Homework 07

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1. Solve the following nonlinear system using Newton's method in Matlab (use `newton.m`):

$$\begin{aligned} 0 &= x^2 - y - \sin(z) + 1, \\ 0 &= x + 1 + \sin(10y) - y, \\ 0 &= (1-x)z - 2. \end{aligned}$$

   Print the norm of the residual to check that your solution is in fact a root.

   Hint: You need to find a suitable starting value for x, y, and z so that the method converges.

   Submit `hw07q1.m` and make sure the definitions of your functions f and ∇f are also included (if you create them in separate files).

2. Compute the interpolating polynomial in a) monomial, b) Lagrange, and c) Newton basis form for the points $(-1, 2)$, $(0, 0)$, $(2, 1)$. Also show that they produce the same polynomial.

   Note: This question is done on paper.

3. Interpolate the function $f(x) = 1/(1 + 25x^2)$ with a polynomial of degree $n - 1$ for $n = 3, 5, 7, 9, 11$ equally spaced points x_i between -1 and 1 (so, for $n = 3$ the points are $x_1 = -1$, $x_2 = 0$, $x_3 = 1$) using monomial representation.

   (a) For each n, compute the error $\|f - p_n\|_\infty = \max_x |f(x) - p_n(x)|$ (you can approximate the maximum by evaluating it for a large number of x values, for example using `linspace(-1,1)`).

   (b) Notice that the error is increasing with n. Confirm this visually, by generating a plot of f and p_n for $n = 5$ and $n = 9$.

   (c) Reconcile your result with the error estimate for polynomial interpolation as discussed in class.