Section 1.4: Linear Functions and Models

A **completely defined model** is a statement that describes the relationship between an output variable and an input variable in context. It includes the following:

- an equation
- an output description, with units
- an input description, with units
- an input data range to describe the interval of data used to find the model

Extrapolation refers to the use of a model to predict an output value for an input value that is *outside* the input data range used to obtain the model.

Interpolation refers to the use of a model to predict an output value for an input value that is *within* the input data range used to obtain the model.

A **Linear Function** has an equation of the form $f(x) = ax + b$. Its graph is a **line,** with **slope** *a*. It has a ***y*-intercept,** or **initial value,** $f(0) = b$.

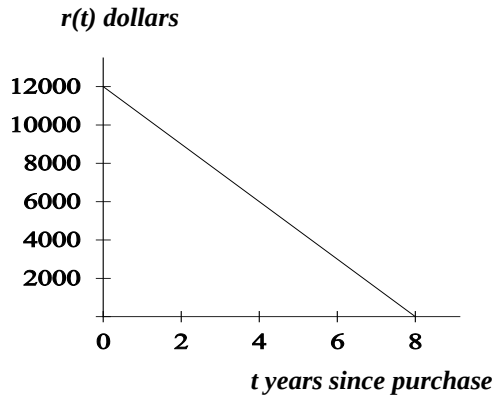
A linear function has a constant slope and therefore has a **constant rate of change**.

A sentence of **interpretation** for the **slope of a line** uses ordinary conversational language to answer the questions:

What? refers to the output description for the function
Increased(es) or Decreased(es)?
By how much? refers to the slope, including slope units
When? refers to the domain of the function

Example 1: (CC5e p. 36)

The linear function $r(t)$, giving the resale value of a car t years after it is purchased, is represented graphically in the figure to the right.



a. Write a sentence of interpretation for the **initial value**.

b. Write a sentence of interpretation for the point at which the output is 0.

c. Find the **slope** of the line and write a sentence of interpretation.

d. Write an equation for $r(t)$.

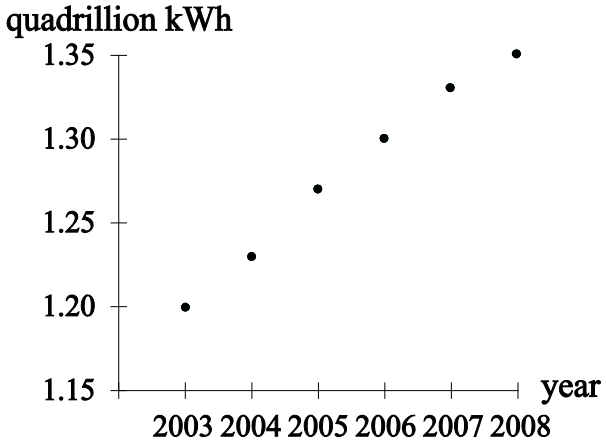
e. Write a completely defined linear model for $r(t)$.

Since a linear function has constant rate of change, a linear function should be considered in modeling a set of data that displays constant (or nearly constant) **first differences**. **First differences** are found by subtracting the output values between successive **evenly spaced** input values. For successive data points (x_1, y_1) *and* (x_2, y_2), the first difference is $y_2 - y_1$.

Example 2: (CC5e p. 39-40)

Retail sales in kilowatt-hours (kWh) of electricity to commercial consumers are shown in both the table and scatter plot.

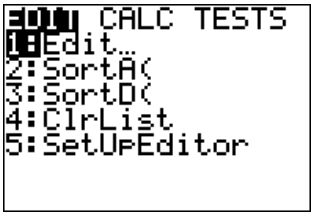
Year	Retail Sales, in Quadrillion kWh
2003	1.20
2004	1.23
2005	1.27
2006	1.30
2007	1.33
2008	1.35

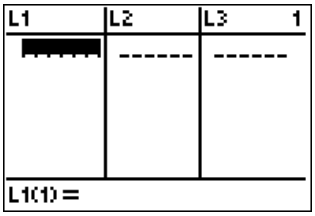


a. View a scatter plot of the data.

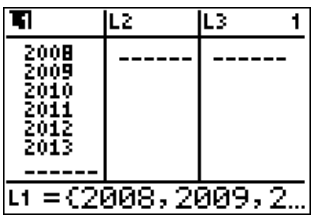
Entering data:
Enter the data from the table into lists L1 and L2.

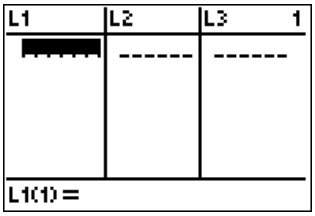
- **STAT** returns options for editing data in lists L1 and L2.
- **1** (or **ENTER**) chooses **1: Edit** enabling data entry/editing



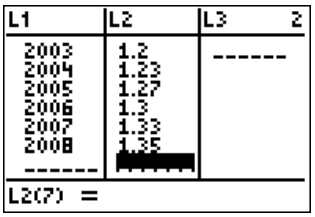


- If there is data in L1 or L2, use the arrows to move the cursor to L1 and then hit **CLEAR** **ENTER** to clear the list.



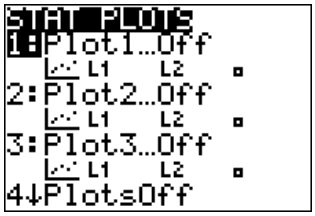


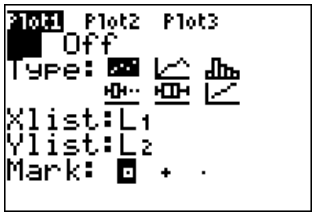
- **2003** enters 2003 as the first input in L1
- **ENTER** (or ▼) moves the cursor to enter **2004 2005 2006 2007 2008**
- ▶ moves the cursor to L2 where output values are entered
- **1.2 1.23 1.27 1.3 1.33 1.35**



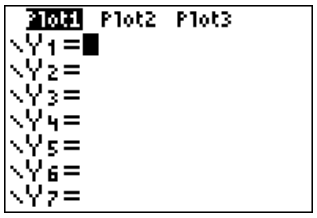
Graphing a scatter plot of the data:

- Use **2ND Y= [STAT PLOT] ENTER** to verify that **1: Plot1** is set up as shown in the near right screen
  - **Plot1** is **On**
  - A scatter plot is shown
  - **Xlist = L1** (**2nd 1**) and **Ylist = L2** (**2nd 2**)
  - The large Mark is selected
  - Make any necessary changes by placing the cursor over the item you wish to select and hit **ENTER**. The selected item should them be highlighted.



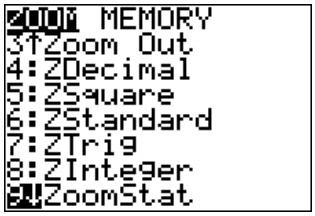


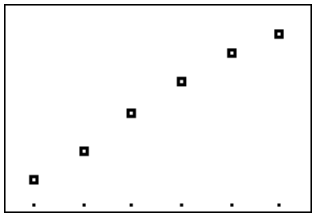
- **Y=** to verify that the Y list is empty (or that any equations are turned off) and Plot1 is turned on.



Note: Plot1 can be turned off and on by placing the cursor on Plot1 and hitting **ENTER**. If there are equations in the Y= list that should not be graphed with the scatter plot, turn the equations off by placing the cursor on each corresponding = and hit **ENTER** to turn off each equation.

- **ZOOM 9** [ZOOMSTAT] to view the scatter plot.





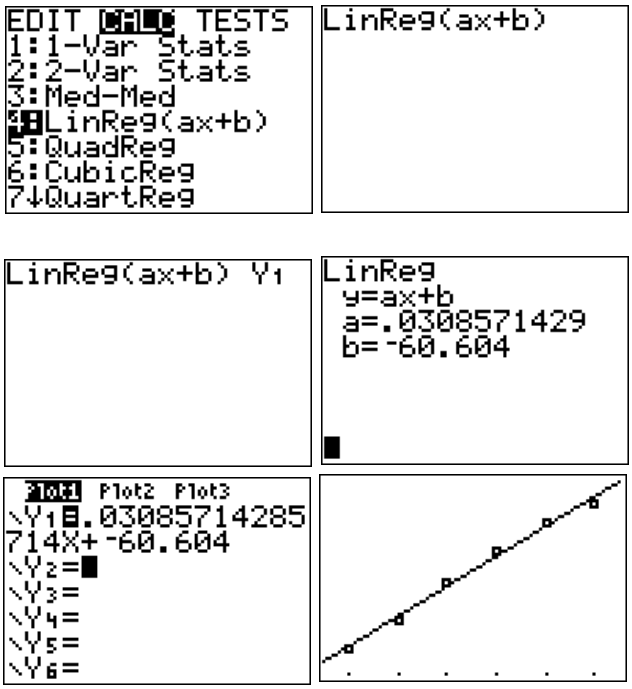
b. Calculate first differences in the data and discuss the amount of concavity suggested by the scatter plot. Verify that it is reasonable to use a linear function to model the data.

Year	2003	2004	2005	2006	2007	2008
Retail Sales, in Quadrillion kWh	1.20	1.23	1.27	1.30	1.33	1.35
First differences	1.23 - 1.20 = 0.03					

c. Find a linear function $r(t)$ that best fits the data. View the graph of the linear function and the scatter plot at the same time on your calculator. Write a completely defined linear model. Round coefficients to three decimal places.

Finding and storing a linear function to model data:

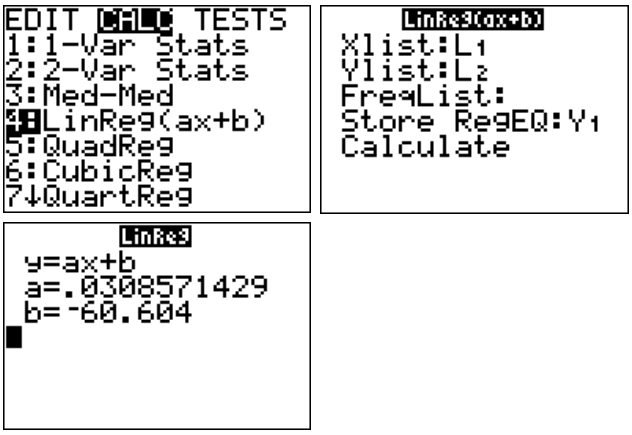
- **STAT** ▸ [CALC] ▾ to 4 [LinReg] **ENTER** returns LinReg(ax+b) on the Home Screen.
- **VARS** ▸ [Y-VARS] **1** or **ENTER** [Function] **1** or **ENTER** [Y1] selects Y1 as the storage position for the equation.
- **ENTER**
  The equation form as well as the values for the parameters appear on the Home Screen and are pasted into Y1.
- **Y=** to verify the unrounded model is stored in Y1.
- **ZOOM 9** [ZOOMSTAT] shows the graph of the function with the scatter plot.



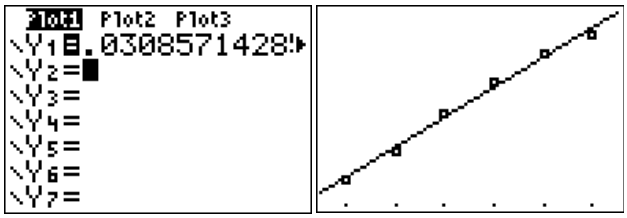
OR

For TI calculators with Stat Wizard ON:

- **STAT** ▸ [CALC] ▾ to 4 [LinReg] **ENTER** returns the LinReg(ax+b) Stat Wizard screen.
- Xlist: **2nd 1** [L1]
  Ylist: **2nd 2** [L2]
  Store ReqEQ:
- **VARS** ▸ [Y-VARS] **1** or **ENTER** [Function] **1** or **ENTER** [Y1] selects Y1 as the storage position for the equation.
  Put the cursor on Calculate and hit **ENTER.** The equation form as well as the values for the parameters appear on the Home Screen and are pasted into Y1.



- **Y=** to verify the unrounded model is stored in Y1.
- **ZOOM 9** [ZOOMSTAT] shows the graph of the function with the scatter plot.



Note: Although equations are generally reported to three decimal places, it's important to use all of the digits found by the calculator when working with a regression equation in a model, to reduce the possibility of round-off error as a result of intermediate rounding. The unrounded equation should always be stored in the Y= list whenever a regression equation is found.

d. Write a sentence of interpretation for the slope of the linear model.

e. According to the model (the unrounded function in Y1), what were retail sales of electricity to commercial customers in 2002? Include units with the answer. Does this use *interpolation or extrapolation*?

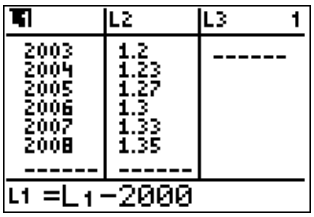
f. According to the model, what were retail sales of electricity to commercial customers in 2005? Include units with the answer. Does this use *interpolation or extrapolation*?

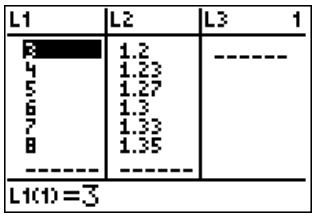
g. According to the model, in what year did retail sales first exceed 1.4 quadrillion kWh?

h. **Align the data** in the table so that t is the number of years since 2000. Find a linear function to model the aligned data (paste it into Y2). Write a completely defined linear model for the aligned data.

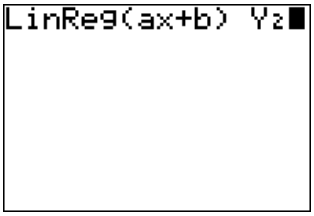
Aligning data:

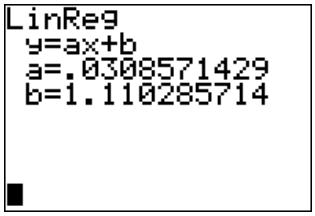
- **STAT ENTER** to edit L1
- Highlight **L1**
- Complete the equation to read **L1= 2nd 1** [L1] – **2000**
- **ENTER**



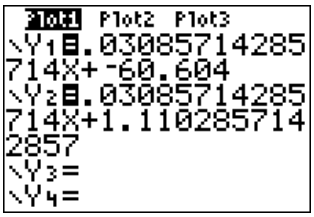


- **STAT** ▶ [CALC] ▼ to 4 [LinReg(ax+b)] **VARS** ▶ [Y-VARS] **ENTER** [Function] **2** [Y2] to find a linear function and paste it in Y2.



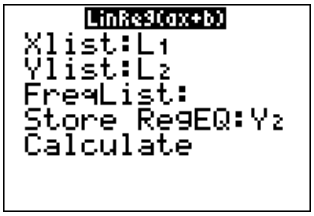


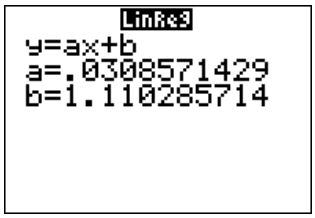
- **Y=** to verify both functions are stored in Y1 and Y2

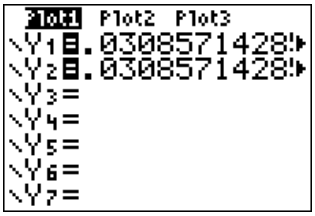


OR

- **STAT** ▶ [CALC] ▼ to 4 [LinReg(ax+b)]
- Xlist: **2nd 1** [L1]
  Ylist: **2nd 2** [L2]
  Store ReqEQ:
  VARS ▶ [Y-VARS] **ENTER** [Function] **2** [Y2]
  Cursor on Calculate and hit **ENTER** to find a linear function and paste it in Y2.
- **Y=** to verify both functions are stored in Y1 and Y2







i. Compare the *linear function* in part *h* to the linear function in part *c*. Compare the *input description* and *input data range* for the models in parts *h* and *c*.

j. According to the model in part *h* (the unrounded function in Y2), what were retail sales of electricity to commercial customers in 2002? Is this the same result as in part e?

Example 3: (CC5e p. 45, Activity 25)

The table shows the number of people in North America who use email as a part of their jobs.

Year	Business Email Users, in millions
2005	125.2
2006	128.7
2007	132.4
2008	136.0
2009	139.8
2010	143.6

a. Find a function to model the number of business email users, aligning the data to the number of years since 2005. Write a completely defined model.

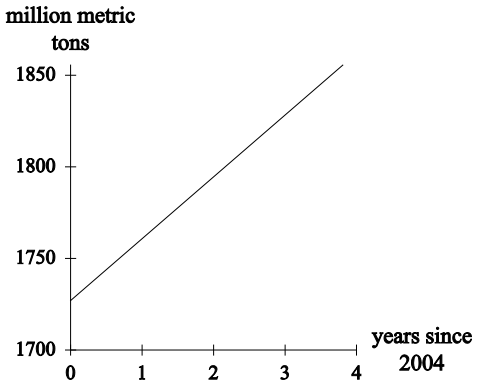
b. What is the **constant rate of change** of the number of North American business email users indicated by the model? Include units with the answer.

c. Use the (unrounded) model to estimate the number of North American business email users in 2013.

d. Is the estimate in part c found by *interpolation or extrapolation*?

Example 4: (CC5e p. 43, Activity 15)

The figure shows a linear function f used to model industrial carbon dioxide emissions.



a. Estimate the slope of the graph.

b. Report the slope in a sentence of interpretation.

c. Are emissions increasing, decreasing, or constant? How is this reflected in the value of the slope?

Trouble-shooting: Restoring a missing column on the list screen

If delete **DEL**, rather than **CLEAR**, is selected to remove data from one of the lists, the entire list will be deleted.

If the lists are incomplete (*as in the near right screen shot*), restore all lists:

- **STAT**
- **5** selects **5: SetUpEditor**
- **ENTER**
- **STAT ENTER** shows the complete set of lists