Test 1 Review

(1) It is very important to be able to work with interval notation. Recall that for real numbers $a, b \in \mathbb{R} \cup \{\pm\infty\}$ such that $a < b$, we define

$$\begin{aligned}
(a,b) &= \{x \in \mathbb{R} \mid a < x < b\} \\
(a,b] &= \{x \in \mathbb{R} \mid a < x \leq b\} \\
[a,b) &= \{x \in \mathbb{R} \mid a \leq x < b\} \\
[a,b] &= \{x \in \mathbb{R} \mid a \leq x \leq b\}.
\end{aligned}$$

Recall that $\cap$ denotes the operation of taking intersections of sets and $\cup$ denotes operation of taking unions of sets. Thus for example, we have

$$(2,4) \cap (3,7) = \{x \in \mathbb{R} \mid 2 < x < 4 \text{ and } 3 < x < 7\} = (3,4)$$

$$(2,4) \cup (3,7) = \{x \in \mathbb{R} \mid 2 < x < 4 \text{ or } 3 < x < 7\} = (2,7)$$

If possible, try to simplify the following sets. If you cannot simplify them, then just write "already in simplified form".

$$\begin{aligned}
(2,3] \cap (3,4) &= (2,4) \\
(2,3) \cap (3,4) &= \\
(-\infty,3) \cup (-3,2) &= \\
[4,8) \cap (4,8] &= \\
(-3,3] \cup (3,8) &= \\
(-3,3] \cup [3,8) &=
\end{aligned}$$

(2) Consider the function $f(x) = 1/\sqrt{x-1}$. State the domain and range of $f(x)$.

(3) Recall the trig identities

$$\begin{aligned}
\cos(a+b) &= \cos a \cos b - \sin a \sin b \\
\sin(a+b) &= \sin a \cos b + \sin b \cos a
\end{aligned}$$

By setting $a = b$, use the identities above to derive the double angle formulas

$$\begin{aligned}
\cos(2a) &= \\
\sin(2a) &=
\end{aligned}$$

Note that you should use the formula

$$\cos^2 a + \sin^2 a = 1$$

to simplify what you get in $\cos(2a)$. Now use the double angle formulas to derive the half angle formulas

$$\begin{aligned}
\cos(a/2) &= \\
\sin(a/2) &=
\end{aligned}$$

Use the half angle formula for $\sin x$ to find the value of $\sin(\pi/12)$. Then use the double angle formula to find the value of $\sin(7\pi/12)$ (hint write $7\pi/12 = \pi/12 + \pi/2$).

(4) Given that $\sin\theta = 5/13$ and θ is in the second quadrant $(\pi/2 < \theta < \pi)$, find the exact value of $\tan\theta$.

(5) Find all solutions to the system of equations given by

$$x^2 + y^2 = 1$$
$$-3x^2 + y = -2$$

Hint: plug in $y = 3x^2 - 2$ into the first equation, and you should get a quartic polynomial of the form

$$ax^4 + bx^2 + c = 0, \tag{1}$$

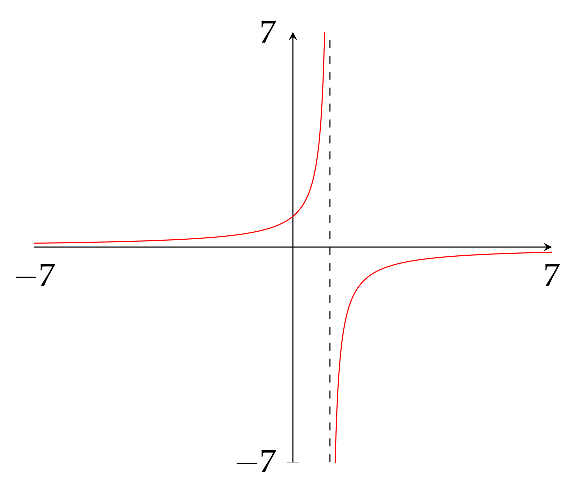
where you need to figure out what a, b, c. Once you have that, rewrite (1) as

$$au^2 + bu + c = 0$$

where $u = x^2$. Now you can use the *quadratic formula* to determine what u needs to be. Then you can determine what $x = \pm\sqrt{u}$ needs to be.

(6) Factor the polynomial $x^4 + 2x^2 + 1$ as much as you can.

(7) Which of the following is a correct equation for the graph below?



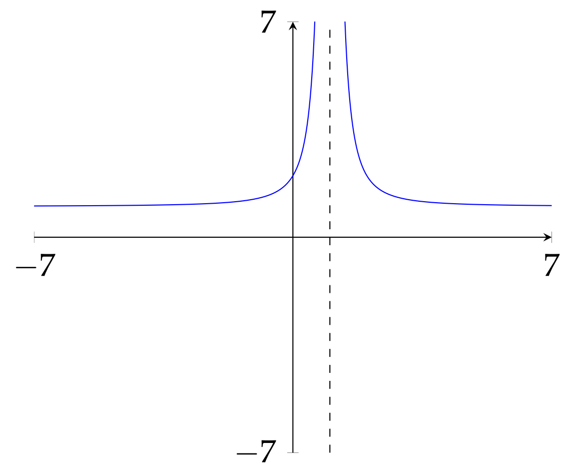
a) $y = 1/x$

b) $y = 1/(1-x)$

c) $y = 1/x^2$

d) $y = 1/(1-x)^2$

(7) Which of the following is a correct equation for the graph below?



a) $y = x^{-1} + 1$

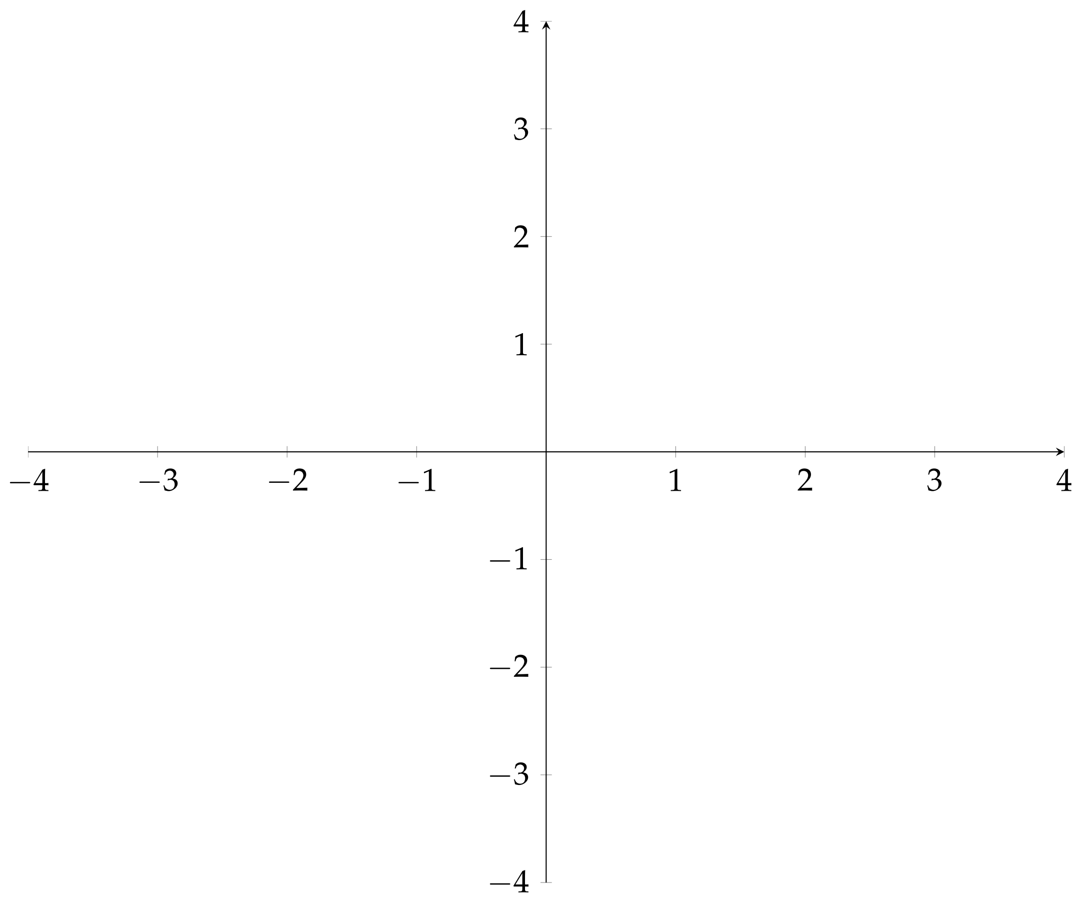
b) $y = x^{-2} + 1$

c) $y = (2-x)^{-1}$

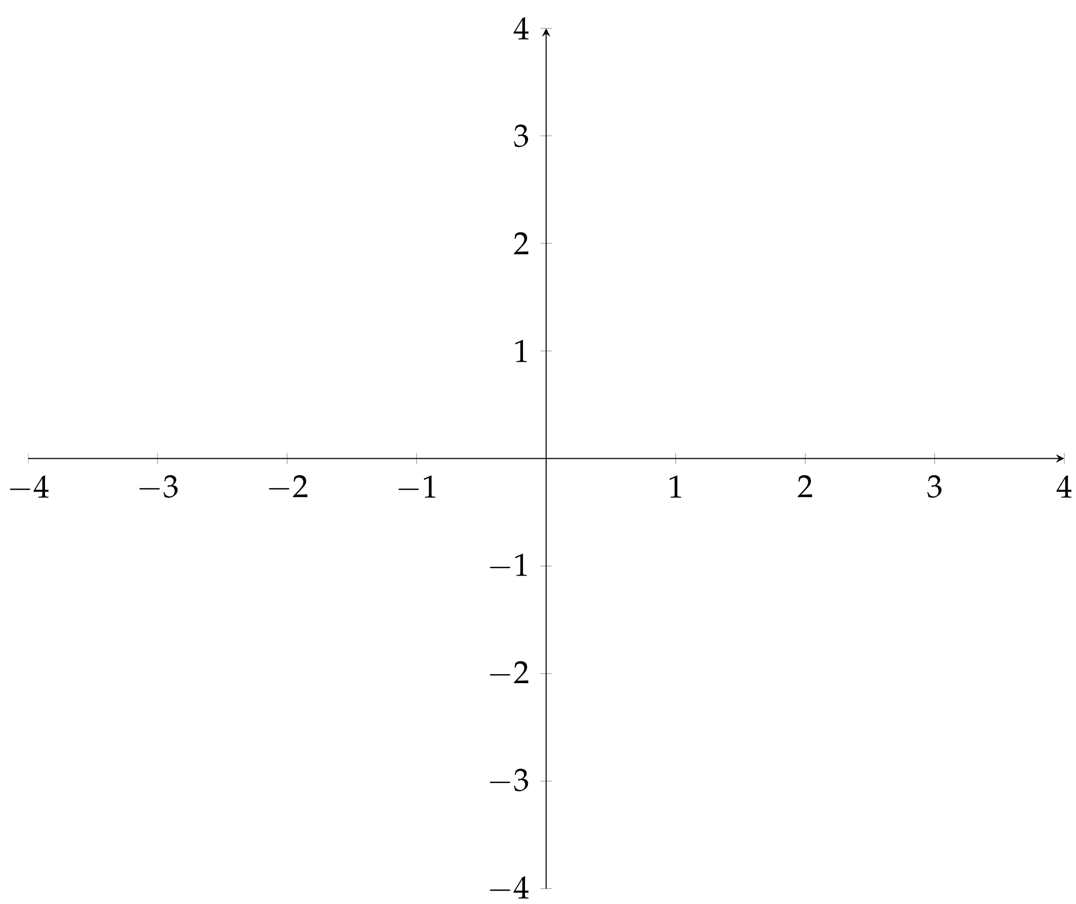
d) $y = (1-x)^{-1} + 1$

e) $y = (1-x)^{-2} + 1$

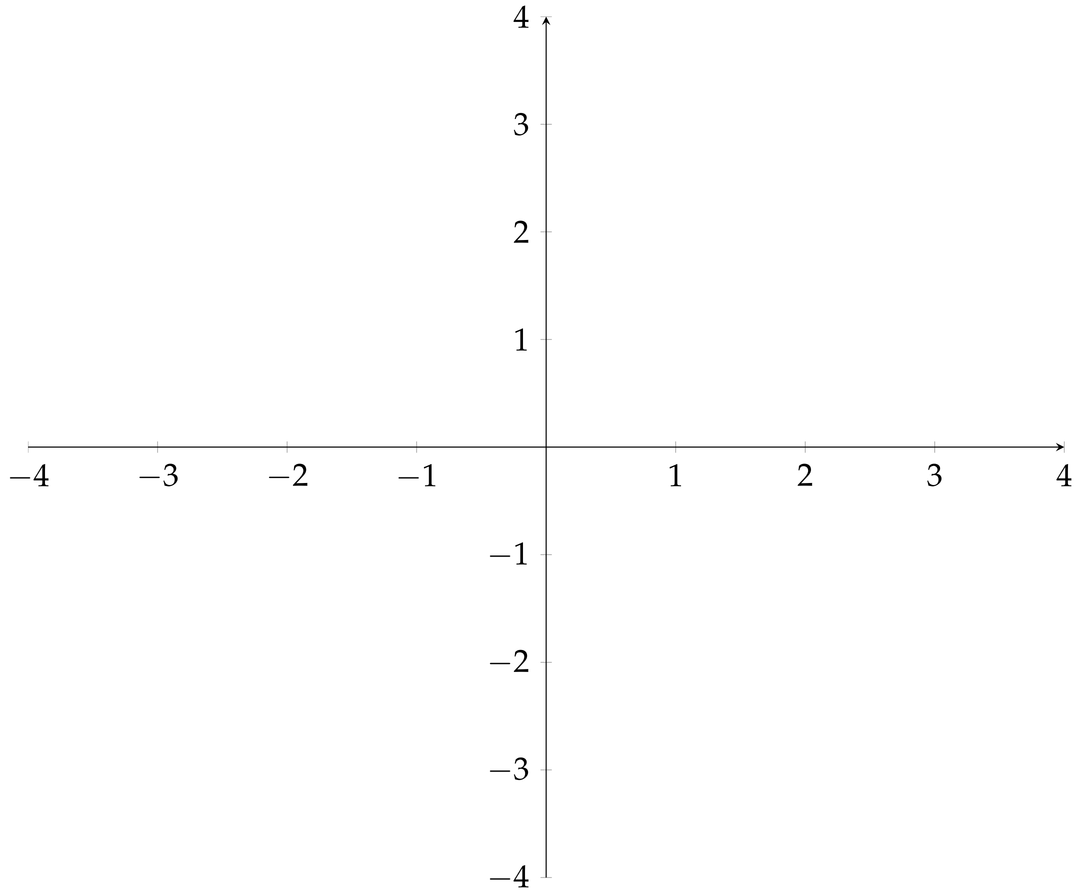
(8) You should definitinely know what the graphs of $\sin x$, $\cos x$, and $\tan x$ look like. You should also know what $\csc x$, $\sec x$, and $\cot x$ look like as well. For this problem, let's focus on $\sin x$. First graph $\sin x$ below



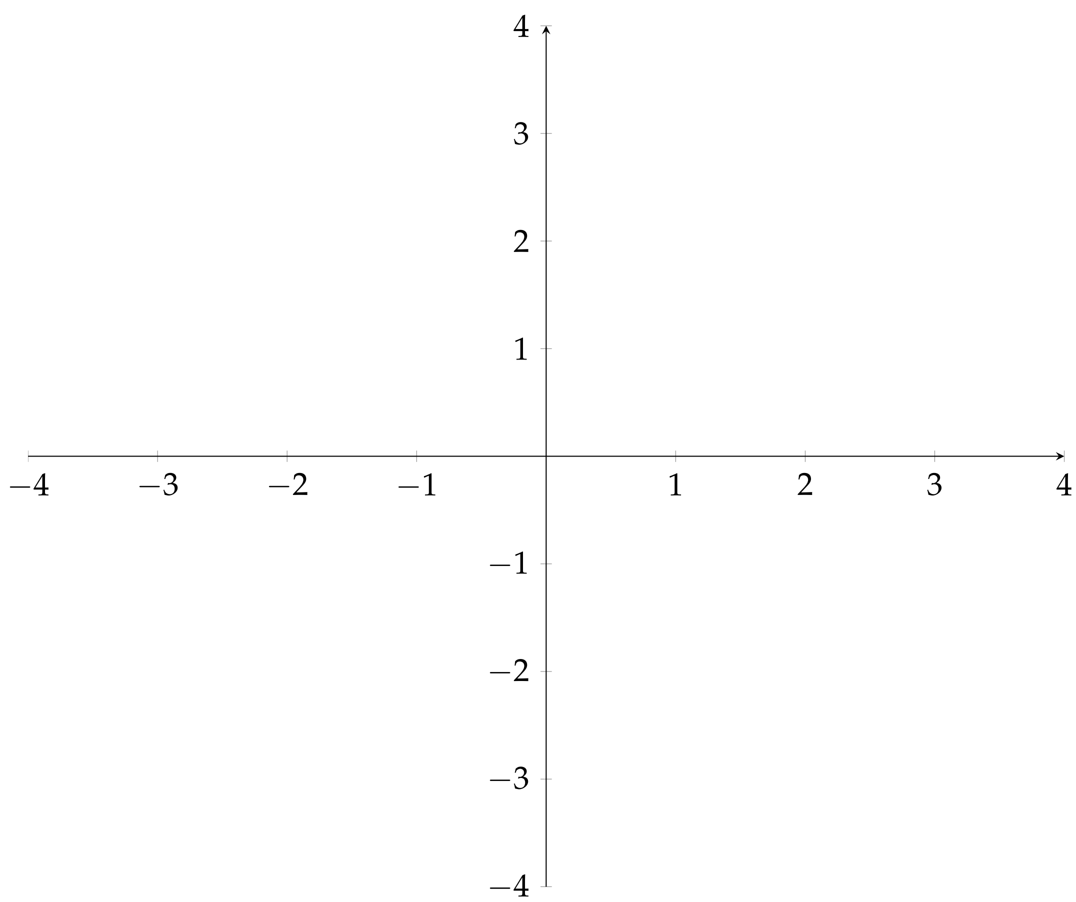
Next graph $3 \sin x$ below



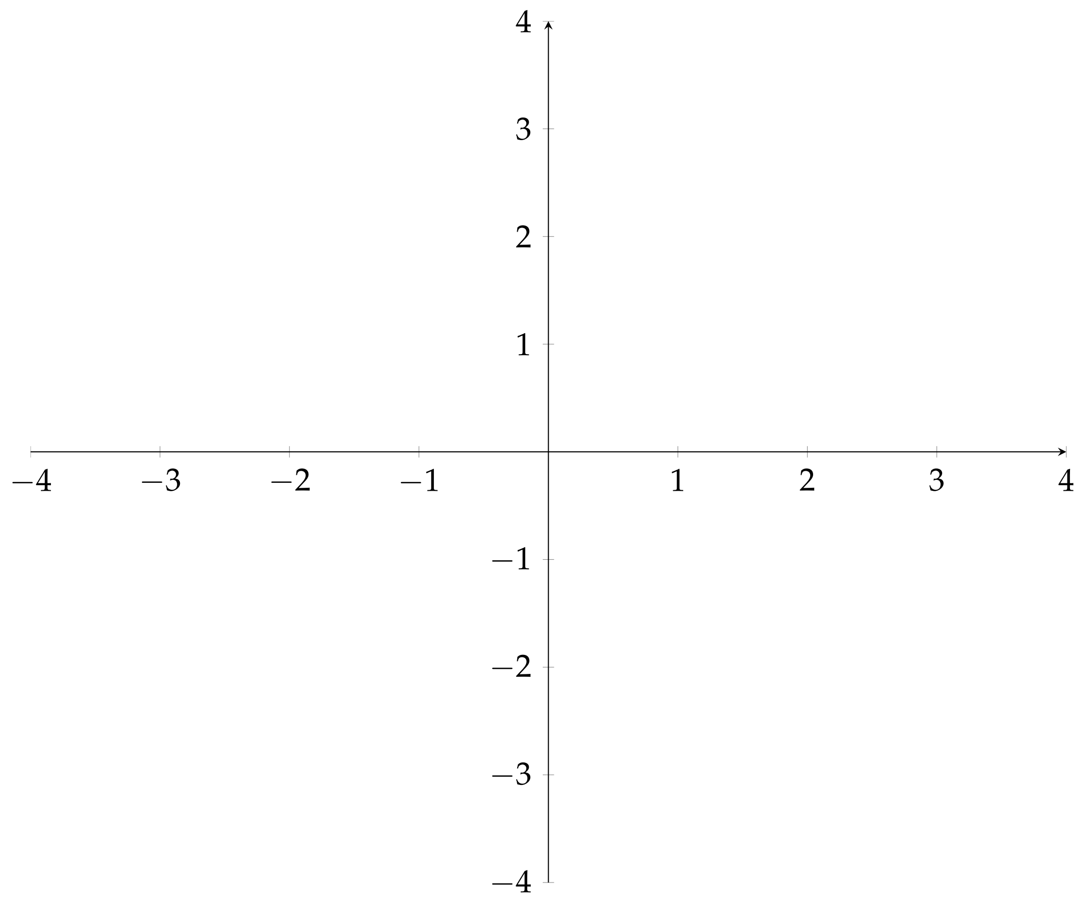
How does the 3 change the graph of $\sin x$? Next graph $-3 \sin x$ below



How does the negative sign change the graph of $3\sin x$? Next graph $-3\sin(2x)$ below



How does the 2 change the graph of $-3\sin(x)$? Finally, graph $-3\sin(2(x+1))$ below



How does the $x + 1$ change the graph of $-3\sin(2x)$? Finally, what is the amplitude, period, and phase shift of $-3\sin(2(x+1))$. You should be able to identify these as this will be on the test.

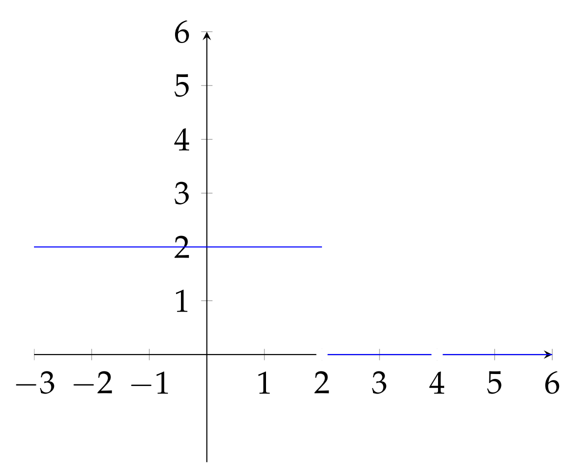
(9) Express the function $f(x) = |x - 2|$ as a piecewise function:

$$|x-2| = \begin{cases} & \text{if } x \\ & \text{if } x \end{cases}$$

(10) Convert 310° to radians (just multiply 310 by $\pi/180$). Convert $2\pi/7$ radians to degrees (plug in $\pi = 180$).

(11) Simplify $\sqrt{x^4 + 2x^2}$.

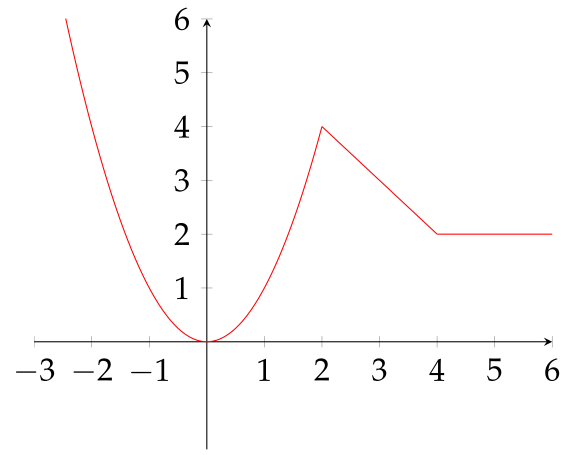
(12) Let $f(x)$ be the function whose graph is given below.



Express the function $f(x)$ as a peicewise function. Note that $f(x)$ is not defined at $x = 2$ nor $x = 4$!

$$f(x) = \begin{cases} & \text{if } x \in (-\infty, 2) \\ & \text{if } x \in \\ & \text{if } x \in \end{cases}$$

(13) Let $f(x)$ be the function whose graph is given below.



Express the function $f(x)$ as a peicewise function. Note that $f(x)$ is not defined at $x = 2$ nor $x = 4$!

$$f(x) = \begin{cases} x^2 & \text{if } x \in (-\infty, 2) \\ & \text{if } x \in \\ & \text{if } x \in \end{cases}$$

(14) Suppose $f(x) = x^2 + ax + b + y^2 + cy + d$ where $a, b, c, d \in \mathbb{R}$ (we are working in a general context). Recall that the set $\{x \in \mathbb{R} \mid f(x) = 0\}$ forms a circle in the plane. In this problem, you need to determine where the circle is centered and what its radius is. Let me help you get started: you need to rewrite $f(x)$ as

$$x^2 + ax + b + y^2 + cy + d = (x - \alpha)^2 + (x - \beta)^2 - r^2. \tag{2}$$

You need to determine what α, β, and r are in terms of the a, b, c, d. In particular, you should expand the righthand side of (2) and compare coefficients. If you do this correctly, you should get $a = -2\alpha$ for example, or in other words, $\alpha = -a/2$. Once you solve α, β, and r in terms of a, b, c, d, you'll be able to state where the center of this circle is ((α, β)) and what its radius is (r).

(15) Find the equation that is perpendicular to the line $8x + 3y = 2$ which passes through the point $(1, 1)$.

(16) Simplify the following expressions

$$\frac{x^2 - x}{\sqrt{x^4 - x^2}} =$$

$$\frac{1}{1 - x} + \frac{1}{1 + x} =$$

$$\frac{3x^2 + 2}{xyz} - \frac{2x - 1}{x^2 y} =$$