Homework #4 Solutions

(1.a) Find the derivative of $f(x) = 5(1+4x)^2 + 3x$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(f(x)) &= \frac{\mathrm{d}}{\mathrm{d}x}(5(1+4x)^2 + 3x) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}(5(1+4x)^2) + \frac{\mathrm{d}}{\mathrm{d}x}(3x) \\
&= 5\frac{\mathrm{d}}{\mathrm{d}x}((1+4x)^2) + 3 \\
&= 5 \cdot 2(1+4x)^{1-1} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(1+4x) + 3 \\
&= 10(1+4x) \cdot \frac{\mathrm{d}}{\mathrm{d}x}(1+4x) + 3 \\
&= 10(1+4x) \cdot 4 + 3 \\
&= 40(1+4x) + 3
\end{aligned}$$

(1.b) Find the derivative of $f(x) = \ln(\ln(x))$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(f(x)) &= \frac{\mathrm{d}}{\mathrm{d}x}(\ln(\ln(x))) \\
&= \frac{1}{\ln(x)} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(\ln(x)) \\
&= \frac{1}{\ln(x)} \cdot \frac{1}{x} \\
&= \frac{1}{x\ln(x)}.
\end{aligned}$$

(1.c) Find the derivative of $f(x) = e^{e^x}$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(f(x)) &= \frac{\mathrm{d}}{\mathrm{d}x}(e^{e^x}) \\
&= e^{e^x} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(e^x) \\
&= e^{e^x} \cdot e^x \\
&= e^{e^x + x}.
\end{aligned}$$

(1.d) Find the derivative of $f(x) = x\ln(3x)$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(f(x)) &= \frac{\mathrm{d}}{\mathrm{d}x}(x\ln(3x)) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}(x) \cdot \ln(3x) + x \cdot \frac{\mathrm{d}}{\mathrm{d}x}(\ln(3x)) \\
&= 1 \cdot \ln(3x) + x \cdot \frac{1}{3x} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(3x) \\
&= \ln(3x) + x \cdot \frac{1}{3x} \cdot 3 \\
&= \ln(3x) + 1.
\end{aligned}$$

(1.e) Find the derivative of $f(x) = \ln(x + 2x^2)$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(f(x)) &= \frac{\mathrm{d}}{\mathrm{d}x}(\ln(x + 2x^2)) \\
&= \frac{1}{x + 2x^2} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(x + 2x^2) \\
&= \frac{1}{x + 2x^2} \cdot (1 + 4x) \\
&= \frac{1 + 4x}{x + 2x^2}.
\end{aligned}$$

(1.f) Find the derivative of $f(x) = \frac{10.2}{1+12.1e^{0.3x}}$. (hint: rewrite $f(x)$ as $10.2(1 + 12.1e^{0.3x})^{-1}$)

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(f(x)) &= \frac{\mathrm{d}}{\mathrm{d}x}(10.2(1 + 12.1e^{0.3x})^{-1}) \\
&= 10.2\frac{\mathrm{d}}{\mathrm{d}x}((1 + 12.1e^{0.3x})^{-1}) \\
&= 10.2 \cdot -1 \cdot (1 + 12.1e^{0.3x})^{-2} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(1 + 12.1e^{0.3x}) \\
&= -10.2(1 + 12.1e^{0.3x})^{-2} \left(\frac{\mathrm{d}}{\mathrm{d}x}(1) + \frac{\mathrm{d}}{\mathrm{d}x}(12.1e^{0.3x})\right) \\
&= -10.2(1 + 12.1e^{0.3x})^{-2} \left(0 + 12.1\frac{\mathrm{d}}{\mathrm{d}x}(e^{0.3x})\right) \\
&= -10.2(1 + 12.1e^{0.3x})^{-2} \cdot 12.1\frac{\mathrm{d}}{\mathrm{d}x}(e^{0.3x}) \\
&= -10.2(1 + 12.1e^{0.3x})^{-2} \cdot 12.1e^{0.3x} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(0.3x) \\
&= -10.2(1 + 12.1e^{0.3x})^{-2} \cdot 12.1e^{0.3x} \cdot 0.3 \\
&= \frac{-37.026e^{0.3x}}{(1 + 12.1e^{0.3x})^2}.
\end{aligned}$$

(1.g) Find the derivative of $f(x) = \sqrt{3x^4 - 2x^3 - 1}$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(f(x)) &= \frac{\mathrm{d}}{\mathrm{d}x}((3x^4 - 2x^3 - 1)^{\frac{1}{2}}) \\
&= \frac{1}{2} \cdot (3x^4 - 2x^3 - 1)^{\frac{1}{2}-1} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(3x^4 - 2x^3 - 1) \\
&= \frac{1}{2} \cdot (3x^4 - 2x^3 - 1)^{-\frac{1}{2}} \left(12x^3 - 6x^2\right) \\
&= \frac{1}{2} \cdot \frac{12x^3 - 6x^2}{\sqrt{3x^4 - 2x^3 - 1}} \\
&= \frac{6x^3 - 3x^2}{\sqrt{3x^4 - 2x^3 - 1}}.
\end{aligned}$$

(1.h) Find the derivative of $f(x) = \frac{e^{3x}}{(5x^4-1)^2}$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(f(x)) &= \frac{\mathrm{d}}{\mathrm{d}x}(e^{3x} \cdot (5x^4 - 1)^{-2}) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}(e^{3x}) \cdot (5x^4 - 1)^{-2} + e^{3x} \cdot \frac{\mathrm{d}}{\mathrm{d}x}((5x^4 - 1)^{-2}) \\
&= e^{3x} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(3x) \cdot (5x^4 - 1)^{-2} + e^{3x} \cdot -2(5x^4 - 1)^{-3} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(5x^4 - 1) \\
&= e^{3x} \cdot 3 \cdot (5x^4 - 1)^{-2} + e^{3x} \cdot -2(5x^4 - 1)^{-3} \cdot 20x^3 \\
&= \frac{3e^{3x}}{(5x^4 - 1)^2} - \frac{40x^3e^{3x}}{(5x^4 - 1)^3}.
\end{aligned}$$

(1.i) Let $f(x) = g(x)h(x)$ and suppose

$$\begin{aligned} g(5) &= 3 \\ g'(5) &= 2 \\ h(5) &= 4 \\ h'(5) &= 6 \end{aligned}$$

Then

$$\begin{aligned} f'(5) &= g'(5)h(5) + g(5)h'(5) \\ &= 2 \cdot 4 + 3 \cdot 6 \\ &= 26. \end{aligned}$$