Section 4.2: Relative Extreme Points

A **relative extreme point** $(c, f(c))$ is a point on a function f at which a relative maximum or a relative minimum occurs.

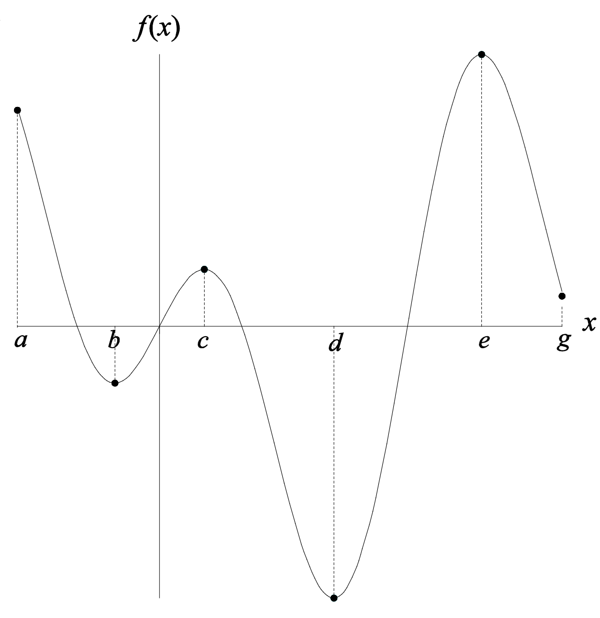
A function has a **relative maximum** at input c if the output $f(c)$ is *greater than or equal to* any other output in some open interval around c. $f(c)$ is referred to as a relative maximum value.

A function has a **relative minimum** at input c if the output $f(c)$ is *less than or equal to* any other output in some open interval around c. $f(c)$ is referred to as a relative minimum value.

If a function $f(x)$ is defined on a closed interval $a \le x \le b$, then a relative extreme point does **not** occur at endpoints $x = a$ or $x = b$.

Example 1: (similar to CC5e p. 267)

a. Identify each of the points at inputs *a, b, c, d, e* and *g* in the graph of $f(x)$ shown to the right as a *relative maximum, a relative minimum, or neither.*
For each relative extreme point, find the slope of the function at that point.



a: ________________________________; _____

b: ________________________________; _____

c: ________________________________; _____

d: ________________________________; _____

e: ________________________________; _____

g: ________________________________; _____

b. What is the slope of the function at each of the relative extreme points?

A **critical point** of a function f is a number c in the domain of f at which $f'(c) = 0$ or $f'(c)$ does not exist.

A critical point identifies the location of all **possible** relative extreme points.
If f has a relative maximum or minimum value at c and $f'(c)$ exists, then $f'(c) = 0$.
However, if $f'(c) = 0$, there is not necessarily an extreme point at $x = c$.

The First Derivative Test (to determine whether a relative extreme point occurs at a critical point):

For a critical point c of a function f that is continuous on some open interval around c :

- If f' changes from positive to negative as x increases through c, then f has a relative maximum at c.
- If f' changes from negative to positive as x increases through c, then f has a relative minimum at c.
- If f' does not change sign as x increases through c (from positive to negative or vice versa), then f does not have a relative extreme point at c.

Graphically, the first derivative test says that for a function that is continuous on an open interval around c with $f'(c) = 0$, if the slope graph **crosses** (not just touches) the input axis at $x = c$, there is a relative extreme point at the critical point $x = c$. For a function that is continuous on an open interval around c with $f'(c)$ not existing, if the slope graph **is on opposite sides** of the input axis at $x = c$, there is a relative extreme point at $x = c$.

If a function f is not continuous at a point c, then $f'(c)$ does not exist. Examine the graph of f to determine whether a relative extreme point exists at the critical point $x = c$.

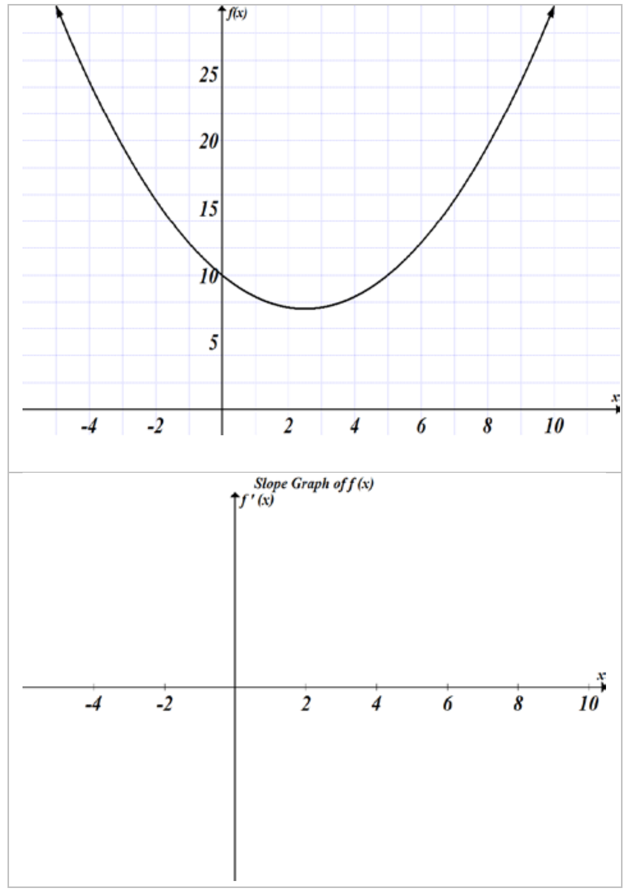
Example 2: (CC5e p. 258)

Given: $f(x) = 0.4x^2 - 2x + 10$

a. Identify the critical point on the graph of $f(x)$ by marking an "X" on the point at which $f'(c) = 0$.

b. Write the equation whose solution identifies the critical point(s).

c. Solve the equation to identify the critical point.



d. Draw the slope graph of $f(x) = 0.4x^2 - 2x + 10$ and verify that the slope graph **crosses** the x-axis at the critical point $x = c$.

e. Circle the terms that correctly complete the statement.

f' is <u>*positive/negative*</u> for $x < c$ and is <u>*positive/negative*</u> for $x > c$.

f. Does the function have a *relative maximum, a relative minimum, or neither* at the critical point in part c)?

Example 3:

a. Identify the critical point on the graph of $f(x)$ by marking an "X" on the point at which $f'(c) = 0.$

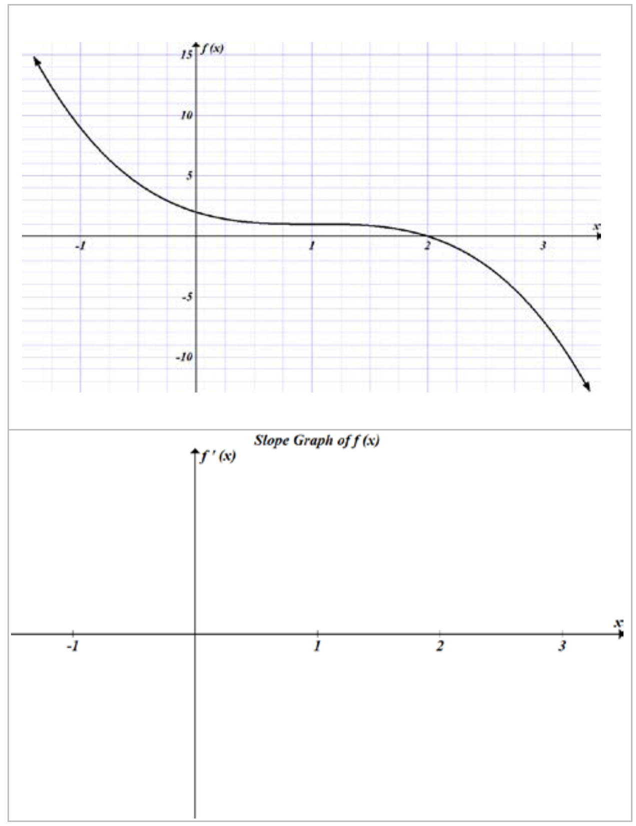
b. Draw the slope graph of $f(x)$.

c. Does the slope graph **cross** the x-axis at the critical point $x = c$?

d. Circle the terms that correctly complete the statement.

f' is <u>positive/negative</u> for $x < c$ and is <u>positive/negative</u> for $x > c$.

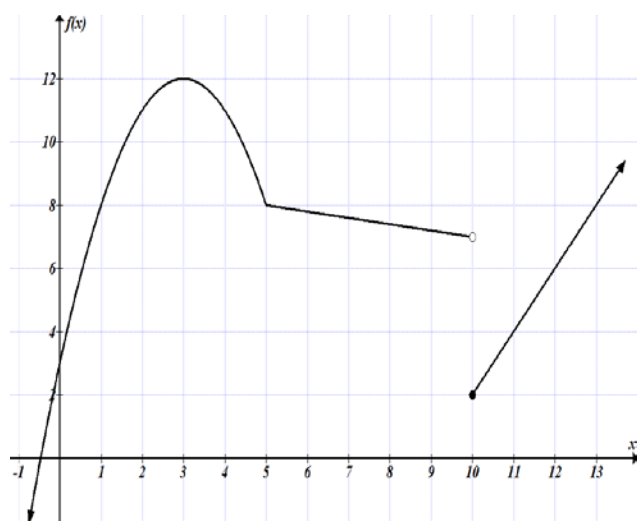
e. Does the function have a *relative maximum, a relative minimum, or neither* at the critical point?



Example 4:

Consider the three critical points on the graph of the function *f:*

a. A critical point of $f(x)$ occurs at $x =$ ___ . If the function is continuous at this point, discuss what happens to the slope graph near this critical point. If the function has a *relative maximum or a relative minimum* at this critical point, mark it on the graph.

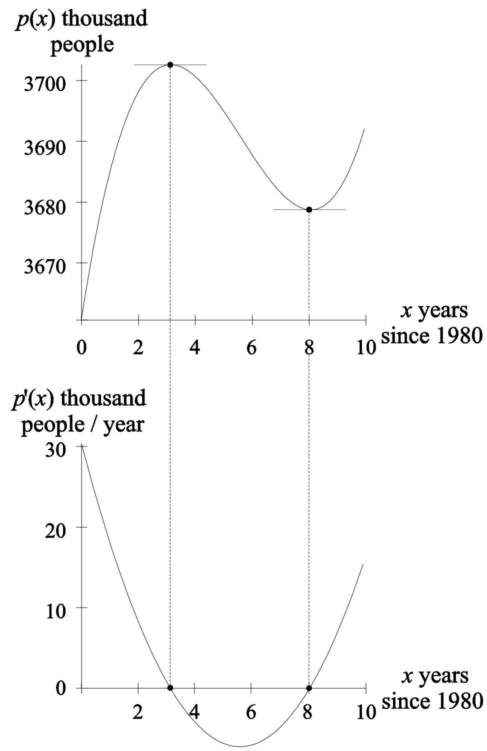


b. A second critical point of $f(x)$ occurs at $x =$ _____ . If the function is continuous at this point, discuss what happens to the slope graph near this critical point. If the function has a *relative maximum or a relative minimum* at this critical point, mark it on the graph.

c. A third critical point of $f(x)$ occurs at $x =$ ___ . If the function is continuous at this point, discuss what happens to the slope graph near this critical point. If the function has a *relative maximum or a relative minimum* at this critical point, mark it on the graph.

Example 5: (CC5e pp. 257-259)

The population of Kentucky can be modeled as $p(x) = 0.395x^3 - 6.67x^2 + 30.3x + 3661$ thousand people where x is the number of years since 1980, $0 \le x \le 10$.



a. Use your calculator to verify that the graph to the right is the graph of $p(x)$ on the interval $0 \le x \le 10$.

b. Label and mark an "X" on the graph of $p(x)$ for the relative maximum and the relative minimum. Does the graph of $p'(x)$ confirm your findings? Explain.

c. Use your calculator to find the following, correct to three decimal places, on the given interval. (See calculator directions on the next page.)

Relative minimum: $x =$ ________________; $p(x) =$ ____________________

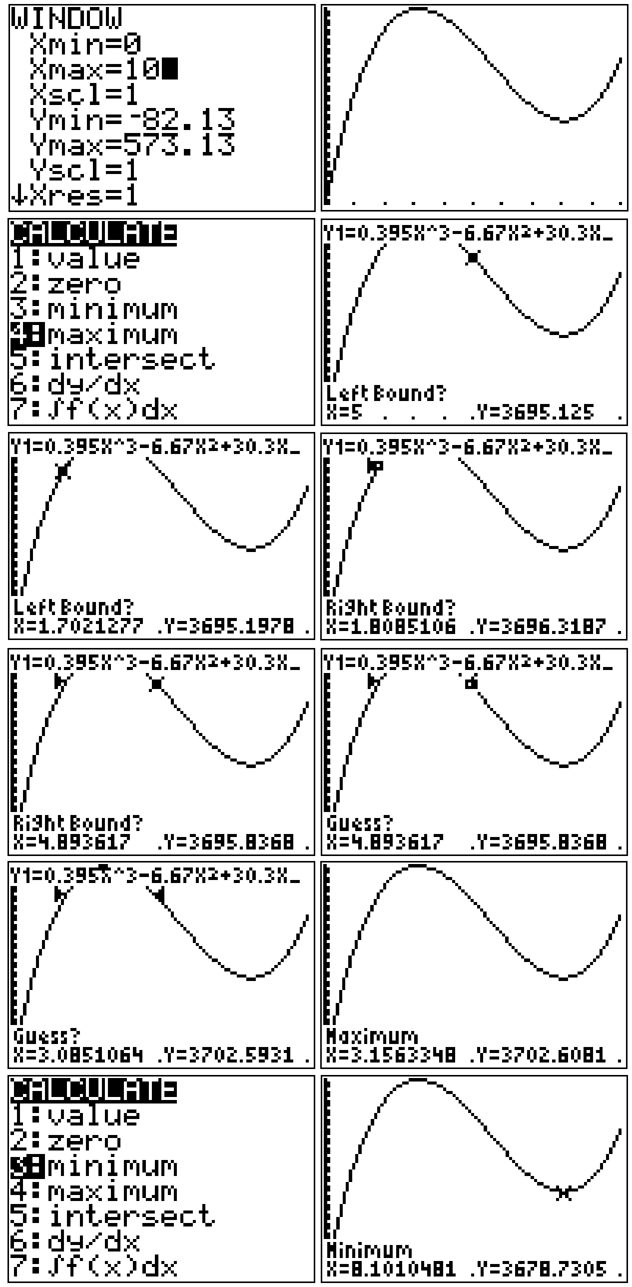
Relative maximum: $x =$ ________________; $p(x) =$ ____________________

d. What was the population of Kentucky at the relative maximum?

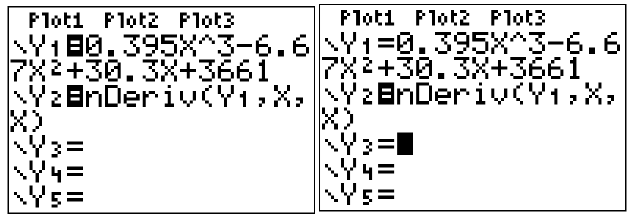
What was the population of Kentucky at the relative minimum?

Finding relative maximum and relative minimum points:

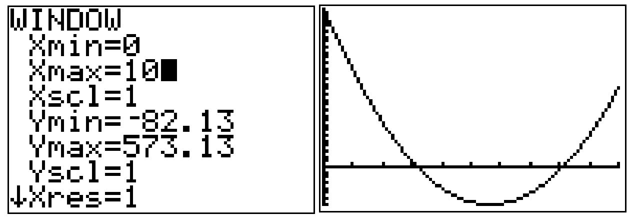
- Enter $p(x)$ into Y1
- Set the window using **WINDOW** Xmin = **0** and Xmax = **10**
- **ZOOM** **0** [ZOOMFIT] returns the graph of p .
- **2ND** **TRACE** [CALC] **4** [maximum] returns the second screen.
- Use ◄ as many times as necessary to position the cursor to the *left* of the relative maximum of p
- **ENTER** marks the left bound
- Use ► as many times as necessary to position the cursor to the *right* of that relative maximum
- **ENTER** marks the right bound
- Use ◄ as many times as necessary to position the cursor at the approximate relative maximum. **ENTER** returns the x and y coordinates of the relative maximum.
- Repeat the process to find the relative minimum using **2ND** **TRACE** [CALC] **3** [minimum].



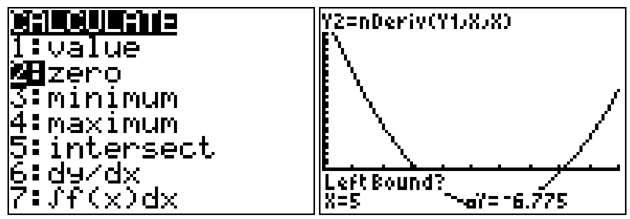
Using critical points as an alternate method for finding relative maximum and relative minimum points:



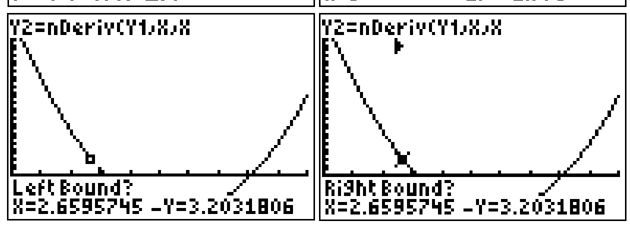
- Enter $p(x)$ into Y1 and $p'(x)$ into Y2 using nDeriv(Y1, X, X). Recall: *nDeriv* is found using MATH 8.
- Move the cursor to Y1 and press ENTER to un-highlight it



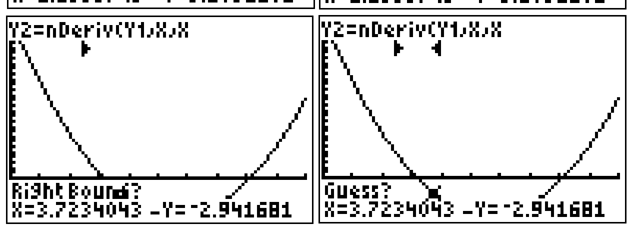
- Set the window using **<u>WINDOW</u>** Xmin = **<u>0</u>** and Xmax = **<u>10</u>**
- **<u>ZOOM</u>** **<u>0</u>** [ZOOMFIT] returns the graph of p'. There are two places where p' crosses the horizontal axis.



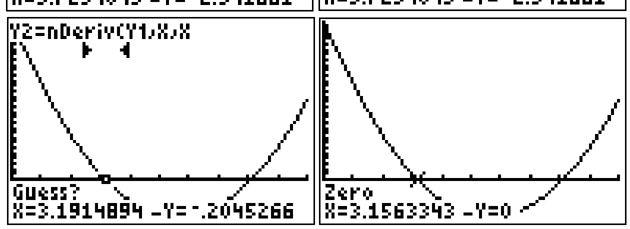
- **<u>2ND</u>** **<u>TRACE</u>** [CALC] **<u>2</u>** [zero] returns the second screen.



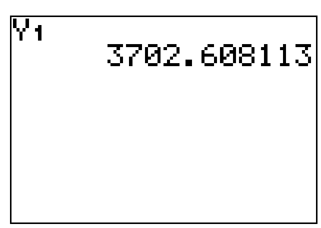
- Use ◄ as many times as necessary to position the cursor to the *left* of the first *x*-intercept (zero) of p'
- **<u>ENTER</u>** marks the left bound.



- Use ► as many times as necessary to position the cursor to the *right* of that *x*-intercept of p'
- **<u>ENTER</u>** marks the right bound.



- Use ◄ as many times as necessary to position the cursor at the approximate zero. **<u>ENTER</u>** returns a solution to $p'(x)=0$.

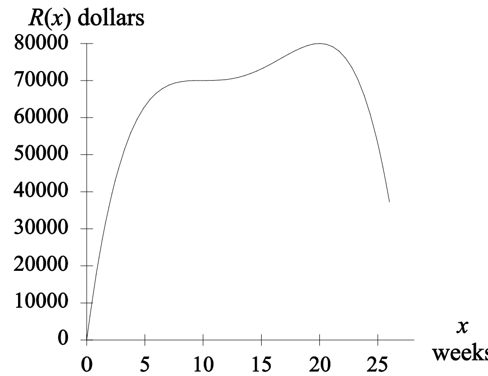


- To eliminate intermediate rounding of the input value, use **MODE** to double-check that the calculator is set to FLOAT the number of decimals. Return to the Home Screen and **<u>Y1</u>** **<u>ENTER</u>** returns the output value of the stored *x-value.* Since the last *x*-value was the location of the relative maximum, $x=3.1563343$, the value returned is the output value (with no intermediate rounding of the input value).

- Repeat the process to find the second critical point and the corresponding minimum.

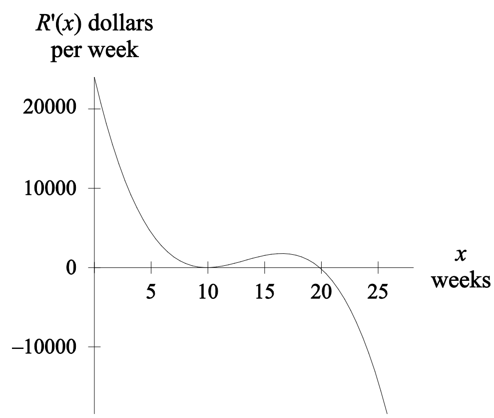
Example 6: (CC5e p. 262)

TW Cable Company actively promoted sales in a town that previously had no cable service. Once TW saturated the market, it introduced a new 50-channel system, raised rates, and began a new sales campaign. As the company began to offer its expanded system, a different company, CC Network, began offering satellite service with more channels than TW and at a lower price. TW Cable's revenue for 26 weeks after it began its sales campaign is given by $R(x) = -3x^4 + 160x^3 - 3000x^2 + 24{,}000x$ dollars, where x is the number of weeks since TW Cable Company began its new sales campaign.



a. Use your calculator to verify that the graph to the right is the graph of $R(x)$ on the interval $0 \le x \le 26$.

b. One critical point occurs at approximately $x = 10$. Explain how the derivative graph shows there is not a relative extreme at this critical point.



c. A relative maximum does occur at a second critical point. Mark the relative maximum with an "X" on the graph of $R(x)$. Also mark the graph of $R'(x)$ by circling the point on $R'(x)$ at which the relative maximum occurs on $R(x)$.

d. Use your calculator to find the second critical point.

When did TW Cable Company's revenue peak during the period shown in the graph?

What was its revenue at that time?