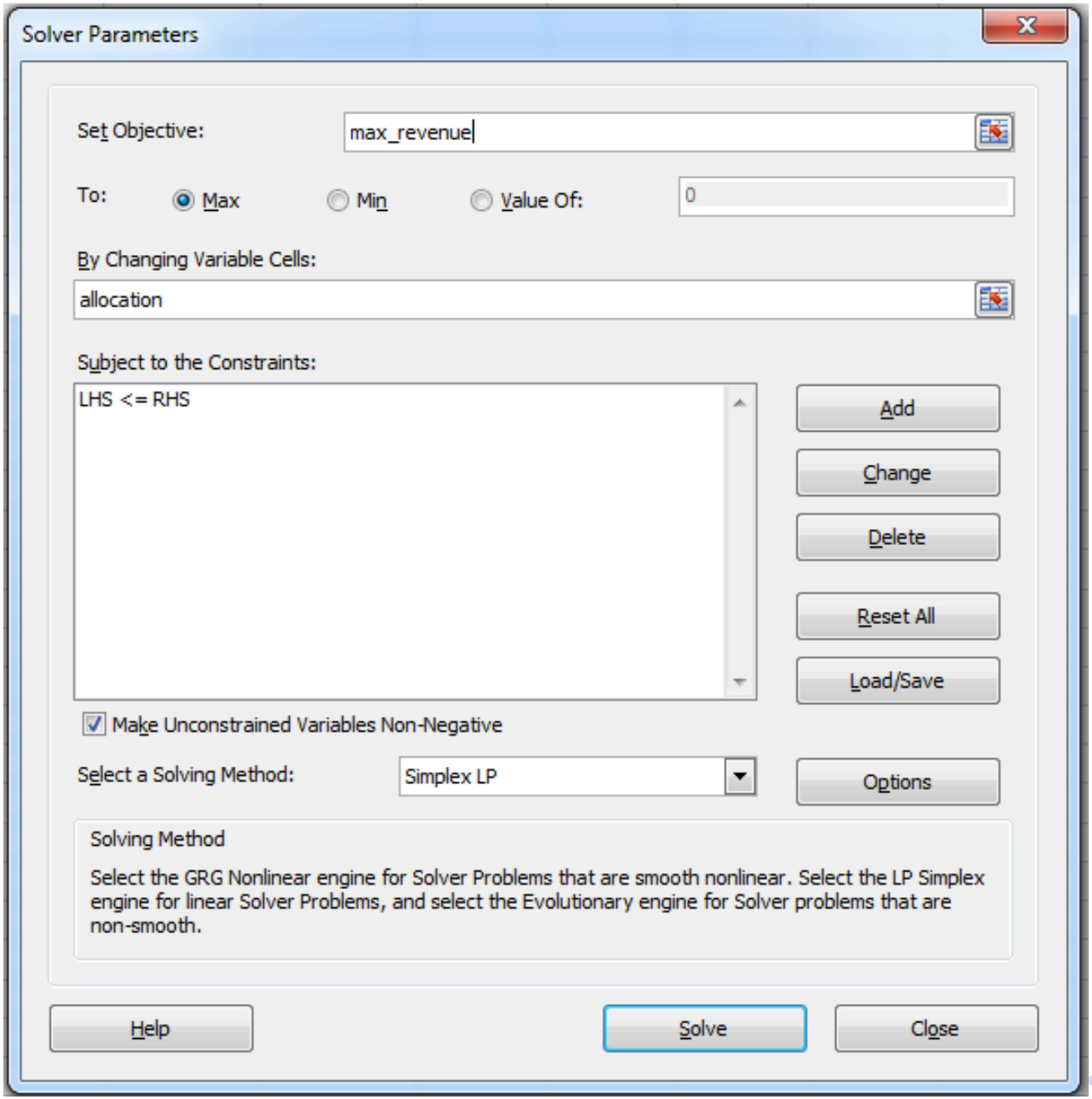
Sample Test Question: What is the Charlotte-Orlando discount demand constraint?

Revenue Management

Here is the optimal allocation.

	A	B	C	D	E	F	G	H
1	**Leisure Air**							
2								
3								
4		**ODIF**	**Origin**	**Destination**	**Fare Class**	**Fare**	**Demand**	**Allocation**
5	Leg 1	**PCQ**	Pittsburgh	Charlotte	Q	$ 178.00	33	**33**
6		**PMQ**	Pittsburgh	Myrtle Beach	Q	$ 268.00	44	**44**
7		**POQ**	Pittsburgh	Orlando	Q	$ 228.00	45	**22**
8		**PCY**	Pittsburgh	Charlotte	Y	$ 380.00	16	**16**
9		**PMY**	Pittsburgh	Myrtle Beach	Y	$ 456.00	6	**6**
10		**POY**	Pittsburgh	Orlando	Y	$ 560.00	11	**11**
11	Leg 2	**NCQ**	Newark	Charlotte	Q	$ 199.00	26	**26**
12		**NMQ**	Newark	Myrtle Beach	Q	$ 249.00	56	**36**
13		**NOQ**	Newark	Orlando	Q	$ 349.00	39	**39**
14		**NCY**	Newark	Charlotte	Y	$ 385.00	15	**15**
15		**NMY**	Newark	Myrtle Beach	Y	$ 444.00	7	**7**
16		**NOY**	Newark	Orlando	Y	$ 580.00	9	**9**
17	Leg 3	**CMQ**	Charlotte	Myrtle Beach	Q	$ 179.00	64	**31**
18		**CMY**	Charlotte	Myrtle Beach	Y	$ 380.00	8	**8**
19	Leg 4	**COQ**	Charlotte	Orlando	Q	$ 224.00	46	**41**
20		**COY**	Charlotte	Orlando	Y	$ 582.00	10	**10**
21								
22	**Plane Capacity**	132						
23								
24	**Max Rev**	**$ 103,103.00**						
25								

Revenue Management



Revenue Management - Sensitivity Analysis

Okay, now let's see what might happen in practice after the initial model solution.

We are going to "tweak" the demands. Remember, we assumed that demand was deterministic, but in fact the demand numbers come from forecasts and are stochastic.

We are going to "tweak" the fares.

Excel and Solver are really good tools for this. Open `revenueMgmt_macro.xlsm`.

Revenue Management - Sensitivity Analysis

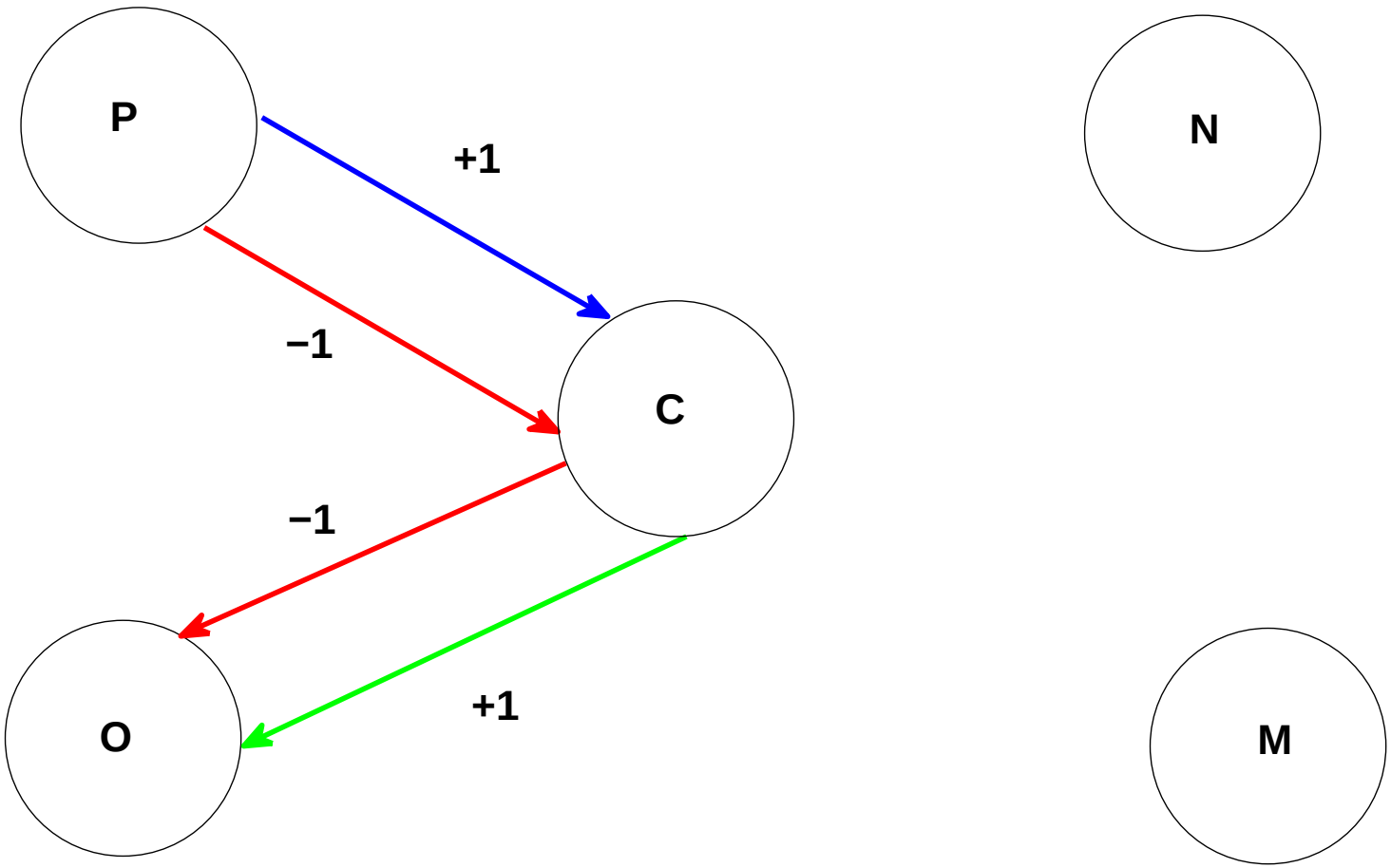
The scenario:

- The perishable inventory consists of the seats in each ODIF (origin-destination-itinerary-fare) on Leisure's two jets for January 23, 2018.
- On January 19, 2018 a user comes to Leisure's Web Site and wants to buy a PCY ODIF – the full fare from Pittsburgh to Charlotte. However, all 16 seats allocated in this PCY ODIF are sold.
- As of January 19, 2018 Leisure has not sold all of the 22 seats allocated to the POQ ODIF.
- The number seats allocated to the COQ ODIF for January 23, 2018 is 41 and the forecasted demand is 46.

What should the revenue management system do?

Revenue Management - Sensitivity Analysis

What is the effect of over booking PCY by 1? Each color represents an itinerary.



Revenue Management - Sensitivity Analysis

What is the effect of over booking PCY by 1?

	A	B	C	D	E	F	G	H
1	**Leisure Air**							
2								
3								
4		**ODIF**	**Origin**	**Destination**	**Fare Class**	**Fare**	**Demand**	**Allocation**
5	Leg 1	**PCQ**	Pittsburgh	Charlotte	Q	$ 178.00	33	**33**
6		**PMQ**	Pittsburgh	Myrtle Beach	Q	$ 268.00	44	**44**
7	-1	**POQ**	Pittsburgh	Orlando	Q	$ 228.00	45	**22**
8	+1	**PCY**	Pittsburgh	Charlotte	Y	$ 380.00	16	**16**
9		**PMY**	Pittsburgh	Myrtle Beach	Y	$ 456.00	6	**6**
10		**POY**	Pittsburgh	Orlando	Y	$ 560.00	11	**11**
11	Leg 2	**NCQ**	Newark	Charlotte	Q	$ 199.00	26	**26**
12		**NMQ**	Newark	Myrtle Beach	Q	$ 249.00	56	**36**
13		**NOQ**	Newark	Orlando	Q	$ 349.00	39	**39**
14		**NCY**	Newark	Charlotte	Y	$ 385.00	15	**15**
15		**NMY**	Newark	Myrtle Beach	Y	$ 444.00	7	**7**
16		**NOY**	Newark	Orlando	Y	$ 580.00	9	**9**
17	Leg 3	**CMQ**	Charlotte	Myrtle Beach	Q	$ 179.00	64	**31**
18		**CMY**	Charlotte	Myrtle Beach	Y	$ 380.00	8	**8**
19	Leg 4 +1	**COQ**	Charlotte	Orlando	Q	$ 224.00	46	**41**
20		**COY**	Charlotte	Orlando	Y	$ 582.00	10	**10**
21								
22	**Plane Capacity**	132						
23								
24	**Max Rev**	**$ 103,103.00**						
25								

Revenue Management - Sensitivity Analysis

Revenue Change

ODIF	Change	Revenue
PCY	+ 1	+ $380
POQ	- 1	- $228
COQ	+ 1	+ $224
		+ $376

Sell the ticket! And adjust the system so that:

- Add one more seat between Pittsburgh and Charlotte in class Y
- Allocate one less seat between Pittsburgh and Orlando in the Q class
- Allocate one more seat between Charlotte and Orlando in the Q class

Revenue Management - Sensitivity Analysis

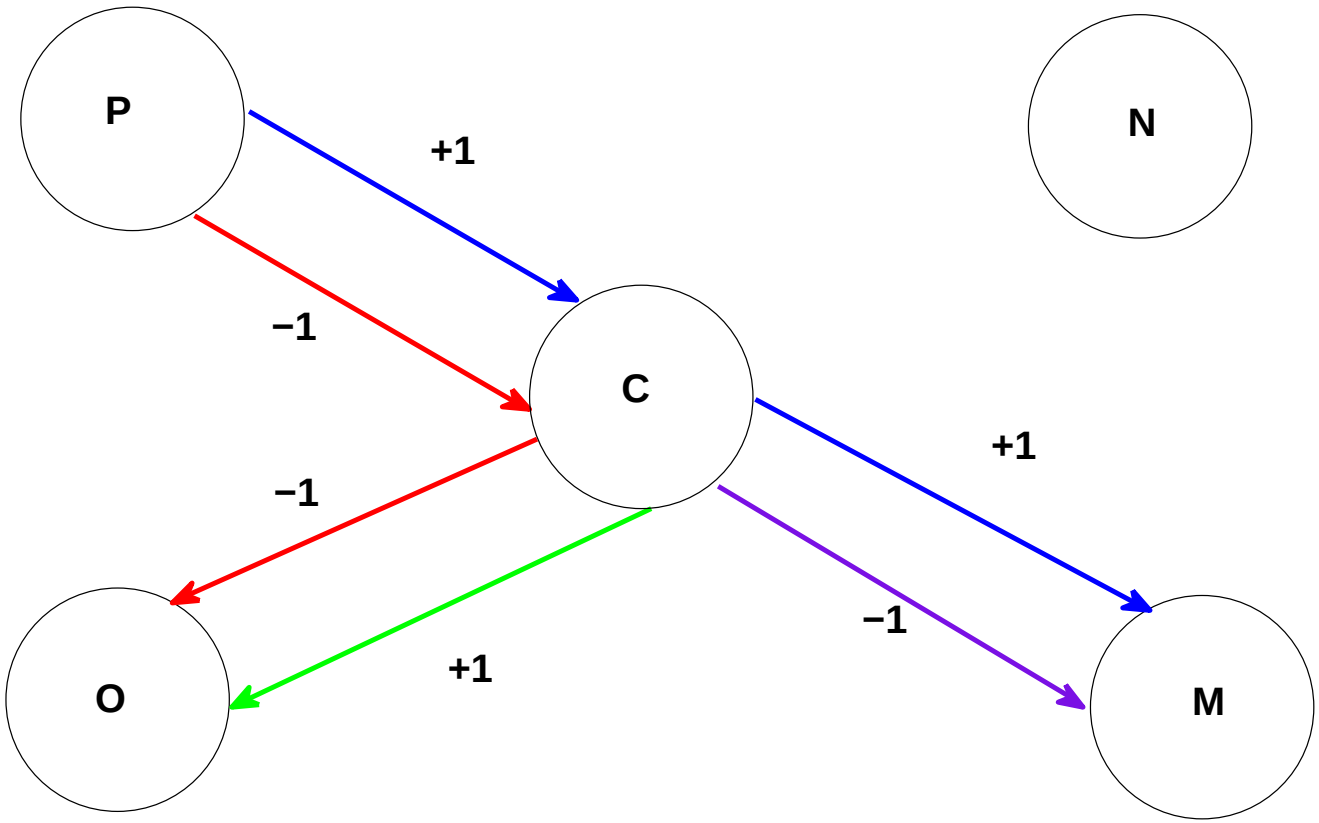
The scenario:

- The perishable inventory consists of the seats in each ODIF (origin-destination-itinerary-fare) on Leisure's two jets for January 23, 2018.
- On January 19, 2018 a user comes to Leisure's Web Site and wants to buy a PMQ ODIF – the cheap Pittsburgh to Myrtle Beach. However, all 44 seats allocated in the PMQ ODIF are sold.
- As of January 19, 2018 Leisure has not sold all of the 22 seats allocated to the POQ ODIF.
- As of January 19, 2018 Leisure has sold not all of the 31 seats allocated to the CMQ ODIF.
- The number seats allocated to the COQ ODIF for January 23, 2018 is 41 and the forecasted demand is 46.

What should the revenue management system do?

Revenue Management - Sensitivity Analysis

What is the effect of over booking PMQ by 1?



Each color represents a distinct ODIF.

Revenue Management - Sensitivity Analysis

What is the effect of over booking PMQ by 1?

	A	B	C	D	E	F	G	H
1	**Leisure Air**							
2								
3								
4		**ODIF**	**Origin**	**Destination**	**Fare Class**	**Fare**	**Demand**	**Allocation**
5	Leg 1	**PCQ**	Pittsburgh	Charlotte	Q	$ 178.00	33	**33**
6	+1	**PMQ**	Pittsburgh	Myrtle Beach	Q	$ 268.00	44	**44**
7	-1	**POQ**	Pittsburgh	Orlando	Q	$ 228.00	45	**22**
8		**PCY**	Pittsburgh	Charlotte	Y	$ 380.00	16	**16**
9		**PMY**	Pittsburgh	Myrtle Beach	Y	$ 456.00	6	**6**
10		**POY**	Pittsburgh	Orlando	Y	$ 560.00	11	**11**
11	Leg 2	**NCQ**	Newark	Charlotte	Q	$ 199.00	26	**26**
12		**NMQ**	Newark	Myrtle Beach	Q	$ 249.00	56	**36**
13		**NOQ**	Newark	Orlando	Q	$ 349.00	39	**39**
14		**NCY**	Newark	Charlotte	Y	$ 385.00	15	**15**
15		**NMY**	Newark	Myrtle Beach	Y	$ 444.00	7	**7**
16		**NOY**	Newark	Orlando	Y	$ 580.00	9	**9**
17	Leg 3 -1	**CMQ**	Charlotte	Myrtle Beach	Q	$ 179.00	64	**31**
18		**CMY**	Charlotte	Myrtle Beach	Y	$ 380.00	8	**8**
19	Leg 4 +1	**COQ**	Charlotte	Orlando	Q	$ 224.00	46	**41**
20		**COY**	Charlotte	Orlando	Y	$ 582.00	10	**10**
21								
22	**Plane Capacity**	132						
23								
24	**Max Rev**	**$ 103,103.00**						
25								

Each color represents a distinct ODIF.

Revenue Management - Sensitivity Analysis

Revenue Change

ODIF	Change	Revenue
PMQ	+ 1	+ $268
POQ	- 1	- $228
CMQ	- 1	- $179
COQ	+ 1	+ $224
		+ $85

Sell the ticket! And adjust the system so that:

- Add one more seat between Pittsburgh and Myrtle Beach in class Q
- Allocate one less seat between Pittsburgh and Orlando in the Q class
- Allocate one less seat between Charlotte and Myrtle Beach in the Q class
- Allocate one more seat between Charlotte and Orlando in the Q class

Revenue Management - Sensitivity Analysis

The dual prices give us this information immediately!

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C27	Leg 1 LHS	132	4	132	23	5
C28	Leg 2 LHS	132	70	132	20	33
C29	Leg 3 LHS	132	179	132	33	31
C30	Leg 4 LHS	132	224	132	5	41
C31	PCQ LHS	33	174	33	5	23
C32	PMQ LHS	44	85	44	5	23
C33	POQ LHS	22	0	45	1E+30	23
C34	PCY LHS	16	376	16	5	16
C35	PMY LHS	6	273	6	5	6
C36	POY LHS	11	332	11	22	11
C37	NCQ LHS	26	129	26	33	20
C38	NMQ LHS	36	0	56	1E+30	20
C39	NOQ LHS	39	55	39	33	5
C40	NCY LHS	15	315	15	33	15
C41	NMY LHS	7	195	7	36	7
C42	NOY LHS	9	286	9	33	5
C43	CMQ LHS	31	0	64	1E+30	33
C44	CMY LHS	8	201	8	31	8
C45	COQ LHS	41	0	46	1E+30	5
C46	COY LHS	10	358	10	41	5

Revenue Management - Sensitivity Analysis

The dual price on the PMQ ODIF is 85.

What happens if you set the right-hand-side on the PMQ ODIF to 45 and solve again?

The dual price on the PCY ODIF is 376.

What happens if you set the right-hand-side on the PCY ODIF to 17 and solve again?

See the Workbook `revenueMgmt_macro.xlsx`.

Key Idea: use the dual prices to adjust the number of seats made available in each ODIF as reservations come in.

Allowable Increase and Decrease

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C27	Leg 1 LHS	132	4	132	23	5
C28	Leg 2 LHS	132	70	132	20	33
C29	Leg 3 LHS	132	179	132	33	31
C30	Leg 4 LHS	132	224	132	5	41
C31	PCQ LHS	33	174	33	5	23
C32	PMQ LHS	44	85	44	5	23
C33	POQ LHS	22	0	45	1E+30	23
C34	PCY LHS	16	376	16	5	16
C35	PMY LHS	6	273	6	5	6
C36	POY LHS	11	332	11	22	11
C37	NCQ LHS	26	129	26	33	20
C38	NMQ LHS	36	0	56	1E+30	20
C39	NOQ LHS	39	55	39	33	5
C40	NCY LHS	15	315	15	33	15
C41	NMY LHS	7	195	7	36	7
C42	NOY LHS	9	286	9	33	5
C43	CMQ LHS	31	0	64	1E+30	33
C44	CMY LHS	8	201	8	31	8
C45	COQ LHS	41	0	46	1E+30	5
C46	COY LHS	10	358	10	41	5

Allowable Increase and Decrease

Consider the constraint on the PMQ ODIF.

The current right-hand-side demand limit is 44, and this implies that we cannot allocate more than 44 seats in this ODIF. We also know that if we sell one more seat, revenue goes up by \$85.

However, the allowable increase is 5. If we observe an increase in demand of six seats, and allocate six seats in this fare class, the sixth seat may not generate \$85 dollars. At the margin, it can never generate **more than** \$85.

Run the model with demands of 49 and 50. What do you observe?

For a maximization problem, the value of the dual price will **decrease** when the right-hand-side is increased by more than the allowable increase.

Okay, so **why is the allowable increase 5**?

Allowable Increase and Decrease

Important: See `revenueMgmt_alt.xlsx`. We formulate the leg capacity and demand limit constraints differently in this workbook. We do not put a constraint on slack. Instead we say:

- The number of seats allocated to a leg cannot exceed the the capacity of the plane on the leg.
- The seats allocated for an ODIF cannot exceed the demand for that ODIF.

The sensitivity report is a bit different. We discussed this last week.

The dual price information on the ODIF constraints appears in the reduced cost information.

Allowable Increase and Decrease

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
H5	Q Allocation	33	174	178	1E+30	174
H6	Q Allocation	44	85	268	1E+30	85
H7	Q Allocation	22	0	228	85	4
H8	Y Allocation	16	376	380	1E+30	376
H9	Y Allocation	6	273	456	1E+30	273
H10	Y Allocation	11	332	560	1E+30	332
H11	Q Allocation	26	129	199	1E+30	129
H12	Q Allocation	36	0	249	55	70
H13	Q Allocation	39	55	349	1E+30	55
H14	Y Allocation	15	315	385	1E+30	315
H15	Y Allocation	7	195	444	1E+30	195
H16	Y Allocation	9	286	580	1E+30	286
H17	Q Allocation	31	0	179	70	55
H18	Y Allocation	8	201	380	1E+30	201
H19	Q Allocation	41	0	224	4	85
H20	Y Allocation	10	358	582	1E+30	358

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C27	Leg 1 LHS	132	4	132	23	5
C28	Leg 2 LHS	132	70	132	20	33
C29	Leg 3 LHS	132	179	132	33	31
C30	Leg 4 LHS	132	224	132	5	41

Allowable Increase and Decrease

We lose the allowable increase and allowable decrease information on the ODIF demand constraints.

The allowable increase and decrease information in the Variable Cells section is on the objective function coefficient, not on the reduced cost.

Allowable Increase and Decrease

The allowable decrease on the PMQ ODIF is 23.

Why 23?

Hint:

- The model wants to fill both planes to capacity.
- If we decrease the number of seats on PMQ, we want to increase seats on some other leg(s).
- What restricts this?

What if we decrease the demand **by more than 23**?

Will the new dual price be smaller or larger than \$85?

Allowable Increase and Decrease

Remember our two important rules:

Rule 1: Helping helps less and less!

Rule 2: Hurting hurts more and more!

These two rules will always tell you what happens as your scarce resource levels increase or decrease.

Allowable Increase and Decrease

Summary: what happens to the optimal objective function value under the following scenarios?

Constraint Type	maximum objective function		minimum objective function	
$\leq$	Increase	Decrease	Increase	Decrease
$\geq$	Increase	Decrease	Increase	Decrease

Objective Function Coefficient Sensitivity

Now, for the rest of the story! In addition to information about the right-hand-sides, Solver provides useful economic information about **objective function coefficients.**

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
I5	Q Allocation	33	0	178	1E+30	174
I6	Q Allocation	44	0	268	1E+30	85
I7	Q Allocation	22	0	228	85	4
I8	Y Allocation	16	0	380	1E+30	376
I9	Y Allocation	6	0	456	1E+30	273
I10	Y Allocation	11	0	560	1E+30	332
I11	Q Allocation	26	0	199	1E+30	129
I12	Q Allocation	36	0	249	55	70
I13	Q Allocation	39	0	349	1E+30	55
I14	Y Allocation	15	0	385	1E+30	315
I15	Y Allocation	7	0	444	1E+30	195
I16	Y Allocation	9	0	580	1E+30	286
I17	Q Allocation	31	0	179	70	55
I18	Y Allocation	8	0	380	1E+30	201
I19	Q Allocation	41	0	224	4	85
I20	Y Allocation	10	0	582	1E+30	358

Objective Function Coefficient Sensitivity

The allowable increase and allowable decrease on an objective function coefficient is the amount the objective function coefficient can increase or decrease without changing the optimal solution.

Consider the POQ ODIF. The associated adjustable cell is `I7` (workbook `revenueMgmt_macro.xlsx`). The current objective coefficient is 228. Let's say we raise the fare on this POQ by 70 dollars. That is, increase the objective function coefficient from 228 to $228 + 70 = 298$.

The optimal solution does not change.

Now exceed the allowable increase of 85. Increase the objective function coefficient from 228 to $228 + 90 = 318$.

Not surprisingly, we increase the seat allocation from 22 to 45.

Objective Function Coefficient Sensitivity

Now consider reducing an objective function coefficient. This makes the ODIF less profitable.

The allowable decrease on the objective function coefficient on the NMQ ODIF is 70.

Decrease the value of the coefficient by 75, that reduce the current 249 to 249 - 75 = 174.

The number seats allocated in the NMQ ODIF is reduced from 36 to 3.

Objective Function Coefficient Sensitivity

Reduced Cost: two ways to define a reduced cost.

- the dual price on the nonnegativity constraint
- for a maximization, the allowable increase on an objective function coefficient if the variable is currently in the solution at 0

For a maximization problem, if a variable is currently at zero in the optimal solution, the reduced cost is the amount by which the objective function coefficient must increase in order for there to be an optimal solution where the variable is positive.

For a minimization problem, if a variable is currently at zero in the optimal solution, the reduced cost is the amount by which the objective function coefficient must be **reduced** in order for there to be an optimal solution where the variable is positive.

Objective Function Coefficient Sensitivity

Below is the sensitivity report when we change the demand on the POY ODIF from 11 to 50.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
I5	Q Allocation	33	0	178	1E+30	89
I6	Q Allocation	27	0	268	89	85
I7	Q Allocation	0	-85	228	85	1E+30
I8	Y Allocation	16	0	380	1E+30	291
I9	Y Allocation	6	0	456	1E+30	188
I10	Y Allocation	50	0	560	1E+30	247
I11	Q Allocation	26	0	199	1E+30	129
I12	Q Allocation	36	0	249	55	70
I13	Q Allocation	39	0	349	1E+30	55
I14	Y Allocation	15	0	385	1E+30	315
I15	Y Allocation	7	0	444	1E+30	195
I16	Y Allocation	9	0	580	1E+30	286
I17	Q Allocation	48	0	179	70	55
I18	Y Allocation	8	0	380	1E+30	201
I19	Q Allocation	24	0	224	55	85
I20	Y Allocation	10	0	582	1E+30	358

In this optimal solution, the POQ ODIF is allocated **zero** seats. It has a reduced cost of 85 (actually, -85, we only worry about the absolute value).

Objective Function Coefficient Sensitivity

In this optimal solution, the POQ ODIF is allocated **zero** seats. It has a reduced cost of 85 (actually, -85, we only worry about the absolute value).

The objective coefficient on the POQ ODIF is \$228.

We would need to raise the fare on the POQ ODIF to \$313 = \$228 + \$85 in order for there to be an optimal solution with a **strictly positive** number of seats allocated to the POQ ODIF.