Section 3.3: Rates of Change for Functions That Can Be Composed

Given two functions $f(t)$ and $t(x)$, the derivative of the composition $f(t(x))$ is given by the First Form of the Chain Rule: $\frac{df}{dx} = \frac{df}{dt} \cdot \frac{dt}{dx}$

Note that $f(t(x))$ can be written in typical function notation as $f(x)$. It follows that the derivative of the composition can be denoted as $\frac{df}{dx}$.

To use the First Form of the Chain Rule:

i. Find the derivative of each of the two given functions.
ii. Multiply the two derivatives together.
iii. Rewrite in terms of a single input variable.

Example 1: (CC5e p.213)

Given $f(t) = 3t^2$ and $t(x) = 4 + 7\ln(x)$, find the derivative of the composition $f(t(x))$.

a. Find: $\frac{df}{dt} =$ and $\frac{dt}{dx} =$

b. Multiply: $\frac{df}{dx} = \frac{df}{dt} \cdot \frac{dt}{dx} =$

c. What is the input variable for the composition function $f(x)$? ________

Rewrite $\frac{df}{dx}$ in terms of x: $\frac{df}{dx} =$

Example 2:

Given $f(t) = 1000e^t$ and $t(x) = 0.025x^2 + 1$, find the derivative of the composition $f(t(x))$.

a. Find: $\frac{df}{dt} =$ and $\frac{dt}{dx} =$

b. Multiply: $\frac{df}{dx} = \frac{df}{dt} \cdot \frac{dt}{dx} =$

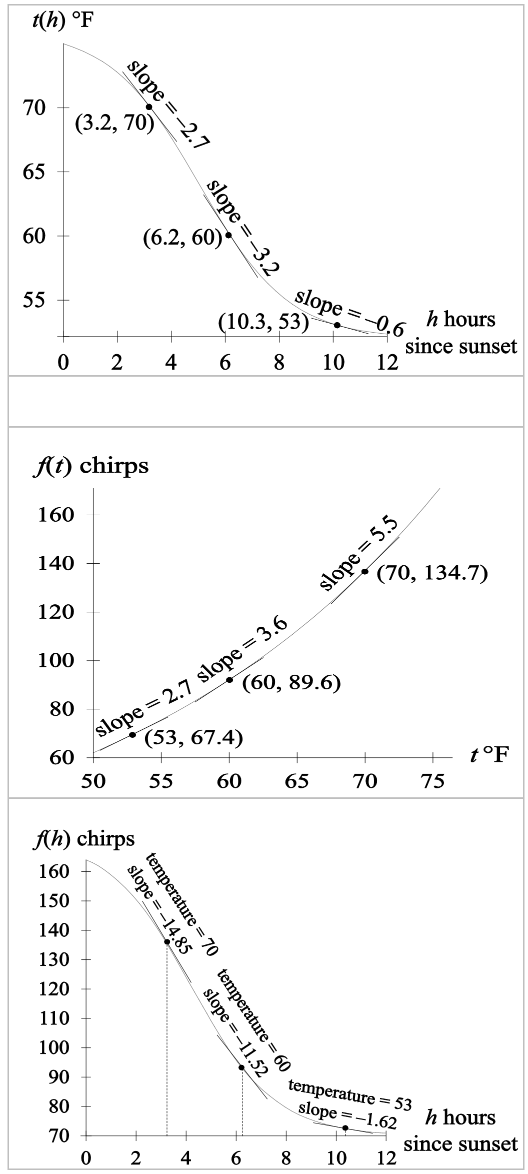
c. What is the input variable for the composition function $f(x)$? ________

Rewrite $\frac{df}{dx}$ in terms of x: $\frac{df}{dx} =$

Example 3: (CC5e pp. 211-212)

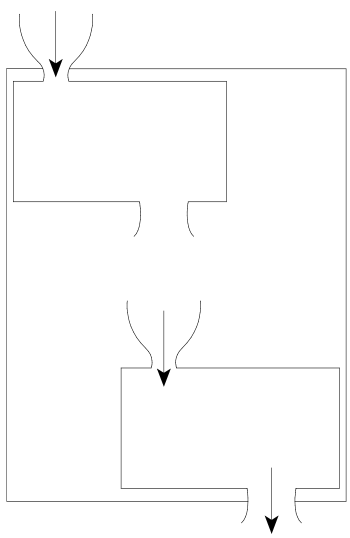
$t(h)$ degrees Fahrenheit gives the temperature h hours after sunset, $0 \le h \le 12$.

$f(t)$ gives the number of cricket chirps in one minute when the temperature is t degrees Fahrenheit, $50 \le t \le 75$.



a. Use the input/output diagram to identify units for the composition $f(t(h))$.
   input units: ____________________

   output units: ___________________

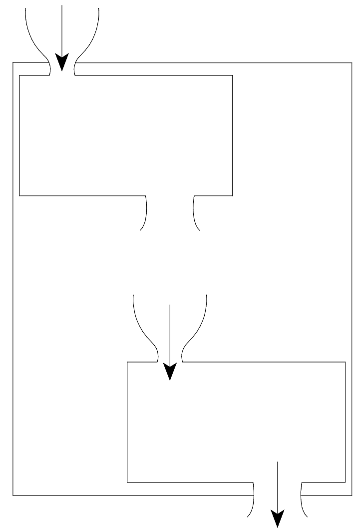


$f(h) =$_____________ **gives the number of cricket chirps in one minute, *h* hours after sunset,** $0 \le h \le 12$**.**

b. Use the graphs to find the number of cricket chirps in one minute 3.2 hours after sunset. Include units with the answer.

c. Use the graphs and the First Form of the Chain Rule to determine how quickly the number of cricket chirps in one minute is changing with respect to time, 3.2 hours after sunset. Include units with each derivative.

Example 4: (CC5e p.213)



$A(v)$ dollars represents the average cost to produce a student violin when v violins are produced.

$v(t)$ represents the number of student violins produced in year t.

In 2011, 10,000 violins are produced and production is increasing by 100 violins per year.

When 10,000 violins are produced,
the average cost to produce a student violin is $142.10, and the average cost is decreasing by $0.15 per violin.

a. Use the input/output diagram to identify units for the composition $A(v(t))$, or $A(t)$.

input units: ________________ output units: ________________

b. Use the given information to find the average cost to produce a student violin in 2011. Include units with your answer.

Since $v(2011) =$ __________ __________ and $A(10{,}000) =$ __________ __________,

the average cost to produce a student violin in 2011 was ____________ ___________.

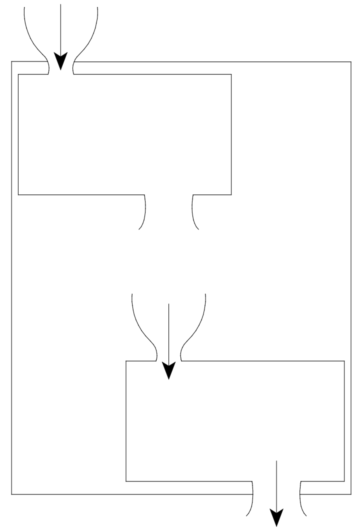
c. Use the given information and the First Form of the Chain Rule to determine how quickly the average cost to produce a student violin is changing with respect to time, in 2011. Include units with each derivative.

$$\left.\frac{dA}{dt}\right|_{t=2011} = \left.\frac{dA}{dv}\right|_{\substack{v=10{,}000\\(t=2011)}} \cdot \left.\frac{dv}{dt}\right|_{t=2011} = \underline{\hspace{6cm}} \cdot \underline{\hspace{6cm}}$$

$$= \underline{\hspace{12cm}}$$

d. Write a sentence of interpretation for your answer to part c.

Example 5: (CC5e p.217, Activity 21)



$f(t) = 0.123t^3 - 3.3t^2 + 22.2t + 55.72$ percent gives the occupancy rate for the month at a motel where $t = 1$ is the end of January, etc.

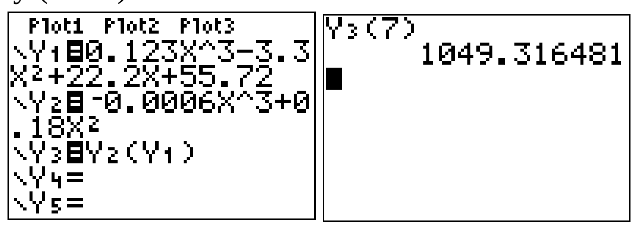
$r(f) = -0.0006f^3 + 0.18f^2$ thousand dollars gives the monthly revenue where f % is the occupancy rate at the motel.

a. Use the input/output diagram to identify units for the composition function $r(f(t))$, or $r(t)$.

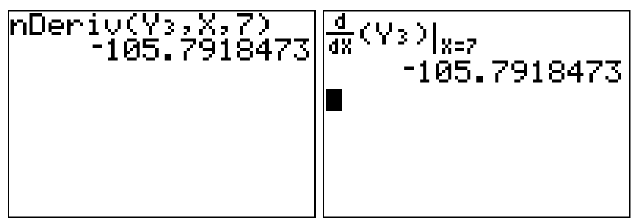
input units: ____________________

output units: ____________________

b. Find the monthly revenue at the end of July ($t = 7$). Include units with the answer.



c. Find the rate of change in monthly revenue at the end of July ($t = 7$). Include units with the answer.



Example 6:

Functions $g(h)$ and $h(x)$ can be composed as $g(h(x))$.

Given $h(-1) = 5$, $g(5) = 0.2$, $\left.\frac{dh}{dx}\right|_{x=-1} = 3$, and $\left.\frac{dg}{dh}\right|_{h=5} = -1.5$, find the following:

a. $g(h(-1)) =$

b. $\left.\frac{dg}{dx}\right|_{x=-1} =$