Homework 06

Math 8600   Timo Heister, `heister@clemson.edu`

1. Consider the Householder transformation

$$P = I - 2vv^T$$

   for a vector v with $\|v\|_2 = 1$.

   (a) Show that P is orthogonal and symmetric.

   (b) Argue why this is also true for the transformation extended by the identity from an m-by-m to an n-by-n matrix as:

   $$\hat{P} = \begin{pmatrix} I & 0 \\ 0 & P \end{pmatrix}.$$

   .

2. Consider the function
   $$f(x) = \sin(10x) - 2x.$$

   (a) How many roots does the function f have? Consider generating a plot in MATLAB to help.

   (b) With this knowledge, find bounds for the bisection method with a sign change and determine all roots with 8 digits of accuracy this way.

   Submit: `hw06q2.m`.

3. For the equation
   $$f(x) = x^2 - 3x + 2 = 0$$

   consider the fixed point problems

   $$
   \begin{aligned}
   g_1(x) &= (x^2 + 2)/3, \\
   g_2(x) &= \sqrt{3x - 2}, \\
   g_3(x) &= 3 - \frac{2}{x}, \\
   g_4(x) &= (x^2 - 2)/(2x - 3).
   \end{aligned}
   $$

   (a) Analyze the convergence properties for each $g_*(x)$ iteration for the root $x = 2$.

   (b) Confirm this by implementing the fixed-point iteration for each $g_*(x)$ and check the convergence and approximate convergence rate.