Section 1.9: Quadratic Functions and Models

A **quadratic** function has an equation of the form $f(x) = ax^2 + bx + c$, $a \neq 0$, *b*, and *c* are constants.

The graph of a quadratic function $f(x) = ax^2 + bx + c$ is a parabola with **one concavity**, determined by the sign of *a:*

- For $a < 0$, f is concave down, with a **maximum value**, and end behavior $\lim_{x \to -\infty} f(x) = -\infty = \lim_{x \to \infty} f(x)$
- For $a > 0$, f is concave up, with a **minimum value**, and end behavior $\lim_{x \to -\infty} f(x) = \infty = \lim_{x \to \infty} f(x)$

Example 1:

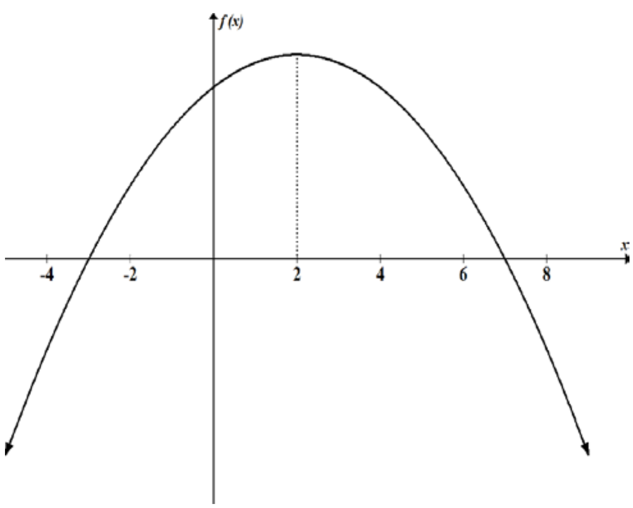
a. Label each of the following graphs as either *concave up* or *concave down*. State the intervals on which the graph is *increasing* or *decreasing*. Mark and label the *absolute maximum* or *absolute minimum* on each graph. Complete the limit statements that describe the end behavior.

b. Identify the graph of $f(x) = -x^2 + 4x + 21$ below.

a =__________. (Note $a < 0$.)

c. Identify the graph of $f(x) = x^2 - 4x - 16$ below.

a =__________. (Note $a > 0$.)

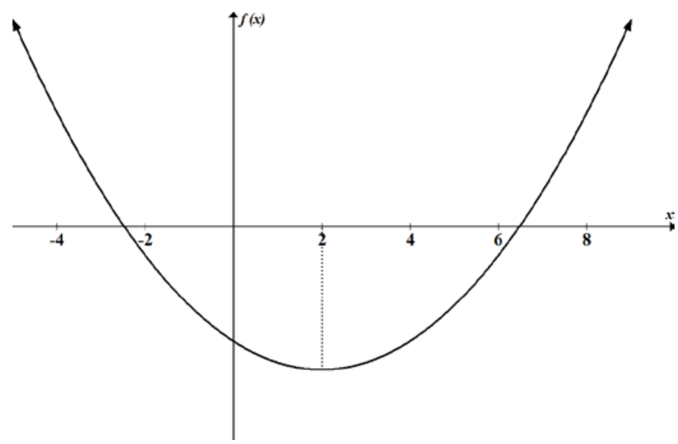


Increasing: ________________

Decreasing: ________________

$\lim\limits_{x\to-\infty} f(x) =$ _________; $\lim\limits_{x\to\infty} f(x) =$ _________

$f(x) =$ ______________________________



Increasing:________________

Decreasing:________________

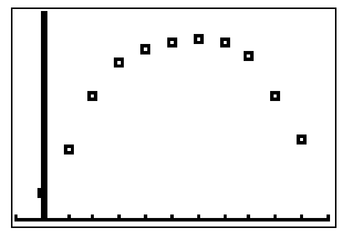
$\lim\limits_{x\to-\infty} f(x) =$ _________; $\lim\limits_{x\to\infty} f(x) =$ _________

$f(x) =$ ______________________________

Example 2: (CC5e p. 86)

The percentage of people in the U.S. over age 14 who are asleep at a given time of night is given in the table.

Hours after 9 pm	0	1	2	3	4	5	6	7	8	9	10
Percentage of people asleep	14.0	36.5	64.4	82.2	89.7	93.0	94.4	91.9	85.2	65.1	41.2



a. Verify that the figure to the right shows a scatter plot of the data.

b. How many concavities are suggested by the scatter plot?

c. The scatter plot indicates that the function is concave _______________ over its entire input interval. The function **changes direction** and has an **absolute maximum value** somewhere between _______________ and _________________hours after 9 pm.

d. Name two other functions that display a single concavity. Discuss why a quadratic is better than these two functions for modeling the data. In particular, how does the end behavior of a concave down logarithmic function compare to the end behavior of a concave down quadratic function?

e. Write a completely defined **quadratic** model for the data.

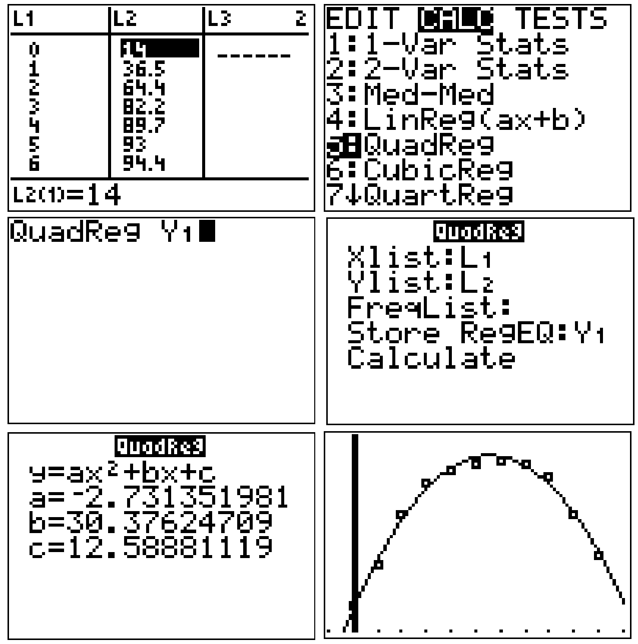
Finding, storing, and viewing a quadratic function:

- With the data in L1 and L2, **<u>STAT</u>** ▸ [CALC] ▾ to 5 [QuadReg] **<u>ENTER</u>** returns QuadReg on the Home Screen **<u>VARS</u>** ▸ [Y-VARS] **<u>1</u>** [Function] **<u>1</u>** [Y1] returns Y1 **<u>ENTER</u>**

  OR

  <u>STAT</u> ▸ [CALC] **<u>5</u>** [QuadReg]
  Xlist: **<u>2nd</u>** **<u>1</u>** [L1]
  Ylist: **<u>2nd</u>** **<u>2</u>** [L2]
  Store RegEQ: **<u>VARS</u>** ▸ [Y-VARS] **<u>1</u>** [Function] **<u>1</u>** [Y1]
  Move cursor to Calculate and hit **<u>ENTER</u>**

- Hit **<u>ZOOM</u>** **<u>9</u>** [ZoomStat] to view the function and the scatter plot



Example 3: (CC5e p. 89)

The percentages of U.S. Internet users who will shop online in a particular year, as reported by *eMarketer Daily*, 6/24/2009 are shown in the table below.

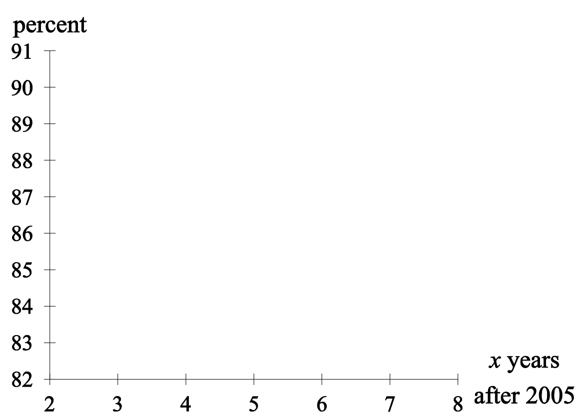
Year	2008	2009	2010	2011	2012	2013
Percentage of Online Shoppers	84.2	86.0	87.5	88.7	89.7	90.5

a. Align the input data to years after 2005. Plot the scatter plot for the data on the set of axes.

b. Describe the scatter plot by completing the following statements:

   The scatter-plot is *<u>increasing/decreasing.</u>*

   The scatter-plot has a single concavity; it is concave *<u>up/down.</u>*



c. Complete the two models that might be used to fit the data.

quadratic: $Q(x) =$ ________________________________ gives the percent of Internet users who shop online, where x is the number of years after 2005, $3 \le x \le 8$.

logarithmic: $L(x) =$ ________________________________ gives the percent of Internet users who shop online, where x is the number of years after 2005, $3 \le x \le 8$.

d. Both functions fit the data well. Which model should be used if the model is to be used for extrapolation past 2013? Explain.

e. According to the model in part d) find the predicted percentage of online shoppers in 2014.

> **Second differences** are found by taking the differences of the first differences. When the second differences are constant or fairly constant, a quadratic function should be considered when finding the best-fit function.

f. Complete the calculations in the tables below to find the **second differences** for the percent of U.S. Internet users who will shop online.

output data	84.2	86.0	87.5	88.7	89.7	90.5
first differences	1.8	1.5				
second differences	-0.3					

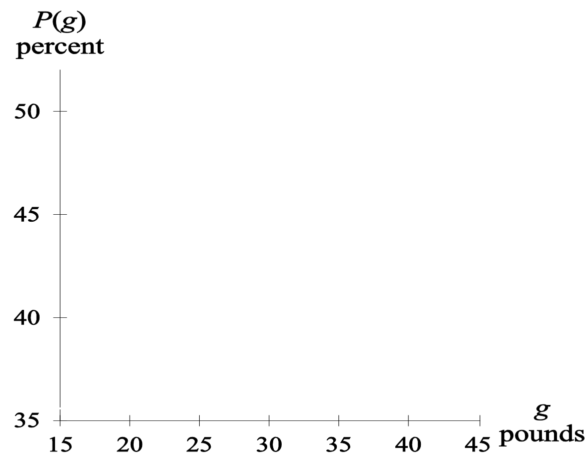
Notice that the second differences are nearly constant. This is one reason we considered a quadratic function in part c.

Example 4: (CC5e p. 88)

The percentage of low birth-weight babies (babies born before 37 weeks of pregnancy and weighing less than 5.5 pounds), and the corresponding prenatal weight gain of the mother is given in the table.

Mother's Weight Gain (pounds)	18	23	28	33	38	43
Low Birth-weight Babies (percent)	48.2	42.5	38.6	36.5	35.4	35.7

a. Plot the scatter plot for the data.



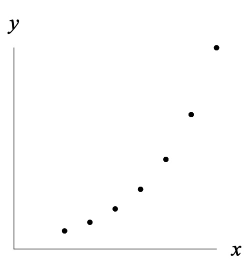
b. Will the leading coefficient of a quadratic function for these data be positive or negative? Explain your answer.

c. Find a quadratic model for the data.

d. Find the percentage of low birth-weight babies born to mothers who gain 45 pounds in pregnancy.

Example 5: (CC5e pp. 91-92, Activities 7, 9, 11, 13)

What type(s) of function(s) might be appropriate to model the data represented by the scatter-plot: linear, exponential, logarithmic, or quadratic?

7.


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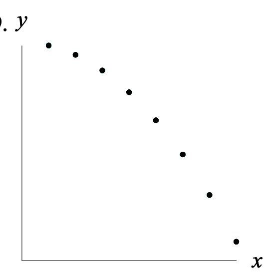

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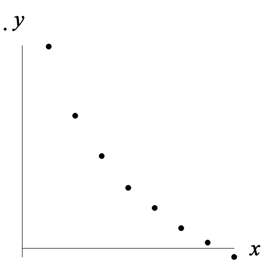

13.