Math 1070 Final Review

Integrals, Derivatives, and Limits

Integrals

Exercise 1. Evaluate the following integrals

1. $\int 12x\left(x-3x^{-2}\right)\mathrm{d}x$
2. $\int \frac{2x+1}{\sqrt{3x+4}}\mathrm{d}x$
3. $\int \frac{\cos\theta}{2-\sin\theta}\mathrm{d}\theta$
4. $\int_{1/4}^{0} \frac{2}{3+48z^2}\mathrm{d}z$
5. $\int e^{2x}\sin(e^{2x}+1)\mathrm{d}x$
6. Find the average value of $f(x)=e^x+1$ on the interval $[0,2]$.

Derivatives

Exercise 2. Evaluate the following derivatives

1. $\frac{\mathrm{d}}{\mathrm{d}x}\left(3^{\ln(x^2+1)}\right)$
2. $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin(\theta)^{\cos(\theta)}\right)$
3. $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\left(\theta^{\cos(\theta)}\right)\right)$
4. $\frac{\mathrm{d}}{\mathrm{d}x}(\sin(\arccos(x))$
5. $\frac{\mathrm{d}}{\mathrm{d}x}\int_{\sqrt{x}}^{x}\ln(1+t^2)\mathrm{d}t$
6. Find the average rate of change of the function $f(x)=e^x+1$ on the interval $[0,2]$.

Derivative Tests

Exercise 3. Use an appropriate derivative test to solve the following:

1. State the intervals on which the function $\sin(e^x)$ is increasing.
2. State the intervals on which the function $F(x)=\int_2^x \frac{1}{\ln t}\mathrm{d}x$ is increasing (this function is a special function studied in number theory).
3. State the intervals on which the function $\ln(\sin x)$ is concave down.

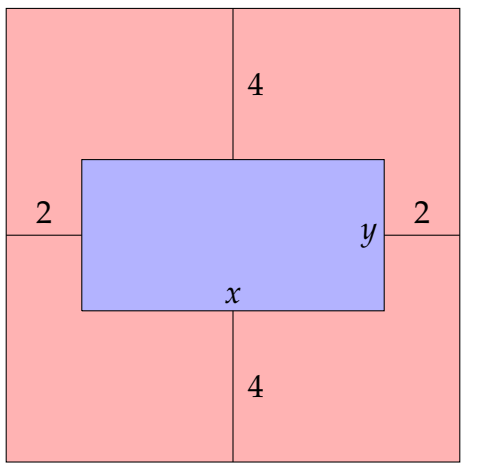
Limits

Exercise 4. Evaluate the following limits

1. $\lim_{x\to 0} \frac{\frac{1}{x+1}-1}{x}$
2. $\lim_{x\to 3} \frac{x^2-5x+6}{x-3}$
3. $\lim_{x\to -2} \frac{-2-x}{1-\sqrt{x+3}}$
4. $\lim_{x\to 1} \frac{\ln(x)}{x-1}$
5. $\lim_{x\to\infty} \left((x^2+1)^{1/x}\right)$

Word Problems

Exercise 5. Consider the situation below where a blue rectangle is contained in a red rectangle as follows:



The numbers in the image below correspond to the distances between the red rectangle and the blue rectangle measure in inches. These distances are fixed in this problem. The variables x and y on the other hand give the dimensions of the blue rectangle. They will vary in this problem. Suppose that the area of the blue rectangle is 16 inches. What dimensions of the blue rectangle will minimize the area of the red rectangle?

Exercise 6. A particle's acceleration after t seconds is given by $a(t) = \sin t$. At time $t = 0$, its velocity was $v(0) = 0$ and its position was $s(0) = 0$. Find the position function $s(t)$ of the particle at t seconds.

Exercise 7. A man 6 ft tall walks at a rate of 5 feet per second away from a streetlight that is 16 feet above the ground. At what rate is the tip of his shadow moving? At what rate is the length of his shadow changing when he is 10 feet from the base of the light?