Section 3.2: Exponential, Logarithmic, and Cyclic Rate-of-Change Formulas

e^x Derivative Rule:

- If $f(x) = e^x$, then $f'(x) = e^x$

Exponential Derivative Rule:

- If $f(x) = b^x$, then $f'(x) = \ln(b) \cdot b^x$

Natural Logarithm Rule for Derivatives:

- If $f(x) = \ln(x)$, $x > 0$, then $f'(x) = \frac{1}{x}$

(*Optional*)
Sine Rule for Derivatives:

- If $f(x) = \sin(x)$, then $f'(x) = \cos(x)$

Cosine Rule for Derivatives:

- If $f(x) = \cos(x)$, then $f'(x) = -\sin(x)$

Note: The e^x Derivative Rule is a special case of the Exponential Derivative Rule. Recall that $\ln(e) = 1$. Thus, $f(x) = e^x$ has derivative $f'(x) = \ln(e) \cdot e^x = 1 \cdot e^x = e^x$.

Example 1:

Find the derivative of each of the following functions using the Exponential Rules.

$y = 2^x + x^2$	$\frac{dy}{dx} =$
$f(x) = 2(1.5^x) - 3x$	$f'(x) =$
$g(t) = 4e^t - 3e^2$	

Example 2:

Find the derivative of each of the following functions using the Logarithmic Rule.

$y = 3\ln(x)$	
$f(x) = 2\ln(x) + 2^x - 3\ln(2)$	
$g(t) = 5 - 8\ln(t)$	
$j(x) = 3.2(0.7^x) + 1.5 - \pi^2$	

Example 3: (CC5e pp. 205-206)

Optimal weekly weight loss can be modeled as a function of the dieter's starting weight. The data in the table shows the optimum weekly weight loss for dieters originally weighing between 140 and 220 pounds.

Body weight in pounds	140	150	180	220
Body weight – 130 pounds	10			
Optimal weight loss, in pounds	1.1	2	3	4

a. Fill in the table by subtracting 130 pounds from each input value to align the input to the amount of weight over 130 pounds.

b. Examine the scatterplot and find a completely defined logarithmic model for the optimum weight loss.

$p(w) =$

c. Write a completely defined rate-of-change model for the optimal weight loss model.

$p'(w) =$

d. What is the optimum weekly weight loss for a person with a body weight of 200 pounds? Include units with the answer.

e. How quickly is the optimum weekly weight loss for a person with a body weight of 200 pounds changing? Include units with the answer.

f. Find the percentage rate of change in optimum weekly weight loss for a person with a body weight of 200 pounds. Include units with the answer.

Example 4 (Optional): (CC5e pp. 209-210, Activities 3, 9, 13)

Find the derivative of the following function using the Sine and Cosine Rules.

$n(x) = 14\sin(x)$	
$n(x) = 6\ln(x) - 13\sin(x)$	
$f(t) = 0.07\cos(t) - 4.7\sin(t)$	
$g(t) = 13\sin(t) + 5\cos(t)$	