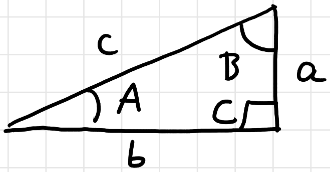
Ch 4 Applications of the Spherical Triangle

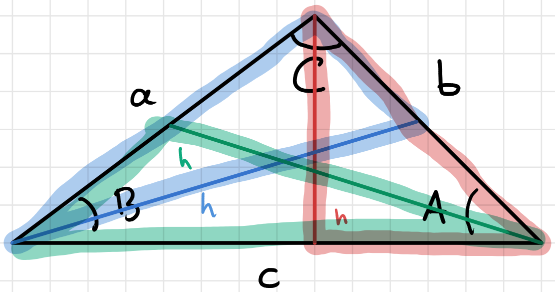
Reminder of facts Euclidean Geometry



- we will use lower case letters a, b, c, ... to be sides of triangle
- sum of interior angles = 180°

$\angle A + B + C = \pi$

Law of Sines



Area of triangle = ½ base × height

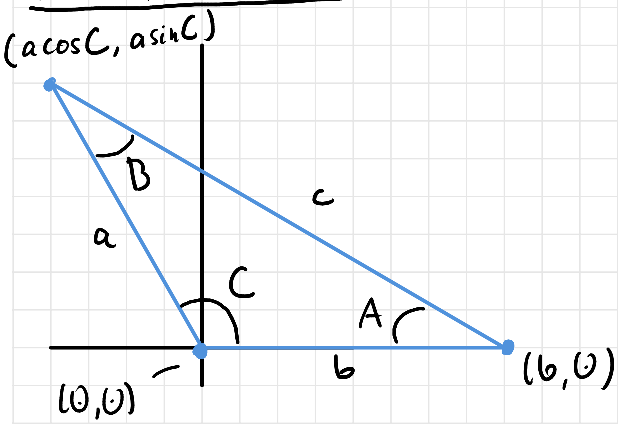
Area = $\frac{1}{2} c \cdot b \sin A$

Area = $\frac{1}{2} b \cdot a \sin C$

Area = $\frac{1}{2} a \cdot c \sin B$

$$\frac{2}{abc}\text{Area} = \boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$ law of sines

Law of Cosines



$|c| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$|c| = \sqrt{(b - a\cos C)^2 + (-a\sin C)^2}$

$c^2 = (b - a\cos C)^2 + (-a\sin C)^2$

$c^2 = b^2 + a^2\cos^2 C - 2ab\cos C + a^2\sin^2 C$

$c^2 = a^2 + b^2 - 2ab\cos C$

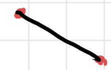
$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= c^2 + a^2 - 2ca\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}}$$

What do our trig rules look like on a spherical surface?

Spherical Geometry is a non-Euclidean geometry.

Axioms of Euclidean Plane Geometry

1) A straight line may be drawn between any two points

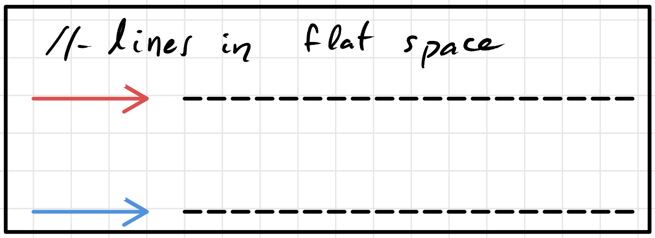


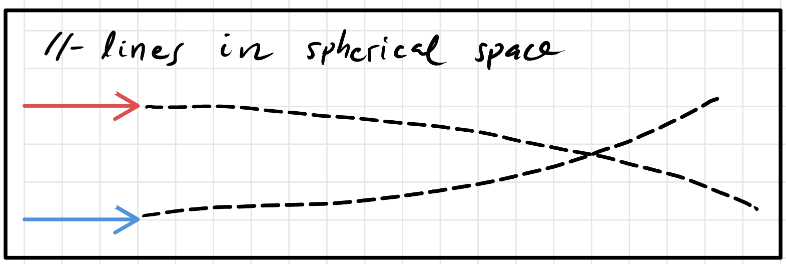
2) Any terminated straight line may be extended indefinately

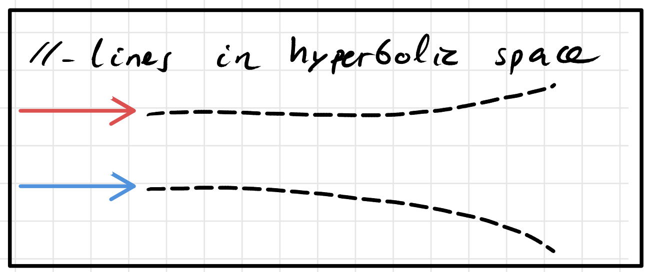
3) A circle can be drawn around any point

4) All right angles are equal

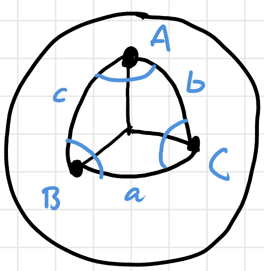
5) Parallel line postulate "parallel rays will never intersect"



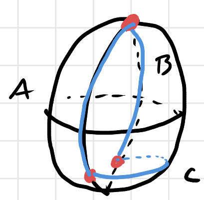




Spherical triangle



- vertices & angles denoted by upper case letters A, B, C
- sum of interior angle greater than 180°

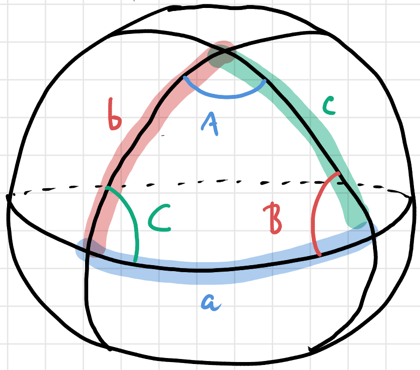


$A = \pi - \varepsilon$

$B = \pi - \varepsilon$

$C = \pi$

$$\boxed{\pi < A + B + C < 3\pi}$$

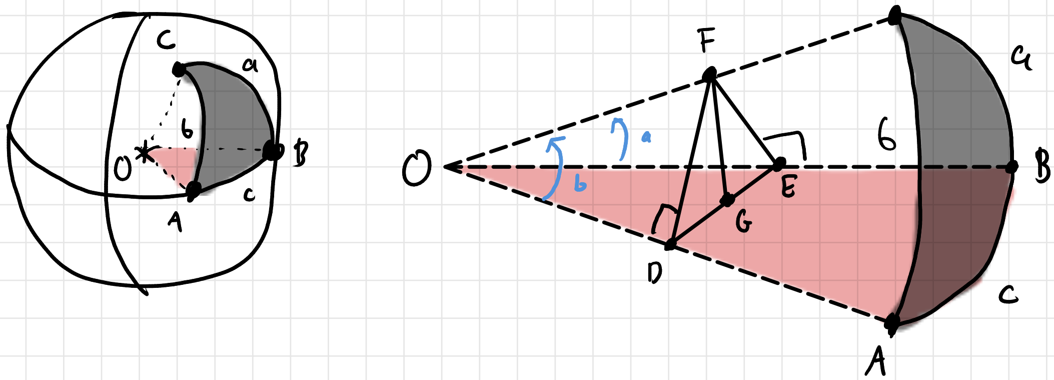


- "sides" denoted by lower case letters a, b, c and measured in <u>radians</u>

  a, b, c are angles measured from center of sphere

• Find the law of sines & cosines for spherical triangles

Consider the following spherical triangle



• $FD \perp OA$

• $FG \perp$ plane AOB

$$\sin \angle FDG = \frac{FG}{DF} = \sin A$$

$$\sin \angle FEG = \frac{FG}{EF} = \sin B$$

$$\Rightarrow \frac{\sin A}{\sin B} = \frac{EF}{DF}$$

$$\sin a = \frac{EF}{OF}$$

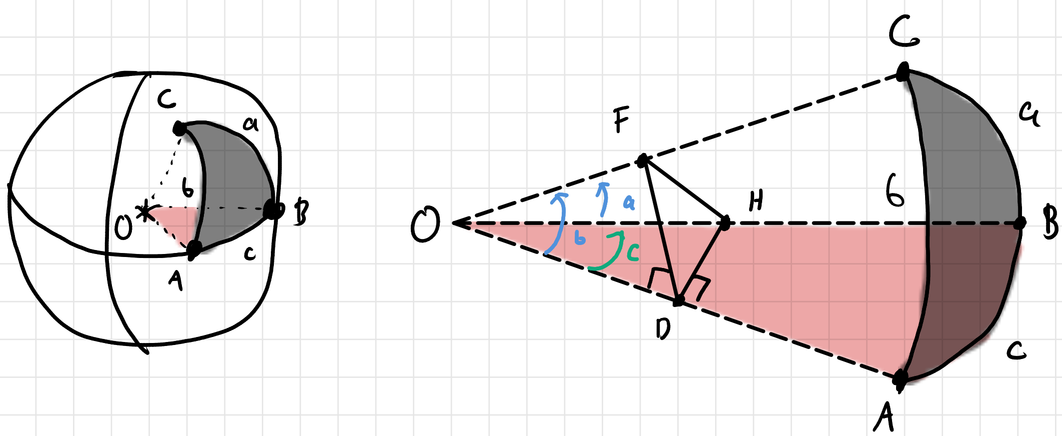
$$\sin b = \frac{DF}{OF}$$

$$\Rightarrow \frac{\sin a}{\sin b} = \frac{EF}{DF}$$

$$\frac{\sin A}{\sin B} = \frac{\sin a}{\sin b}$$

Law of Sines: (spherical triangles) $\boxed{\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}}$

similar trick to get law of cosisines



• $DF \perp OA$

• $DH \perp OA$

apply (Euclidean) law of cosines to triangles $a^2 = b^2 + c^2 - 2bc\cos A$

$$OHF: (HF)^2 = (OH)^2 + (OF)^2 - 2(OH)(OF)\cos \overbrace{\angle FOH}^{a}$$

$$DHF: (HF)^2 = (DH)^2 + (DF)^2 - 2(DH)(DF)\cos \underbrace{\angle FDH}_{A}$$

• subtract these two equations

$$0 = (OH)^2 + (OF)^2 - 2(OH)(OF)\cos a - (DH)^2 + (DF)^2 + 2(DH)(DF)\cos A$$

• right triangle DOF : $(OF)^2 = (OD)^2 + (DF)^2$

• right triangle DOH : $(OH)^2 = (DH)^2 + (OD)^2$

$$0 = (OD)^2 - 2(OH)(OF)\cos a + (OD)^2 + 2(DH)(DF)\cos A$$

$$\cos a = \frac{(OD)^2}{(OH)(OF)} + \frac{(DH)(DF)}{(OH)(OF)}\cos A$$

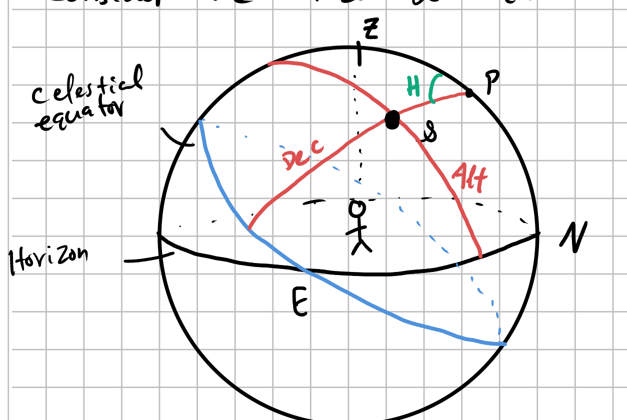
$$\cos a = \frac{(OD)}{(OF)}\frac{(OD)}{(OH)} + \frac{(DF)}{(OF)}\frac{(DH)}{(OH)}\cos A$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

Law of cosines : spherical triangles	$\cos a = \cos b \cos c + \sin b \sin c \cos A$ $\cos b = \cos c \cos a + \sin c \sin a \cos B$ $\cos c = \cos a \cos b + \sin a \sin b \cos C$

Hour Angle & coordinate conversions (text fig 1.6 b)

Consider the horizon & equatorial systems simultaneously



ZPN = observer's celestial meridian

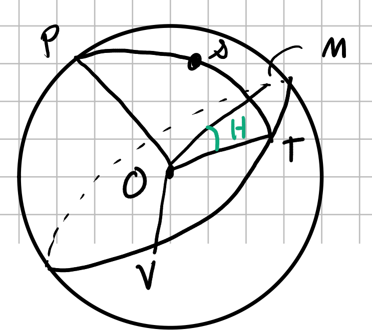
└ divides sky into east half & west half

$\angle ZPS = H =$ hour angle

H negative if in east half

H positive if in west half

Hour angle defined to be the angle between observer's meridian & hour circle through star.



$H = \angle MOT$