Section 1.5: Exponential Functions and Models

An **exponential** function has an equation of the form $f(x) = ab^x$, with **initial value** a and **constant multiplier** b.

Percentage change between two points of a function $f(x)$ is calculated as $\frac{f(x_2) - f(x_1)}{f(x_1)} 100\%$ where $(x_1, f(x_1))$ and $(x_2, f(x_2))$ are two points on the function.
An exponential function has a **constant percentage change**. This **constant percentage change** over a single unit input interval is equal to $(b-1)\cdot 100\%$.

The graph of an exponential function $f(x) = ab^x$ has **one concavity**, determined by the sign of *a:*

- for $a > 0$, f is concave up
- for $a < 0$, f is concave down

The graph of an exponential function $f(x) = ab^x$ has a **horizontal asymptote** at $y = 0$. The values of b and a determine the end behavior of f, as x increases without bound:

- for $0 < b < 1$, $\lim_{x\to\infty} f(x) = 0$
- for $b > 1$, $\begin{cases} \text{if } a > 0, \text{ then } \lim_{x\to\infty} f(x) = \infty \\ \text{if } a < 0, \text{ then } \lim_{x\to\infty} f(x) = -\infty \end{cases}$

Example 1:

a. Label each of the following graphs as either *increasing* or *decreasing* and as either *concave up* or *concave down*. Complete the limit statements that describe the end behavior.

b. Find the following for the function $f(x) = 3(2^x)$.

- a = b = (Note that $b > 1$.)
- percentage change =
- Does graph I or graph II look like the graph of $f(x) = 3(2^x)$?

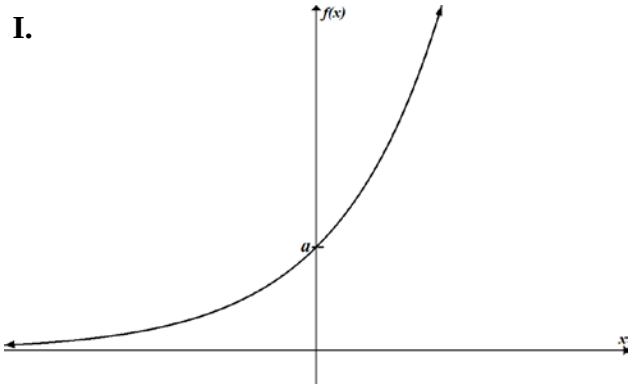
This is an example of **exponential growth**.

c. Find the following for the function $f(x) = 3(0.35^x)$.

- a = b = (Note that $0 < b < 1$.)
- percentage change =
- Does graph I or graph II look like the graph of $f(x) = 3(0.35^x)$?

This is an example of **exponential decay**.

I.

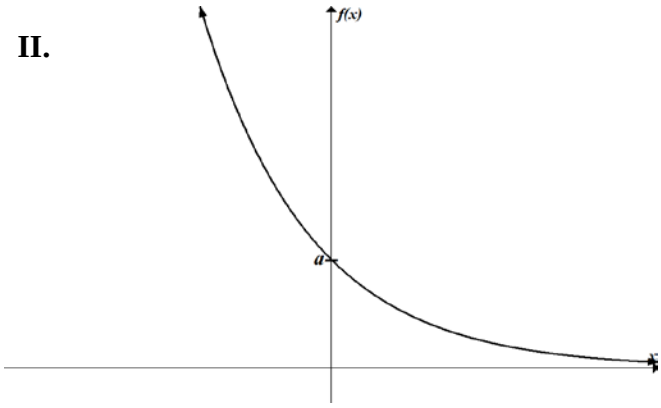


Increasing or decreasing?

Concave up or concave down?

$\lim_{x \to -\infty} f(x) =$ _____; $\lim_{x \to \infty} f(x) =$ _____

II.



Increasing or decreasing?

Concave up or concave down?

$\lim_{x \to -\infty} f(x) =$ _____; $\lim_{x \to \infty} f(x) =$ _____

Example 2: (CC5e p. 49)

Apple introduced the iPod in 2001. iPod sales were 7.68 million units in 2006 and increased approximately 9.1% each year between 2006 and 2008.

a. Why is an exponential model appropriate to describe iPod sales between 2006 and 2008?

b. Find an exponential model of the form $f(x) = ab^x$ for iPod sales between 2006 and 2008. Align the input data to the number of years since 2006. (*Aligned input is generally necessary for exponential model*s.)

- a = b =

- model:

c. Use the model to estimate iPod sales in 2010.

d. Was *interpolation or extrapolation* used to answer part *c*? Explain.

Example 3: (CC5e p. 54)
According to the Social Security Advisory Board, the number of workers per beneficiary of the Social Security program was 3.3 in 1996 and is projected to decline by 1.45% each year through 2030.

a. Find a model for the number of workers per beneficiary from 1996 through 2030.

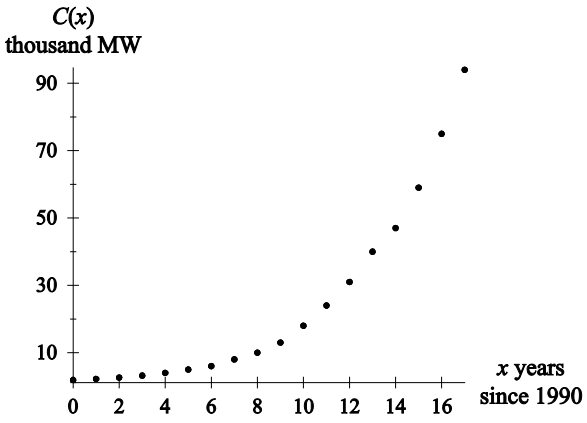
b. What does the model predict that the number of workers per beneficiary will be in 2030?

Example 4: (CC5e p. 51)

Over the past 30 years, wind power has been harnessed by wind turbines to produce a low-cost, green alternative for electricity generation. The table gives the cumulative capacity in thousand megawatts (MW) for wind power worldwide.

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998
Wind Power thousand MW	1.9	2.2	2.6	3.2	4	5	6	8	10
Year	1999	2000	2001	2002	2003	2004	2005	2006	2007
Wind Power thousand MW	13	18	24	31	40	47	59	75	94

a. Verify that the figure to the right shows a scatter plot of the data.



b. How does the end behavior and suggested concavity of the scatter plot indicate that an exponential function is appropriate for the wind power capacity data?

- end behavior

- concavity

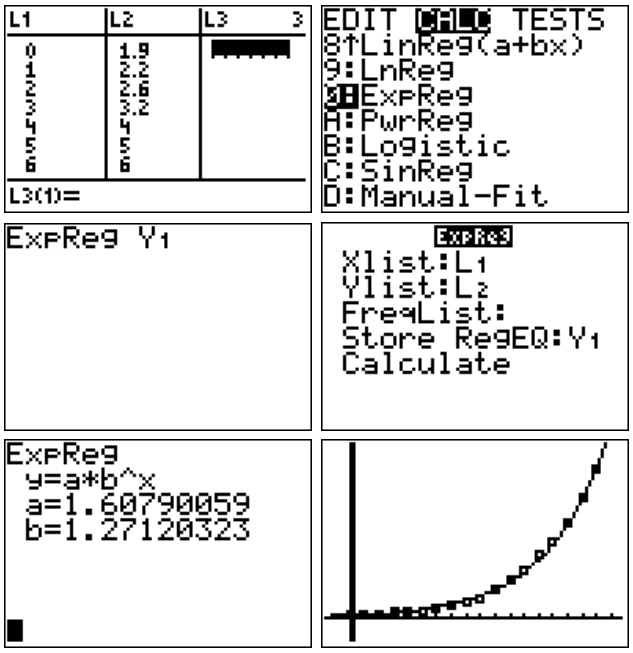
c. Align the data so that $x = 0$ in 1990 and find an exponential function to model the data in the table. Write a completely defined model.

Finding, storing, and viewing an exponential function:

- With the data in L1 and L2, **STAT** ▸ [CALC] ▾ to 0 [ExpReg] **ENTER** returns ExpReg on the Home Screen. **VARS** ▸ [Y-VARS] **1** [Function] **1** [Y1] returns Y1 **ENTER** finds and stores the function

  OR

  STAT ▸ [CALC] ▾ to 0 [ExpReg] **ENTER** returns the ExpReg Screen
  Xlist: **2nd** **1** [L1]
  Ylist: **2nd** **2** [L2]
  Store RegEQ: **VARS** ▸ [Y-VARS] **1** [Function] **1** [Y1]
  Move cursor to Calculate and hit **ENTER**



- **Y=** to verify the unrounded model is stored in Y1.

- View the function and the scatter plot at the same time: **ZOOM** **9** [ZoomStat]

d. What is the percentage change for the exponential function?

Since an exponential function has constant percentage change, an exponential function should be considered in modeling a set of data that displays constant (or nearly constant) **percentage change.** For successive evenly spaced data points (x_1, y_1) and (x_2, y_2), percentage change is calculated by $\frac{y_2 - y_1}{y_1} \cdot 100\%$.

Example 5: (CC5e p. 49)

The data in the table represent the estimated population of northern Canadian cod over a specific range.

Decades (since 1963)	0	1	2	3	4
Population (billions)	1.72	0.63	0.24	0.085	0.032
Percentage Change	$\frac{0.63-1.72}{1.72} \cdot 100$ = -63.372	-61.905	-64.583	-62.353	

a. Do the percentage changes appear to be nearly constant?

b. Discuss the end behavior and concavity of the scatter plot.

c. Write a completely defined model for the data.

d. Find the percentage change of the exponential model.

Example 6: (CC5e p. 55, Activity 21)

The data in the table gives the percentage of MySpace users who are a certain age.

Age (years)	17	19	21	23	25	27	29	31	33	35
Female (percent)	9.6	7.8	6.1	5.1	4.3	3.8	2.4	2.1	1.2	1.1

a. Align the input data to the number of years after 17. Write an exponential model for the female MySpace user data.

b. Use the model found in part a to find the constant percentage change in the percentage of female MySpace users.

c. What percentage of female MySpace users are 18 years old? 20 years old? Are these answers found using extrapolation or interpolation?