Section 1.7: Constructed Functions

New functions can be formed by combining known functions using **addition, subtraction, multiplication, or division.** A new function can also be formed using **function composition** or by finding the **inverse of a function.**

Terms from Business and Economics:

- **Total Cost =** fixed costs + variable costs, where **fixed costs** are costs that do not depend on the number of units produced and **variable costs** are costs that vary according to the number of units produced. **Cost** (without a modifier) is assumed to be Total Cost unless the context indicates otherwise.

- **Average Cost** $= \dfrac{\text{total cost}}{\text{number of items produced}}$

- **Revenue** $= \left(\dfrac{\text{selling price}}{\text{unit}}\right) \cdot (\text{number of units sold})$

- **Profit** = Revenue – Cost. (Equivalently, Revenue = Profit + Cost.)

- **Break-even point** is the point at which total cost is equal to total revenue, or the point at which profit is zero.

A new model may be created from existing models when the input and output units of the functions in the existing models can be combined in such a way that the new function makes sense.

Operation used to form new function:	First check:	Then check:
Addition $(f+g)(x) = f(x)+g(x)$ Subtraction $(f-g)(x) = f(x)-g(x)$	$f(x)$ and $g(x)$ must have identical input descriptions and units for x	If so, then $f(x)$ and $g(x)$ must have identical output units
Multiplication $(f \cdot g)(x) = f(x) \cdot g(x)$ or Division $(f \div g)(x) = \dfrac{f(x)}{g(x)}, g(x) \neq 0$	$f(x)$ and $g(x)$ must have identical input descriptions and units for x	If so, then output units must be compatible
Composition* $(f \circ g)(x) = f(g(x))$	Output description and units of $g(x)$ must be identical to input description and units of $f(x)$	

***Function composition** is a method of constructing a new function by using the output of one function as the input of a second function.

Example 1: (CC5e p. 66)

The number of student tickets sold for a home basketball game at State University is represented by $S(w)$ tickets when w is the winning percentage of the team. The number of nonstudent tickets sold for the same game is represented by $N(w)$ hundred tickets when the winning percentage of the team is w.

a. Write the input units and description and output units of measure for functions $S(w)$ and $N(w)$.

Function	$S(w)$	$N(w)$
Input units and description	$w =$	$w =$
Output units	$S =$	$N =$

b. **Function addition** requires the output units of the two functions be identical. Multiplying by a factor of 100 changes the second function's units to "tickets".

$N(w)$ *hundred* tickets can be rewritten as ______•$N(w)$ tickets.

c. A new function, T, giving total tickets sold for a home basketball game at State University is modeled as:

$T(w)$ =______________________________ tickets gives the total number of tickets sold for a

home basketball game at State University, when w is ___________________________________.

d. Suppose more nonstudent tickets than student tickets are sold for a home basketball game at State University. Find a new function, D, giving the number by which nonstudent tickets exceeded student tickets sold, and use it to complete the model.

$D(w) =$ __________________________ tickets gives the number by which nonstudent

tickets exceeds student tickets sold, when w is _____________________________________.

Example 2: (CC5e pp. 67-68)

Sales of 12-ounce bottles of sparkling water are modeled as $D(x) = 287.411(0.266^x)$ million bottles when the price is x dollars per bottle. Find and write a completely defined model for the *revenue* from the sale of 12-ounce bottles of sparkling water.

a. To find a revenue equation, recall that revenue can be found by multiplying $(\textit{price, in dollars per item}) \cdot (\textit{number of items sold})$. Locate the variables for price and the number of items sold in the given model to write the revenue equation:

$$\text{Revenue} = (\_\_\_\_\_) \cdot (\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)$$

Output units for revenue can be found by multiplying units for price x, times output units for number of items sold $D(x)$. Find the output units for revenue.

$$\text{Revenue} = \left(x \ \frac{\text{dollars}}{\text{bottle}} \right) \cdot \left(D(x) \text{ million bottles} \right) = x \cdot D(x) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .$$

b. Write a completely defined model for the *revenue* from the sale of 12-ounce bottles of sparkling water.

c. Find the revenue if bottles of sparkling water are priced at \$2.50 per bottle.

Example 3: (CC5e pp. 68-69)

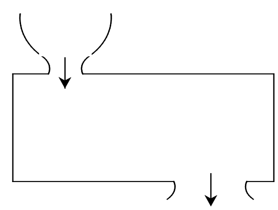
The level of contamination in a certain lake is $f(p)=\sqrt{p}$ parts per million when the population of the surrounding community is p people. The population of the surrounding community is modeled as $p(t)=400t^2+2500$ people where t is the number of years since 2000.

a. Write the input and output description and units of measure for functions $f(p)$ and $p(t)$.

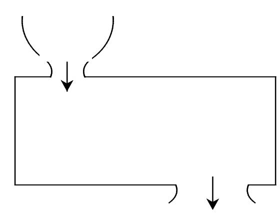
Function	$f(p)$	$p(t)$
Input description and units	$p=$	$t=$
Output description and units	$f=$	$p=$

b. Why do these two functions satisfy the criterion for **composition** of functions?

c. Which function is used as the input for the new function?



Complete the input output diagrams to demonstrate the composition. Find the new composition function.

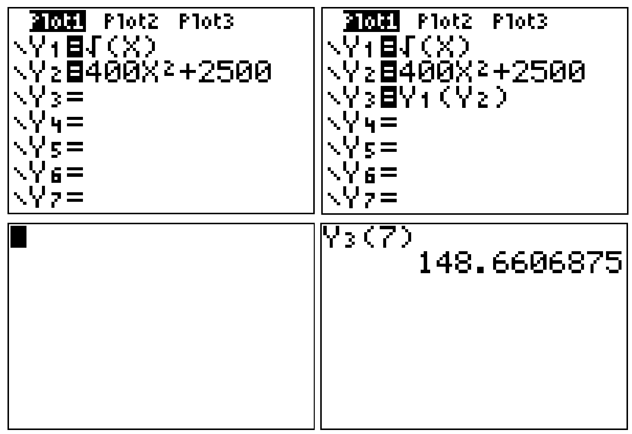


d. Write a completely defined model for the new function.

e. Calculate the level of contamination in the lake in 2007.

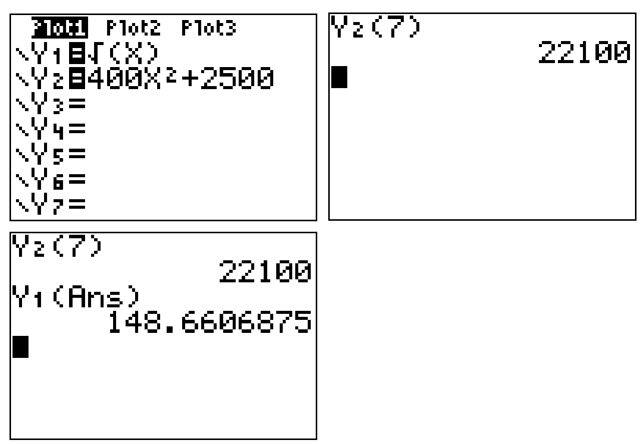
Evaluating a function constructed using function composition:

- Enter $f(p)$ in Y1 and $p(t)$ in Y2
- Enter $f(p(t))$ in Y3 as **<u>Y1(Y2)</u>**
- Return to the Home Screen
  <u>2nd</u> <u>MODE</u> [Quit]
- **<u>Y3(7)</u> <u>ENTER</u>** evaluates Y1(Y2(7))



Alternate method: **Use two steps to evaluate a function constructed by composition:**

- Enter $f(p)$ in Y1 and $p(t)$ in Y2 and then return to the Home Screen
- To evaluate $f(p(7))$, first evaluate $p(7)$ by finding **<u>Y2(7)</u>**
- To evaluate *f* at $p(7)$,
  <u>Y1</u> <u>(</u> <u>2nd</u> <u>(-)</u> [ANS] **<u>)</u> <u>ENTER</u>**
  which evaluates Y1 at the previous answer



If the input and output values of a function $f(x)$ are reversed, a new relation is created. If the new relation satisfies the definition of a function, that new relation is called the **inverse function** to $f(x)$.

Example 4: (CC5e p. 70)

Underwater pressure (measured in atm), d feet below the surface is shown in the table below.

Depth below surface, in feet	Surface (0)	33	66	99	132
Underwater pressure, in atm	1	2	3	4	5

a. Verify that $p(d) = 0.030d + 1$ atm (atmospheres) gives the underwater pressure, d feet below the surface of the water, $0 \le d \le 132$.

b. Reverse the input and output data:

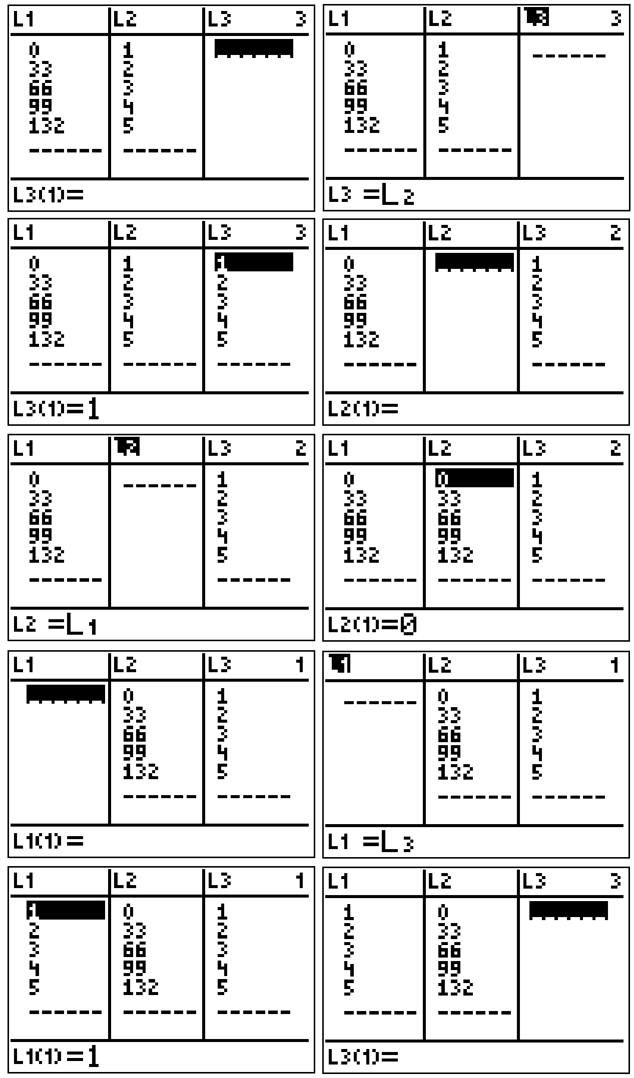
Underwater pressure, in atm	1				
Depth below surface, in feet	Surface (0)				

c. Find the **inverse** function to the linear function, $p(d)$, and write a completely defined model.

$d(p) =$

Swapping input and output data:

- **STAT** **ENTER** [Edit…] to view the previously entered data
- With L3 highlighted, complete the equation as L3 = L2 by hitting **2nd** **2** [L2]
- **ENTER** copies L2 into L3
- With L2 highlighted, **CLEAR** **ENTER** clears the data from L2
- With L2 highlighted, complete the equation as L2 = L1 by hitting **2nd** **1** [L1]
- **ENTER** copies L1 into L2
- With L1 highlighted, **CLEAR** **ENTER** clears the data from L1
- With L1 highlighted, complete the equation as L1 = L3 by hitting **2nd** **3** [L3]
- **ENTER** copies L3 into L1
- With L3 highlights, **CLEAR** **ENTER** clears the data from L3

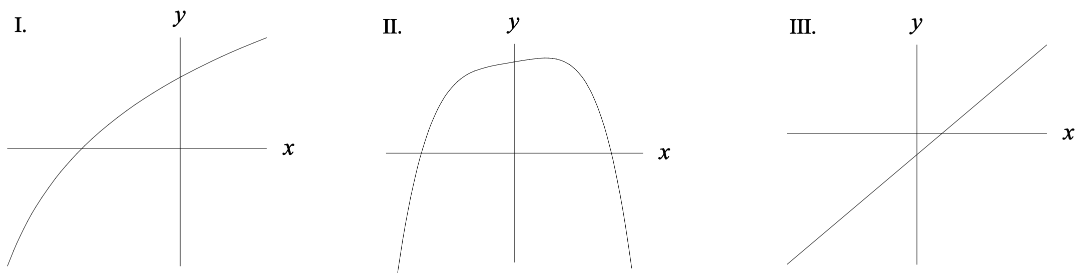


A **horizontal line test** can be used to determine graphically whether a function is *one-to-one*. A one-to-one function is a function for which every output value corresponds to exactly one input value.

If a function is one-to-one, then it has an **inverse function**.

If $f(x)$ and $g(x)$ are inverse functions, then $f(g(x)) = x$ and $g(f(x)) = x$

Example 5: (CC5e p. 69)



a. Use the horizontal line test to determine which of the functions shown above are one-to-one.

b. Which of the functions have an inverse function?

Example 6:

Show that $p(x)=\frac{1}{33}x+1$ and $d(x)=33x-33$ are inverse functions.

a. $p(d(x))=$

b. $d(p(x))=$