Test 2 Review Solutions

Limits

Exercise 1. Let $f(x)$ be a function defined in a neighborhood of a real number a. Suppose $L = \lim_{x \to a^-} f(x)$ and $R = \lim_{x \to a^+} f(x)$. When does $\lim_{x \to a} f(x)$ exist?

Exercise 2. Evaluate the following limits. If the limit does not exist, then write DNE and explain why it does not exist.

$$\lim_{x \to -1} \sqrt{x^2} + 1 =$$

$$\lim_{t \to 4} \frac{t^2 - 7t + 12}{t - 4} =$$

$$\lim_{x \to -3^-} \frac{3x}{\sqrt{x+3}} =$$

$$\lim_{x \to -3^+} \frac{3x}{\sqrt{x+3}} =$$

$$\lim_{x \to 0} \frac{x}{\sin x} =$$

(note that $1 \leq \frac{x}{\sin x} \leq 3 - 2\cos x$)

$$\lim_{x \to \infty} \frac{\sqrt{x} - x^2}{2x + 1} =$$

$$\lim_{x \to 0^+} \sqrt{\sec x} =$$

$$\lim_{x \to 1} \frac{x - 1}{|x| - 1} =$$

$$\lim_{x \to 1} \frac{x - 1}{|x - 1|} =$$

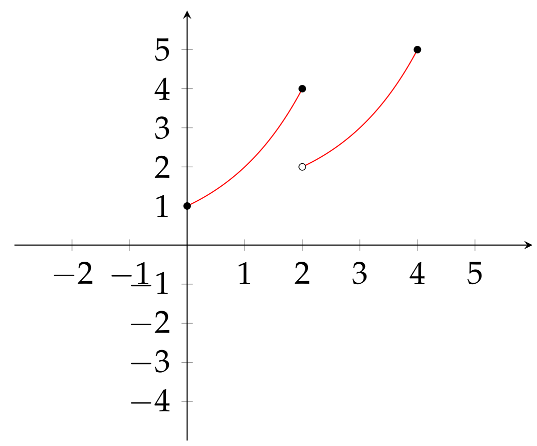
(make sure you get this one right)

$$\lim_{t \to \infty} \frac{1}{\ln t} =$$

Exercise 3. Suppose $\lim_{x \to 3} f(x) = 6$ and $\lim_{x \to 3} g(x) = 2$. Evaluate

$$\lim_{x \to 3} \left(\frac{(f(x) + g(x))^{1/3}}{f(x)g(x)} \right) =$$

Exercise 4. Consider the function $f(x)$ defined on the closed interval $[0,4]$ whose graph is given below

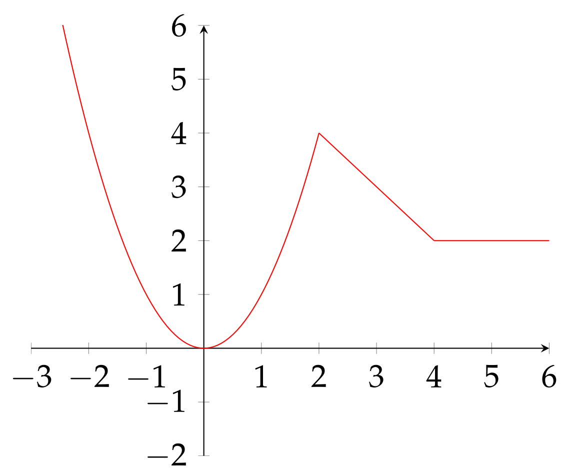


(3.a) Evaluate the following

$$\lim_{x\to 1^+} f(x) =$$
$$\lim_{x\to 2^-} f(x) =$$
$$\lim_{x\to 2^+} f(x) =$$
$$\lim_{x\to 2} f(x) =$$
$$f(2) =$$
$$\lim_{x\to 4^-} f(x) =$$

(3.b) Is $f(x)$ continuous at $x = 2$? Why or why not?
(3.c) Is this function 1-1? Why or why not.

Exercise 5. Consider the function $f(x)$ defined on the whole real line whose graph is given below

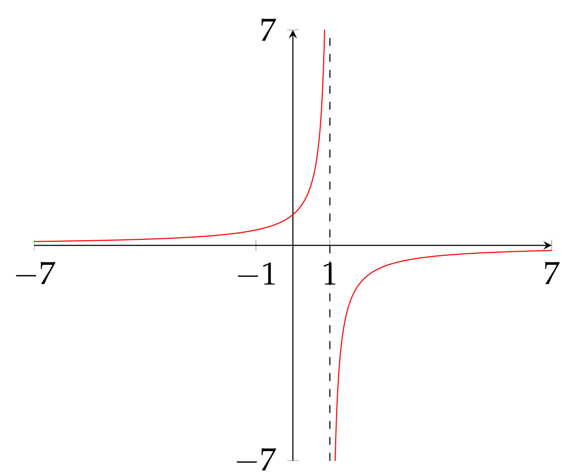


(4.a) Evaluate the following

$$\lim_{x\to 2^-} f(x) =$$
$$\lim_{x\to 2^+} f(x) =$$
$$\lim_{x\to 2} f(x) =$$
$$f(2) =$$
$$\lim_{x\to 4^-} f(x) =$$
$$\lim_{x\to 4^+} f(x) =$$
$$\lim_{x\to 4} f(x) =$$
$$f(4) =$$

(4.b) Is $f(x)$ continuous at $x = 2$? Why or why not?
(4.c) Is this function 1-1? Why or why not.

Exercise 6. Let $f(x)$ be the function whose graph is given below



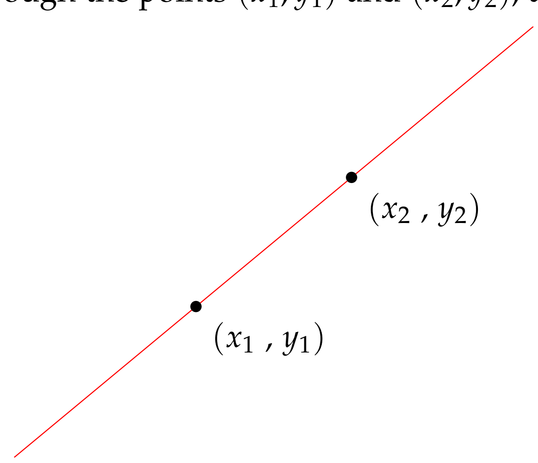
(5.a) Evaluate the following

$$\lim_{x\to 1^-} f(x) =$$
$$\lim_{x\to 1^+} f(x) =$$
$$\lim_{x\to 1} f(x) =$$

(5.b) Is $f(x)$ continuous at $x = 2$? Why or why not?
(5.c) Is this function 1-1? Why or why not.

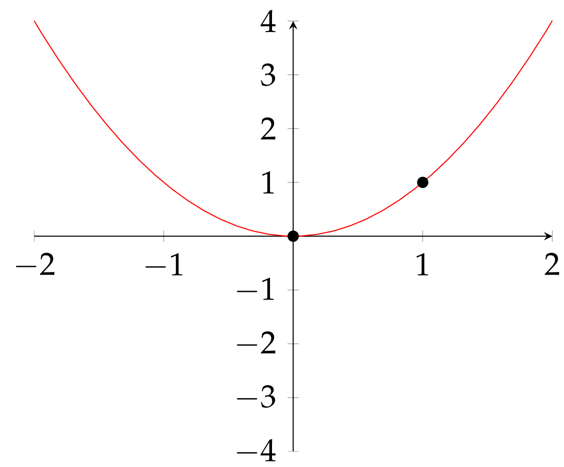
Graphs

Exercise 7. Suppose a line passes through the points (x_1, y_1) and (x_2, y_2), as shown below:



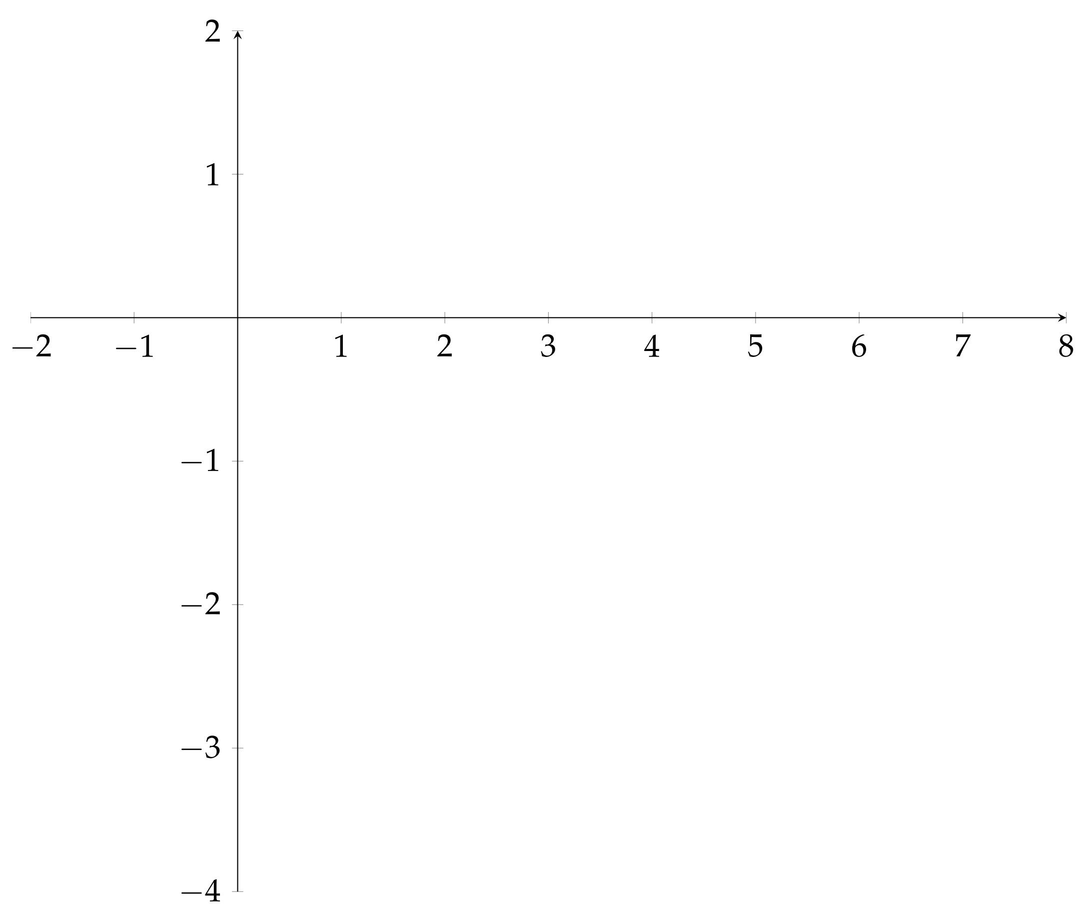
What is the slope of this line?

Exercise 8. Let $f(x)$ be the function given by the graph below:

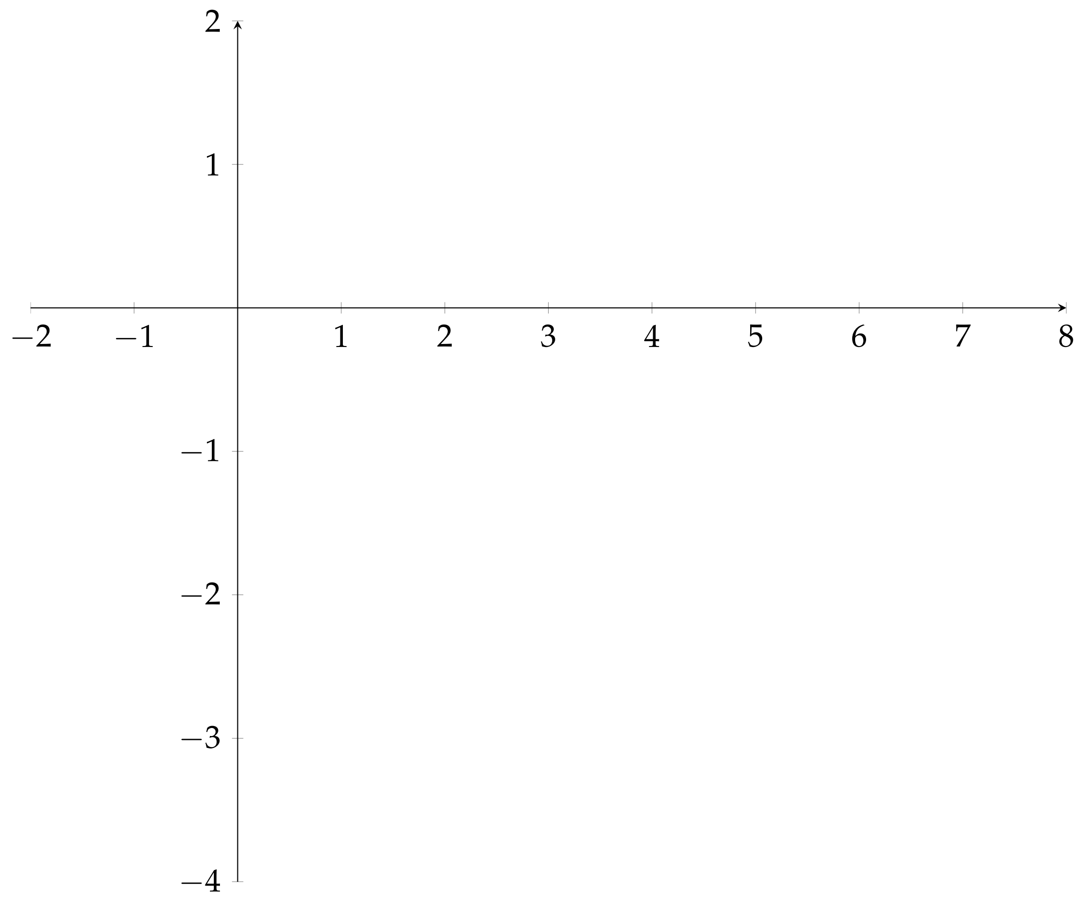


(2.a) Draw the tangent line to the graph of the function at the point $(1, 1)$ and find the slope of this line.
(2.b) Draw the tangent line to the graph of the function at the point $(0, 0)$ and find the slope of this line.
(2.c) Draw the secant line through the points $(0, 0)$ and $(1, 1)$ and find the slope of this line.
(2.d) What is the average velocity of this function from $x = 0$ to $x = 1$?
(2.e) What is the difference between a tangent line and a secant line?

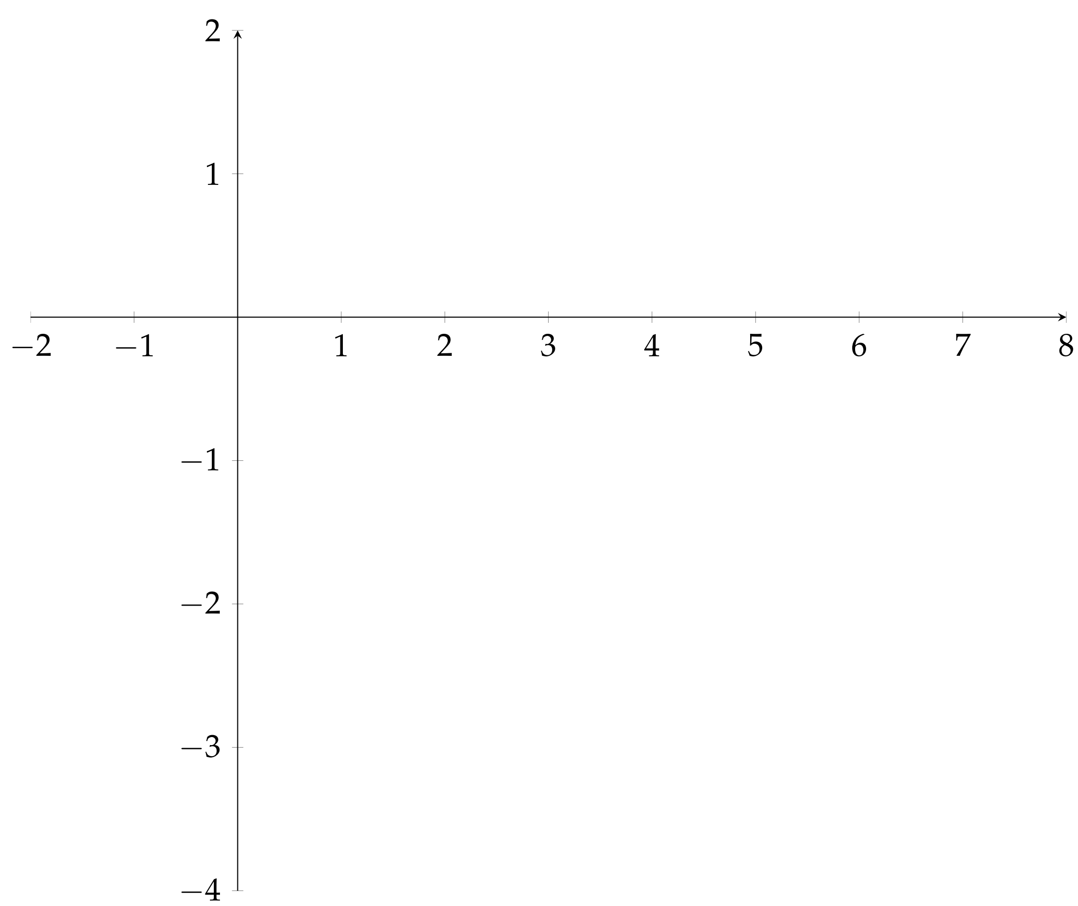
Exercise 9. You should definitinely know what the graphs of $\ln x$ and e^x look like. For this problem, let's focus on $\ln x$. First graph $\ln x$ below



Next graph $\ln(x - 1)$



Next graph $\ln(x-1)-2$



What is the vertical asymptote of the function $\ln(x-1)-2$? Draw the vertical asymptote where it needs to be in the graph above.

Exercise 10. You should definitinely know what the graphs of $\ln x$ and e^x look like. For this problem, let's focus on e^x. First graph e^x below

Next graph e^{x+1}.

Next graph $e^{x+1} - 1$.

What is the horizontal asymptote of the function $e^{x+1}-1$? Draw the horizontal asymptote where it needs to be in the graph above.

Algebra

Exercise 11. Let a, b, c be real numbers. Simplify the following expressions.

$$\frac{1}{x^a} = x^{-a}$$

$$x^a x^b =$$

$$(x^a)^b =$$

$$\sqrt{x} = x^{1/2}$$

$$\sqrt[3]{x} =$$

$$\frac{1}{\sqrt[3]{x^2}} =$$

$$\log_a(bc) =$$

$$\log_a(b^c) =$$

$$e^{a \ln b} =$$

Exercise 12. Suppose $f(x) = 2 - \ln x$ and $g(x) = x + \cos x$. Find

$$(f \circ g)(x) =$$
$$(g \circ f)(x) =$$

Now suppose $f(x) = 3^x - x$ and $g(x) = \sec x + \tan x$. Find

$$(f \circ g)(x) =$$
$$(g \circ f)(x) =$$

Now suppose $f(x) = e^{x+3}$. Find

$$f^{-1}(x) =$$

Exercise 13. Solve for x in the following equations

$$3^x = 9^{x^2}$$

$$\log_{125}(x+1) = \frac{1}{3}$$

$$\log_4(x^2+1) = \frac{1}{2}$$

$$e^{\log(2x+1)} = x^2 + 2$$

$$2^{2x} - 2^{x+1} + 1 = 0$$