Section 1.8: Logarithmic Functions and Models

A **logarithmic function** has an equation of the form $f(x) = a + b\ln(x)$, where a and $b \neq 0$ are constants, and input values $x > 0$.

The graph of a logarithmic function $f(x) = a + b\ln(x)$ is an **inverse** function to an exponential function. It has **one concavity**, determined by the sign of *b:*

- for $b < 0$, f is concave up
- for $b > 0$, f is concave down

The graph of $f(x) = a + b\ln x$ has a **vertical asymptote** at $x = 0$.

As *x* increases without bound, output values show an increasingly slow increase or decrease.

- for $b < 0$, $\lim\limits_{x\to\infty} f(x) = -\infty$
- for $b > 0$, $\lim\limits_{x\to\infty} f(x) = \infty$

Example 1:

a. Label each of the following graphs of $y = a + b\ln(x)$ as either *increasing* or *decreasing* and as either *concave up* or *concave down*. Complete the limit statements that describe the end behavior.

b. Find the following for the function $f(x) = -1 + \ln(x)$.

- a = b = (Note that $b > 0$.)

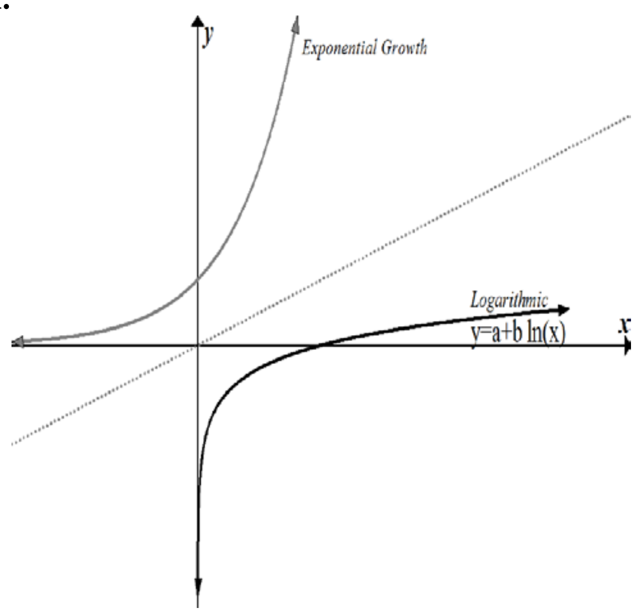
- Does graph I or graph II look like the graph of $f(x) = -1 + \ln(x)$?

c. Find the following for the function $f(x) = 1 - \ln(x)$.

- a = b = (Note that $b < 0$.)

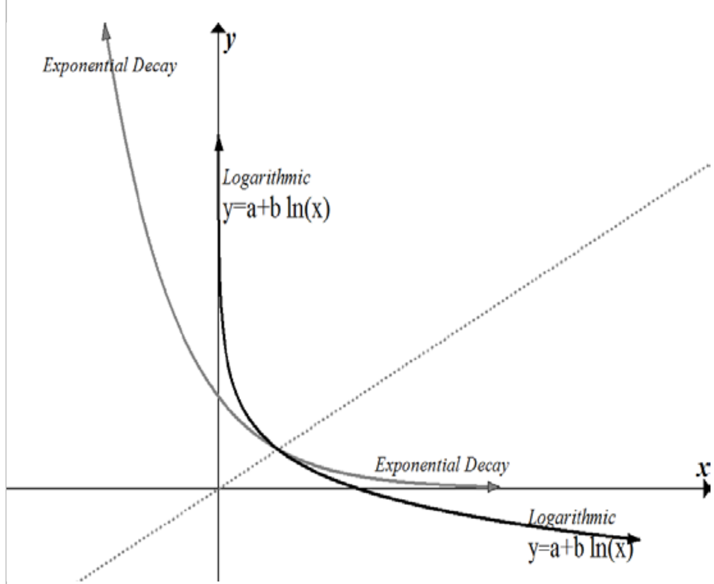
- Does graph I or graph II look like the graph of $f(x) = 1 - \ln(x)$?

I.



Increasing or decreasing?
Concave up or concave down?
$\lim_{x\to 0^+} f(x) =$ ________; $\lim_{x\to\infty} f(x) =$ ________

II.



Increasing or decreasing?
Concave up or concave down?
$\lim_{x\to 0^+} f(x) =$ ________; $\lim_{x\to\infty} f(x) =$ ________

Example 2: (CC5e p. 78)

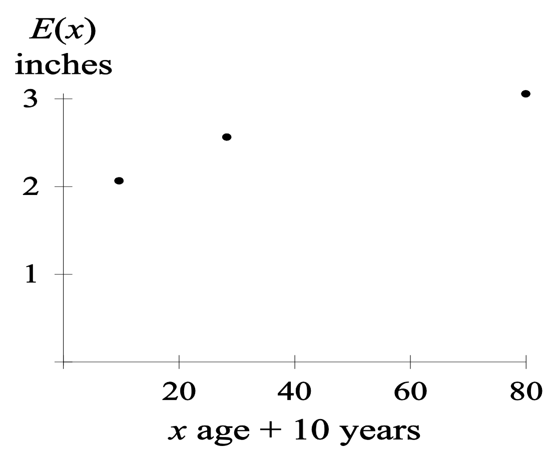
The outer ear in humans continues to grow throughout life even when other organs have stopped growing. The table shows the average length of the outer ear for men at different ages.

Age, in years	0 (at birth)	20	70
Age + 10			
Outer Ear Length, in inches	2.04	2.55	3.07

a. Find a scatter plot for the data given in the table.

b. How does the suggested concavity of the scatter plot indicate that a logarithmic function is appropriate for modeling human male outer ear length?

c. **Since 0 cannot be used for input in a logarithmic equation**, the input data must be aligned. Complete the second row of the table.



d. Find the logarithmic function to model the aligned data. Is it a good fit? _______*(yes/no)* Complete the model below, paying attention to the input description.

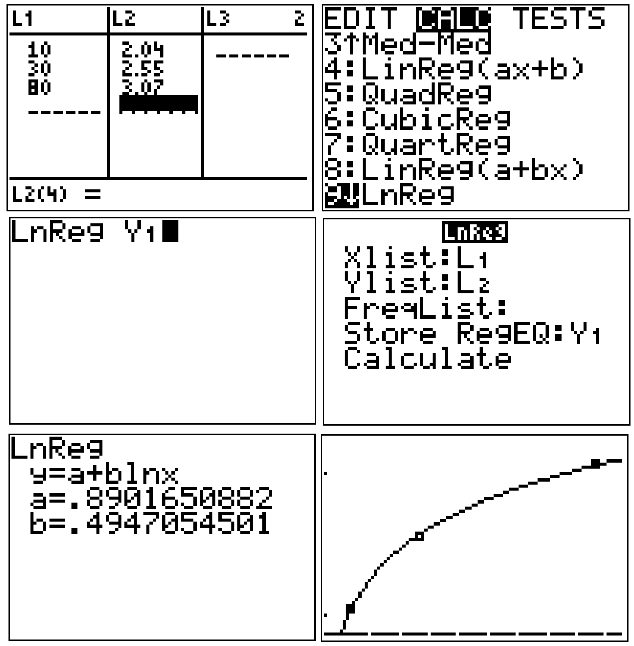
$E(x) =$ ______________________________ inches is the average outer ear length for human males, where $x-10$ is the age in years, $10 \le x \le 80$.

Finding, storing, and viewing a logarithmic function:

- With the aligned data in L1 and L2, **STAT** ▸ [CALC] ▾ to 9 [LnReg] **ENTER** returns LnReg on the Home Screen. **VARS** ▸ [Y-VARS] **1** [Function] **1** [Y1] returns Y1 **ENTER**

  OR

  Xlist: **2nd** **1** [L1]
  Ylist: **2nd** **2** [L2]
  Store RegEQ: **VARS** ▸ [Y-VARS] **1** [Function] **1** [Y1]
  Move cursor to Calculate and hit **ENTER**
- Hit **ZOOM** **9** [ZoomStat] to view the function and the scatter plot

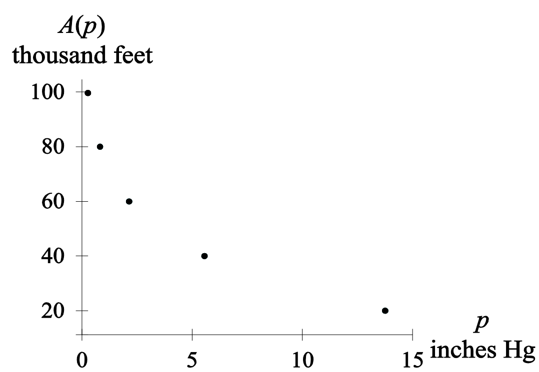


e. Use the (unrounded) model found in part d to find the average outer ear length for 60-year-old men.

Examples 3: (CC5e p. 77, 79)

A pressure altimeter determines altitude in thousand feet by measuring the air pressure in inches of mercury (“inHg” or “Hg”).

Air Pressure, in inHg	Altitude, in thousand feet
0.33	100
0.82	80
2.14	60
5.56	40
13.76	20



a. Describe the concavity of the scatter plot (no concavity, one concavity, two concavities). What two types of functions might fit the data?

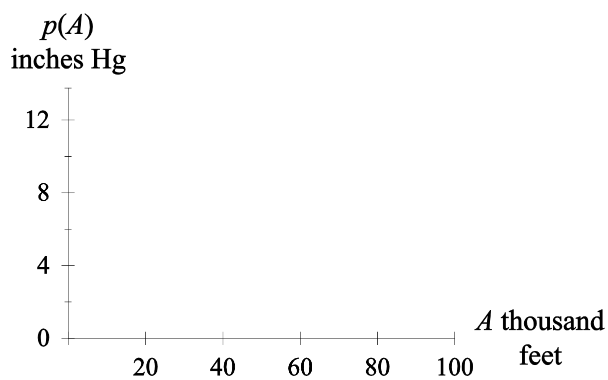
b. Find the best-fitting exponential function and store it in Y1. Find the best-fitting logarithmic function and store it in Y2. View the graphs of both functions over the scatter plot. Which function fits the data best? Explain why.

c. Write a completely defined logarithmic model for altitude, given air pressure.

$A(p) =$

d. Exchange the input and output data to complete the table. Graph the scatter plot of the inverse function on the axes provided.

Altitude (thousand feet)	Air Pressure (inches Hg)



e. Write a completely defined exponential model for air pressure, given altitude. View the graph of the exponential function over the scatter plot and comment on the fit of the exponential function.

$p(A) =$

Example 4: (based on CC5e p. 82, Activity 11)

The table gives estimated concentrations in the bloodstream, in micrograms per milliliter (μg / ml), of the drug piroxicam taken in 20 mg doses once a day.

Days	1	3	5	7	9	11	13	15	17
Concentration (μg / ml)	1.5	3.2	4.5	5.5	6.2	6.5	6.9	7.3	7.5

a. What features of the scatter plot indicate that a logarithmic function may be a good fit for the data?

b. Write a completely defined logarithmic model for the data.

c. According to the unrounded logarithmic model, what is the concentration of piroxicam in the bloodstream after 20 days?

d. According to the unrounded logarithmic model, when will the concentration of piroxicam in the bloodstream reach 8.5 μg / ml?