Section 1.6: Models in Finance

The **future value** of an investment (or a loan) is given by $F(t)$, where t represents the amount of time since the investment was made (or the loan was taken), in years. P represents the **present value** (called the *principal* of an investment, or *face value* of a loan). The decimal form r of the **annual percentage rate (APR)** is called the **nominal rate** of interest.

Simple Interest

- The *simple interest, I*, accumulated after t years at an interest rate r (in decimal form) on a present value of P dollars is calculated as
$$I(t) = Prt \text{ dollars}$$
- The *future value* in t years, is obtained by adding the interest to the present value
$$F_s(t) = P + Prt = P(1+rt) \text{ dollars}$$

Compound Interest Investments (*n* times per year)

- *Compounding* of interest occurs when interest is earned (or charged) on previous interest.

- The *future value* in t years of an investment (or loan) with present value of P dollars is calculated as $F_c(t) = P\left(1+\frac{r}{n}\right)^{nt}$ dollars, where n is the number of compoundings per year, nt is the total number of compounding periods in t years and r is the nominal rate of interest (in decimal form).

Continuously Compound Interest Investments

- *Continuous compound interest* occurs when the number of compoundings in a year, *n,* is allowed to increase without bound.

- When interest is compounded continuously, the *future value* in t years of an investment (or loan) with present value of P dollars is $F_c(t) = Pe^{rt}$ dollars, where r is the nominal rate of interest (in decimal form) compounded continuously.

Example 1:

Suppose $1,000 is invested at 4.2% APR.

a. What is the (future) value of the investment after 19 months, if interest is **compounded quarterly**?

i. Choose the appropriate interest formula to use in this situation: *simple interest, compound interest n times per year, or continuously compound interest.*

ii. Write the formula for $F(t)$, using the given information.

- $P =$ $r =$ $n =$

- $F(t) =$

iii. The shaded cells in the table show the months in which interest is calculated when compounding quarterly. To find the value of the investment after 19 months, one must evaluate the function after __________________ months.

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24

Find the future value of the investment after 19 months.

b. What is the (future) value of the investment after 19 months, if interest is **compounded continuously**?

i. Choose the appropriate interest formula to use in this situation: *simple interest, compound interest n times per year, or continuously compound interest.*

ii. Write the formula for $F(t)$, using the given information.

- $P =$ $r =$

- $F(t) =$

iii. Find the value of the investment after 19 months.

Example 2:

Suppose $1,000 is invested at 4.2% APR.

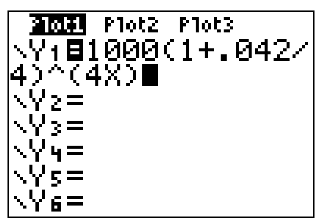
a. Find the time it takes the investment to double, if interest is compounded quarterly.

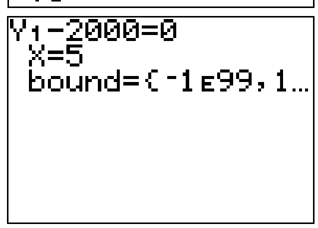
i. Write an equation to be solved for *t.*

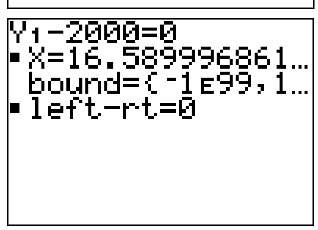
ii. Solve the above equation using the calculator's Math Solver or the calculator's Table. To interpret the result, consider the months in which compounding occurs. Answer by stating the number of years and months it takes the investment to reach double its present value.

Using the solver to find the doubling time of an investment:

- **Y= 1000 (1 + .042 /4) ^ (4X)** enters the right side of the doubling time equation for the investment into Y_1
- **MATH 0 [SOLVER]** OR **MATH B [SOLVER]** accesses the Equation Solver
- Complete the equation to be solved as 0 = **Y_1 – 2000**
- Enter a guess for X, the number of years it will take the investment to double (any positive number will work).
- With your cursor in the X = row, hit **ALPHA ENTER** [SOLVE] to solve.
- Don't forget to consider how often interest is compounded before answering the question.







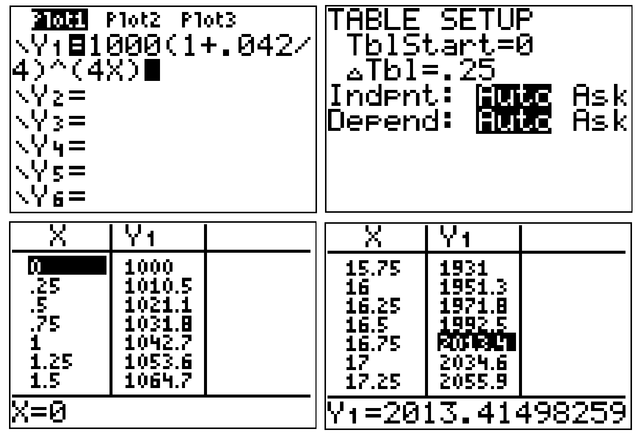
Using the <u>table</u> to find the doubling time of an investment:

- **<u>Y=</u> <u>1000</u> <u>(1</u> <u>+</u> <u>.042</u> <u>/4)</u> <u>^</u> <u>(4X)</u>** enters the right side of the doubling time equation for the investment into Y1

- **<u>2ND</u> <u>WINDOW</u>** [TBLSET] accesses the Table Setup; Use the options as shown in the 2nd screen shot (Note: Δ Tbl = .25 because interest is compounded quarterly)

- **<u>2ND</u> <u>GRAPH</u>** [TABLE]

- Hold down ▼ to scroll down the X column until the value in the Y1 column is at least 2000.



b. Find the time it takes the investment to double, if interest is compounded continuously.

i. Write an equation to be solved for *t.*

ii. Solve the equation using the calculator's Math Solver. (The calculator's TABLE is not useful in this case.) Answer in years. Round the answer to three decimal places.

Annual percentage yield (APY) is the percentage change in the value of an investment (or loan) over a one-year period. It is used to compare investments or loans.
The decimal form of the APY is called the **effective rate** of interest.

For compounding *n times each year,* $APY = \left(\left(1 + \frac{r}{n} \right)^n - 1 \right) \cdot 100\%$.

For compounding *continuously,* $APY = \left(e^r - 1 \right) \cdot 100\%$.

Example 3:

Compare two investments, one offering 4.2% APR compounded quarterly, and another offering 4.15% APR compounded continuously.

a. Find the APY of a $1000 investment with 4.2% APR compounded quarterly.

b. Find the APY of a $1000 investment, with 4.15% APR compounded continuously.

c. Which is the better investment?

d. Does the answer to part c change if the principal is $10,000? Why or why not?

Example 4: (CC5e p. 60)

An investment at 2.8% APR compounded quarterly has a future value of $5000 payable in five years. Find the present value, the amount that must be invested into this account today to obtain $5000 in 5 years.

a. Write the equation that can be used to solve for P.

b. Solve the equation using the calculator's Math Solver. Include units with your answer.

Example 5: (CC5e p. 61)

Suppose $50,000 is invested at 5% APR. Find the value of the investment after 23 months in various situations. Then find the time it takes the investment to double.

Compounding	Formula	Value of t	Future Value	Doubling Time
Annually				___yrs___mos
Semiannually				___yrs___mos
Quarterly				___yrs___mos
Monthly				___yrs___mos
Continuously				__________ yrs

Example 6: (CC5e p.59)

A student borrows $1000 at an APR of 4% compounded monthly. What is the future value of the loan if the borrower repays the loan after 1 year? 2 years? 3 years? 4 years? 5 years?

a. Write the formula for $F(t)$, using the given information.

- $P =$ $r =$ $n =$

- $F(t) =$

b. Find the future value of the loan at the various times indicated in the table.

Year	0	1	2	3	4	5
Value	$1000					

Example 7: (CC5e p. 61)

Which is better for a borrower, a loan with an APR of 7.2% compounded monthly or a loan with an APY of 7.4%? Compare the effective rates for a one-year loan. Assume all other conditions are equal.

	APR	APY
Loan 1	7.2% compounded monthly	
Loan 2	------------	7.4%

Example 8: (CC5e p. 63)

Consider two investment offers: an APR of 6.9% compounded quarterly (Investment A) or an APR of 6.7% compounded monthly (Investment B).

a. Determine the better investment by calculating the effective rate (APY) for each.

- Investment A

- Investment B

b. Compare the time it will take an investment to double for each offer. Which investment doubles more quickly and is therefore the better investment?

- Investment A

- Investment B