Homework 03

Math 8600

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1. Implement a banded version of Gauss Elimination (without pivoting) that takes an additional argument k denoting the number of off-diagonal entries in each direction that are possibly non-zero ($a_{ij} = 0$ if $i > j + k$ or $j > i + k$ and takes advantage of this fact. Create `BandedGaussElimination.m` with the function
`function x = BandedGaussElimination(A,b,k)`.

   Test your algorithm with the matrix
`A=diag(2*ones(1,n))+diag(-1*ones(1,n-1),1)+diag(-1*ones(1,n-1),-1);`
and RHS `b=ones(n,1);` by comparing against MATLAB's backslash for example for n=10.

2. Based on question 1, consider the linear systems defined there for different values of n (maybe $2^8, 2^9, \ldots, 2^13$).

   Compare the speed (using `tic` and `toc`) of the following methods to solve the linear system:

   (a) MATLAB's backslash
   (b) Gauss Elimination (with partial pivoting)
   (c) Banded Gauss Elimination (you just implemented it here).
   (d) MATLAB's backslash with the sparse matrix `B=sparse(A);`

   For example:

```
A= ...;
b= ...;
fprintf('a) backslash:\n')
tic;
x=A\b;
toc;
...

fprintf('d) sparse backslash:\n')
B=sparse(A);
tic;
x=B\b;
toc;
```

   Create a log-log plot of time vs n with a line for each method and also include lines for $O(n)$, $O(n^2)$ and $O(n^3)$. Do the results match your expectations?

   Submit: `hw03q3.m` and include a picture of your plot in your .pdf submission.

3. What is the LU factorization of the matrix
$$A = \begin{pmatrix} 1 & a \\ c & b \end{pmatrix}$$
and under what condition is the matrix A singular?

4. Compute the LU factorization of the matrix
$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}.$$