# MATH 9500
# FALL 2020
# HOMEWORK 6

*Due Monday, November 9, 2020*

1. (5 pts) Let $R$ be an integral domain. Show that $R$ is a Prüfer domain if and only if every overring of $R$ is integrally closed. (Hint: consider $R_M$ for some maximal ideal and if $x, y$ in $R_M$, consider $R_M[\frac{y^2}{x^2}]$).

2. (5 pts) Show that if $K$ is a field then any maximal ideal of $K[x_1, x_2, \cdots, x_n]$ can be generated by $n$ elements.

3. Let $R$ be a commutative ring with 1 and $S \subset R$ a multiplicatively closed subset not containing 0.

a) (5 pts) Show that $R[x]_S = R_S[x]$.
b) (5 pts) Show that $R[[x]]_S \subseteq R_S[[x]]$.
c) (5 pts) Let $S = R \setminus \{0\}$. Show that equality in b) holds if and only if for every countable collection $\{a_i\}_{i \in \mathbb{N}}$ of elements of $S$, $\bigcap_{i \in \mathbb{N}} (a_i) \neq 0$.
d) (5 pts) Show that if $R$ is a PID then every $S \subseteq R$ satisfies the above property if and only if $R$ is a field.

4. (5 pts) Show that if $R$ is a $0-$dimensional rings then any nonunit of $R$ is a zero-divisor.