Homework #5

(1) $f(x) = x^3 - 5.85x^2 + 5.27x + 13.56$ hundred dollars gives the stock price of Company A, x hours after 9:30 am on a given day, on the closed interval $0 \le x \le 5$.

(1.a) At what x value(s) does the function have a relative maximum?

(1.b) At what x value(s) does the function have a relative minimum?

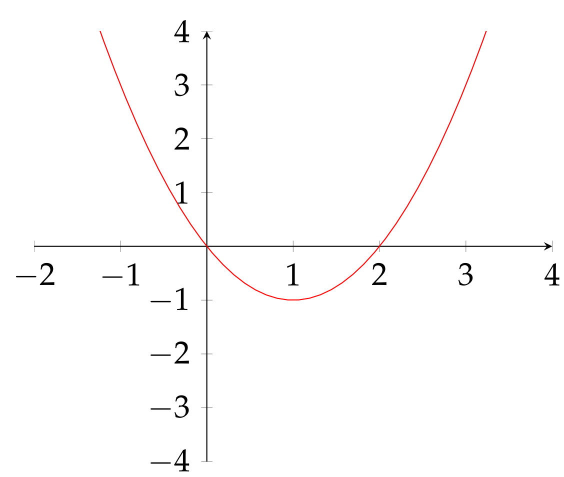
(1.c) At what x value(s) does the function have an inflection point?

(1.d) On the closed interval $0 \le x \le 5$, the stock price was *highest* _______ hours after 9:30 am, at which the price of the stock was ________________________ (make sure to include units!).

(1.e) On the closed interval $0 \le x \le 5$, the stock price was *lowest* _______ hours after 9:30 am, at which the price of the stock was ________________________ (make sure to include units!).

(1.f) On the closed interval $0 \le x \le 5$, the stock price was *decreasing most rapidly* _______ hours after 9:30 am, at a rate of ________________________ (make sure to include units!).

(2) Consider the function $f(x)$ defined on the whole real line below whose *slope* graph (i.e. the graph of $f'(x)$) is given below



(2.a) Where are the relative maxes of $f(x)$ located? (If there are none, just say "there are none")

(2.b) Where are the absolute maxes of $f(x)$ located? (If there are none, just say "there are none")

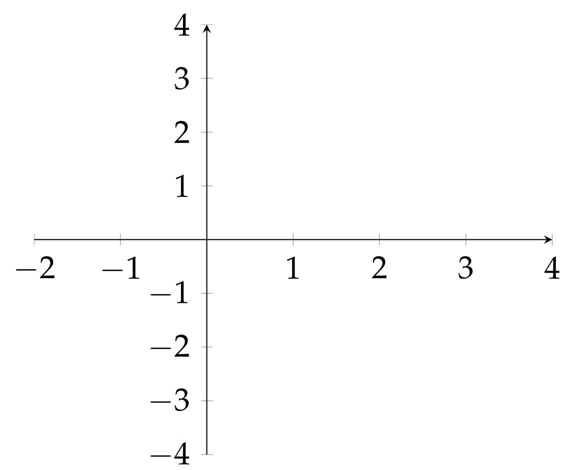
(2.c) Where are the relative mins of $f(x)$ located? (If there are none, just say "there are none")

(2.d) Where are the absolute mins of $f(x)$ located? (If there are none, just say "there are none")

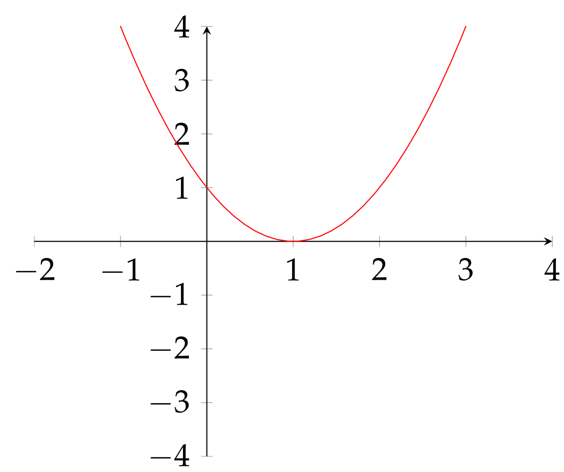
(2.e) Where are the critical points of $f(x)$ located? (If there are none, just say "there are none")

(2.f) Where are the inflection points of $f(x)$ located? (If there are none, just say "there are none")

(2.g) Sketch how the shape of the graph of $f(x)$ should look (there's actually many different functions whose slope graph corresponds to the one above but they all have the same shape):



(3) Consider the function $f(x)$ defined on the whole real line below whose *slope* graph is given below



(3.a) Where are the relative maxes of $f(x)$ located? (If there are none, just say "there are none")

(3.b) Where are the absolute maxes of $f(x)$ located? (If there are none, just say "there are none")

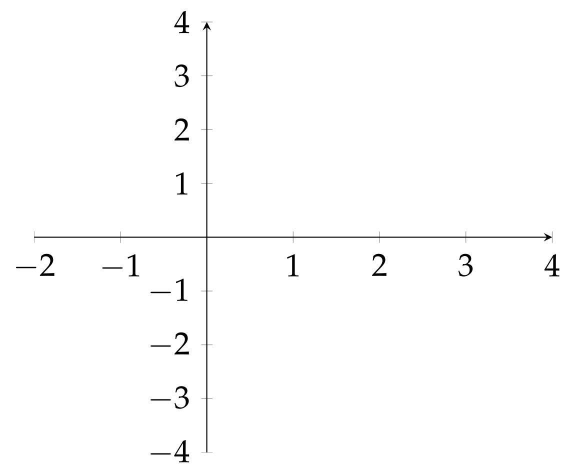
(3.c) Where are the relative mins of $f(x)$ located? (If there are none, just say "there are none")

(3.d) Where are the absolute mins of $f(x)$ located? (If there are none, just say "there are none")

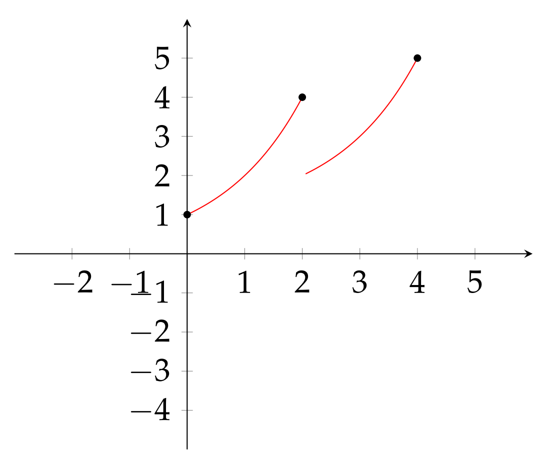
(3.e) Where are the critical points of $f(x)$ located? (If there are none, just say "there are none")

(3.f) Where are the inflection points of $f(x)$ located? (If there are none, just say "there are none")

(3.g) Sketch how the shape of the graph of $f(x)$ should look:



(4) Consider the function $f(x)$ defined on the closed interval $[0,4]$ whose graph is given below



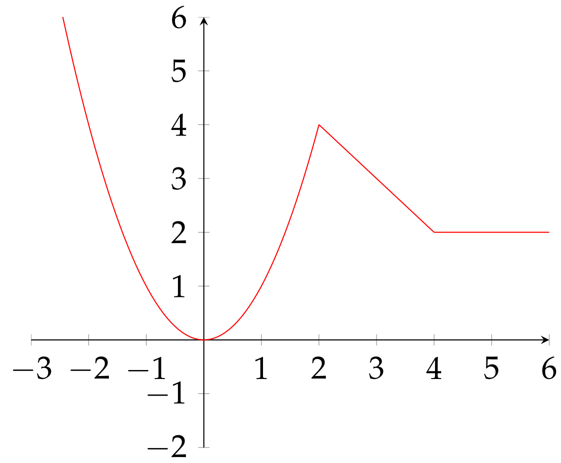
(4.a) Where are the relative maxes of $f(x)$ located? (If there are none, just say "there are none")

(4.b) Where are the absolute maxes of $f(x)$ located? (If there are none, just say "there are none")

(4.c) Where are the relative mins of $f(x)$ located? (If there are none, just say "there are none")

(4.d) Where are the absolute mins of $f(x)$ located? (If there are none, just say "there are none")

(5) Consider the function $f(x)$ defined on the whole real line whose graph is given below



(5.a) Where are the critical points of $f(x)$ located? (If there are none, just say "there are none")

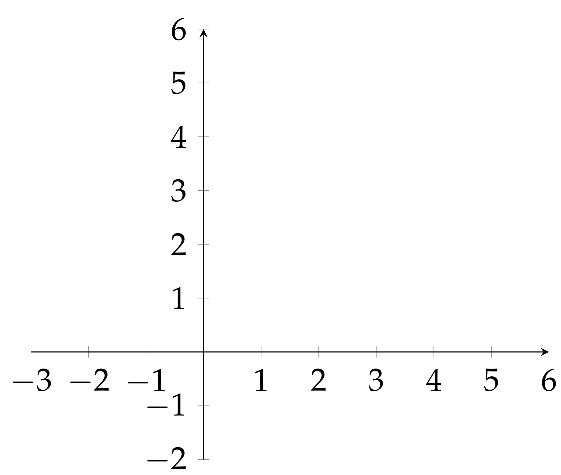
(5.b) Where are the relative maxes of $f(x)$ located? (If there are none, just say "there are none")

(5.c) Where are the absolute maxes of $f(x)$ located? (If there are none, just say "there are none")

(5.d) Where are the relative mins of $f(x)$ located? (If there are none, just say "there are none")

(5.e) Where are the absolute mins of $f(x)$ located? (If there are none, just say "there are none")

(5.f) Sketch the graph of $f''(x)$ below (hint: keep in mind that there are two places where $f''(x)$ is not defined):



(6) Let $f(x)$ a continuous differentiable function on the whole real line and let a be a real number.

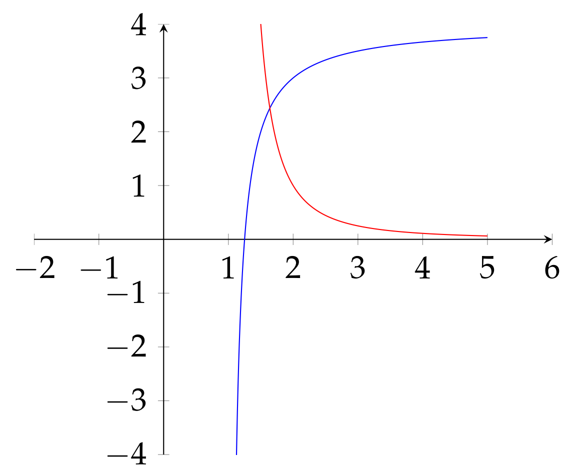
(6.a) True or False: If $f''(a) \neq 0$, then the function may have an inflection point at $x = a$.

(6.b) True or False: If $f''(a) = 0$, then the function has an inflection point at $x = a$. (this one is a little more subtle than you think)

(6.c) True or False: If $f''(a) > 0$, then the function is concave up at $x = a$.

(6.d) True or False: If $f'(a) = 0$ and $f''(a) > 0$, then the function has a relative max at $x = a$.

(7) Consider the graphs of the two functions below:



One of them is $f(x)$ and one of them is $f'(x)$. Which is which?

(8.a) Let $h(x) = 11(1.8^x)$ and $g(h) = h^3$. Find the derivative of $g(h(x))$.

(8.b) Write $f(x) = e^{\sqrt{x^2+1}}$ as a composition $g(h(x))$.

$$g(h) =$$
$$h(x) =$$

(8.c) Write $f(x) = \sqrt{x^3 + 1}$ as a composition $g(h(x))$.

$$g(h) =$$
$$h(x) =$$

(8.d) Find the derivative of $f(x) = (x^2 + 2)e^{3x^2+1}$.

(8.e) Find the derivative of $f(x) = \frac{2+x}{3+\ln(x)}$.

(8.f) Find the *second* derivative of $\ln(3x + 1)$.

(8.g) Find the derivative of $2^{(x^3-1)}e^{2x}$.

(9) Let f be a differentiable function and suppose that $f(3) = 5$ and $f'(3) = -2$.

(9.a) Find the linearization of f at $a = 3$.

(9.b) Estimate the change between $x = 3$ and $x = 4$.

(9.c) Estimate $f(4)$. If $f(x)$ is concave up, then is this an overestimate or an underestimate?

(9.d) $p(t)$ cents is the average retail price of a pound of salted, graded A butter, t years since 1990. Suppose that in 1998, the average retail price of salted, graded A butter was 296 cents and was increasing by 54 cents per year. Use a linear estimation to find the average retail price of salted, graded A butter in 1999.