MATH 8650: FUNDAMENTAL GROUP HOMEWORK

DUE WEDNESDAY, MARCH 30TH

You may work on homework together, but you must write up your solutions individually and write the names of the individuals with whom you worked. If you use any materials outside of the course materials, e.g., the internet, a different book, or discuss the problems with *anyone other than me*, make sure to provide a short citation.

PROBLEMS:

(1) On Page 14 of the online version of Hatcher, there is a diagram of a genus three surface as a quotient of a 12-gon. Compute the fundamental group of this surface in the following two ways:
   (a) As a quotient of a free group on 6 elements via the attached disk.
   (b) Using the Seifert-van Kampen theorem by splitting the genus three surface into a punctured genus two and a punctured genus one surface (please let me know if you need a sketch of this setup).

(2) Let a and b be the generators of $\pi_1(S^1 \vee S^1)$ corresponding to the two summands.
   (a) Draw a picture of the covering space of $S^1 \vee S^1$ with fundamental group $\langle a^2, b^2, (ab)^2 \rangle$ and explain why this covering space corresponds to the given group. Does this covering space have any deck transformations?
   (b) Draw a picture of the covering space of $S^1 \vee S^1$ with fundamental group the normal group generated by a^2, b^2, and $(ab)^2$ and explain why this covering space corresponds to the given group. Find all deck transformations of this covering space.