Math 9853: Homework #5

Due Thursday, September 26, 2019

(a) Let $f\colon V \to W$ be any linear map of vector spaces over a field K.

(a.1) Suppose f is surjective. Describe all possible injective linear maps $s\colon W \to V$ so that $f \circ s = I_W$ (the identity map on W, that is, $f(s(u)) = u$ for all $u \in W$). (**Hint**: Fix a basis for W and note that, for two such maps s_1 and s_2, $f(s_1(u)) = u = f(s_2(u))$ iff $s_1(u) - s_2(u) \in \mathrm{Ker}(f)$.)

(a.2) Suppose f is injective. Describe all possible surjective linear maps $\tau\colon W \to V$ so that $\tau \circ f = I_V$.

(b) Let $U = \mathrm{Span}(u_1, u_2, u_3)$ and $V = \mathrm{Span}(v_1, v_2)$ where

$$u_1 = (1,1,1,1,1), u_2 = (1,0,1,0,1), u_3 = (0,0,1,1,1), v_1 = (0,1,0,1,1), v_2 = (0,1,1,0,1)$$

all computed modulo 2. Decide if the sum $U + V$ is a direct sum. (**Hint**: Form the matrix M whose rows consists of the following vectors:

$$(u_1, 0), \quad (u_2, 0), \quad (u_3, 0), \quad (v_1, v_1), \quad (v_2, v_2),$$

all with length 10. Then reduce M to a reduced row Echelon form where the row vectors have the form (w_1, w_2) and $(0, w_3)$ where each w_i has length 5. Explain why all the nonzero w_1 form a basis for $U + V$ and all the nonzero w_3 form a basis for $U \cap V$.)

(c) Problem 6.4, page 173 of the book.