Section 3.5: Rates of Change for Functions That Can Be Multiplied

If two functions $g(x)$ and $h(x)$ are multiplied together, the derivative of their product $f(x) = (g \cdot h)(x) = g(x) \cdot h(x)$ is given by the Product Rule: $f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$

The Product Rule says that the derivative of a product function is found by:

$$\begin{pmatrix} \text{derivative of} \\ \text{the first function} \end{pmatrix} \bullet \begin{pmatrix} \text{second} \\ \text{function} \end{pmatrix} + \begin{pmatrix} \text{first} \\ \text{function} \end{pmatrix} \bullet \begin{pmatrix} \text{derivative of} \\ \text{the second function} \end{pmatrix}$$

Example 1:

Given $g(x) = 5x^6$ and $h(x) = \ln(x)$, find the derivative of their product.

a. The product function is $(g \cdot h)(x) = g(x) \cdot h(x) =$

b. The derivative of the product function is

$$(g \cdot h)'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

$$= (\underline{\hspace{4cm}}) \cdot (\underline{\hspace{3cm}}) + (\underline{\hspace{3cm}}) \cdot (\underline{\hspace{3cm}})$$

$$= \underline{\hspace{14cm}}$$

Note: The functions and their derivatives can be organized in a table.

First function: $g(x) = 5x^6$	Second function: $h(x) = \ln(x)$
Derivative of first function: $g'(x) = 30x^5$	Derivative of second function: $h'(x) = \frac{1}{x}$

Example 2:

Given $g(x) = 2\left(3^x\right)$ and $h(x) = 3x^2 - 2x + 1$, find the derivative of their product.

a. The product function is $(g \cdot h)(x) = g(x) \cdot h(x) =$

b. The derivative of the product function is

$$(g \cdot h)'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

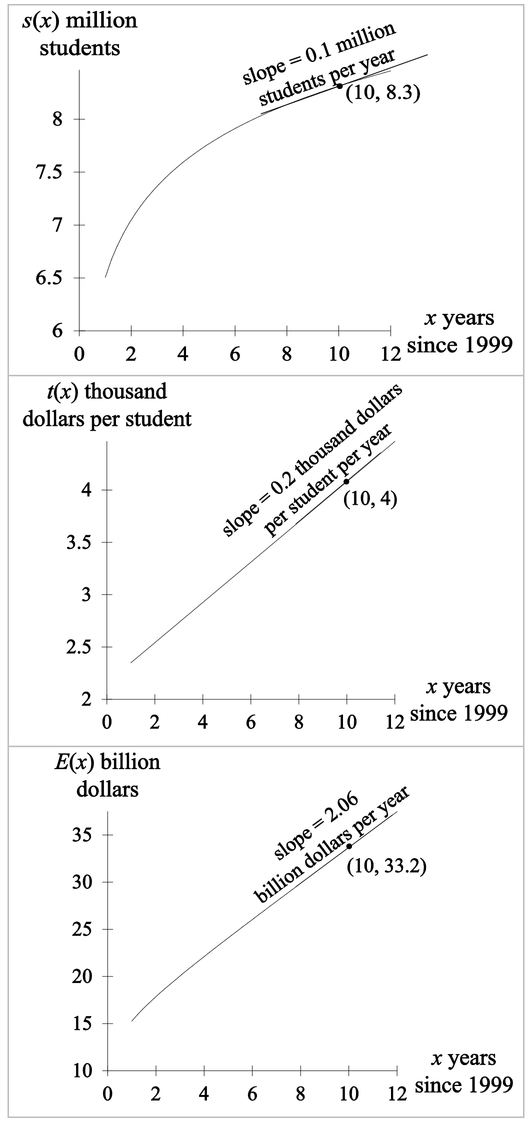
$$= (\underline{\hspace{4cm}}) \cdot (\underline{\hspace{3cm}}) + (\underline{\hspace{3cm}}) \cdot (\underline{\hspace{3cm}})$$

$$= \underline{\hspace{14cm}}$$

Note: The functions and their derivatives can be organized in a table.

First function:	Second function:
Derivative of first function:	Derivative of second function:

Example 3: (CC5e pp. 226-227)



$s(x)$ million students gives the number of full time students enrolled in American public colleges and universities, where x is the number of years since the fall semester, 1999.

$t(x)$ thousand dollars per student gives the average tuition paid by a full-time student in an American public colleges and universities, where x is the number of years since the fall semester, 1999.

a. Identify the input and output units for the product $E(x) = s(x) \cdot t(x)$.

input units:

output units:

b. Use the graphs to find the total amount spent on tuition by students enrolled full time in American public colleges and universities in the fall semester of 2009. Include units with the answer.

c. Use the graphs and the Product Rule to determine how quickly the amount spent on tuition by students enrolled full time in American public colleges and universities was changing in the fall semester of 2009. Include units with each factor and in the answer.

Example 4: (CC5e p.228)

$f(m)$ layers gives the number of laying hens in month m, where $m = 1$ is January, etc.

$g(m)$ eggs per layer gives the laying capacity for one laying hen in month m, where $m = 1$ is January, etc.

In March 2010, a poultry farmer had 30,000 laying hens and he was increasing his flock by 500 laying hens per month.

In March 2010, the monthly laying capacity was 21 eggs per hen and the laying capacity was increasing by 0.2 eggs per hen per month.

a. Identify units for the product $(f \cdot g)(m)$.

input units: output units:

b. Use the given information to find the egg production in March 2010. Include units in each blank.

Since $f(3) =$ ______________________ and $g(3) =$ ______________________ ,

egg production in March 2010 was ________________________________.

c. Use the given information and the Product Rule to determine how quickly egg production was changing in March 2010. Include units with each factor and in the answer.

$$\left.\frac{d(f \cdot g)}{dm}\right|_{m=3} = f'(3) \cdot g(3) + f(3) \cdot g'(3) =$$

d. Write a sentence of interpretation for the answer to part c.

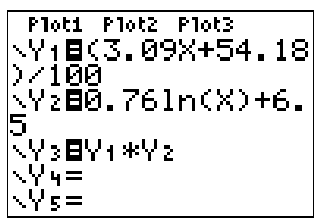
Example 5: (CC5e p.230-231)

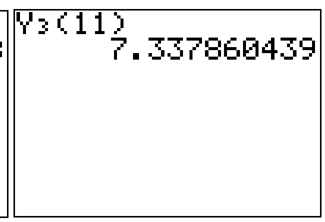
$f(x) = 3.09x + 54.18$ gives the percent of post-secondary students with some form of financial aid where $x = 1$ represents fall 2000, $x = 2$ represents fall 2001, etc.

$s(x) = 0.76\ln(x) + 6.5$ million students gives the full-time enrollment in American public colleges and universities where $x = 1$ represents fall 2000, $x = 2$ represents fall 2001, etc.

a. Write a model for the number of full-time students with some form of financial aid. (*Convert the units for function* $f(x)$ *to a decimal*.)

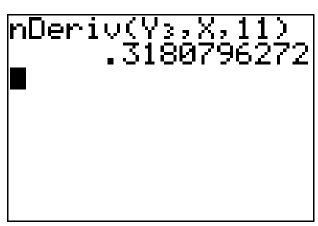
b. How many students had financial aid in fall 2010? Include units with the answer.

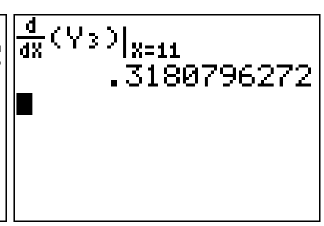




Note: $Y_1 * Y_2$ denotes a product of the functions, whereas $Y_1(Y_2)$ denotes a composition of the functions and cannot be used in this case.

c. How quickly was the number of students with financial aid changing in fall 2010? Include units with the answer.





Example 6: (CC5e p.232, Activity 5)

$s(x) = 15 + \frac{2.6}{x+1}$ dollars gives the value of one share of a company's stock where x is the number of weeks after the stock is first offered.

An investor buys some of the stock each week and owns $n(x) = 100 + 0.25x^2$ shares after x weeks.

The value of the investor's stock is $v(x) = s(x) \cdot n(x)$ dollars x weeks after the stock is first offered.

a. What is the value of the investor's stock 10 weeks after the stock was first offered?

b. How quickly is the value of the investor's stock changing 10 weeks after the stock was first offered?

Example 7:

Suppose $f(x) = g(x) \cdot h(x)$. Use the table below to answer the following. Include work.

	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
$x = -1$	5	0.02	-8	-0.025

a. $f(-1) =$

b. $f'(x) =$

c. $f'(-1) =$