MONASH University

MONASH BUSINESS SCHOOL

# ETC3550: Applied forecasting for business and economics

Ch9. Dynamic regression models

OTexts.org/fpp2/

# Outline

# Regression with ARIMA errors

## Regression models

$$y_t = \beta_0 + \beta_1 x_{1,t} + \dots + \beta_k x_{k,t} + \varepsilon_t,$$

- $y_t$ modeled as function of $k$ explanatory variables $x_{1,t}, \dots, x_{k,t}$.
- In regression, we assume that $\varepsilon_t$ was WN.
- Now we want to allow $\varepsilon_t$ to be autocorrelated.

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- Now we want to allow $\varepsilon_t$ to be autocorrelated.

## Example: ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

where $\varepsilon_t$ is white noise.

# Residuals and errors

**Example: $\eta_t$ = ARIMA(1,1,1)**

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# Residuals and errors

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- Be careful in distinguishing $\eta_t$ from $\varepsilon_t$.
- Only the errors $\eta_t$ are assumed to be white noise.
- In ordinary regression, $\eta_t$ is assumed to be white noise and so $\eta_t = \varepsilon_t$.

# Estimation

If we minimize $\sum \eta_t^2$ (by using ordinary regression):

1. Estimated coefficients $\hat{\beta}_0, \ldots, \hat{\beta}_k$ are no longer optimal as some information ignored;
2. Statistical tests associated with the model (e.g., t-tests on the coefficients) are incorrect.
3. *p*-values for coefficients usually too small ("spurious regression").
4. AIC of fitted models misleading.

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3. *p*-values for coefficients usually too small ("spurious regression").
4. AIC of fitted models misleading.

- Minimizing $\sum \varepsilon_t^2$ avoids these problems.
- Maximizing likelihood is similar to minimizing $\sum \varepsilon_t^2$.

# Stationarity

**Regression with ARMA errors**

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$

where $\eta_t$ is an ARMA process.

- All variables in the model must be stationary.
- If we estimate the model while any of these are non-stationary, the estimated coefficients can be incorrect.
- Difference variables until all stationary.
- If necessary, apply same differencing to all variables.

# Stationarity

## Model with ARIMA(1,1,1) errors

$$
\begin{aligned}
y_t &= \beta_0 + \beta_1 x_{1,t} + \dots + \beta_k x_{k,t} + \eta_t, \\
(1-\phi_1 B)(1-B)\eta_t &= (1+\theta_1 B)\varepsilon_t,
\end{aligned}
$$

# Stationarity

## Model with ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \dots + \beta_k x_{k,t} + \eta_t,$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

## Equivalent to model with ARIMA(1,0,1) errors

$$y'_t = \beta_1 x'_{1,t} + \dots + \beta_k x'_{k,t} + \eta'_t,$$
$$(1 - \phi_1 B)\eta'_t = (1 + \theta_1 B)\varepsilon_t,$$

where $y'_t = y_t - y_{t-1}$, $x'_{t,i} = x_{t,i} - x_{t-1,i}$ and $\eta'_t = \eta_t - \eta_{t-1}$.

# Regression with ARIMA errors

Any regression with an ARIMA error can be rewritten as a regression with an ARMA error by differencing all variables with the same differencing operator as in the ARIMA model.

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**Original data**

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t$$

$$\text{where} \quad \phi(B)(1-B)^d \eta_t = \theta(B)\varepsilon_t$$

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$$\text{where} \quad \phi(B)(1 - B)^d \eta_t = \theta(B)\varepsilon_t$$

## After differencing all variables

$$y'_t = \beta_1 x'_{1,t} + \cdots + \beta_k x'_{k,t} + \eta'_t.$$

$$\text{where} \quad \phi(B)\eta_t = \theta(B)\varepsilon_t$$

$$\text{and} \quad y'_t = (1 - B)^d y_t$$

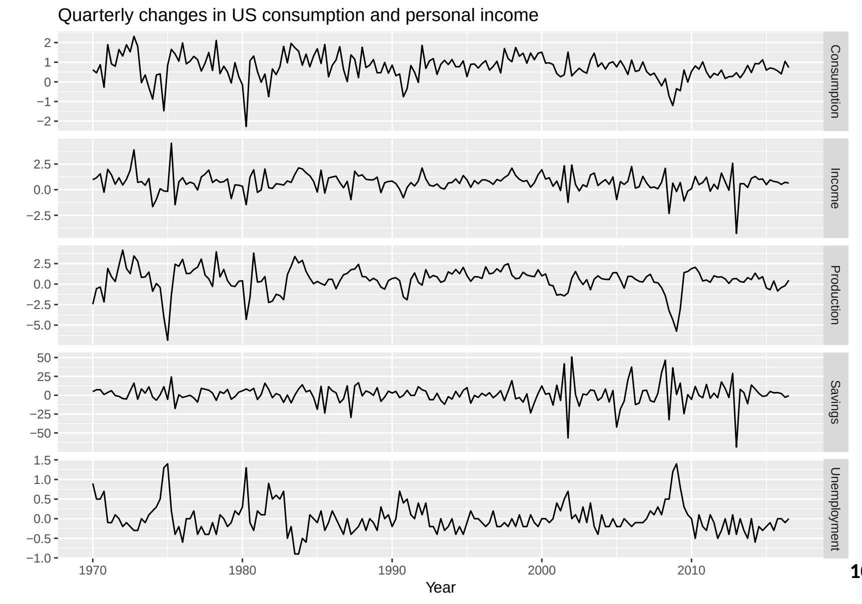
# Model selection

- Check that all variables are stationary. If not, apply differencing. Where appropriate, use the same differencing for all variables to preserve interpretability.
- Fit regression model with automatically selected ARIMA errors.
- Check that $\varepsilon_t$ series looks like white noise.

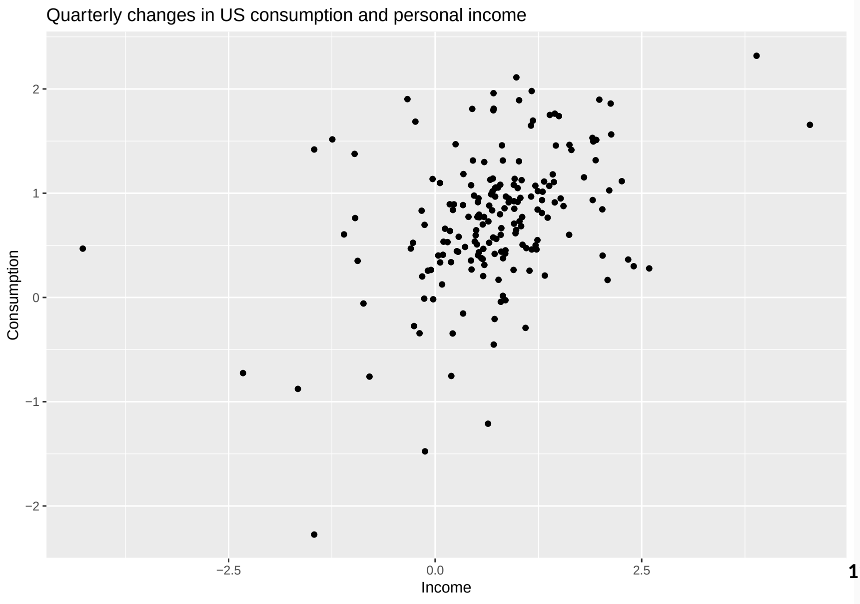
### Selecting predictors

- AICc can be calculated for final model.
- Repeat procedure for all subsets of predictors to be considered, and select model with lowest AIC value.

# US personal consumption and income



# US personal consumption and income



# US personal consumption and income

- No need for transformations or further differencing.
- Increase in income does not necessarily translate into instant increase in consumption (e.g., after the loss of a job, it may take a few months for expenses to be reduced to allow for the new circumstances). We will ignore this for now.

# US personal consumption and income

```r
(fit <- auto.arima(uschange[,1], xreg=uschange[,2]))
```

```
## Series: uschange[, 1]
## Regression with ARIMA(1,0,2) errors
##
## Coefficients:
##          ar1      ma1     ma2  intercept    xreg
##       0.6922  -0.5758  0.1984     0.5990  0.2028
## s.e.  0.1159   0.1301  0.0756     0.0884  0.0461
##
## sigma^2 estimated as 0.3219:  log likelihood=-156.95
## AIC=325.91   AICc=326.37   BIC=345.29
```

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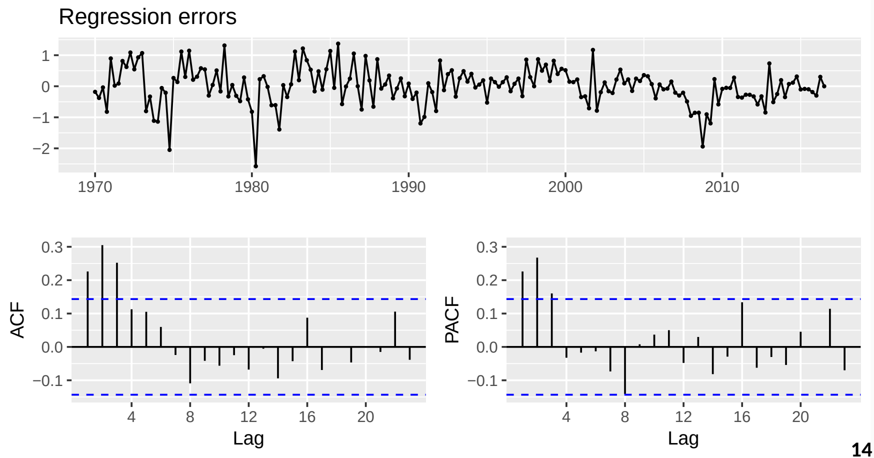
```

Write down the equations for the fitted model.

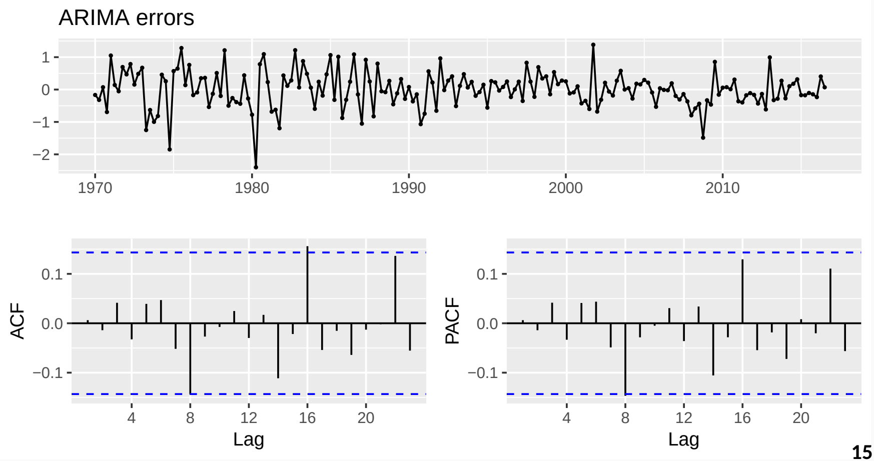
# US personal consumption and income

```r
ggtsdisplay(residuals(fit, type='regression'),
  main="Regression errors")
```



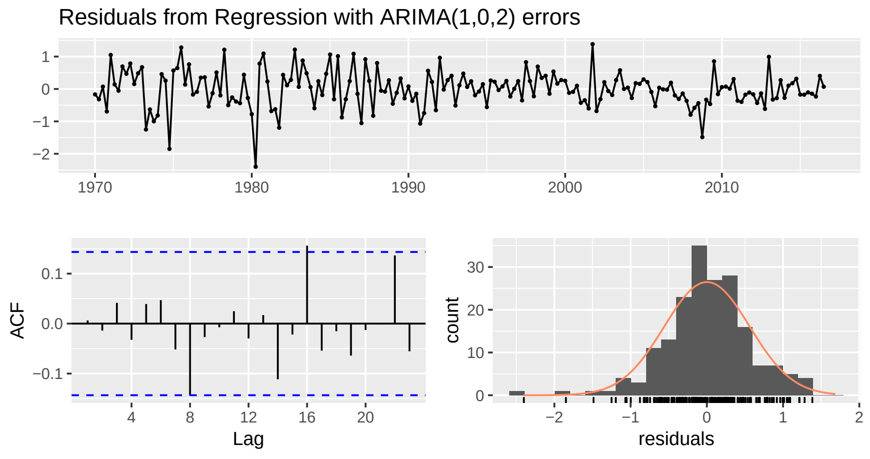
# US personal consumption and income

```r
ggtsdisplay(residuals(fit, type='response'),
  main="ARIMA errors")
```



# US personal consumption and income

```
checkresiduals(fit, test=FALSE)
```



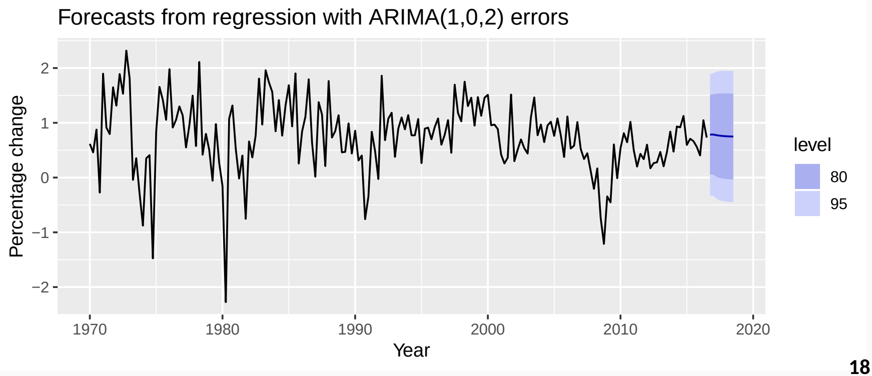
# US personal consumption and income

```r
checkresiduals(fit, plot=FALSE)
```

```
##
##  Ljung-Box test
##
## data:  Residuals from Regression with ARIMA(1,0,2) errors
## Q* = 5.8916, df = 3, p-value = 0.117
##
## Model df: 5.   Total lags used: 8
```

# US personal consumption and income

```
fcast <- forecast(fit,
  xreg=rep(mean(uschange[,2]),8), h=8)
autoplot(fcast) + xlab("Year") +
  ylab("Percentage change") +
  ggtitle("Forecasts from regression with ARIMA(1,0,2) errors")
```



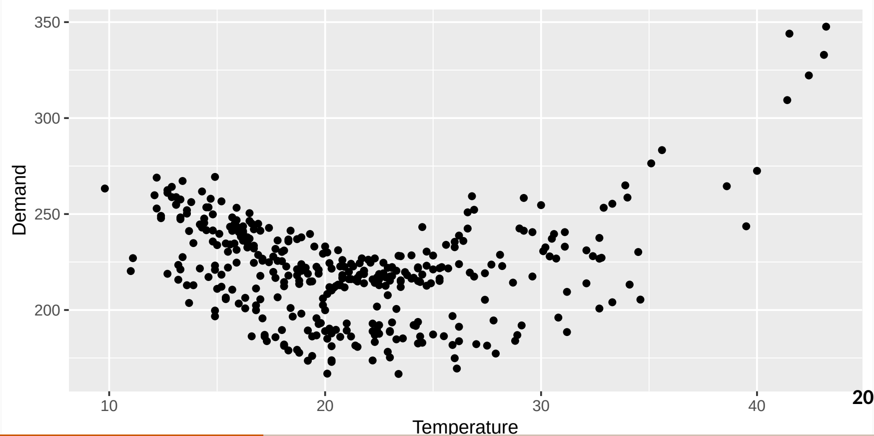
# Forecasting

- To forecast a regression model with ARIMA errors, we need to forecast the regression part of the model and the ARIMA part of the model and combine the results.
- Some predictors are known into the future (e.g., time, dummies).
- Separate forecasting models may be needed for other predictors.
- Forecast intervals ignore the uncertainty in forecasting the predictors.

# Daily electricity demand

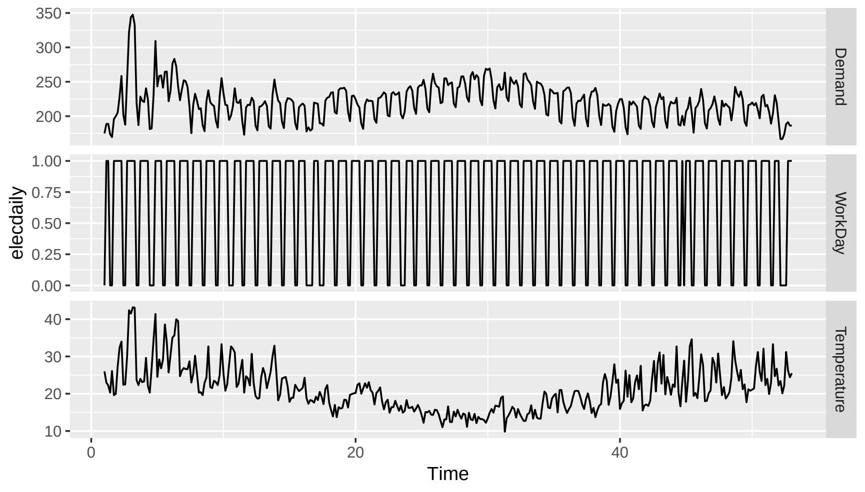
Model daily electricity demand as a function of temperature using quadratic regression with ARMA errors.

```r
qplot(elecdaily[,"Temperature"], elecdaily[,"Demand"]) +
  xlab("Temperature") + ylab("Demand")
```



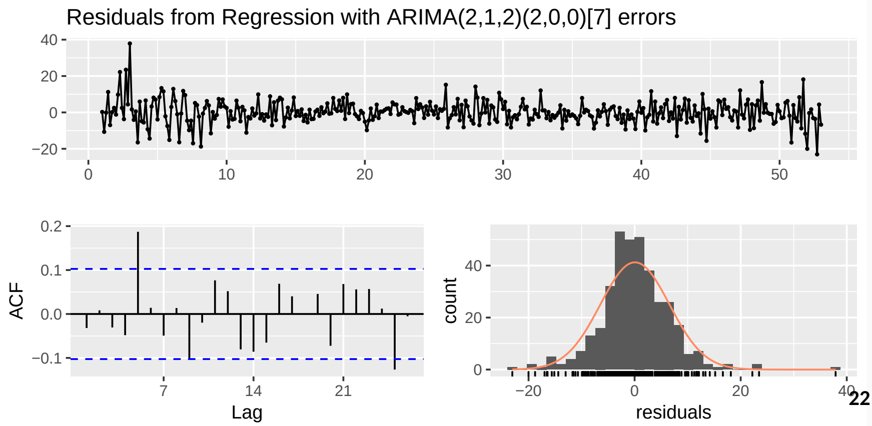
# Daily electricity demand

```r
autoplot(elecdaily, facets = TRUE)
```



# Daily electricity demand

```
xreg <- cbind(MaxTemp = elecdaily[, "Temperature"],
              MaxTempSq = elecdaily[, "Temperature"]^2,
              Workday = elecdaily[, "WorkDay"])
fit <- auto.arima(elecdaily[, "Demand"], xreg = xreg)
checkresiduals(fit)
```



# Daily electricity demand

```
##
##  Ljung-Box test
##
## data:  Residuals from Regression with ARIMA(2,1,2)(2,0,0)[7] errors
## Q* = 28.229, df = 4, p-value = 1.121e-05
##
## Model df: 10.   Total lags used: 14
```

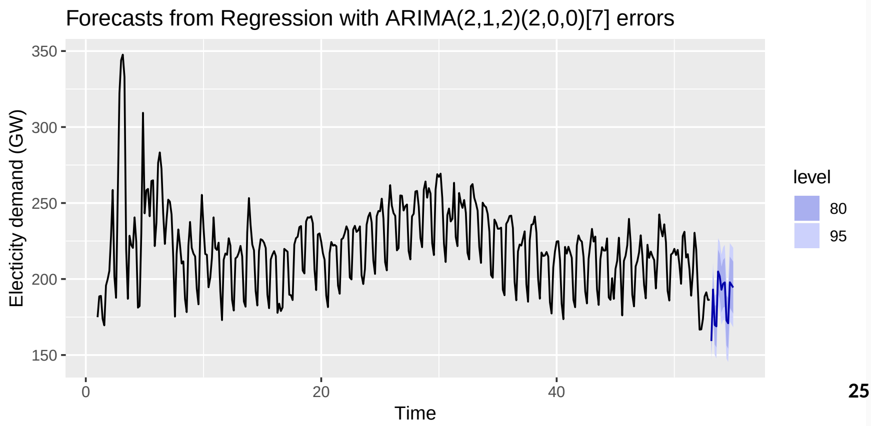
# Daily electricity demand

```r
# Forecast one day ahead
forecast(fit, xreg = cbind(26, 26^2, 1))
```

```
##          Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 53.14286        189.769 181.2954 198.2427 176.8096 202.7284
```

# Daily electricity demand

```r
fcast <- forecast(fit,
  xreg = cbind(rep(26,14), rep(26^2,14),
    c(0,1,0,0,1,1,1,1,1,0,0,1,1,1)))
autoplot(fcast) + ylab("Electicity demand (GW)")
```



# Outline

# Stochastic & deterministic trends

## Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where $\eta_t$ is ARMA process.

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where $\eta_t$ is ARIMA process with $d \geq 1$.

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## Stochastic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

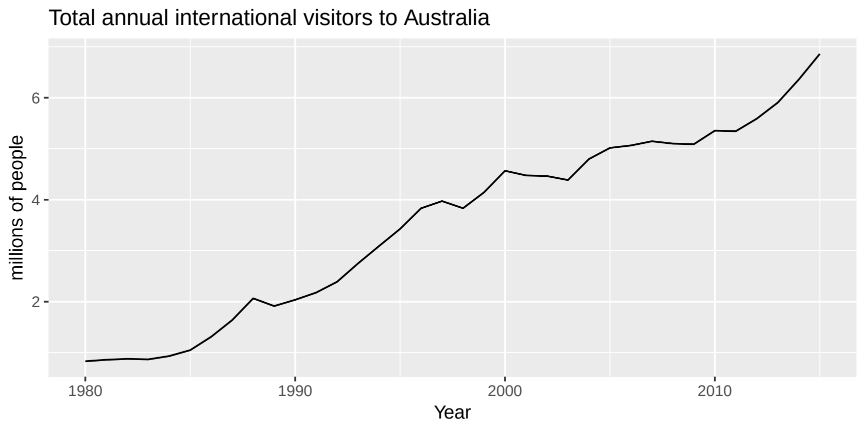
where $\eta_t$ is ARIMA process with $d \geq 1$.

Difference both sides until $\eta_t$ is stationary:

$$y'_t = \beta_1 + \eta'_t$$

where $\eta'_t$ is ARMA process.

# International visitors



# International visitors

**Deterministic trend**

```
trend <- seq_along(austa)
(fit1 <- auto.arima(austa, d=0, xreg=trend))
```

```
## Series: austa
## Regression with ARIMA(2,0,0) errors
##
## Coefficients:
##          ar1      ar2  intercept    xreg
##       1.1127  -0.3805     0.4156  0.1710
## s.e.  0.1600   0.1585     0.1897  0.0088
##
## sigma^2 estimated as 0.02979:  log likelihood=13.6
## AIC=-17.2   AICc=-15.2   BIC=-9.28
```

# International visitors

### Deterministic trend

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trend <- seq_along(austa)
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##
## sigma^2 estimated as 0.02979:  log likelihood=13.6
## AIC=-17.2   AICc=-15.2   BIC=-9.28
```

$$
\begin{aligned}
y_t &= 0.42 + 0.17t + \eta_t \\
\eta_t &= 1.11\eta_{t-1} - 0.38\eta_{t-2} + \varepsilon_t \\
\varepsilon_t &\sim \text{NID}(0, 0.0298).
\end{aligned}
$$

# International visitors

**Stochastic trend**

```
(fit2 <- auto.arima(austa,d=1))
```

```
## Series: austa
## ARIMA(0,1,1) with drift
##
## Coefficients:
##          ma1   drift
##       0.3006  0.1735
## s.e.  0.1647  0.0390
##
## sigma^2 estimated as 0.03376:  log likelihood=10.62
## AIC=-15.24   AICc=-14.46   BIC=-10.57
```

# International visitors

### Stochastic trend

```r
(fit2 <- auto.arima(austa,d=1))
```

```
## Series: austa
## ARIMA(0,1,1) with drift
##
## Coefficients:
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##       0.3006  0.1735
## s.e.  0.1647  0.0390
##
## sigma^2 estimated as 0.03376:  log likelihood=10.62
## AIC=-15.24   AICc=-14.46   BIC=-10.57
```
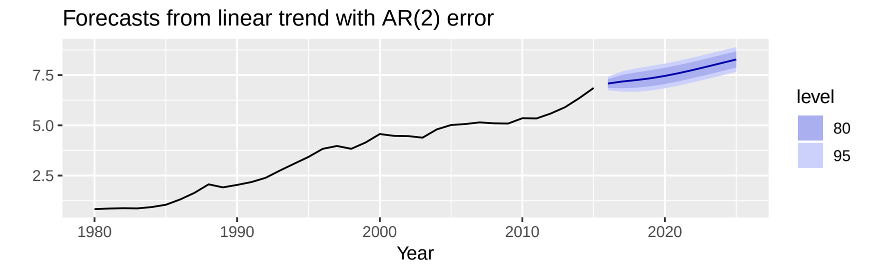
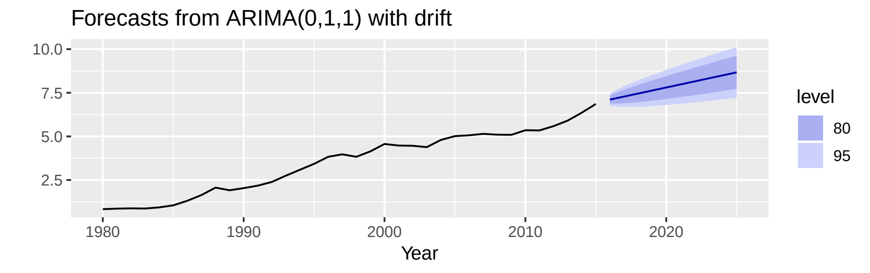

$$
\begin{aligned}
y_t - y_{t-1} &= 0.17 + \varepsilon_t \\
y_t &= y_0 + 0.17t + \eta_t \\
\eta_t &= \eta_{t-1} + 0.30\varepsilon_{t-1} + \varepsilon_t \\
\varepsilon_t &\sim \text{NID}(0, 0.0338).
\end{aligned}
$$

# International visitors



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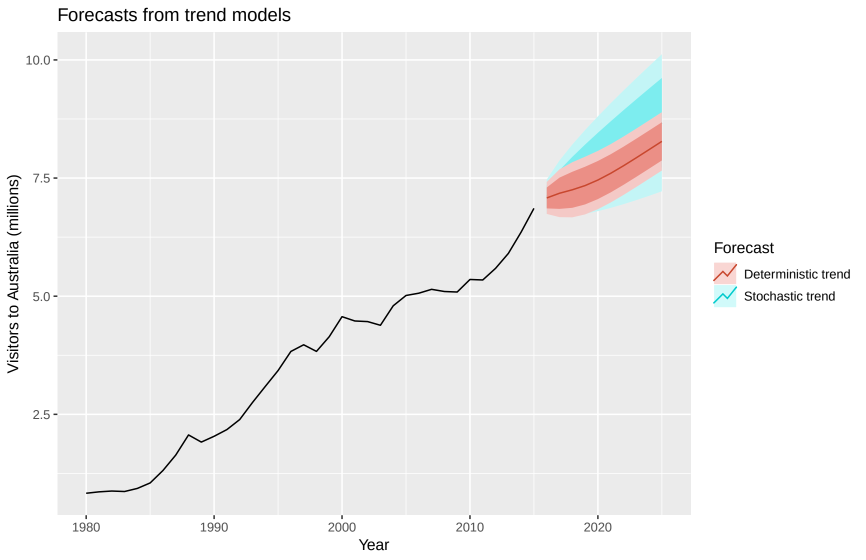


# Forecasting with trend

- Point forecasts are almost identical, but prediction intervals differ.
- Stochastic trends have much wider prediction intervals because the errors are non-stationary.
- Be careful of forecasting with deterministic trends too far ahead.

# Outline

# Dynamic harmonic regression

## Combine Fourier terms with ARIMA errors

### Advantages

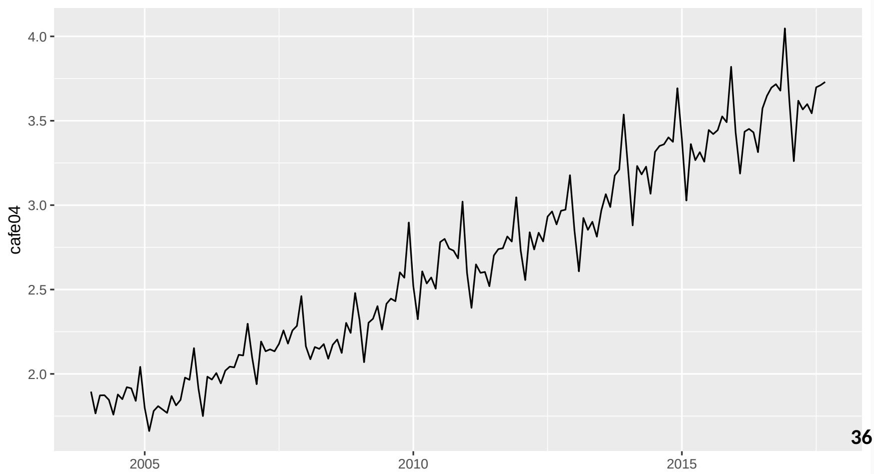
- it allows any length seasonality;
- for data with more than one seasonal period, you can include Fourier terms of different frequencies;
- the seasonal pattern is smooth for small values of $K$ (but more wiggly seasonality can be handled by increasing $K$);
- the short-term dynamics are easily handled with a simple ARMA error.

### Disadvantages

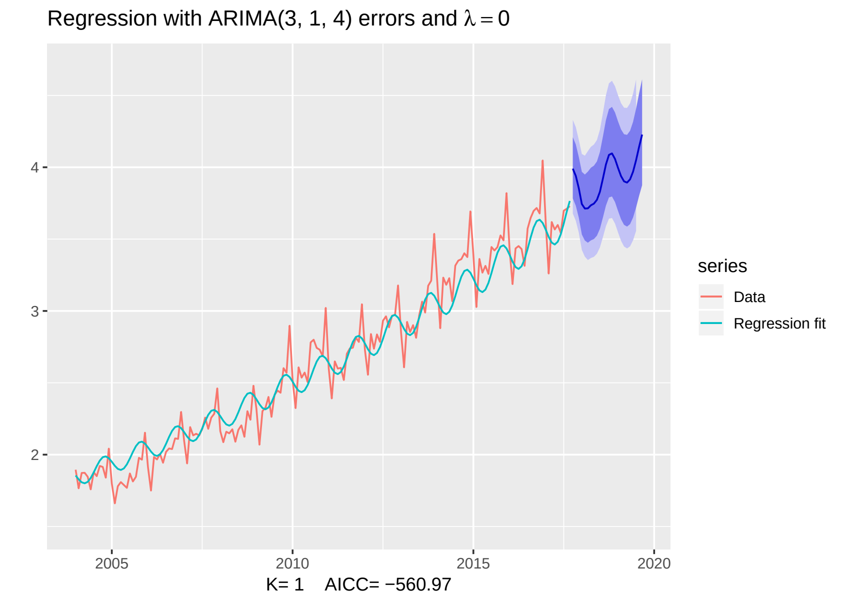
- seasonality is assumed to be fixed

# Eating-out expenditure

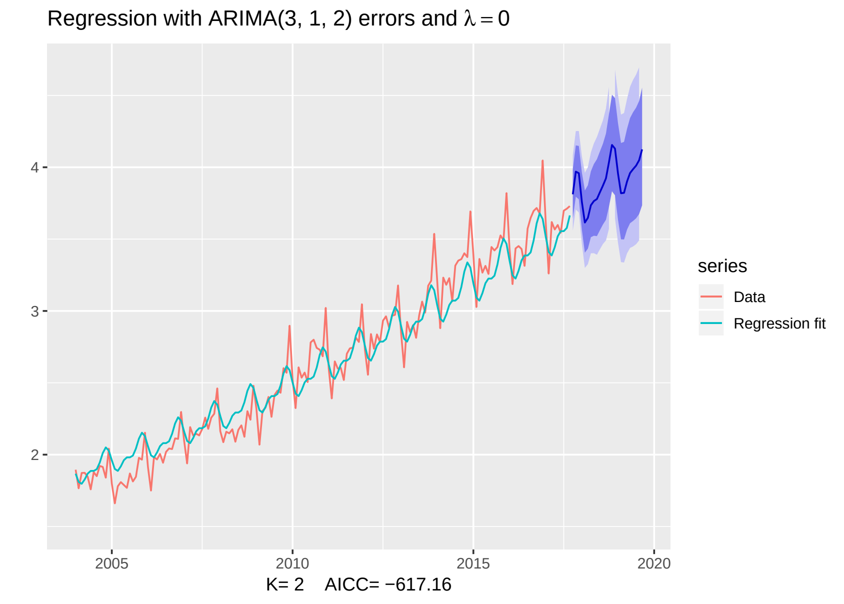
```r
cafe04 <- window(auscafe, start=2004)
autoplot(cafe04)
```



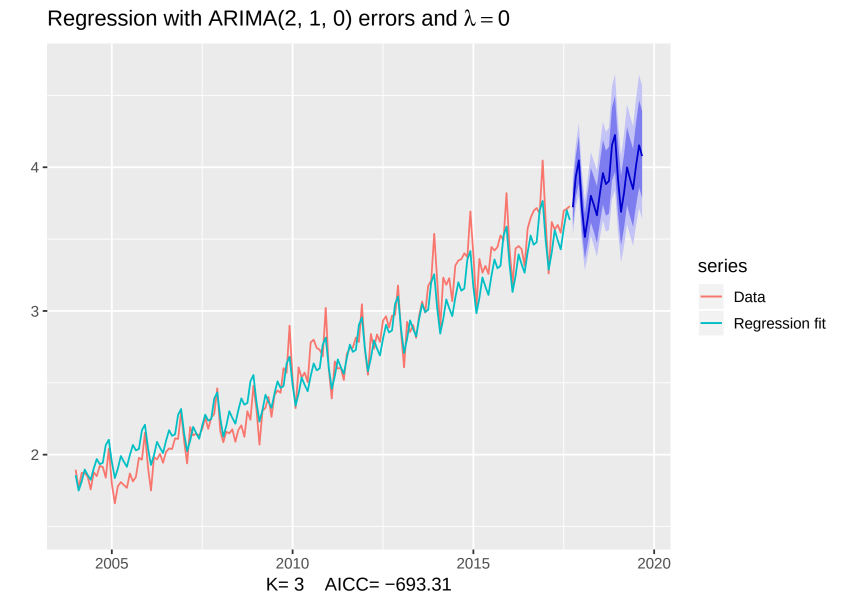
# Eating-out expenditure



# Eating-out expenditure



# Eating-out expenditure



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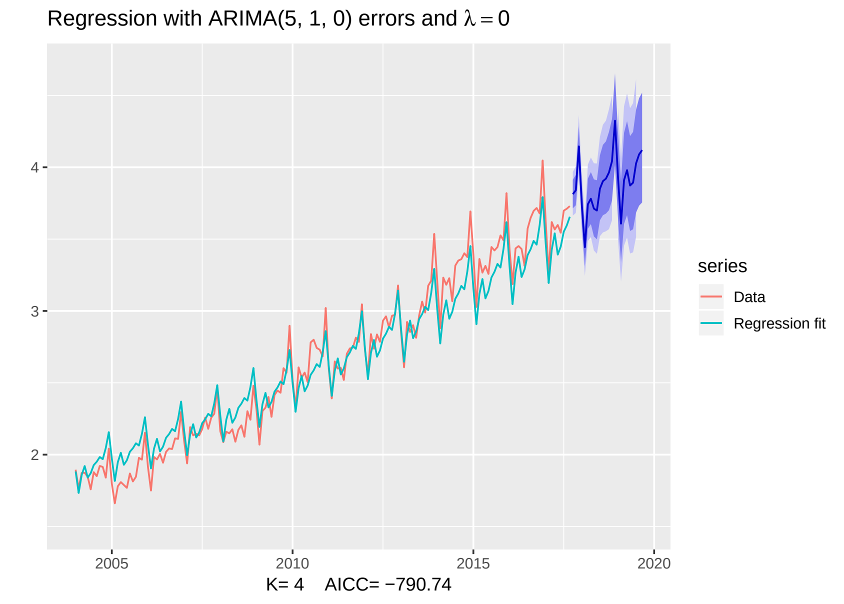

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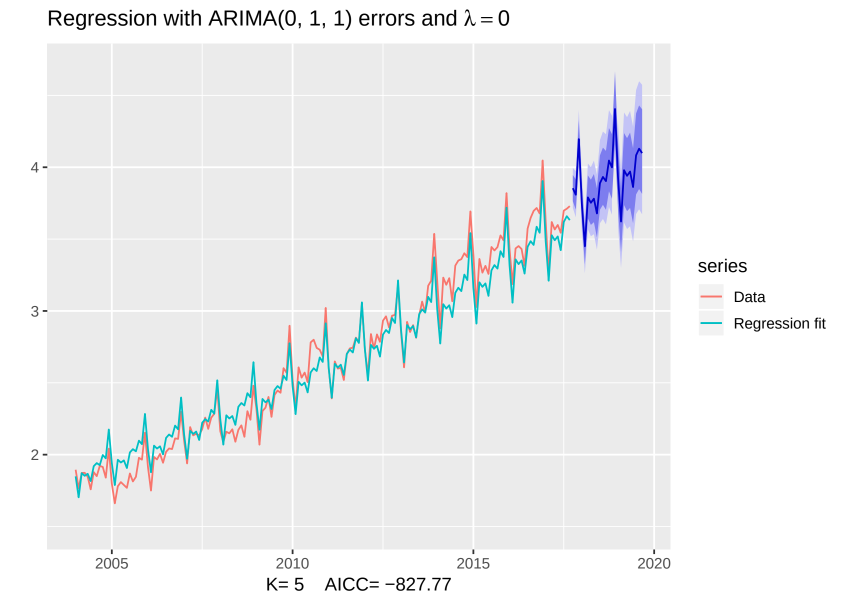

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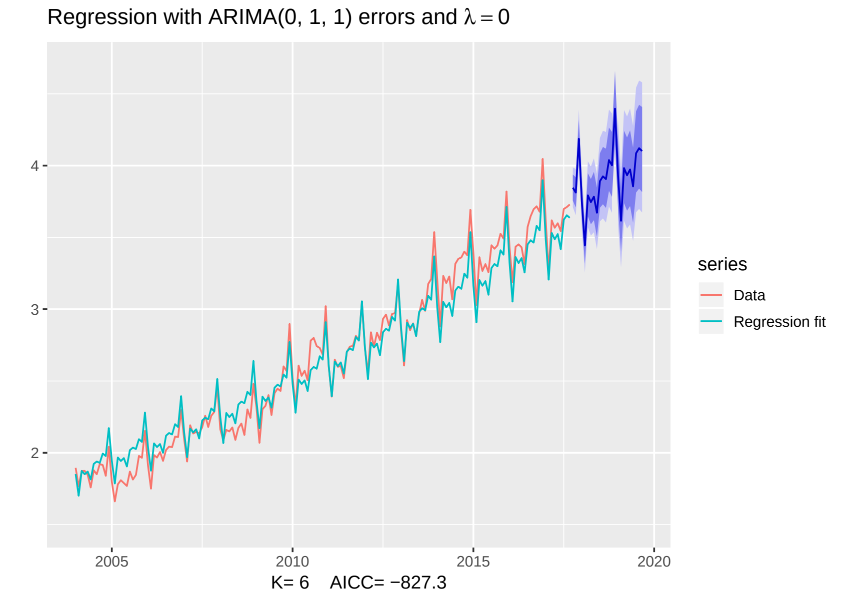


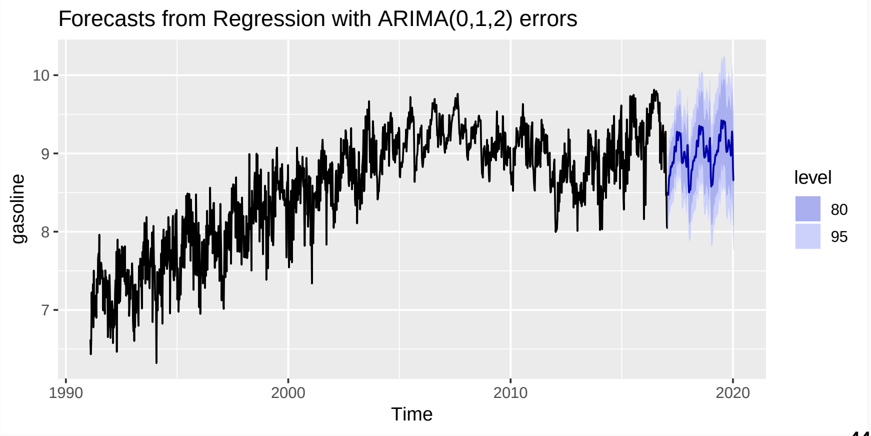
# Example: weekly gasoline products

```r
harmonics <- fourier(gasoline, K = 13)
(fit <- auto.arima(gasoline, xreg = harmonics, seasonal = FALSE))
```

```
## Series: gasoline
## Regression with ARIMA(0,1,2) errors
##
## Coefficients:
##           ma1     ma2   drift   S1-52    C1-52    S2-52    C2-52   S3-52
##       -0.9612  0.0936  0.0014  0.0315  -0.2555  -0.0522  -0.0175  0.0242
## s.e.   0.0275  0.0286  0.0008  0.0124   0.0124   0.0090   0.0089  0.0082
##         C3-52   S4-52    C4-52    S5-52    C5-52   S6-52    C6-52   S7-52
##       -0.0989  0.0321  -0.0257  -0.0011  -0.0472  0.0580  -0.0320  0.0283
## s.e.   0.0082  0.0079   0.0079   0.0078   0.0078  0.0078   0.0078  0.0079
##        C7-52   S8-52   C8-52    S9-52   C9-52   S10-52  C10-52  S11-52
##       0.0368  0.0238  0.0139  -0.0172  0.0119  -0.0236  0.0230  0.0001
## s.e.  0.0079  0.0079  0.0079   0.0080  0.0080   0.0081  0.0081  0.0082
##        C11-52   S12-52   C12-52  S13-52   C13-52
##       -0.0191  -0.0288  -0.0177  0.0012  -0.0176
## s.e.   0.0082   0.0083   0.0083  0.0084   0.0084
##
## sigma^2 estimated as 0.05603:  log likelihood=43.66
## AIC=-27.33   AICc=-25.92   BIC=129
```

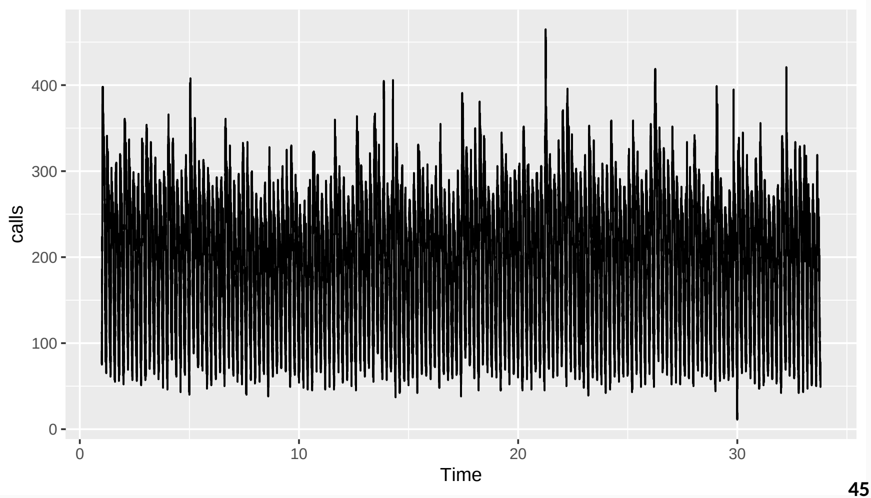
# Example: weekly gasoline products

```r
newharmonics <- fourier(gasoline, K = 13, h = 156)
fc <- forecast(fit, xreg = newharmonics)
autoplot(fc)
```



# 5-minute call centre volume

```r
autoplot(calls)
```



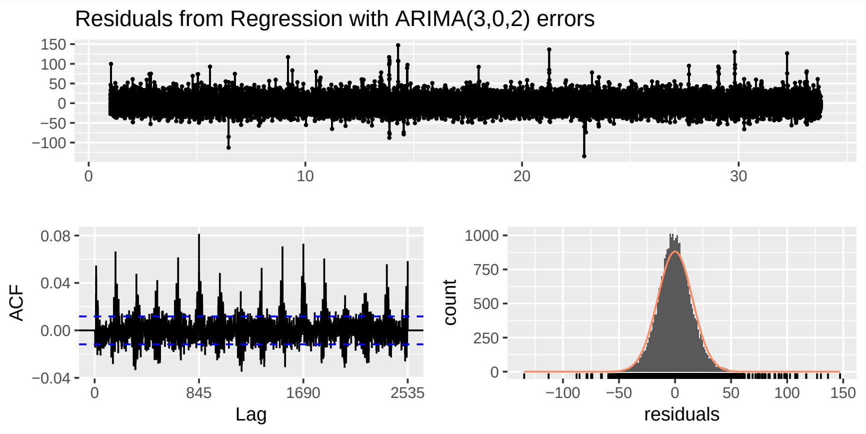
# 5-minute call centre volume

```r
xreg <- fourier(calls, K = c(10,0))
(fit <- auto.arima(calls, xreg=xreg, seasonal=FALSE, stationary=TRUE))
```

```
## Series: calls
## Regression with ARIMA(3,0,2) errors
##
## Coefficients:
##           ar1     ar2      ar3      ma1      ma2  intercept   S1-169
##        0.8406  0.1919  -0.0442  -0.5896  -0.1891   192.0697  55.2447
## s.e.   0.1692  0.1782   0.0129   0.1693   0.1369     1.7638   0.7013
##           C1-169   S2-169    C2-169    S3-169   C3-169   S4-169   C4-169
##         -79.0871  13.6738  -32.3747  -13.6934  -9.3270  -9.5318  -2.7972
## s.e.      0.7007   0.3788    0.3787    0.2727   0.2726   0.2230   0.2230
##          S5-169  C5-169  S6-169  C6-169  S7-169  C7-169  S8-169   C8-169
##         -2.2393  2.8934  0.1730  3.3052  0.8552  0.2935  0.8575  -1.3913
## s.e.     0.1956  0.1956  0.1788  0.1788  0.1678  0.1678  0.1602   0.1601
##          S9-169   C9-169  S10-169  C10-169
##         -0.9864  -0.3448  -1.1964   0.8010
## s.e.     0.1546   0.1546   0.1504   0.1504
##
## sigma^2 estimated as 242.5:  log likelihood=-115411.5
## AIC=230877   AICc=230877.1   BIC=231099.3
```

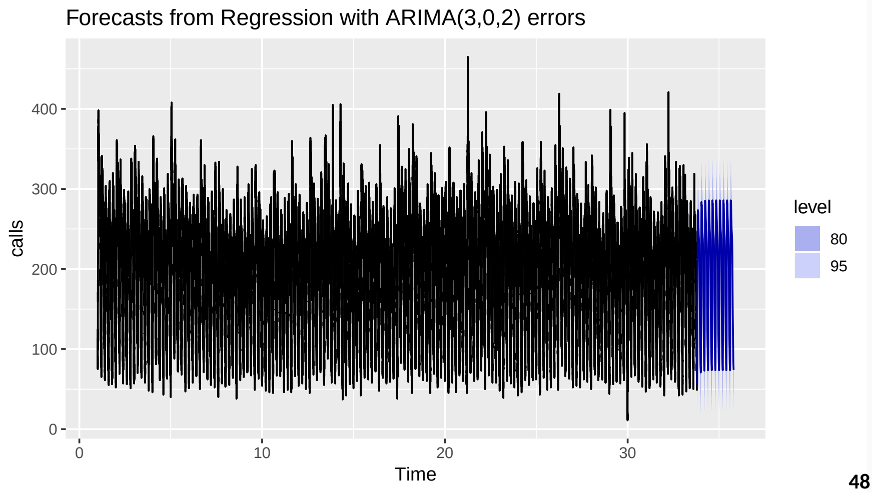
# 5-minute call centre volume

```
checkresiduals(fit, test=FALSE)
```



# 5-minute call centre volume

```r
fc <- forecast(fit, xreg = fourier(calls, c(10,0), 1690))
autoplot(fc)
```



# Outline

# Lagged predictors

**Sometimes a change in $x_t$ does not affect $y_t$ instantaneously**

# Lagged predictors

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- $y_t$ = sales, $x_t$ = advertising.
- $y_t$ = stream flow, $x_t$ = rainfall.
- $y_t$ = size of herd, $x_t$ = breeding stock.

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- $y_t$ = stream flow, $x_t$ = rainfall.
- $y_t$ = size of herd, $x_t$ = breeding stock.

- These are dynamic systems with input ($x_t$) and output ($y_t$).
- $x_t$ is often a leading indicator.
- There can be multiple predictors.

# Lagged predictors

The model include present and past values of predictor: $x_t, x_{t-1}, x_{t-2}, \dots$.

$$y_t = a + \nu_0 x_t + \nu_1 x_{t-1} + \cdots + \nu_k x_{t-k} + \eta_t$$

where $\eta_t$ is an ARIMA process.

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where $\eta_t$ is an ARIMA process.

**Rewrite model as**

$$\begin{aligned} y_t &= a + (\nu_0 + \nu_1 B + \nu_2 B^2 + \cdots + \nu_k B^k) x_t + \eta_t \\ &= a + \nu(B) x_t + \eta_t. \end{aligned}$$

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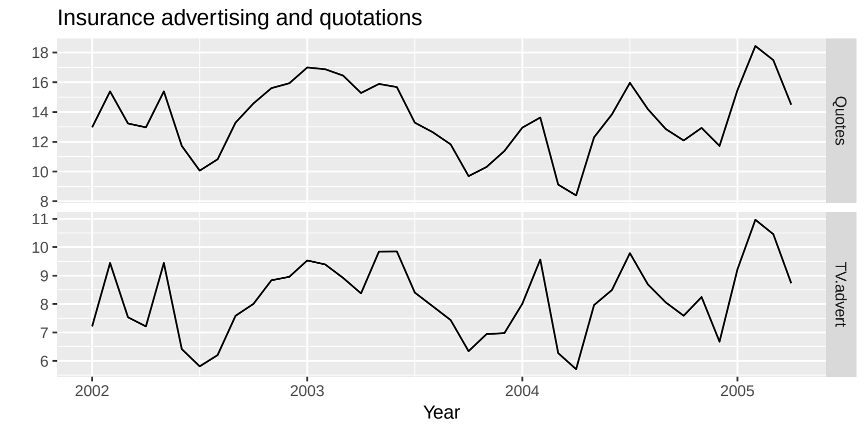
**Rewrite model as**

$$\begin{aligned} y_t &= a + (\nu_0 + \nu_1 B + \nu_2 B^2 + \cdots + \nu_k B^k) x_t + \eta_t \\ &= a + \nu(B) x_t + \eta_t. \end{aligned}$$

- $\nu(B)$ is called a *transfer function* since it describes how change in $x_t$ is transferred to $y_t$.
- $x$ can influence $y$, but $y$ is not allowed to influence $x$.

# Example: Insurance quotes and TV adverts



# Example: Insurance quotes and TV adverts

```r
Advert <- cbind(
    AdLag0 = insurance[,"TV.advert"],
    AdLag1 = lag(insurance[,"TV.advert"],-1),
    AdLag2 = lag(insurance[,"TV.advert"],-2),
    AdLag3 = lag(insurance[,"TV.advert"],-3)) %>%
  head(NROW(insurance))

# Restrict data so models use same fitting period
fit1 <- auto.arima(insurance[4:40,1], xreg=Advert[4:40,1],
  stationary=TRUE)
fit2 <- auto.arima(insurance[4:40,1], xreg=Advert[4:40,1:2],
  stationary=TRUE)
fit3 <- auto.arima(insurance[4:40,1], xreg=Advert[4:40,1:3],
  stationary=TRUE)
fit4 <- auto.arima(insurance[4:40,1], xreg=Advert[4:40,1:4],
  stationary=TRUE)
c(fit1$aicc,fit2$aicc,fit3$aicc,fit4$aicc)
```

```
## [1] 68.49968 60.02357 62.83253 68.01684
```

# Example: Insurance quotes and TV adverts

```r
(fit <- auto.arima(insurance[,1], xreg=Advert[,1:2],
  stationary=TRUE))
```

```
## Series: insurance[, 1]
## Regression with ARIMA(3,0,0) errors
##
## Coefficients:
##          ar1      ar2     ar3  intercept  AdLag0  AdLag1
##       1.4117  -0.9317  0.3591     2.0393  1.2564  0.1625
## s.e.  0.1698   0.2545  0.1592     0.9931  0.0667  0.0591
##
## sigma^2 estimated as 0.2165:  log likelihood=-23.89
## AIC=61.78   AICc=65.28   BIC=73.6
```

# Example: Insurance quotes and TV adverts

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##
## sigma^2 estimated as 0.2165:  log likelihood=-23.89
## AIC=61.78   AICc=65.28   BIC=73.6
```

$$
\begin{aligned}
y_t &= 2.04 + 1.26x_t + 0.16x_{t-1} + \eta_t, \\
\eta_t &= 1.41\eta_{t-1} - 0.93\eta_{t-2} + 0.36\eta_{t-3} + \varepsilon_t,
\end{aligned}
$$

# Example: Insurance quotes and TV adverts

```r
fc <- forecast(fit, h=20,
  xreg=cbind(c(Advert[40,1],rep(10,19)), rep(10,20)))
autoplot(fc)
```

# Example: Insurance quotes and TV adverts

```r
fc <- forecast(fit, h=20,
  xreg=cbind(c(Advert[40,1],rep(8,19)), rep(8,20)))
autoplot(fc)
```

# Example: Insurance quotes and TV adverts

```
fc <- forecast(fit, h=20,
  xreg=cbind(c(Advert[40,1],rep(6,19)), rep(6,20)))
autoplot(fc)
```

# Transfer function models

$$y_t = a + \nu(B)x_t + \eta_t$$

where $\eta_t$ is an ARMA process. So

$$\phi(B)\eta_t = \theta(B)\varepsilon_t \qquad \text{or} \qquad \eta_t = \frac{\theta(B)}{\phi(B)}\varepsilon_t = \psi(B)\varepsilon_t.$$

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$$y_t = a + \nu(B)x_t + \psi(B)\varepsilon_t$$

- ARMA models are rational approximations to general transfer functions of $\varepsilon_t$.
- We can also replace $\nu(B)$ by a rational approximation.
- There is no R package for forecasting using a general transfer function approach.