MATH 8000

Homework Assignment 1

Problem 1. For n events A_1, A_2,..., A_n, prove that the Inclusion-Exclusion Principle holds; i.e, prove that

$$\begin{aligned} P\left(\bigcup_{i=1}^{n} A_j\right) &= \sum_{i=1}^{n} P(A_i) - \sum_{i_1<i_2} P(A_{i_1} \cap A_{i_2}) + \sum_{i_1<i_2<i_3} P(A_{i_1} \cap A_{i_2} \cap A_{i_3}) - \\ &\quad \cdots + (-1)^{n+1} P(A_1 \cap A_2 \cap \cdots \cap A_n). \end{aligned}$$

The proof here should be the more general version rather than the special case examined in class.

Problem 2. The rules for the game of craps are as follows. A player rolls two dice and computes the total of the spots showing. If the player's first toss is a 7 or 11 the player wins the game. If the first toss is a 2, 3, or 12, the player looses the game. If the player rolls anything else (4, 5, 6, 8, 9, or 10) on the first toss that value becomes the players point. If the player does not win or lose on the the first toss he tosses the dice repeatedly until he obtains either his point or a 7. He wins if he tosses his point before tossing a 7 and looses if he tosses a 7 before his point.

(a) Consider the situation in which a player decides to play 10 games of craps. Define the random variable

$$X = \{\text{number of games won}\}.$$

Derive the PDF and CDF of X, be sure to list any and all assumptions that you make.

(b) Consider the stubborn player who will not give up until he has won a game. Define the random variable

$$X = \{\text{number of games played until players goal is reached}\}.$$

Derive the PDF and CDF of X, be sure to list any and all assumptions that you make.

(c) Further, consider the stubborn player's even more stubborn brother who will not quit playing until he wins 6 times. Define the random variable

$$X = \{\text{number of games played until players goal is reached}\}.$$

Derive the PDF and CDF of X, be sure to list any and all assumptions that you make.

Problem 3. Suppose that n men at a party throw their hats in the center of the room. Each man then randomly selects a hat. Define the random variable

$$X = \{\text{the number of men who selected their own hat}\}.$$

(a) Derive the probability mass function (PMF) of X.

(b) Now express the PMF of X as we let n grow large without bound (i.e., as $n \to \infty$).

(c) Consider the case in which n is large (e.g., $n = 100$) suggest an approximation to the PMF of X that could be calculated easily, then using computer software evaluate how well your approximation works. (Hint: You should consider for a fixed n how well the approximation works at every point in $\mathcal{X}$, suggest a way to summarize this information, then expand across n, your summary should be concise, to the point, and informative.)

Additional problems from text: 1.6, 1.7, 1.8, 1.11, 1.23, 1.24, 1.37, 1.38, 1.39, 1.41, 1.43, 1.45, 1.51, 1.52, 1.53, 1.55