## Section 4.5: Marginal Analysis

In **marginal analysis,** a decision to increase production is analyzed by weighing potential benefits against potential costs.

In marginal analysis the decision is based on **a unit change in input.**

Marginal analysis uses **derivatives**. Recall from section 4.1 that an **estimate of change** between $(q, f(q))$ and a nearby point $(q+h, f(q+h))$ on a function $f(x)$ is given by $f'(q)\cdot h$. Using $h=1$ in this formula finds an estimate in the amount of change between $(q, f(q))$ and a point $(q+1, f(q+1))$ and is given by $f'(q)\cdot 1 = f'(q)$.

If $C(q)$ is a continuous function that gives the cost of producing $q$ units, then an estimate of the cost of producing **one** additional unit is given by $C'(q)$ and is called the **Marginal Cost of producing the *q+1st* unit**.

If $R(q)$ is a continuous function that gives the revenue from selling $q$ units, then an estimate of the revenue generated by selling **one** additional unit is given by $R'(q)$ and is called the **Marginal Revenue from selling the *q+1st* unit**.

A sentence of **interpretation** for **marginal cost or marginal revenue** does not use the word "marginal" and may be written in the same manner as a sentence of interpretation for a derivative. Or, more commonly in economics, it may take the following form:

> *The additional cost of producing the **q+1st** unit is $C'(q)$ dollars (or output units of C).*

*or*

> *The additional revenue from selling the **q+1st** unit is $R'(q)$ dollars (or output units of R).*

Marginal cost or marginal revenue is measured in cost or revenue units per **single** production unit.

**Example 1:** (CC5e pp. 285)

$C(x)$ hundred dollars gives the cost of producing $x$ thousand microwave ovens. When 2.4 thousand microwave ovens are produced, the total cost of production is 96 hundred dollars and costs are increasing by 13 hundred dollars per thousand microwave ovens.

a. Write the following sentence using derivative notation.
When 2.4 thousand microwave ovens are produced, the total cost of production is increasing by 13 hundred dollars per thousand microwave ovens.

b. Marginal cost is measured in cost units per **single** production unit.

Convert the derivative to the new units:

$C'(2.4) =$ _____dollars per microwave oven

c. What is the marginal cost for the 2401st microwave oven?

d. Write a sentence of interpretation for marginal cost, using the form commonly used in economics.

The _____cost of producing the _____*st* microwave

oven is _____dollars.

**Example 2:** (CC5e pp. 289, based on Activity 15)

$C(x) = 0.16x^3 - 8.7x^2 + 172x + 69.4$ dollars gives the hourly production costs where $x$ television sets are produced each hour, $5 \le x \le 35$.

a. What are the hourly production costs when five television sets are produced each hour? Include units with the answer.

b. How quickly are production costs changing when five television sets are produced each hour? Include units with the answer.

c. Find the linearization for $C(x)$ with respect to $x = 5$.

$C_L(x) =$

d. Write a sentence of interpretation for the marginal cost at an hourly production level of five television sets, using the form commonly used in economics.

The _____ cost of producing the _____*th* television

set each hour is _____dollars.

e. Complete the following statements using marginal analysis.

The *additional* cost of producing the *21st* television set each hour is _____*dollars*.

When 30 television sets are produced each hour, the hourly cost of production is increasing

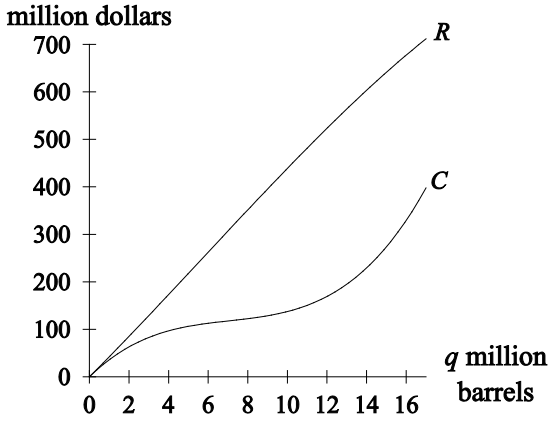
by _____ dollars per additional television set.

**Example 3:** (CC5e p. 284)

$C(q)$ million dollars gives the cost of producing $q$ million barrels of crude oil at a certain oil field, $0 < q < 17$.

$R(q)$ million dollars gives the revenue from the sale of $q$ million barrels of crude oil at a certain oil field, $0 < q < 17$.

The graphs of $C$ and $R$ are shown to the right.




a. $C'(10) = 10.720$. Convert the units before filling in the blanks in the following sentences of interpretation for marginal cost.

When _____ million barrels of crude oil are produced, the cost of production is increasing by _____**dollars per barrel**.

Or, using the form commonly used in economics,

The _____ cost to produce the _____*st* barrel of crude oil is _____ **dollars**.

b. $R'(10) = 43.090$. Convert the units before filling in the blanks in the following sentences of interpretation for marginal revenue.

When _____ million barrels of crude oil are sold, revenue is increasing by _____**dollars per barrel**.

Or, using the form commonly used in economics,

The _____ revenue from the sale of the _____*st* barrel of crude oil is _____ **dollars**.

c. If the revenue from the sale of 10 million barrels of crude oil is 438.9 million dollars, estimate the revenue from the sale of 10,000,001 barrels of oil. Include units with your answer.

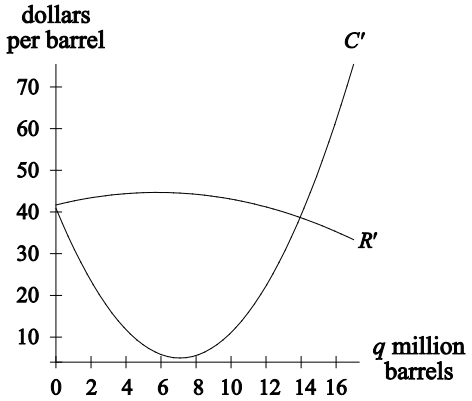
**Profit Maximization Rule (optional):**

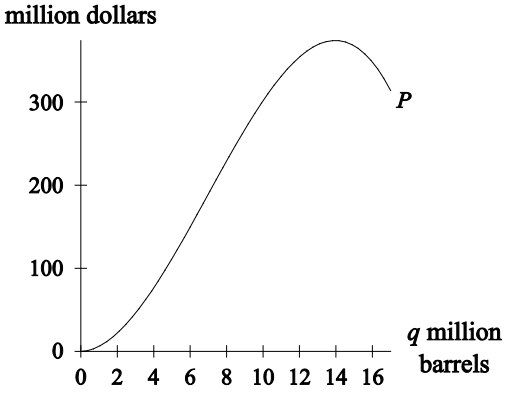
For a cost function $C(q)$ and a revenue function $R(q)$, a profit function is given by $P(q) = R(q) - C(q)$, when producing and selling $q$ units.

Profit is maximized when $P'(q) = R'(q) - C'(q) = 0$ or $R'(q) = C'(q)$.
Profit is maximized when **marginal revenue equals marginal cost.**

Since a solution to this equation may produce a minimum profit, check the graph of the profit function to verify that profit is maximized.

d. (Optional) The graph showing both marginal revenue and marginal cost is shown to the left below and the graph of the profit function, $P(q) = R(q) - C(q)$ is shown to the right below.




Estimate the input value where the profit is maximized by finding the input value at the intersection of the marginal cost and the marginal revenue. Include units with the answer.

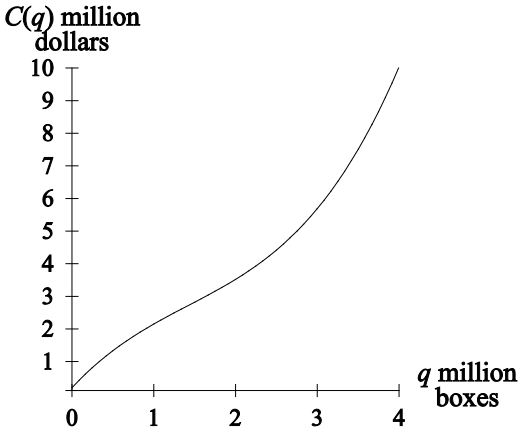
Use the graph of $P(q)$ to verify that the input value occurs where the profit is maximized.

**Example 4:** (CC5e pp. 285-286)

Each council of Girl Scouts has costs associated with their annual cookie sales. The council sets the price for cookies sold in their region.

During a recent campaign, one council set the sales price at $4.00 per box.

$C(q) = 0.23q^3 - 0.98q^2 + 2.7q + 0.2$ million dollars gives the Girl Scout council's total cost associated with this campaign when $q$ million boxes are sold, $0 < q < 4$.




a. Complete the derivative model:

$C'(q)$ = _____ dollars per box gives the rate of change in the costs associated with Girl Scout cookie sales in one council's region when $q$ million boxes are sold, $0 < q < 4$.

b. Complete the model for revenue:

$R(q)$ = _____ million dollars gives the revenue from the sale of Girl Scout cookie sales in one council's region when $q$ million boxes are sold, $0 < q < 4$.

c. Complete the derivative model:

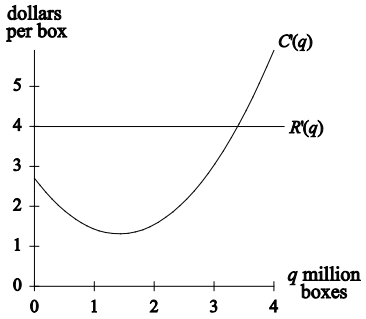
$R'(q)$ = _____dollars per box gives the rate of change in the revenue from the sale of Girl Scout cookies in one council's region when $q$ million boxes are sold, $0 < q < 4$.

d. Complete the model for profit:

$P(q)$ = _____ million dollars gives the profit from the sale of Girl Scout cookie sales in one council's region when $q$ million boxes are sold, $0 < q < 4$.

e. What is the marginal cost when 2 million box of Girl Scout cookies are produced (the marginal cost for the 2,000,001st box)?

f. (Optional) Profit is maximized at _____ million dollars when _____ boxes are sold.




(Optional)
A **Production function** $Q(k)$ gives the quantity of output that can be produced by a given amount of input $k$, called a **production factor**, when all other production factors are held constant. Production factors may be labor costs, capital expenditures, natural resources, etc.

**Marginal Product** refers to the change in production level resulting from **one** additional unit of the variable production factor $k$.

**Example 5** (optional)**:** (CC5e pp. 287)

$Q(k)$ automobiles represents the output for a Honda automobile plant in Dongfeng, China when its production factor of capital investment is $k$ million U.S. dollars.

Current production is $Q(200) = 119,200$ and $Q'(200) = 600$.

a. When _____ million U.S. dollars are invested, the Honda automobile plant in Dongfeng produces _____ automobiles and its output is increasing by _____ automobiles per million U.S. dollars in capital investment.

b. By increasing investment by one million U.S. dollars, production will increase to _____ automobiles at the Dongfeng plant.

c. The **marginal product** with respect to capital is _____automobiles per U.S. dollar in capital investment.