Teaching Statement

Michael Nelson

As a passionate mathematician and educator, I am thrilled to have the opportunity to share my knowledge and love for mathematics with students. My teaching philosophy centers around fostering a deep understanding of mathematical concepts, nurturing critical thinking skills, and promoting an inclusive and engaging learning environment. Through my teaching experiences at Clemson University and my dedication to continuous improvement, I have developed a pedagogical approach that encourages student exploration and empowers them to become confident problem solvers.

During my time at Clemson University, I had the privilege of teaching a range of undergraduate mathematics courses, including Business Calculus (Math 1020), Pre-Calculus and Introductory Differential Calculus (Math 1040), and Differential and Integral Calculus (Math 1070).

- In my Business Calculus classes, I recognized the importance of connecting mathematical concepts to real-world applications. By emphasizing the practical relevance of the subject matter, I motivated students to see the direct impact of calculus in their chosen fields of study. I incorporated relevant examples and case studies to demonstrate the applicability of the concepts, enabling students to develop a deeper appreciation for the subject and its relevance to their academic and professional pursuits.
- Teaching Pre-Calculus and Introductory Differential Calculus further allowed me to scaffold students' mathematical understanding and bridge the gap between high school and college-level mathematics. I focused on building a strong foundation in fundamental concepts while simultaneously introducing new topics and problem-solving techniques. Through clear explanations, guided practice, and targeted feedback, I helped students develop the necessary skills to excel in subsequent calculus courses and beyond.
- In teaching Differential and Integral Calculus, I aimed to cultivate students' analytical thinking and problem-solving abilities. I encouraged active participation through group discussions, in-class exercises, and thought-provoking questions that prompted students to explore different approaches to problem-solving. By fostering an environment that valued collaboration and open communication, I witnessed the growth of students' mathematical reasoning and their ability to think critically about complex mathematical problems.

My teaching approach also extends beyond the classroom. I am committed to providing support and mentorship to students outside of formal instructional settings. I actively engage in office hours, one-on-one meetings, and online platforms to ensure that students have opportunities to seek clarification, receive individualized guidance, and discuss their mathematical inquiries. By establishing a welcoming and supportive learning environment, I strive to inspire confidence in students and foster a sense of belonging in the mathematical community.

Math 1070 Test Review Sample

Something that I believe I excel at is writing things down for students in the form of expository notes. In this section, I provide an example of a test review that I wrote for my Math 1070 students.

L'Hospital Rule

Exercise 1. Is 0^0 considered an indeterminate form? Explain why or why not.

Solution 1. Yes. Essentially, this is because we can take log on both sides to turn it into a $-\infty/\infty$ situation. Very informally, this looks like:

$$\begin{aligned}\ln(0^0) &= 0\ln(0)\\ &= 0\cdot -\infty\\ &= \frac{-\infty}{\frac{1}{0}}\\ &= \frac{-\infty}{\infty}\end{aligned}$$

For instance, if we wanted to compute $\lim_{x\to 0} x^x$. Then we'd first try to do it naively as follows:

$$\begin{aligned}\lim_{x\to 0}(x^x) &= (\lim_{x\to 0} x)^{(\lim_{x\to 0} x)}\\ &= 0^0.\end{aligned}$$

Arriving at the indeterminate form 0^0, we see that we should probably consider L'Hopital's rule. To do so though, we first need to translate the indeterminate form 0^0 to the indeterminate form $-\infty/\infty$:

$$\begin{aligned}\ln\left(\lim_{x\to 0}(x^x)\right) &= \lim_{x\to 0}(\ln(x^x)) && \text{we can do this because log is continuous}\\ &= \lim_{x\to 0}(x\ln x)\\ &= \lim_{x\to 0}\left(\frac{\ln x}{\frac{1}{x}}\right) && \text{this has indeterminate form } -\infty/\infty\\ &= \lim_{x\to 0}\left(\frac{\frac{1}{x}}{-\frac{1}{x^2}}\right) && \text{L'Hopital's rule}\\ &= \lim_{x\to 0}(-x)\\ &= 0.\end{aligned}$$

It follows that

$$\begin{aligned}\lim_{x\to 0}(x^x) &= e^{\ln(\lim_{x\to 0}(x^x))}\\ &= e^0\\ &= 1.\end{aligned}$$

Exercise 2. Is $\infty/0$ considered an indeterminate form? Explain why or why not.

Solution 2. No. For instance, consider

$$\lim_{x\to\infty}\frac{x}{(\sin x)/x}.$$

The limit in the denominator goes towards 0 since $|\sin x| \le 1$, whereas the limit in the numerator goes towards ∞. So one could say this limit has the form $\infty/0$. However, we cannot use L'Hopital's rule to evaluate this limit. In fact, this limit doesn't even exist: look at its graph of wolframalpha to see why:

https://www.wolframalpha.com/input/?i=%28x%2F%28sin%28x%29%2Fx%29

Exercise 3. Evaluate the limit $\lim_{x\to 1}\frac{\ln x}{x-1}$.

Solution 3. First we try to do this naively by taking the limit of the numerator and denominator separately as follows:

$$\begin{aligned}\lim_{x\to 1}\frac{\ln x}{x-1} &= \frac{\lim_{x\to 1}(\ln x)}{\lim_{x\to 1}(x-1)}\\ &= \frac{0}{0}.\end{aligned}$$

We get the indeterminate form $0/0$. Thus we should use L'Hospital's rule:

$$\begin{aligned}
\lim_{x\to 1}\frac{\ln x}{x-1} &= \lim_{x\to 1}\frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(x-1)} \\
&= \lim_{x\to 1}\frac{\frac{1}{x}}{1} \\
&= \lim_{x\to 1}\frac{1}{x} \\
&= 1.
\end{aligned}$$

Exercise 4. Does L'Hospital's rule apply to $\lim_{x\to 0} e^{\sin x}/x$?

Solution 4. No, because when we try to do the naive limit we do not get an indeterminate form:

$$\begin{aligned}
\lim_{x\to 0}\frac{e^{\sin x}}{x} &= \frac{\lim_{x\to 0}(e^{\sin x})}{\lim_{x\to 0}(x)} \\
&= \frac{(e^{\lim_{x\to 0}(\sin x)})}{\lim_{x\to 0}(x)} && \text{why am I allowed to bring lim inside } e^x\text{?} \\
&= \frac{e^0}{0} \\
&= \frac{1}{0}.
\end{aligned}$$

L'Hospital's rule doesn't apply here, but it's easy to see that this limit is ∞.

Exercise 5. Evaluate the limit

$$\lim_{x\to\infty}\left((x^2+1)^{1/x}\right).$$

Solution 5. We try the naive way first:

$$\begin{aligned}
\lim_{x\to\infty}\left((x^2+1)^{1/x}\right) &= \left(\lim_{x\to\infty}(x^2+1)\right)^{\left(\lim_{x\to\infty} 1/x\right)} \\
&= \infty^0.
\end{aligned}$$

We arrive at the indeterminate form ∞^0. So we need to apply L'Hopital's rule, but first we need to bring the $1/x$ down from the exponent using the log function:=

$$\begin{aligned}
\ln\left(\lim_{x\to\infty}(x^2+1)^{1/x}\right) &= \lim_{x\to\infty}\left(\ln\left((x^2+1)^{1/x}\right)\right) \\
&= \lim_{x\to\infty}\left(\frac{\ln(x^2+1)}{x}\right) \\
&= \lim_{x\to\infty}\left(\frac{\frac{2x}{x^2+1}}{1}\right) && \text{L'Hopital's rule} \\
&= \lim_{x\to\infty}\left(\frac{2x}{x^2+1}\right) \\
&= \lim_{x\to\infty}\left(\frac{2}{2x}\right) && \text{L'Hopital's rule} \\
&= \lim_{x\to\infty}\left(\frac{1}{x}\right) \\
&= 0.
\end{aligned}$$

Therefore we have

$$\begin{aligned}
\lim_{x\to\infty}\left((x^2+1)^{1/x}\right) &= e^{\ln\left(\lim_{x\to\infty}(x^2+1)^{1/x}\right)} \\
&= e^0 \\
&= 1.
\end{aligned}$$

Exercise 6. Evaluate the limit

$$\lim_{x\to 1}(x-1)\sin(\pi x).$$

Solution 6.

Sums, Integrals, and Antiderivatives

Exercise 7. Suppose $f'(x) = \frac{1}{1+3x} + 2e^x$ and $f(0) = 1$. Find $f(x)$.

Solution 7. First we'll find the general antiderivative; $f(x)$ has the form

$$f(x) = \frac{1}{3}\ln(1+3x) + 2e^x + C \tag{1}$$

where C is some constant to be determined. You should check that (1) really does differentiate to $f'(x)$. Now we need to determine what C is; we do this by using the fact that $f(0) = 1$:

$$\begin{aligned} f(0) = 1 &\implies \frac{1}{3}\ln(1+3\cdot 0) + 2e^0 + C = 1 \\ &\implies \frac{1}{3}\ln(1) + 2 + C = 1 \\ &\implies 2 + C = 1 \\ &\implies C = -1. \end{aligned}$$

Therefore

$$f(x) = \frac{1}{3}\ln(1+3x) + 2e^x - 1.$$

Exercise 8. Verify that $\int(\sqrt{4-x^2})\mathrm{d}x = \frac{1}{2}x\sqrt{4-x^2} + 2\arcsin(x/2) + C$.

Solution 8. This is left for you to do. Basically you just need to make sure that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}x\sqrt{4-x^2} + 2\arcsin(x/2) + C\right) = \sqrt{4-x^2}.$$

Exercise 9. Evaluate $\sum_{k=0}^{4} k^2 - k + 1$.

Solution 9. We'll compute the manually as follows:

$$\begin{aligned} \sum_{k=0}^{4} k^2 - k + 1 &= (0^2 - 0 + 1) + (1^2 - 1 + 1) + (2^2 - 2 + 1) + (3^2 - 3 + 1) + (4^2 - 4 + 1) \\ &= 1 + 1 + 3 + 7 + 13 \\ &= 25. \end{aligned}$$

Exercise 10. Suppose that $\int_0^1 (2f(x) + x)\mathrm{d}x = 3$ and $\int_2^1 f(x)\mathrm{d}x = 2$. Find $\int_0^2 f(x)\mathrm{d}x$.

Solution 10. Recall that integrals satisfy the linearity property. In particular, this means that if $\alpha, \beta \in \mathbb{R}$ and f, g are functions, then

$$\int_a^b (\alpha f(x) + \beta g(x))\mathrm{d}x = \alpha\int_a^b f(x)\mathrm{d}x + \beta\int_a^b g(x)\mathrm{d}x. \tag{2}$$

You should think about how this is related to the derivative satisfying the linear property:

$$\frac{\mathrm{d}}{\mathrm{d}x}(\alpha f(x) + \beta g(x)) = \alpha \frac{\mathrm{d}}{\mathrm{d}x}(f(x)) + \beta \frac{\mathrm{d}}{\mathrm{d}x}(g(x)).$$

Or how the limit operator satisfying the linear property:

$$\lim_{x\to a}(\alpha f(x) + \beta g(x)) = \alpha \lim_{x\to a}(f(x)) + \beta \lim_{x\to a}(g(x)).$$

There are many operators like these in mathematics which satisfy the linear property, and this helps simplify computations.

The integral satisfies other properties which are not shared by $\mathrm{d}/\mathrm{d}x$ or $\lim_{x\to a}$ though. For instance, the integral satisfies the following additivity property:

$$\int_a^b f(x)\mathrm{d}x + \int_b^c f(x)\mathrm{d}x = \int_a^c f(x)\mathrm{d}x. \tag{3}$$

It also satisfies the following sign change property:

$$\int_a^b f(x)\mathrm{d}x = -\int_b^a f(x)\mathrm{d}x. \tag{4}$$

There are many other properties which integrals satisfy, but we will only need (2), (3), and (4) to solve this problem. First, we use linearity of integration to obtain

$$\begin{aligned} 3 &= \int_0^1 (2f(x) + x)\mathrm{d}x \\ &= 2\int_0^1 f(x)\mathrm{d}x + \int_0^1 x\mathrm{d}x \\ &= 2\int_0^1 f(x)\mathrm{d}x + \frac{x^2}{2}\Big|_0^1 \\ &= 2\int_0^1 f(x)\mathrm{d}x + \frac{1}{2} - 0 \\ &= 2\int_0^1 f(x)\mathrm{d}x. \end{aligned}$$

It follows that $\int_0^1 f(x)\mathrm{d}x = 2/3$. Therefore

$$\begin{aligned} \int_0^2 f(x)\mathrm{d}x &= \int_0^1 f(x)\mathrm{d}x + \int_1^2 f(x)\mathrm{d}x \\ &= \int_0^1 f(x)\mathrm{d}x - \int_2^1 f(x)\mathrm{d}x \\ &= \frac{2}{3} - 2 \\ &= -\frac{4}{3}. \end{aligned}$$

Exercise 11. Evaluate $\int \frac{3x^3+1}{x}\mathrm{d}x$.

Solution 11. Remember that the notation $\int f(x)\mathrm{d}x$ means the family of antiderivatives of f. In this case, we have

$$\begin{aligned} \int \frac{3x^3+1}{x}\mathrm{d}x &= \int (3x^2 + x^{-1})\mathrm{d}x \\ &= 3\int x^2\mathrm{d}x + \int x^{-1}\mathrm{d}x \\ &= 3\left(\frac{x^3}{3} + C\right) + (\ln x + D) && C, D \text{ constants} \\ &= x^3 + \ln x + 3C + D \\ &= x^3 + \ln x + E && \text{where } E = 3C + D. \end{aligned}$$

Notice that we didn't really need to introduce the constants C and D at the fourth line, since at the end of our computation, we'd get another constant anyways. Thus every antiderivative of $\frac{3x^3+1}{x}$ has the form $x^3 + \ln x + E$ where E is a constant.

Exercise 12. Evaluate $\int_0^1 (\sin(2\pi x) - x^3 + e^x)\mathrm{d}x$.

Solution 12. Remember that the notation $\int_a^b f(x)\mathrm{d}x$ is a number! In particular, it represents an area. The way we solve this integral is by choosing one antiderivative of f and then evaluating it at the endpoints. In this case, we have

$$\begin{aligned}\int_0^1 (\sin(2\pi x) - x^3 + e^x)\mathrm{d}x &= \left(-\frac{1}{2\pi}\cos(2\pi x) - \frac{x^4}{4} + e^x\right)\Bigg|_0^1 \\ &= \left(-\frac{1}{2\pi}\cos(2\pi \cdot 1) - \frac{1^4}{4} + e^1\right) - \left(-\frac{1}{2\pi}\cos(2\pi \cdot 0) - \frac{0^4}{4} + e^0\right) \\ &= -\frac{1}{2\pi} - \frac{1}{4} + e + \frac{1}{2\pi} \\ &= e - \frac{1}{4}.\end{aligned}$$

Make sure that $-\frac{1}{2\pi}\cos(2\pi x) - \frac{x^4}{4} + e^x$ really does differentiate to $\sin(2\pi x) - x^3 + e^x$.

Exercise 13. Differentiate the following functions:

1. $F_1(x) = \int_0^x \cos(t^2 - 1)\mathrm{d}t$
2. $F_2(x) = \int_2^{x^2-x} e^{t^2-1}\mathrm{d}t$
3. $F_3(x) = \int_0^2 e^{2t-1}\mathrm{d}t$
4. $F_4(x) = \int_0^x x\mathrm{d}t$

Solution 13. For the first function, we just use the fundamental theorem of calculus

$$\begin{aligned}F_1'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}\int_0^x \cos(t^2 - 1)\mathrm{d}t \\ &= \cos(x^2 - 1).\end{aligned}$$

For the second, we'll use the fundamental theorem of calculus together with the chain rule

$$\begin{aligned}F_2'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}\int_2^{x^2-x} te^{t^2-1}\mathrm{d}t \\ &= (x^2 - x)e^{(x^2-x)^2-1} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(x^2 - x) \\ &= (x^2 - x)(2x - 1)e^{(x^2-x)^2-1}.\end{aligned}$$

Notice that in both the first and second function, we had something of the form $\frac{\mathrm{d}}{\mathrm{d}x}\int_a^{h(x)} \cdots$ where a is a number and where $h(x)$ is a function in x. We can apply the fundamental theorem of calculus (together with the chain rule if necessary) to cancel out the $\frac{\mathrm{d}}{\mathrm{d}x}$ and the $\int_a^{h(x)}$. For the third function, we do not have this situation. In particular, note that $F_3(x) = \int_0^2 e^{2t-1}\mathrm{d}t$ is just a constant function! In particular

$$F_3'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^2 e^{2t-1}\mathrm{d}t\right) = 0,$$

since the derivative of a constant function is 0. Finally, the fourth function requires a little bit of thought. Notice that we can pull the x out of the integrand since we are integrating over t:

$$\int_0^x x\mathrm{d}t = x\int_0^x \mathrm{d}t.$$

Since $\int_0^x \mathrm{d}t = x$, we see that

$$\begin{aligned} F_4'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^x x\mathrm{d}t\right) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}\left(x\int_0^x \mathrm{d}t\right) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}(x^2) \\ &= 2x. \end{aligned}$$

Exercise 14. Use the limit definition of integration to show that $\int_0^1 (3x^2-2)\mathrm{d}x = -1$. Do not use the fundamental theorem of calculus to solve this problem.

Solution 14. We have

$$\begin{aligned} \int_0^1 (3x^2-2)\mathrm{d}x &= \lim_{n\to\infty}\left(\sum_{i=1}^n f(x_i)\Delta x\right) \\ &= \lim_{n\to\infty}\left(\sum_{i=1}^n f(x_0+i\Delta x)\Delta x\right) \\ &= \lim_{n\to\infty}\left(\sum_{i=1}^n f\left(\frac{i}{n}\right)\frac{1}{n}\right) \\ &= \lim_{n\to\infty}\left(\sum_{i=1}^n \left(3\left(\frac{i}{n}\right)^2-2\right)\frac{1}{n}\right) && \text{since } f(x)=3x^2-2 \\ &= \lim_{n\to\infty}\left(\frac{1}{n}\sum_{i=1}^n \left(\frac{3i^2}{n^2}-2\right)\right) && \text{pulling out } 1/n \text{ out of sum} \\ &= \lim_{n\to\infty}\left(\frac{1}{n}\left(\frac{3}{n^2}\sum_{i=1}^n i^2-\sum_{i=1}^n 2\right)\right) && \text{apply linearity if } \Sigma_{i=1}^n \\ &= \lim_{n\to\infty}\left(\frac{1}{n}\left(\frac{3}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)-2n\right)\right) && \text{evaluate } \sum_{i=1}^n i^2 \text{ and } \sum_{i=1}^n 2 \\ &= \lim_{n\to\infty}\left(\frac{6n^3+\cdots}{6n^3+\cdots}-\frac{2n+\cdots}{n+\cdots}\right) && \text{find lead terms} \\ &= \frac{6}{6}-\frac{2}{1} && \text{evaluate limit} \\ &= -1. \end{aligned}$$

Make sure you're able to do this! Use different numbers to practice on your own.

Exercise 15. Find the x-value that satisfies the MVT for integrals for the function $f(x)=3x^2+1$ on the interval $[0,2]$.

Solution 15. The MVT for integrals states the following: if f is continuous on the closed interval $[a,b]$, then there exists a point c in the open interval (a,b) such that

$$f(c) = \frac{1}{b-a}\int_a^b f(t)\mathrm{d}t. \tag{5}$$

Recall that the quantity $\frac{1}{b-a}\int_a^b f(t)\mathrm{d}t$ is called the **average value** of an integrable function f on the interval $[a,b]$ and is sometimes denoted by $\overline{f}$. Thus the mean value theorem for integrals says there exists a $c \in (a,b)$ such that $f(c)$ equals the average value of f on $[a,b]$.

In this problem, we have $f = 3x^2 + 1$ and $[a,b] = [0,2]$. Let us first calculate the average value of the integrable function f on the interval $[0,2]$. We have

$$\begin{aligned}\overline{f} &= \frac{1}{2}\int_0^2 (3t^2 + t)\mathrm{d}t \\ &= \frac{1}{2}\left(\left(t^3 + t\right)\Big|_0^2\right) \\ &= \frac{1}{2}\left(\left(2^3+2\right) - \left(0^3+0\right)\right) \\ &= 5.\end{aligned}$$

Thus we need to find $c \in (0,2)$ such that $f(c) = 5$. Given $c \in (0,2)$, we have

$$\begin{aligned}f(c) = 5 &\iff 3c^2 + 1 = 5 \\ &\iff 3c^2 = 4 \\ &\iff c^2 = 4/3 \\ &\iff c = 2/\sqrt{3}.\end{aligned}$$

Notice that $c = -2/\sqrt{3}$ is another solution to $c^2 = 4/3$, but this x-value does not count since $-2/\sqrt{3} \notin (0,2)$.

Exercise 16. Find the average value of the function $f(x) = e^x + x + 1$ on the interval $[-2,2]$.

Solution 16. Recall that the average value of an integrable function f on the closed interval $[a,b]$ is given by

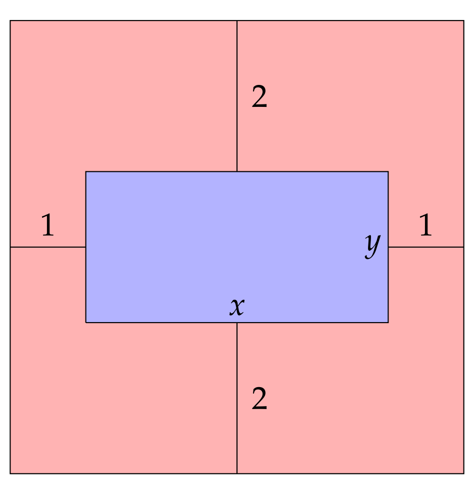
$$\overline{f} = \frac{1}{b-a}\int_a^b f(t)\mathrm{d}t. \tag{6}$$

In this problem, we have $f = e^x + x + 1$ and $[a,b] = [-2,2]$. Thus we have

$$\begin{aligned}\overline{f} &= \frac{1}{2-(-2)}\int_{-2}^2 (e^t + t + 1)\mathrm{d}t \\ &= \frac{1}{4}\left(\left(e^t + \frac{t^2}{2} + 1\right)\Big|_{-2}^2\right) \\ &= \frac{1}{4}\left(\left(e^2+2+1\right) - \left(e^{-2}+2+1\right)\right) \\ &= \frac{1}{4}\left(e^2 + 2 + 1 - e^{-2} - 2 - 1\right) \\ &= \frac{e^2 + e^{-2}}{4}.\end{aligned}$$

Maximization/Minimization Word Problems

Exercise 17. Consider the situation below where a blue rectangle is contained in a red rectangle as follows:



The numbers in the image below correspond to the distances between the red rectangle and the blue rectangle measure in inches. These distances are fixed in this problem. The variables x and y on the other hand give the dimensions of the blue rectangle. They will vary in this problem. Suppose that the area of the blue rectangle is 8 inches. What dimensions of the blue rectangle will minimize the area of the red rectangle?

Solution 17. Let A denote the area of the red rectangle. The first step is to find a formula of A in terms of one of the variables. Note that in this problem, we are told that the area of the blue rectangle is 10 inches. Thus we automatically have $xy = 8$. This allows us to express y in terms of x as $y = 8/x$. Thus essentially x is the only variable in this problem. So let's try to write A in terms of the variable x. We do this by breaking the red rectangle into five rectangles where one of these rectangles is the blue rectangle. We obtain

$$\begin{aligned} A &= 2(x+2) + y \cdot 1 + 2(x+2) + y \cdot 1 + xy \\ &= 2(x+2) + \frac{8}{x} + 2(x+2) + \frac{8}{x} + x\left(\frac{10}{x}\right) \\ &= 4(x+2) + \frac{16}{x} + 10 \\ &= 4x + \frac{16}{x} + 18. \end{aligned}$$

Note that the domain of A is $(0, \infty)$. Indeed, as $x \to 0$, we get blue rectangles with smaller and smaller width, and as $x \to \infty$, we get blue rectangles with smaller and smaller height (since $y = 1/x$). So in order minimize A, we need to find its critical points in $(0, \infty)$. The only critical points of A in this domain are the $c \in (0, \infty)$ such that $A'(c) = 0$. Thus we have

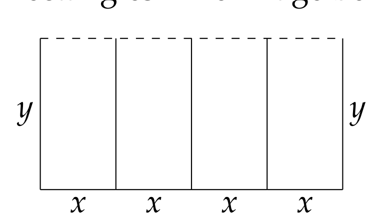
$$\begin{aligned} c \text{ is a critical point of } A &\iff A'(c) = 0 \\ &\iff \left(\frac{\mathrm{d}}{\mathrm{d}x}\left(4x + \frac{16}{x} + 18\right)\right)\Big|_{x=c} = 0 \\ &\iff \left(4 - \frac{16}{x^2}\right)\Big|_{x=c} = 0 \\ &\iff 4 - \frac{16}{c^2} = 0 \\ &\iff 4 = \frac{16}{c^2} \\ &\iff c^2 = 4 \\ &\iff c = 2. \end{aligned}$$

Notice that $c = -2$ is another solution to $c^2 = 4$, but this is not a critical point of A since it does not belong to the domain $(0, \infty)$. Now since $c = 2$ is the only critical point of A and since A is differentiable everywhere in $(0, \infty)$, we see that $c = 2$ must either be an absolute max or an absolute min. To determine this, we just choose a point to the left of 2 and to the right of 2 and evaluate A at each of these points:

$$\begin{aligned} A(1) &= 38 \\ A(2) &= 34 \\ A(4) &= 38 \end{aligned}$$

Thus $c = 2$ corresponds to an absolute minimum of A. We finish the problem in sentence form: **The area of the red rectangle will be minimized when the blue rectangle has dimensions 2 inches by 1/2 inches**.

Exercise 18. A farmer wants to set up fencing in the following way: he needs the fencing to form four rectangles, each having the same dimensions. He wants these four rectangles stacked next to each other, and he does not need any fencing on the northside of these rectangles. The image below illustrates this configuration:



Suppose he only has 100 feet of fencing to work with. What are the dimensions of each rectangle which would maximize the total enclosed area (the combined area of all rectangles added together).

Solution 18. It's easy to see that the total enclosed area is given by $A = 4xy$, but we'd like to think of A as a function in one variable only. In order to do this, we need an equation which relates x with y. Such an equation is given to us in the problem, in partiuclar we are told that the total amount of material we can work with is 100 feet. Thus we have the equation

$$2y + 4x = 100. \tag{7}$$

In other words, by rearranging (7), we have $y = 50 - 2x$. Therefore A can be thought of as a function in just one variable:

$$\begin{aligned} A &= 4xy \\ &= 4x(50 - 2x) \\ &= 200x - 8x^2. \end{aligned}$$

Now that we have A as a function in one variable, we can try to maximize it: first we note that the domain of A (when thinking of it as a function in x) is given by $(0, 25)$. Now let's find the ciritical points of A in $(0, 25)$. Given $c \in (0, 25)$, we have

$$\begin{aligned} A'(c) = 0 &\iff 200 - 16c = 0 \\ &\iff 200 = 16c \\ &\iff c = 25/2. \end{aligned}$$

Note that

$$\begin{aligned} A(0) &= 0 \\ A(25/2) &= 1250 \\ A(25) &= 0. \end{aligned}$$

Thus since there is only one critical point of A in $(0, 25)$, we see that A is maximized when $x = 25/2$ (and hence when $y = 25$). We finish the problem in sentence form: **The area is maximized when the dimension of each rectangle is $25/2$ feet by 25 feet.**

Exercise 19. A particle moves along a line. Suppose its acceleration after t seconds is given by $a(t) = e^{2t}$. Also suppose that its velocity at time $t = 0$ is $v(0) = 2$, and that its position at time $t = 0$ is $s(0) = 1$. Find the position function $s(t)$ of the particle at t seconds.

Solution 19. Recall that $v(t)$ is an antiderivative of $a(t)$. Thus it has the form

$$v(t) = \frac{1}{2}e^{2t} + C,$$

where C is a constant to be determined. Since $v(0) = 2$, we have

$$\begin{aligned} 2 &= v(0) \\ &= \frac{1}{2}e^{2\cdot 0} + C \\ &= \frac{1}{2} + C. \end{aligned}$$

Therefore $C = 3/2$, and hence $v(t) = (1/2)e^{2t} + (3/2)$. Recall that $s(t)$ is an antiderivative of $v(t)$. Thus is has the form

$$s(t) = \frac{1}{4}e^{2t} + \frac{3}{2}t + D$$

where D is a constant to be determined. Since $s(0) = 1$, we have

$$\begin{aligned} 1 &= s(0) \\ &= \frac{1}{4}e^{2\cdot 0} + \frac{3}{2} \cdot 0 + D \\ &= \frac{1}{4} + D. \end{aligned}$$

Therefore $D = 3/4$, and hence $s(t) = (1/4)e^{2t} + (3/2)t + (3/4)$.