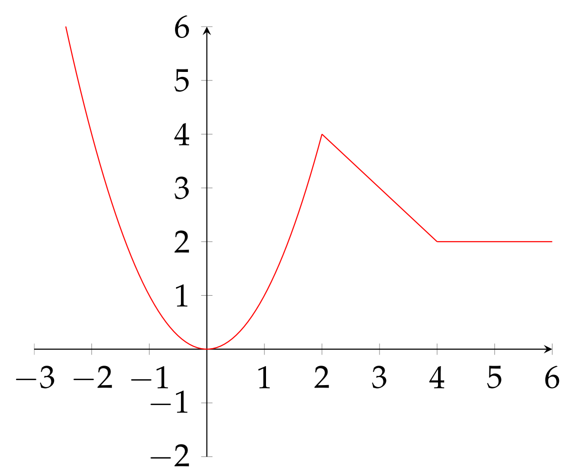
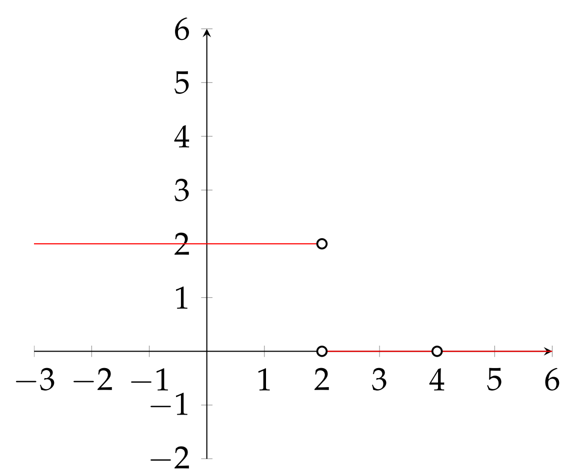
Test 3 Study Guide

Exercise 1. Consider the function $f(x)$ defined on the whole real line whose graph is given below



State all x-values where this function is not differentiable. State all x-values where this function is not continuous.

Exercise 2. Consider the function $f(x)$ defined on the whole real line whose graph is given below



State all x-values where this function is not differentiable. State all x-values where this function is not continuous.

Exercise 3. You should know the derivatives of the trigonemetric functions by heart. Write them down below

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) &= \\
\frac{\mathrm{d}}{\mathrm{d}x}(\cos x) &= \\
\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) &= \\
\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) &= \\
\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) &= \\
\frac{\mathrm{d}}{\mathrm{d}x}(\csc x) &=
\end{aligned}$$

Exercise 4. Find the derivative of the following functions (I'll do the first one for you)

$$p(x) = x^3 + 3x^2$$

$$\begin{aligned}
p'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}(p(x)) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}(x^3 + 3x^2) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}(x^3) + \frac{\mathrm{d}}{\mathrm{d}x}(3x^2) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}(x^3) + 3\frac{\mathrm{d}}{\mathrm{d}x}(x^2) \\
&= 3x^{3-1} + 3 \cdot 2x^{2-1} \\
&= 3x^2 + 6x
\end{aligned}$$

You don't have to do every step as I did above. If you feel comfortable, you can skip some steps like this:

$$p(x) = x^3 + 3x^2$$

$$\begin{aligned}
p'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}(p(x)) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}(x^3 + 3x^2) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}(x^3) + 3\frac{\mathrm{d}}{\mathrm{d}x}(x^2) \\
&= 3x^2 + 6x.
\end{aligned}$$

Now you do the rest

$$h(t) = e^t - t^2 + 5\sqrt{t} + \frac{2}{t^3}$$

$$\begin{aligned}
h'(t) &= \frac{\mathrm{d}}{\mathrm{d}t}(h(t)) \\
&= \frac{\mathrm{d}}{\mathrm{d}t}\left(e^t - t^2 + 5\sqrt{t} + \frac{2}{t^3}\right)
\end{aligned}$$

$$g(t) = \frac{3t^3 - t^2 + 1}{t}$$

$$\begin{aligned} g'(t) &= \frac{\mathrm{d}}{\mathrm{d}t}(g(t)) \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{3t^3 - t^2 + 1}{t}\right) \end{aligned}$$

Using what you found above, find $g'(1)$.

$$f(x) = 5e^x + e^e + x^e \qquad f'(x) =$$

$$f(x) = \frac{\cos x}{\tan x} \qquad f'(x) =$$

$$f(x) = \frac{x^2 e^x}{\sec x} \qquad f'(x) =$$

Exercise 5. At what x-values does $f(x) = e^x - 2x$ have a horizontal tangent line?

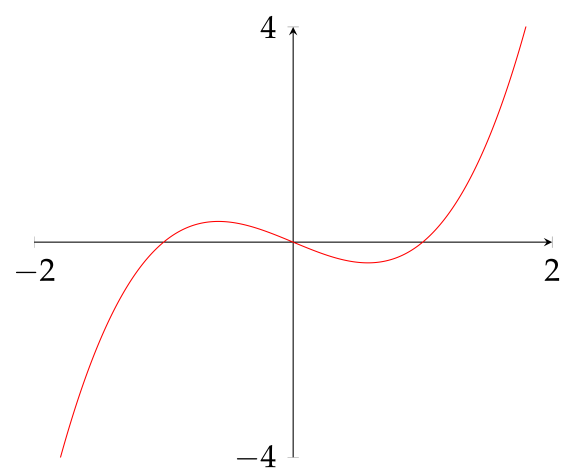
Exercise 6. Find the first second and third derivatives of $f(x) = \sec x - \tan x + e^x$.

Exercise 7. Find the derivative of $f(x) = 2x^2 + 3x - 2$ using the *limit* definition. I'll get you started

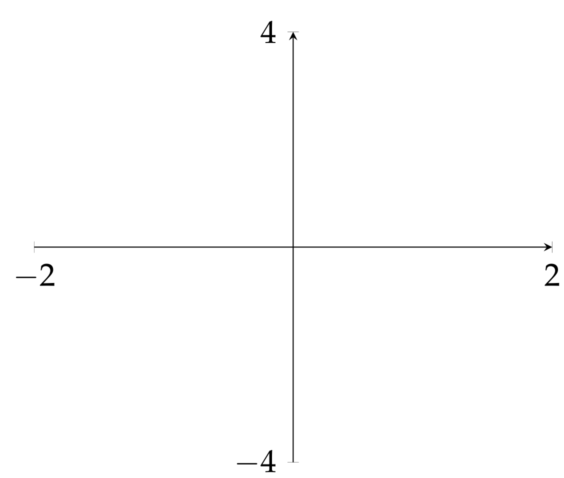
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

You'll be asked this kind of question on the test, so make sure you get it right. For instance, keep the $\lim_{h \to 0}$ operator until the very end when you can set $h = 0$.

Exercise 8. Let $f(x)$ be the function whose graph is given below:



Sketch the graph of $f'(x)$ below



Exercise 9. Evaluate $\lim_{x \to 0} \frac{5x}{\sin 3x}$.

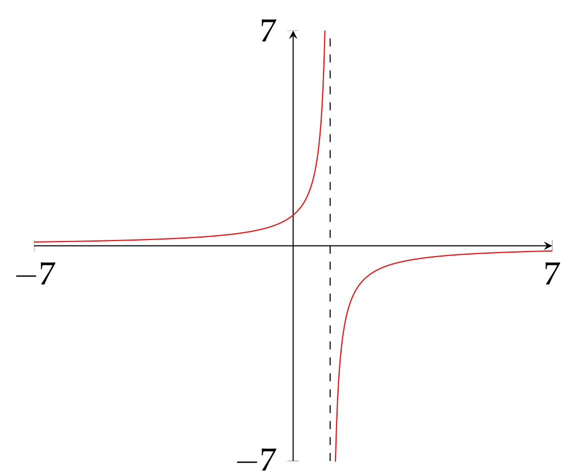
Exercise 10. Find the line normal to $f(x) = 5x^3 - x + 1$ at $a = 0$.

Exercise 11. Find the line tangent to $f(x) = 5x^3 - x + 1$ at $a = 0$.

Exercise 12. Let $f(x)$ be a function and suppose $f(1) = 4$ and $f'(1) = 3$. Find $\frac{d}{dx}(f(x)/x)\Big|_{x=1}$.

Exercise 13. An equation of the line tangent to the graph of $f(x)$ at the input $x = 3$ is $y = -5x + 2$. Find $f(3)$ and $f'(3)$.

Exercise 14. Make sure you know the different types of discontinuities a function can have, namely jump, infinite, removable, and oscilatting discontinuities. What type of discontinuity is presented in the graph below?



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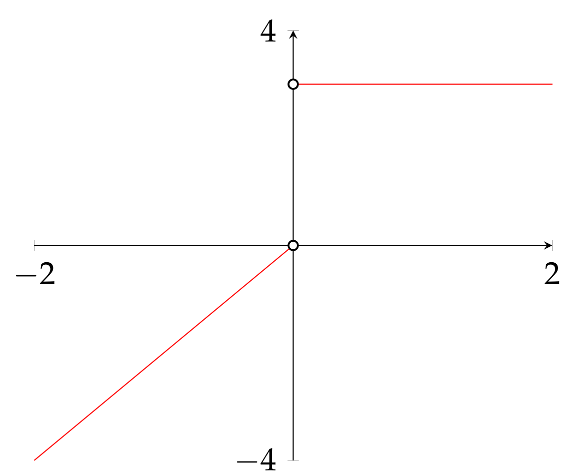
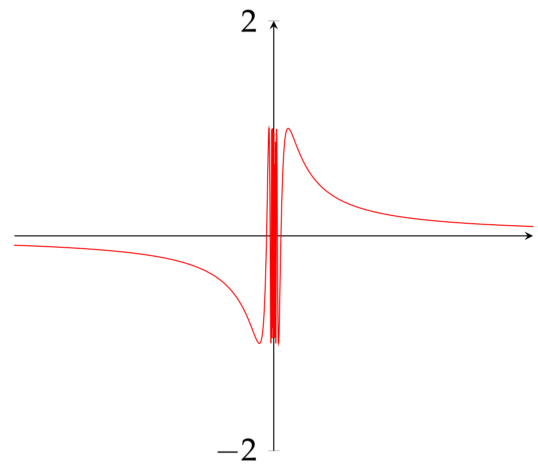


What type of discontinuity is presented in the graph below?



What type of discontinuity is given by the function below?

$$f(x) = \frac{(x-3)(x+2)}{x+2}.$$

Can you redefine this function in a way so that it is continuous everywhere?

Exercise 15. Find the value of a that would make

$$f(x) = \begin{cases} -3x^2 + 1 & \text{if } x \leq 0 \\ x + a & \text{if } x > 0 \end{cases}$$

continuous.

Exercise 16. Find an interval in which the function $f(x) = x^3 + x + 1$ has a zero.