MATH 9500
FALL 2020
HOMEWORK 3

Due Monday, September 28, 2020

Let R be a commutative ring (maybe without identity). We say that R is *von Neumann regular* if for every $x \in R$, there exists $y \in R$ such that $x = xyx$.

1. (5 pts) Show that any direct product or direct sum of fields is von Neumann regular.

2. (5 pts) Suppose that R has identity. Show that if R is von Neumann regular, then R is $0-$dimensional.

3. (5 pts) Again, assume that R, a von Neumann regular ring, has identity and let $\mathfrak{P} \subset R$ be a prime ideal. Show that

$$R_{\mathfrak{P}} \cong R/\mathfrak{P}.$$

4. (5 pts) Let R be an integral domain. Show that R is a UFD if and only if $R[x]$ is a UFD.

5. (5 pts) Let R be a commutative ring with 1. Characterize $U(R[x])$ in general.

Let R be a commutative ring with identity and $\mathfrak{C} = \{I_\alpha\}_{\alpha \in \Lambda}$ be a chain of ideals between the ideals $I \subseteq J$. We say that this chain is *maximal* if any ideal $A \subseteq R$ that is comparable to every $I_\alpha \in \mathfrak{C}$ is in $\mathfrak{C}$.

6. (5 pts) Show that for any ideals $I \subseteq J$, there is a maximal chain of ideals between I and J (inclusive of I and J).