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ME140 Final Report

1. Introduction

In this final lab, Team Rocket combined theory with practice to develop a nozzle / fuel grain combination that would result in the highest possible total impulse. To do this, we focused on proven rocketry concepts and precision manufacturing to develop a complex nozzle geometry, while incorporating our test fire data into careful calculations and simulations to guide the evolution of our fuel grains.

2. Nozzle Design and Machining

To design the converging section of our nozzle, Team Rocket drew back on knowledge gained during our Compressible Fluids course (ME131B). In that course, we learned that a good way to design a converging nozzle was to use a 5th-order polynomial:

$$y = a_o + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$$

And to fully define the polynomial with 6 known end conditions, we used:

$$\left.\frac{dy}{dx}\right|_{x=0} = \left.\frac{dy}{dx}\right|_{x=L} = 0 \qquad \left.\frac{d^2y}{dx^2}\right|_{x=0} = \left.\frac{d^2y}{dx^2}\right|_{x=L} = 0$$

$$L = D \qquad y(L) = 0$$

Thus, after solving for the unknown constants, we obtained a function for the desired converging geometry (as seen in the left half of Figure 1 below).

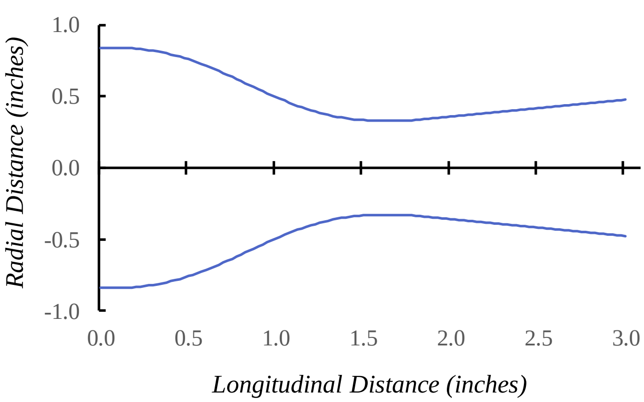

Figure 1. Cross-sectional view of the final nozzle geometry.

We then used this data to extract the radial distance from the centerline to the edge geometry at steps every 0.025 inches in the longitudinal direction. This allowed us to use a boring bar on the machine lathe to turn to the given radial and longitudinal dimensions to machine the complex geometry in a simple and fast manner.

As for the diverging section, we initially explored creating a bell-shaped geometry using the Method of Characteristics, a solution that would involve solving several complex partial differential equations (PDEs). Although we found a MATLAB simulation that would help us in solving these PDEs to design the proper diverging section (see Figure 2 below), it required the inputting of more parameters than we had access to or control over. We then decided that it would be sufficient to design the diverging section as a conical geometry. Knowing that our chamber pressure to atmospheric pressure ratio was about 10, we used the thrust coefficient to area ratio graph presented in class to select an expansion ratio of approximately 2.2.

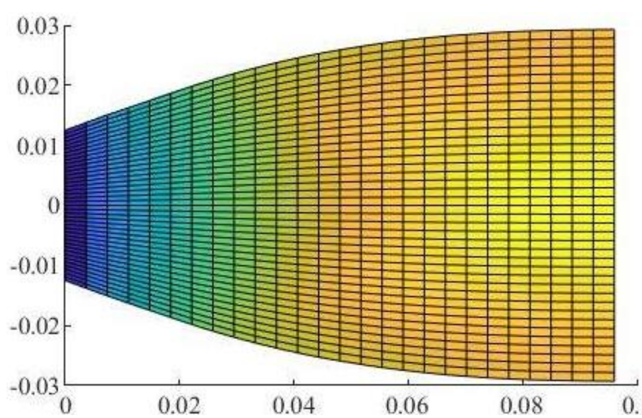

Figure 2. Plot of a proposed diverging bell using the Method of Characteristics simulation.

In regard to the throat section that would connect the converging-diverging sections, we initially chose a diameter of 0.606 inches as a starting point. However, when we realized that our grain design would foul with this small throat diameter (as we had fouled with the 0.65-inch TA nozzle), we then decided to increase the diameter of our throat to 0.656 inches (based on an available 21/32-inch drill bit).

3. Fuel Grain Design and Machining

Prior to designing our first fuel grains, we looked for a way to simulate the grain burn without having to expend limited fuel on test fires. We found a solution in a simulation made by Ian Berve, a former ME140 student[1]. We used his simulation as the starting point to our own simulations, which grew more accurate as we incorporated our own test data. The simulation allowed us to iterate quickly and efficiently through port designs before firing. Based on these simulations, we would take the best performing design and machines the grains accordingly. However, this method was not perfect, as the simulation was limited in that it assumed constant burn throughout the grain.

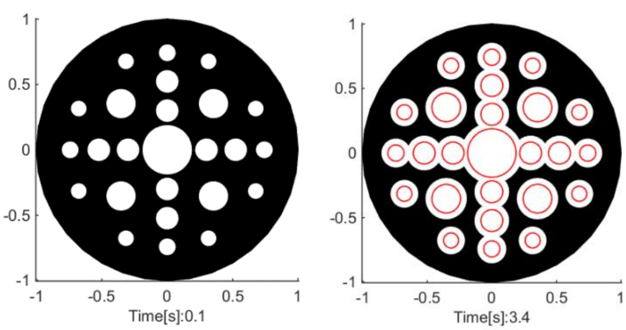

Figure 3. The fuel grain regression simulation we adapted could approximately give us the amount of fuel burned using an aggregate experimental regression rate.

Ultimately, we chose a "cross-shaped" pattern based on maximizing the available cross-sectional surface area. Once our port pattern was finalized, decided to increase our potential for a clean burn by incorporating a "tiered-cake" design, in which each section of the fuel grain would have progressively larger port holes, with the final holes being 25% bigger than the initial port holes. We then proceeded to laser-cut Duron template caps that contained our port patterns for the grains, allowing us to slip a cap onto the grain to ensure precise port drilling.

4. Grain Surface Area vs. Mixture Ratio

To refine our port pattern further, we wanted to optimize the total surface area used. Using the MATLAB fuel grain simulation, we simulated the combustion of this design using a range of port sizes, iterating as we gathered new data. In doing so, we changed the overall longitudinal surface area while maintaining the same design. Using this simulation, we gained an understand of how the surface area for this specific design is related to mixture ratio. As shown in Figure 4 below, the mixture ratio reaches its lowest theoretical point with a total port internal surface area of approximately 0.045 m^2. We used this optimized design for our second, third, and final test fires, helping to reduce our mixture ratio as well as reduce the chance of our design fouling.

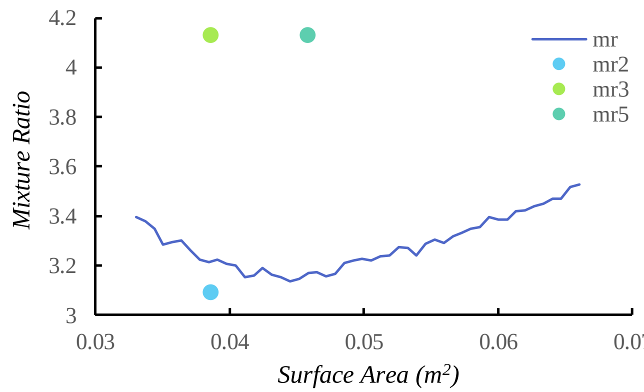

Figure 4. Mixture ratio as a function of grain surface area for our 'X' fuel grain design.

5. C* vs. Mixture Ratio

After each of our tests we compared our c* data with our predicted results based off our Cantera analysis performed in Project 7. By using Cantera, we were able to create a rough over-and-under estimate for c* by assuming the flow was either still equilibrating or frozen through the nozzle. We then calculated our c* from our tests using the following equation and then compared to these theoretical results.

$$c^* = \frac{P_{chamber} \times A_{throat}}{mdot_{fuel} + mdot_{O2}}$$

As shown in Figure 5 below, most of our tests were near the theoretical curves. As we continued our tests we successfully reduced our mixture ratio however we also slow a small drop in our c* value as well.

[1] http://cargocollective.com/irberve/Hybrid-Rocket-Fuel-Grain-and-Nozzle.

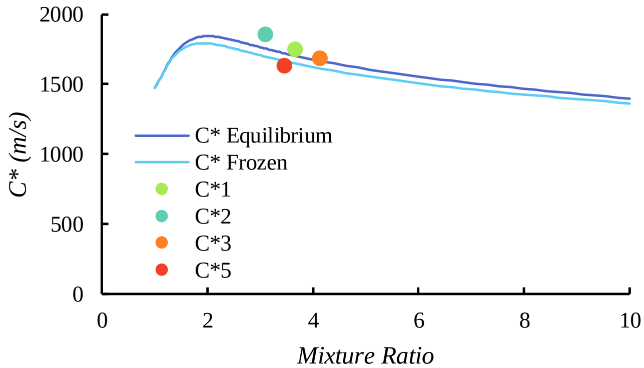

Figure 5. Theoretical and experimental characteristic velocity as a function of mixture ratio.

6. Thrust Coefficient vs. Mixture Ratio

To further guide our design direction, we also produced theoretical coefficients of thrust from our Cantera analysis to compare to our actual data. Using this coefficient as well as our exit velocity, we were able to get a good idea about how well our nozzle was accelerating our flow. For both the theoretical and experimental values we used the following equation:

$$C_F = \frac{V_{exit}}{C_{star}}$$

As shown in Figure 6 below, our tests were all far below the theoretical values that we calculated could be obtained. However, as with our c* coefficient, we did see improvement in our coefficient of thrust after each new iteration of our design, as well as a small reduction in our mixture ratio.

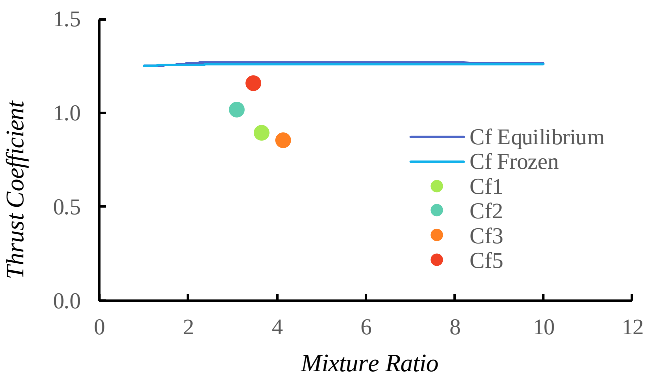

Figure 6. Theoretical and experimental coefficient of thrust as a function of mixture ratio.

7. Exit Velocity vs. Mixture Ratio

Figure 7 below shows a similar trend as our plot of coefficient of thrust vs. mixture ratio. This is expected, as we see that the coefficient of thrust is directly proportional to the exit velocity. We see that our final test was our closest point to our theoretical values and our highest velocity from all our fires.

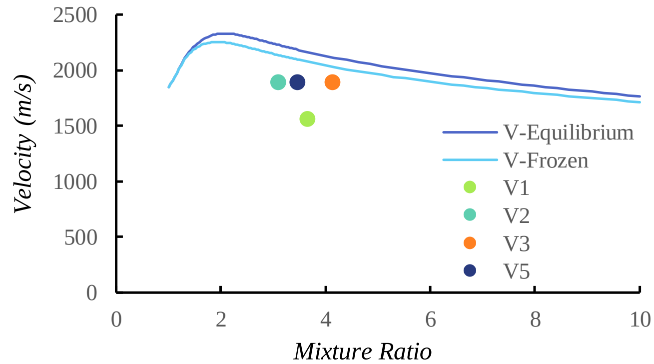

Figure 7. Theoretical and experimental exit velocity as a function of mixture ratio.

8. Results

The results of our test fires, accompanied by sample data that the TA's graciously provided, are provided in the table below.

Test	Mixture Ratio	Total Impulse	Specific Impulse	Foul?
TA Test Data 1	10.944	221.89	75.83	-
TA Test Data 2	25.359	176.01	63.09	-
Test Fire 1	3.662	561.20	159.73	No
Test Fire 2	3.091	736.29	193.10	Yes
Test Fire 3	4.131	543.88	146.96	No
Test Fire 4	50.104	40.49	13.48	No
Test Fire 5	3.461	742.38	196.09	No

For our first fire with the TA nozzle, we used a dual concentric ring pattern with an omitted port in each concentric ring to avoid slivering. In addition, we incorporated a straight-walled mixing chamber to increase our combustor pressure and to induce better burns in the outer ports. Our second fuel grain had more ports with a smaller diameter in a cross pattern (as stated in section 3) and a conical mixing chamber. These changes increased total impulse greatly but unfortunately also led us to foul. Our third grain was similar to our second pattern, with the main improvement being an increase in port diameter. It was also the first test fire that used our own machined nozzle. Unfortunately, the total impulse dropped greatly from our previous fire to our dismay. As we saw in the thrust versus time graph, this test initially failed to ignite and therefore decreased total impulse. We decided to fire test 4 using our nozzle and the TA fuel grain (a single ¼ in center hole) just as a sanity check, which proved to be unproductive. Our final modification to the grain design was a “tiered-cake” design, in which the port sizes increased by 8% from tier to tier from combustor to nozzle. To our relief, our last fire was our best as we achieved a total impulse of 742 Ns, validating our design choices both for the nozzle and fuel grains. Figure 8 below shows a combined plot of our test fires.

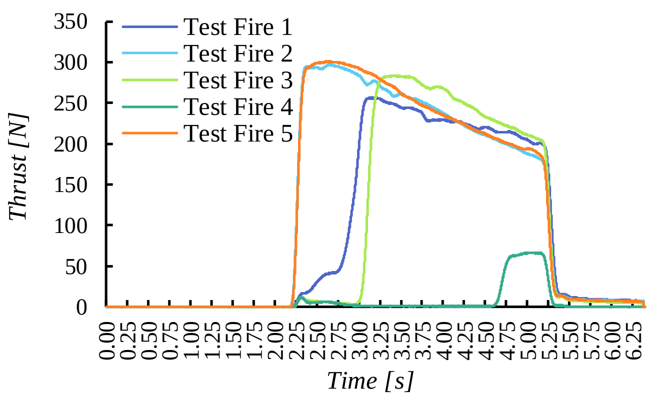

Figure 8. Plot of thrust vs. time for test fires.

9. Next Steps

Given the opportunity to repeat this experiment with gained experience, there is quite a bit that we would change. For example, we would fire each of our designs twice, to account for lab equipment discrepancies, as well as to build a more complete data set. Given our limited data, it was often difficult to iterate our designs properly or to discern the successful features of each fire. Additionally, while we implemented a “tiered-cake” design into our grain to induce turbulence, we would also like to introduce tapped holes in our fuel grain design to increase turbulence and mixing, as well as tapered ports to allow for a cleaner burn further from the combustion chamber. Lastly, assessing our data, the chamber pressure was well below orifice pressure in our last test fire, indicating that our fuel grain port design could potentially be more aggressive.

As for our nozzle, a potential increase in thrust could be obtained by increasing our expansion ratio by either decreasing the throat diameter or increasing the exit diameter. Furthermore, we would probably choose to invest more time into designing the diverging section of the nozzle using the method of characteristics to allow for a better conditioned flow at the exit, one that had more thrust parallel to the axis of the nozzle.

10. Team Reflection

To optimize our team performance, we divided into two small subgroups to work in parallel on the machining and analysis components of the project. This allowed us to fire a total of 5 times with varying combinations of fuel grains and our nozzle. Ultimately, this project was a good exercise in identifying real-world anomalies and validating computational models with practical experimentation. As for the total time spent on this project, in the past two weeks we spent approximately 40 hours in the product realization lab machining our fuel grain and nozzle, and about the same again on analysis.

11. Conclusion

This project served as a fitting capstone to the aggregation of our thermodynamics and fluids knowledge, as well as validation of the fundamentals of engineering that we have had instilled in us. By working through all phases of this project, from modelling and design to machining and testing, we were able to develop and refine our intuition as to how different aspects of rocketry affect each other. In the end, we were able to discover and understand the critical roles that nozzle geometry, port design, and mixing chambers held in successful firings, and design a fuel grain / nozzle combination that not only expanded our knowledge but led us to placing first in the category of total impulse.

12. Acknowledgements

Team Rocket wishes to thank Dr. Reggie Mitchell and his amazing team of TA’s for their guidance and support throughout this project, and for keeping us motivated through the many late hours that led to our successful test fires.