Homework #4

Due Monday (October 18th)

This homework is designed to be used as a study guide for the upcoming test. The questions you see in this homework will be closely related to the questions you will see on the test. I am grading this homework based on your effort alone. On Monday, I will post the solutions to this homework on canvas, so that you can compare your answers to the solutions I give.

Sentences of Interpretations

(1) $T(x)$ degrees Fahrenheit gives the temperature of an oven x minutes after it's been turned.

(1.a) Write a sentence of interpretation for $\frac{dT}{dx}|_{x=2} = 26$.

(1.b) Write a sentence of interpretation for $T(2) = 120$.

(1.c) The table below gives the temperature of the oven at various minutes after it's been turned on.

x minutes	$T(x)$ degree Fahrenheit
0	70
2	120
5	200
9	400

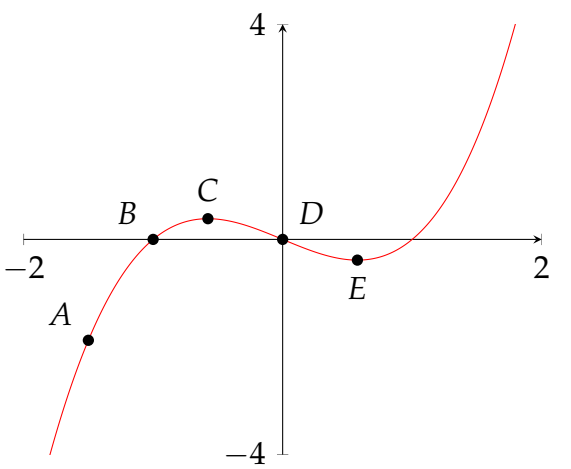
Write a sentence of interpretation of the average rate of change of $T(x)$ between 2 minutes and 9 minutes.

(1.d) Write a sentence of interpretation for the percent change of $T(x)$ between 2 minutes and 9 minutes.

(1.e) Write a sentence of interpretation for the percent rate of change of $T(x)$ at $x = 2$.

Slope Graphs

(2) Let $f(x)$ be the function whose graph is given below:



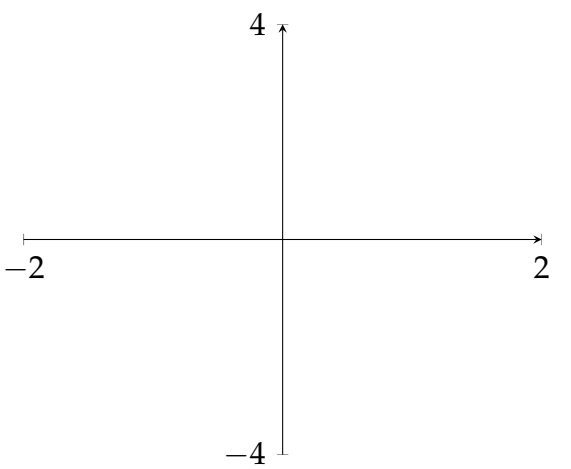
(2.a) List the points A, B, C, D, E from least to greatest steepness.

(2.b) List the points A, B, C, D, E from least to greatest slope.

(2.c) How many x-intercepts will the slope graph of $f(x)$ have? Which points out of A, B, C, D, E gives us this information.

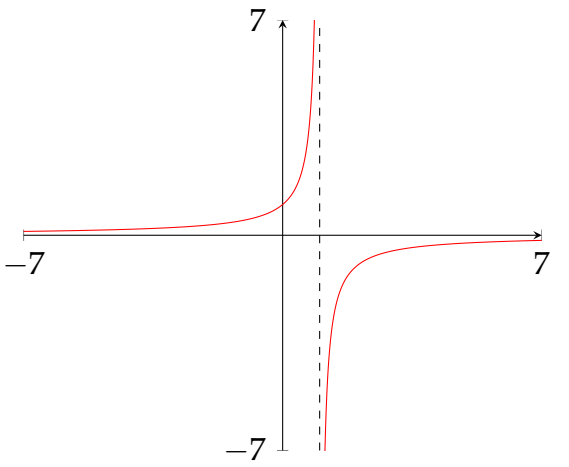
(2.d) How many relative mins/maxes will the slope graph of $f(x)$ have? Which points out of A, B, C, D, E gives us this information.

(2.e) Sketch the slope graph below

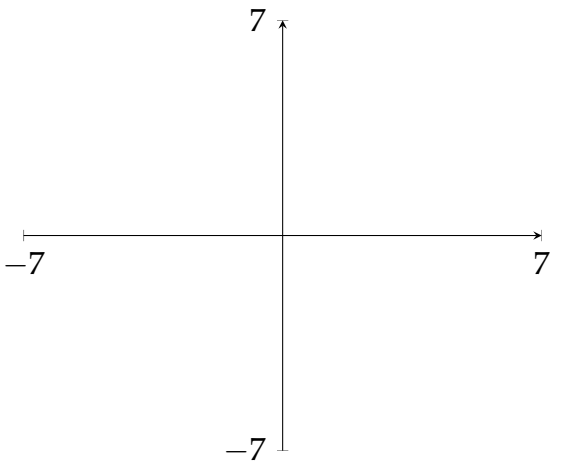


(2.f) Let $g(x)$ be a function and let a be a real number (say $a = 0$ if you want). List three reasons why $g'(a)$ may not exist.

(2.g) Let $h(x)$ be the function whose graph is given below



Give a sketch of the slope graph of $h(x)$. (You don't need to be completely accurate here. Just draw tangent lines of various points on the graph of $h(x)$. Are the slopes of these tangent lines positive/negative? If positive, then plot a positive output at the same input in the slope graph of $h(x)$)



Compounding Interest

(3.a) How much will a $100 investment, compounded quarterly at 5% APR yield in 10 months? How long will it take to double your investment?

(3.b) How much will a $100 investment, compounded continuously at 5% APR yield in 10 months? How long will it take to double your investment?

(3.c) What is the APY for an investment that pays 3% APR compounded quarterly? *(*If you need to, go to section 1.6 in the book and look up the formula for APY. This type of question will be on the test, so make sure you know it.)

(3.d) What is the APY for an investment that pays 3% APR compounded continuously? (The formula for APY is different for continuous compounding than the one for quarterly compounding. Again, look up the formula in section 1.6 if you don't know it.)

Algebra

(4.a) Let a, b be real numbers (i.e. constants). Simplify the following expressions. You need to write each as $x^{\text{something}}$ (I'll do the first for you).

$$\frac{1}{x^a} = x^{-a}$$

$$x^a x^b =$$

$$(x^a)^b =$$

$$\frac{1}{x^a} =$$

$$\sqrt{x} =$$

$$\sqrt[3]{x} =$$

$$\frac{1}{\sqrt[3]{x^2}} =$$

(4.b) It's important to keep track of notation. Write three ways of expressing (in terms of notation, i.e. symbols) the derivative of a function $f(x)$. Then write two ways of expressing the derivative of a function $f(x)$ *evaluated* at a number a.

(4.c) Let $f(x)$ and $g(x)$ be functions and let a and b be real numbers (i.e. constants). Complete the expressions below (I'll do the first two for you).

$$\frac{d}{dx}(a) = 0$$

$$\frac{d}{dx}(af(x)) = a\frac{d}{dx}(f(x))$$

$$\frac{d}{dx}(f(x) + g(x)) =$$

$$\frac{d}{dx}(x^a) =$$

$$\frac{d}{dx}(a^x) =$$

$$\frac{d}{dx}(e^x) =$$

$$\frac{d}{dx}(\ln x) =$$

(4.d) Find the derivative of the following functions (I'll do the first one for you)

$$p(x) = x^3 + 3x^2$$

$$\begin{aligned} p'(x) &= \frac{d}{dx}(p(x)) \\ &= \frac{d}{dx}(x^3 + 3x^2) \\ &= \frac{d}{dx}(x^3) + \frac{d}{dx}(3x^2) \\ &= \frac{d}{dx}(x^3) + 3\frac{d}{dx}(x^2) \\ &= 3x^{3-1} + 3 \cdot 2x^{2-1} \\ &= 3x^2 + 6x \end{aligned}$$

$$h(t) = t^2 + 5\sqrt{t} + \frac{2}{t^3}$$

$$\begin{aligned} h'(t) &= \frac{d}{dt}(h(t)) \\ &= \end{aligned}$$

$$h(x) = \sqrt{\frac{1}{\sqrt[3]{x}} + \sqrt{x^3}}$$

$$\begin{aligned} h'(x) &= \frac{d}{dx}(h(x)) \\ &= \end{aligned}$$

$$g(t) = \frac{3t^3 - t^2 + 1}{t}$$

$$\begin{aligned} g'(t) &= \frac{d}{dt}(g(t)) \\ &= \end{aligned}$$

$$g(x) = 2\ln(x) + 3^x$$

$$\begin{aligned} g'(x) &= \frac{d}{dx}(g(x)) \\ &= \end{aligned}$$

$$f(x) = 5e^x + e^e$$

$$\begin{aligned} f'(x) &= \frac{d}{dx}(f(x)) \\ &= \end{aligned}$$

(4.e) In the limit definition of the derivative, what does h represent? What does $\frac{f(x+h)-f(x)}{h}$ represent? What does $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ represent?