Section 1.10: Logistic Functions and Models

A quantity with exponential growth may have limiting factors that cause that quantity to level off. A **logistic function** either *increases* towards an upper limit or *declines* toward a lower limit as the input values increase.

A **logistic function** has an equation of the form $f(x) = \dfrac{L}{1 + Ae^{-Bx}}$, where A and B are nonzero constants and $L > 0$ is the **limiting value** of the function.

The graph of a logistic function has **two concavities**, with an inflection point. An inflection point is a point at which a function is increasing or decreasing the most or least rapidly on an interval around that inflection point.

The direction of a logistic function is determined by the sign of B*:*

- for $B < 0$, f is decreasing
- for $B > 0$, f is increasing

The graph of a logistic function of the form $f(x) = \dfrac{L}{1 + Ae^{-Bx}}$ is bounded by the horizontal axis and its limiting value. It has two horizontal asymptotes: $y = 0$ and $y = L$

- for $B < 0$, $\lim_{x \to -\infty} f(x) = L$ and $\lim_{x \to \infty} f(x) = 0$
- for $B > 0$, $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = L$

Example 1:

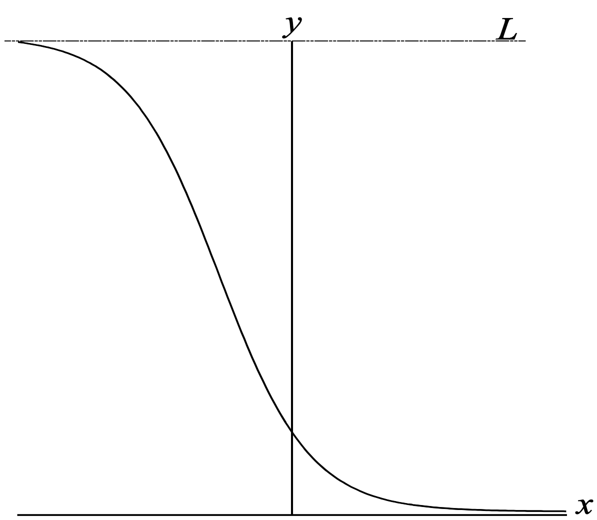
a. Label each of the following graphs of $f(x)=\dfrac{L}{1+Ae^{-Bx}}$ as either *increasing* or *decreasing*. Identify the *inflection point* by marking an "X" on the graph. Complete the limit statements that describe the end behavior.

b. Find the following for the function $f(x)=\dfrac{3.143}{1+2.251e^{0.466x}}$. Identify its graph below.

- B = (Note that $B<0$.)
- The equations of its two horizontal asymptotes are:

c. Find the following for the function $f(x)=\dfrac{2.458}{1+3.331e^{-0.688x}}$. Identify its graph below.

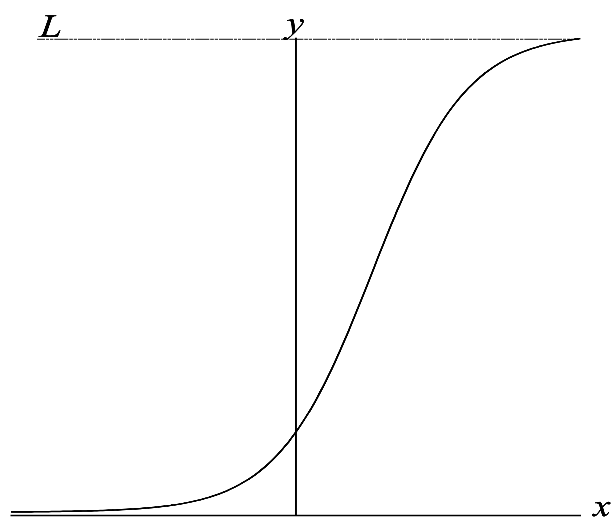
- B = (Note that $B>0$.)
- The equations of its two horizontal asymptotes are:



Increasing or decreasing?

$\lim\limits_{x\to-\infty} f(x)=$ ________; $\lim\limits_{x\to\infty} f(x)=$ ________

$f(x)=$ ________________________



Increasing or decreasing?

$\lim\limits_{x\to-\infty} f(x)=$ ________; $\lim\limits_{x\to\infty} f(x)=$ ________

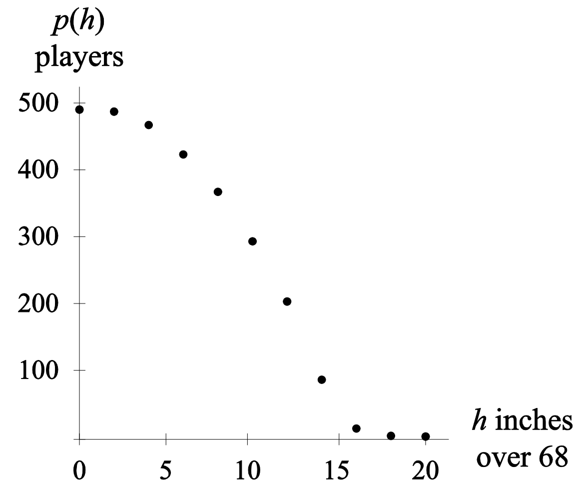
$f(x)=$ ________________________

Example 2: (CC5e p. 96)

The number of NBA players on the 2009-2010 roster who are taller than a given height are given in the table.

Height, in inches	68	70	72	74	76	78	80	82	84	86	88
Number of NBA players	490	487	467	423	367	293	203	86	13	2	1

a. Align the data to the number of inches over 68. Verify the scatter-plot of the aligned data is shown to the right.



b. Answer the following about the scatterplot.

The scatterplot is *increasing* or *decreasing.*

The scatterplot shows *zero, one,* or *two* concavities.

The scatterplot appears to be concave *up* or *down* near $h = 0$ and to be concave *up* or *down* near $h = 20$.

There appears to be an inflection point near $h =$ ______.

c. Write a completely defined model for the (aligned) data.

$p(h) =$

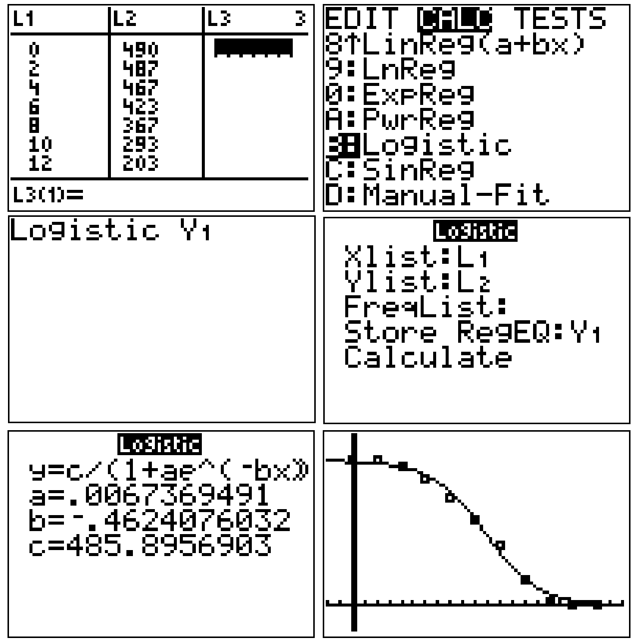
Finding, storing, and viewing a logistic function:

- With the data in L1 and L2, **STAT** ▸ [CALC] ▾ to B [Logistic] **ENTER** returns Logistic on the Home Screen **VARS** ▸ [Y-VARS] **1** [Function] **1** [Y1] returns Y1 **ENTER**

  OR

  STAT ▸ [CALC] ▾ to B [Logistic] **ENTER** returns the Logistic Screen
  Xlist: **2nd** **1** [L1]
  Ylist: **2nd** **2** [L2]
  Store RegEQ: **VARS** ▸ [Y-VARS] **1** [Function] **1** [Y1]
  Move cursor to Calculate and hit **ENTER**

- Hit **ZOOM** **9** [ZoomStat] to view the function and the scatter plot



d. Write the equations of the two horizontal asymptotes for the function.

e. Describe the end behavior of the function model in part c).

$\lim_{h\to-\infty} p(h) =$ ________________; $\lim_{h\to\infty} p(h) =$ ________________

f. As the heights of NBA players increase, the number of players taller than a particular height approaches ______________.

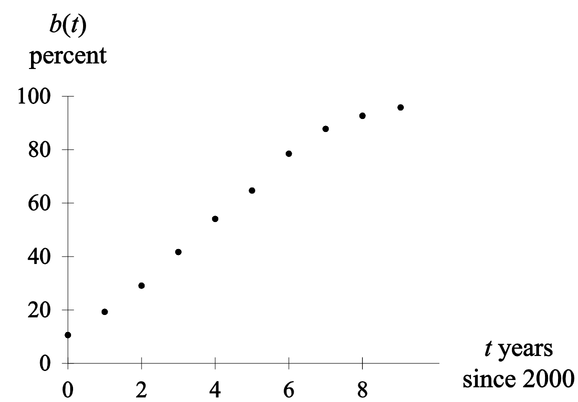
g. Sketch the function model and the two horizontal asymptotes over the scatter plot given in part a.

Example 3: (CC5e p. 97)

The total residential broadband (high-speed) access as a percentage of Internet access for specific years is shown in the table.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Broadband (percent)	10.6	19.3	29.1	41.7	54.1	64.7	78.5	87.8	92.7	95.8

a. What does the context suggest about the end behavior as the input values increase?



b. Align the input data to years after 2000. Find an appropriate model for the data.

c. State the equations of the two horizontal asymptotes.

d. In what year does the model predict that 99.9% of residences with Internet access will have broadband access?

e. In approximately what year is the percentage of residences with internet access who have broadband access increasing most rapidly?

Example 4: (CC5e pp. 94-95)

Suppose that a computer virus has attacked the computers of an international corporation. The worm is first detected on 100 computers. The corporation has 10,000 computers. As time passes, the number of infected computers can approach, but never exceed 10,000.

Hours	Infected Computers (thousand computers)
0	0.1
0.5	0.597
1	2.851
1.5	7.148
2	9.403
2.5	9.900

a. Does the scatter plot of the data indicate one or two concavities?

b. Write a completely defined logistic model for the data.

c. Write the equations of the two horizontal asymptotes for the logistic function.

d. According to the model, how many computers will be infected by the computer virus after 3 hours and 15 minutes?

e. How long would it take for the computer virus to infect 5500 computers?