

Homework #4

Due Monday (October 18th)

This homework is designed to be used as a study guide for the upcoming test. The questions you see in this homework will be closely related to the questions you will see on the test. I will be grading it based on your effort. Thus, you just need to show me on Monday that you tried solving the problems on your own, and then I'll give you a 10/10. On Monday, I will post the solutions to this homework on canvas, so that you can compare your answers to the solutions I give.

1. $T(x)$ degrees Fahrenheit gives the temperature of an oven t minutes after it's been turned.

1.a. (1 point) Write a sentence of interpretation for $\frac{dT}{dx}|_{x=1} = 2$.

Sentence of Interpretation: The temperature of the oven is *increasing* by 2 degrees Fahrenheit *per* minute, at 1 minute after it has been turned on.

What: The temperature of the oven

I/D: Increasing (with an "ing" at the end because we are considering only one input, namely 1 minute).

By How Much: by 2 degree Fahrenheit per minute

When: 1 minute after it has been turned on.

1.b (1 point) Write a sentence of interpretation for $T(1) = 120$.

Sentence of Interpretation: The temperature of the oven is 120 degrees Fahrenheit, 1 minute after it has been turned on.

What: The temperature of the oven

I/D: There is no I/D when writing a sentence of interpretation for an ordered pair.

How Much: is 120 degree Fahrenheit

When: 1 minute after it has been turned on.

1.c (1 point) The table below gives the temperature of the oven at various minutes after its been turned on.

| x minutes | $T(x)$ degree Fahrenheit |
|-------------|--------------------------|
| 0 | 70 |
| 1 | 120 |
| 5 | 200 |
| 9 | 400 |

Write a sentence of interpretation of the average rate of change of $T(x)$ between 1 minute and 9 minutes.

Calculation: We first calculate average rate of change of $T(x)$ between 1 minute and 9 minutes. This is given by

$$\begin{aligned}\frac{T(9) - T(1)}{9 - 1} &= \frac{400 - 120}{9 - 1} \\ &= \frac{280}{8} \\ &= 35.\end{aligned}$$

Sentence of Interpretation: The temperature of the oven increased on average by 35 degrees Fahrenheit per minute between 1 minute and 10 minutes after it has been turned on.

What: The temperature of the oven

I/D: Increased on average (with an “ed” at the end because we are considering two input points, namely 1 minute and 10 minutes).

By How Much: by 120 degrees Fahrenheit per minute

When: Between 1 minute and 10 minutes after it has been turned on.

1.d (1 point) Write a sentence of interpretation for the percent rate of change of $T(x)$ at $x = 1$.

Calculation: We first calculate the percent rate of change of $T(x)$ at $x = 1$. This is given by

$$\frac{T'(1)}{T(1)} = \frac{2}{120} \\ \approx 0.017$$

Sentence of Interpretation: The temperature of the oven is *increasing* by 0.017 percent per minute, 1 minute after it has been turned on.

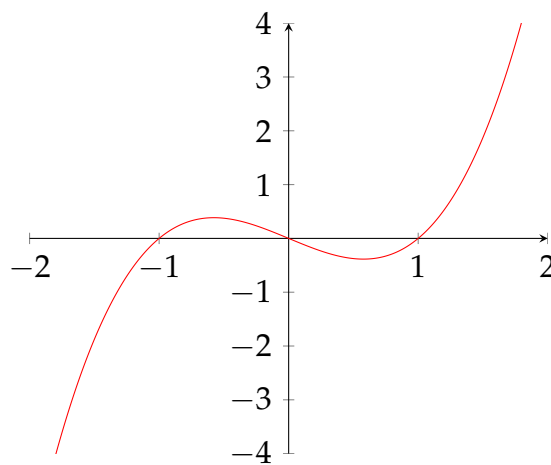
What: The temperature of the oven

I/D: Increasing (with an “ing” at the end because we are considering only one input, namely 1 minute).

By How Much: by 0.017 percent per minute

When: 1 minute after it has been turned on.

2 Let $f(x)$ be the function given by the graph below:

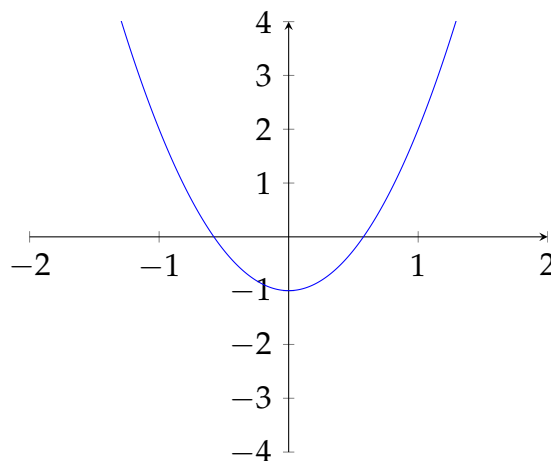


2.a How many x -intercepts will the slope graph of $f(x)$ have?

2.b How many relative mins / relative maxes will the slope graph of $f(x)$ have?

2.c Sketch the slope graph below

Solution:



2.d Let $f(x)$ be a function and let a be a real number (say $a = 0$ if you want). List three reasons why $f'(a)$ may not exist.

The three reasons that we learned about in class are

1. $f(x)$ is not continuous at a (remember this means either $f(a)$ does not exist or $\lim_{x \rightarrow a^+} f(x) \neq \lim_{x \rightarrow a^-} f(x)$);
2. the tangent line at the point $(a, f(a))$ on the graph of $f(x)$ is a vertical line;
3. the graph of $f(x)$ at the point $(a, f(a))$ has a sharp corner.

3.a How much will a \$100 investment, compounded quarterly at 5% APR yield in 10 months? How long will it take to double your investment?

3.b How much will a \$100 investment, compounded continuously at 5% APR yield in 10 months? How long will it take to double your investment?

3.c What is the APY for an investment that pays 3% APR compounded quarterly? (If you need to, go to section 1.6 in the book and look up the formula for APY)

3.d What is the APY for an investment that pays 3% APR compounded continuously? (If you need to, go to section 1.6 in the book and look up the formula for APY)

4. Find the derivative of the following functions

$$h(t) = t^2 + t + t^{-1}$$

$$h(x) = \sqrt{\frac{1}{x^{1/3}}}$$

$$g(t) = \frac{3t^2 - t + 1}{t}$$

$$g(x) = 2\ln(x) + e^x$$

$$f(x) = 5e^x + e^2$$

$$f(t) = 3^t + t$$

$$g'(t) = 3 - \frac{1}{t^2}$$

$$g'(x) = \frac{2}{x} + e^x$$

$$f'(x) = 5e^x$$

$$f'(t) = \ln(3)3^t + 1$$