Let $R = K[x,y,z]$ and let $I = \langle xy^2z^3, x^2yz^3, x^3yz^2, x^3y^2z, x^2y^3z, xy^3z^2 \rangle$. We describe two free resolutions of R/I. The first is given by

$$0 \longrightarrow R(-9) \xrightarrow{\varphi_3} R(-7)^6 \xrightarrow{\varphi_2} R(-6)^6 \xrightarrow{\varphi_1} R \longrightarrow 0 \tag{1}$$

where

$$\varphi_3 = \begin{pmatrix} xy \\ y^2 \\ yz \\ z^2 \\ xz \\ x^2 \end{pmatrix}, \qquad \varphi_2 = \begin{pmatrix} -x & 0 & 0 & 0 & 0 & y \\ y & -x & 0 & 0 & 0 & 0 \\ 0 & z & -y & 0 & 0 & 0 \\ 0 & 0 & z & -y & 0 & 0 \\ 0 & 0 & 0 & x & -z & 0 \\ 0 & 0 & 0 & 0 & x & -z \end{pmatrix}, \qquad \varphi_1 = \begin{pmatrix} xy^2z^3 & x^2yz^3 & x^3yz^2 & x^3y^2z & x^2y^3z & xy^3z^2 \end{pmatrix}.$$

This resolution was constructed using the permutohedron $\mathcal{P}(1,2,3)$[1]. In this case, the graded Betti numbers look like

$$\begin{aligned} \beta_{0,0} &= 1 \\ \beta_{1,6} &= 6 \\ \beta_{2,7} &= 6 \\ \beta_{3,9} &= 1 \end{aligned}$$

The second is given by

$$0 \longrightarrow R(-9) \xrightarrow{\psi_3} R(-7) \oplus R(-8) \oplus R(-7)^2 \oplus R(-8) \oplus R(-7) \xrightarrow{\psi_2} R(-6)^6 \xrightarrow{\psi_1} R \longrightarrow 0 \tag{2}$$

where

$$\psi_3 = \begin{pmatrix} xy \\ x \\ z^2 \\ yz \\ z \\ x^2 \end{pmatrix}, \qquad \psi_2 = \begin{pmatrix} -x & 0 & 0 & 0 & 0 & y \\ y & -y^2 & 0 & 0 & 0 & 0 \\ 0 & z^2 & -x & 0 & 0 & 0 \\ 0 & 0 & y & -z & 0 & 0 \\ 0 & 0 & 0 & y & -y^2 & 0 \\ 0 & 0 & 0 & 0 & x^2 & -z \end{pmatrix}, \qquad \psi_1 = \begin{pmatrix} xy^2z^3 & x^2yz^3 & x^2y^3z & x^3y^2z & x^3yz^2 & xy^3z^2 \end{pmatrix},$$

note that ψ_1 differs from φ_1 only by a swap of position of the generators x^3yz^2 and x^2y^3z. This resolution was constructed using the Cayley graph of the symmetric group S_3. In this case, the graded Betti numbers look like

$$\begin{aligned} \beta_{0,0} &= 1 \\ \beta_{1,6} &= 6 \\ \beta_{2,7} &= 4 \\ \beta_{2,8} &= 2 \\ \beta_{3,9} &= 1 \end{aligned}$$

[1]Recall that $\mathcal{P}(1,2,3)$ is defined to be the convex hull of $\{(\pi(1), \pi(2), \pi(3)) \mid \pi \in S_3\}$ in $\mathbb{R}^3$.