Multicriteria Optimization Notes

Definition 0.1. A **feasible solution** $\widehat{x} \in X$ is called **efficient** (or **Pareto optimal**) if there is no $x \in X$ such that $f(x) \leq f(\widehat{x})$. If $\widehat{x}$ is efficient, then $\widehat{y} = f(\widehat{x})$ is called a **nondominated point**. If $a, b \in X$ and $f(a) \leq f(b)$, then we say a **domiantes** b and $f(a)$ dominates $f(b)$. The set of all efficient soluitons is denoted X_E and the set of all nondominated points is denoted Y_N.

Definition 0.2. Consider the following MOP:

$$\min_{x \in X}(f_1(x), \ldots, f_p(x)).$$

1. The point $y^{\mathrm{I}} = (y_1^{\mathrm{I}}, \ldots, y_p^{\mathrm{I}})$ given by
$$y_k^{\mathrm{I}} := \min_{x \in X} f_k(x) = \min_{y \in Y} y_k,$$
is called the **ideal point** of the MOP.
2. The point $y^{\mathrm{N}} = (y_1^{\mathrm{N}}, \ldots, y_p^{\mathrm{N}})$ given by
$$y_k^{\mathrm{I}} := \max_{x \in X_E} f_k(x) = \max_{y \in Y_N} y_k,$$
is called the **nadir point** of the MOP.
3. The point $y^{\mathrm{U}} = (y_1^{\mathrm{U}}, \ldots, y_p^{\mathrm{U}})$ given by
$$y_k^{\mathrm{U}} = y_k^{\mathrm{I}} - \varepsilon$$
where $\varepsilon > 0$ is a small constant, is claled the **utopian point** of the MOP.

Remark 1. See page 35 in Ehrgott for illustration.