Math 1070 Test 1 Review

This study guide is deisgned to help you be prepared for the test 1.

Inverse Trig Functions

Exercise 1. For this exercise, you need to complete the table below. I have filled in all of the information regarding the trig functions for you, however you should also verify that what I wrote down is correct. For instance, why is the domain of $\cot x$ equal to $\mathbb{R} \setminus \{n\pi \mid n \in \mathbb{Z}\}$? This is because $\cot x = \cos x / \sin x$, so $\cot x$ is defined whenever $\sin x \neq 0$, and $\sin x \neq 0$ whenever x is any real number except those of the form $n\pi$ where n is an integer. It is very important that *understand* why the domains and ranges for these functions are the way that they are. Finally, when filling this table out, please try to derive your answers on your own (rather than simply copying them from another source). To see what I mean, I'll show you how to derive the derivative of $\arcsin x$: All you have to do is use the fact that $\sin(\arcsin x) = x$ and take derivatives on both sides.

$$\begin{aligned}
\sin(\arcsin x) = x \implies & \frac{\mathrm{d}}{\mathrm{d}x}(\sin(\arcsin x)) = \frac{\mathrm{d}}{\mathrm{d}x}(x) \\
\implies & \cos(\arcsin x)\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin x) = 1 \\
\implies & \sqrt{1-x^2}\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin x) = 1 \\
\implies & \frac{\mathrm{d}}{\mathrm{d}x}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}.
\end{aligned}$$

Here we used the fact that $\cos(\arcsin x) = \sqrt{1-x^2}$. In fact, you may want to fill in the table in Exercise (4) before calculating these derivatives.

Function	Domain	Range	Derivative
$\sin x$	$\mathbb{R}$	$[-1,1]$	$\cos x$
$\cos x$	$\mathbb{R}$	$[-1,1]$	$-\sin x$
$\tan x$	$\mathbb{R} \setminus \left\{\frac{\pi}{2} + n\pi \mid n \in \mathbb{Z}\right\}$	$\mathbb{R}$	$\sec^2 x$
$\csc x$	$\mathbb{R} \setminus \{n\pi \mid n \in \mathbb{Z}\}$	$(-\infty, 1] \cup [1, \infty)$	$-\csc x \cot x$
$\sec x$	$\mathbb{R} \setminus \left\{\frac{\pi}{2} + n\pi \mid n \in \mathbb{Z}\right\}$	$(-\infty, 1] \cup [1, \infty)$	$\sec x \tan x$
$\cot x$	$\mathbb{R} \setminus \{n\pi \mid n \in \mathbb{Z}\}$	$\mathbb{R}$	$-\csc^2 x$
$\arcsin x$			$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$			
$\arctan x$			
$\operatorname{arccsc} x$			
$\operatorname{arcsec} x$			
$\operatorname{arccot} x$			

Remark. Note that $\arcsin x$ and $\sin^{-1} x$ denote the *same* function. Technically speaking, $\arcsin x$ is the more accurate notation here, however in this class you'll often see both notations used, so keep this in mind. Also make sure you understand that $\sin^{-1} x$ is the *inverse* function – it does not denote the function $(\sin x)^{-1} = 1/\sin x$. This notation is admittedly confusing since $\sin^2 x$ means $(\sin x)^2 = \sin x \cdot \sin x$.

Exercise 2. Evaluate the following:

$$\begin{aligned}
\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) &= \\
\arcsin\left(\frac{-\sqrt{3}}{2}\right) &= \\
\sec^{-1}(1) &= \\
\operatorname{arccot}(1) &= \\
\operatorname{arcsec}\left(\sqrt{2}\right) &= \\
\arcsin\left(\frac{1}{2}\right) &=
\end{aligned}$$

Exercise 3. Suppose $a \in (-\infty, -1] \cup [1, \infty)$. Explain why $\operatorname{arccsc}(a) = \arcsin(1/a)$. Similarly explain why $\operatorname{arcsec}(a) = \arccos(1/a)$.

Exercise 4. Complete the table below. I've filled in the first two rows for you (check that what I wrote down is accurate!). You should only have to use the two formulas

$$\sin^2 x + \cos^2 x = 1 \quad \text{and} \quad \sec^2 x - \tan^2 x = 1$$

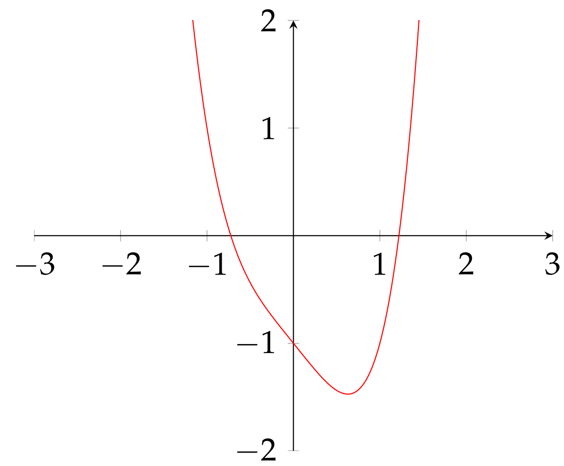
to fill in what's left. Again, it's very important that you are able to derive this stuff on your own (rather than copying from another source).

Composite Function	Alternate Form	Domain	Range
$\cos(\arcsin x)$	$\cos(\arcsin x) = \sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$
$\tan(\arcsin x)$	$\tan(\arcsin x) = x/\sqrt{1-x^2}$	$(-1, 1)$	$\mathbb{R}$
$\sec(\arcsin x)$			
$\csc(\arcsin x)$			
$\cot(\arcsin x)$			
$\sin(\arccos x)$			
$\tan(\arccos x)$			
$\tan(\operatorname{arcsec} x)$			
$\cot(\operatorname{arcsec} x)$			

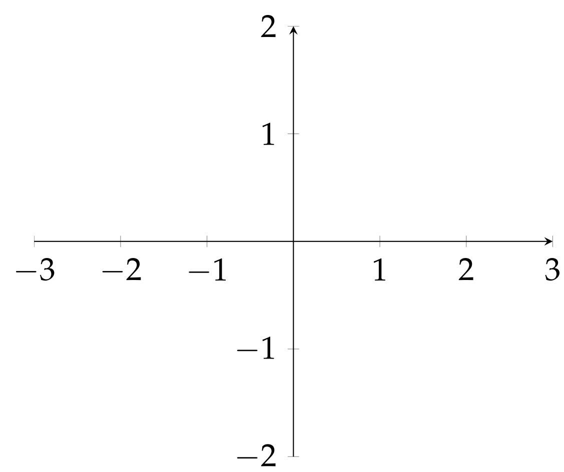
Note that there is a subtle issue that you need to consider when filling in the last two rows. You should look up the answer on wolfram.com and try to understand why they express these composite functions in the way that they do.

Graphs

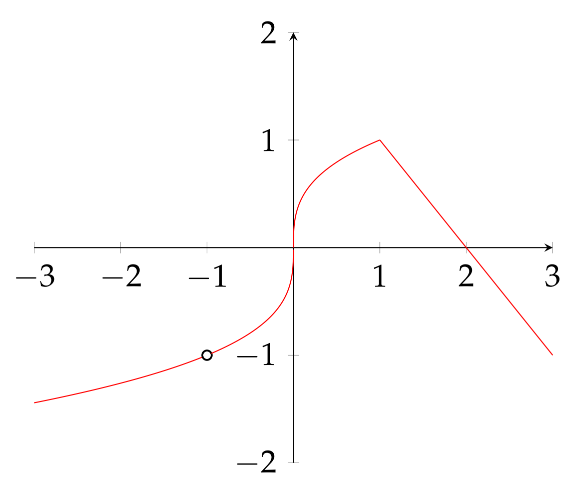
Exercise 5. Consider the function $f(x)$ whose graph is given below



Draw the graph of the derivative function $f'(x)$ below



Exercise 6. Consider the function $f(x)$ whose graph is given below

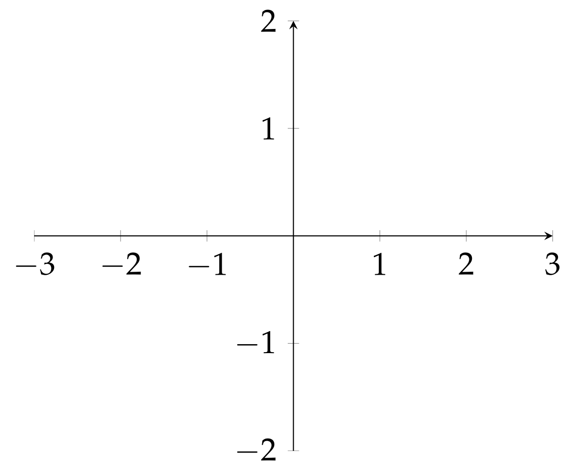


State all x-values where this function is not differentiable. State all x-values where this function is not continuous.

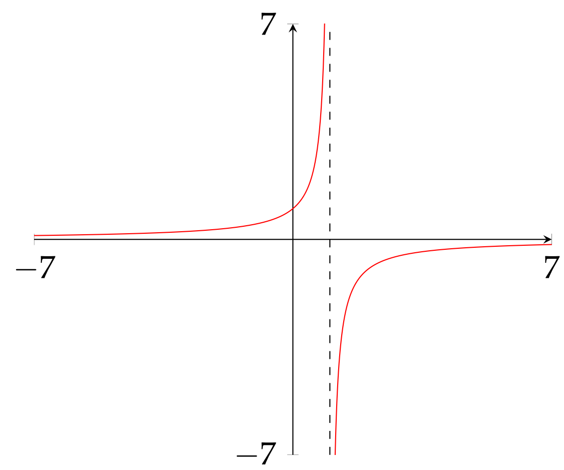
It is not differentiable at:

It is not continuous at:

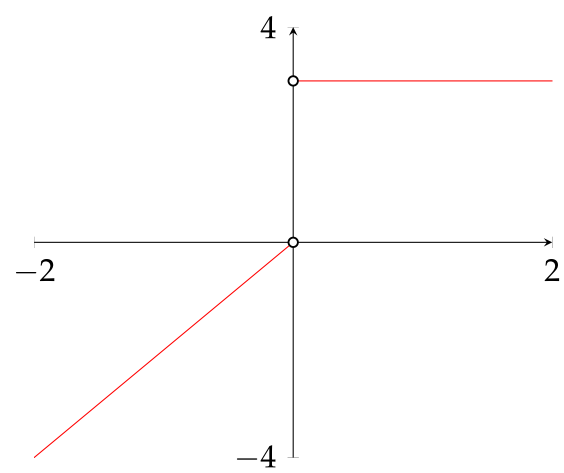
Now draw the graph of the derivative function $f'(x)$ below



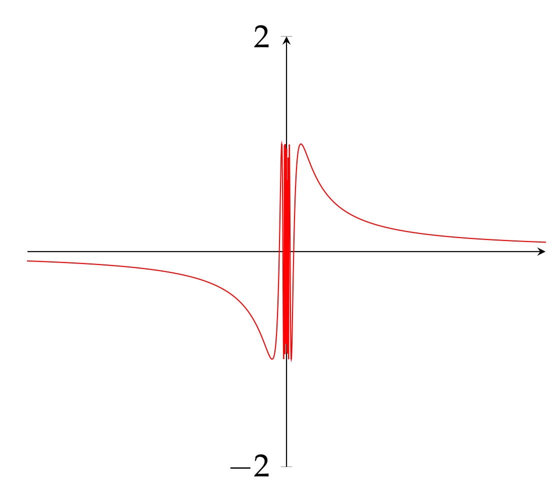
Exercise 7. Make sure you know the different types of discontinuities a function can have, namely jump, infinite, removable, and oscilatting discontinuities. What type of discontinuity is presented in the graph below?



What type of discontinuity is presented in the graph below?



What type of discontinuity is presented in the graph below?



What type of discontinuity is given by the function below?

$$f(x) = \frac{(x-3)(x+2)}{x+2}.$$

Derivatives and Limits

Exercise 8. List three reasons why a funciton $f(x)$ may not be differentiable at a point $x = a$.

Exercise 9. Find the derivative of $f(x) = -4x^2 + 3x - 2$ using the *limit* definition. Note that you'll see this type of question on the test. **Make sure you do this right and do not skip any steps!** I'll show you how to do it when it comes time to review this question in class, but you need to be able to write it on your own almost *exactly* the way that I will show you.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Exercise 10. Find the derivative of the following functions (I'll do the first one for you).

$$p(x) = x^3 + 3x^2$$

$$\begin{aligned} p'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}(p(x)) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}(x^3 + 3x^2) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}(x^3) + \frac{\mathrm{d}}{\mathrm{d}x}(3x^2) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}(x^3) + 3\frac{\mathrm{d}}{\mathrm{d}x}(x^2) \\ &= 3x^{3-1} + 3 \cdot 2x^{2-1} \\ &= 3x^2 + 6x \end{aligned}$$

You don't have to do every step as I did above. If you feel comfortable, you can skip some steps like this:

$$p(x) = x^3 + 3x^2 \qquad \begin{aligned} p'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}(p(x)) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}(x^3 + 3x^2) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}(x^3) + 3\frac{\mathrm{d}}{\mathrm{d}x}(x^2) \\ &= 3x^2 + 6x. \end{aligned}$$

Now you do the rest

$$h(t) = e^{\sqrt{t}} - t^e + 5e \qquad \begin{aligned} h'(t) &= \frac{\mathrm{d}}{\mathrm{d}t}(h(t)) \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\left(e^{\sqrt{t}} - t^e + 5e\right) \end{aligned}$$

$$f(x) = \tan(2e^{3x+1}) \qquad f'(x) =$$

$$g(x) = \sqrt[5]{2x}\sin(x^4) + \sec^3(x+1) \qquad f'(x) =$$

Exercise 11. Evaluate the following limits. Do *not* use L'Hospital's rule! Note that some of these limits are secretly derivatives of simple functions in disguise.

$$\lim_{h \to 0} \frac{e^{x+h} - e^x}{h} =$$

$$\lim_{x\to 3} \frac{x^2-5x+6}{x-3} =$$

$$\lim_{x\to -2} \frac{-2-x}{1-\sqrt{x+3}} =$$

$$\lim_{x\to 0} \frac{\frac{1}{x+1}-1}{x} =$$

$$\lim_{x\to 1} \frac{\ln(x)}{x-1} =$$