Homework #3 Solutions
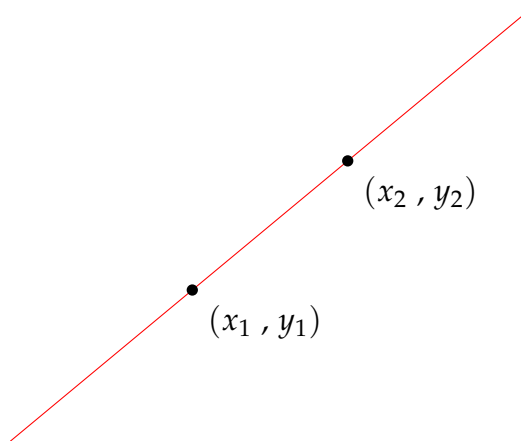
1. (1 point) Suppose a line passes through the points (x_1, y_1) and (x_2, y_2), as shown below:

(x_2 , y_2)

(x_1 , y_1)
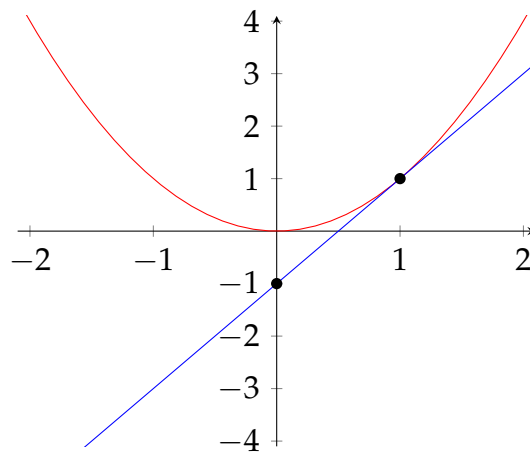
What is the slope of this line?

Solution: The slope is given by $\frac{y_2 - y_1}{x_2 - x_1}$.

2. Let $f(x)$ be the function given by the graph below:
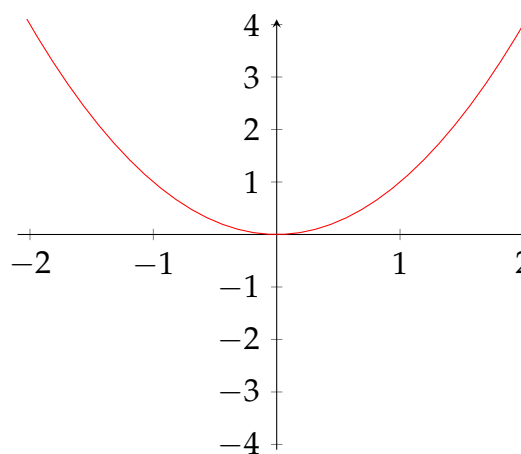
2.a. (1 point) Use a ruler to draw the tangent line to the graph of the function at the point $(1, 1)$. Find the slope of this line (Hint: You need to find two points this line goes through. Then you need to use the slope formula you wrote down above).
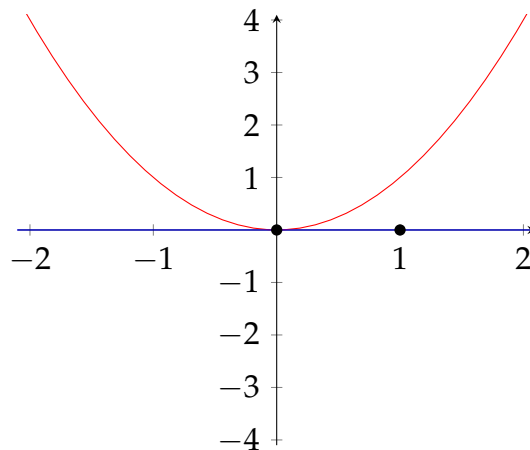
Solution:

The tangent line to the graph of the function at the point $(1,1)$ goes through the points $(1,1)$ and $(0,-1)$. Therefore, the slope of this line is $\frac{-1-1}{0-1} = 2$.

2.b. (1 point) Use a ruler to draw the tangent line to the graph of the function at the point $(0,0)$. Find the slope of this line.
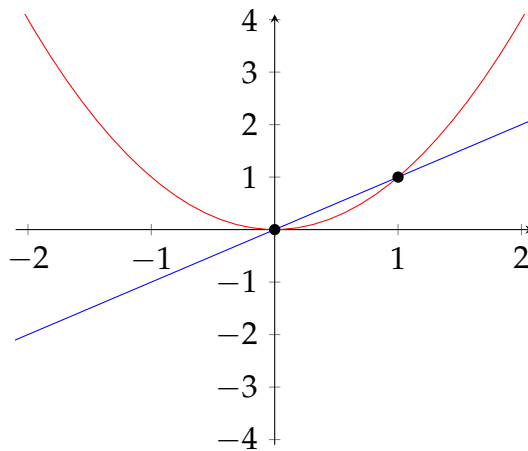
Solution:

The tangent line to the graph of the function at the point $(0,0)$ goes through the points $(0,0)$ and $(1,0)$. Therefore, the slope of this line is $\frac{0-0}{1-0} = 0$.

2.c. (1 point) Use a ruler to draw the secant line through the points $(0,0)$ and $(1,1)$. Find the slope of this line.

Solution:

The secant line to the graph of the function through the points $(0,0)$ and $(1,1)$ goes through the points $(0,0)$ and $(1,1)$. Therefore, the slope of this line is $\frac{1-0}{1-0} = 1$.

2.d. (1 point) With the information above, evaluate $f(0)$, $f'(0)$, $f(1)$, and $f'(1)$. What is the average rate of change between the points $(0,0)$ and $(1,1)$?
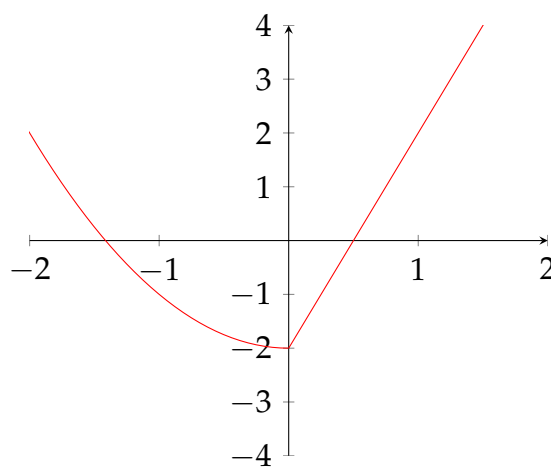
Solution: We have

$$f(0) = 0$$
$$f'(0) = 0$$
$$f(1) = 1$$
$$f'(1) = 2$$

The average rate of change between the points $(0,0)$ and $(1,1)$ is given by the slope of the secant line which goes through these two points. Thus the average rate of change is 1.[1]

3. (5 points) Let $f(x)$ be the function given by the graph below:

Sketch the graph of $f'(x)$: **Solution:**

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[1]On the test, you'll have to worry about units here. However in this problem, there are no units; we're just talking about functions.
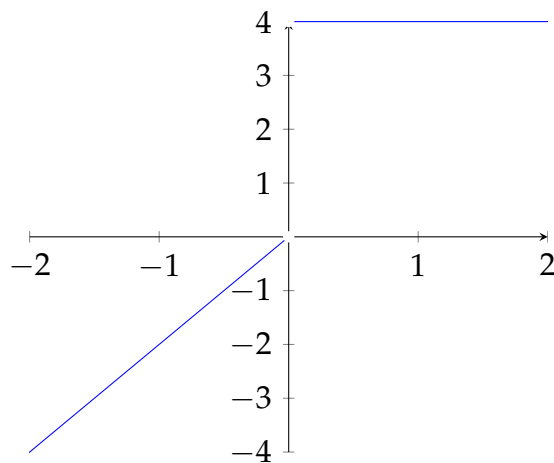
(Hint: The function goes through the point $(0.5, 0)$. Find the tangent line to the graph at this point, then find the slope of this line (this is $f'(0.5)$). Now plot the point $(0.5, f'(0.5))$. The function also goes through the point $(1, 2)$. Repeat the same procedure and plot the point $(1, f'(1))$. The function also goes through the point $(-2, 2)$. Repeat the same procedure and plot the point $(-2, f'(-2))$. You should be able to construct $f'(x)$ from this).

4. The function in the graph below is given by $f(x) = x^2$.
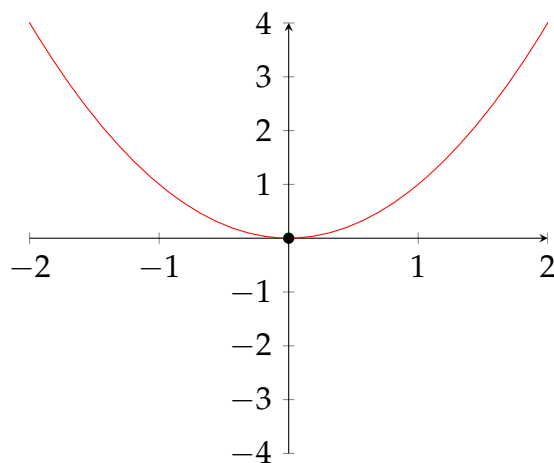
From problem 2, we know what the slope of the tangent line at $(0, 0)$ is (i.e. $f'(0)$). In this exercise, we want to compute $f'(0)$ by taking the limit of slopes of secant lines[2]

(4.a,4.b,4.c) Use your ruler to draw the secant line through the points $(0, 0)$ and $(2, 4)$. Then use your ruler to draw the secant line through the points $(0, 0)$ and $(1, 1)$. Then use your ruler to draw the secant line through the points $(0, 0)$ and $\left(\frac{1}{2}, \frac{1}{4}\right)$.

Solution:

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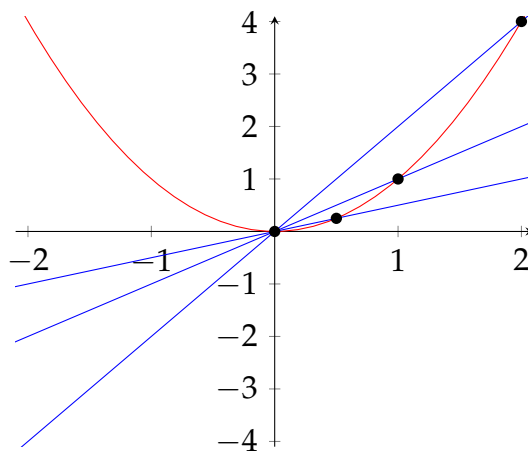
[2]In Calculus, almost everything is defined in terms of limits.

4.d Complete the table below. **Solution:**

x	Slope of secant line from $(0,0)$ to $(x, f(x))$
2	2
1	1
0.5	0.5
0.1	0.1

What does this table suggest? (Hint use $\lim_{x \to 0^+} f(x)$ notation). **Solution:** The table suggests

$$\lim_{x \to 0^+} f(x) = 0.$$