Printed Name: _____

Section #: _____ Instructor:_____

***Please do not ask questions during this exam**. If you consider a question to be ambiguous, state your assumptions in the margin and do the best you can to provide the correct answer*

- Any communication with any person (other than the instructor or the designated proctor) during this exam in any form, including written, signed, verbal or digital, is understood to be a violation of academic integrity.
- All devices, such as computers, cell phones, cameras, watches, and PDAs must be turned off while the student is in the testing room.
- The only calculators to be used are TI-83, TI-83+, TI-84 or TI-84+. You may NOT borrow or share a calculator with another person taking this test.
- **Statement of Academic Integrity**: I have not and will not give or receive improper aid on this test.

In signing below, I acknowledge that I have read, understand, and agree to these testing conditions.

Student's Signature: _____

(This test will not be accepted for grading unless it bears the signature of the student.)

- **General Directions and Calculator Hints are located on the next page.**

	FR#1a	FR#1b	FR #2	FR#3	FR#4	FR#5	Free Response Total	Multiple Choice Total	Total
Possible points	6	5	3	7	7	10	38	62	100
Points Earned									

Formula: $f_L(x) = f(c) + f'(c)(x-c)$

General Directions:

- Show work where possible. Answers without supporting work (where work is appropriate) may receive little credit.
- **Do not round intermediate calculations**.
- Answers in context ALWAYS require **units.**
- Round your answers to **3 decimal places** UNLESS the answer needs to be rounded differently to make sense in the context of the problem OR the directions specify another type rounding OR the complete answer has less than 3 decimal places.
- When you are asked to write a model, include all components of a model: an equation, a description of the input including units, a description of the output including units, and the interval when known
- When asked to write a sentence of practical interpretation, answer the questions: **when?, what?, and how much?** using ordinary, conversational language. DO NOT use math words, terms, or unnecessary phrases.
- Always use a ruler when estimating values of a graph.

HINTS FOR TROUBLESHOOTING YOUR CALCULATOR:

- If you lose your L1, L2, etc., you may reinsert them using STAT 5 (set-up editor) enter.
- The SCATTER PLOT will not show unless Plot 1 has been turned on and there is data in L1 and L2.
- ZOOM 0 *may* not work for graphing if Plot 1 is turned on.
- DIM MISMATCH error usually means that the lists in L1 and L2 are not of equal length.
- DATA TYPE error usually means that you already have something in Y1 and you need to clear it before you can paste a new equation.
- INVALID DIM error usually means that your plot(s) are on, but that you have no data in the lists. Refer to the second hint above.
- If your batteries die, raise your hand and hold up your calculator. If your instructor has an extra calculator available, he/she will loan it to you for a few minutes.
- SYNTAX ERROR: Try GO TO. This will happen if you use a subtraction minus sign when you should use a negative sign.
- MATH SOLVER only works if there is a variable "x" in Y1.
- If you need to CLEAR MEMORY, use 2^{nd} +, 7:Reset, 1:All Ram, 2:Reset

MULTIPLE CHOICE: 62 points

Use a #2 pencil and completely fill each bubble on your scantron to answer each multiple choice question. (For future reference, circle your answers on this test paper.) Each question is worth 3 points, unless noted otherwise. There is no penalty for guessing on multiple choice questions. If you indicate more than one answer, or you leave a blank, the question will be marked as incorrect.

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1. Which one of the following is a **FALSE** statement?

   a. An absolute extreme value may occur at an endpoint of a closed interval.

   b. A relative extreme value may occur at an endpoint of a closed interval.

   c. A function may have an absolute maximum and a relative maximum at the same input c.

   d. It's possible that a function defined on a closed interval may not have a relative minimum.

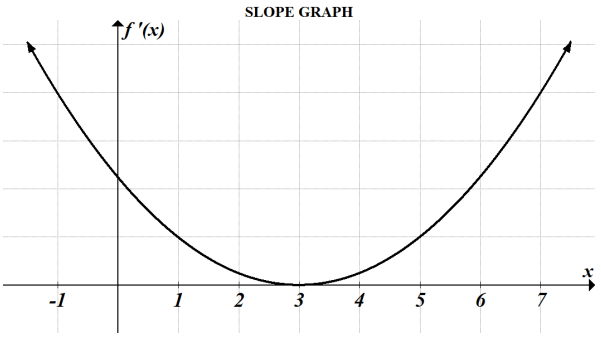
2. Consider the **slope graph** $f'(x)$ below. Which one of the following statements is **TRUE**?

   a. $f(x)$ has a relative maximum at $x = 3$.

   b. $f(x)$ has a relative minimum at $x = 3$.

   c. $f(x)$ does not have a relative extreme point at $x = 3$.

   d. $f(x)$ has an absolute maximum at $x = 3$.

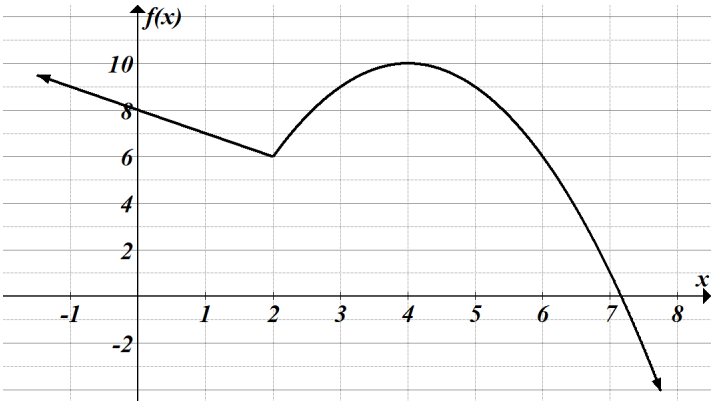


3. The function f, given below, has a *relative minimum at* _____.

   a. $x = 1$ and $x = 4$ b. $x = 1$ only

   c. $x = 4$ only d. $x = 2$ and $x = 5$

Use the graph below to answer the next three questions.



4. [2 pts] Find the input value(s) for any critical points for the function f.

   a. $x = 2$ and $x = 4$

   b. $x = 4$ only

   c. $x = 2$ only

   d. There are no critical points.

5. [2 pts] At $x = 2$, f has _____.

   a. neither a relative maximum nor a relative minimum

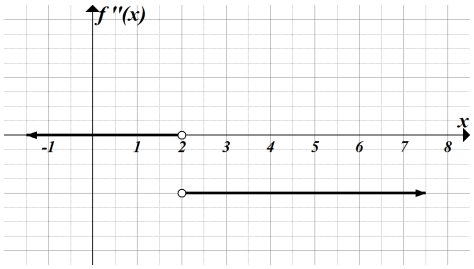
   b. both a relative maximum and a relative minimum

   c. a relative maximum

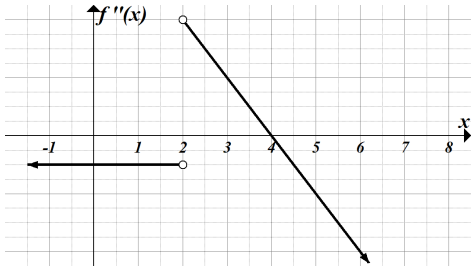
   d. a relative minimum

6. Find the **second derivative** graph for the function f graphed above.

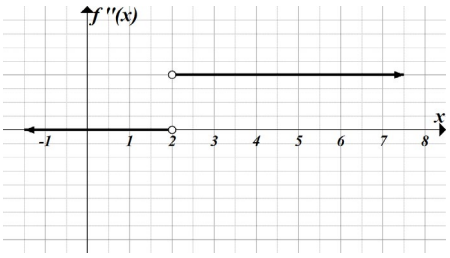
   a.



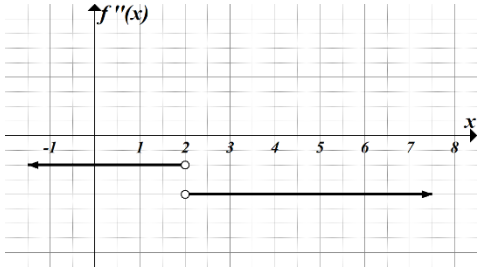
   b.



   c.



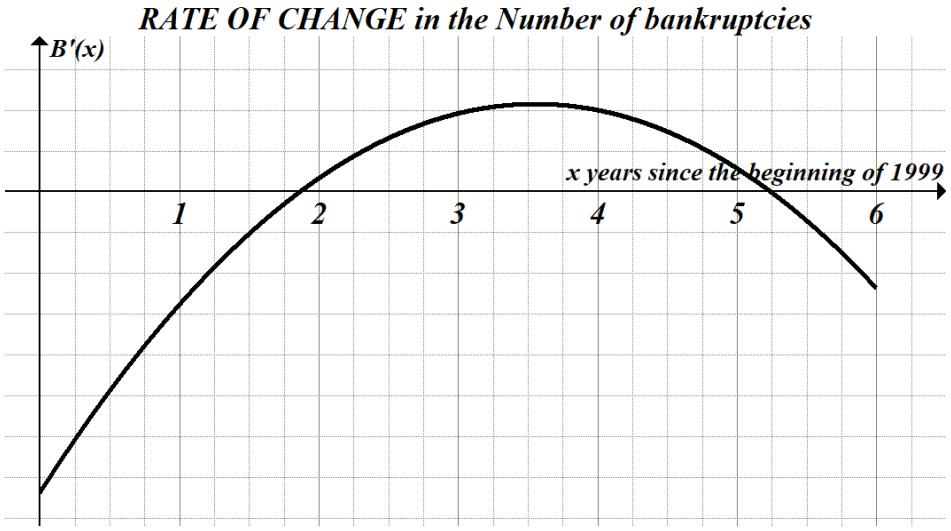
   d.



Use the following to answer the next three questions. [2 pts each]

The **rate of change** for the number of bankruptcies is described in the following model.

$B'(x)$ bankruptcies per year gives the **rate of change** in the number of non-business Chapter 11 bankruptcies, x years since the *beginning* of 1999, $0 \le x \le 6$.



7. On the interval number $0 \le x \le 6$, the number of non-business Chapter 11 bankruptcies was at a *relative maximum* approximately _____ years after the beginning of 1999.

   a. $x = 0$ b. $x = 1.85$ c. $x = 3.6$ d. $x = 5.25$

8. On the interval number $0 \le x \le 6$, the number of non-business Chapter 11 bankruptcies was at a *relative minimum* approximately _____ years after the beginning of 1999.

   a. $x = 0$ b. $x = 1.85$ c. $x = 5.25$ d. there is no relative minimum

9. On the interval number $0 \le x \le 6$, the number of non-business Chapter 11 bankruptcies was at an *inflection point* approximately _____ years after the beginning of 1999.

   a. $x = 1.85$ b. $x = 3.6$ c. $x = 5.25$ d. there is no inflection point

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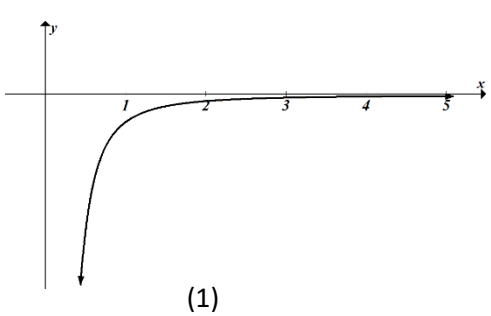
10. For a continuous differentiable function $f(x)$, in which $f'(2) = 0$ and $f''(2) > 0$, which one of the following statements about $f(x)$ must be true?

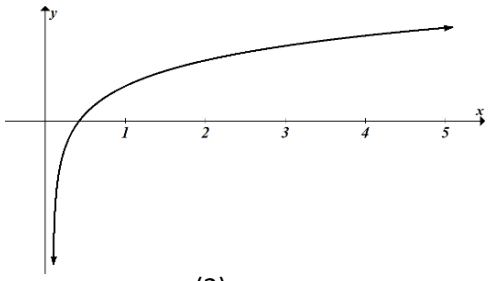
    a. $f(x)$ has an inflection point at $x = 2$. b. $f(x)$ has a zero at $x = 2$.

    c. $f(x)$ has a relative minimum at $x = 2$. d. $f(x)$ has a relative maximum at $x = 2$.

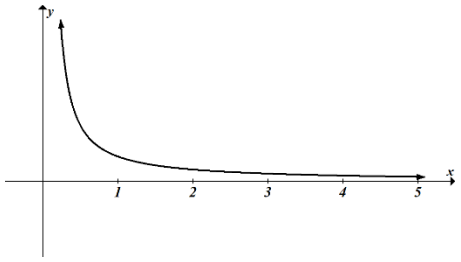
11. A mathematics instructor draws the graphs of $f(x)$, $f'(x)$, and $f''(x)$ on the board, but forgets to label which one is which. Which of the following three graphs must be the graph of $f''(x)$?



(1)



(2)



(3)

a. Graph (1) is $f''(x)$

b. Graph (2) is $f''(x)$

c. Graph (3) is $f''(x)$

d. Impossible to determine

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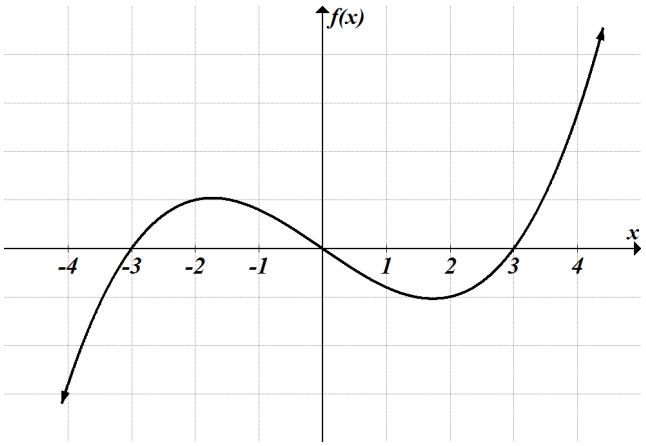
12. The following is a graph of a function $f(x)$. For which x - values is the **second derivative** negative, in other words, for which x - values is $f''(x) < 0$?



a. $(-2, 2)$

b. $(0, \infty)$

c. $(-\infty, -3)$ and $(0, 3)$

d. $(-\infty, 0)$

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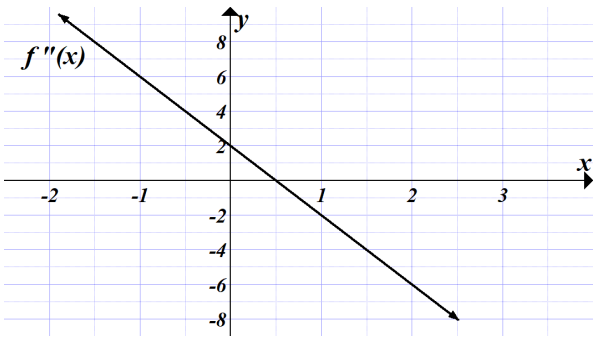
13. A function $f(x)$ has a **second derivative graph** $f''(x)$ as shown below.
At $x = 0.5$, the graph of $f(x)$ will have a/an _____.



a. x-intercept

b. relative maximum

c. inflection point

d. relative minimum

14. $P(t) = \dfrac{11.498}{1+12.812e^{-0.027t}}$ billion people gives the world population, t years since the end of 1900, $0 \le t \le 150$. *Check point*: $P(2) = 0.875$

In what year was the world population increasing most rapidly?

a. 1975 b. 1985 c. 1995 d. 2005

15. Consider the function $f(x) = e^{3x}$. Find the **second derivative**, $f''(x)$.

a. $f''(x) = 9e^{3x}$ b. $f''(x) = e^{3x}$ c. $f''(x) = 3e^{3x}$ d. $f''(x) = (3x)(3x-1)e^{(3x-2)}$

Use the following for the next two questions.

A pancake restaurant manager calculated that $p(h)$ dozen pancakes was the total number of pancakes sold on a Sunday, h hours since opening the restaurant at 8 am, $0 \le h \le 10$.

$s(p)$ bottles was the number of 10-oz bottles of syrup consumed when p dozen pancakes were sold, $p \ge 0$.

The manager found the following: $p(4) = 116.75$, $s(116.75) = 19.45$, $\left.\dfrac{dp}{dh}\right|_{h=4} = 8.25$, $\left.\dfrac{ds}{dp}\right|_{p=116.75} = 0.3$

16. How quickly was the number of bottles of syrup used changing, with respect to time, 4 hours after the restaurant opened?

a. 0.3 b. 2.475 c. 19.45 d. 195.488

17. [2 pts] What are the output units for the answer to the previous question?

a. bottles per dozen pancakes b. dozen pancakes per hour

c. dozen bottles d. bottles per hour

Use the following to answer the next two questions.

In 2017, the median price of electric vehicles in the US was 64.300 thousand dollars/car and 199.826 thousand cars were sold. Also in 2017, the median price of electric vehicles in the US was **de**creasing by 8.700 thousand dollars/car per year and sales were increasing by 167. 481 thousand cars per year.

18. What was the rate of change of the revenue generated by electric vehicle sales in the US in 2017?

a. 12,848.13 b. -1,457.085 c. 9,030.542 d. 167.481

19. [2 pts] What are the units for the previous question?

a. million dollars

b. million dollars per year

c. million cars per year

d. thousand dollars per year

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20. Given $h(x) = 3x^2 - 2e^x$ and $g(h) = \sqrt{h}$, find the **derivative of** $g(h(x))$**.**

a. $\frac{dg}{dx} = \frac{1}{2}\left(3x^2 - 2e^x\right)^{-1/2}\left(6x - 2e^x\right)$

b. $\frac{dg}{dx} = \frac{1}{2}x^{-1/2}\left(6x - 2e^x\right)$

c. $\frac{dg}{dx} = \frac{1}{2}\left(6x - 2e^x\right)^{-1/2}$

d. $\frac{dg}{dx} = \left(6x - 2e^x\right)x^{1/2} + \left(3x^2 - 2e^x\right)\cdot\frac{1}{2}x^{-1/2}$

21. Find $f'(x) =$ _____ for $f(x) = 2\ln(4x^3 - 7x)$.

a. $\frac{2}{x}\cdot(12x^2 - 7)$

b. $\frac{2}{12x^2 - 7}$

c. $\frac{2}{4x^3 - 7x}$

d. $\frac{2}{4x^3 - 7x}\cdot(12x^2 - 7)$

22. If $g(x) = 3^{(x^2-1)} \cdot \ln(x)$, find $g'(x) =$ _____ .

a. $\ln(3)3^{(x^2-1)}(2x) \cdot \frac{1}{x}$

b. $\ln(3)3^{(x^2-1)}(2x) \cdot \ln(x) + 3^{(x^2-1)} \cdot \frac{1}{x}$

c. $\frac{\ln(3)3^{(2x)}}{x}$

d. $\ln(3)3^{(2x)} \cdot \ln(x) + 3^{(x^2-1)} \cdot \frac{1}{x}$

23. Jack is driving his car from his home to a nearby city on a straight road. 1.5 hours after his departure from home, he has traveled 75 miles. At that moment, his distance from home is increasing by 45 miles per hour. Using the linear approximation, estimate how far Jack has traveled from home 2 hours after his departure.

a. 97.5 miles b. 165 miles c. 120 miles d. 82.5 miles

FREE RESPONSE: 38 points

Show work where possible. Read the directions on rounding, inclusion of units, and writing models and sentences at the back of the test.

1. Find the derivative of the following functions. Use exact numbers. It is not necessary to simplify. All work and the final answer should be clearly presented with proper use of equal signs and proper notation.

a. $f(x) = (x^3 - 5x) \cdot \ln(2x^2 - x + e^3)$

$(\_\_\_\_ / 6\,pts)$

b. $f(x) = \frac{3}{1 + 8e^{0.5x}}$

$(\_\_\_\_ / 5\,pts)$

2. $f(x)=\sqrt[3]{2x^2+x-1}$ is a composition $g(h(x))$ of two functions $g(h)$ and $h(x)$. Identify the two functions. (Neither answer should be the same as the original function.)
Do **not** find the derivative.

$g(h)=$ _____

$h(x)=$ _____

$(\_\_\_\_/3\,pts)$

3. $T(x)$ thousand trees is the number of apple trees in a Washington State apple orchard in year x.

$A(x)$ apples per tree gives the apple production for a typical tree in the apple orchard, in year x.

In 2012, the apple orchard had 2.5 thousand trees and with additional trees being planted, the number of trees in the orchard was increasing by 0.019 thousand trees per year. In the same year, the orchard's production level was 875 apples per tree and was **de**creasing by 6.2 apples per tree per year.

a. The two functions $T(x)$ and $A(x)$ can be used to construct a new meaningful function using

____function multiplication / function composition__(circle one)_____.

b. Use the given information to determine the following values:

$T(2012)=$ _____ , $A(2012)=$ _____ , $\left.\frac{dT}{dx}\right|_{x=2012}=$ _____ , $\left.\frac{dA}{dx}\right|_{x=2012}=$ _____

c. How quickly was the apple production changing in the Washington State apple orchard in 2012?
For full credit, show work. Include units in the answer and round the correctly to three decimal places.

$(\_\_\_\_/7\,pts)$

4. Let f be a differentiable function such that $f(1) = -2$ and $f'(1) = 5$.

a. Find the linearization of f at the point $c = 1$.

$f_L(x) =$ _____

b. Estimate the *change* in f between $x = 1$ and $x = 1.75$. _____

c. Estimate $f(1.75)$. _____

d. If $f(x)$ is concave down on an interval about $x = 1$, the estimate in part c) is an

____ underestimate / overestimate__(circle one)_____.

$(\_\_\_\_ / 7\,pts)$

5. $f(x) = x^3 - 5.65x^2 + 4.77x + 13.52$ dollars gives the stock price for "I ❤ Business Calc, Merchandise and Co.", x hours after 9:30 am on a given day, $0 \le x \le 5$. *Checkpoint:* $f(2) = 8.460$

Answer the questions below for the function $f(x)$ on the given interval $0 \le x \le 5$. Show <u>exactly</u> 3 decimal places of accuracy in each answer blank. Answers will <u>not</u> receive credit unless reported to <u>exactly</u> 3 decimal places.

On the given interval, the stock price was **highes**t _____ hours after 9:30 am, at which time the price of the stock was _____ dollars.

On the given interval, the stock price was **lowest** _____ hours after 9:30 am, at which time the price of the stock was _____dollars.

On the given interval, the stock price was **decreasing most rapidly** _____ hours after 9:30 am, at which time the price of the stock was decreasing by _____dollars per hour and the price of the stock was _____ dollars.

$(\_\_\_\_ / 10\,pts)$

END OF TEST