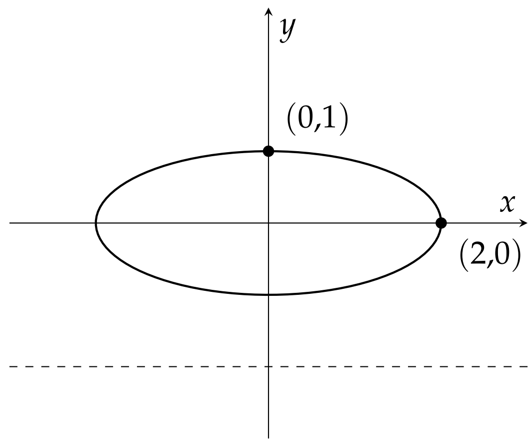
Morning Exam - 2019 CCC - Version A

(*) 1. Consider the ellipse E defined by the set of all points (x, y) in the plane such that $\frac{x^2}{4} + y^2 = 1$:



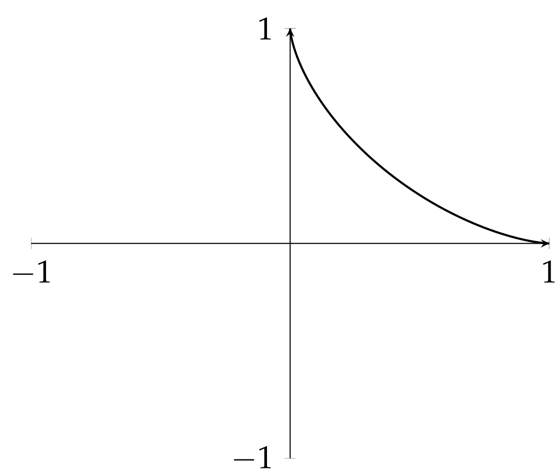
Find the volume of the elliptic torus $\widetilde{E}$ obtained by rotating E around the $y = -2$ line.

(A) 8π (B) $4\pi^2$ (C) $8\pi^2$ (D) 4π (E) none of these

2. Let Γ be the curve in the plane parametrized by $\gamma\colon [0,1] \to \mathbb{R}^2$ be given by

$$\gamma(t) = \left(t, \left(1 - t^{2/3}\right)^{3/2}\right)$$

for all $t \in [0,1]$:



Compute the arclength of Γ.

(A) $\frac{3}{2}$ (B) $\frac{\pi}{2}$ (C) $\frac{5\pi}{2}$ (D) $\frac{5}{2}$ (E) none of these

3. Compute $\int_0^{\sin x} \arcsin t \, dt$

(A) $x \sin x + \cos x + C$ (B) $\sin x + x \cos x + C$ (C) $-x \sin x + \cos x + C$ (D) $\sin x - x \cos x + C$ (E) none of these

4. Compute $\sum_{n=1}^{\infty} \frac{4}{n^2+2n}$

(A) 2 (B) 3 (C) 4 (D) does not converge (E) none of these

6. Compute $-\frac{13}{2} \int_0^{\pi} e^{2x} \cos(3x) dx$

(A) $1 + 3\pi$ (B) $1 + 2\pi$ (C) $-1 + 2\pi$ (D) $-1 + \pi$ (E) none of these

7. Compute $\int \frac{\ln(\ln x)}{x \ln x} dx$

(A) $\ln^2(\ln x) + C$ (B) $\frac{1}{2}\ln^2(\ln x) + C$ (C) $\ln(\ln^2 x) + C$ (D) $\frac{1}{2}\ln(\ln^2 x) + C$ (E) none of these

(*) 8. Define $\varphi \colon \mathbb{N} \to \mathbb{N}$ by

$$\varphi(m) = \min\{n \in \mathbb{N} \mid 2^m < 3^n\}.$$

for all $m \in \mathbb{N}$. Thus $\varphi(1) = 1$, $\varphi(2) = 2$, $\varphi(3) = 2$, $\varphi(4) = 3$, and so on. The function φ can be described more explicitely by

(A) $\lceil m \ln(2) / \ln(3) \rceil$ (B) $\lceil m \ln(3) / \ln(2) \rceil$ (C) $\lfloor m \ln(3) / \ln(2) \rfloor$ (D) $\lfloor m \ln(2) / \ln(3) \rfloor$ (E) none of these

(**) 9. Determine whether the following series converges. Justify your answer.

$$\sum_{n=1}^{\infty} (-1)^n \sum_{i=n^2}^{(n+1)^2-1} \frac{1}{i}.$$

If the series converges, then provide an upper bound for it.