MATH 8650: HOMOLOGY HOMEWORK

DUE WEDNESDAY, APRIL 27TH

You may work on homework together, but you must write up your solutions individually and write the names of the individuals with whom you worked. If you use any materials outside of the course materials, e.g., the internet, a different book, or discuss the problems with *anyone other than me*, make sure to provide a short citation.

PROBLEMS:

(1) Let $X = S^1 \times S^1$ and $Y = S^1 \vee S^1 \vee S^2$.
   (a) Compute the homology of X and Y and confirm that the homology is the same in every dimension.
   (b) Describe the universal covering spaces of X and Y.
   (c) Show that the universal covering spaces of X and Y do not have the same homology.

(2) Compute the homology groups $H_n(X, A)$ in the following cases
   (a) X is S^2 and A is a finite set of points in X
   (b) X is $S^1 \times S^1$ and A is a finite set of points in X
   (c) X is a surface of genus 2 and A is a loop that separates the two holes (see Loop A in the figure on Page 132 of Hatcher - Page 141 of the pdf doucment)
   (d) X is a surface of genus 2 and A is a loop that goes through one of the two holes (see Loop B in the figure on Page 132 of Hatcher - Page 141 of the pdf doucment)

(3) Compute the homologies of the following spaces:
   (a) The quotient of S^2 by identifying the north and south poles to a point.
   (b) The space $S^1 \times (S^1 \vee S^1)$. This space looks somewhat like a torus, but each of the radial slices is a figure-eight.
   (c) The quotient space formed from deleting two disjoint open disks in the interior of D^2 and identifying all three boundaries, preserving the clockwise orientations of the circles.

(4) A map $f : S^n \to S^n$ satisfying $f(x) = f(-x)$ for all x is an *even map.* For this problem, you may assume that f has the property that there is some point $y \in S^n$ with finitely many preimages.
   (a) Prove that an even map $S^n \to S^n$ must have even degree.
   (b) Prove that when n is even, the degree of an even map must be 0.
   (c) Prove that when n is odd, there exist even maps of any given even degree.