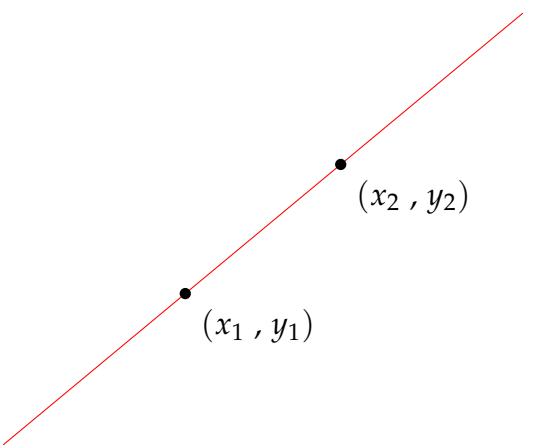
Homework #3

Due Friday (October 11th)

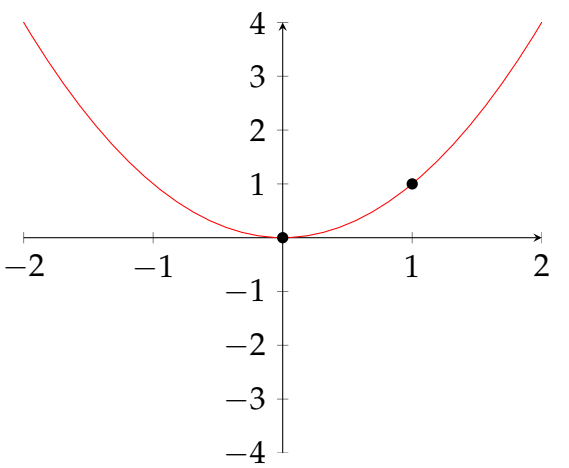
Name _____

1. (1 point) Suppose a line passes through the points (x_1, y_1) and (x_2, y_2), as shown below:



What is the slope of this line?

2. Let $f(x)$ be the function given by the graph below:



2.a. (1 point) Use a ruler to draw the tangent line to the graph of the function at the point $(1, 1)$. Find the slope of this line (Hint: You need to find two points this line goes through. Then you need to use the slope formula you wrote down above).

2.b. (1 point) Use a ruler to draw the tangent line to the graph of the function at the point $(0, 0)$. Find the slope of this line .

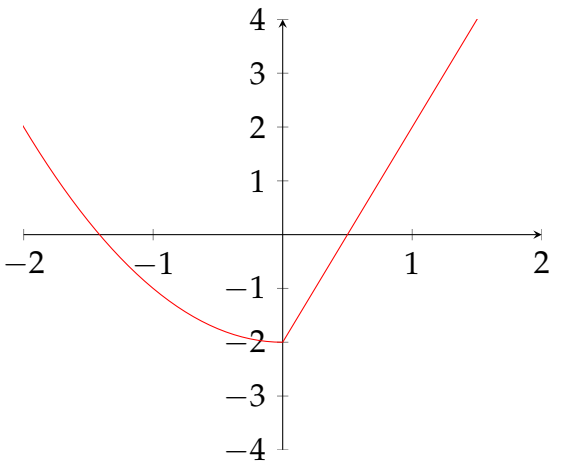
2.c. (1 point) Use a ruler to draw the secant line through the points $(0,0)$ and $(1,1)$. Find the slope of this line (Hint: You are given two points which lie on this line, namely $(0,0)$ and $(1,1)$. This is all you need to calculate the slope of this line).

2.d. (1 point) With the information above, evaluate

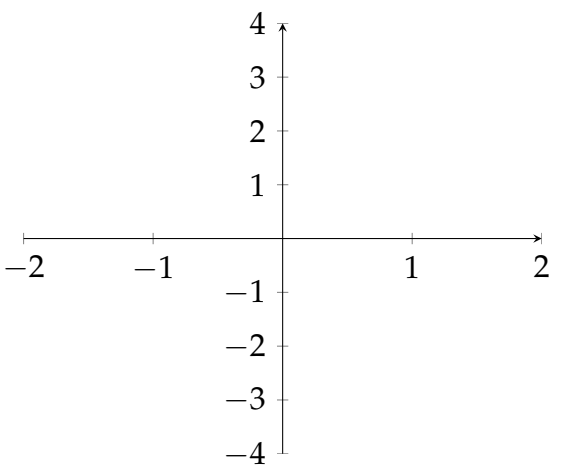
$$f(0) =$$
$$f'(0) =$$
$$f(1) =$$
$$f'(1) =$$

What is the average rate of change between the points $(0,0)$ and $(1,1)$?

3. (6 points) Let $f(x)$ be the function given by the graph below:

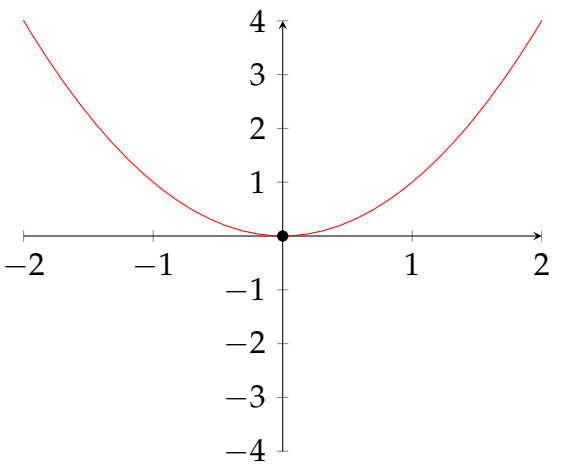


Sketch the graph of $f'(x)$:



(Hint: The function $f(x)$ goes through the point $(0.5,0)$. Find the tangent line to the graph of $f(x)$ at this point, then find the slope of this line (the slope of this line is by definition the value $f'(0.5)$). Now plot the point $(0.5, f'(0.5))$. The function also goes through the point $(1,2)$. Repeat the same procedure as above and plot the point $(1, f'(1))$. The function also goes through the point $(-2,2)$. Repeat the same procedure as above and plot the point $(-2, f'(-2))$. You should be able to construct $f'(x)$ from this).

4. Let $f(x)$ be the function given by the graph below:



From problem 2, we know what the slope of the tangent line at $(0,0)$ is (i.e. $f'(0)$). In this exercise, we want to compute $f'(0)$ by taking the limit of slopes of secant lines[1]

4.a. (1 point) Use a ruler to draw the secant line through the points $(0,0)$ and $(2,4)$. Find the slope of this line.

4.b. (1 point) Use a ruler to draw the secant line through the points $(0,0)$ and $(1,1)$. Find the slope of this line.

4.c. (1 point) Use a ruler to draw the secant line through the points $(0,0)$ and $\left(\frac{1}{2},\frac{1}{4}\right)$. Find the slope of this line.

4.d (1 point) Complete the table below

x	Slope of secant line from $(0,0)$ to $(x,f(x))$
2	
1	
0.5	
0.1	

What does this table suggest? (Hint use $\lim_{x\to 0^+} f(x)$ notation).

[1] In Calculus, almost everything is defined in terms of limits.