Section 3.4

h.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(3\sqrt{x^2+1}\right) &= 3\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x^2+1}\right) \\
&= 3\frac{\mathrm{d}}{\mathrm{d}x}\left((x^2+1)^{\frac{1}{2}}\right) \\
&= 3\cdot\frac{1}{2}(x^2+1)^{\frac{1}{2}-1}\cdot\frac{\mathrm{d}}{\mathrm{d}x}(x^2+1) \\
&= 3\cdot\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot\left(\frac{\mathrm{d}}{\mathrm{d}x}(x^2)+\frac{\mathrm{d}}{\mathrm{d}x}(1)\right) \\
&= 3\cdot\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot(2x+0) \\
&= 3\cdot\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x.
\end{aligned}$$

g.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(2^{\ln(x)}\right) &= \ln(2)\cdot 2^{\ln(x)}\cdot\frac{\mathrm{d}}{\mathrm{d}x}(\ln(x)) \\
&= \ln(2)\cdot 2^{\ln(x)}\cdot\frac{1}{x}.
\end{aligned}$$

e.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{5}{7(x^3-x)^2}\right) &= \frac{5}{7}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{(x^3-x)^2}\right) \\
&= \frac{5}{7}\frac{\mathrm{d}}{\mathrm{d}x}\left((x^3-x)^{-2}\right) \\
&= \frac{5}{7}\cdot -2(x^3-x)^{-3}\cdot\frac{\mathrm{d}}{\mathrm{d}x}(x^3-x) \\
&= \frac{5}{7}\cdot -2(x^3-x)^{-3}\cdot(3x^2-1)
\end{aligned}$$

b.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left(2e^{(3t^2)}\right) &= 2\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{(3t^2)}\right) \\
&= 2e^{(3t^2)}\cdot\frac{\mathrm{d}}{\mathrm{d}t}(3t^2) \\
&= 2e^{(3t^2)}\cdot 6t.
\end{aligned}$$

Example 9.

a. To write a completely defined rate of change of model we first need calulate the derivative $f'(t)$.

$$\begin{aligned}
f'(t) &= \frac{\mathrm{d}}{\mathrm{d}t}(7.8e^{(0.0407t)}) \\
&= 7.8\frac{\mathrm{d}}{\mathrm{d}t}(e^{(0.0407t)}) \\
&= 7.8e^{0.0407t}\cdot\frac{\mathrm{d}}{\mathrm{d}t}(0.0407t) \\
&= 7.8e^{0.0407t}\cdot 0.0407 \\
&= 0.31746e^{0.0407t}.
\end{aligned}$$

$f'(t) = 0.31746e^{0.0407t}$ chirps per Fahrenheit gives the rate of change for the average number of chirps when the termperature is t degree Fahrenheit.

b. To write a completely defined rate of change of model we first need calulate the derivative $t'(h)$.

$$
\begin{aligned}
t'(h) &= \frac{\mathrm{d}}{\mathrm{d}h}\left(\frac{24}{1+0.04e^{(0.6h+0.02)}}+52\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}h}\left(\frac{24}{1+0.04e^{(0.6h+0.02)}}\right)+\frac{\mathrm{d}}{\mathrm{d}h}(52) \\
&= \frac{\mathrm{d}}{\mathrm{d}h}\left(\frac{24}{1+0.04e^{(0.6h+0.02)}}\right) \\
&= 24\frac{\mathrm{d}}{\mathrm{d}h}\left(\frac{1}{1+0.04e^{(0.6h+0.02)}}\right) \\
&= 24\frac{\mathrm{d}}{\mathrm{d}h}\left(\left(1+0.04e^{(0.6h+0.02)}\right)^{-1}\right) \\
&= -24\left(1+0.04e^{(0.6h+0.02)}\right)^{-2}\cdot\frac{\mathrm{d}}{\mathrm{d}h}(1+0.04e^{(0.6h+0.02)}) \\
&= -24\left(1+0.04e^{(0.6h+0.02)}\right)^{-2}\cdot\left(\frac{\mathrm{d}}{\mathrm{d}h}(1)+0.04\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{(0.6h+0.02)}\right)\right) \\
&= -24\left(1+0.04e^{(0.6h+0.02)}\right)^{-2}\cdot 0.04\frac{\mathrm{d}}{\mathrm{d}h}\left(e^{(0.6h+0.02)}\right) \\
&= -24\left(1+0.04e^{(0.6h+0.02)}\right)^{-2}\cdot 0.04\frac{\mathrm{d}}{\mathrm{d}h}\left(e^{(0.6h+0.02)}\right) \\
&= -24\left(1+0.04e^{(0.6h+0.02)}\right)^{-2}\cdot 0.04\cdot e^{(0.6h+0.02)}\cdot\frac{\mathrm{d}}{\mathrm{d}h}(0.06h+0.02) \\
&= -24\left(1+0.04e^{(0.6h+0.02)}\right)^{-2}\cdot 0.04\cdot e^{(0.6h+0.02)}\cdot 0.06 \\
&= -0.0576\left(1+0.04e^{(0.6h+0.02)}\right)^{-2}e^{(0.6h+0.02)} \\
&= \frac{-0.0576e^{(0.6h+0.02)}}{\left(1+0.04e^{(0.6h+0.02)}\right)^{2}}.
\end{aligned}
$$

$t'(h) = \frac{-0.0576e^{(0.6h+0.02)}}{\left(1+0.04e^{(0.6h+0.02)}\right)^{2}}$ Fahrenheit per hours gives the rate of change for temperature on an average late-summer evening in south-central Michigan h hours after sunset.