Section 4.3: Absolute Extreme Points

A function f has an **absolute maximum** at input c if the output $f(c)$ is greater than (or equal to) every other output value on the domain of the function.

A function f has an **absolute minimum** at input c if the output $f(c)$ is less than (or equal to) every other output value on the domain of the function.

The output $f(c)$ is referred to as the *maximum* (value) or the *minimum* (value) of f.

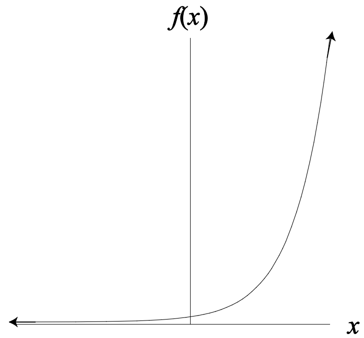
If a function f is defined on a closed interval $a \le x \le b$, the absolute maximum or absolute minimum may occur at either endpoint $x = a$ or $x = b$ or an absolute extreme value may occur where a relative extreme value occurs.

To find an absolute extreme on a closed interval $a \le x \le b$, compare the relative extreme values in the interval with the output values at the endpoints $f(a)$ and $f(b)$.
The largest of these values is the absolute maximum and the smallest of these values is the absolute minimum.

Example 1: (CC5e p. 269)

Identify absolute extreme points for the following functions on the domain of all real numbers.

a.

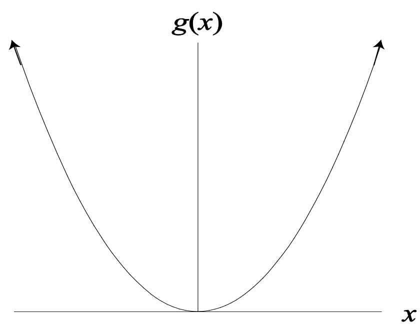


$f(x) = e^x$

Absolute maximum? (*yes/no*);
If yes, where? __________

Absolute minimum? (*yes/no*);
If yes, where? __________

b.



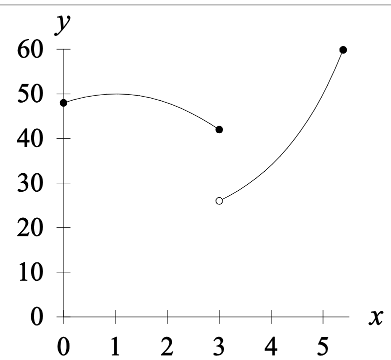
$g(x) = x^2$

Absolute maximum? (*yes/no*);
If yes, where? __________

Absolute minimum? (*yes/no*);
If yes, where? __________

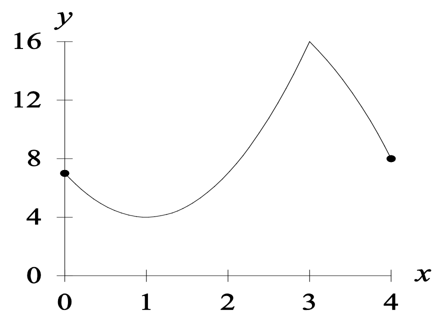
Example 2: (CC5e p. 271, Activities 3, 4)

Identify absolute extreme points for the following functions on the given domain.
For each extreme point, indicate whether the derivative at that point is zero or does not exist.



Absolute Maximum: (*yes/no*)
If yes, Input value: ______
Output value: _____
Nature of derivative:
(*zero/does not exist*)

Absolute Minimum: (*yes/no*)
If yes, Input value: ______
Output value: ______
Nature of derivative:
(*zero/does not exist*)



Absolute Maximum: (*yes/no*)
If yes, Input value: _____
Output value: _____
Nature of derivative:
(*zero/does not exist*)

Absolute Minimum: (*yes/no*)
If yes, Input value: ______
Output value: ______
Nature of derivative:
(*zero/does not exist*)

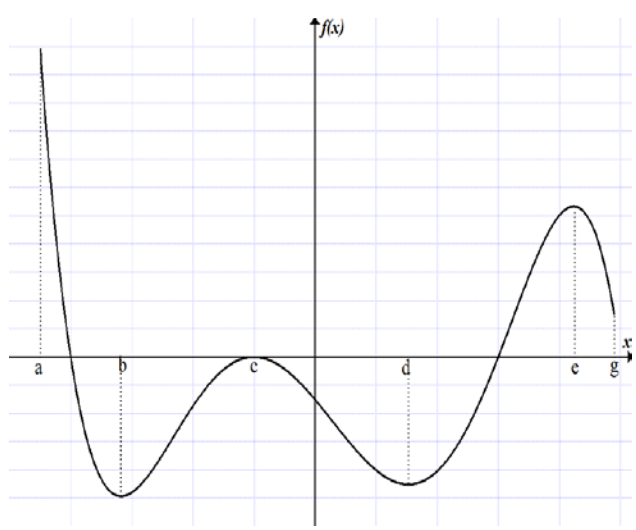
Example 3: (similar to CC5e p. 267)

The function *f*, defined on a closed interval, is shown to the right.

a. Label each of the points at inputs *a, b, c, d, e,* and *g* with all that apply*: relative maximum, relative minimum, absolute maximum, absolute minimum.*

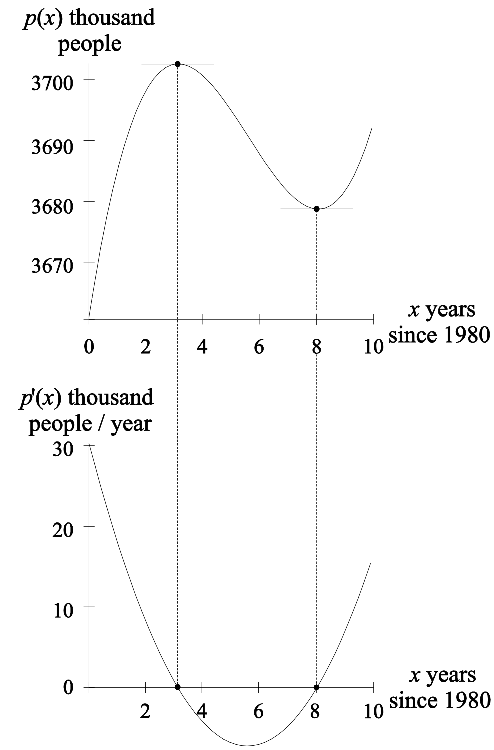
b. Can an *absolute extreme* value occur at an endpoint of a closed interval?

c. Can a *relative extreme* value occur at an endpoint of a closed interval?



Example 4:

The population of Kentucky can be modeled as
$p(x) = 0.395x^3 - 6.67x^2 + 30.3x + 3661$
thousand people where x is the number of years since 1980, $0 \le x \le 10$.



a. Label and mark an “X” on the graph of $p(x)$ for the absolute maximum and the absolute minimum.

b. In section 4.2, we found the following:

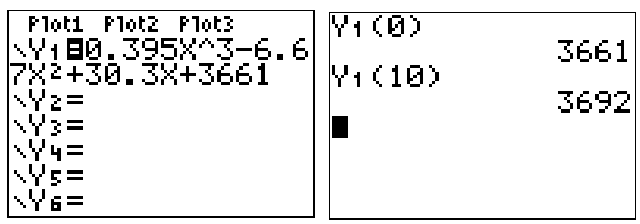
Relative minimum:

$x =$ ______________; $p(x) =$ ________________

Relative maximum:

$x =$ ______________; $p(x) =$ ________________

c. Find $p(0) =$ _________ and $p(10) =$ _________ .



d. Comparing the relative extreme points and the endpoints, determine the absolute extreme points:

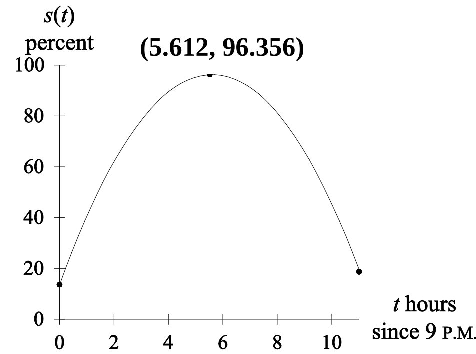
Absolute minimum: $x =$ ________________; $p(x) =$ __________________

Absolute maximum: $x =$ ________________; $p(x) =$ __________________

e. Between the years 1980 and 1990, Kentucky’s population was *lowest* in ________, at which time the population was ____.

Between the years 1980 and 1990, Kentucky’s population was *highest* ________ years after 1980, at which time the population was ______________________________.

Example 5: (CC5e pp. 268-269)



$s(t) = -2.63t^2 + 29.52t + 13.52$ percent gives the percentage of people aged 15 and older in the United States who are sleeping t hours after 9:00 pm, $0 \le t \le 11$.

a. Write a sentence of interpretation for the ordered pair $(5.612, 96.356)$.

b. Identify the absolute maximum on the closed interval $0 \le t \le 11$. Interpret the answer.

Absolute maximum: $t =$ _________________; $s(t) =$ ________________

The highest percentage of people, 15 years and older in the U.S., who are sleeping occurs _______ hours after 9 pm, at which time __________ percent of people 15 years and older are sleeping.

c. Find the absolute minimum on the closed interval $0 \le t \le 11$. Interpret the answer.

Absolute minimum: $t =$ _________________; $s(t) =$ ________________

The lowest percentage of people, 15 years and older in the U.S., who are sleeping occurs _______ hours after 9 pm, at which time __________ percent of people 15 years and older are sleeping.

Example 6: (CC5e pp. 271-272, Activity 15)

$f(h) = -0.865h^3 + 12.05h^2 - 8.95h + 123.02$ cubic feet per second (cfs) gives the flow rate of a river in the first 11 hours after the beginning of a severe thunderstorm, h hours after the storm began.

a. What are the flow rates for $h = 0$ and $h = 11$?

b. Identify the absolute maximum on the closed interval $0 \le h \le 11$. (Hint: Graph the function on the interval $-1 \le h \le 11$ to more easily find the absolute maximum on $0 \le h \le 11$.)

Absolute maximum: $h =$ ________________; $f(h) =$ ________________

In the first eleven hours after a severe thunderstorm, the flow rate for a river was *highest*

__________ hours after the storm began. At that time, the flow rate was _________ cfs.

c. Find the absolute minimum on the closed interval $0 \le t \le 11$. Hint: Compare the relative minimum to $f(0)$. Interpret the answer.

Absolute minimum: $h =$ ________________; $f(h) =$ ________________

In the first eleven hours after a severe thunderstorm, the flow rate for a river was *lowest*

__________ hours after the storm began. At that time, the flow rate was _________ cfs.

Example 7:

A clothing manufacturer determines the cost of producing x jackets is $C(x) = 2500 + 0.25x^2$ dollars, and sets a sales price of $p(x) = 150 - 0.5x$ dollars per jacket, $0 \le x \le 200$.

a. Find the total revenue from the sale of x jackets.

b. Find the total profit from the sale of x jackets.

c. How many jackets must the manufacturer produce and sell to maximize profit?

d. What is the maximum profit?