Homework #1 Solution

1. (10 points) Suppose \$50000 is invested at 5% APR. Find the value of the investment after 23 months in various situations. I'll do quarterly for you.

Solution:

Compounding	Formula	Value of t	Future Value	Doubling Time
Annually	$F_1(t) = 50000\,(1 + 0.05)^t$	$t = 1$	52500 dollars	15 years
Semiannually	$F_2(t) = 50000\left(1 + \frac{0.05}{2}\right)^{2t}$	$t = 1.5$	$\approx$ 53844.53 dollars	14 years and 6 months
Quarterly	$F_4(t) = 50000\left(1 + \frac{0.05}{4}\right)^{4t}$	$t = 1.75$	$\approx$ 54542.53 dollars	14 years
Monthly	$F_{12}(t) = 50000\left(1 + \frac{0.05}{12}\right)^{12t}$	$t = 23/12$	$\approx$ 55017.83 dollars	13 years and 11 months
Continuously	$F_\infty(t) = 50000 e^{0.05t}$	$t = 23/12$	$\approx$ 55028.78 dollars	$\approx$ 13.863 years

2. (5 points) How much will a \$100 investment, compounded quarterly at 5% APR yield in 10 months? How long will it take to double your investment?

Solution: We first insert the function

$$F_4(t) = 100\left(1 + \frac{0.05}{4}\right)^{4t}$$

into our calculator. Then we go to tblset (2nd window) and set ΔTbl=1/4. We also make sure Indpnt and Depend are set to auto. Now we go to table (2nd graph). Since 10 months corresponds to $10/12 \approx 0.833$ years, we will have 103.80 dollars (rounding to two decimal places) in 10 months.

To find the time it takes for our intitial investment to double, we scroll down the table until the output value reaches a number ≥ 200. In this case, it reaches 200.50 dollars in 14 years. Thus the doubling time is 14 years.

3. (5 points) How much will a \$100 investment, compounded continuously at 5% APR yield in 10 months? How long will it take to double your investment?

Solution: We first insert the function

$$F_\infty(t) = 100 e^{0.05t}$$

into our calculator. Then we evaluate[1] $F_\infty(10/12)$ in our calculator (don't do table method for continuous compounding). We find that our investment will yield 104.25 dollars.

To find the time it takes for our intitial investment to double, we solve[2] the equation

$$F_\infty(t) = 200.$$

We find that $t \approx 13.863$ years (rounded to three decimal places).

4. (5 points) What is the APY for an investment that pays 3% APR compounded quarterly? (If you need to, go to section 1.6 in the book and look up the formula for APY. This type of question will be on the test, so make sure you know it.)

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[1] To do this you enter vars $\rightarrow$ yvars $\rightarrow$ function $\rightarrow$ y_1 then enter $(10/12)$ in calculator.

[2] You can either use the intersect method or the math solver method. The math solver method goes like this: math $\rightarrow$ solver $\rightarrow$ $0 = y_1 - 200 \rightarrow$ enter $\rightarrow$ solve (alpha enter). If you aren't sure how to solve an equation, then go back to section 1.1 in the book where they explain how to do so in more detail.

Solution: Using the formula given in the book, we have

$$\text{APY}_4 = \left(\left(1 + \frac{0.03}{4} \right)^4 - 1 \right) \cdot 100\% \approx 3.034\%$$

5. (5 points) What is the APY for an investment that pays 3% APR compounded continuously? (The formula for APY is different for continuous compounding than the one for quarterly compounding. Again, look up the formula in section 1.6 if you don't know it.)

Solution: Using the formula given in the book, we have

$$\text{APY}_\infty = \left(e^{0.03} - 1 \right) \cdot 100\% \approx 3.045\%$$