

Practice Exam

Math 8600

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1. This is an open book exam, you are allowed to use our textbook, your handwritten notes, and all material available on Canvas.
2. The exam period is 50 minutes (class time). The pdf of the exam is available to you on canvas 3 minutes before start time.
3. The exam is done on paper, starting a new sheet for every question. You have 15 minutes to scan and upload a pdf at the end of the exam.
4. You will not need to use MATLAB or a calculator, all computations are to be done by hand.
5. You are expected to be visible on Zoom with your camera on until your exam is submitted. Try to push your laptop back and point the camera down, so that you are visible while working.
6. Searching online, communicating with your peers or anyone else, or using any other resource not mentioned above is not permitted.

1. Show that 0.1 can not be represented as a finite binary number.
2. You are given a vector $x = (x_1, \dots, x_n)$ and are asked to compute $\|x\|_1$. How would you try to minimize error accumulation?
3. For computing the midpoint m of an interval $[a, b]$, which of the two formulas is preferable in floating-point arithmetic? $m = (a + b)/2.0$, or $m = a + (b - a)/2.0$? Try to construct examples where the answer m will be outside the interval $[a, b]$.
4. Give the smallest number x where the next largest number in 64-bit floating point format y has $y > x + 1.0$.
5. The equation $x^3 + 4x^2 - 10 = 0$ has a unique root in $[1, 2]$. Verify that that a fixed-point of

$$g(x) = x - x^3 - 4x^2 + 10$$

is a solution of the original equation. Is the fixed-point iteration guaranteed to converge to the solution for any starting value in $[1, 2]$? If not, can we reduce the interval $[1, 2]$ to make this work (choose it as large as possible)?

6. Find the LU decomposition for

$$A = \begin{pmatrix} 2 & 2.1 \\ 2 & 1.9 \end{pmatrix}.$$

Compute the condition number of A . How can one find the inverse of a matrix A given L , U ?

7. Given a QR decomposition (full or economy) of a matrix A , how do you solve the linear least squares problem $\min \|b - Ax\|_2$?
8. Explain (precisely, defining every expression you use) how to solve the nonlinear system

$$\begin{aligned} x^2 &= 1, \\ xy &= 2 \end{aligned}$$

using Newton's method.

9. Construct the interpolating polynomial $p(x)$ using Lagrange basis for the points $(0, 3)$, $(1, 4)$, $(3, 1)$.
10. Consider polynomial interpolation of $f(x) = \sin(x)$ on the interval $[0, 2\pi]$ with equidistant points. How many points are required to achieve a maximum error of 10^{-8} ?