Chapter 2: Matrix Algebra

DA 410

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Problem 2.1

Let A =

```
##      [,1] [,2] [,3]
## [1,]    4    2    3
## [2,]    7    5    8
```

and B =

```
##      [,1] [,2] [,3]
## [1,]    3   -2    4
## [2,]    6    9   -5
```

(a) Find A+B and A-B

A+B=

```
##      [,1] [,2] [,3]
## [1,]    7    0    7
## [2,]   13   14    3
```

A-B=

```
##      [,1] [,2] [,3]
## [1,]    7    0    7
## [2,]   13   14    3
```

(b) Find A'A and AA'

A'A =

```
crossprod(A) #t(A) %*% A
```

```
##      [,1] [,2] [,3]
## [1,]   65   43   68
## [2,]   43   29   46
## [3,]   68   46   73
```

AA' =

```r
tcrossprod(A) #A %*% t(A)
```

```
##      [,1] [,2]
## [1,]   29   62
## [2,]   62  138
```

Problem 2.2

(a) Find (A+B)' and A'+B' and compare them, thus illustrating (2.15)

(A+B)' =

```r
t(A+B)
```

```
##      [,1] [,2]
## [1,]    7   13
## [2,]    0   14
## [3,]    7    3
```

A'+B'=

```r
t(A)+t(B)
```

```
##      [,1] [,2]
## [1,]    7   13
## [2,]    0   14
## [3,]    7    3
```

The output for (A+B)' and A'+B' are the same.

(b) Show that (A')' = A, thus illustrating 2.6

(A')' =

```r
t(t(A))
```

```
##      [,1] [,2] [,3]
## [1,]    4    2    3
## [2,]    7    5    8
```

(A')' == A

```
##      [,1] [,2] [,3]
## [1,] TRUE TRUE TRUE
## [2,] TRUE TRUE TRUE
```

Problem 2.3

Let A =

```
##      [,1] [,2]
## [1,]    1    3
## [2,]    2   -1
```

and B =

```
##      [,1] [,2]
## [1,]    2    0
## [2,]    1    5
```

(a) Find AB and BA

AB =

```
A %*% B
```

```
##      [,1] [,2]
## [1,]    5   15
## [2,]    3   -5
```

BA =

```
B %*% A
```

```
##      [,1] [,2]
## [1,]    2    6
## [2,]   11   -2
```

Problem 2.14

Let A =

```
##      [,1] [,2] [,3]
## [1,]    1    3    2
## [2,]    2    0   -1
```

B =

```
##      [,1] [,2]
## [1,]    1    2
## [2,]    0    1
## [3,]    1    0
```

C =

```
##      [,1] [,2] [,3]
## [1,]    2    1    1
## [2,]    5   -6   -4
```

Find AB and CB.

AB =

```
A %*% B
```

```
##      [,1] [,2]
## [1,]    3    5
## [2,]    1    4
```

CB =

```
C %*% B
```

```
##      [,1] [,2]
## [1,]    3    5
## [2,]    1    4
```

Are they equal? AB is equal to CB

AB == CB

```
AB <- A %*% B
CB <- C %*% B
AB == CB
```

```
##      [,1] [,2]
## [1,] TRUE TRUE
## [2,] TRUE TRUE
```

What is the rank for A, B and C?

```
qr(A)$rank
```

```
## [1] 2
```

```
qr(B)$rank
```

```
## [1] 2
```

```
qr(C)$rank
```

```
## [1] 2
```

Problem 2.18

The columns of the following matrix are mutually orthogonal:

A =

```
##      [,1] [,2] [,3]
## [1,]    1   -1    1
## [2,]    2    1    0
## [3,]    1   -1   -1
```

(a) Normalize the columns of A by dividing each column by its length; denote the resulting matrix by C

C =

```
##           [,1]       [,2]       [,3]
## [1,] 0.4082483 -0.5773503  0.7071068
## [2,] 0.8164966  0.5773503  0.0000000
## [3,] 0.4082483 -0.5773503 -0.7071068
```

(b) Show that C is orthogonal matrix, that is, C'C = CC' = I

C'C =

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

CC' =

```
##              [,1] [,2]         [,3]
## [1,] 1.000000e+00    0 2.220446e-16
## [2,] 0.000000e+00    1 0.000000e+00
## [3,] 2.220446e-16    0 1.000000e+00
```

C is orthogonal.