Section 1.2: Function Behavior and End Behavior Limits

The **direction** of a function is described as:
Increasing if output values increase as input values increase,
Decreasing if output values decrease as input values increase, and
Constant if output values remain the same as input values increase.

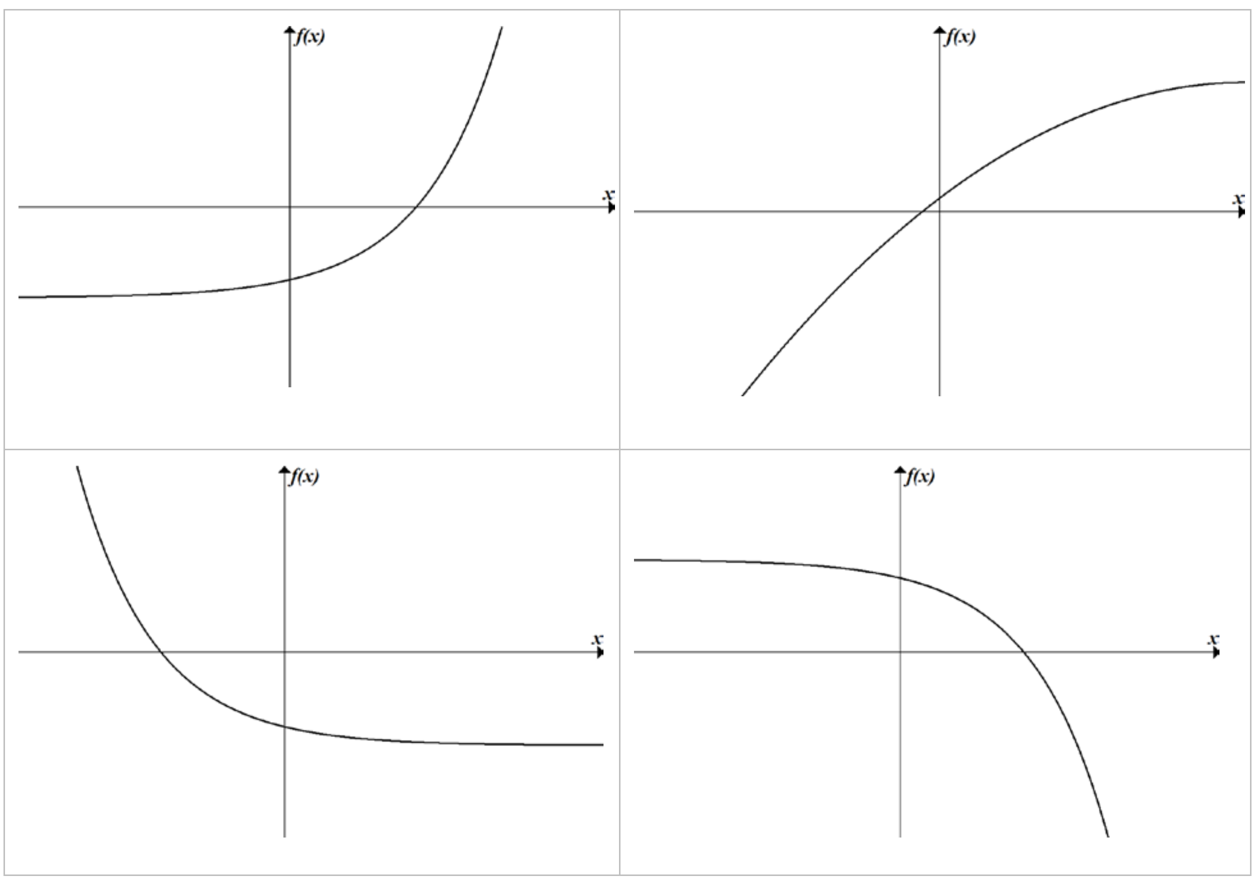
The **curvature** of a function is described as **concave up** on an interval where the graph appears as a portion of an arc that opens upward. The curvature of a function is described as **concave down** on an interval where the graph appears as a portion of an arc opening downward. A line has no concavity.

Any point on a continuous function where the concavity changes is called an **inflection point**.

Example 1:

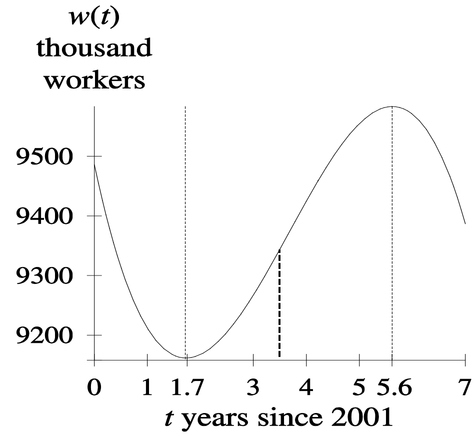
Identify each function as *increasing, decreasing, or constant* on the given interval.

Identify each function as *concave up or concave down* on the given interval.



Example 2: (CC5e p. 15)

The figure shows the graph of function w that models the number of 20- to 24-year-olds employed full time for years between 2001 and 2008.



a. State the interval(s) on which w is increasing.

b. State the interval(s) on which w is decreasing.

c. State the interval(s) on which w is concave up.

d. State the interval(s) on which w is concave down.

e. At what input value does w have an inflection point?

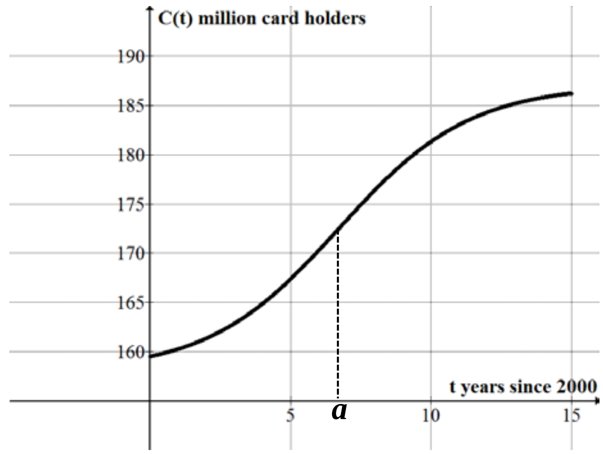
The **end behavior** of a function describes output values of a function as input values either increase or decrease without bound. It can be estimated by evaluating the function at increasingly large or decreasingly small input values. This process is called **numerical estimation**.

The notation $\lim\limits_{x\to\pm\infty} f(x) = L$ indicates that the output values of a function f have a **limiting value** L as x increases or decreases without bound. When a function has a limiting value L, the line with equation $y = L$ is called a **horizontal asymptote**.

The notations $\lim\limits_{x\to\pm\infty} f(x) = \infty$ and $\lim\limits_{x\to\pm\infty} f(x) = -\infty$ indicate that the output values of a function f do not have a limiting value, but instead increase or decrease indefinitely as x increases or decreases without bound.

Example 3: (CC5e p. 18)

The number of credit card holders in the United States can be modeled as $C(t) = \dfrac{29}{1+18e^{-0.43t}} + 158$ million credit card holders where t is the number of years since 2000, $0 \le t \le 15$.



a. Describe the behavior of C over the intervals $0 < t < a$ and $a < t < 10$ using the terms increasing or decreasing and concave up or concave down.

b. How does the behavior of C change at the point with input $t = a$?

c. What is the mathematical name for the point with input $t = a$?

d. Complete the tables, stopping when the end behavior can be estimated. *Show rounding to three decimal places in the table.*

$t \to \infty$	$C(t)$
10	
30	
90	
270	
810	

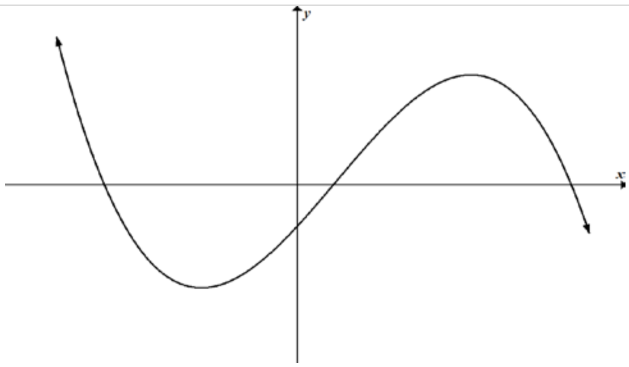
$t \to -\infty$	$C(t)$
–40	
–80	
–120	

e. Using *limit notation*, describe the end behavior of C as t increases without bound and as t decreases without bound.

f. Write the equations for the two horizontal asymptotes of C and draw them on the graph.

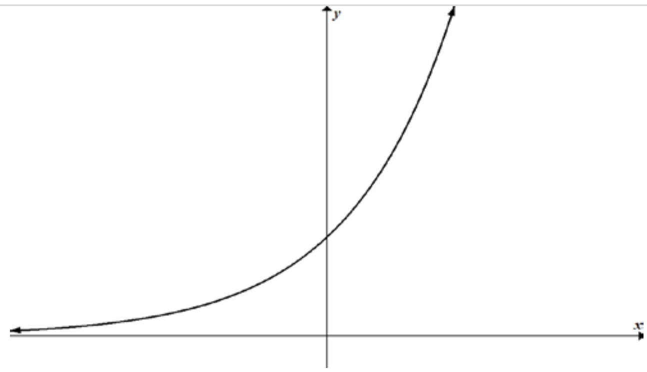
Example 4:

Using *limit notation*, describe the end behavior of each function as x decreases without bound and as x increases without bound. How many horizontal asymptotes does each function have?



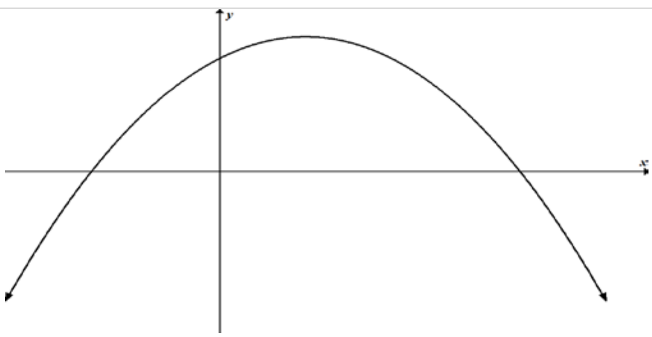
$\lim\limits_{x\to-\infty} f(x) =$ ________; $\lim\limits_{x\to\infty} f(x) =$ ________

Number of horizontal asymptotes:



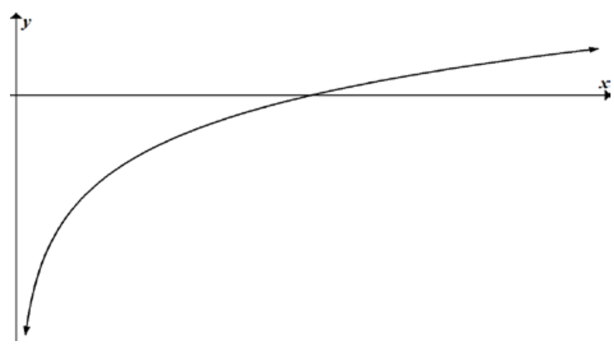
$\lim\limits_{x\to-\infty} f(x) =$ ________; $\lim\limits_{x\to\infty} f(x) =$ ________

Number of horizontal asymptotes:



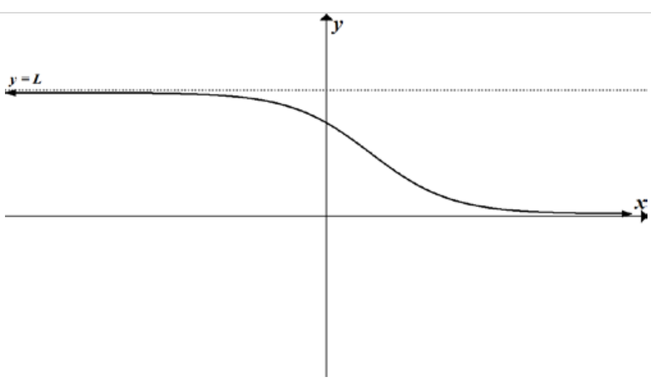
$\lim\limits_{x\to-\infty} f(x) =$ ________; $\lim\limits_{x\to\infty} f(x) =$ ________

Number of horizontal asymptotes:



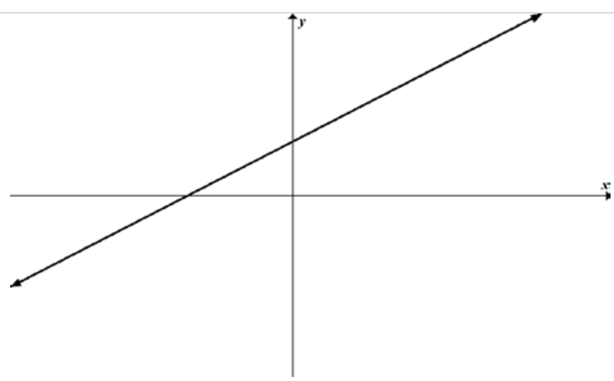
$\lim\limits_{x\to\infty} f(x) =$ ________

Number of horizontal asymptotes:



$\lim\limits_{x\to-\infty} f(x) =$ ________; $\lim\limits_{x\to\infty} f(x) =$ ________

Number of horizontal asymptotes:



$\lim\limits_{x\to-\infty} f(x) =$ ________; $\lim\limits_{x\to\infty} f(x) =$ ________

Number of horizontal asymptotes: