Ch6 Lecture Notes

Perry Nerem

Contents

1 Intro

- Creation of eyeglasses ($\sim$ 1285 Italy)
- Telescope invented 1608 (Hans Lippershey) for military purposes
- Galileo in 1609 starts building his own telescopes. Discovers
  - mountains and craters of the Moon
  - moons of Jupiter
  - new stars in the Milky Way
  - phases of Venus (definitive proof Venus orbits Sol)

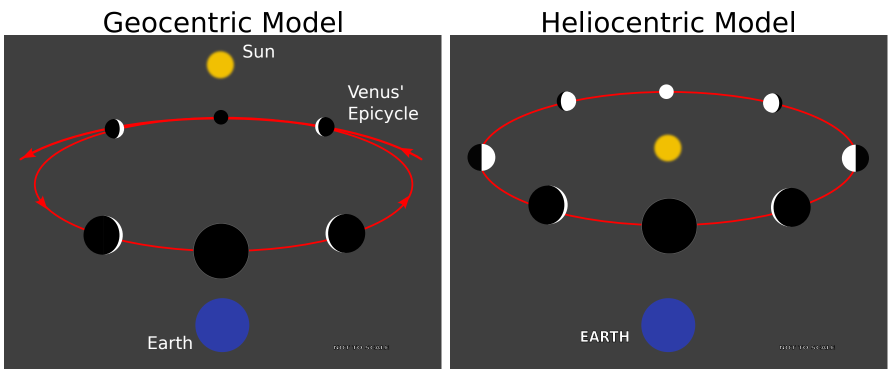

Figure 1: Only crescent phase possible in geocentric model.

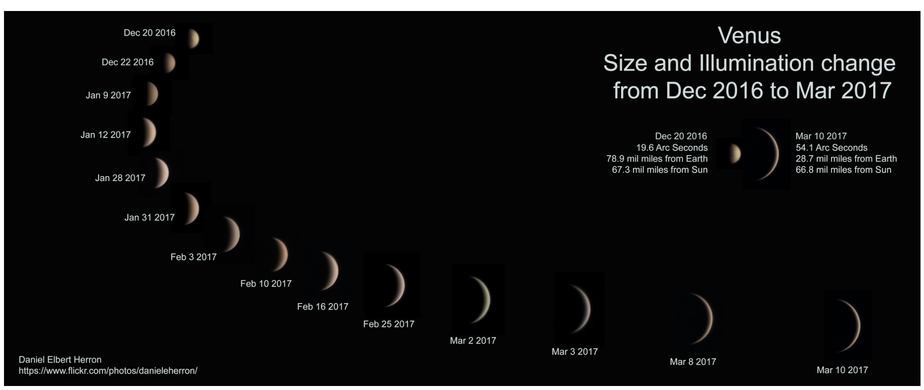

Figure 2: APOD Phases of Venus

- **Telescopes have three main functions for the observer**
  1) Collect light
  2) Magnify an image
  3) Resolve fine details.

  The *light collecting power* (aka light grasp) of a telescope is the most important aspect.

2 Fermat's Principle

What is the "fastest" path from point $A \to B$?

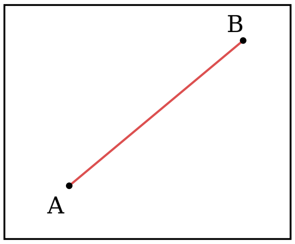

Figure 3: The shortest path in time is not necessarily the shortest path in space

The simplest answer to this question is that fastest path is the *shortest* geometric path. But what if your speed depends on the terrain?

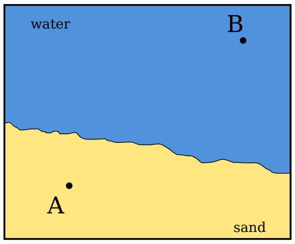

Figure 4: Your speed now depends on material you run through.

Assume that you run faster in the sand than swim in the water. You want to maximize time on the sand!

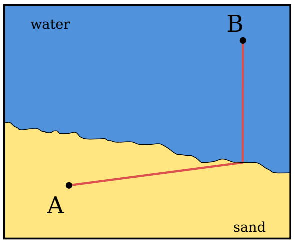

Figure 5: This minimizes time on the sand, but the time in water increases

You can immediately see that this cannot be an optimal path. The "fastest" path is some intermediate one that optimizes the amount time on sand versus water.

The path taken is the one that is the *least amount of time*!

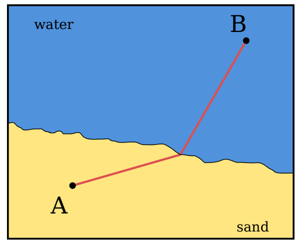

Figure 6: This path is the least amount of time from $A \rightarrow B$.

Consider the following beam path of a light ray bending between two mediums (presume air to water). Let's use this principle of least time to quantify the ray path from S to P.

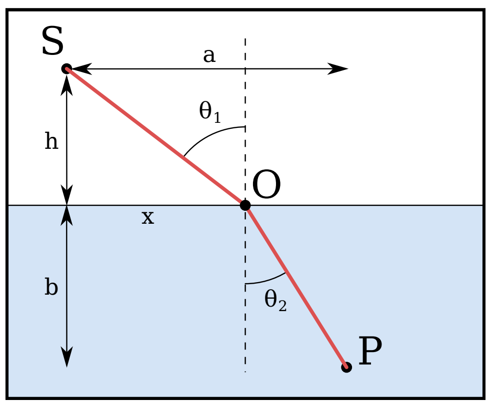

Figure 7: The light ray refracts when changing mediums.

The time for the ray to travel from S to P is

$$t = \frac{\bar{SO}}{v_1} + \frac{\bar{OP}}{v_2}$$

$$t = \frac{\sqrt{x^2 + h^2}}{v_1} + \frac{\sqrt{(a-x)^2 + b^2}}{v_2}$$

Minimize the time by setting $dt/dx = 0$.

$$\frac{dt}{dx} = \frac{1}{v_1}(\tfrac{1}{2})\big(x^2 + h^2\big)^{-1/2}(2x) + \frac{1}{v_2}(\tfrac{1}{2})\big((a-x)^2 + b^2\big)^{-1/2}(2)(a-x)(-1)$$

$$\frac{1}{v_1}\underbrace{\frac{x}{\sqrt{x^2+h^2}}}_{\text{geometry}}=\frac{1}{v_2}\underbrace{\frac{(a-x)}{\sqrt{(a-x)^2+b^2}}}_{\text{geometry}}$$

$$\frac{\sin\theta_1}{v_1}=\frac{\sin\theta_2}{v_2}$$

Define the index of refraction

$$n_i=\frac{c}{v_i}$$

$$\boxed{\text{Snell's Law } n_1\sin\theta_1=n_2\sin\theta_2}$$

2.1 Image formation: Refraction

Consider a material with a refractive index n_2 and a spherical surface.

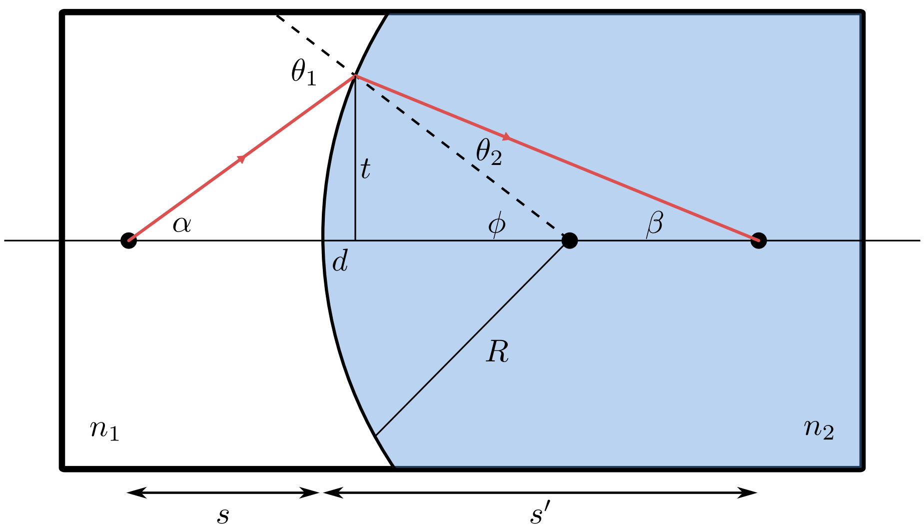

Figure 8: Draw a parallel line where ray hits surface to show that $\theta_1=\alpha+\phi$

R = radius of curvature of the surface.

From the geometry we can see that

$$\theta_1=\alpha+\phi,\qquad \theta_2=\phi-\beta$$

$$\tan\alpha = \frac{t}{s+d}$$
$$\tan\beta = \frac{t}{s'-d}$$
$$\tan\phi = \frac{t}{R-d}$$

Now apply Snell's law and make the approximation that θ_i is small. We can assume that by considering only **paraxial rays**. Rays that are *nearly parallel* to the optical axis.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1\theta_1 = n_2\theta_2$$

$$n_1(\alpha + \phi) = n_2(\phi - \beta)$$

Note that $\tan x \approx x$ for small angles, $\therefore$

$$n_1\left(\frac{t}{s+d} + \frac{t}{R-d}\right) = n_2\left(\frac{t}{R-d} - \frac{t}{s'-d}\right)$$

For **paraxial** rays $d \to 0$

$$n_1\left(\frac{1}{s} + \frac{1}{R}\right) = n_2\left(\frac{1}{R} - \frac{1}{s'}\right)$$

$$\boxed{\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}}$$

2.1.1 Thin-Lens Equation

Let us determine an equation for a thin lens by using our previous result. Treat $n_1 = 1$ for vacuum (or air where n air $= 1.0003$), and label n_2 as just n for the index of refraction of the lens.

Consider a lens that has a **thickness** t and a radius of curvature R_1 on one side and R_2 on the other.

The rays from an object refract from the first edge of the lens and create a virtual image behind the object.

For the 1st edge we have

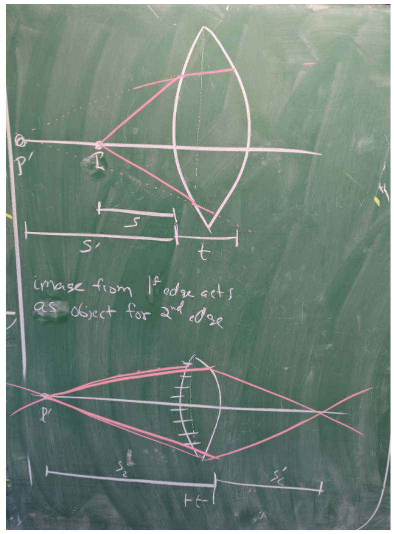

Figure 9: Image from 1st edge acts as object for 2nd edge

$$\boxed{\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}}$$

and the second edge we have

$$\frac{n}{s_2} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

$$s_2 = t + |s_1'|$$

For this case we have a virtual image $\therefore$ $s_1' < 0$.

$$s_2 = t - s_1'$$

and our 2nd edge equation becomes

$$\frac{n}{t - s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

Thin lens approximation: $t \to 0$. Our 2nd edge equation becomes

$$-\frac{n}{s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

Add the 1st edge equation to this result

$$\frac{1}{s_1} + \frac{1}{s_2'} = \frac{n-1}{R_1} + \frac{1-n}{R_2}$$

$$\frac{1}{s_1} + \frac{1}{s_2'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

We define the right hand side of this equation as the focal length. If the object is infinitely far away than s_2' equals the focal length of the lens. We drop the 1,2 notation since we only care about the object and the resulting final image.

$$\boxed{\text{Lens maker's equation: } \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$$\boxed{\text{thin-lens equation: } \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

3 Telescope optics: Basic Principles

Refraction and reflections are primary principles behind telescopes

Grab knight and work out examples of compound lens problem

4 Collecting light