MATH 9850, Free Resolutions
Fall 2019
Exercises 4
Due date: Thu 31 Oct 4:30PM

Let k be a field, and set $R = k[W, X, Y, Z]$. Consider the ideals $I = \langle WX, XY, YZ\rangle$ and $J = \langle WZ, WX, XY, YZ\rangle$.

Exercise 1. (a) Compute the following Taylor resolutions, describing each differential as a matrix: $T = T^R(WX, XY, YZ)$ and $U = T^R(WZ, WX, XY, YZ)$.

(b) Verify directly (without invoking Theorem II.D.8) that T is a resolution of R/I and that U is a resolution of R/J.

Exercise 2. Consider the natural surjection $\pi\colon R/I \to R/J$ induced by the inclusion $I \subseteq J$.

(a) Explicitly construct a chain map $\Phi^+\colon T^+ \to U^+$ such that $\Phi_{-1} = \pi$, describing the maps Φ_i for $i \geq 0$ as matrices, and verifying that Φ^+ is a chain map.

(b) Explicitly compute $\mathrm{Cone}(\Phi)$, describing each differential as a matrix.

(c) Is $\mathrm{Cone}(\Phi)$ a resolution? Justify your answer.