

Section 2.1: Measures of Change over an Interval

The amount of **change**, the **percentage change**, or the **average rate of change** can be calculated between two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ by using the formulas in the table below.

Change finds the difference in two output values. Graphically, this is the amount of vertical change between the two points.

Percentage change expresses the amount of **change** as a percentage of the first output value.

Average rate of change divides the change by the length of the interval and expresses how quickly a quantity is changing on average over an interval. Graphically, this is the slope of the **secant line**, a line that connects the two points.

A sentence of **interpretation** for **change, percentage change, or average rate of change over an interval** uses ordinary conversational language to answer the questions:

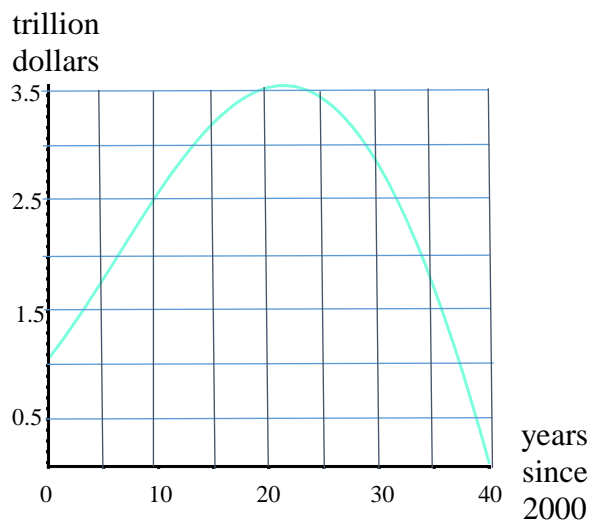
- *When?* refers to the input interval.
- *What?* refers to the output description for the function.
- *Increased(es) or Decreased(es)?*
For average rate of change, use the phrase “increased on average by” or “decreased on average by”.
- *By how much?* refers to the change calculation, and includes its corresponding units.

	Formula (assume $x_1 < x_2$)	Units
Change	$f(x_2) - f(x_1)$	output units
Percentage change	$\frac{f(x_2) - f(x_1)}{f(x_1)} \cdot 100\%$	percent
Average rate of change	$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$	output units per input unit

Example 1: (CC5e p. 132)

The following graph shows the Social Security Advisory Board's estimates of the federal government's Social Security assets between 2002 and 2040 (in constant 2005 dollars).

- a. Use the graph to estimate the output values for the points that correlate to 2012 and 2033. Write the two ordered pairs.



- b. Calculate and interpret in a sentence the **change** in Social Security assets between 2012 and 2033.
- c. Calculate and interpret in a sentence the **percentage change** in Social Security assets between 2012 and 2033.
- d. Draw the **secant line** that cuts through the graph at input values 12 and 33. What is the slope of the secant line?
- e. Calculate and interpret in a sentence the **average rate of change** in Social Security assets between 2012 and 2033.

Example 2: (CC5e p. 131)

The table shows temperatures on a typical day in May, in a certain Midwestern city.

Time, in hours after 7 am	0	1	2	3	4	5	6	7	8	9	10
Temperature, in degrees Fahrenheit ($^{\circ}\text{F}$)	49	58	66	72	76	79	80	80	78	74	69

- Find and interpret in a sentence the **change** in temperature between 8 A.M. and 5 P.M.
- Find and interpret in a sentence the **percentage change** in temperature between 8 A.M. and 5 P.M.
- Find and interpret in a sentence the **average rate of change** in temperature between 8 A.M. and 5 P.M.
- How might the average rate of change in temperature between 8 A.M. and 5 P.M. be a misleading measure?

Example 3: (CC5e p. 133)

$f(t) = -0.8t^2 + 10t + 49$ degrees Fahrenheit gives the temperature of a certain Midwestern city on a day in May, t hours after 7 am.

- a. Calculate the **change** in temperature between 11:00 am and 4:30 pm. Include units with your answer.

Plot1 Plot2 Plot3	$Y_1(9.5) - Y_1(4)$
$\sqrt{Y_1 = -0.8X^2 + 10X + 49}$	
$\sqrt{Y_2 =}$	
$\sqrt{Y_3 =}$	
$\sqrt{Y_4 =}$	
$\sqrt{Y_5 =}$	
$\sqrt{Y_6 =}$	

- b. Calculate the **percentage change** between 11:00 am and 4:30 pm. Include units with your answer.

$(Y_1(9.5) - Y_1(4)) / Y_1(4)$	Ans*100

- c. Calculate the **average rate of change** between 11:00 am and 4:30 pm. Include units with your answer.

$(Y_1(9.5) - Y_1(4)) / (9.5 - 4)$

Example 4: (CC5e p. 135, similar to Activity 11)

Kelly Services, Inc., a leading global provider of staffing services, had a revenue of 4850 million dollars in 2004 and a revenue of 5700 million dollars in 2007.

- Write a sentence of interpretation for the amount of change in Kelly Services' revenue between 2004 and 2007.
- Write a sentence of interpretation for the percentage change in Kelly Services' revenue between 2004 and 2007.
- Write a sentence of interpretation for the average rate of change in Kelly Services' revenue between 2004 and 2007.

Example 5:

$W(x) = 1.608(1.271^x)$ thousand megawatts gives the cumulative capacity for wind power worldwide over the past x years since 1990, $0 \leq x \leq 17$.

- a. Since W is an exponential function, it has constant percentage change. Find the constant percentage change and use it to complete the following sentence:

Between 1990 and 2007, the cumulative capacity for wind power worldwide increased by _____ % each year.

- b. Calculate the percentage change in the cumulative capacity between 1990 and 2007 and complete the following sentence:

Between 1990 and 2007, the cumulative capacity for wind power worldwide increased by _____ %.

- c. Explain the difference in the answers in part a and part b.