Section 3.6: Rates of Change of Product Functions

The derivative of a product $f(x)=g(x)\cdot h(x)$ is given by the Product Rule:

$$f'(x)=g'(x)\cdot h(x)+g(x)\cdot h'(x)$$

The Product Rule says that the derivative of a product function is found by:

$$\begin{pmatrix}\text{derivative of}\\ \text{the first function}\end{pmatrix}\bullet\begin{pmatrix}\text{second}\\ \text{function}\end{pmatrix}+\begin{pmatrix}\text{first}\\ \text{function}\end{pmatrix}\bullet\begin{pmatrix}\text{derivative of}\\ \text{the second function}\end{pmatrix}$$

Example 1:

$f(x)=x^{2/3}(x^3-5x^2)$ is a product $f(x)=g(x)\cdot h(x)$.

a. Identify the first and second functions.

First function: $g(x)=$ Second function: $h(x)=$

b. Find the derivatives:

$g'(x)=$ $h'(x)=$

First function: $g(x)=x^{2/3}$	Second function: $h(x)=x^3-5x^2$
Derivative of first function: $g'(x)=\frac{2}{3}x^{-1/3}$	Derivative of second function: $h'(x)=3x^2-10x$

c. Use the Product Rule to find the derivative of $f(x)=x^{2/3}(x^3-5x^2)$.

$f'(x)=g'(x)\cdot h(x)+g(x)\cdot h'(x)=$

Example 2:

$f(x)=e^{2x}\sqrt{x^3-5x^2}$ is a product $f(x)=g(x)\cdot h(x)$.

a. Identify the first and second functions.

First function: $g(x)=$ Second function: $h(x)=$

b. Find the derivatives. Note that the first and second functions are compositions and finding their derivatives will require the Chain Rule.

$g'(x)=$ $h'(x)=$

First function:	Second function:
Derivative of first function:	Derivative of second function:

c. Use the Product Rule to find the derivative of $f(x)=e^{2x}\sqrt{x^3-5x^2}$.

$f'(x)=g'(x)\cdot h(x)+g(x)\cdot h'(x)=$

Example 3:

Use the Product Rule to find the derivative of each function. Verify that each function is a product by identifying the two functions that are being multiplied (call the first function g and the second function h). Use proper notation.

a. $f(x)=\left(4x^2-x+1.5\right)\left[2\left(5^x\right)\right]$

$f'(x)=$

b. $f(x) = \dfrac{-2\left(3^x\right)}{\sqrt{x}} =$ ________________

$f'(x) =$

c. $f(x) = 2.5x\,\sqrt{x^3 - x} =$ ______________

d. $f(x) = (6x-4)^5(2x+1)$

e. $f(x) = \dfrac{2x^3 + 7x}{3x-5} =$ _______________

f. $f(x) = 2(5^x)\ln(x)$

Example 4: (CC5e pp. 234-235, 235-336)

Write a rate of change model for each of the following models.

a. $f(t) = 110\,t\,e^{-0.7t}$ ng/mL gives the concentration levels of the active ingredient in Ambien in the bloodstream t hours after a single 5 mg dose is taken orally, $0 \le t \le 12$.

b. The production level at a plant manufacturing radios can be modeled as

$f(x) = 10.54x^{0.5}\left(2 - 0.13x\right)^{0.3}$ thousand radios where x thousand dollars has been spent on modernizing plant technology, $0 \le x \le 12$.

Example 5: (CC5e pp. 236-237)

Kish Industries develops and produces deck laminates for naval vessels. $C(q) = 500 + 190\ln(q)$ thousand dollars gives the production costs to develop and produce q thousand gallons of deck laminate, $0 \le q \le 20$.

a. Write an equation for the average cost to produce a gallon of deck laminate when q thousand gallons of deck laminate are produced.

b. Rewrite the average cost function in part a) as a product (instead of a quotient).

c. Find the derivative of the average cost function. Include the output units.