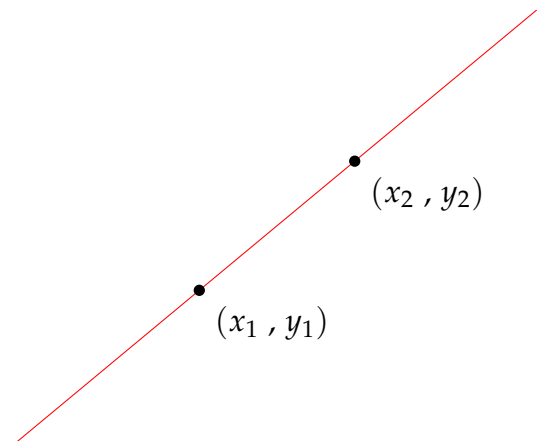
Homework #3

Due Wednesday (March 4th)

Name _____

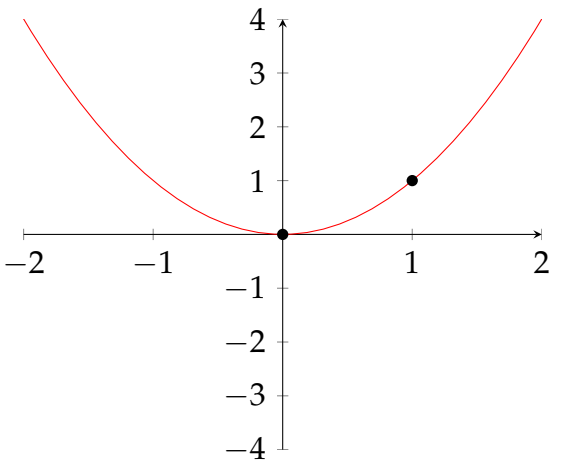
This homework will be graded out of 30 points based on effort alone. Of course you must put serious effort into each problem for you to be given full credit on that problem. I will post the solutions on Wednesday and we will go over them to review for the test.

1. Suppose a line passes through the points (x_1, y_1) and (x_2, y_2), as shown below:



What is the slope of this line?

2. Let $f(x)$ be the function given by the graph below:



2.a. Use a ruler to draw the tangent line to the graph of the function at the point $(1, 1)$. Find the slope of this line (Hint: You need to find two points this line goes through. Then you need to use the slope formula you wrote down above).

2.b. Use a ruler to draw the tangent line to the graph of the function at the point $(0,0)$. Find the slope of this line .

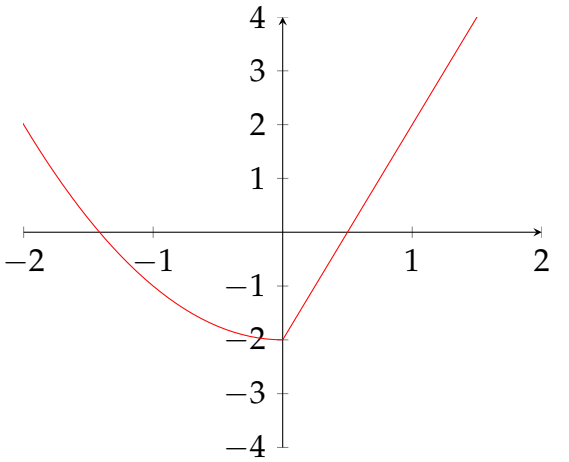
2.c. Use a ruler to draw the secant line through the points $(0,0)$ and $(1,1)$. Find the slope of this line (Hint: You are given two points which lie on this line, namely $(0,0)$ and $(1,1)$. This is all you need to calculate the slope of this line).

2.d. With the information above, evaluate

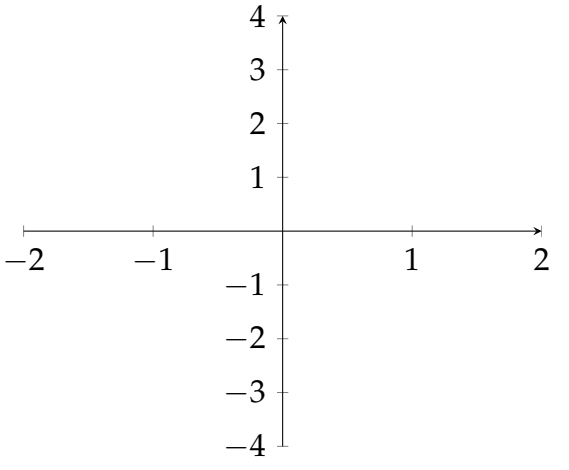
$$f(0) =$$
$$f'(0) =$$
$$f(1) =$$
$$f'(1) =$$

What is the average rate of change between the points $(0,0)$ and $(1,1)$?

3. Let $f(x)$ be the function given by the graph below:



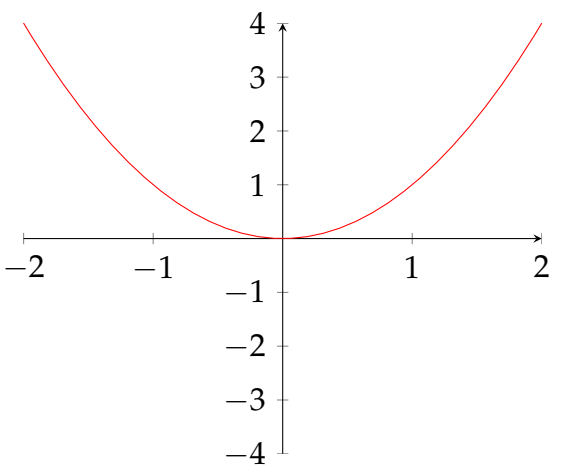
Sketch the graph of $f'(x)$:



(Hint: The function $f(x)$ goes through the point $(0.5,0)$. Find the tangent line to the graph of $f(x)$ at this point, then find the slope of this line (the slope of this line is by definition the value $f'(0.5)$). Now plot the point $(0.5, f'(0.5))$. The function also goes through the point $(1,2)$. Repeat the same procedure as above and plot the

point $(1, f'(1))$. The function also goes through the point $(-2, 2)$. Repeat the same procedure as above and plot the point $(-2, f'(-2))$. You should be able to construct $f'(x)$ from this).

4. The function in the graph below is given by $f(x) = x^2$.



From problem 2, we know what the slope of the tangent line at $(0, 0)$ is (i.e. $f'(0)$). In this exercise, we want to compute $f'(0)$ by taking the limit of slopes of secant lines[1]

4.a. Use a ruler to draw the secant line through the points $(0, 0)$ and $(2, 4)$. Find the slope of this line.

4.b. Use a ruler to draw the secant line through the points $(0, 0)$ and $(1, 1)$. Find the slope of this line.

4.c. Use a ruler to draw the secant line through the points $(0, 0)$ and $\left(\frac{1}{2}, \frac{1}{4}\right)$. Find the slope of this line.

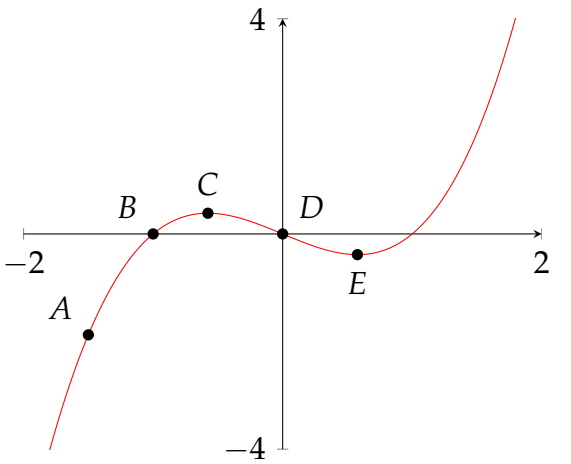
4.d Complete the table below

x	Slope of secant line from $(0, 0)$ to $(x, f(x))$
2	
1	
0.5	
0.1	

What does this table suggest? (Hint use $\lim_{x \to 0^+} f(x)$ notation).

[1] In Calculus, almost everything is defined in terms of limits.

5. Let $f(x)$ be the function whose graph is given below:



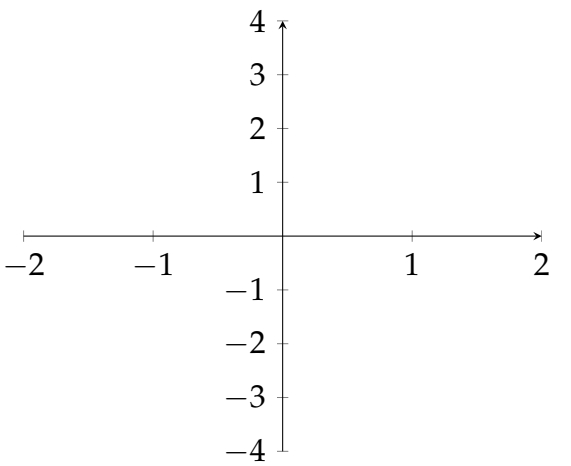
(5.a) List the points A, B, C, D, E from least to greatest steepness.

(5.b) List the points A, B, C, D, E from least to greatest slope.

(5.c) How many x-intercepts will the slope graph of $f(x)$ have? Which points out of A, B, C, D, E gives us this information.

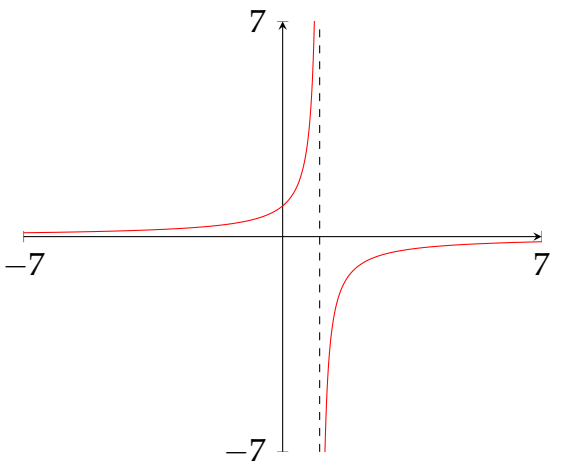
(5.d) How many relative mins/maxes will the slope graph of $f(x)$ have? Which points out of A, B, C, D, E gives us this information.

(5.e) Sketch the slope graph below

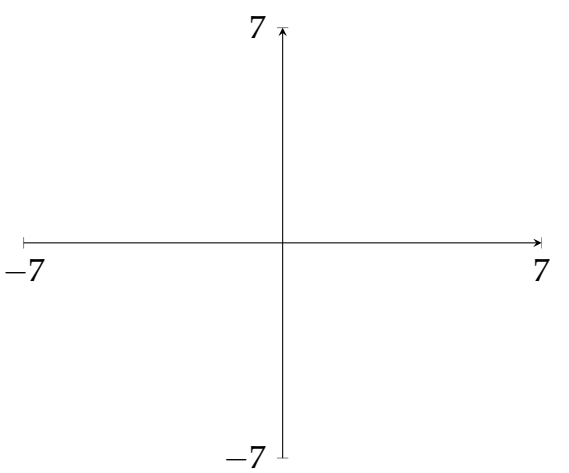


6. Let $g(x)$ be a function and let a be a real number. List three reasons why $g'(a)$ may not exist.

7. Let $h(x)$ be the function whose graph is given below



Give a sketch of the slope graph of $h(x)$. (You don't need to be completely accurate here. Just draw tangent lines of various points on the graph of $h(x)$. Are the slopes of these tangent lines positive/negative? If positive, then plot a positive output at the same input in the slope graph of $h(x)$)



8. Find the derivative of $f(x) = 2x^2 + 0.1x - 2$ using the *limit* definition. Show all steps.

9. Let a, b be real numbers (i.e. constants). Simplify the following expressions. You need to write each as

$x^{\text{something}}$ (I'll do the first for you).

$$\frac{1}{x^a} = x^{-a}$$

$$x^a x^b =$$

$$(x^a)^b =$$

$$\frac{1}{x^a} =$$

$$\sqrt{x} =$$

$$\sqrt[3]{x} =$$

$$\frac{1}{\sqrt[3]{x^2}} =$$

10. It's important to keep track of notation. Write three ways of expressing (in terms of notation, i.e. symbols) the derivative of a function $f(x)$. Then write two ways of expressing the derivative of a function $f(x)$ *evaluated* at a number a.

11. Let $f(x)$ and $g(x)$ be functions and let a and b be real numbers (i.e. constants). Complete the expressions below (I'll do the first two for you).

$$\frac{\mathrm{d}}{\mathrm{d}x}(a) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(af(x)) = a\frac{\mathrm{d}}{\mathrm{d}x}(f(x))$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(f(x)+g(x)) = \frac{\mathrm{d}}{\mathrm{d}x}(f(x)) + \frac{\mathrm{d}}{\mathrm{d}x}(g(x))$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^a) = ax^{a-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(a^x) = \ln(a)a^x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(e^x) = e^x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\ln x) = \frac{1}{x}$$

12. Find the derivative of the following functions (I'll do the first one for you)

$$p(x) = x^3 + 3x^2$$

$$\begin{aligned} p'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}(p(x)) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}(x^3 + 3x^2) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}(x^3) + \frac{\mathrm{d}}{\mathrm{d}x}(3x^2) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}(x^3) + 3\frac{\mathrm{d}}{\mathrm{d}x}(x^2) \\ &= 3x^{3-1} + 3 \cdot 2x^{2-1} \\ &= 3x^2 + 6x \end{aligned}$$

$$h(t) = t^2 + 5\sqrt{t} + \frac{2}{t^3}$$

$$h'(t) = \frac{\mathrm{d}}{\mathrm{d}t}(h(t))$$

$$h(x) = \sqrt{\frac{1}{\sqrt[3]{x}} + \sqrt{x^3}}$$

$$h'(x) = \frac{\mathrm{d}}{\mathrm{d}x}(h(x))$$

$$g(t) = \frac{3t^3 - t^2 + 1}{t}$$

$$g'(t) = \frac{\mathrm{d}}{\mathrm{d}t}(g(t))$$

$$g(x) = 2\ln x + 3^x$$

$$g'(x) = \frac{\mathrm{d}}{\mathrm{d}x}(g(x))$$

$$f(x) = 5e^x + e^e$$

$$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}(f(x))$$