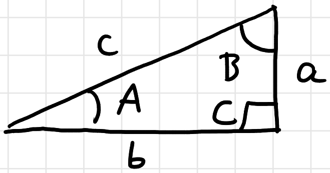
# Ch4 Applications of the Spherical Triangle

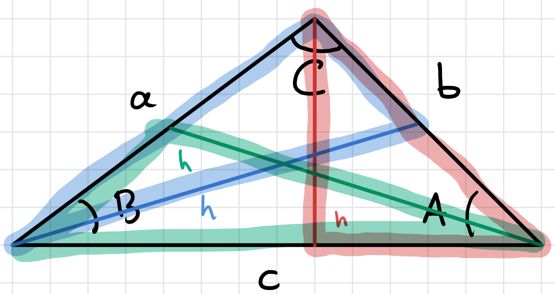
Reminder of facts    Euclidean Geometry




- we will use lower case letters $a, b, c, \ldots$ to be sides of triangle
- sum of interior angles = $180°$

$$\angle A + B + C = \pi$$

## Law of Sines




Area of triangle = $\frac{1}{2}$ base × height

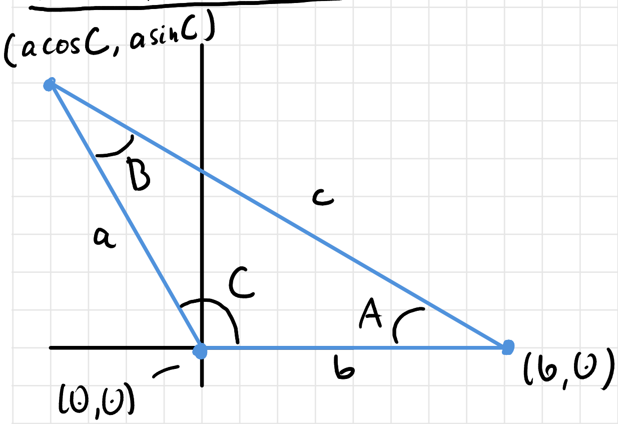
$\text{Area} = \frac{1}{2} c \cdot b \sin A$

$\text{Area} = \frac{1}{2} b \cdot a \sin C$

$\text{Area} = \frac{1}{2} a \cdot c \sin B$

$$\frac{2}{abc}\text{Area} = \boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$ law of sines

## Law of Cosines




$$|c| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$|c| = \sqrt{(b - a\cos C)^2 + (-a\sin C)^2}$$

$$c^2 = (b - a\cos C)^2 + (-a\sin C)^2$$

$$c^2 = b^2 + a^2\cos^2 C - 2ab\cos C + a^2\sin^2 C$$

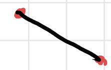
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= c^2 + a^2 - 2ca\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}}$$

What do our trig rules look like on a spherical surface?

Spherical Geometry is a non-Euclidean geometry.

# Axioms of Euclidean Plane Geometry

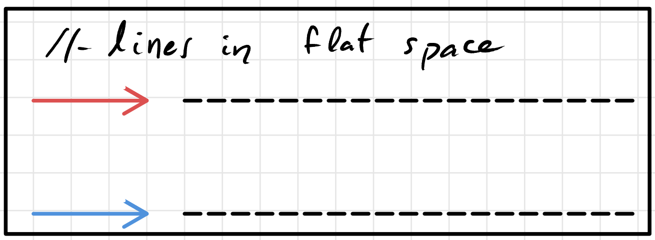
1) A straight line may be drawn between any two points 

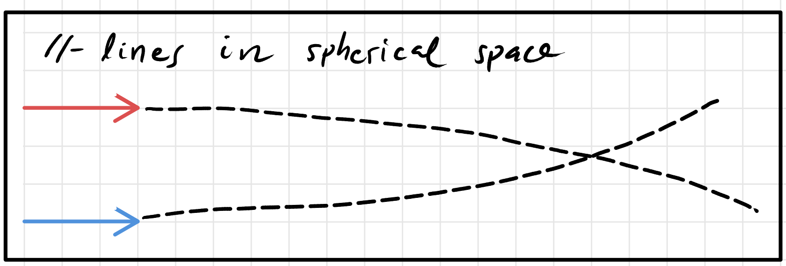
2) Any terminated straight line may be extended indefinately

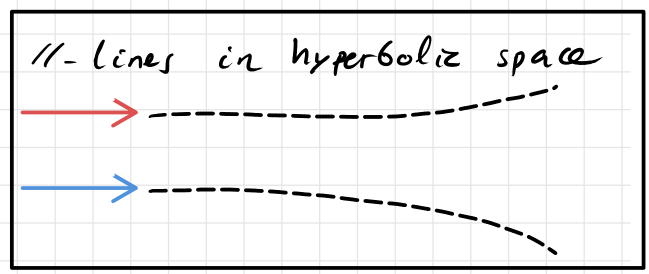
3) A circle can be drawn around any point

4) All right angles are equal

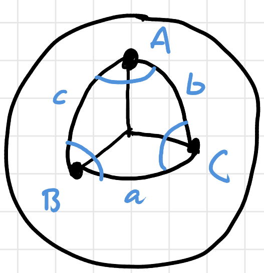
5) Parallel line postulate "parallel rays will never intersect"



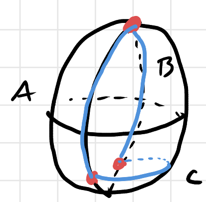




Spherical triangle




- vertices & angles denoted by upper case letters A, B, C
- sum of interior angle greater than 180°

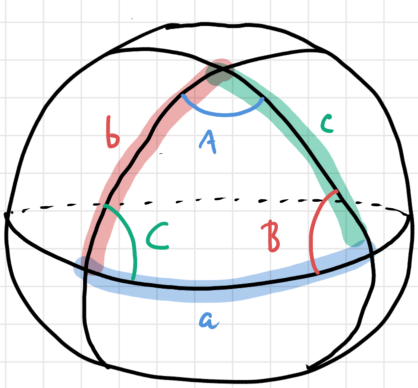



$A = \pi - \varepsilon$

$B = \pi - \varepsilon$

$C = \pi$

$$\boxed{\pi < A + B + C < 3\pi}$$




- "sides" denoted by lower case letters a, b, c and measured in <u>radians</u>

  a, b, c are angles measured from center of sphere

Class had a few questions about //-lines in spherical geometry.

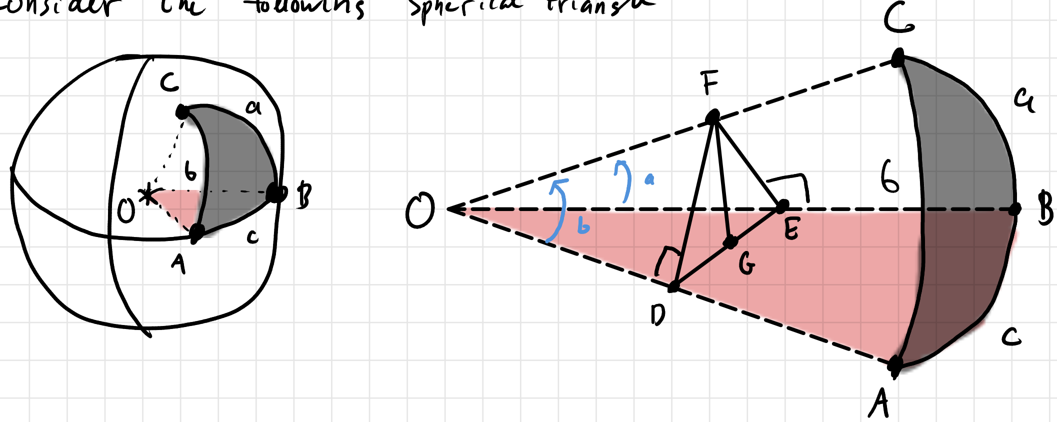
- on spherical geometry <u>lines</u> are defined as great circles
- all great circles intersect
- No parallel lines exist
- What about a small circle? Not a line

See wiki article on "map projections"

Projections by preservation of a metric property

- Find the law of sines & cosines for spherical triangles

Consider the following spherical triangle




- $FD \perp OA$
- $FG \perp$ plane $AOB$

$$\sin \angle FDG = \frac{FG}{DF} = \sin A$$

$$\sin \angle FEG = \frac{FG}{EF} = \sin B$$

$$\Rightarrow \frac{\sin A}{\sin B} = \frac{EF}{DF}$$

$$\sin a = \frac{EF}{OF}$$

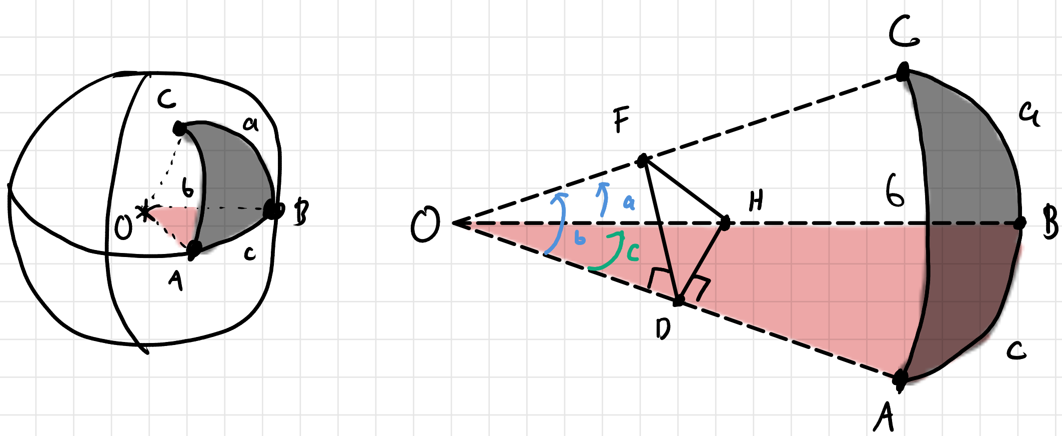
$$\sin b = \frac{DF}{OF}$$

$$\Rightarrow \frac{\sin a}{\sin b} = \frac{EF}{DF}$$

$$\frac{\sin A}{\sin B} = \frac{\sin a}{\sin b}$$

$$\boxed{\text{Law of Sines (spherical triangles):} \quad \frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}}$$

similar trick to get law of cosines




- $DF \perp OA$
- $DH \perp OA$

apply (Euclidean) law of cosines to triangles $a^2 = b^2 + c^2 - 2bc\cos A$

$$OHF: \ (HF)^2 = (OH)^2 + (OF)^2 - 2(OH)(OF)\overbrace{\cos \angle FOH}^{a}$$

$$DHF: \ (HF)^2 = (DH)^2 + (DF)^2 - 2(DH)(DF)\underbrace{\cos \angle FDH}_{A}$$

- subtract these two equations

$$0 = (OH)^2 + (OF)^2 - 2(OH)(OF)\cos a - (DH)^2 + (DF)^2 + 2(DH)(DF)\cos A$$

- right triangle DOF: $(OF)^2 = (OD)^2 + (DF)^2$
- right triangle DOH: $(OH)^2 = (DH)^2 + (OD)^2$

$$0 = (OD)^2 - 2(OH)(OF)\cos a + (OD)^2 + 2(DH)(DF)\cos A$$

$$\cos a = \frac{(OD)^2}{(OH)(OF)} + \frac{(DH)(DF)}{(OH)(OF)}\cos A$$

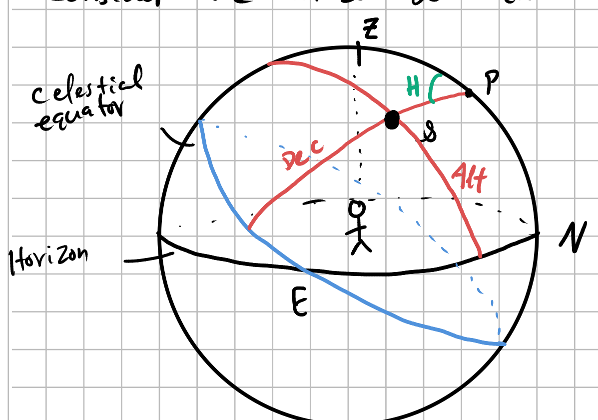
$$\cos a = \frac{(OD)}{(OF)}\frac{(OD)}{(OH)} + \frac{(DF)}{(OF)}\frac{(DH)}{(OH)}\cos A$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\boxed{\begin{array}{ll} \text{Law of cosines:} & \cos a = \cos b \cos c + \sin b \sin c \cos A \\ \text{spherical triangles} & \cos b = \cos c \cos a + \sin c \sin a \cos B \\ & \cos c = \cos a \cos b + \sin a \sin b \cos C \end{array}}$$

## Hour Angle & coordinate conversions (text fig 1.6 b)

Consider the horizon & equatorial systems simultaneously



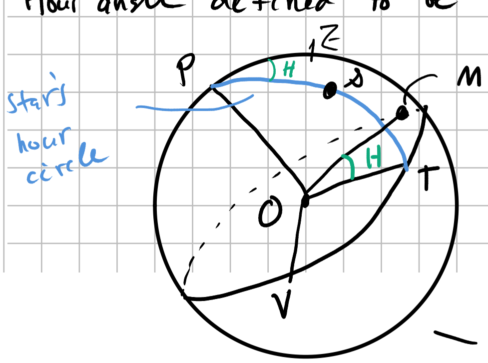

ZPN = observer's celestial meridian

↳ divides sky into east half & west half

$\angle ZP\star = H =$ hour angle

H negative if in east half
H positive if in west half

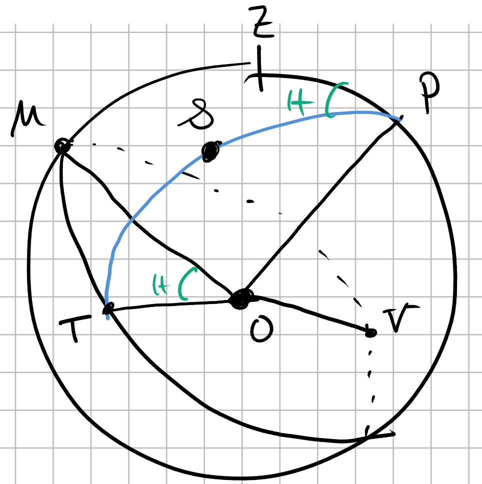
Hour angle defined to be the angle between observer's meridian & hour circle through star




— star in western half of sky

$H = \angle MOT$, M = point where your meridian crosses celestial equator

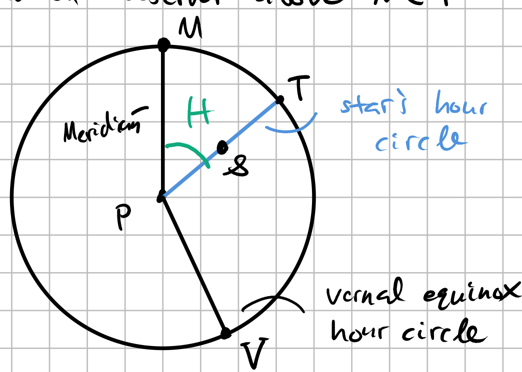
T = great circle from N-S pole crossing through star of interest




$H = \angle MOT = \angle ZPS$

Star in Eastern half of sky

View of observer above NCP




- ideal observations is when H is small (star close to your meridian)

- if $H = 12^h$, star is below the horizon or between pole & horizon

* hour angle = local sidereal time − right ascension *

## Coordinate Conversion

- Determine the altitude & azimuth of a star at a specific time.
- Must know
  - (celestial) coordinates of star
  - local sidereal time
  - observer's latitude
- $H = RA - LST$ (right ascension − local sidereal time)

DEC

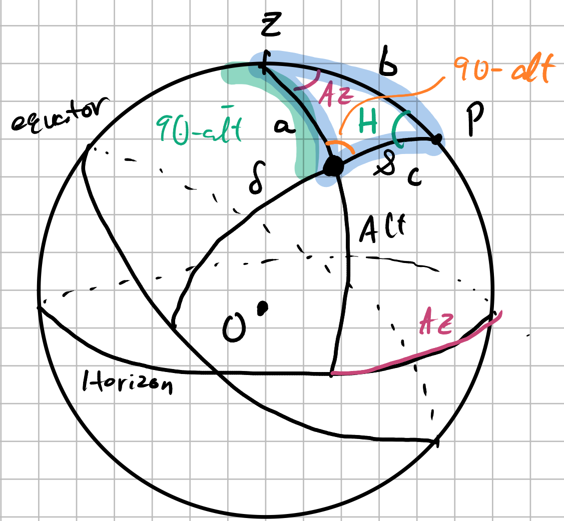
Example : Iota Herculis  RA = $17^h 40^m 5.7^s$ , $\delta = 45° 59' 19.2''$

- pick time of observation to be ~ 10am (EST)
- local sidereal time = 19.22 hr ⇒ $H = RA - LST \approx -1.5h$

$H = 1^h 30^m$ east

$= -1.5^h \cdot \frac{360°}{24^h} = -22.5°$

- $\phi$ = our latitude = 36.84°




$H = \angle ZPS$

$PZ = 90° - \phi = 53.16°$

$Z\delta = 90° - alt$

$P\delta = 90° - \delta \approx 44°$

Find alt w/ law of cosines

$\cos a = \cos b \cos c + \sin b \sin c \cos A$

$$\cos(90° - alt) = \cos(90° - \phi)\cos(90° - \delta) + \sin(90° - \phi)\sin(90° - \delta)\cos(H)$$

Trig ID: $\cos(90° - x) = \sin(x)$

$\sin(90° - x) = \cos(x)$

$$\boxed{\sin(alt) = \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(H)}$$

$$alt = \arcsin[\sin(36.84°)\sin(46°) + \cos(36.84°)\cos(46°)\cos(22.5)] = 70.9°$$

Get the azimuth w/ law of sines

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

text book says

$$\boxed{\frac{\sin 90° - alt}{\sin H} = \frac{\sin 90° - \delta}{\sin Az}} \Rightarrow Az = \arcsin\left[\frac{\cos\delta}{\cos alt}\sin H\right] = -54.3°$$