Math 9853: Homework #9

Due Tuesday, November 26, 2019

Reading: Sections 9.1, 12.1, 12.2, 12.4, 12.8, 15.2, 23.1.

New Reading: Sections 22.1, 22.2, 22.4.

(a) Problem 12.18. (Review Problem 12.17 where $E = K^{(I)}$ is defined, and check the proof of Prop. 12.11 and see how finite dimension is used. Note that F^0 and F^{00} in part (5) should be $F^\perp$ and $F^{\perp\perp}$.)

(b) We define an inner product on $\mathbb{R}^3$ as: $x^t \cdot y = x^t H y$ where

$$H = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

Let $(e_1, e_2, e_3) = I_3$. Use this inner product in (b.1), (b.2), (b.3) and (b.5) below.

(b.1) Compute the length of e_i, $1 \leq i \leq 3$.

(b.2) Compute (a basis for) the subspace of $\mathbb{R}^3$ that is orthogonal to e_1.

(b.3) Apply the Gram-Schmidt orthonormalization procedure to e_1, e_2, e_3 to find an orthogonal basis for $\mathbb{R}^3$ (See Prop. 12.10 and the example in class).

(b.4) Find the QR-decomposition of H. (See Prop. 12.16 and Apply the Gram-Schmidt orthonormalization procedure to the columns of H under the usual inner product of $\mathbb{R}^3$.)

(b.5) Decide if $\mathbb{R}^3$ is an Euclidean space under the above inner product.

(c) Problem 22.7.

(d) Problem 22.4 (Follow the proof of Theorem 22.5).