(100 points; Show all work to get full credit.)

Guidance for writing your assignment:
a) make sure that your writing is legible and clear
b) wherever appropriate, underline or rewrite the final answer
c) clearly separate your work for subsequent questions
d) submit your work on Canvas as one pdf file saved as <LastName_H#.pdf>, for example, <Smith_H1.pdf>

1. Consider the function $f(x) = x_1^3 + 5x_1^2 x_2 + 7x_1 x_2^2 + 2x_2^3$. Let $x^0 = (-2, 3)^T$.

a) (2 points) Calculate the gradient vector $\nabla f(\mathbf{x})$ at $\mathbf{x} = \mathbf{x}^0$.

b) (3 points) Calculate the Hessian matrix $H(\mathbf{x})$ at $\mathbf{x} = \mathbf{x}^0$.

c) (5 points) Using the point $\mathbf{x}^0$ write the Taylor series expansion with three terms. Derive the resulting quadratic function.

d) (5 points) Find the approximate value of the function f at $\mathbf{x} = (-1.9; 3.2)$ using your work above.

e) (5 points) Calculate the true value of the function f at $\mathbf{x} = (-1.9; 3.2)$ and compare it with the approximate value. What do you observe?

2. Consider the function $f(\mathbf{x}) = (1/4)(x_1 - 2)^2 + (1/9)(x_2 - 3)^2$.

a) (5 points) Write the defining complete statement of the level curve of value 1. Clearly draw this level curve.

b) (5 points) Calculate the gradient vector $\nabla f(\mathbf{x})$ at $\mathbf{x}^0 = (3, 3 + (3/2) \, 3^{1/2})^T$.

c) (10 points) Derive the equation of the tangent line to the level curve of value 1 at $\mathbf{x}^0$. Show your work.

3. Consider the univariate function $f(x) = xe^{-2x}$. Giving analytical justification for your claims

a) (10 points) find all local/global minimizers and maximizers;

b) (10 points) find all inflections points.

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4. (10 points) Problem 9.2 c) page 334.

5. Problem 9.23 page 338. Give analytical justification for your claims.

a) (10 points)

b) (5 points)

6.

a) (10 points) Problem 9.25 page 338. Give analytical justification for your claims.

b) (5 points) Is f strictly convex? Explain why.