Section 1.1: Functions - Four Representations

A **relation** is a rule that links one input value (x) to an output value (y). If any particular input value corresponds to more than one output, the relation is not a function.

A **function** is a rule that assigns exactly one output value (y) to each input value (x). An input value can be referred to as a member of the function's domain; an output value can be referred to as a member of the function's range. **Function notation** $f(x)$ is used to describe a function with input variable x and output $f(x)$.

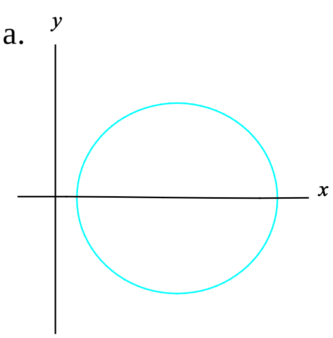
A function can be represented **numerically** (*in table form*), **algebraically** (*as an equation*), **verbally** (*a description*), or **graphically**.

The **Vertical Line Test** can be used to determine whether a relation satisfies the definition of a function. If there is a vertical line that crosses the graph of a relation in two or more places, then the graph does not represent a function.

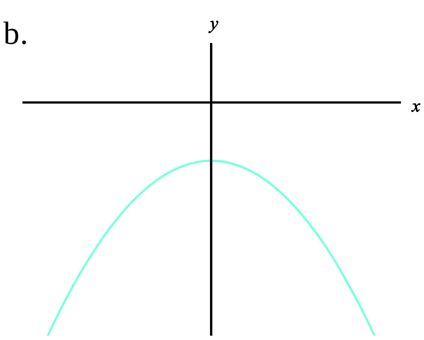
Example 1: (CC5e pp. 11-12)

Determine whether each of the following graphs of a relation is also a function. If the relation is not a function, explain why not. (The horizontal axis is the input axis.)

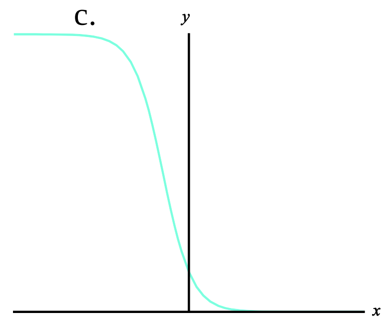
a.



b.



c.



____________________ ____________________ ____________________

____________________ ____________________ ____________________

Example 2: (CC5e pp. 3-4)

Determine whether the following functions are represented *numerically*, *algebraically*, *verbally*, *or graphically*.

a. The function g can be represented by either of the tables shown below.

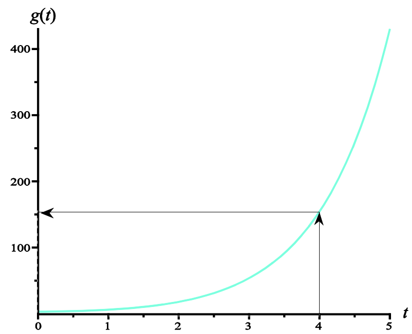
It has a(n) ______________________________ representation.

t	$g(t)$
2	18
3	54
4	156
5	435

t	2	3	4	5
$g(t)$	18	54	156	435

b. The function g can be represented by the graph shown below.

It has a(n) ____________________________________ representation.



c. The function g can be represented as $g(t) = 3e^t - 2t$.

It has a(n) ____________________________ representation.

d. The function g can be represented by the statement:

The area of a square equals the length of a side squared.

It has a(n) ______________________________________ representation.

e. The resident population of the United States between 1900 and 2000 can be modeled as $g(t) = 80(1.013^t)$ million people where t is the number of years since the end of 1900.

The function g has a(n) _________________________ representation.

Example 3: (CC5e pp. 5-6)

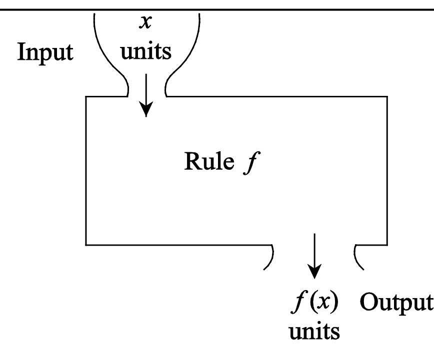
$p(t) = 80(1.013^t)$ million people gives the US population, where t is the number of years since the end of 1900, between the years 1900 and 2000.

a. Explain why p is a function.

b. Identify the notation, unit of measure, and description for the input and output.

	Input	**Output**
Notation (variable)		
Unit of measure		
Description		

An **input/output** diagram is a drawing that uses a box and arrows to identify the notation representing the input, output, and rule for a particular function.



c. Draw an input/output diagram for *p*.

d. The ordered pair $(0, 80)$ lies on the graph of $p(t)$. Rewrite the ordered pair using *function notation.*

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A **sentence of interpretation** for an ordered pair uses ordinary conversational language to answer the questions *when*?, *what*?, and *how much*?
When? refers to the input value and does not necessarily involve time.
What? refers to the output description for the function.
How much? refers to the output value.

e. Write a sentence of interpretation for part *d*.

Given a function, when an input value is given, the output value is found by evaluating the function at the specified input value.

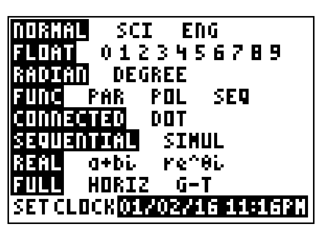
When an output value is given, the function is used to write an equation which is **solved** to find the necessary input value or values.

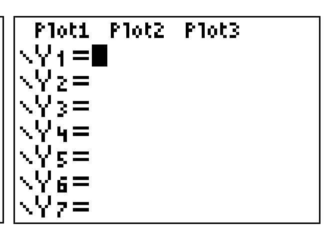
Example 4: (CC5e p. 4, p. 6)

Consider the function: $g(t) = 3e^t - 2t$. The input variable to the function is *t* and the output variable is *g*.

a. Find the corresponding output value for an input of 4, using your calculator. In other words, evaluate $g(4)$. Round the answer to 3 decimal places.

Using a TI-83/84 calculator, check that the calculator is set to FLOAT and TURN PLOTS OFF:

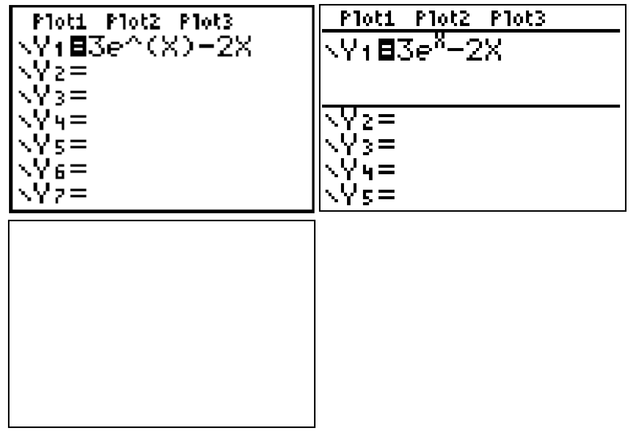




- **MODE**
  If FLOAT isn't highlighted, hit ▼ (down arrow) **ENTER** to change the mode to FLOAT
- **Y=**
  If Plot1, Plot 2, or Plot 3 is highlighted, hit ▲ (up arrow) on the name of the plot you want to turn off and hit **ENTER**. Repeat if needed.

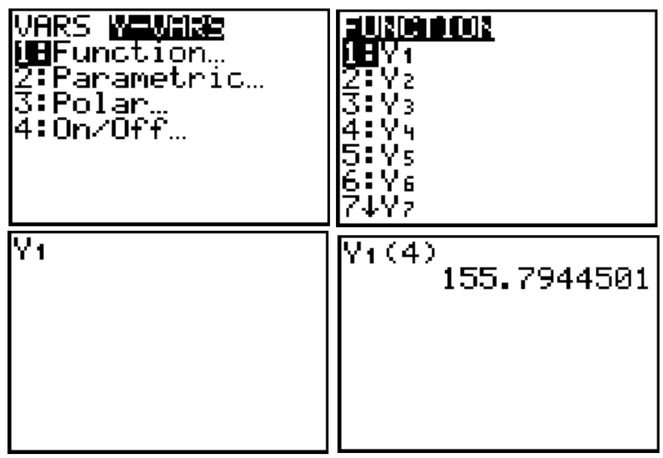
Enter a function into the equation editor:

- **<u>Y=</u>** (located directly under the calculator screen) opens the equation editor
- Enter the right side of the function equation into Y1 using **<u>2ND</u> <u>LN</u> [e^x]** for *e* and **<u>X,T,θ,n</u>** for *x*
- **<u>2ND</u> <u>MODE</u> [QUIT]** returns to the home screen



Evaluate the output value for the function entered in Y1:

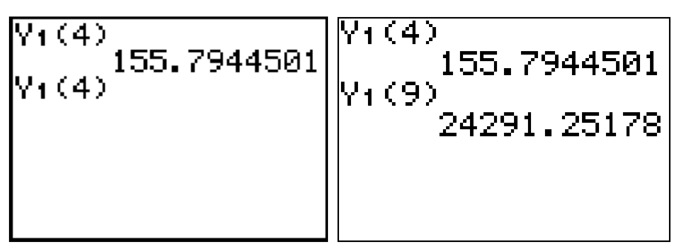
- **<u>VARS</u>** ▶ (right arrow) obtains the VARS Y-VARS options
- **<u>ENTER</u>** or **<u>1</u>** chooses **1: Function**
- **<u>ENTER</u>** or **<u>1</u>** chooses **1: Y1**
- **<u>(4) ENTER</u>** evaluates *g* at *t* = 4



b. Find $g(9)$. Round the answer to 3 decimal places.

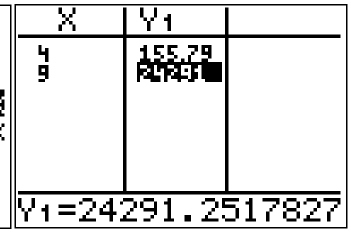
Re-evaluating a function:

- **<u>2ND</u> <u>ENTER</u> [ENTRY]** copies a previous command (that has not yet been deleted)
- Use ◀ to move the cursor over the 4 then change 4 to **<u>9</u>**
- **<u>ENTER</u>** re-evaluates the function



Using the table to evaluate a function (an alternate method):

- **2ND** **WINDOW** **[TBLSET]** opens options for the Table feature
- Use ▼ and ▶ to move the cursor over the word Ask
- **ENTER** sets up the Table to accept input values to be evaluated



- **2ND** **GRAPH [TABLE]** opens the Table. In the X column type **4** **ENTER** **9** **ENTER**.

- Use ▶ and ▼ to move the cursor over a y-value to view its full decimal expansion.

c. Find all input values corresponding to an output of 10. In other words, solve the equation $3e^t - 2t = 10$.

Using the equation solver to solve an equation:

- **MATH:SOLVER** (0: Solver or B: Solver) returns the solver

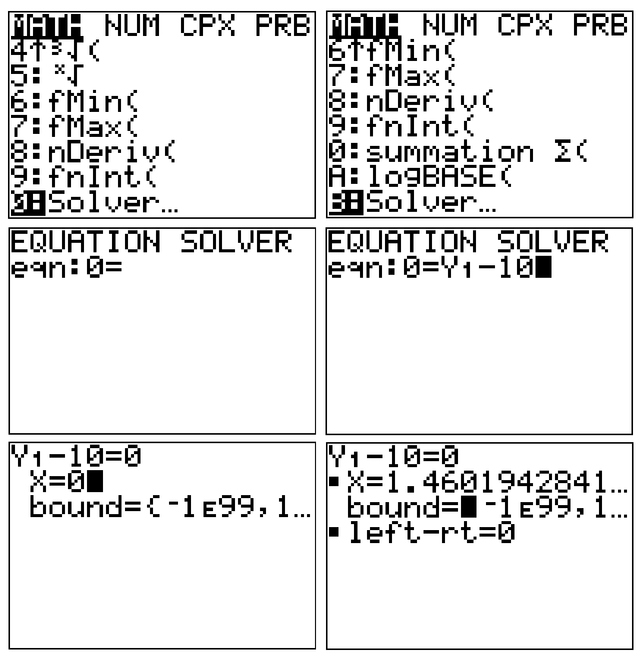
- ▲ (up arrow) returns the **EQUATION SOLVER** screen

- Complete the equation to be solved so that it reads 0 = Y1 – 10

- ▼ (down arrow) returns a new screen

- Position the cursor on the X = line (shown as X = 0 in the screenshot to the right)

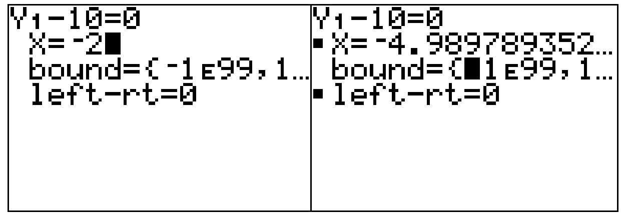
- **ALPHA** **ENTER** **[SOLVE]** returns a solution to the equation



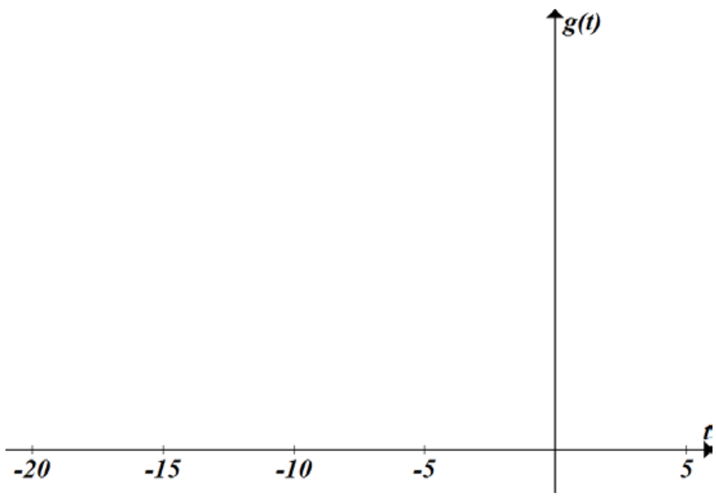
For an equation with only one solution, any value in the domain of the function will return the solution. For an equation with multiple solutions, the solver usually returns the solution that is closest to the initial x-value provided by the user, so several guesses may be necessary.

Finding a second solution:

- Use ▲ to return to the X = line and type a different *x*-value, such as **<u>-2</u>**
- **<u>ALPHA</u> <u>ENTER</u>** returns a second solution

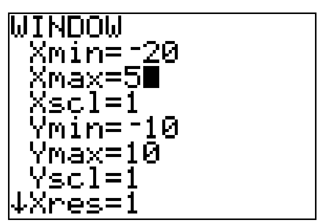


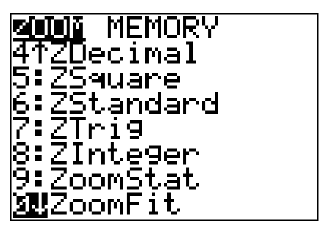
d. Sketch a graph of $g(t)=3e^{t}-2t$ on $-20\le t\le 5$.

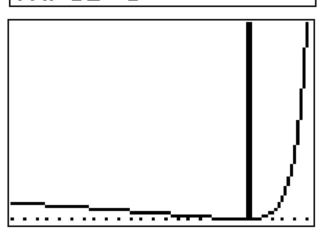


Graphing on a specific domain*:*

- Set the **<u>WINDOW</u>** (*x*-values only): Xmin = **<u>-20</u>** and Xmax = **<u>5</u>**
- **<u>ZOOM</u> <u>0</u>** (0:ZoomFit)





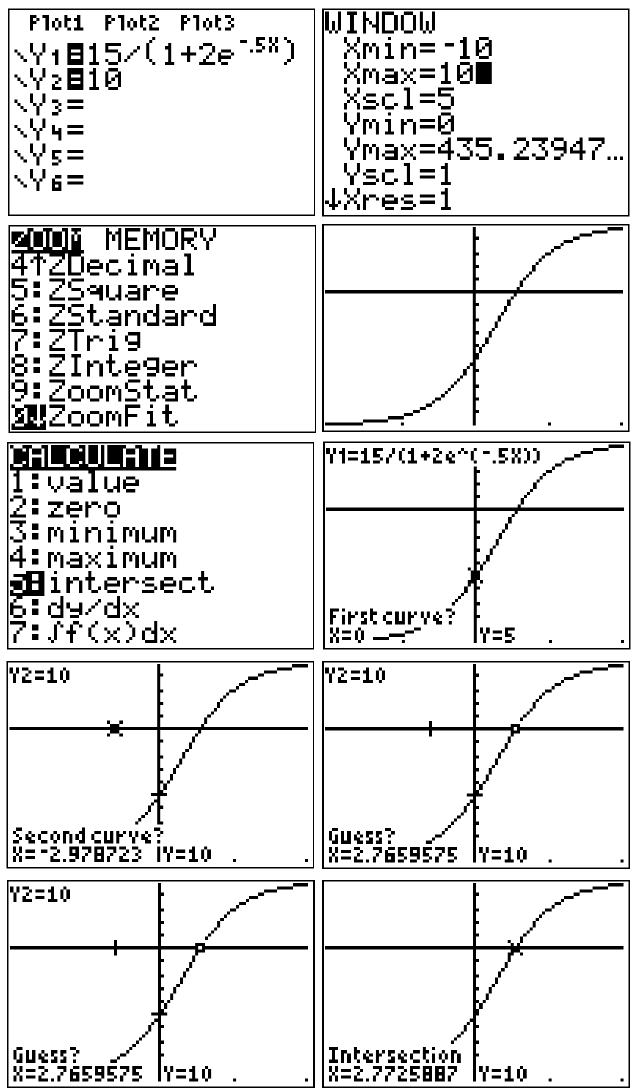


Example 5:

Consider $f(x)=\dfrac{15}{1+2e^{-0.5x}}$ on the interval $-10 \le x \le 10$. Solve for the input value that corresponds to output value $f(x)=10$.

Using a graph and the intersect function to solve an equation (an alternate method):

- In the Y= list, enter $f(x)$ in Y1 and the output value in Y2
- **WINDOW**
- Xmin=**-10** and Xmax=**10**
- **ZOOM** **0** [ZoomFit] returns the graph
- **2ND** **TRACE** **5** [intersect]
- Place the cursor on one of the functions and hit **ENTER**
- Place the cursor on the other function and hit **ENTER**
- Use the ▸ to move the cursor to the intersection point and hit **ENTER**
- **2ND** **MODE** [QUIT] to return to the home screen



Example 6: (CC5e pp. 5-6)

$p(t) = 80(1.013^t)$ million people gives the US population, where *t* is the number of years since the end of 1900, between the years 1900 and 2000.

a. Calculate the values of $p(t)$ for the given values of *t*. Report the answers correct to three decimal places.

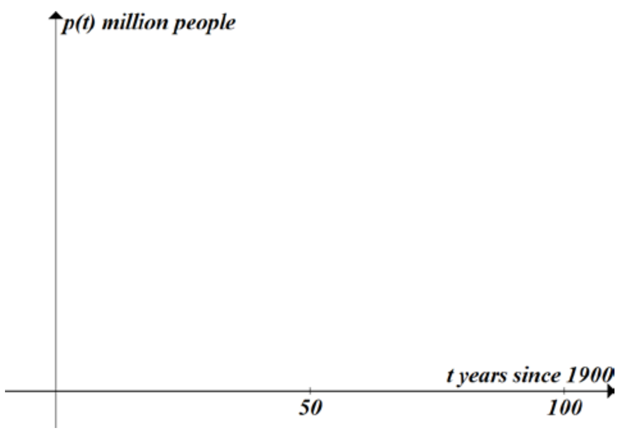
t years since 1900	0	20	40	60	80	100
$p(t)$ million people						

b. Use *p* to calculate the U.S. population at the end of 1945. Include units with the answer.

c. Write the answer to part *b* using *function notation*.

d. Write a sentence of interpretation for parts *b* and *c*.

e. Sketch a graph of p on the interval $0 \le t \le 100$. Note the variable and unit of measure on each axis.



f. According to the function *p*, in what year did the U.S. population reached 250,000,000 people?

Example 7: (CC5e p. 10, Activity 17)

Consider the function $t(n)=15e^{0.5n}-5n^2$. If an input value is given, find its corresponding output value. If an output value is given, find its corresponding input value(s). Round answers to three decimal places.

a. $n=3$ $t(3)=$ ______________________________

b. $n=0.2$ $t(0.2)=$ ______________________________

c. $t(n)=200$ $n=$ ______________________________

Example 8: (CC5e p. 10, Activity 25)

Consider the function $t(n)=\dfrac{15}{1+2e^{-0.5n}}$. If an input value is given, find its corresponding output value. If an output value is given, find its corresponding input value(s). Round answers to three decimal places.

a. $n=-2.5$ $t(-2.5)=$ ________________________

b. $t(n)=7.5$ $n=$ ______________________________

c. $t(n)=1.8$ $n=$ ______________________________

Example 9: (CC5e p. 10, Activity 31)

Consider the function $g(x)=4x^2+32x-13$. If an input value is given, find its corresponding output value. If an output value is given, find its corresponding input value(s). Round answers to three decimal places.

a. $x=-3$ $g(-3)=$ ______________________________

b. $g(x)=247$ $x=$ ______________ or ______________