Section 4.1: Linearization and Estimates

For a point $(c, f(c))$ and a nearby point $(c+h, f(c+h))$ on a differentiable function *f*, the rate of change $f'(c)$ can be used to **approximate the amount of change** between the two points. $f'(c)$ can also be used to **approximate the value** of $f(c+h)$**.**

Recall from section 2.1 that **change** between $(c, f(c))$ and a nearby point $(c+h, f(c+h))$ is defined as the difference between output values: $f(c+h) - f(c)$, for $h > 0$.

An **estimate of change** between $(c, f(c))$ and a nearby point $(c+h, f(c+h))$ is given by: $f'(c) \cdot h$. Graphically, $f'(c) \cdot h$ is the vertical distance between points at $x = c$ and $x = c+h$ on the tangent line to *f* at $x = c$.

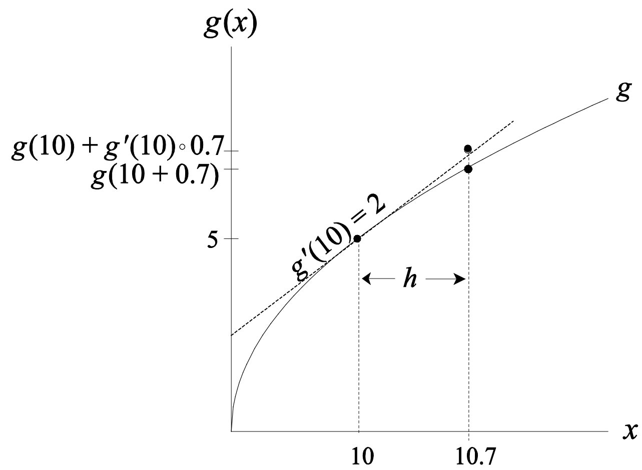
$f(c+h) \approx f(c) + f'(c) \cdot h$ gives an **estimate of the output value** $f(c+h)$ by finding the output value for $x = c+h$ on the tangent line to *f* at $x = c$. It is found by adding the estimated amount of change to $f(c)$**.**

The **linearization of a function *f* at a point *c*** is the equation of the line tangent to the function at point *c*. This equation can be found using the point-slope formula of a line, with a point on the tangent line $(c, f(c))$ and its slope $f'(c)$**.**

The linearization of f **at point** c is given by the formula $f_L(x) = f(c) + f'(c) \cdot (x - c)$.

Example 1: (CC5e p. 252)

As shown in the graph to the right, $g(10) = 5$ and $g'(10) = 2$.



a. Approximate the amount of **change** between $g(10)$ and $g(10.7)$.

The amount of change, $g(10.7) - g(10)$, can be estimated by $g'(10) \cdot 0.7 =$

b. Approximate the output value of $g(10.7)$.

$g(10.7) \approx [g(10) + \textit{estimated change}] =$

c. The estimate in part b is an *underestimate/overestimate* because the function is *concave up/concave down* and the tangent line lies *below/above* the graph.

d. Write a linearization for $g(x)$ at $c = 10$.

$g_L(x) =$

e. Use the linearization in part d to approximate $g(10.7)$. Verify that the answer is the same as found in part b.

Example 2: (CC5e p. 255, Activity 5)

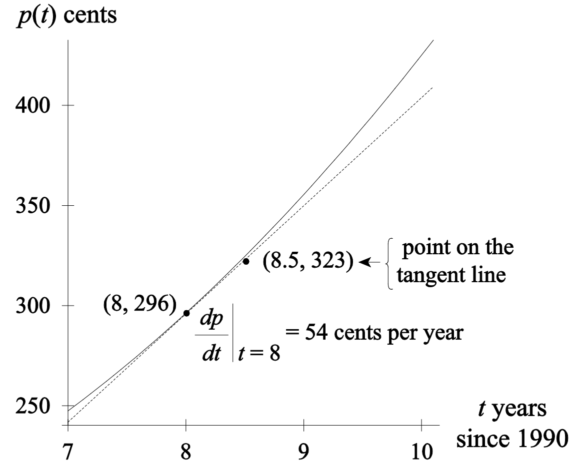
a. If $f(3) = 17$ *and* $f'(3) = 4.6$, write a linearization for f at input $c = 3$.

b. Use the linearization to estimate $f(3.5)$.

Example 3: (CC5e pp. 250-251)

$p(t)$ cents is the average retail price of a pound of salted, grade A butter, t years since 1990.

Use the graph of p along with a tangent line at $t = 8$ to complete the following statements.



a. $p(8) =$ __________ and $\left.\frac{dp}{dt}\right|_{t=8} =$ ___________

b. Estimate the **change** in the price of butter from the end of December, 1998 until the end of June, 1999.

$p'(8) \cdot 0.5 =$

c. Estimate the price of butter at the end of June, 1999.

$p(8.5) \approx p(8) + p'(8) \cdot 0.5 =$

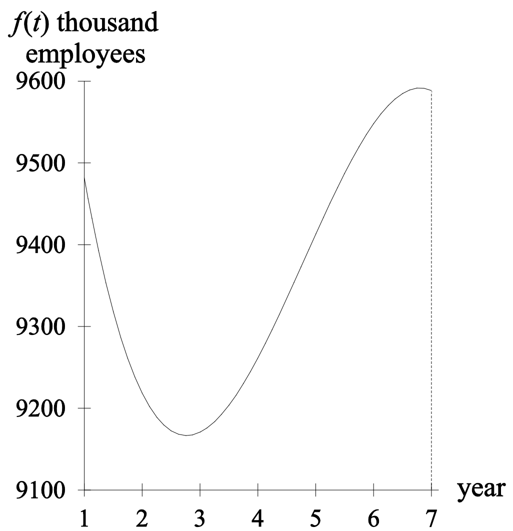
d. Is the estimated price of butter found in part c an *underestimate or overestimate*?

e. Write a **linearization** for the function p at $c = 8$.

Example 4: (CC5e pp. 253-254)

$f(t) = -12.92t^3 + 185.45t^2 - 729.35t + 10038.57$ thousand employees gives the number of 20-24 year old full-time employees over a six-year period, $1 \le t \le 7$. Assume that the six-year period represents the *previous* six years, so that $t = 7$ represents **current** full-time employment.

The graph to the right shows $f(t)$ on the interval $1 \le t \le 7$.



a. Find $f(2)$ and $f'(2)$. Include units.

b. Find the linearization $f_L(t)$ at $c = 2$.

$$f_L(t) = f(c) + f'(c) \cdot (t - c) =$$

c. Estimate $f(3)$ using the linearization found in part b. Is this an underestimate or an overestimate of the model's prediction of the number of 20-24 year old full-time employees when $t = 3$?

d. Find $f(3)$ using the given function $f(x)$. Compare your answer to your answer to part c.