The University of Chicago
Booth School of Business
36106 Sensitivity Analysis Clarification
Kipp Martin

Assume you have a constraint that is a $\leq$ constraint. If you increase the right-hand-side of the constraint, you will *always make the constraint easier to satisfy.* This statement is true regardless of where your variables and constants are. You can have the variables on one side or both sides, constants on one side or both sides. Consider the arbitrary constraint

$$5x_1 - 10 \leq 17 - x_2 + 5x_3.$$

If I increase the right-hand-side by 3 units it becomes

$$5x_1 - 10 \leq 17 - x_2 + 5x_3 + 3$$

and is easier to satisfy. Given a $\leq$ constraint, increasing the right-hand-side helps, end of story. No, ifs, ands, or buts. It helps! It helps! It helps!

Unfortunately, a problem may arise if you want to do sensitivity analysis and look at the allowable increase/decrease. Consider the constraint

$$2x_1 + 5x_2 \leq 3. \tag{1}$$

Interpret the constraint as a capacity constraint with current capacity of 3. I often like to see an explicit slack calculation so I might write this capacity constraint as the slack is nonnegative. That is, I write the constraint as

$$3 - 2x_1 - 5x_2 \geq 0. \tag{2}$$

Obviously the same thing from a purely algebraic standpoint. Now let's say I want to know the effect of increasing my capacity from 3 units to 5 units. Solver does not know your original constraint was (1). The Solver sensitivity analysis is always with respect to the right-and-side. *No matter where your variables and constants are, Solver assumes you are increasing the right-hand-side.* This means that if you input the constraint in the form (2), Solver will give a sensitivity report with respect to (2). Therefore, if I increase the right-hand-side of (2) by 2 units it becomes

$$3 - 2x_1 - 5x_2 \geq 2 \tag{3}$$

which is equivalent to

$$2x_1 + 5x_2 \leq 1 \tag{4}$$

Argh! I actually reduced capacity from 3 to 1 when I wanted to investigate the change from 3 to 5. I would need to reduce (2) by 2 units, i.e. write

$$3 - 2x_1 - 5x_2 \geq -2 \tag{5}$$

which is equivalent to

$$2x_1 + 5x_2 \leq 5 \tag{6}$$

to investigate the effect of increasing the capacity from 3 to 5.

Bottom Line: when I rewrite (1) as (2) the allowable increase and decrease values in the Solver sensitivity analysis are reversed from what I actually want. Note however, that if I rewrite (1) as

$$2x_1 + 5x_2 - 3 \leq 0 \tag{7}$$

and require the *slack to be nonpositive* then I do not encounter the problem.