Efficient Data Structures and Graph Width Parameters

Marek Sokołowski

25 February 2025

Featured works

T. Korhonen, K. Majewski, W. Nadara, Mi. Pilipczuk, M. Sokołowski [FOCS '23]
Dynamic Treewidth

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Almost-Linear Time Parameterized Algorithm for Rankwidth via Dynamic Rankwidth

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Graphs of Bounded Twin-Width Are Quasi-Polynomially χ-Bounded

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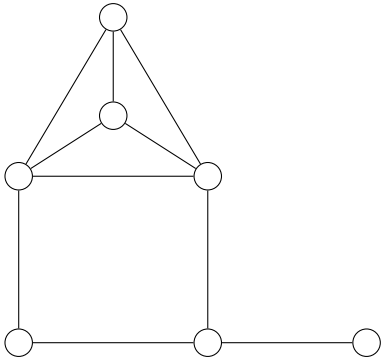

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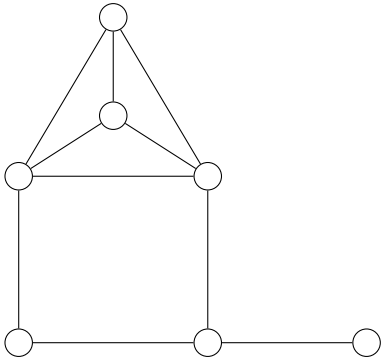

n vertices, m edges

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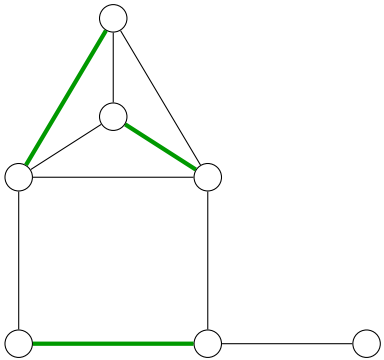

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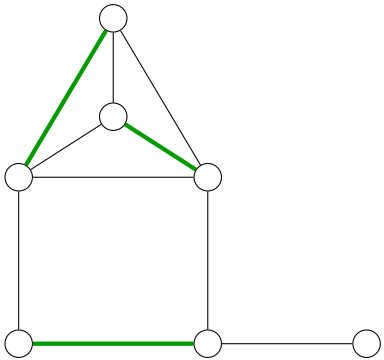
Maximum Matching

Graphs & Graph problems



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[Edmonds '61]

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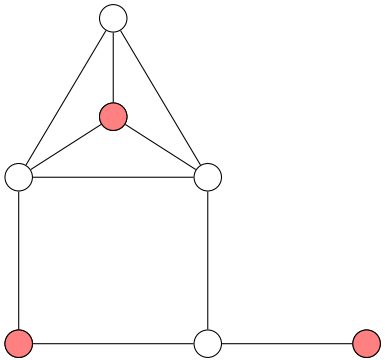

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Maximum Independent Set

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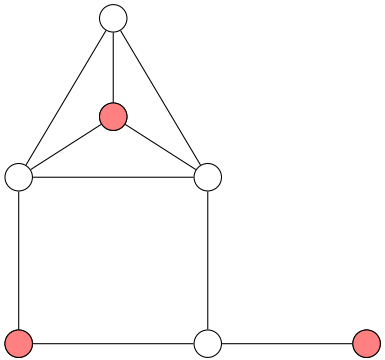

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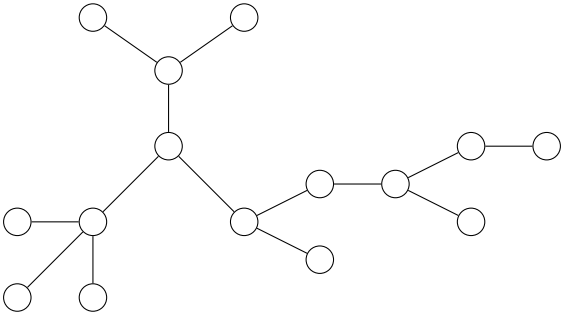
Maximum Independent Set

NP-hard!

[Cook '71, Karp '72, Levin '73]

Trees

Maximum Independent Set is NP-hard in general... But becomes easy on **trees**!



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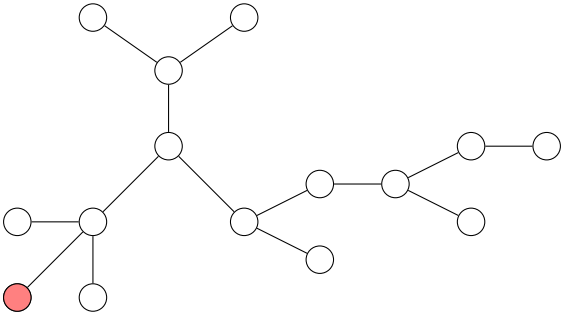

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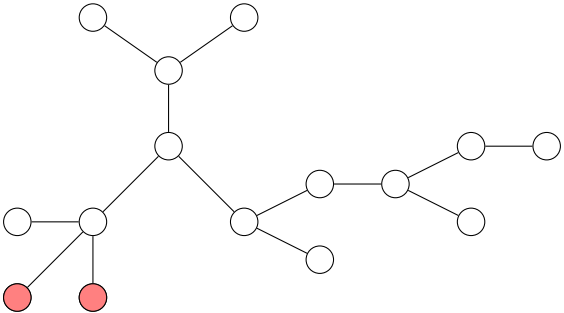

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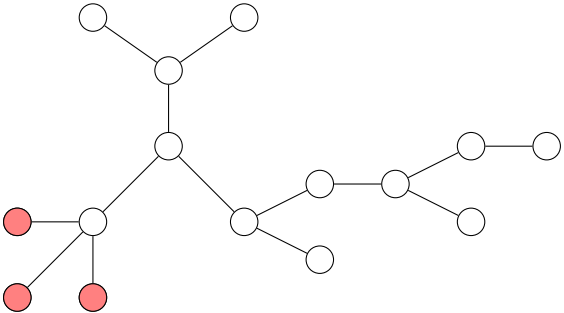

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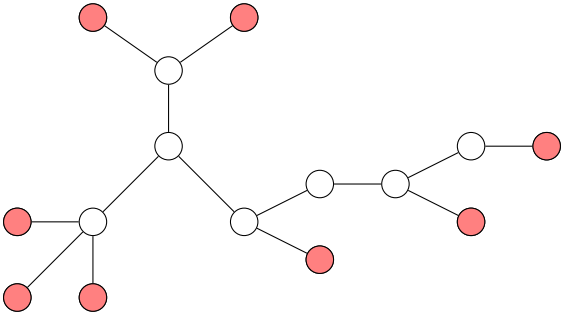

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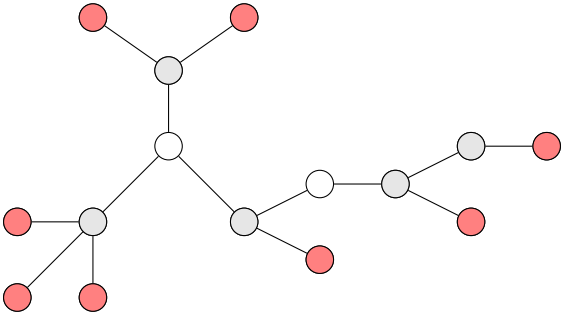

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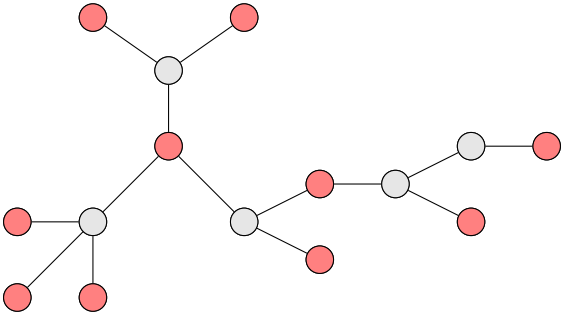

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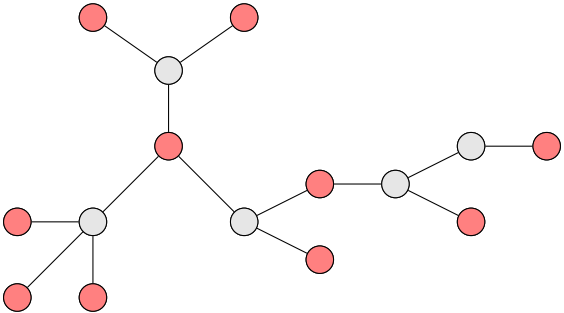
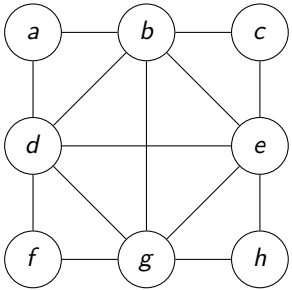


Question

Maybe some hard problems can be solved efficiently on **more general tree-like** graphs?

Treewidth



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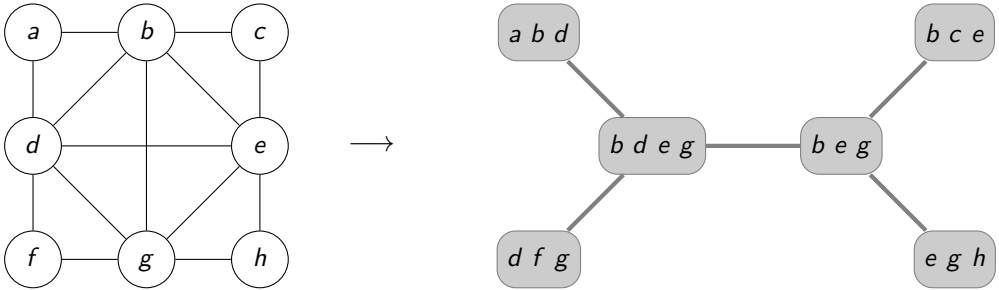
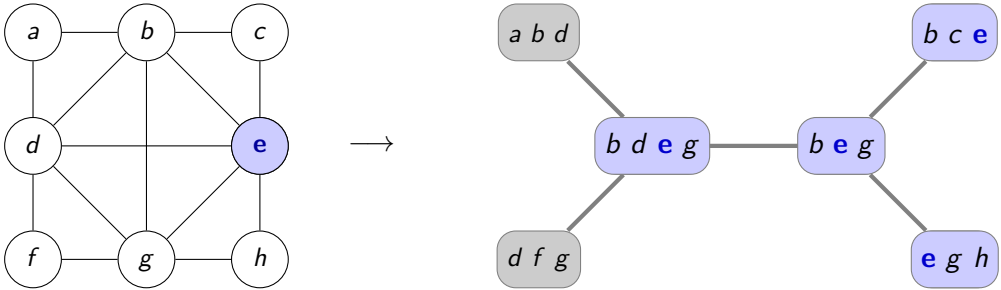


tree decomposition

Treewidth



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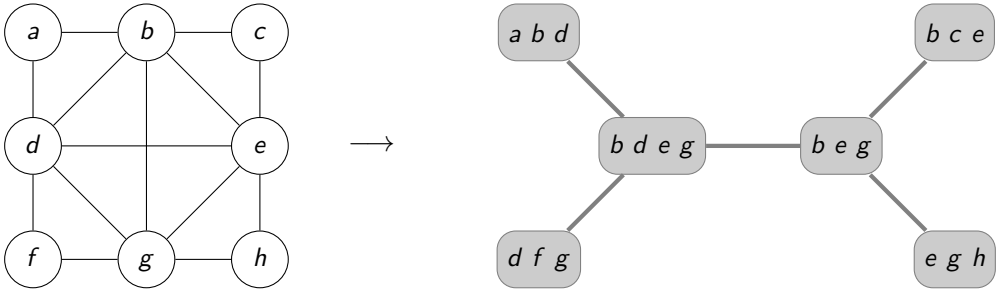

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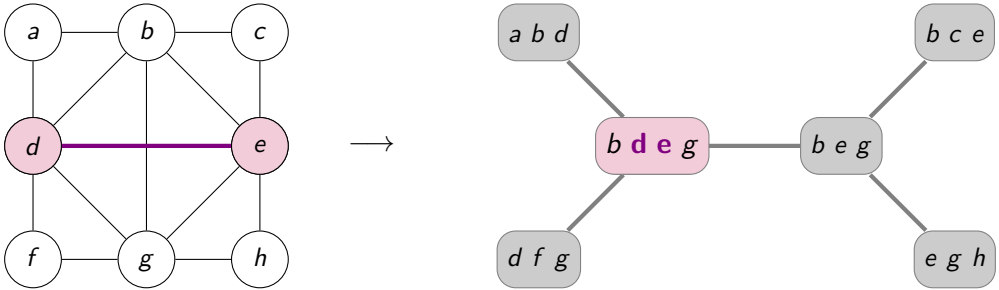

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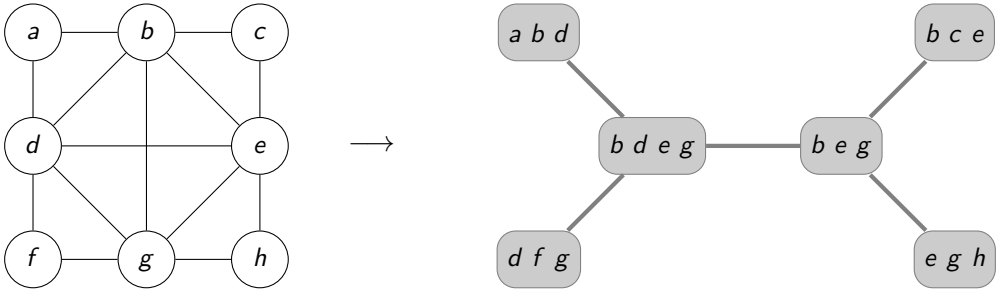

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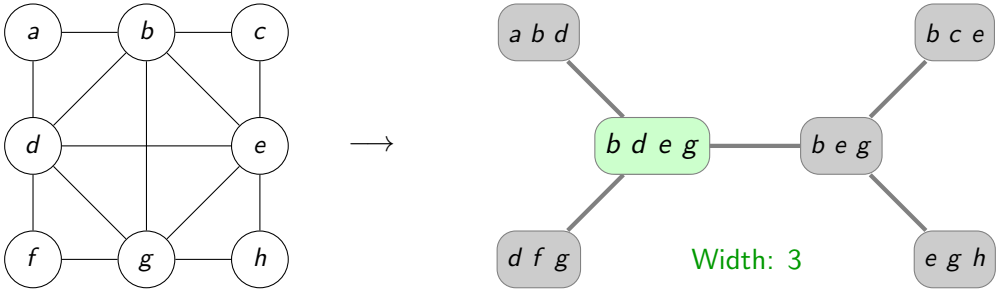

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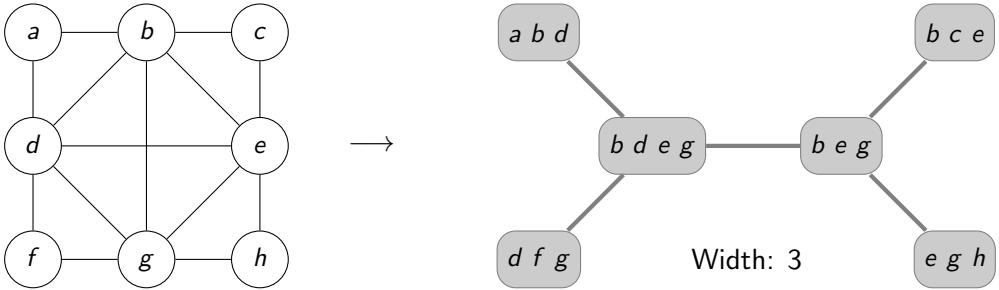

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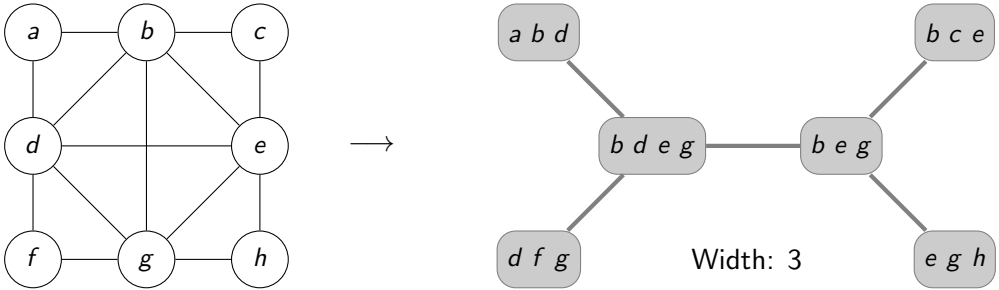

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- **Treewidth:** minimum possible width of a tree decomposition

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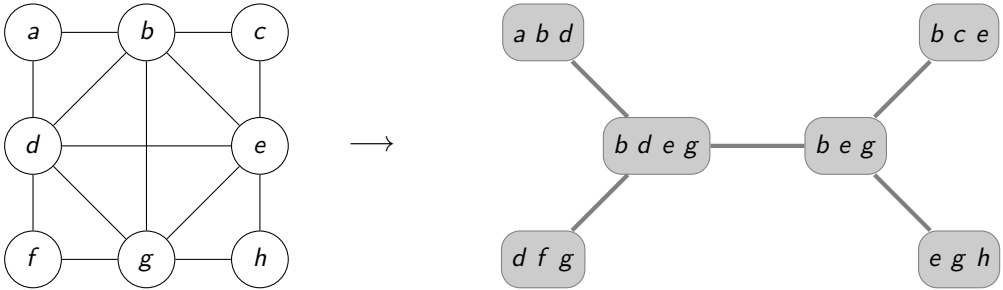

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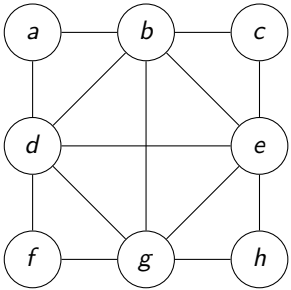

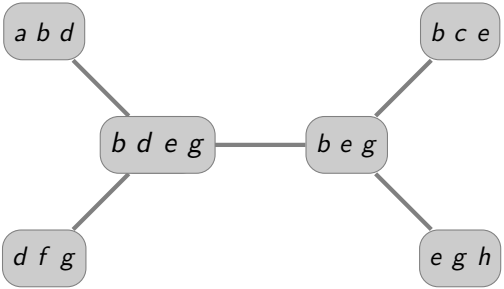

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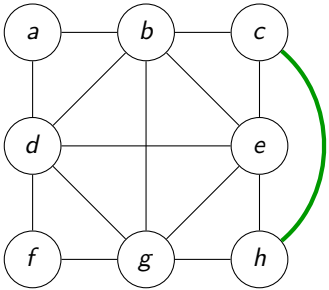

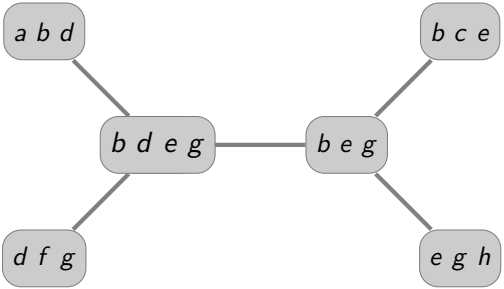

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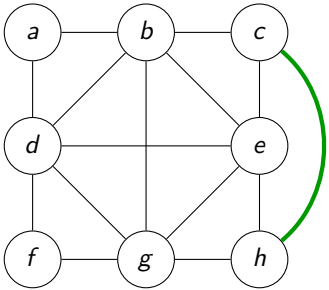

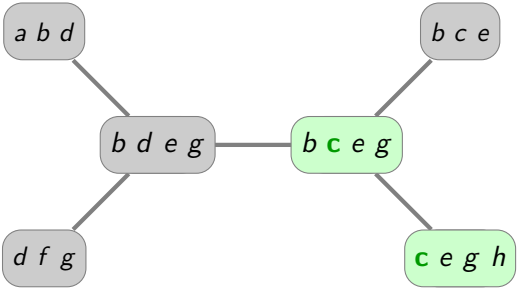

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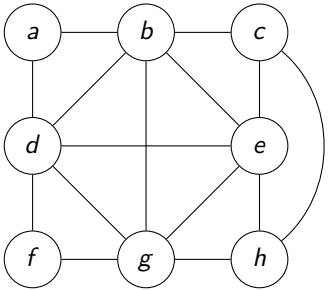

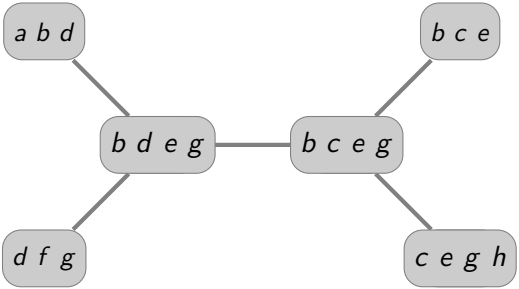

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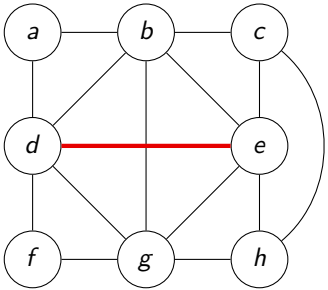

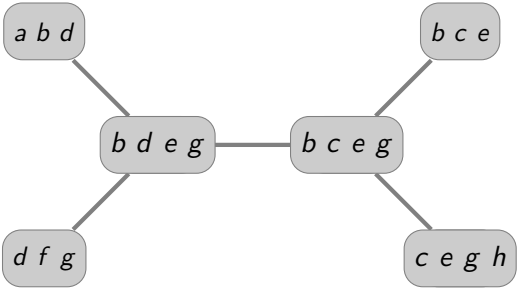

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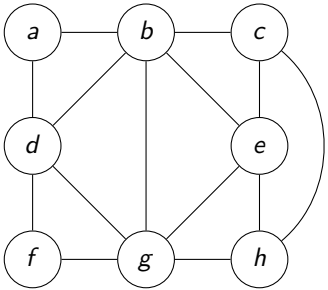

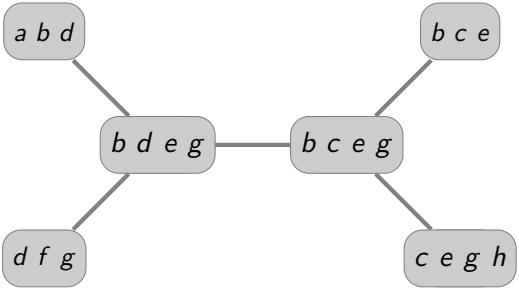

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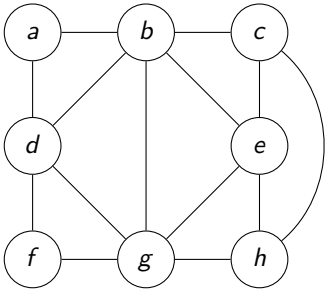

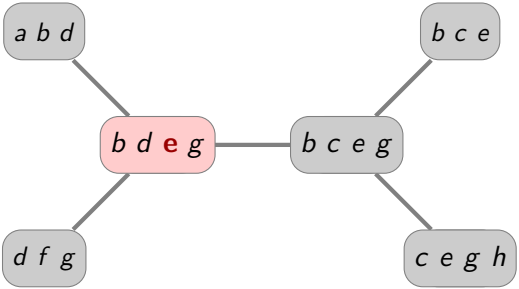

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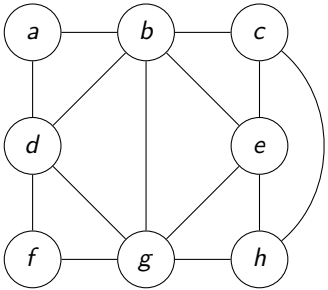

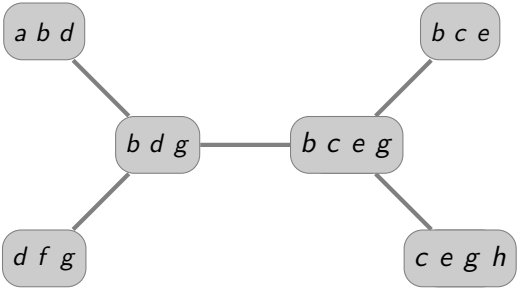

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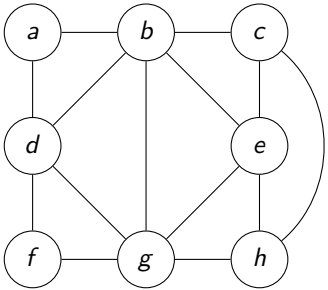

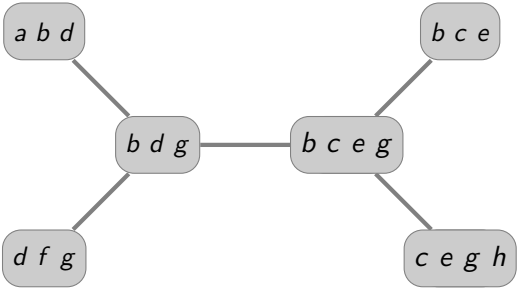

Problem

How to maintain tree decompositions of **dynamic graphs**?

Dynamic Treewidth

Korhonen, Majewski, Nadara, Pilipczuk, **Sokołowski** [FOCS '23]

Dynamic Treewidth

Main result

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In a **dynamic graph** G with n vertices of treewidth w ...

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We maintain: a tree decomposition of G of width at most $6w + 5$ ...

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$$\log^{1000} n \quad \ll \quad \mathbf{2}^{\sqrt{\log \mathbf{n} \log\log \mathbf{n}}} \quad \ll \quad n^{0.001}$$

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We can also dynamically solve any decision/optimization problem expressible in CMSO_2 logic.

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Extension

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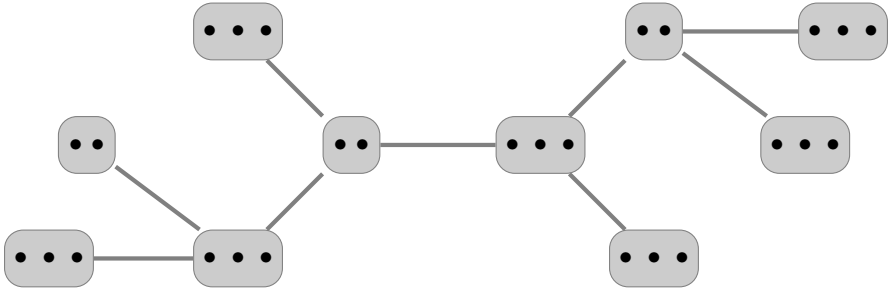
Max Matching, Max Independent Set, Longest Path, Hamiltonian Cycle...

Approach



- Maintain a **low-diameter** tree decomposition of the graph ($2^{\mathcal{O}(\sqrt{\log n \log\log n})}$)

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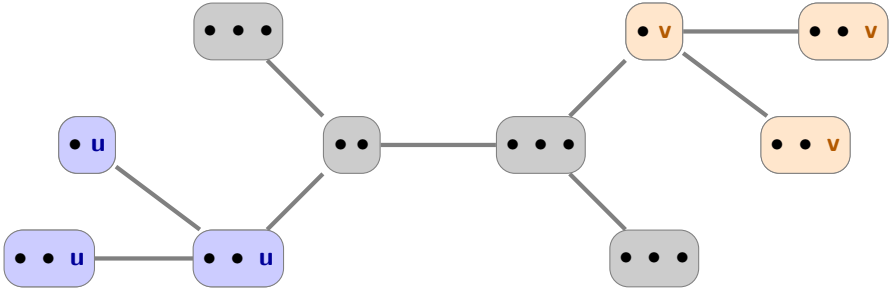

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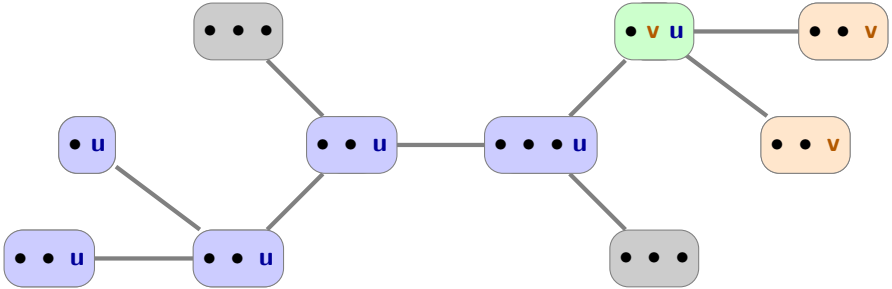
- Edge **uv** added?

Approach



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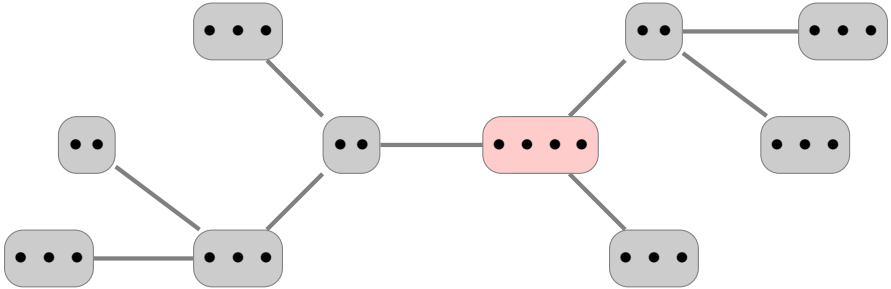

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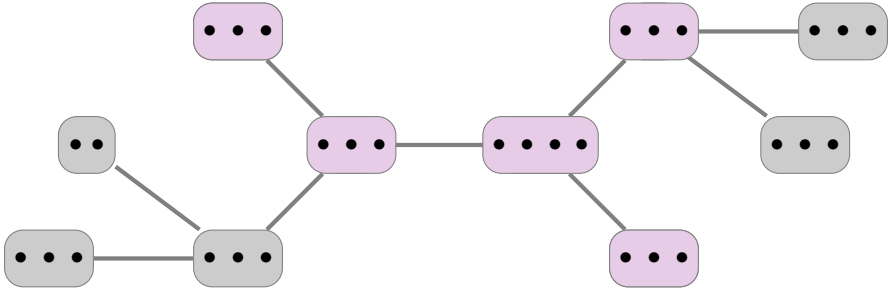

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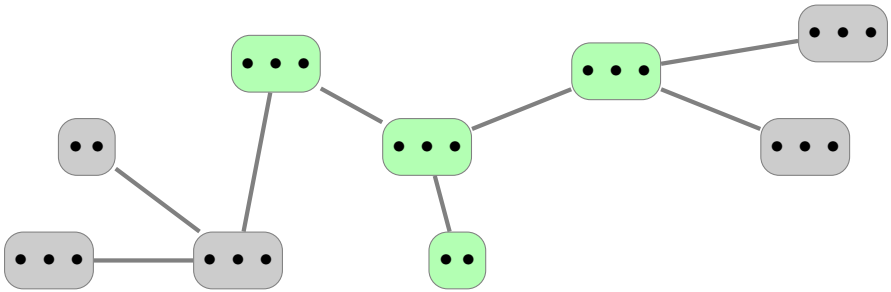

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Treewidth, but for denser graphs

Issue: treewidth applicable only to **sparse** graphs...

But there also exist **dense** tree-like graphs!

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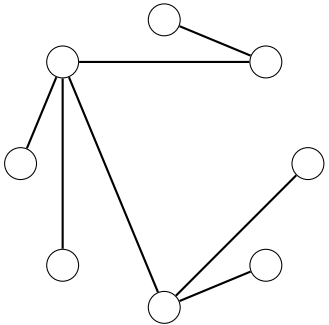

trees

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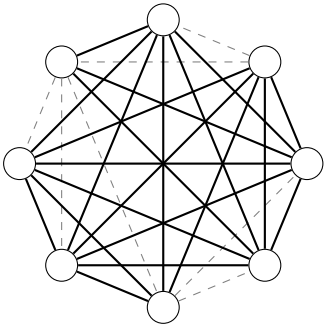

complements of trees

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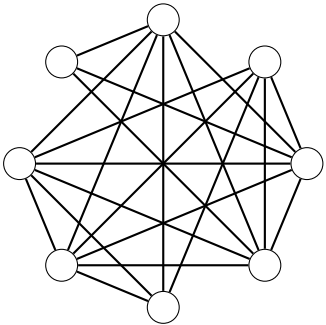

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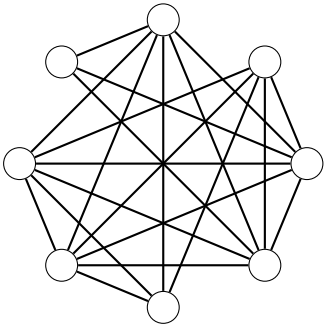

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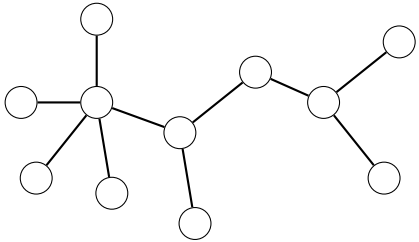

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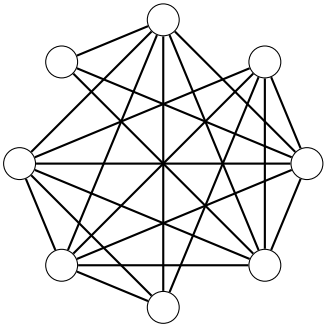

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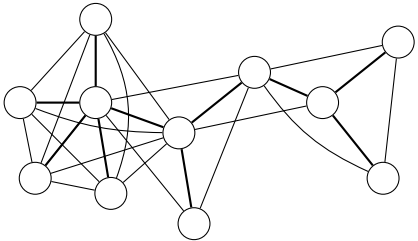

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Solution

Equivalent notions of **cliquewidth** [Courcelle et al. '93] and **rankwidth** [Oum, Seymour '06].

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Given: n-vertex graph G and its **rank decomposition** of width w
Then: Maximum Independent Set can be solved in time $2^{f(w)} \cdot n$

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Also Max Clique, Min Dominating Set, Longest Induced Path, ...

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Same problem: Need to compute a rank decomposition.

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Korhonen, **Sokołowski** [STOC '24]

Almost-Linear Time Parameterized Algorithm for Rankwidth via Dynamic Rankwidth

Main result

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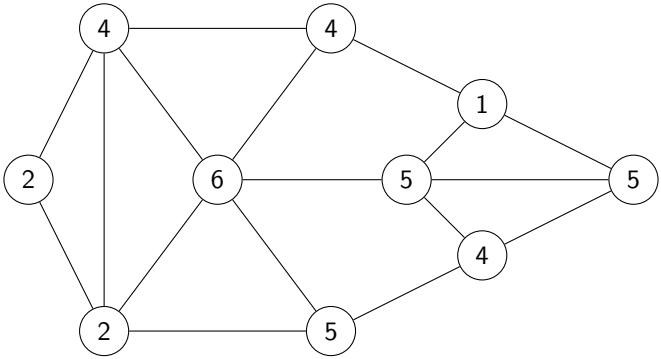
Dynamic Planar Maximum Weighted Independent Set



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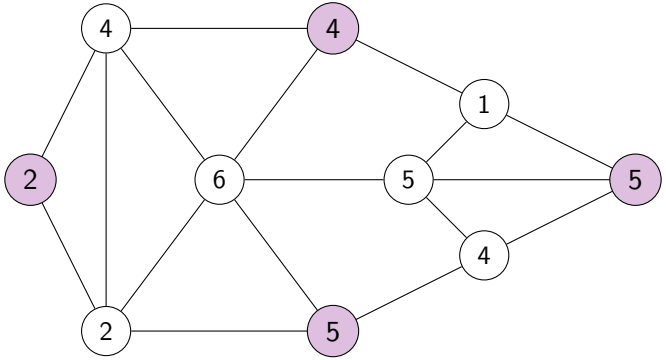

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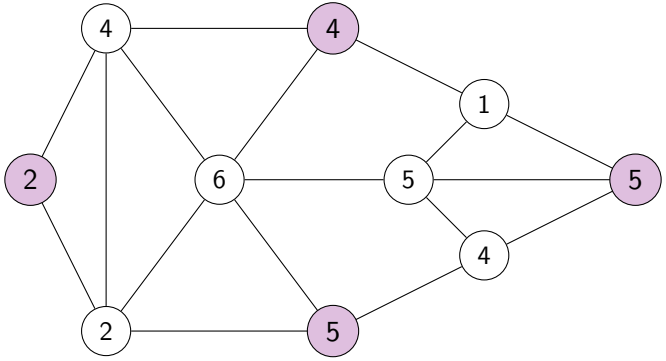

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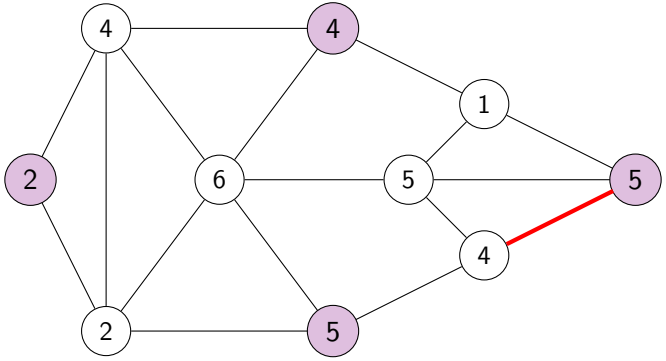

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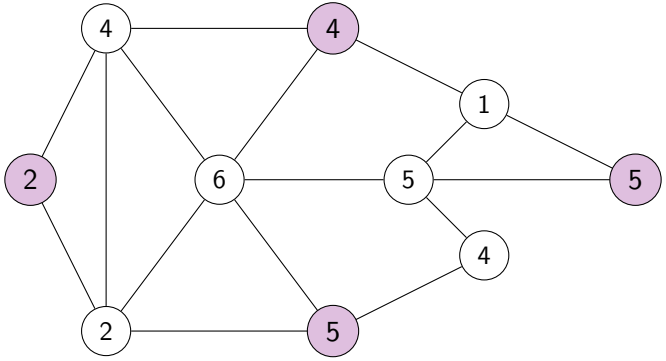

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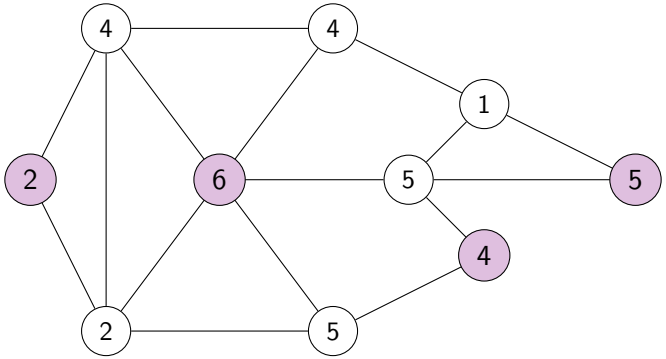

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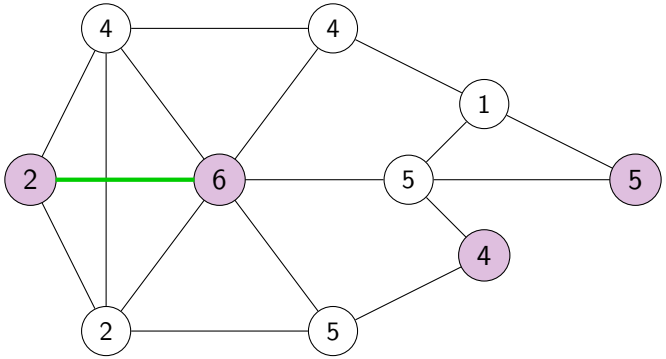

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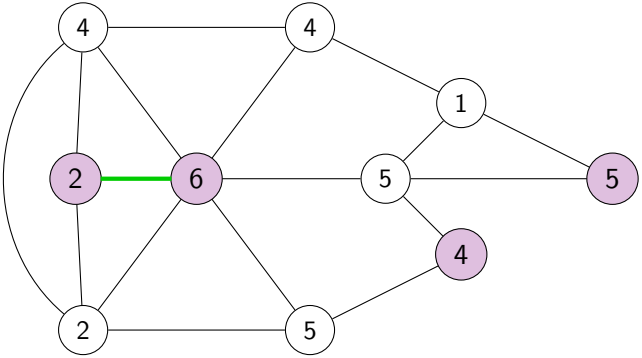

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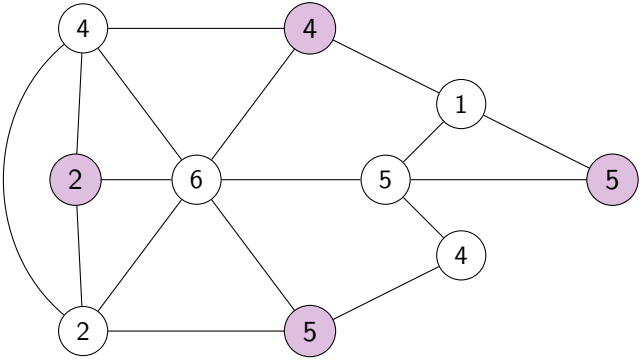

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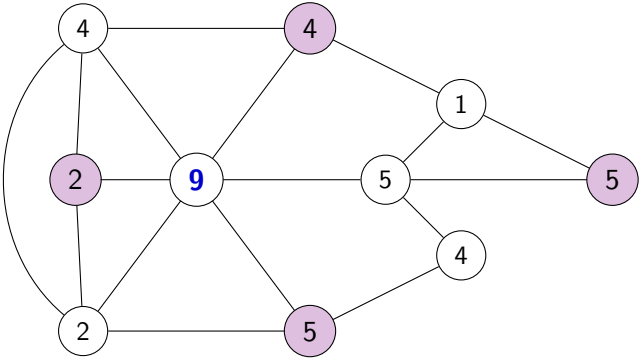

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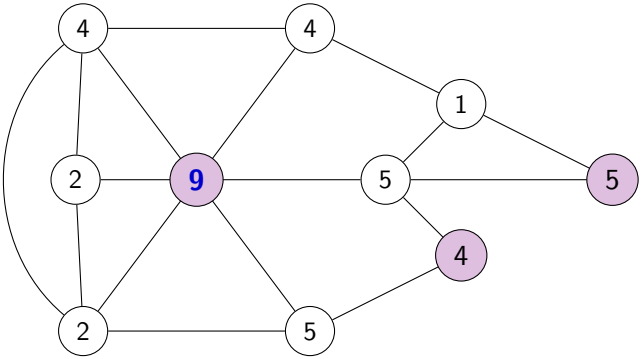

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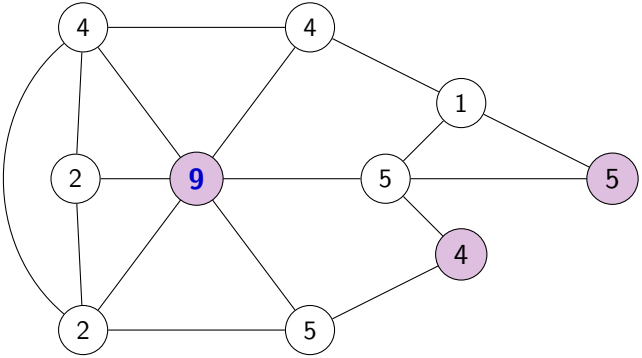

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Question: What about **dynamic** approximation schemes?

Dynamic Baker scheme

Korhonen, Nadara, Pilipczuk, **Sokołowski** [SODA '24]

Fully Dynamic Approximation Schemes on Planar and Apex-Minor-Free Graphs

Main result

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Fully Dynamic Approximation Schemes on Planar and Apex-Minor-Free Graphs

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Given: Initially edgeless, vertex-weighted dynamic **planar** graph G.

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Fully Dynamic Approximation Schemes on Planar and Apex-Minor-Free Graphs

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Given: Initially edgeless, vertex-weighted dynamic **planar** graph G. Let also $\varepsilon > 0$.

Dynamic Baker scheme

Korhonen, Nadara, Pilipczuk, **Sokołowski** [SODA '24]

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Update time: $f(\varepsilon) \cdot n^{o(1)}$ *(amortized)*.

Also generalizations to wider classes of graphs, Min Dominating Set...

Featured works

T. Korhonen, K. Majewski, W. Nadara, Mi. Pilipczuk, M. Sokołowski [FOCS '23]
Dynamic Treewidth

T. Korhonen, M. Sokołowski [STOC '24]
Almost-Linear Time Parameterized Algorithm for Rankwidth via Dynamic Rankwidth

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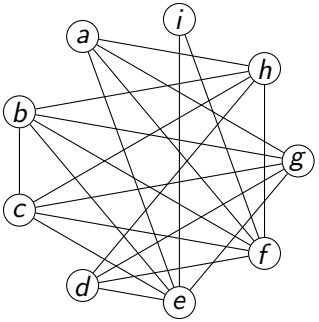
Thank you!

Appendix

Definition of rankwidth

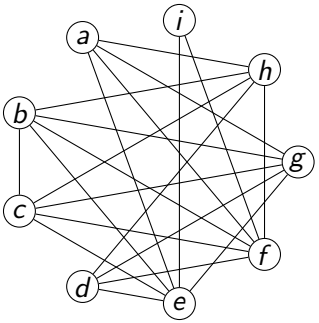
Rankwidth

Aim: recursively vertex-decompose a graph G...



Rankwidth

Aim: recursively vertex-decompose a graph $G\ldots$



$\{a, b, c, d, e, f, g, h, i\}$

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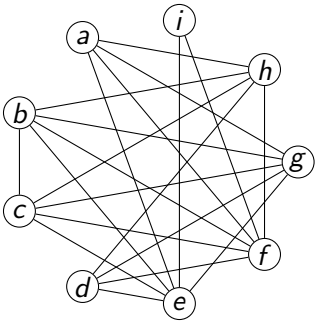

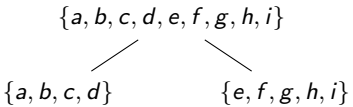

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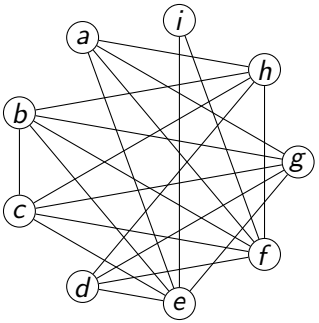

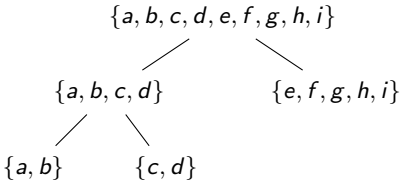

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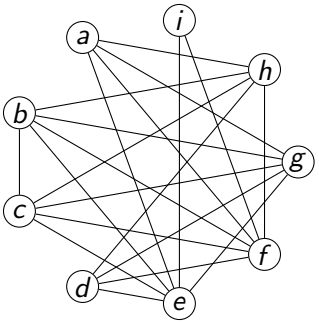

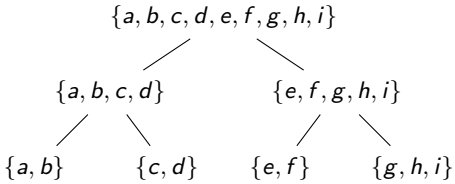

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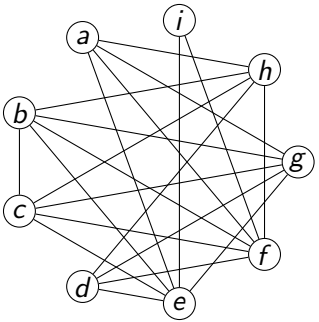

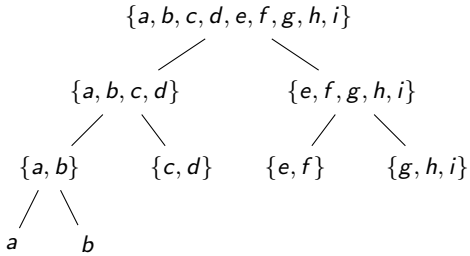

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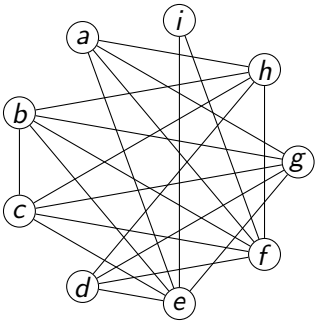

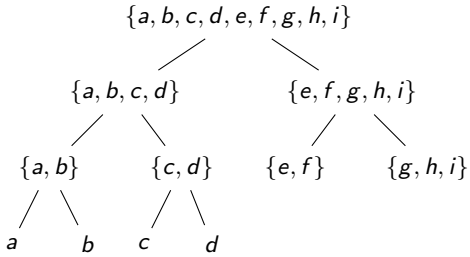

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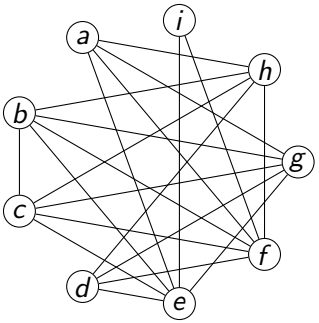

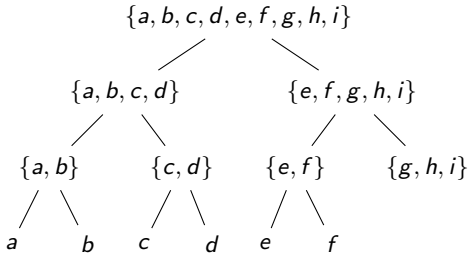

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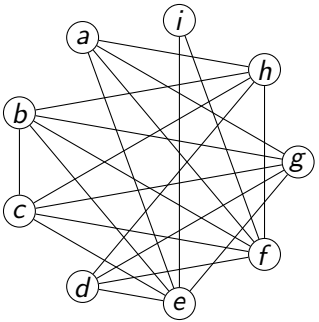

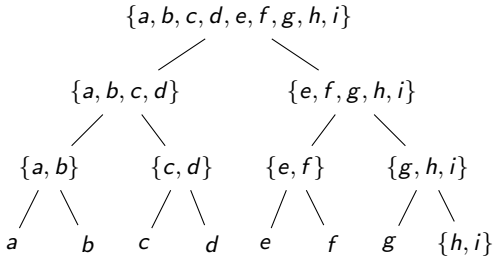

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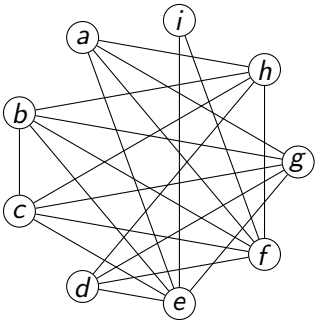

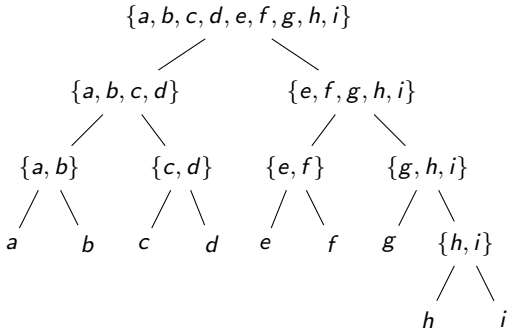

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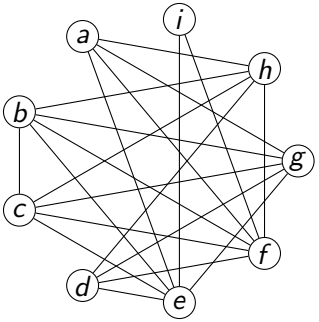

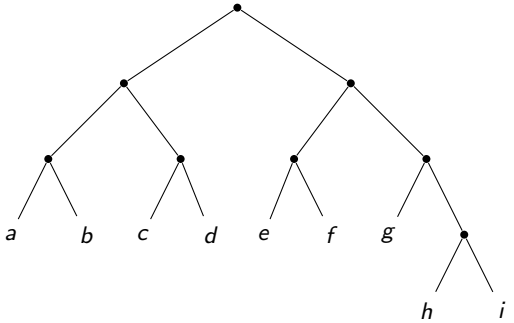

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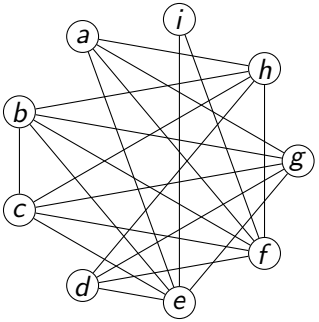

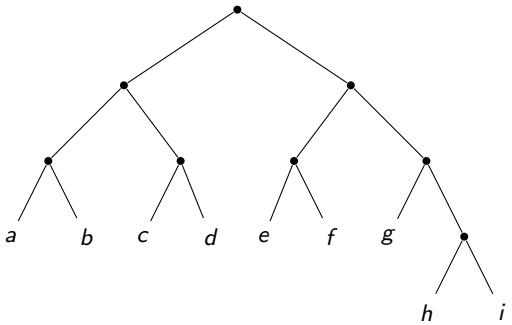

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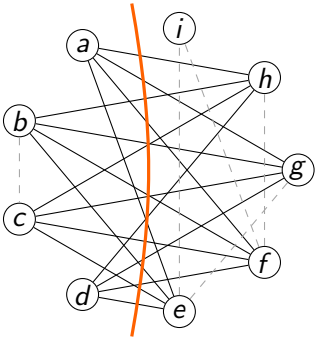

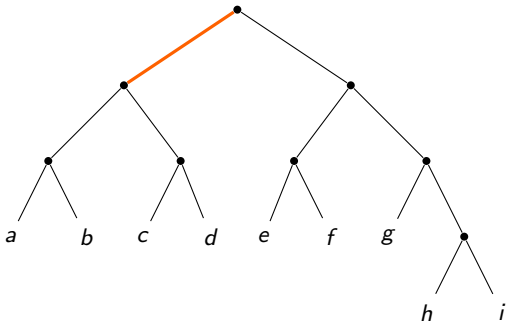

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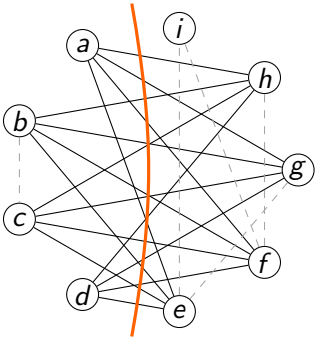
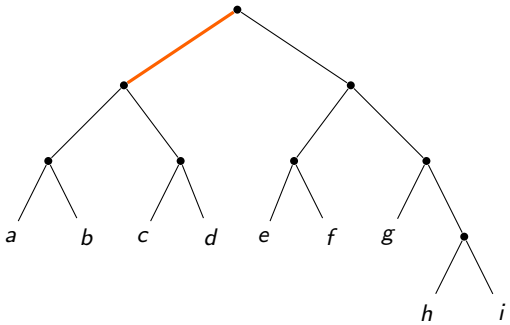
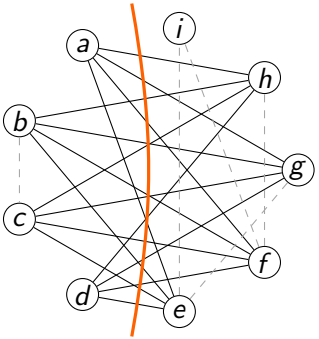




Simple for rankwidth: adjacency matrix of the cut has **small** $\mathrm{GF}(2)$ **rank**

Rankwidth

Aim: recursively vertex-decompose a graph G... so that each **cut** is **simple**



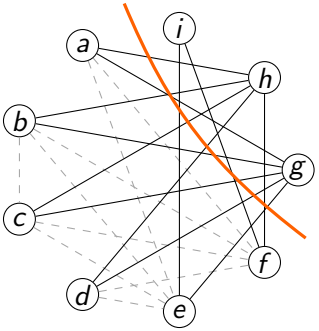
	e	f	g	h	i
a	●	●	●	●	
b	●	●	●	●	
c	●	●	●	●	
d	●	●	●	●	

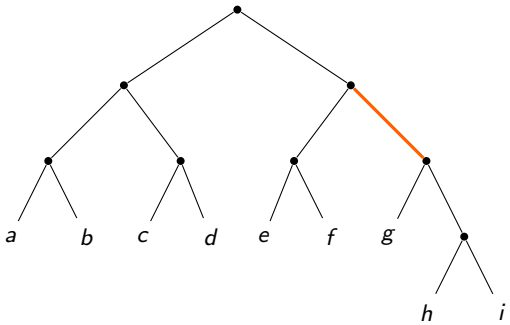
Rank: 1

Simple for rankwidth: adjacency matrix of the cut has **small** $\mathrm{GF}(2)$ **rank**

Rankwidth

Aim: recursively vertex-decompose a graph G... so that each **cut** is **simple**





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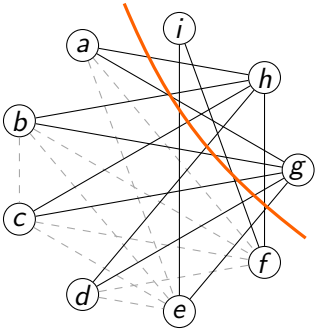


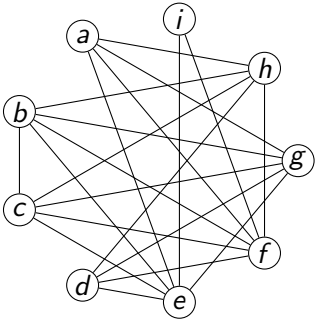
	g	h	i
a	●	●	
b	●	●	
c	●	●	
d	●	●	
e	●		●
f		●	●

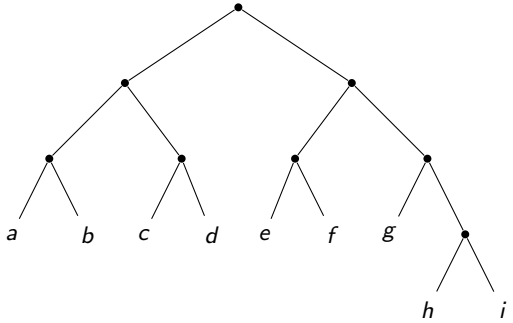
Rank: 2

Simple for rankwidth: adjacency matrix of the cut has **small** $\mathrm{GF}(2)$ **rank**

Rankwidth

Aim: recursively vertex-decompose a graph G... so that each **cut** is **simple**

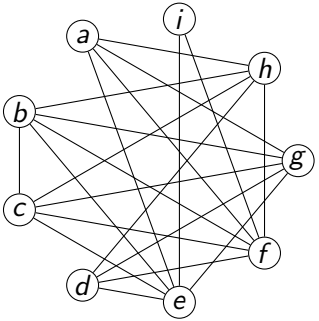


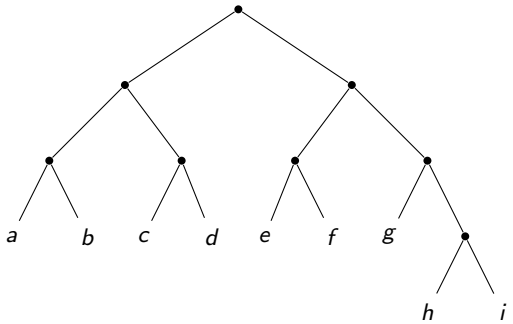


Width of a cut $(X, \overline{X})$: $\mathrm{GF}(2)$ rank of the adjacency matrix of $G[X, \overline{X}]$

Rankwidth

Aim: recursively vertex-decompose a graph G... so that each **cut** is **simple**

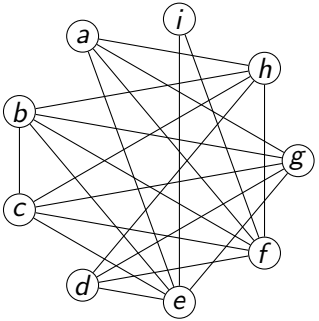


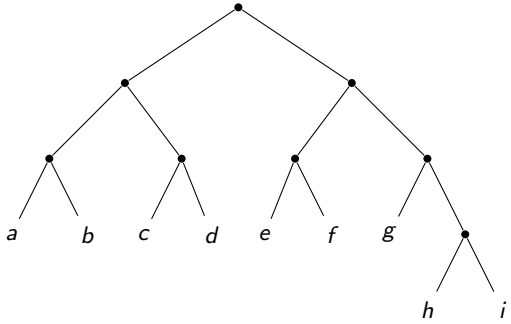


Width of a cut $(X, \overline{X})$: $\mathrm{GF}(2)$ rank of the adjacency matrix of $G[X, \overline{X}]$
Width of a **rank decomposition** $\mathcal{T}$ of G: maximum width of a cut given by $\mathcal{T}$

Rankwidth

Aim: recursively vertex-decompose a graph G... so that each **cut** is **simple**





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Rankwidth of G: minimum width of a rank decomposition of G