$$\bar{E}(\nu) = \frac{h\nu}{e^{h\nu/k_bT} - 1}$$

↓ recast as a flux

$$B_\nu = \frac{2h\nu}{c^2} \frac{1}{e^{h\nu/k_bT} - 1} \quad \& \quad B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_bT} - 1}$$

if $h\nu << k_bT$ than $h\frac{c}{\lambda} << k_bT$ $\quad c = \lambda\nu$

$$\frac{hc}{k_bT\lambda} << 1$$

$$B_\lambda = \frac{2hc}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{k_bT}\frac{1}{\lambda}\right) - 1}$$

let $x = \frac{hc}{k_bT} << 1$

$$= \frac{2hc}{\lambda^5} \frac{1}{e^x - 1}$$

$e^x \approx 1 + x$ for $x << 1$

$$\approx \frac{2hc}{\lambda^5 x}$$

$$\approx \frac{2hc}{\lambda^5} \frac{k_bT\lambda}{hc}$$

$$\approx \frac{2k_bt}{\lambda^4}$$

— Rayleigh Jeans Law is back

---

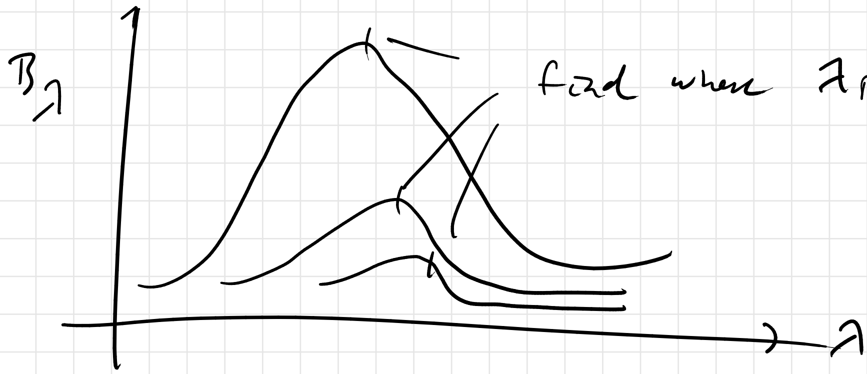
if $h\nu >> k_bT$

$$B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_bT} - 1}$$

$$\approx \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_bT}}$$

Wein approximation

Derivation of Wein's displacement law



find when λ_{peak} occurs!

- to find a local max of a function do

$$\frac{\partial f}{\partial \xi} = 0$$

& solve for ξ. technically need to check concavity to lazy

$$\frac{\partial}{\partial \lambda}\left[B_\lambda\right] = 0 \qquad \frac{d}{dx} e^{f(x)} = f'(x) e^{f(x)}$$

$$\frac{\partial}{\partial \lambda}\left[\frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T}-1}\right]$$

$$0 = (-5)2\frac{hc^2}{\lambda^6}\frac{1}{e^{hc/\lambda k_B T}-1} + \frac{2hc^2}{\lambda^5}(-1)\left[e^{hc/\lambda k_B T}-1\right]^2\left((-1)\frac{hc}{\lambda^2 k_B T} e^{hc/\lambda k_B T}\right)$$

let $x = \frac{hc}{\lambda k_B T}$

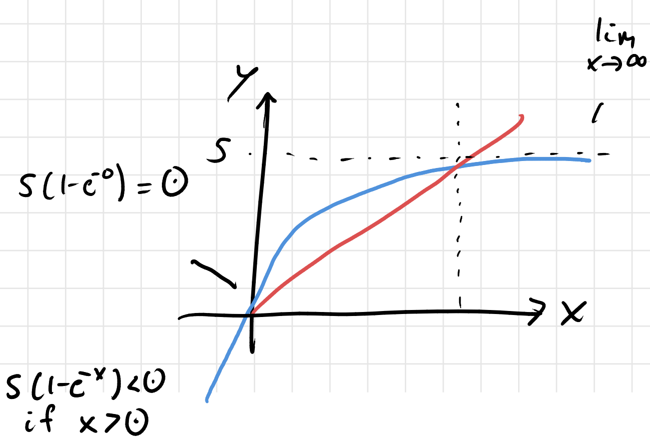
$$0 = -5\frac{2hc^2}{\lambda^6}\frac{1}{e^x-1} + \frac{2hc^2}{\lambda^7}\frac{1}{(e^x-1)^2}\frac{hc}{k_B T}e^x$$

$$0 = -5\frac{1}{e^x-1} + \frac{1}{\lambda}\frac{e^x}{(e^x-1)}\frac{hc}{k_B T}$$

$$0 = -5 + \frac{e^x}{e^x-1}x$$

$$xe^x = 5(e^x-1)$$

$$\boxed{x = 5(1-e^{-x})}$$

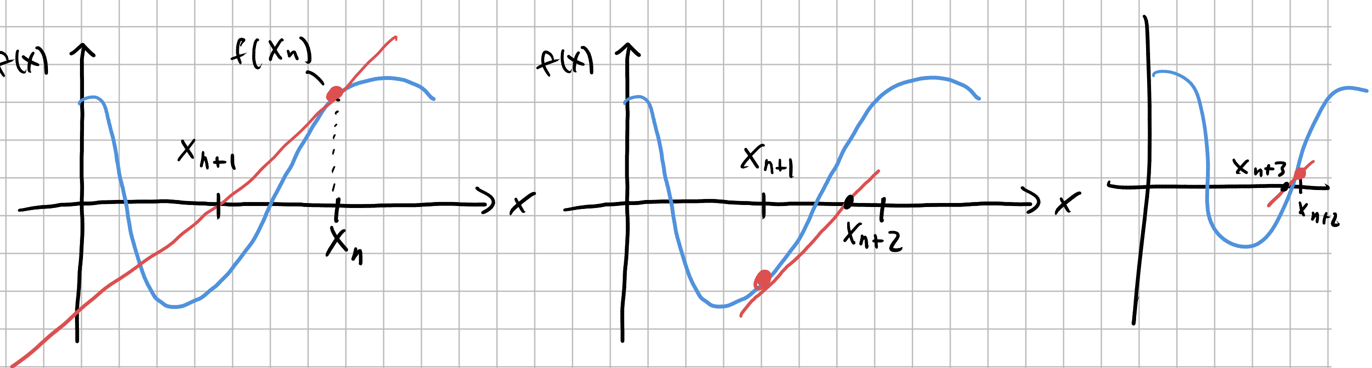


we'll try numerical approach to solution

x	$5(1-e^{-x})$
1	negative #
2	4.32...
3	4.75
4	4.91
5	4.96

solution is between $x=4$ & $x=5$

Use Newton's method to get roots of this function



each subsequent step is closer to a root

tangent line (red curve)

$$f'(x_n) = \frac{\Delta y}{\Delta x} = \frac{f(x_n) - (0)}{x_n - x_{n+1}} \Rightarrow \boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$$f(x) = x - 5(1-e^{-x})$$

$$\boxed{f(x) = 5e^{-x} + x - 5}$$

$$\boxed{f'(x) = -5e^{-x} + 1}$$ take $x_0 = 4$

python code to run Newton's method

;

$$x = 4.96511423$$

$$x = \frac{hc}{\lambda k_b t}$$

$$\lambda = \frac{hc}{x k_b} \frac{1}{T}$$

$$\lambda = \frac{0.002897 \text{ mK}}{T}$$ Wein's Displacement law

We can easily model stars as spherical black bodies & find their luminosity

$$dL = B_\lambda(T)\, d\lambda\, \underbrace{dA}_{\text{surface area}}\, \underbrace{d\Omega}_{\text{solid angle}}$$

$$L = \int_0^{4\pi} \int_0^{4\pi R^2} \int_0^\infty 2\frac{hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_b T} - 1}\, d\lambda\, dA\, d\Omega$$

$$= (4\pi R)^2 \int_0^\infty 2\frac{hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_b T} - 1}\, d\lambda$$

let $x = \frac{hc}{\lambda k_b T}$, $dx = (-1)\frac{hc}{\lambda^2 k_b T}\, d\lambda$

$$= (4\pi R)^2 \int_\infty^0 2\frac{hc^2}{\lambda^5} \frac{1}{e^x - 1} (-1)\lambda^2 \frac{k_b T}{hc}\, dx$$

$$= (4\pi R)^2 \int_0^\infty 2 \frac{c k_b T}{\lambda^3} \frac{1}{e^x - 1}\, dx$$

$$= (4\pi R)^2 \int_0^\infty c k_b T \left(\frac{x k_b T}{hc}\right)^3 \frac{1}{e^x - 1}\, dx$$

$$= (4\pi R)^2 \frac{1}{c^2} (k_b T)^4 \int_0^\infty x^3 \frac{1}{e^x - 1}\, dx$$

$$= (4\pi R)^2 \frac{1}{c^2} k_b^4 T^4 \frac{\pi^4}{15}$$

$$= 4\pi R^2 \frac{4\pi^5 k_b^4 T^4}{15 c^2}$$

$$dL = B_\lambda(T)\, d\lambda\, dA\, d\Omega$$

surface area

solid angle

why solid angle of π

$$L = \int_0^{\pi} \int_0^{4\pi R^2} \int_0^{\infty} 2 \frac{hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_b T} - 1}\, d\lambda\, dA\, d\Omega$$

$$= \pi (4\pi R^2) \int_0^{\infty} 2 \frac{hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_b T} - 1}\, d\lambda$$

let $x = \frac{hc}{k_b T} \frac{1}{\lambda}$, $dx = (-1) \frac{hc}{k_b T} \frac{1}{\lambda^2} d\lambda$

$\frac{1}{\lambda} = x \frac{k_b T}{hc}$, $dx = (-1) \frac{x}{\lambda} d\lambda$

$d\lambda = (-1) \frac{\lambda}{x} dx$

$$= \pi (4\pi R^2) \int_{\infty}^{0} 2 \frac{hc^2}{\lambda^5} \frac{1}{e^x - 1} (-1) \frac{\lambda}{x}\, dx$$

$$= \pi (4\pi R^2) \int_0^{\infty} 2hc^2 \frac{1}{\lambda^4} \frac{1}{x} \frac{1}{e^x - 1}\, dx$$

$$= \pi (4\pi R^2) \int_0^{\infty} 2hc^2 \left(x \frac{k_b T}{hc}\right)^4 \frac{1}{x} \frac{1}{e^x - 1}\, dx$$

$$= \pi (4\pi R^2) \int_0^{\infty} 2 \frac{k_b^4 T^4}{h^3 c^2} x^3 \frac{1}{e^x - 1}\, dx$$

$$= \pi (4\pi R^2)\, 2 \frac{k_b^4 T^4}{h^3 c^2} \int_0^{\infty} x^3 \frac{1}{e^x - 1}\, dx$$

$$= \pi (4\pi R^2)\, 2 \frac{k_b^4 T^4}{h^3 c^2} \left(\frac{\pi^4}{15}\right)$$

$$= 4\pi R^2 \underbrace{\left(\frac{2\pi^5 k_b^4}{h^3 c^2}\right)}_{\text{Stefan-Boltzmann constant}} T^4$$

Stefan-Boltzmann constant $\sigma = 5.670 \ldots \cdot 10^{-8} \frac{W}{m^2 K^4}$

$$\boxed{L = 4\pi R^2 \sigma T^4}$$