FLOW THROUGH A COMPRESSIVE-EXPANSIVE DUCT

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INTRODUCTION

This work has as purpose the study of a flow behavior through a compressive-expansive duct, as observing the path the streamlines make when sliding a perpendicular section.

In order to achieve this, we have programmed a MATLAB code that, by iterating, shows us an approximate plot of the real streamline variation through the nozzle.

When finishing this research, we have been able to appreciate the difficulty of getting a similar result to the exact solution using this type of approximation and, also, the useful they can be if the exact problem's resolution is hard to find.

THEORETICAL BACKGROUND OF THE PROBLEM

In our study we want to know how the streamlines look like through a compressive-expansive duct. To solve it, we have to do some suppositions that will help us when approximating.

First of all, we suppose that the inlet velocity and the outlet velocity are the same, therefore the value of ψ in both inlet and outlet is also equal. In order to find the velocity in the middle of our duct, we are going to use the continuity equation

$$A_1 V_1 = A_2 V_2$$

We also suppose that ψ remains constant where the flow is in contact with the walls of the duct. At the top, ψ is going to be equal to the inlet velocity value by the inlet perpendicular section, and at the bottom is going to be zero.

In this way, we establish the boundary conditions in order to be able to analyze how ψ varies inside the duct.

NUMERICAL DISCRETIZATION

We have used the Finite-Difference Method to reach our results. This method finality is to approximate the partial derivatives in a physical equation calculating the differences between nodal values, separated a finite distance from each other.

This distances will be called Δx and Δy. If we approximate the derivatives $\partial \psi/\partial x$ and $\partial \psi/\partial y$:

$$\frac{\partial \psi}{\partial x} \approx \frac{1}{\Delta x}(\psi_{i+1,j} - \psi_{i,j})$$

$$\frac{\partial \psi}{\partial y} \approx \frac{1}{\Delta y}(\psi_{i,j+1} - \psi_{i,j})$$

$$\frac{\partial^2\psi}{\partial x^2} \approx \frac{1}{\Delta x^2}(\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j})$$

$$\frac{\partial^2\psi}{\partial y^2} \approx \frac{1}{\Delta y^2}(\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1})$$

And introduce them into the stream-function form of the two-dimensional Laplace equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$$

We obtain the equation we will have to iterate to find the nodal values:

$$2(1+\beta)\psi_{i,j} \approx \psi_{i-1,j} + \psi_{i+1,j} + \beta(\psi_{i,j-1} + \psi_{i,j+1})\ , \quad \text{where } \beta = \left({}^{\Delta x}/_{\Delta y}\right)^2$$

In order we could use the discretization mentioned above, we have to create a mesh that fits our duct efficiently. Therefore, we have had to impose a relation between Δx and Δy.

$$\Delta y = \Delta x \tan\alpha$$

Where α is the angle between the compression- expansion of the duct and the ground, as it could be seen in Figure 1.

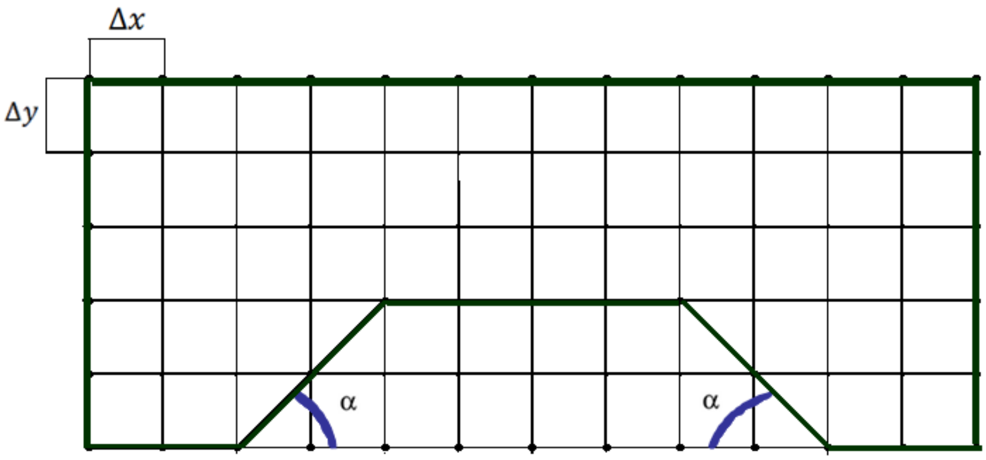

Figure 1.- Mesh scheme

We, also, have to take into account four more parameters when creating the mesh: inlet and outlet area (a), intermediate part area (b), intermediate part length (c), inlet and outlet length (d). (See Figure 2.)

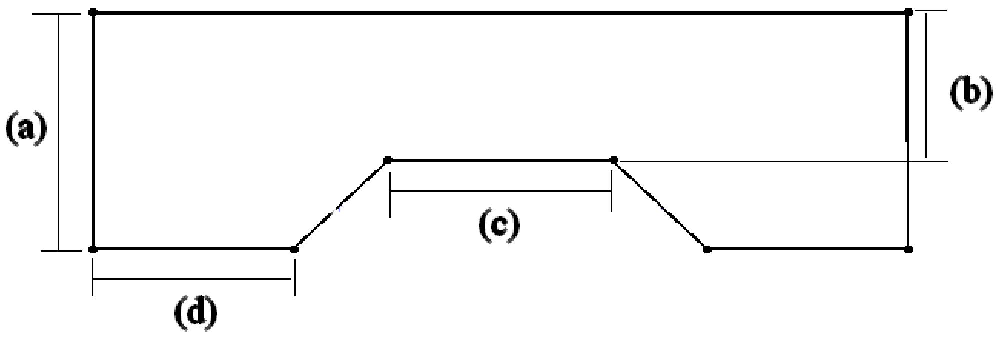

Figure 2.- Compressive-Expansive duct parameters

With these six parameters, we define our duct completely and, at the same time, the mesh that will allow us making the approximation for this duct. Precisely for this reason, in order to reproduce the flow through different ducts, our code allows the user to change the variables effortless.

The inlet and outlet velocity, which we need to impose the boundary conditions at the inlet (see following equation), is the last parameter we will have to introduce.

$$\psi_{i,j} = \psi_{i-1,j} + V_\infty \Delta y$$

NUMERICAL SIMULATIONS

Once all the variables are defined, we are able to simulate and see the approximation of the streamlines through our compressive-expansive duct.

Below, some of the simulations we have obtained by varying the angle and letting the other parameters remain constant.

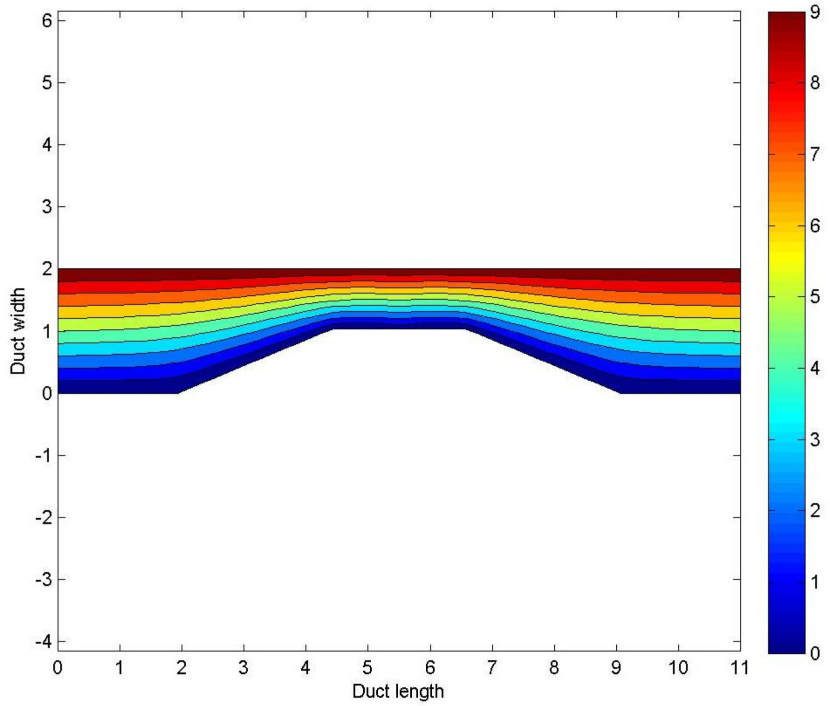

Figure 3.- Simulation 1. $\alpha = 20°$

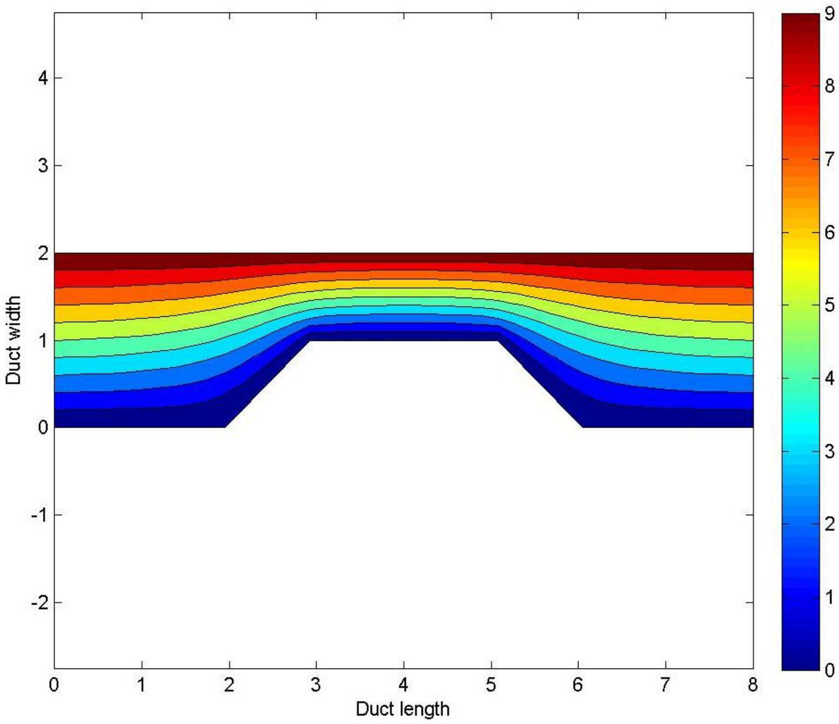

Figure 4.- Simulation 2. $\boldsymbol{\alpha = 45°}$

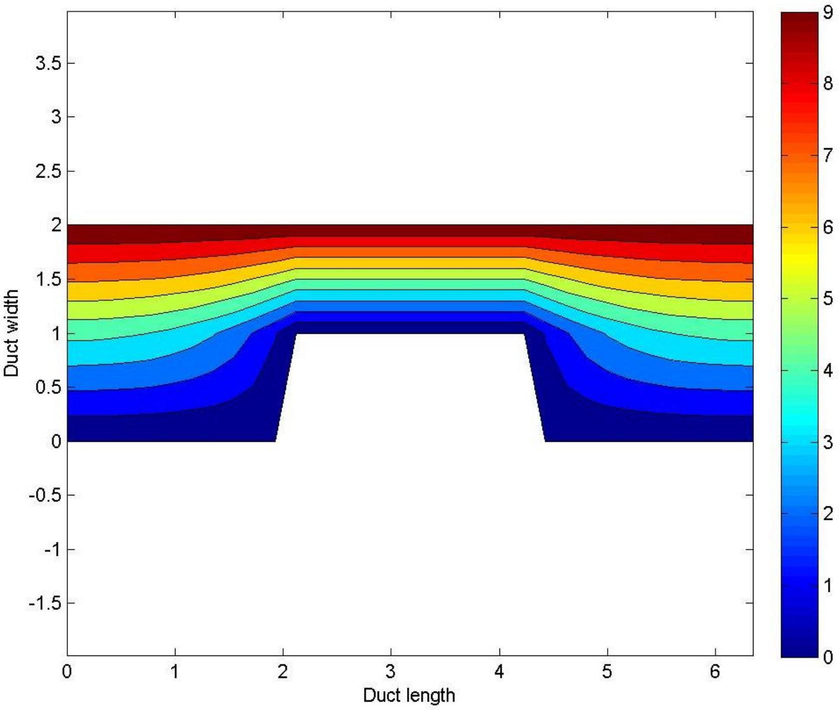

Figure 5.- Simulation 3. $\boldsymbol{\alpha = 80°}$

REFERENCES

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