MATH 8650: COHOMOLOGY HOMEWORK

DUE FRIDAY, MAY 6TH

You may work on homework together, but you must write up your solutions individually and write the names of the individuals with whom you worked. If you use any materials outside of the course materials, e.g., the internet, a different book, or discuss the problems with *anyone other than me*, make sure to provide a short citation.

PROBLEMS:

(1) Let T be the torus, K the Klein bottle, and P the real projective plane.
   (a) Use the universal coefficient theorem to compute the cohomology of T, K, and P over $\mathbb{Z}$.
   (b) Use the definition to compute the simplicial cohomology of T, K, and P over $\mathbb{Z}$ using the Δ-complex structure on a square formed from two triangles.

(2) Show that if $f : S^n \to S^n$ has degree d, then $f^* : H^n(S^n; G) \to H^n(S^n; G)$ is multiplication by d.

(3) Use cup products over $\mathbb{Z}/2\mathbb{Z}$ to show that $\mathbb{R}P^3$ is not homotopy equivalent to $\mathbb{R}P^2 \vee S^3$.