EE23BTECH11217 - Prajwal M*

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Exercise 9.5

25) Find the sum of the following series up to n terms:

$$\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \dots$$

Solution:

Given:

$$S_n = \frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \dots + a_n$$

let a_r be the general term, where

$$a_r = \frac{\sum_{i=1}^{r} i^3}{\sum_{j=1}^{r}(2j - 1)}$$

$$= \frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2}$$

$$= \frac{(r + 1)^2}{4}$$

let S_n be sum of series upto n terms, where

$$S_n = \sum_{r=1}^{n} a_r$$

$$= \sum_{r=1}^{n} \frac{(r + 1)^2}{4}$$

$$= \frac{\sum_{r=1}^{n+1} r^2 - 1}{4}$$

$$= \frac{n(n + 1)(n + 2)}{24} - \frac{1}{4}$$