MATH 4420/6420 HW 4 BOLP due March 16, 2022

(100 points; Show all work to get full credit.)

Guidance for writing your assignment:
a) make sure that your writing is legible and clear
b) wherever appropriate, underline or rewrite the final answer
c) clearly separate your work for subsequent questions
d) submit your work on Canvas as one pdf file saved as <LastName_H#.pdf>, for example, <Smith_H1.pdf>

In questions 1, 2, and 3 consider the following biobjective linear program (BOLP):

$$\begin{aligned} \min \ & [\, f_1(\mathbf{x}) = 5x_1 - x_2,\ f_2(\mathbf{x}) = x_1 + 4x_2 \,] \\ \text{s.t.} \ & -5x_1 + 2x_2 \le 10 \\ & x_1 + x_2 \ge 3 \\ & x_1 + 2x_2 \ge 4 \\ & x_1,\ x_2 \ge 0 \end{aligned}$$

1.
a) (3 points) Draw the feasible set X in the decision space $\mathbb{R}^2$.

b) (11 points) Find and draw the outcome set Y in the objective space $\mathbb{R}^2$. (Include all details when you construct Y.)

c) (3 points) Identify and mark the Pareto set Y_P in Y.

d) (3 points) Identify and mark the efficient set X_E in X.

2.
a) (2 points) Formulate the weighted-sum problem P(w) for this BOLP.

b) (5 points) Let w = 1/3. Geometrically solve P(w) and report the optimal solution(s) in X and Y.

c) (3 points) Did you find efficient solutions to BOLP in part b)? Explain why.

d) (2 points) Find all values of w for which the optimal solutions to P(w) are efficient to this BOLP.

3.
a) (2 points) Formulate the epsilon-constraint problem $P(\varepsilon_1)$ for this BOLP.

b) (5 points) Let $\varepsilon_1 = 0$. Geometrically solve $P_2(\varepsilon_1)$ and report the optimal solution(s) in X and Y.

c) (3 points) Did you find efficient solutions to BOLP in part b)? Explain why.

d) (2 points) Find all values of ε_1 for which the optimal solutions to P(ε_1) are efficient to BOLP.

e) (2 points) Consider P(ε_2) and find all values of ε_2 for which the optimal solutions to P(ε_2) are efficient to BOLP.

4. Consider the following resource allocation problem:

A manufacturer of solar technology considers mass-producing two types of photovoltaic solar cells for energy conversion: (a) a cell module (product 1) for application in weather satellites, and (b) a cell chip (product 2) for use in pocket calculators. Unfortunately, the production process results in emissions of a pollutant into the atmosphere. The company would like to know what total volume of each type of cells should be manufactured so that the pollutant emissions are minimized but profits are maximized.

One unit of volume of each product requires the following inputs and results in the following sales prices:

	Product 1	Product 2
Units of gallium arsenide required	1.0	5.0
Machine time required (hour)	0.5	0.25
Assembly time required (man-hour)	0.2	0.2
Direct material costs ($)	0.25	0.75
Direct labor costs ($)	2.75	1.25
Sales price ($)	4.0	5.0

The following assumptions are made:

1. One production period is involved.
2. Production is limited to two absolute restrictions:
   (a) machine capacity is 8 hours per period, and
   (b) assembly capacity is 4 man-hours per period.
3. Raw material is limited to 72 units.
4. The only variable costs are those incurred for material and labor.
5. The amount of pollutant emitted is three units per unit of t product 1 and two units per unit of product 2.

a) (6 points) Model this problem as a biobjective linear program (BOLP).

b) (2 points) Formulate the weighted-sum problem P(w) for this BOLP.

c) (15 points) Use any software of your choice and solve P(w) for ALL efficient extreme points (EEPs) and the associated Pareto outcomes of this BOLP. Report your findings in the table given below.

#	EEP		Pareto point	
	x_1	x_2	$-f_1(x_1)$	$f_2(x_2)$
1				
…	…	…	…	…

d) (5 points) How can you guarantee that you have found all EEPs? Explain.

e) (12 points) For each EEP you reported in the table above, find all weights w in [0,1] such that this EEP is an optimal solution to P(w). Present the weights in the table below.

#	EEP		weight w
	x_1	x_2	
1			
…	…	….	…

5. Consider the following Goal Programming Problem (GPP):

goal $f_1(\mathbf{x}) = 5x_1 - x_2 \in [0, 6]$
goal $f_2(\mathbf{x}) = x_1 + 2x_2 = 4$
s.t. $\mathbf{x} \in X = \{(x_1, x_2) \in R^2\colon\ 3x_1 + 2x_2 \le 12,\ x_1 + 2x_2 \le 1,\ x_1 \le 3,\ x_1 \ge 0,\ x_2 \ge 0\}$.

a) (4 points) Reformulate GPP into a Goal Multiobjective Problem (Goal MOP). Clearly define all variables.

b) Geometrically find <u>all</u> $\mathbf{x} \in X$ for which

i) **(5 points)** <u>all</u> deviational variables are <u>zero</u>;

ii) **(5 points)** <u>all</u> deviational variables are <u>not zero</u>.