Math 9853: Homework #8

Due Thursday, November 7 , 2019

Reading: Sections 9.1, 12.1 and 12.2.

(a)(a.1) Problem 15.2 on page 550.

(a.2) Let V be the vector space of polynomials over K of degree $\leq n-1$ and $f\colon V \to V$ the map defined by derivative:

$$f(u(x)) = u'(x), \quad u(x) \in V.$$

Show that $f^n = 0$ and describe all invariant subspaces of V. Is V decomposable? (**Hint**: Consider two cases: K contains $\mathbb{Q}$ (so all nonzero integers are nonzero in K), or K contains $\mathbb{Z}_p$ where $p > 1$ is a prime (so $p = 0$ in K and $i \neq 0$ for every integer i with $0 < i < p$)).

(b) Let $v_1, v_2, \ldots, v_n$ be a basis of a vector space V. Suppose $f\colon V \to V$ is a linear map so that

$$f(v_i) = \begin{cases} v_i + v_{i-1} & \text{if } 1 \leq i \leq m, \\ v_i & \text{if } m < i \leq n, \end{cases}$$

where $1 \leq m \leq n$ and $v_0 = 0$.

(b.1) Find the matrix and the characteristic polynomial of f.

(b.2) Find a basis for the eigenspace of $\lambda = 1$.

(b.3) Show that $(X-1)^m$ is the minimal polynomial of f.

(c) Problem 12.10 on page 457.