MATH 9500
FALL 2020
HOMEWORK 8

Due Thursday, December 10, 2020

1. (5 pts) Show that if R is a Dedekind and $I \subseteq R$ is an ideal than I can be generated with two elements.

2. (5 pts) Show that if d is a square-free integer, then the integral closure of $\mathbb{Z}$ in $\mathbb{Q}(\sqrt{d})$ is given by

$$\overline{\mathbb{Z}}_{\mathbb{Q}(\sqrt{d})} = \begin{cases} \mathbb{Z}[\sqrt{d}] \text{ if } d \equiv 2, 3 \bmod(4) \text{ and} \\ \mathbb{Z}[\frac{1+\sqrt{d}}{2}] \text{ if } d \equiv 1 \bmod(4). \end{cases}$$

3. Let R be a domain with quotient field K. We say that $\omega \in K$ is almost integral over R if there is a nonzero $x \in R$ such that $x\omega^n \in R$ for all $n \in \mathbb{N}$. We say that R is completely integrally closed if it contains all of its almost integral elements.

a) (5 pts) Show that if $\omega \in K$ is integral, then ω is almost integral over R.
b) (5 pts) Show that if R is Noetherian and $\omega \in K$ is almost integral over R, then ω is integral over R.
c) (5 pts) Give an example of a domain R and an element $\omega \in K$ (where K is the quotient field of R) that is almost integral over R, but not integral over R.
d) (5 pts) Show that any UFD is completely integrally closed.