Section 3.1: Simple Rate-of-Change Formulas

Constant Rule for Derivatives:

- If $f(x)=b$, where b is a constant, then $f'(x)=0$.

Simple Power Rule for Derivatives:

- If $f(x)=x^n$, then $f'(x)=nx^{n-1}$, where x is any non-zero real number and n is a constant.

Constant Multiplier Rule for Derivatives:

- If $f(x)=c\cdot g(x)$ where c is a constant, then $f'(x)=c\cdot g'(x)$

Sum and Difference Rules for Derivatives:

- If $h(x)=\left(f+g\right)(x)=f(x)+g(x)$ then $h'(x)=f'(x)+g'(x)$
- If $h(x)=\left(f-g\right)(x)=f(x)-g(x)$ then $h'(x)=f'(x)-g'(x)$

Example 1:

Find the derivative of each of the following functions using the Constant Rule.

$y=5$	$\dfrac{dy}{dx}=$
$f(x)=\pi$	$f'(x)=$
The speed of a car with its cruise control set is $s(t)=65$ mph where t is time in minutes.	$s'(t)=$ _____ mph per minute

Rules of Exponents

For integer constants $m>0$, $n>0$, and $x\neq 0$,

$$x^{\frac{1}{n}}=\sqrt[n]{x} \qquad x^{\frac{m}{n}}=\sqrt[n]{x^m} \qquad x^{-n}=\frac{1}{x^n}$$

Example 2:

Find the derivative of each of the following functions using the Power Rule. If necessary, first use the Rules of Exponents to rewrite the function in the form $f(x) = x^n$.

$y = x^2$	$\frac{dy}{dx} =$
$f(x) = x^5$	$f'(x) =$
$g(x) = \sqrt{x} =$ ________	$g'(x) =$
$p(t) = \sqrt[3]{t} =$ ________	$p'(t) =$
$s(t) = \frac{1}{t^2} =$ ________	
$s(r) = \frac{1}{r} =$ ________	
$f(x) = x$	
$t(x) = \frac{1}{\sqrt{x}} =$ ________	
$g(x) = \frac{1}{\sqrt[3]{x^2}} =$ ________	

Example 3:

Identify "c" and "$g(x)$" for each of the following functions and find the derivative of each using the Constant Multiplier Rule.

$f(x)=5x^3$ $c=$ ______ ; $g(x)=$ ___________	$f'(x)=$
$j(x)=\dfrac{1}{2x^5}=$ _________ $c=$ ______ ; $g(x)=$ ___________	

Example 4:

Find the derivative of each of the following functions using the Sum and Difference Rules.

$p(x)=5x^3-3.5x^2+9x-6\pi^2$	
$f(x)=\dfrac{3x^2-4x+8}{2x}=$ _______________________ (*Hint*: Rewrite as 3 terms)	

Example 5: (CC5e p. 199, Activity 31**)**

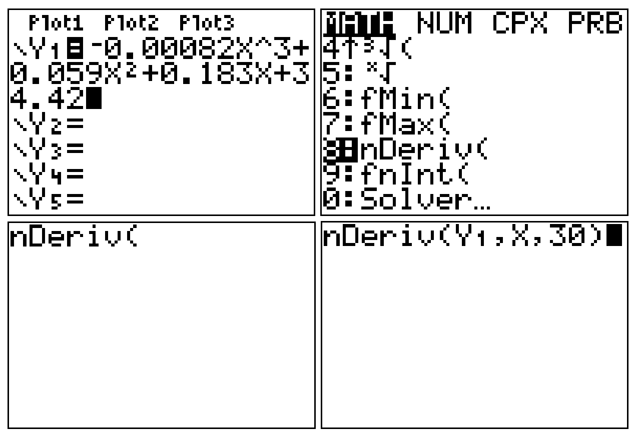
$n(x) = -0.00082x^3 + 0.059x^2 + 0.183x + 34.42$ million people gives the number of Americans age 65 or older, x years after 2000, with projections through 2050.

a. What is the projected number of Americans 65 years of age and older in 2030? Include units with the answer.

b. How quickly is the projected number of Americans age 65 or older changing in 2030? Include units with the answer.

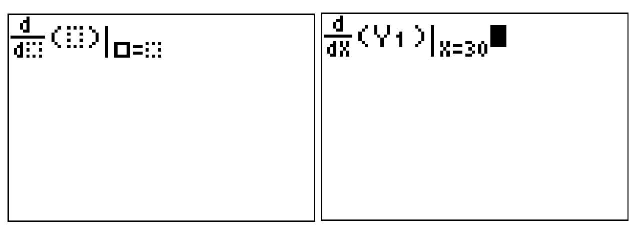
Evaluating the derivative function at a point:

- Enter $n(x)$ in Y1
- **2nd** **QUIT** returns to the Home Screen
- **MATH** ▼ to 8 [nDeriv(] **ENTER** returns nDeriv(on the Home Screen
- Complete the nDeriv statement with **Y1 , X , 30).** **ENTER** returns the value of the derivative at $x = 30$.

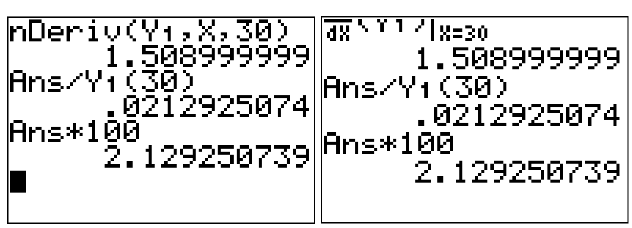


OR

- **MATH** ▼ to 8 [nDeriv(] **ENTER** returns the first screen on the Home Screen.
- Fill in the blanks as shown in the second screen. **ENTER** returns the value of the derivative at $x = 30$.



c. Calculate the percentage rate of change of $n(x)$ in 2030. Include units in the answer.



A **rate-of-change model** is a statement that describes the relationship between an output variable and an input variable of a derivative equation in context.
It includes the following:

- a derivative equation
- an output description, with (derivative) units
- an input description, with units
- an input data range

d. Find the derivative of $n(x) = -0.00082x^3 + 0.059x^2 + 0.183x + 34.42$ and write a rate-of - change model.

$n'(x) =$ ____ million people per ________

gives the **rate of change** in the number of Americans 65 years of age and older, x years after 2000, with projections through 2050.

Example 6:

The number of student tickets sold for a home basketball game at State University is represented by $S(w)$ tickets when w is the winning percentage of the team.

The number of nonstudent tickets sold for the same game is represented by $N(w)$ hundred tickets when the winning percentage of the team is w.

a. $T(w)$ =______________________________ tickets gives the total number of tickets sold for a home basketball game at State University, when w is the winning percentage of the team.

b. Use simple rate-of-change formulas to express the derivative of $T(w)$ and write a rate-of-change model for the total number of tickets sold.

$T'(w) =$ ____ tickets per ______________
gives the ______________________________ in the total number of tickets sold for a home basketball game at State University, when w is the winning percentage of the team.

Example 7: (CC5e p. 197)

The data in the table give the maintenance costs for vehicles driven for 15,000 miles in the U.S. from 1993 through 2000. The maintenance costs given are yearly averages.

Year	1993	1994	1995	1996	1997	1998	1999	2000
Maintenance costs, in dollars	360	375	390	420	420	465	540	585

a. Write a quadratic model, where t is the number of years since 1993, for the maintenance cost for a vehicle driven 15,000 miles.

b. Write a rate-of-change model for the maintenance cost model.

c. How rapidly were maintenance costs changing in 1998? Write a sentence of interpretation.

Example 8: (CC5e p. 198, Activities 1, 5, 9, 13, 17, 21, 25)

For each function given in the first column, write the derivative formula in the second column.

$y = 17.5$	
$f(x) = x^5$	
$x(t) = t^{2\pi}$	
$f(x) = -0.5x^2$	
$f(x) = 5x^3 + 3x^2 - 2x - 5$	
$g(x) = \dfrac{-9}{x^2}$	
$f(x) = \dfrac{3x^2 + 1}{x}$	
$s(x) = x + \dfrac{1}{x}$	
$r(x) = x^e - \pi x^3$	
$g(x) = \sqrt{x^3} - \dfrac{2}{3\sqrt{x}}$	