Section 2.3: Rates of Change – Notation and Interpretation

The **(instantaneous) rate of change** of a function *f* at a point *x* is also referred to as the **derivative of f at *x*.** It is denoted by $f'(x)$, read as "*f prime of x*", or $\dfrac{df}{dx}$, and read as "*d-f d-x*".

The rate of change, or derivative, at a specific input *a* can be denoted as $f'(a)$, read as "*f prime evaluated at a*", or $\left.\dfrac{df}{dx}\right|_{x=a}$, read as "*d-f d-x, evaluated at a*".

The unit of measure for a rate of change or a derivative f' is **output units of f per input unit of** f.

Recall:
A sentence of **interpretation** for a **derivative** or **percentage rate of change** at a point uses ordinary conversational language to answer the questions:

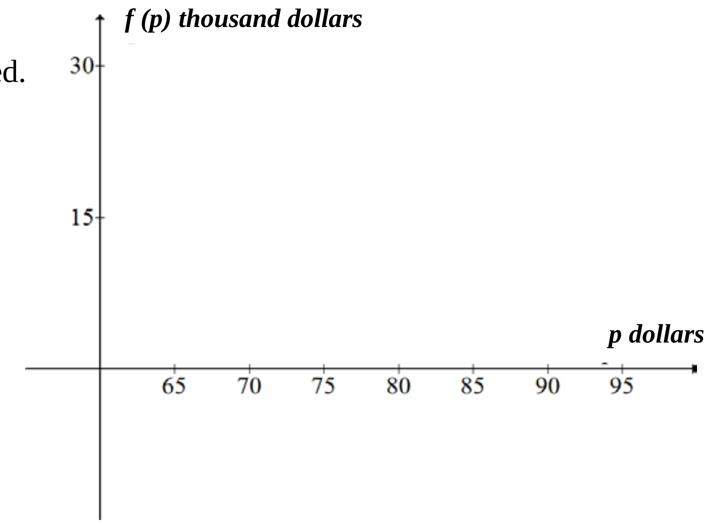
- *When?* refers to the single point.
- *What?* refers to the output description for the function.
- *Increas**ing** or Decreas**ing**?*
- *By how much?* refers to the rate of change or percentage rate of change calculation, and includes its corresponding units.

Example 1: (CC5e p. 157, Activity 3)

The function f gives the weekly profit, in thousand dollars, that an airline makes on its flights from Boston to Washington D.C. when the ticket price is p dollars.

Given: $f(65) = 15$, $f'(65) = 1.5$, and $f'(90) = -2$.

a. On the basis of the given information, sketch a graph of f on the axes provided.



b. Write a sentence of interpretation for each of the following:

- $f(65) = 15$

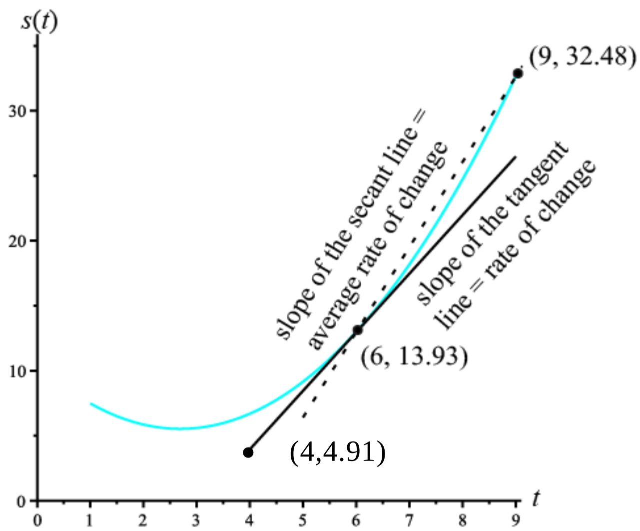
- $f'(65) = 1.5$

c. Find the percentage rate of change for the function at $p = 65$. Include units with the answer.

Average Rate of Change	Rate of Change/Derivative
Measures how rapidly a quantity *changes* on average between two points.	Measures how rapidly a quantity *is changing* at a single point.
Graphically finds the slope of the secant line between two points.	Graphically finds the slope of the tangent line at a single point on a continuous and smooth graph.
Requires two points	Requires a continuous function at a point that is not a sharp corner and does not have a vertical tangent.

Example 2: (CC5e, p. 153)

The figure to the right shows Apple Corporation's annual net sales, in trillion dollars, over an eight year period.



a. Find and interpret the average rate of change between year 6 and year 9.

b. Find the slope of the graph at $(6,\ 13.93)$. Write the answer using both notations for the derivative.

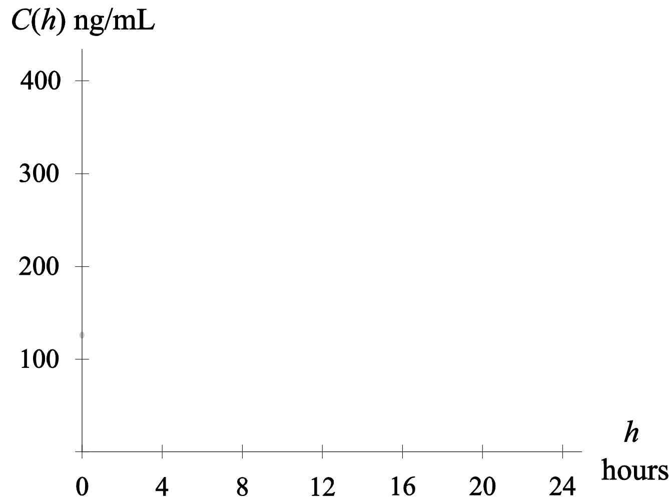
c. Write a sentence of interpretation for the derivative found in part b.

d. Write a sentence of interpretation for the percentage rate of change at $(6,13.93)$.

Example 3: (CC5e p. 156)

$C(h)$ is the average concentration, in ng/mL, of a drug in the bloodstream h hours after the administration of a dose of 360 mg. On the basis of the following information sketch a graph of C:

- $C(0) = 124$ ng/mL
- $C(4) = 252$ ng/mL and $C'(4) = 48$ ng/mL per hour
- The concentration after 24 hours is 35.9 ng/mL higher than it was when the dose was administered.
- The concentration of the drug is increasing most rapidly after 4 hours.
- The maximum concentration of 380 ng/mL occurs after 8 hours.

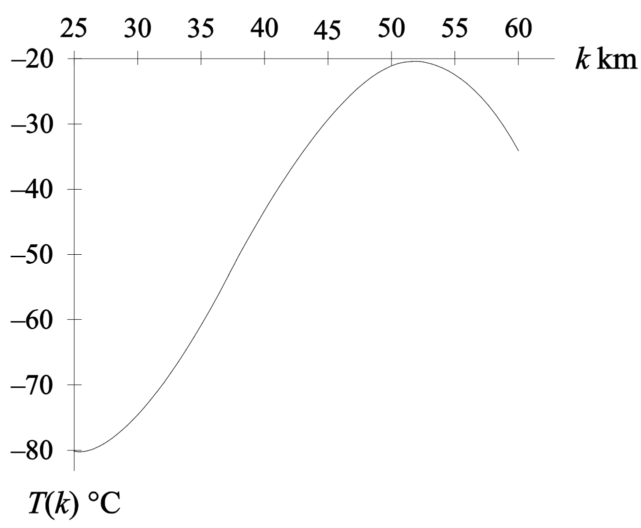


- Between $h = 8$ and $h = 24$, the concentration declines at a constant rate of 14 g/mL.

Example 4: (CC5e p. 155)

The graph shows the temperature T, in $^\circ\mathrm{C}$, of the polar night region as a function of k, the number of kilometers above sea level.

a. Sketch a tangent line and estimate its slope at 45 km. Include units with the answer.



b. Use derivative notation to express the slope of the graph of T when $k = 45$.

c. Write a sentence interpreting the rate of change of T at 45 km.

Summary of Measures of Change		
	Formula (assume $x_1 < x_2$)	**Units**
Change	$f(x_2) - f(x_1)$	output units of f
Percentage change	$\dfrac{f(x_2) - f(x_1)}{f(x_1)} \cdot 100\%$	percent
Average rate of change	$\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$	output units of f per input unit of f
Instantaneous rate of change *or* rate of change *or* derivative at *x* = a	$f'(a)$	output units of f per input unit of f
Percentage rate of change at *x* = a	$\dfrac{f'(a)}{f(a)} \cdot 100\,\%$	% per input unit of f