Name: ____

Disclaimer:
This document is meant to be a friendly aid. It cannot be guaranteed to be inclusive or exclusive of all that is covered on the test. Any advice cannot be guaranteed to work for all persons.

Mrs. Simms' suggested study plan:

Read this document section by section alongside your L&LAs and the keys to the L&LAs.
Work the problems. Check the key for accuracy and presentation.

Discuss your concerns with others (your group, Mrs. Simms, PAL leaders, ASC tutors, a study group, friends, etc.).

Use my solutions to textbook problems to help you review specific HW problems that cover topics giving you difficulty.

Look at old tests posted on Department web site with the WARNING that the curriculum for some previous semesters was from a different text and in a somewhat different order previous to this semester.

You need to consider the objectives list for each section and be able to identify what type of problem you are presented with. You need to know how to do every type of problem that was in a L&LA or HW. This means you need to understand the meaning and structure of the problems enough to know when you are presented with another of that type. It may be helpful to consider the wording of the directions and the variety of presentations of similar problems. For this purpose, it may be helpful to do/consider problems other than the specific HW problem you did. Generally similar problems are grouped together in the textbook exercises.
Remember, generally I have solutions posted to more problems than just the HW problems.

The test is Wednesday, November 18, 2020 from 7:30 – 9:00. You will test with Lockdown Browser and Respondus Monitor. You need your ID. You will scan & post your solutions to Gradescope.

Format of Test:

- Roughly 30-35% Multiple Choice
- Rest Free Response
- 90 minutes
- No Calculators allowed on Tests.

You will not get back a list of answers or right/wrong when you get your grade, so be sure to mark your answers in the test also. When it comes to studying for the final exam you probably will want to know what your answers were.
Your work will not be graded on multiple choice. It is important to work carefully on multiple choice because one arithmetic error could cost you 3 points at a one time since there is no partial credit on multiple choice. I personally work multiple choice questions without looking at the answers. Work through the problem as if it was free response, follow normal steps for doing the problem, get your answer, then find it. If your answer is not listed look back through your work to figure out what you did wrong. This method may help keep you from being distracted by the distracter answers.

Presentation of your work is important.

Work needs to be legible.

Work needs to be logical.

Proper mathematic structure and notation are important. (Your work needs to read properly in math translated to English in the grader's mind.)

You may be penalized for not following any of these principles.

My posted solutions for learning activities and textbook problems model the solutions structures you should use.

It is department policy during common testing that all electronic devices be OFF during the testing period. Any electronic device discovered to be on will be considered a possible indication of academic dishonesty. This includes smart watches – safest thing to do it to remove and stow it in your stuff.

You should plan to have and use 5-6 sheets of paper for the test. Suggested page breaks are now included in the test file. The 1040 instructors ask that you please follow the suggested page breaks to facilitate proper and complete presentations of your work in the Free Response section. Remember the work (proper, complete mathematical thoughts with proper notation) in the Free Response is graded – not just the final answers.

To present your work in an organized manner with the page breaks as requested,
you will need 1 or 2 sheets of paper for the multiple choice and 4 sheets of paper for the free response. (5-6 sheets of paper in total)

This means you will have 5-6 pages in your pdf scan of your test. After you upload to Gradescope, please take the time to match up your pages with the problems in Gradescope.

Overall comment:

You **simplify** an expression. Factor, cancel, etc.

You can't multiply or divide an expression by something w/o changing it.

You CAN add 0 in special forms.

You CAN multiply by 1 in special forms.

You **solve** an equation (has equal sign and two sides). Perform operations to both sides.

Multiply or Divide by something. Add or Subtract something from both sides. etc.

** Problems included in this review set are only roughly grouped by section. You may need a "future" section to do some problems.

I am not including review questions here but as carry over from Unit 1 you must be able to graph $y=|x|$

$y=x^2$ $y=x^3$ $y=\frac{1}{x}$ $y=\frac{1}{x^2}$ $y=\sqrt{x}$ $y=\sqrt[3]{x}$ $y=\sin x$ $y=\cos x$ $y=\tan x$

and now $y=b^x$ $y=e^x$ $y=\log_a x$ $y=\ln x$

Know their domains and ranges. Be able to translate and reflect.

For trig, also change amplitude and period.

Be able to evaluate all six trig functions at all the axes 0, $\frac{\pi}{2}$ π $\frac{3\pi}{2}$

the important angles in the first quadrant $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$

and at reflections and rotations of those angles, etc.

Calculus Section 2.6 : Continuity

Objective	Correlated MyLab Math Problems	Calculus
Answer conceptual questions involving continuity.	2, 9	
Find points of discontinuity or intervals of continuity.	7, 13, 15, 30, 39, 43, 47	
Determine if functions are continuous at given values.	19, 21, 23, 87	
Evaluate limits using continuity principles.	32, 49, 51, 65	53, 88
Use the Intermediate Value Theorem to show equations have solutions on given intervals.		67a, 71a
Sketch graphs of continuous functions given information about their points of discontinuity.	85	
Solve applications involving continuity principles.		93
Classify discontinuities.	95, 99	
(*Review: Evaluate two-sided limits using limit laws and theorems.)	(2.3.27)	

- continuity graphically should be straightforward as long as you know how to read the graph well – holes, breaks, vertical asymptotes, and oscillations
- intervals of continuity: be able to use parentheses and brackets correctly to show continuity from left and right or lack thereof.
- 3 step continuity checklist implied by continuity definition
  be specific as to what part of test fails

- where is a function continuous/discontinuous? Support answer with limits
- what type of discontinuity?
  Removable: hole in graph at c, removed with the value of $\lim_{x\to c} f(x)$
  Denominator is zero but the "issue" can be cancelled away
  Infinite: vertical asymptote $x = a$, proved with $\lim_{x\to \pm a^{\pm}} f(x) = \pm\infty$
  Denominator is zero and the "issue' cannot be cancelled
  Jump: in a piecewise function, $\lim_{x\to c} f(x)$ dne because $\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$
- Know the classes of functions that are continuous or those that are continuous on their domains.
  Be able to state where a function is continuous in interval notation (must have supporting work)
- Be able to do the problems where you find a value for a constant to make the function continuous.

- There were some limits in this section (some involving trig functions and powers).
  See the L&LA and HW.

- Be able to use Intermediate Value Theorem to show there is at least one root/solution on an interval.
  Model the structure shown in the L&LA key. Always state the name in your work of any theorem that has a special name.

1. Give the three conditions that must be satisfied by a function to be continuous at a point.

(1) $f(a)$ defined (2) $\lim_{x\to a} f(x)$ exists (3) $\lim_{x\to a} f(x) = f(a)$

2. Complete the following sentences:

a. A function is continuous from the left at a if $\lim_{x\to a^-} f(x) = f(a)$.

b. A function is continuous from the right at a if $\lim_{x\to a^+} f(x) = f(a)$.

3. Where is $f(x) = \sqrt{1-x^2}$ continuous? Show supporting work.

root functions are continuous on their domain

$1 - x^2 \geq 0$

$-x^2 \geq -1$

$x^2 \leq 1$

continuous on $[-1, 1]$

4.

Let

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leqslant x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$$

(a) Evaluate each limit, if it exists.

(i) $\lim\limits_{x \to 0^+} f(x)$ (ii) $\lim\limits_{x \to 0^-} f(x)$ (iii) $\lim\limits_{x \to 0} f(x)$

(iv) $\lim\limits_{x \to 3^-} f(x)$ (v) $\lim\limits_{x \to 3^+} f(x)$ (vi) $\lim\limits_{x \to 3} f(x)$

(b) Where is f discontinuous?

(c) Sketch the graph of f.

(a) $\lim\limits_{x \to 0^+} (3-x) = 3$

$\lim\limits_{x \to 0^-} \sqrt{-x} = 0$

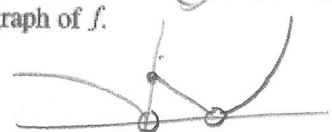
$\lim\limits_{x \to 0} f(x)$ dne

$\lim\limits_{x \to 3^-} (3-x) = 0$

$\lim\limits_{x \to 3^+} (x-3)^2 = 0$

$\lim\limits_{x \to 3} f(x) = 0$

(b) not continuous at $x = 0$ b/c left $\neq$ right



5. There were some limits in this section (some involving trig functions and powers). See the L&LA and HW.

6. Use the Intermediate Value Theorem to show that the equation $x^5 + 7x + 5 = 0$ has a root/solution in the interval $(-1, 0)$.

$f(x) = x^5 + 7x + 5$

$f(x)$ is continuous b/c it is a polynomial

$f(-1) = -1 - 7 + 5 = -3 < 0$ $f(0) = 5 > 0$

0, the desired output value, is between -3 and 5

so IVT says there is at least one $c \in (-1, 0)$ where $f(c) = 0$

7. Use the Intermediate Value Theorem to show that the equation $\cos x = x$ has a solution in the interval $\left(0, \frac{\pi}{2}\right)$.

Let $f(x) = \cos x - x$

$f(x)$ is continuous b/c it is $\cos x$ and a monomial

$f(0) = \cos 0 - 0 = 1 > 0$ $f\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} - \frac{\pi}{2} = 0 - \frac{\pi}{2} < 0$

0, the desired output value, is between $-\frac{\pi}{2}$ and 1

so IVT says there is at least one $c \in \left(0, \frac{\pi}{2}\right)$ where $f(c) = 0$

8. Prove that the equation $2x^3 - 11x^2 + 10x + 8 = 0$ has at least one root on the interval $(1,3)$. As part of your proof, be sure to show that the conditions of the theorem are met and be sure you state the name of the theorem you are using.

Let $f(x) = 2x^3 - 11x^2 + 15x + 8$

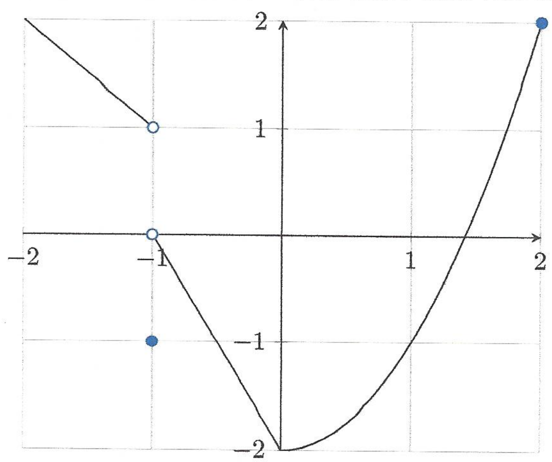
$f(x)$ is continuous b/c it is a polynomial

$f(1) = 2 - 11 + 15 + 8 = 14 > 0$ $\quad f(3) = 16 - 99 + 30 + 8 = -45 < 0$

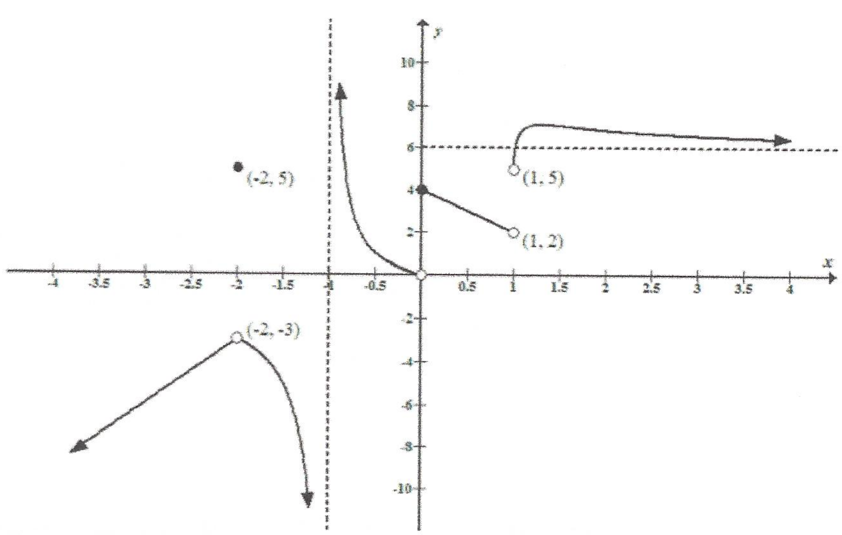
0, the desired output value, is between -45 and 14

so IVT says there is at least one $c \in (1,3)$ where $f(c) = 0$

9. Where are the graphs not continuous? Classify the discontinuities. What part of the continuity checklist is violated? Also, state intervals of continuity



$x = -1$ jump discontinuity



$x = -2$ removable discontinuity

$x = -1$ infinite discontinuity

$x = 0$ jump discontinuity

$x = 1$ jump discontinuity

10. Determine whether the function $g(x) = \begin{cases} \dfrac{x^2-16}{x-4} & \text{if } x \neq 4 \\ 9 & \text{if } x = 4 \end{cases}$ is continuous at $a=4$ using the continuity checklist to justify your answer.

$g(4) = 9$

$\lim_{x\to4} \frac{x^2-16}{x-4} = \lim_{x\to4} \frac{(x-4)(x+4)}{x-4} = \lim_{x\to4}(x+4) = 8$

$g(4) \neq \lim_{x\to4} g(x)$
so not continuous at $x=4$

11. Determine the intervals on which the function $f(z) = (z-1)^{\frac{3}{4}}$ is continuous. Use proper brackets/parentheses to specify left or right continuity or not.

root functions are continuous on domain

$(z-1)^{3/4} = \sqrt[4]{(z-1)^3}$ requires $(z-1)^3 \geq 0$
$z-1 \geq 0$
$z \geq 1$

continuous on $[1, \infty)$

12. Find a value of the constant k for which the function $f(x) = \begin{cases} x^2, & x \leq 7 \\ x+k, & x > 7 \end{cases}$ is continuous at $x=7$

$f(7) = 7^2 = 49$

$\lim_{x\to7^-} x^2 = 49$

$\lim_{x\to7^+} (x+k) = 7+k$

$\lim_{x\to7} f(x)$ exists when $49 = 7+k$
$k = 42$

13. Let $f(x)$ be the piecewise function defined by $f(x) = \begin{cases} k\tan\left(\dfrac{\pi x}{3}\right) & x \geq 1 \\ x-2 & x < 1 \end{cases}$ where k is a constant.

Find the value of k that will make the function continuous at $x=1$. Support your answer with limits.

$f(1) = k \cdot \tan\frac{\pi}{3} = k \cdot \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = k\cdot\sqrt{3}$

$\lim_{x\to1^-}(x-2) = -1$

$\lim_{x\to1^+} k\tan\left(\frac{\pi x}{3}\right) = k\cdot\sqrt{3}$

$\lim_{x\to1} f(x)$ exists when $\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x)$

$-1 = k\cdot\sqrt{3}$

$k = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

14. Define $f(2)$ in a way that extends $f(x) = \dfrac{x^2+x-6}{x^2-4}$ to be continuous at $x=2$. Support your answer with limits.

$\lim_{x\to2} \frac{x^2+x-6}{x^2-4} = \lim_{x\to2} \frac{(x+3)(x-2)}{(x-2)(x+2)} = \lim_{x\to2} \frac{x+3}{x+2} = \frac{5}{4}$

so $f(x) = \begin{cases} \dfrac{x^2+x-6}{x^2-4} & x \neq 2 \\ \dfrac{5}{4} & x = 2 \end{cases}$

Calculus section 3.1: Introducing the Derivative

Objective	Correlated MyLab Math Problems	Calculus
Answer conceptual questions involving tangent lines and derivatives.	5	
Solve applications involving the use of limits to calculate derivatives.	13, 51	
Use limit definitions to find equations of tangent lines.	15, 21, 25, 27, 29, 31, 35, 37, 41, (44)	
Use limit definitions to evaluate derivatives at given points.	44	
Compute average and instantaneous rates of change from graphs and tables. (Review of skill from Section 2.1)	52	
Determine functions given limits of difference quotients.	57, 60	
(*Review: Evaluate limits at infinity.)	(2.5.20)	
(*Review: Find horizontal and vertical asymptotes of functions.)	(2.5.77)	

- Find the slope of the tangent line at a given point P (answers are numbers)
  - you must do the long limit definition when the directions tell you to.
  - realize that there are 2 forms. You need to be able to use both.
  - variety of limit methods used within these problems: expansion and simplification, conjugation, common denominator
- Find the equation of tangent line to a curve at a given point

- Understand that these long definitions for finding the slope of the tangent at a point P are finding the derivative value at that point.
- Be able to find $f'(a)$ with both forms.

- Understand and be able to use the parallels with these definitions for slope of a tangent and average velocity and instantaneous velocity.

1. Given a function $f(x)$ and a point a in its domain, what does $f'(a)$ represent?

$f'(a)$ is value of derivative at the point $x=a$

2. Explain the relationships among the slope of a tangent line, the instantaneous rate of change, and the value of the derivative at a point.

they are the same

3. Find the equation of the tangent line to the function $f(x) = -3x^2 - 5x + 1$ at $(1,-7)$ using the definition (both versions)

$$① \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{-3x^2-5x+1 - -7}{x-1} = \lim_{x\to 1} \frac{-(3x^2+5x-8)}{x-1}$$

$$= \lim_{x\to 1} \frac{-(3x+8)(x-1)}{x-1} = \lim_{x\to 1} -(3x+8) = -11$$

$$② \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{-3(1+h)^2-5(1+h)+1 - -7}{h}$$

$$= \lim_{h\to 0} \frac{-3(1+2h+h^2)-5-5h+8}{h} = \lim_{h\to 0} \frac{-11h-3h^2}{h}$$

$$= \lim_{h\to 0} (-11-3h) = -11$$

4. Find the equation of the tangent line to the function $f(x) = -\frac{1}{x^2}$ at $\left(3, -\frac{1}{9}\right)$ using the definition (both versions)

① $\lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = \lim_{x\to 3} \frac{-\frac{1}{x^2} - -\frac{1}{9}}{x-3} = \lim_{x\to 3} \frac{-9+x^2}{9x^2} \div (x-3)$

$= \lim_{x\to 3} \frac{(x-3)(x+3)}{9x^2} \cdot \frac{1}{x-3} = \lim_{x\to 3} \frac{(x+3)}{9x^2} = \frac{6}{81} = \frac{2}{27}$

② $\lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{-\frac{1}{(3+h)^2} - -\frac{1}{9}}{h} = \lim_{h\to 0} \frac{-9+(3+h)^2}{9(3+h)^2} \div h$

$= \lim_{h\to 0} \frac{-9+9+6h+h^2}{9(3+h)^2 \cdot h} = \lim_{h\to 0} \frac{6h+h^2}{9(3+h)^2 h} = \lim_{h\to 0} \frac{6+h}{9(3+h)^2}$

$= \frac{6}{9\cdot 9} = \frac{6}{81} = \frac{2}{27}$

5. a. Use $\lim_{x\to 3} \frac{f(x)-f(3)}{x-3}$ to find the slope of the tangent to the graph of $f(x) = x^2+3x+1$ at the point $(3,19)$.

$\lim_{x\to 3} \frac{x^2+3x+1-19}{x-3} = \lim_{x\to 3} \frac{x^2+3x-18}{x-3} = \lim_{x\to 3} \frac{(x+6)(x-3)}{x-3}$

$= \lim_{x\to 3} (x+6) = 9$

so equation of tangent at $x=3$ is

$y-19 = 9(x-3)$

b. Find an equation of the line tangent to the graph of $f(x) = x^2+3x+1$ at the point $(3,19)$.

6. Look back at 3.1 L&LA #10 (3.1:#7-10) and 3.1 L&LA #13 (3.1:#48-51)

Calculus section 3.2: The Derivative as a Function

Objective	Correlated MyLab Math Problems	Calculus
Answer conceptual questions involving the derivative as a function.	1, 7, 8	
Obtain the graphs of derivative functions from graphs of functions.	15, 17, 51	
Find points where functions are continuous and differentiable.	19, 54, 77	71
Find derivatives of functions using limits.	29, 35, 37, 39	43
Solve applications involving derivatives as functions.	55	41
Use graphs of functions to analyze slopes of tangent lines.	45, (73)	47, 48, 49
Obtain graphs of functions from graphs of their derivative function.	60	62
Find equations of normal lines.	63, 66	
Find vertical tangent lines from graphs.	73	
(*Review: Find points of discontinuity or intervals of continuity.)	(2.6.28)	
(*Review: Evaluate limits using continuity principles.)	(2.6.57)	

• find the derivative FUNCTION – use "x" instead of a specific value – you must do the long limit definition when the directions say "use the definition . . . "

  • variety of limit methods used within these problems: expansion and simplification, conjugation, common denominator

• Find the equation of tangent line to a curve at a given point.
• Find the equation of normal line to a curve at a given point.

• be sure you are aware of variety of notations and wordings for derivative. You should understand and be able to use all of them.

• Sketch derivative graphs or match functions to their derivatives
  know properties of functions and corresponding properties of the derivative graphs

• Differentiability
  graphically – not differentiable when not continuous, sharp corner, cusp, vertical tangent
  analytically – not differentiable when derivative domain "has an issue"

1. Find the derivative of the function $f(x) = x^2 - 2x + 3$ using the definition.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - 2(x+h) + 3 - (x^2-2x+3)}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-2x-2h+3-x^2+2x-3}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2-2h}{h} = \lim_{h\to 0}(2x+h-2) = 2x-2$$

2. Find the derivative of the function $f(x)=\frac{1}{\sqrt{x}}$ using the definition.

$$f'(x)=\lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}=\lim_{h\to 0}\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\div h$$

$$=\lim_{h\to 0}\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}\cdot h}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}=\lim_{h\to 0}\frac{x-(x+h)}{\sqrt{x}\sqrt{x+h}\cdot h(\sqrt{x}+\sqrt{x+h})}$$

$$=\lim_{h\to 0}\frac{-h}{\sqrt{x}\sqrt{x+h}\cdot h(\sqrt{x}+\sqrt{x+h})}=\lim_{h\to 0}\frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

$$=\frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x}+\sqrt{x})}=\frac{-1}{x(2\sqrt{x})}=\frac{-1}{2x^{3/2}}$$

3. Differentiate $g(z)=1+\sqrt{4-z}$ using the definition. Then find an equation for the tangent line at P(3,2)

$$g'(z)=\lim_{h\to 0}\frac{1+\sqrt{4-z-h}-(1+\sqrt{4-z})}{h}=\lim_{h\to 0}\frac{\sqrt{4-z-h}-\sqrt{4-z}}{h}\cdot\frac{\sqrt{4-z-h}+\sqrt{4-z}}{\sqrt{4-z-h}+\sqrt{4-z}}$$

$$=\lim_{h\to 0}\frac{4-z-h-(4-z)}{h(\sqrt{4-z-h}+\sqrt{4-z})}=\lim_{h\to 0}\frac{-h}{h(\sqrt{4-z-h}+\sqrt{4-z})}$$

$$=\lim_{h\to 0}\frac{-1}{\sqrt{4-z-h}+\sqrt{4-z}}=\frac{-1}{2\sqrt{4-z}}$$

pt (3,2)

$$m_{tan}\big|_{x=3}=\frac{-1}{2\sqrt{4-3}}=\frac{-1}{2}$$

equation of tangent to $g(z)$ at $x=3$ is

$$y-2=-\frac{1}{2}(x-3)$$

4. Use the limit definition of the derivative to find the derivative of $f(x)=\frac{2x-1}{x+3}$. Then find the equation of the tangent line at $x=-2$

$$f'(x)=\lim_{h\to 0}\frac{\frac{2x+2h-1}{x+h+3}-\frac{2x-1}{x+3}}{h}=\lim_{h\to 0}\frac{(2x+2h-1)(x+3)-(2x-1)(x+h+3)}{(x+h+3)(x+3)h}$$

$$=\lim_{h\to 0}\frac{2x^2+2xh-x+6x+6h-3-(2x^2+2xh+6x-x-h-3)}{(x+h+3)(x+3)h}$$

$$=\lim_{h\to 0}\frac{\cancel{2x^2}+\cancel{2xh}+\cancel{5x}+6h-\cancel{3}-\cancel{2x^2}-\cancel{2xh}-\cancel{5x}+h+\cancel{3}}{(x+h+3)(x+3)h}$$

$$=\lim_{h\to 0}\frac{7h}{(x+h+3)(x+3)h}=\lim_{h\to 0}\frac{7}{(x+h+3)(x+3)}=\frac{7}{(x+3)^2}$$

5. If $f(x)$ is differentiable at $x = a$, must $f(x)$ be continuous at $x = a$? Give an example.

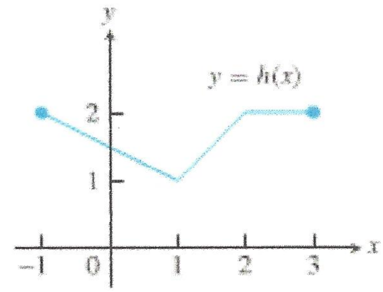
yes

6. If $f(x)$ is continuous at $x = a$, must $f(x)$ be differentiable at $x = a$? Give an example.

no ex: $y=|x|$  continuous at $x=0$ but not differentiable at $x=0$

7. Where is this function
a. continuous? (use interval notation)

$[-1,3]$



b. NOT differentiable?

sharp points $x=1$
$x=2$

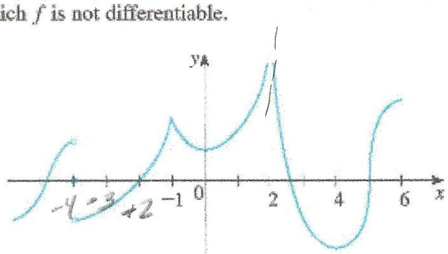
8.

Is it differentiable? Is $f(x) = \dfrac{x^2 - 5x + 6}{x - 2}$ differentiable at $x = 2$? Justify your answer.

$f(x)$ is not differentiable at $x=2$
b/c it is not continuous at $x=2$
b/c $f(2)$ is undefined

9.

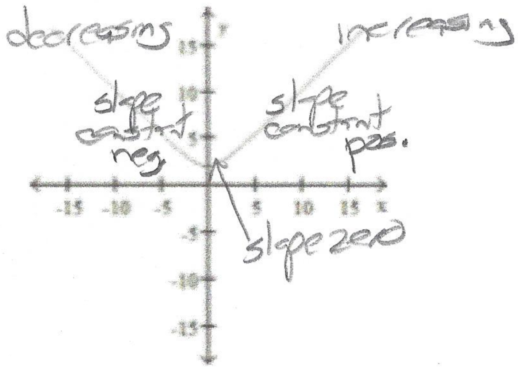
The graph of f is shown. State, with reasons, the numbers at which f is not differentiable.



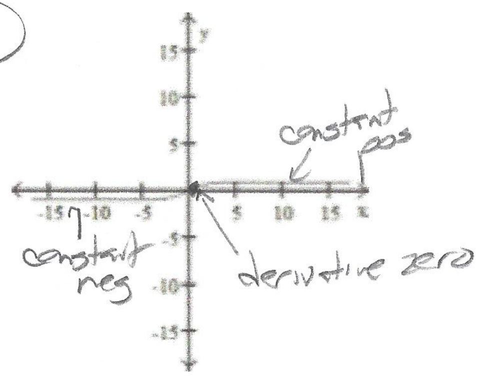
$x=-4$ jump discontinuity (pieces do not connect)
$x=-1$ cusp (sharp point, no concavity change)
$x=2$ infinite discontinuity (vertical asymptote)
$x=5$ vertical tangent (suddenly steep)

10.

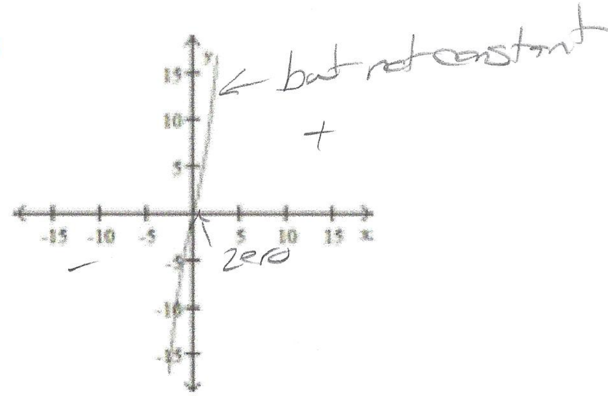
The graph of a function is given. Choose the answer that represents the graph of its derivative.



a)



b)



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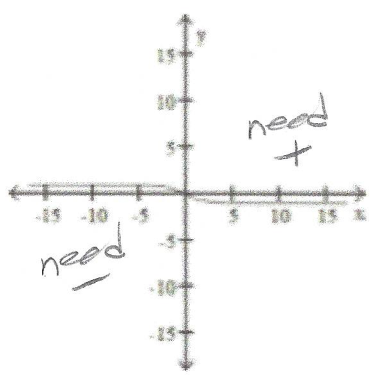

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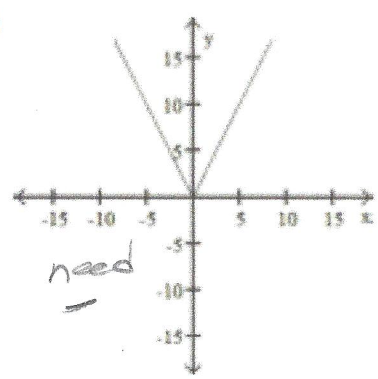
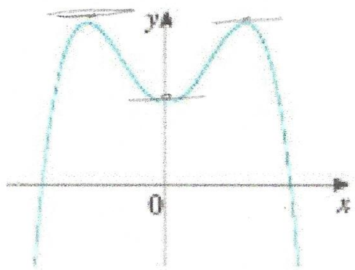
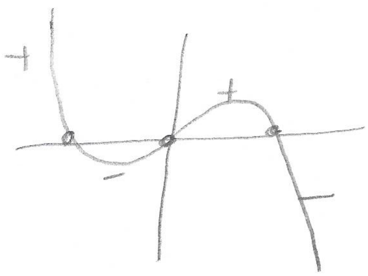


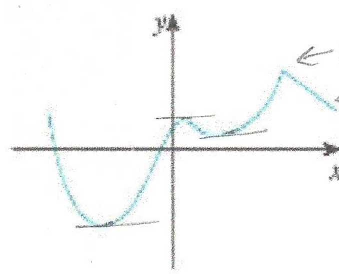
11. Sketch the derivative graph

a. (Ch. 2 Rev. #43)

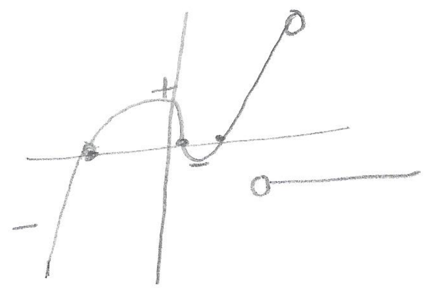




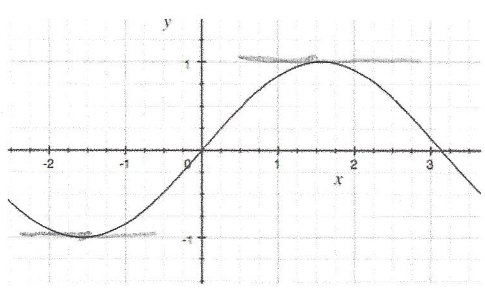
b. (Ch. 2 Rev. #44)

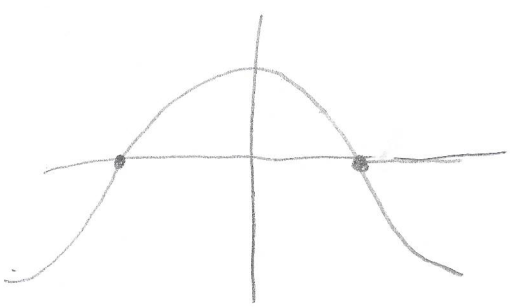


← not differentiable
← negative constant slope

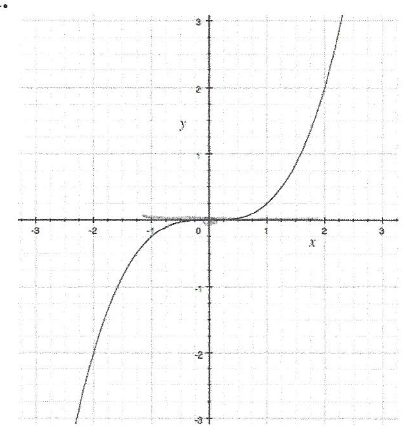


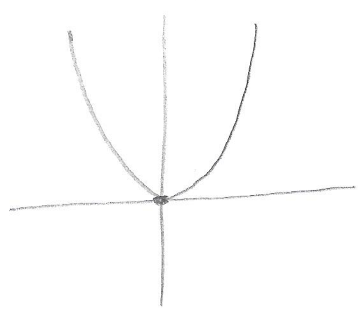
c.





d.





Calculus section 3.3 & 3.4: Rules of Differentiation

Objective	**Correlated MyLab Math Problems**	**Calculus**
Answer conceptual questions involving rules of differentiation.	4, 5	
Use graphs and tables to find derivatives.	11, 14	
Find derivatives using rules of differentiation.	21, 28, 29, 33, 35	
Solve applications involving rules of differentiation.	44	
Simplify products and quotients to find their derivatives.	47, 49, 51, 57	
Use derivatives to find slope locations and equations of tangent lines.	61, 63	67
Find higher order derivatives of functions.	68, 72	78, 79, 80, 81
Use derivatives to evaluate limits.	82, 85	
Use a calculator to approximate limits. (Review of skill from Section 2.1.)		91
(*Review: Evaluate two-sided limits using limit laws and theorems.)	(2.3.33)	
Objective	Correlated MyLab Math Problems	Calculus
Answer conceptual questions involving the product and quotient rules.	1, 2	
Find derivatives using two different methods.	10, 11, 16	
Find derivatives of products and quotients of functions involving exponentials.	20, 24, 25, 37, 48	
Find derivatives of products and quotients of algebraic expressions.	21, 22, 29, 43, 53	56
Find derivatives using the extended power rule.	39, 41	
Find slopes and equations of tangent lines of functions involving products and quotients.	63, 74, 92	
Solve applications involving the product rule and quotient rule.	65	
Find higher order derivatives of products and quotients.	71, 73	
Find derivatives of products and quotients using given values or graphs.	78, 81	
(*Review: Evaluate limits analytically.)	(2.4.27)	

The more derivatives you can practice the better.

- memorize the differentiation rules and know how to use them WELL
  - incorrect product rule or quotient rule will likely get you NO points (i.e. f'/g')
  - notate your derivative function or you will lose half your points

  (don't let it look like you think the function & derivative are the same)

- be sure you are aware of variety of notations and wordings for derivative. You should understand and be able to use all of them.

- for the power rule on negative exponents be sure you truly "take one off the power"

  ie $\frac{d}{dx}(x^{-4}) = -4x^{-5}$ NOT $-4x^{-3}$

- You may want to memorize $\frac{d}{dx}(\frac{1}{x}) = -\frac{1}{x^2}$ but be sure you understand why

- Think about whether you can simplify the function before you use product rule and/or quotient rule. Might make derivative much easier if you simplify first.

MATH 1040 Topical Review with problem set 2.6, 2.7, 3.1 - 3.6

DMS

- look at the problems where you are given values but not a specific function (or a table of values) to test your understanding of rules and notation.

Questions that get asked quite often:

- find equation of tangent line, also equation of normal line
- The tangent is horizontal when slope (derivative) is zero
  remember "point" should be presented as an ordered pair
- Also be prepared for problems that ask where slope has a specific value.
- Find a tangent line parallel to another line.

- Higher Order Derivatives - Take derivatives as many times as asked. Use/understand notation well.

- position, velocity, acceleration application

1a. How do you find the derivative of a constant multiplied by a function?

$y = c \cdot f(x)$
$y' = c \cdot f'(x)$

hold the multiplying constant times the derivative of the function

b. How do you find the derivative of the sum of two functions?

$y = f + g$
$y' = f' + g'$

derivative first + derivative second

c. How do you find the derivative of the product of two functions that are differentiable at a point?

$y = f \cdot g$
$y' = f \cdot g' + f' \cdot g$

keep the first times derivative of second + derivative first times keep second

d. How do you find the derivative of the quotient of two functions that are differentiable at a point?

$y = \frac{f}{g}$ $y' = \frac{g \cdot f' - f \cdot g'}{g^2}$

keep bottom times derivative top minus keep top times derivative bottom over bottom squared

e. How do you find the fifth derivative of a function?

derivative, its der = 2nd der, its der = 3rd der, it's der = 4th der, its der = 6th der.

2.

47. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. The derivative $\frac{d}{dx}(10^5)$ equals $5 \cdot 10^4$. F $y = 10^5$ $y' = 0$

b. The slope of a line tangent to the curve $y = 4x + 1$ is never 0. T $y' = 4$ so slope of line tangent to $y = 4x+1$ is always 4 never 0

c. The nth derivative $\frac{d^n}{dx^n}(5x^3 + 2x + 5)$ equals 0 for any integer $n \geq 3$.

$y = 5x^3 + 2x + 5$
$y' = 15x^2 + 2$
$y'' = 30x$
$y''' = 30$
$y^{(n)} = 0$ for $n \geq 4$ F

3 a. Give two ways to differentiate $f(x)=\dfrac{1}{x^{10}}$.

rewrite $f(x)=\dfrac{1}{x^{10}}=x^{-10}$ or quotient $f'(x)=\dfrac{x^{10}(0)-1\cdot 10x^9}{(x^{10})^2}=\dfrac{-10x^9}{x^{20}}=\dfrac{-10}{x^{11}}$

power $f'(x)=-10x^{-11}=\dfrac{-10}{x^{11}}$

b. Give two ways to differentiate $f(x)=(x-3)(x^2+4)$.

multiply first $f(x)=x^3-3x^2+4x-12$ or product $f'(x)=(x-3)(2x)+(1)(x^2+4)$

derivative $f'(x)=3x^2-6x+4$

$=2x^2-6x+x^2+4$

$=3x^2-6x+4$

4. Find derivatives with differentiation rules (not the limit definition of the derivative).

a. Find f' if $f(x)=-4x\sqrt[3]{x}-\dfrac{7}{5x}+x$

$f(x)=-4x^{4/3}-\frac{7}{5}x^{-1}+x$

$f'(x)=-\frac{16}{3}x^{1/3}+\frac{7}{5}x^{-2}+1$

$=-\frac{16\sqrt[3]{x}}{3}+\frac{7}{5x^2}+1$

b. Find $\dfrac{d}{dx}g(x)$ if $g(x)=\dfrac{2x^2-11x+12}{x-4}$

$g(x)=\dfrac{(2x-3)\cancel{(x-4)}}{\cancel{x-4}}=2x-3$

$g'(x)=2$

c. Find y'' if $y=\dfrac{x^3-2x}{17}=\frac{1}{17}x^3-\frac{2}{17}x$

$y'=\frac{3}{17}x^2-\frac{2}{17}$

$y''=\frac{6}{17}\cdot x$

d. Find $\dfrac{dp}{dx}$ if $p(x)=\dfrac{4x^3+3x+1}{2x^5}=2x^{-2}+\frac{3}{2}x^{-4}+\frac{1}{2}x^{-5}$

$p'(x)=-4x^{-3}-6x^{-5}-\frac{5}{2}x^{-6}$

$=-\frac{4}{x^3}-\frac{6}{x^5}-\frac{5}{2x^6}$

5. Find $h''(t)$ when $h(t)=\pi-\sqrt{t}+\pi e^t-e^7-\frac{1}{t^e}$

$h(t)=\pi-t^{1/2}+\pi e^t-e^7-t^{-e}$

$h'(t)=0-\frac{1}{2}t^{-1/2}+\pi e^t+0-e\cdot t^{e-1}$

$h''(t)=+\frac{1}{4}t^{-3/2}+\pi e^t-e(e-1)\cdot t^{e-2}$

6. Find $f'(x)$ if $f(x)=(7x^2-4x+2)(8x^5+\pi^4)$ Do NOT simplify.

$f'(x)=(7x^2-4x+2)(40x^4+0)+(14x-4)(8x^5+\pi^4)$

7 a. Find $g'(-1)$ if $g(x)=(x^3)^5+(2x)^4$

$g(x) = x^{15} + 16x^4$

$g'(x) = 15x^4 + 64x^3$

$g'(-1) = 15(-1)^4 + 64(-1)^3$

$= 15 - 64 = -49$

b. Find $\left.\frac{dy}{dx}\right|_{x=2}$ if $y=\frac{1}{x^4}+\frac{2x^{\frac{3}{2}}}{7}+4$ $= x^{-4} + \frac{2}{7}x^{3/2} + 4$

$y' = -4x^{-5} + \frac{6}{14}x^{1/2} + 0$

$= -\frac{4}{x^5} + \frac{3}{7}\sqrt{x}$

$y'(2) = -\frac{4}{32} + \frac{3}{7}\sqrt{2} = -\frac{1}{8} + \frac{3}{7}\sqrt{2}$

8. If $f(3)=4$, $g(3)=2$, $f'(3)=-6$, $g'(3)=5$ Find

a. $(f+g)'(3)$

$y = f(x) + g(x)$

$y' = f'(x) + g'(x)$

$y'(3) = f'(3) + g'(3)$

$= -6 + 5 = -1$

b. $\left(\frac{f}{g}\right)'(3)$

$y = \frac{f(x)}{g(x)}$ $y' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$y'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{[g(3)]^2}$

$= \frac{5 \cdot -6 - 4 \cdot 5}{2^2} = \frac{-30-20}{4} = -8$

9. If $f(x)=\sqrt{x}\cdot g(x)$ where $g(4)=8$ and $g'(4)=7$, find $f'(4)$

$f'(x) = \sqrt{x}\cdot g'(x) + \frac{1}{2}x^{-1/2}g(x)$

$= \sqrt{x}\cdot g'(x) + \frac{g(x)}{2\sqrt{x}}$

$f'(4) = \sqrt{4}\cdot g'(4) + \frac{g(4)}{2\sqrt{4}}$

$= 2\cdot 7 + \frac{8}{4} = 14 + 2 = 16$

10 a. Find the equation of the tangent line to $f(x)=x+\sqrt{x}$ at $x=1$. $f(1) = 1 + \sqrt{1} = 2$

$f'(x) = 1 + \frac{1}{2}x^{-1/2} = 1 + \frac{1}{2\sqrt{x}}$

$m_{tan}\big|_{x=1} = f'(1) = 1 + \frac{1}{2\sqrt{1}} = \frac{3}{2}$

equation of tangent at $x=1$ is

$y - 2 = \frac{3}{2}(x-1)$

b. Find the equation of the normal line to $f(x)=x+\sqrt{x}$ at $x=1$.

$m_{nor}\big|_{x=1} = -\frac{2}{3}$

equation of normal at $x=1$ is

$y - 2 = -\frac{2}{3}(x-1)$

11. If $g(x)=x\cdot f(x)$, where $f(3)=4$ and $f'(3)=-2$, find an equation of the **normal** line to the graph of $g(x)$ at the point where $x=3$. $g(3) = 3\cdot f(3) = 3\cdot 4 = 12$

$g'(x) = x\cdot f'(x) + 1\cdot f(x)$

$g'(3) = 3\cdot f'(3) + f(3) = 3(-2) + 4 = -6 + 4 = -2$

$m_{nor} = +\frac{1}{2}$

so equation of tangent at $x=3$ is

$y - 12 = \frac{1}{2}(x-3)$

12.

Consider the following table of values for $f(x), g(x), f'(x)$, and $g'(x)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
−2	3	1	−5	8
−1	−9	7	4	1
0	5	9	9	−3
1	3	−3	2	6
2	−5	3	8	0

a. Let $r(x) = x^2 \cdot g(x)$. Find $r'(-2)$.

$r'(x) = x^2 \cdot g'(x) + 2x\, g(x)$

$r'(-2) = 4 \cdot g'(-2) + 2(-2) \cdot g(-2)$

$= 4 \cdot 8 - 4 \cdot 1 = 32 - 4 = 28$

b. Let $p(x) = 3f(x) - 2g(x)$. Find $p'(0)$.

$p'(x) = 3f'(x) - 2g'(x)$

$p'(0) = 3 \cdot f'(0) - 2g'(0)$

$= 3 \cdot 9 - 2(-3) = 27 + 6$

$= 33$

c. Find $\left.\dfrac{d}{dx}\sqrt{3f(x)}\right|_{x=1}$ $\quad y = \sqrt{3f(x)}$

w/ chain rule $y' = \frac{1}{2}(3f(x))^{-1/2} \cdot 3f'(x) = \dfrac{3f'(x)}{2\sqrt{3f(x)}}$

$y'(1) = \dfrac{3f'(1)}{2\sqrt{3f(1)}} = \dfrac{3 \cdot 2}{2\sqrt{3 \cdot 3}} = \dfrac{6}{2 \cdot 3} = 1$

d. Let $h(x) = \dfrac{-2f(x)}{g(x)}$. Find $h'(-1)$.

$h'(x) = \dfrac{g(x)(-2f'(x)) - (-2f(x))g'(x)}{[g(x)]^2}$

$h'(-1) = \dfrac{-2g(-1)f'(-1) + 2f(-1)g'(-1)}{[g(-1)]^2}$

$= \dfrac{-2 \cdot 7 \cdot 4 + 2 \cdot -9 \cdot 1}{7^2} = \dfrac{-56 - 18}{49}$

$= \dfrac{-74}{49}$

13. For what values of x does the graph of $y = x^3 - x^2 - x + 1$ have a horizontal tangent?

horizontal tangents when $y' = 0$

$y' = 3x^2 - 2x - 1$

$3x^2 - 2x - 1 = 0$

$(3x + 1)(x - 1) = 0$

$x = -\frac{1}{3} \quad x = 1$

14. For what values of x does the graph of $y = \frac{1}{3}x^3 - 5x^2 + 28x$ have tangent lines with slope of 3.

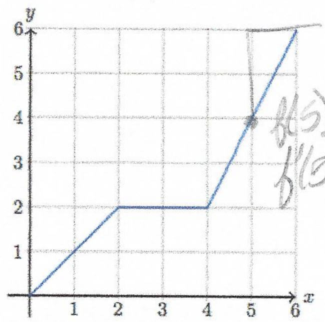
$y' = \frac{1}{3} \cdot 3x^2 - 10x + 28$

$x^2 - 10x + 28 = 3$

$x^2 - 10x + 25 = 0$

$(x-5)^2 = 0 \qquad x = 5$

15. Let $f(x)$ be the function shown below defined on $[0,6]$.



$f(5) = 4$

$f'(5) = 2$

a. Find $h'(5)$ if $h(x) = \frac{5}{x} + \sqrt{x}f(x) - \pi^2$

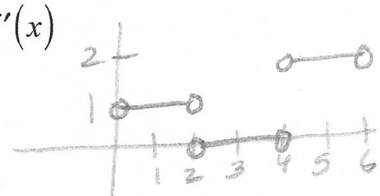
$h'(x) = -\frac{5}{x^2} + \sqrt{x}\, f'(x) + \frac{1}{2}x^{-1/2} f(x) + 0$

$= \frac{-5}{x^2} + \sqrt{x}\, f'(x) + \frac{f(x)}{2\sqrt{x}}$

$h'(5) = \frac{-5}{5^2} + \sqrt{5} \cdot f'(5) + \frac{f(5)}{2\sqrt{5}}$

$= -\frac{1}{5} + \sqrt{5} \cdot 2 + \frac{4}{2\sqrt{5}} = -\frac{1}{5} + 2\sqrt{5} + \frac{2}{\sqrt{5}}$

b. Sketch $f'(x)$



16. Let $f(x) = \frac{4e^x}{1+e^x}$. Find all the x-values where $f(x)$ has a slope of 1.

$f'(x) = \frac{(1+e^x)4e^x - 4e^x(e^x)}{(1+e^x)^2} = \frac{4e^x + 4e^{2x} - 4e^{2x}}{(1+e^x)^2} = \frac{4e^x}{(1+e^x)^2}$

$f'(x) = 1$ when $\frac{4e^x}{(1+e^x)^2} = 1$

$4e^x = (1+e^x)^2$

$4e^x = 1 + 2e^x + e^{2x}$

$0 = e^{2x} - 2e^x + 1$

$0 = (e^x - 1)(e^x - 1)$

$e^x - 1 = 0$

$e^x = 1$

$x = \ln 1 = 0$

Calculus section 3.5: Derivatives of Trigonometric Functions

Objective	Correlated MyLab Math Problems	Calculus
Answer conceptual questions involving derivatives of trigonometric functions.	1, 8, 54, 72, 31, 78	79
Find limits involving trigonometric functions.	12, 13, 14, 16, 21, 66, 70	
Find derivatives of basic trigonometric functions.	23, 43	
Find derivatives of products, quotients, and powers of functions with trigonometric expressions.	25, 29, 31, 37, 40, 44, 48	50
Solve applications involving derivatives of trigonometric functions.	55	
Find higher order derivatives of functions involving trigonometric functions.	59	
(*Review: Find derivatives of functions using limits.)	(3.2.25)	
(*Review: Evaluate limits at infinity.)	(2.5.17)	
(*Review: Find horizontal and vertical asymptotes of functions.)	(2.5.75)	

- Memorize the differentiation rules for the trig functions and know how to use them well

Questions that get asked quite often:
- find equation of tangent line, also equation of normal line
- The tangent is horizontal when slope (derivative) =zero
  remember "point" should be presented as an ordered pair
- Also be prepared for problems that ask where slope has a specific value.
- Find a tangent line parallel to another line.

- Higher Order Derivatives – Take derivatives as many times as asked. Use/understand notation well. Higher order derivatives for sine and cosine are cyclic.

- Trig Limits – remember the basic $\lim_{x\to 0}\frac{\sin x}{x}=1$ and $\lim_{x\to 0}\frac{\cos x-1}{x}=0$
  - the rest of the trig limit problems presented in this section relate back to these theorems.
  - Show work and good notation (only 1 pt for final answer)
  - if you have the limits of a multiplication break it into two limits multiplied
  - remember you can take a constant through the limit but not variables.
  - remember you CANNOT put things into or pull out of a trig function's argument.
  - remember $\sin 2x \neq 2\sin x$. (2 inside changes period, 2 outside changes amplitude)

Please remember that you cannot separate a trig function from its argument. Three letters can not stand alone as a trig function.

1. Where does $f(x)=\sin x$ have a horizontal tangent line(s)?

horizontal tangents when $f'(x)=0$

$f'(x)=\cos x$ $\qquad \cos x = 0$

$x=\frac{\pi}{2}, \frac{3\pi}{2}$, etc

$x=\frac{\pi}{2}+\pi k$ $\quad k:$ integer

2.a. Can you state the 6 trig derivatives in less than 30 seconds?

answer should be yes

b. Can you state the sine, cosine, tangent of 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π in less than 30 seconds?

answer should be yes

c. How about sine, cosine, tangent for $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \ldots$??

maybe not 30 seconds but should be able to do relatively easily

3.

37. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. $\frac{d}{dx}(\sin^2 x) = \cos^2 x$ F $y = \sin^2 x = (\sin x)^2$ $y' = 2\sin x \cos x$

b. $\frac{d^2}{dx^2}(\sin x) = \sin x$ F $y = \sin x$ $y' = \cos x$ $y'' = -\sin x$

c. $\frac{d^4}{dx^4}(\cos x) = \cos x$ T $y = \cos x$ $y' = -\sin x$ $y'' = -\cos x$ $y''' = +\sin x$ $y^{(4)} = \cos x$

d. The function $\sec x$ is not differentiable at $x = \pi/2$.

T b/c $\frac{\pi}{2}$ is not in domain of $\sec x$

4. Find the first derivative of the following

a. $s(t) = t^4 - \sec t + e^5$

$s'(t) = 4t^3 - \sec t \tan t + 0$

b. $s(t) = \frac{1+\csc t}{1-\csc t}$

$s'(t) = \frac{(1-\csc t)(-\csc t \cot t) - (1+\csc t)(+\csc t \cot t)}{(1-\csc t)^2}$

$= \frac{-2\csc t \cot t}{(1-\csc t)^2}$

c. $f(x) = \frac{3\cot x}{x}$

$f'(x) = \frac{-x 3\csc^2 x - 3\cot x}{x^2}$

$= \frac{-3x\csc^2 x - 3\cot x}{x^2}$

d. $y = 4x^5 \cos x + 3e^6 + \tan \pi$

$y' = 4x^5(-\sin x) + 20x^4 \cos x + 0 + 0$

$y' = -4x^5 \sin x + 20x^4 \cos x$

e. $y = \pi^2 \tan x - \frac{\sin x}{x}$

$y' = \pi^2 \sec^2 x - \left(\frac{x\cos x - \sin x \cdot 1}{x^2}\right)$

f. Find $y^{(72)}$ for $y = \cos x$

$72 \div 4 = 18$ R0

$y^{(72)} = y^{(4)}$

$y = \cos x$

$y' = -\sin x$

$y'' = -\cos x$

$y''' = +\sin x$

$y^{(4)} = \cos x$

5. If $g(\theta)=\theta\sin\theta$, find $g''\left(\frac{\pi}{6}\right)$.

$g'(\theta)=\theta\cos\theta+1\cdot\sin\theta$

$g''(\theta)=-\theta\sin\theta+1\cdot\cos\theta+\cos\theta$

$=-\theta\sin\theta+2\cos\theta$

$g''\left(\frac{\pi}{6}\right)=-\frac{\pi}{6}\cdot\sin\left(\frac{\pi}{6}\right)+2\cos\left(\frac{\pi}{6}\right)$

$=-\frac{\pi}{6}\cdot\frac{1}{2}+2\cdot\frac{\sqrt{3}}{2}$

$=-\frac{\pi}{12}+\sqrt{3}$

6. Find the equation of the tangent line to $y=\tan x$ at $x=\frac{\pi}{4}$ $\quad y\left(\frac{\pi}{4}\right)=\tan\frac{\pi}{4}=1$

$y'=\sec^2 x$

$y'\left(\frac{\pi}{4}\right)=m_{tan}\big|_{x=\frac{\pi}{4}}=\left(\sec\frac{\pi}{4}\right)^2=\frac{1}{\left(\cos\frac{\pi}{4}\right)^2}=\left(\frac{1}{\frac{\sqrt{2}}{2}}\right)^2=\left(\frac{2}{\sqrt{2}}\right)^2=\sqrt{2}^2=2$

equation of tangent at $x=\frac{\pi}{4}$ is

$y-1=2\left(x-\frac{\pi}{4}\right)$

7. Give an equation of the tangent line to $f(x)=2\cos x-1$ at $x=\frac{2\pi}{3}$. $\quad f\left(\frac{2\pi}{3}\right)=2\cos\frac{2\pi}{3}-1$

$=2\left(-\frac{1}{2}\right)-1=-2$

$f'(x)=-2\sin x$

$f'\left(\frac{2\pi}{3}\right)=-2\sin\frac{2\pi}{3}=-2\left(\frac{\sqrt{3}}{2}\right)=-\sqrt{3}$

so equation of tangent at $x=\frac{2\pi}{3}$ is

$y+2=-\sqrt{3}\left(x-\frac{2\pi}{3}\right)$

8.

65. At what points on the curve $y=\sin x+\cos x$, $0\leqslant x\leqslant 2\pi$, is the tangent line horizontal?

$y'=\cos x-\sin x$ $\quad$ horizontal tangent when $y'=0$

$\cos x-\sin x=0$

$\cos x=\sin x$

both positive $\quad$ both negative

$x=\frac{\pi}{4}$ $\left(\frac{\pi}{4},\sqrt{2}\right)$ $\quad$ $x=\frac{5\pi}{4}$ $\left(\frac{5\pi}{4},-\sqrt{2}\right)$

$y=\sin\frac{\pi}{4}+\cos\frac{\pi}{4}$

$=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=\sqrt{2}$

$y=\sin\frac{5\pi}{4}+\cos\frac{5\pi}{4}$

$=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}=-\sqrt{2}$

9. Find the x-values of the point(s) where the graph of $f(x)=x+2\sin x$ has horizontal tangent lines on the interval $[0,2\pi]$.

horizontal tangent when $f'(x)=0$ $\quad f'(x)=1+2\cos x$

$1+2\cos x=0$

$2\cos x=-1$

$\cos x=-\frac{1}{2}$

$x=\frac{2\pi}{3},\ \frac{4\pi}{3}$

10. Evaluate $\lim_{t \to 0} \frac{\tan 6t}{5 \sin 2t}$

$$\lim_{t\to 0} \frac{\tan 6t}{\sin 2t} = \lim_{t\to 0} \frac{\sin 6t}{\cos 6t} \div \sin 2t$$

$$= \lim_{t\to 0} \frac{\sin 6t}{\cos 6t \cdot \sin 2t} \cdot \frac{t}{t}$$

$$= \lim_{t\to 0} \frac{\sin 6t}{t} \cdot \lim_{t\to 0} \frac{1}{\cos 6t} \cdot \lim_{t\to 0} \frac{t}{\sin 2t}$$

$$= \frac{6}{2} \lim_{t\to 0} \frac{\sin 6t}{6t} \cdot \lim_{t\to 0} \frac{1}{\cos 6t} \cdot \lim_{t\to 0} \frac{2t}{\sin 2t}$$

$$= 3 \cdot 1 \cdot \frac{1}{\cos 0} \cdot 1 = 3$$

11. Evaluate $\lim_{t\to 0} \frac{t^3}{\tan^3 2t}$.

$$= \left(\lim_{t\to 0} \frac{t}{\tan 2t}\right)^3 = \left(\frac{1}{2} \cdot \lim_{t\to 0} \frac{2t}{\tan 2t}\right)^3$$

$$= \left(\frac{1}{2} \cdot 1\right)^3 = \frac{1}{8}$$

Look back at 3.5b L&LA for more of these special trig limits.