MATHEMATICAL PRINCIPLES FOR COMPUTER SCIENCE
WORKBOOK 2021

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Module Resources	
Prescribed Book for this Module	The IIE. 2021. *Mathematical Principles for Computer Science Module Manual*. Independent Institute of Education: Sandton.

Module Purpose	
The purpose of this module is to provide students with a foundational knowledge of the basic mathematical principles and logical skills to solve Application Development and Networking problems.	
Module Outcomes	
MO1	Demonstrate knowledge and understanding of the basic mathematical calculations and principles.
MO2	Demonstrate knowledge and understanding of logical operations, logical gates and relevant calculations.
MO3	Apply mathematical problem solving skills to given hypothetical scenarios.
MO4	Demonstrate knowledge and understanding of the application of the different number conversions and skills that apply to programming and networking.

Must-Know Facts and Formulas

Concept	Fact
Integers	*. . . , -3, -2, -1, 0, 1, 2, 3, . . .*
Rational	Fractions, that is, anything that can be expressed as a ratio of integers
Real	integers plus rational plus special numbers such as $\sqrt{2}$, $\sqrt{3}$ and π
Factors	The factors of a number divide into that number without a remainder. Example: the factors of *52* are *1, 2, 4, 13, 26*, and *52*
Prime Factorization	break up a number into prime factors (2, 3, 5, 7, 11, . . .) $200 = 4 \times 50 = 2 \times 2 \times 2 \times 5 \times 5$ $52 = 2 \times 26 = 2 \times 2 \times 13$
Greatest Common Factor	multiply common prime factors $200 = 2 \times 2 \times 2 \times 5 \times 5$ $60 = 2 \times 2 \times 3 \times 5$ $GCF(200, 60) = 2 \times 2 \times 5 = 20$
Least Common Multiple	check multiples of the largest number *LCM (200, 60): 200 (no), 400 (no), 600 (yes!)*
Multiples	The multiples of a number are divisible by that number without a remainder. Example: the positive multiples of 20 are 20, 40, 60, 80, . . .
Powers, Exponents, Roots	• $x^a \cdot x^b = x^{a+b}$ • $(x^a)^b = x^{a \cdot b}$ • $x^0 = 1$ • $x^a / x^b = x^{a-b}$ • $(xy)^a = x^a \cdot y^a$ • $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$ • $1/x^b = x^{-b}$ • $(-1)^n = \{ +1, \text{ if } n \text{ is even}; -1, \text{ if } n \text{ is odd}\}$ • If $0 < x < 1$, then $0 < x^3 < x^2 < x < \sqrt{x} < \sqrt[3]{x} < 1$.

Terminology

Term	Description/ Definition
integers	Integers are numbers without a fractional part (and that is why they are often called the whole numbers). Integers include 1, 2, 3, . . . (the counting numbers) along with 0, −1, −2, −3, . . .
remainder	When an integer is divided by another, the remainder is the integer amount that is left over. For example, when 66 is divided by 7, the remainder is 3, since 7 goes into 66 a total of 9 times, with 3 left over: $66 = 7 \times 9 + 3$.
even integers	Even integers can be divided by two without a remainder. The even integers include 0, 2, 4, 6, 8, 10, 12, . . . , 2753 , . . . along with −2, −4, −6, . . . , −37954, . . .
odd integers	Odd integers cannot be divided by two without a remainder. The odd integers include 1, 3, 5, 7, 9, 11, . . . , $2^{452} + 1$, . . . along with −1, −3, −5, . . . , −37955, . . .
positive, negative	A positive number is greater than zero, and a negative number is less than zero. Zero is neither positive nor negative. Note that a negative number raised to an even power is positive, and when raised to an odd power is negative. For example, $(-1)^{374} = 1$ but $(-1)^{373} = -1$.
multiple	A multiple of a number is the result of multiplying that number by any integer. For example, the multiples of 15 include 15, 30, 45, 60, . . . but also 0, −15, −30, . . .
factor	A factor of a number is any integer that can divide that number without a remainder. For example, the factors of 12 are 1, 2, 3, 4, 6, and 12; the factors of 29 are just 1 and 29.
prime	A prime number is a positive integer that has only two factors: itself and 1. The prime numbers include 2, 3, 5, 7, 11, . . . but do not include 1 (the number 1 only has one factor, not two). The prime factors of a number are the factors of the number that also are prime. For example, the prime factors of 12 are 2 and 3 and the only prime factor of 29 is 29.

Term	Description/ Definition
in terms of	You are often to solve for some variable “in terms of” another variable or variables. For example, if 6a + 12b = 3a + 6b − 9c + 15, and you are asked to solve for a in terms of b and c, then simply solve for a with all other variables and numbers on the other side of the equation. Here, you would get 3a = 15 − 6b − 9c so that a = 5 − 2b − 3c.
less, fewer	Some questions involve translating from words into an algebraic equation that you can solve. When you see “less” or “fewer” you should think subtraction. For example, “y is three less than twice x” is equivalent to y = 2x − 3. Another example: “Aubrey has 6 fewer cabbages than Bill does” could be written in equation form as A = B − 6. Note that the number or expression that comes before “less” or “fewer” appears after the minus sign in the equivalent expression.
rational	A rational number is any number that can be written as a fraction: a ratio of two integers. Rational numbers include 1/2, 3/4, 5 (since 5 = 5/1), 22/7, 1/3, and so on. These numbers can always be written as a finite decimal or as an infinite decimal that repeats. For example, 2/5 = 0.4, 7/11 = $0.63\overline{63}$, and 22/7 = $3.\overline{142857}$.
real	The real numbers are all the numbers on the number line, including the integers, the rational numbers, and everything else, which includes for example the irrational numbers such as $\sqrt{2}$ and π. Not to be confused with the fake numbers

Learning Unit 1: An Introduction to Arithmetic	
Material used for this learning unit: • Module Manual Learning Unit 1; • Workbook; • IIE Learn.	**My notes**
Selected Online Resources: Exponents: • http://mcckc.edu/tutoring/docs/br/math/expon_logar/Exponent_Rules_Practice.pdf • https://magoosh.com/gmat/2014/challenging-gmat-problems-with-exponents-and-roots/ Numbers: • https://www.mathtutordvd.com/worksheets/prealgebra_vol1/a_Pre-Algebra_Vol1_Worksheet_1_Real_Numbers.pdf Scientific Notation: • http://ieer.org/wp/wp-content/uploads/2005/07/ScientificNotationWorksheet+Answers.pdf	

1 Exercise A: Numbers

1. In a faraway galaxy, there are 8,768,351,202 stars. What does the digit 3 represent in this problem?

2. The development of a city over the past twenty years cost R962,234,532,274,312. What is the value of the digit 6 in this number?

3. A company had a new office building constructed. The final cost was seventy-four million, three hundred sixty-two dollars. Write this number in standard form.

4. A record number of 23,386 people voted in a city election. Round this number to the nearest hundred.

5. Which expression can be used to correctly compare the numbers 85 and 19?

6. A farmer has produced 230 pumpkins for the autumn harvest. Last year, he produced 198. Write an expression that compares these two numbers.

7. A local company built a playground at a park. It took the company 124 hours to plan out the playground, 243 hours to prepare the site, and 575 hours to build the playground. Find the total number of hours the company spent on the project.

8. A woman preparing an outdoor market is setting up a stand with 321 papayas, 45 peaches, and 213 mangos. How many pieces of fruit in total does the woman have on her stand?

9. Each year, John is out of the U.S. on business for 142 days, including travel time. The number of days per year he is in the U.S. is the difference of 365 days and 142 days. How many days during the year is John in the U.S.?

10. Simplify $60 - 30 \div 3 \cdot 5 + 7$.

11. Simplify $4 - 3[20 - 3 \cdot 4 - (2 + 4)] \div 2$.

12. Simplify $40 - (4 + 6) \div 2 + 3$.

13. Identify the constant and variable in the expression $24 – x$. John is planning a rectangular garden that is 2 feet wide. He hasn't decided how long to make it, but he's considering 4 feet, 5 feet, and 6 feet. He wants to put a short fence around the garden. Using x to represent the length of the rectangular garden, he will need $x + x + 2 + 2$, or $2x + 4$, feet of fencing. How much fencing will he need for each possible garden length? Evaluate the expression when $x = 4$, $x = 5$, and $x = 6$ to find out.

14. The number 0 belongs to which of the following sets of numbers? [natural numbers and/ or whole numbers and/ or integers]

15. Find $|x|$ when $x = −7$.

16. After Bangor reached a low temperature of −13°, the temperature rose only 4 degrees higher for the rest of the day. What was the high temperature that day?

17. Find $22\frac{1}{3} - x$, when $x = -\frac{3}{5}$

18. Everett paid several bills without balancing his cheque book first! When the last cheque he wrote was still to be deducted from his balance, Everett's account was already overdrawn. The balance was −R201.35. The final cheque was for R72.66, and another R25 will be subtracted as an overdraft charge. What will Everett's account balance be after that last cheque and the overdraft charge are deducted?

19. What is the reciprocal, or multiplicative inverse, of −12?

20. Find $\frac{6}{7} \div \frac{3}{10}$. Write the answer in lowest terms.

21. Find $4x \div (-6)$ when $x = -\frac{2}{3}$

22. During a storm, the temperature dropped by $\frac{1}{2}$ degree every minute. At the beginning of the storm, the temperature was 83°F. An expression giving the temperature t minutes after the storm began is $-\frac{1}{2}t + 83$. What was the temperature after 8 minutes?

23. In solving the algebraic equation $2(x - 5) = 2x + 10$, you end up with $-10 = 10$. What does this mean?

24. How many solutions are there for the equation:

$$2\left[\frac{1}{4}(4y-8)+3y\right]+7=3(y+1)+5y$$

25. Amanda's dad is twice as old as she is today. The sum of their ages is 66. Find the ages of Amanda and her dad.

26. Gina has found a great price on paper towels. She wants to stock up on these for her cleaning business. Paper towels cost R1.25 per package. If she has R60 to spend, how many packages of paper towels can she purchase? Write an equation that Gina could use to solve this problem and show the solution.

27. Levon and Maria were shopping for candles to decorate tables at a restaurant. Levon bought 5 packages of candles plus 3 single candles. Maria bought 11 single candles plus 4 packages of candles. Each package of candles contains the same number of candles. After finishing shopping, Maria and Levon realized that they had each purchased the same exact number of candles. How many candles are in a package?

28. The money from two vending machines is being collected. One machine contains 30 dollar bills and a bunch of dimes. The other machine contains 38 dollar bills and a bunch of nickels. The number of coins in both machines is equal, and the amount of money that the machines collected is also equal. How many coins are in each machine?

29. Albert and Bryn are buying candy at the corner store. Albert buys 5 bags and 3 individual pieces; Bryn buys 3 bags and then eats 2 pieces of candy from one of the bags. Each bag has the same number of pieces of candy.
After Bryn eats the 2 pieces, she has exactly half the number of pieces of candy as Albert. How many pieces of candy are in each bag?
Pick the equation that could be used to solve the problem above. Use the variable b to represent the number of pieces of candy in one bag.

30. Solve the equation $d = r \cdot t$ for t.

2 Exercise B: Inequalities

1. Check that $x < 2$ is the solution to $x + 3 < 5$.

2. Solve for x.

$$\frac{15}{2} + x > -\frac{37}{4}$$

3. Solve for x.

$$-\frac{1}{3}x > -12x$$

4. Solve for *a:* $-\frac{a}{5} < \frac{35}{8}$

5. Solve for *x*.

$$\frac{1}{3}(x+3)+\frac{1}{2}>-\frac{9}{2}$$

6. A student is solving the inequality $\frac{5}{2}+7x \leq 4x-\frac{7}{2}$. If she combines like terms, which of the following inequalities could she see?

A) $-6 \leq 3x$

B) $3x \leq -6$

C) $\frac{19x}{2} \leq \frac{x}{2}$

D) $x \leq \frac{-6x}{7}$

7. Solve for *d*.

$$\frac{3}{5}(2d-5)\leq 4\left(7-\frac{1}{5}d\right)$$

8. Solve for *x*. $5\left[2(3-x)-1\right]\leq 27$

9. Solve for *y*.

$2y + 7 < 13$ or $-3y - 2 \leq 10$

10. Solve for h.

    $h + 3 > 12$ *or* $3 - 2h > 9$

11. Solve for w.

    $3|4w - 1| - 5 = 10$

3 Exercise C: Estimation

1. Estimate the sum 1,472 + 398 + 772 + 164 by rounding each number to the nearest hundred.

2. In three months, a freelance graphic artist earns R1,290 for illustrating comic books, R2,612 for designing logos, and R4,175 for designing web sites. Estimate how much she earned in total by first rounding each number to the nearest hundred.

3. Estimate the difference of 474,128 and 262,767 by rounding to the nearest thousand.

4. When buying a new computer, you find that the computer tower and keyboard cost R1,295, the monitor costs R679, the printer costs R486, the 2-year warranty costs R196, and a software package costs R374. Estimate the total cost by first rounding each number to the nearest hundred.

5. A space shuttle traveling at 17,581 miles per hour decreases its speed by 7,412 miles per hour. Estimate the speed of the space shuttle after it has slowed down by rounding each number to the nearest hundred.

6. A factory produces 58 packages of cookies in one hour. There are 32 cookies in each package. Which is the best estimate of the number of cookies the factory produces in one hour?

4 Exercise D: Properties and Laws of Whole Numbers

1. Write the expression 10 + 25 in a different way, using the commutative law of addition, and show that both expressions result in the same answer.

2. Rewrite 15 + 12 = 27 in a different way, using the commutative law of addition.

3. Write the expression 30 • 50 in a different way, using the commutative law of multiplication, and show that both expressions result in the same answer.

4. Problem: Rewrite 52 • 46 in a different way, using the commutative law of multiplication.

5. Rewrite (5 + 8) + 3 using the associative law of addition. Show that the rewritten expression yields the same answer.

6. Rewrite 10 + (5 + 6) using the associative property.

7. Rewrite (10 • 200) • 24 using the associative law of multiplication and show that the rewritten expression yields the same answer.

8. 10 • 2 = 20 is rewritten as 2 • 10 = 20. Was this expression rewritten using the commutative law or the associative law?

9. 12 • (6 • 2) = 144 is rewritten as (12 • 6) • 2 = 144. Was this expression rewritten using the commutative law or the associative law?

10. 17 • 3 = 51 is rewritten as 3 • 17 = 51. Was this expression rewritten using the commutative law or associative law?

11. Jim is buying 8 pears, 7 apples, and 2 oranges. He decided the total number of fruit is 8 + 7 + 2. Use the commutative property to write this expression in a different way. Then find the total.

12. Rewrite the expression 5(8 + 4) using the distributive property of multiplication over addition. Then simplify the result.

13. Rewrite the expression 30(2 + 4) using the distributive property of addition.

14. Rewrite $\frac{1}{2} \bullet \left(\frac{5}{6} \bullet 6\right)$ using only the associative property.

15. Use the distributive property to evaluate the expression 5(2*x* – 3) when *x* = 2.

16. Solve $\frac{1}{2}x - 3 = 2 - \frac{3}{4}x$ by clearing the fractions in the equation first.

17. Solve for *a*: $\frac{1}{4}(a+3)=2-a$

5 Exercise E: Exponents

1. Write 10 • 10 • 10 • 10 • 10 • 10 in exponential notation.

2. Find $\sqrt{36}$.

3. Simplify: $100-5^2 \bullet 4$

4. Simplify $\frac{5-[3+(2\bullet(-6))]}{3^2+2}$

5. Simplify $\left[\dfrac{3^3+3}{(-2)(-3)}\right]^2+1$.

6. Simplify: $(5|3-4|)^3$.

7. Solve $\dfrac{1}{2}(2+a)=\dfrac{3a+4}{3^2}$.

6 Exercise F: Revision

1. If $3^{n-3} + 3^2 = 18$, what is the value of n?

2. Three consecutive integers are such that four times the least integer is three times the greatest. What is the greatest of these three integers?

3. If $2^x + 2^{x+2} = 40$, then the value of x is which of the following?

4. When each side of a particular square is lengthened by 2 inches, the area of the square increases by 32 square inches. What is the length in inches of a side of the original square?

5. If $x < y < 0$, which of the following is greatest in value?
   A. $2x + y$
   B. $x + 2y$
   C. $x - 2y$
   D. $y - 2x$
   E. $2y - x$

6. When the positive integer m is divided by 7, the remainder is 4. What is the remainder when 2m is divided by 7?

7. If a and b are positive integers such that a + b = 9, then what is the value of $\frac{b-9}{4a}$?

8. If $\frac{x}{3} = \frac{y}{3}$, which of the following is equivalent to $\frac{y}{3}$?

   A. $\frac{x}{6}$
   B. $\frac{2x}{9}$
   C. $\frac{x}{3}$
   D. $\frac{2x}{3}$
   E. X

9. If w is a positive integer and $(2^w)^w = 2^8 \cdot 2^8$, then what is the value of w?

10. If $-1 < y < 0$, which of the following is *not* between -1 and 0?

A) $-1 - y$ B) $\frac{y}{2}$ C) $-\sqrt{y+1}$ D) $\frac{1}{y}$ E) $-y^2$

11. If y is 12 less than the product of a and b, then which of the following is an expression for y in terms of a and b?

A. $12 - (a + b)$
B. $(a + b) - 12$
C. $12 - ab$
D. $a/b - 12$
E. $ab - 12$

12. Consider the equation below:

$$\frac{7}{3x+4} = \frac{7}{6x-2}$$

If x satisfies the equation above, then what is the value of x?

13. At a fancy vegetable stand, Jim bought purple tomatoes for R8 each and Aubrey bought organic giant cabbages for R12 each. In total, they paid R104 for 10 vegetables.
How many cabbages did Aubrey buy?

14. A caterer will make cabbage sandwiches for special occasions by charging a one-time fee of R50, plus R8 for each sandwich. Which of the following is the total cost in rands for an order of c cabbage sandwiches?

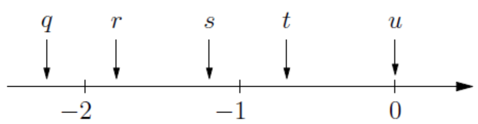
A. $42c$
B. $50c + 8$
C. $58 + c$
D. $50 + 8c$
E. $50 + c + 8$

15. If $4^{n-2} + 4^2 = 32$, then what is the value of n?

16. If $5500 > m+2000 > 5000$, and m is an integer, then what is the least possible value of m?

17. The total cost of 30 identical erasers is y rands. At this rate, what is the total cost in dollars of 70 of these erasers in terms of y?

18. Consider the number line below:



The numbers q, r, s, t, and u are indicated in the number line above. Which of the following is the largest in value?

19. Keisha has a collection of dimes and nickels worth a total of R4. If she has 50 coins in all, how many nickels does Keisha have?

20. If $2^{p+2} + 2^{p+1} = 96$, then what is the value of p?

21. If $1 < 6a - 1 < 2$, what is one possible value of a?

22. If x percent of y is z, then z is what percent of xy?

23. If $a^b = 4 - ab$ and $b^a = 1$, where a and b are positive integers, what is the value of a?

24. When a number x is subtracted from 36 and the difference is divided by x, the result is 2. What is the value of x?

25. The senior class spent 20% of its budget on a big box of tomatoes. It then spent one-fourth of the remaining funds on a helium blimp. Finally, it spent 1/3 of the remaining funds on an extra-large helicopter for the prom. What fraction of the original funds were not spent?

26. The sum of the integers p and q is 495. If p is divided by 10, the result is equal to q. What is the value of p?

27. In a box containing only purple and green marshmallows, 6 marshmallows are purple. If the probability of choosing a purple marshmallow from the box Is 1/3, how many green marshmallows are in the box?

28. If m and n are prime numbers greater than 2, which of the following could also be a prime number?
   A. $m + n$
   B. $m + n + 1$
   C. $m + n + 2$
   D. mn
   E. $mn + 1$

29. You are given the equation below:

$$\frac{a}{3}+\frac{b}{6}=1$$

If a and b are positive integers in the equation above, then what is the value of $a \cdot b$?

30. A meter is a measure of length, and 10 decimetres is equal in length to one meter. How many decimetres are equal in length to 12.5 meters?

31. Let *m* be an even integer. How many possible values of *m* satisfy $\sqrt{m+7} \leq 3$?

6.1 Exercise F: Revision – Solutions

1. Isolate the term with n:
$3^{n-3} = 18 - 32 = 9$, so that $3^{n-3} = 32$.
Since the bases are the same (both 3), the exponents must be equal:
$n-3 = 2$ which means that $n = 5$.

2. Let x be the greatest of the three integers. Then, the three integers from least to greatest is $x - 2$, $x - 1$, and x. Since four times the least is three times the greatest, $4(x - 2) = 3(x)$, or $4x - 8 = 3x$. Solving for x gives $x = 8$.

3. There is a common factor of 2x in the left-hand side of the equation:
$2^x + 2^{x+2} = 2^x(1+22) = 2x(5) = 40$.
Dividing both sides by 5 gives $2^x = 8$, which means that $x = 3$.

4. Let the length of one side of the original square be s.
Then, the original square's area is s^2.
When the length of the side increases to s+2, the area increases to $(s + 2)^2$.
The increase in area is $(s + 2)^2 - s^2 = s^2 + 4s + 4 - s^2 = 4s + 4 = 32$.
Solving for s gives $4s = 28$ so that $s = 7$.

5. **Answer: D**

   Strategy Solution: Plug in easy numbers for *x* and *y*! Do not forget to use numbers that satisfy the requirement $x < y < 0$. Good choices here would be $x = -2$ and $y = -1$. Next, go through the answers, plugging in the numbers you chose, until you find the largest number. In this case, you will get −5 for answer A, −4 for B, 0 for C, 3 for D, and 0 for E, so answer D is correct.

   Alternative Solution: Since *x* and *y* are negative, answers A and B are negative. However, answer C is the negative of answer E, so either C or E must be positive or zero, making A and B incorrect. Also, whenever $x = 2y$, answers C and E are both zero. Therefore, answer D must be correct since there can only be one correct answer for any *x* and *y*.

6. The remainder is 4 when *m* is divided by 7, so it must be true that $m = 7n + 4$, where *n* is an integer. Therefore, $2m = 14n + 8$ and $2m/7 = 2n + 8/7 = 2n + 1 + 1/7$. So, when 2*m* is divided by 7, the quotient is $2n + 1$ and the remainder is 1.

7. Since $a+b = 9$, $b = 9-a$ so that $b- 9 = -a$. Then, $(b- 9)/4a = (-a)/4a = -1/4$, making answer C the correct one.

8. **Answer B:**

   Strategy Solution: Plug in easy numbers for *x* and *y*, making sure that $x/3 = y/2$. Good choices would be $x = 3$ and $y = 2$ since both sides of the equation are then equal to 1. We are looking for the answer that is equal to $y/3 = 2/3$. Go through the answers, plugging in 3 for *x* until you get 2/3. You will find that only answer B equals 2/3, so it must be the correct one.

   Alternative Solution: Since $x/3 = y/2$, $y = 2x/3$ and $y/3 = 2x/9$, so answer B is correct.

9. You may also realize that the equation can be written: $2^{w \cdot w} = 2^{16}$, so that $w \cdot w = 16$, or $w = 4$.

10. Solution: The question is asking you to identify which of the answers is *not* between −1 and 0 whenever y is between −1 and 0. **Plug in a real number** for y to make this problem more concrete. In this question, any easy number will do as long as it is between −1 and 0. For example, try $y = -0.5$, and check each expression in the answers until you get a number that is *not* between −1 and 0. If $y = -0.5$, you will find that answer D is −2, so D is the correct answer. Use an *easy* number; in this case, −0.5 is better than, say, −0.29.
    The "plug in real numbers" strategy is very powerful: it can also be used on the SAT when the answers are not given, i.e., in the student-produced response questions (also known as the "grid-in" questions).

11. E

    Strategy: plug in real numbers. Put in easy numbers for a and b. For example, if $a = 3$ and $b = 5$, then $ab = 15$ and $y = 15-12 = 3$. Now go through each answer, putting in the same numbers for a and b, until you get an answer matching the value you obtained for y. For this question, you will get $y = 3$ only for answer E.
    Alternative Solution: The product of a and b is ab, and 12 less than ab is $ab - 12$, so the answer is E.

12. Cross-multiply the given equation: $7(6x-2) = 7(3x+4)$ so that $6x - 2 = 3x + 4$. Solving for x gives $3x = 6$, so that $x = 2$.

13. Let t be the number of tomatoes and c be the number of cabbages. Set up two equations: $8t+12c = 104$ and $t+c = 10$. Solve for t and c by solving one equation for one variable and substituting into the other equation. Here, $t = 10-c$ so that $8(10-c)+12c = 104$. Simplifying, $80-8c+12c = 80+4c = 104$ so that $c = 6$.

14. **D**

    Strategy: plug in real numbers. Suppose 5 sandwiches were ordered (i.e., plug in 5 for c. These sandwiches would cost R8 each, or R40 in all. With the fee, the total cost would be R50 + R40 = R90. Go through the answers, plugging in 5 for c until you get R90. This occurs only for answer D.

    Alternative Solution: If R8 is the cost per sandwich (excluding the fee), then $8c$ is the cost in dollars for c sandwiches. Since the fee is a one-time charge, the total cost in dollars is just $50 + 8c$.

15. $4^{n-2} + 16 = 32$ so that $4^{n-2} = 16 = 4^2$. If $4^{n-2} = 4^2$, the bases are equal, so the exponents must be equal: $n - 2 = 2$ so that $n = 4$.

16. **Strategy: plug in real numbers.** The numbers in the problem are in the thousands, so let's try $m = 2000$. Then, $m + 2000 = 4000$, which is less than 5500 but not greater than 5000. Try a bigger number: $m = 3000$. Now, $m+2000 = 5000$, which is less than 5500, but still not bigger than 5000 (but almost!). Since m is an integer, the next number to try is 3001, making $m + 2000 = 5001$, which is both bigger than 5000 and less than 5500. Any other number that works will be bigger than 3001, so that is the correct answer.

    Alternative Solution: Simplify the inequality by subtracting 2000 from all sides: $3500 > m > 3000$. Since m is an integer, the smallest value of m which is less than 3500 and greater than 3000 is 3001.

17. Let x be the total cost of 70 erasers. Then we set up a proportion:

$$\frac{30}{y} = \frac{70}{x}$$

    and solve for x by cross-multiplying. This gives: $30x = 70y$ so that $x = 7y/3$.

18. Since the numbers are negative except for u, multiplying two of them will result in a positive number. We can therefore eliminate any answers with u since those answers will all be equal to 0. To get the biggest positive number, we need to multiply the two smallest (most negative) numbers marked in the diagram: qr is the correct answer.

19. Let n be the number of nickels. Then, the number of dimes is $50 - n$, and we need $0.05n + 0.10(50 - n) = 4$. Solving for n gives $5 - 0.05n = 4$ so that $0.05n = 1$, or $n = 20$.

20. Rewrite the left-hand side of the equation using the rule that $x^a \cdot x^b = x^{a+b}$. We get: $2_2{}^p \cdot 2^2 + 2^p \cdot 2^1 = 96$. Since 2^p is common to both terms on the left, we have $2^p(2 + 2) = 2^p(6) = 96$ so that $2^p = 16$, or $p = 4$.

21. $1/3 < a < 1/2$ or $.333 < a < .5$

    Strategy: plug in real numbers. Plug in real numbers for a until the inequality works! Notice that $a = 0$ is too small and $a = 1$ is too big. Also, $a = 0.5$ is too big since $6(0.5)-) - 1 = 2$ and our answer needs to be strictly *less* than 2. A number such as $a = 0.4$ works just fine: $6(0.4) - 1 = 1.4$ and $1 < 1.4 < 2$.

Alternative Solution: Adding 1 to all sides of the inequality gives $2 < 6a < 3$ so that $1/3 < a < 1/2$. You could grid any number between 0.333 and 0.5.

22. 1%

Strategy: plug in real numbers. Plug in easy, real numbers for x, and y in order to determine z. Then, plug those numbers into the second equation. For example, if $x = 20$ and $y = 100$, then 20% of 100 is 20 so that $z = 20$. Now, since $xy = 2000$, we have to answer: 20 is what percent of 2000? The answer is just $20/2000 \cdot 100\% = 1\%$.

Math Teacher Solution: We need to convert the phrase "x percent of y is z" into an equation:

$$\frac{x}{100} \cdot y = z.$$

However, this equation is the same as:

$$\frac{1}{100} \cdot xy = z,$$

which means that z is just 1% of xy.

23. **C**

Strategy: plug in real numbers. Plug in real numbers for a and b. Since it isn't clear what numbers to plug in to satisfy the first equation, look at the second equation instead. First, realize that a cannot be 0 since a is a positive integer. (This is a difficult question, so it isn't unusual to have a "trap" answer; for this question, the trap is answer A.) Since $a = 06$, the only way to get $b^a = 1$ is if $b = 1$ (1 to any power is 1). Plugging $b = 1$ into the first equation, we get $a = 4 - a$ so that $2a = 4$ and $a = 2$.

Alternative Solution: For this question, the above solution *is* the math teacher solution. There really isn't any good algebraic solution here, if one exists at all. Let me know if you find one; if so, you can claim the prize (currently, a stuffed Erik the Red plush toy).

24. Convert the words into an equation for x:

$$\frac{36 - x}{x} = 2.$$

Multiplying both sides by x gives $36 - x = 2x$ so that $x = 12$.

25. Let x be the original budget. After the tomatoes, the seniors have $x - 0.2x = 0.8x$.
After the blimp, they have $0.8x - (-\ (1/4)(0.8x) = 0.6x$.
After the helicopter, they have $0.6x - (1/3)(0.6x) = 0.4x$, so the answer is 40%.

26. Set up two equations:

$p + q = 495$

and

$$\frac{p}{10} = q$$

Substituting the second equation into the first, we get $p + p/10 = (1.1)p = 495$ so that $p = 450$.

27. Let x be the number of green marshmallows. Then, $6 + x$ is the total number of marshmallows. The probability of choosing a purple marshmallow is the number of purple marshmallows (6) divided by the total number of marshmallows ($6 + x$). So, we need:

$$\frac{6}{6+x} = \frac{1}{3}.$$

Cross-multiplying, $18 = 6 + x$ so that $x = 12$.

28. **B**

Strategy: plug in real numbers. Try easy prime numbers for m and n and plug them in, checking to see which of the answers is prime. For example, let $m = 3$ and $n = 5$. For this choice, *none* of the answers is prime! When you are plugging in numbers, you need to be able to eliminate all answers except for one. If you can only eliminate a few answers, or none at all, try different numbers to plug in. However, if you do eliminate some answers, you do not need to check those particular answers again. For this question, if you try $m = 3$ and $n = 7$ next, you will find that only answer B is prime, so it must be correct.

Alternative Solution: If m and n are prime numbers, then answer D is a composite number and cannot be prime. Answers A and C are even: since m and n are odd numbers, $m + n$ is even, and $m + n + 2$ is even as well. In addition, mn is odd, so that $mn + 1$ (answer E) is even. By process of elimination, answer B is correct.

29. Combine the fractions using a common denominator of 6:

$$\frac{a}{3} + \frac{b}{6} = \frac{2a}{6} + \frac{b}{6} = \frac{2a+b}{6} = 1$$

so that $2a + b = 6$. Since a and b are positive integers, either $a = 1$ and $b = 4$, or $a = 2$ and $b = 2$. In either case, $a \cdot b = 4$.

30. If one meter equals 10 decimeters, then 12.5 meters equals $12.5 \times 10 = 125$ decimeters.

31. Make up some even integers, plug them in for m, and see if they work! Since we can't take the square root of a negative number, m can't be less than −6. Also, if $m = 2$, then $\sqrt{m + 7} = 3$, but any larger value of m won't work. So, the possible values are −6, −4, −2, 0, and 2.

Learning Unit 2: Equations and Expressions	
Material used for this learning unit: • Module Manual Learning Unit 2; • Workbook; • IIE Learn.	**My notes**

1 Exercise A: Proprieties of Equality

Provide 2 examples of each of the following properties of equity:

For all real numbers of a, b, and c where applicable,

Property	Example 1	Example 2
1. If $a = b$, then $a + c = b + c$.		
2. If $a = b$, then $a - c = b - c$.		
3. If $a = b$, then $a \bullet c = b \bullet c$		
4. $a(b + c) = ab + ac$.		
5. If $a > b$, then $a + c > b + c$		

Property	Example 1	Example 2
6. If $a > b$, then $a - c > b - c$		
7. If $a > b$, then $ac > bc$, if $c > 0$		
8. If $a > b$, then $ac < bc$, if $c < 0$		
9. If $a > b$, then $\frac{a}{c} > \frac{b}{c}$, if $c > 0$		
10. If $a > b$, then $\frac{a}{c} < \frac{b}{c}$, if $c < 0$		

1.1 Exercise A: Proprieties of Equality – Solutions

Examples may vary.

2 Exercise B: Absolute Values

Provide 2 examples of each of the following properties of absolute values:

For any positive number *a*, the solution of

Property	Example 1	Example 2
1. $\|x\| = a$ *is* $x = a$ or $x = -a$		
2. $\|x\| \leq a$ is equivalent to $-a \leq x \leq a$		
3. $\|x\| < a$ is equivalent to $-a<x<a$		
4. $\|x\| \geq a$ is equivalent to $x \leq -a$ or $x \geq a$		
5. $\|x\| > a$ is equivalent to $x<-a$ or $x>a$		

2.1 Exercise B: Absolute Values – Solutions

Examples may vary.

3 Exercise C: Equations and Inequalities

1. What would you do to isolate the variable in the equation below, using only one step?

$$x + 10 = 65$$

2. What would you do to isolate the variable in the equation below, using only one step?

$$x - \frac{1}{4} = \frac{7}{2}$$

3. Identify the step that will *not* lead to a correct solution to the problem.

$$3a - \frac{11}{2} = -\frac{3a}{2} + \frac{25}{2}$$

A) Multiply both sides of the equation by 2.

B) Add $\frac{11}{2}$ to both sides of the equation.

C) Add $\frac{3a}{2}$ to the left side, and add $-3a$ to the right side.

D) Rewrite $3a$ as $\frac{6a}{2}$.

4. In which of the following equations is the distributive property properly applied to the equation 2(y +3) = 7?

A) $y + 6 = 7$
B) $2y + 6 = 14$
C) $2y + 6 = 7$
D) $2y + 3 = 7$

5. Solve for a: $\frac{1}{4}(a+3) = 2 - a$

6. In solving the algebraic equation $2(x – 5) = 2x + 10$, you end up with $-10 = 10$. What does this mean?

A) $x = -10$ and 10
B) There is no solution to the equation.
C) You must have made a mistake in solving the equation.
D) x = all real numbers

7. How many solutions are there for the equation:

$$2\left[\frac{1}{4}(4y-8)+3y\right]+7=3(y+1)+5y$$

A) There is one solution.
B) There are two solutions.
C) There are an infinite number of solutions.
D) There are no solutions.

8. Amanda’s dad is twice as old as she is today. The sum of their ages is 66. Find the ages of Amanda and her dad.

9. A landscaper wants to rent a tree stump grinder to prepare an area for a garden. The rental company charges a R26 one-time rental fee plus R48 for each hour the machine is rented.

   Write an expression for the rental cost for any number of hours.

10. A landscaper wants to rent a tree stump grinder to prepare an area for a garden. The rental company charges a R26 one-time rental fee plus R48 for each hour the machine is rented.

    What is the maximum number of hours the landscaper can rent the tree stump grinder, if he can spend no more than R290? (The machine cannot be rented for part of an hour.)

11. Gina has found a great price on paper towels. She wants to stock up on these for her cleaning business. Paper towels cost R1.25 per package. If she has R60 to spend, how many packages of paper towels can she purchase? Write an equation that Gina could use to solve this problem and show the solution.

12. Levon and Maria were shopping for candles to decorate tables at a restaurant. Levon bought 5 packages of candles plus 3 single candles. Maria bought 11 single candles plus 4 packages of candles. Each package of candles contains the same number of candles. After finishing shopping, Maria and Levon realized that they had each purchased the same exact number of candles. How many candles are in a package?

13. The money from two vending machines is being collected. One machine contains 30 dollar bills and a bunch of dimes. The other machine contains 38 dollar bills and a bunch of nickels. The number of coins in both machines is equal, and the amount of money that the machines collected is also equal. How many coins are in each machine?

14. Albert and Bryn are buying candy at the corner store. Albert buys 5 bags and 3 individual pieces; Bryn buys 3 bags and then eats 2 pieces of candy from one of the bags. Each bag has the same number of pieces of candy.
After Bryn eats the 2 pieces, she has exactly half the number of pieces of candy as Albert. How many pieces of candy are in each bag?
Pick the equation that could be used to solve the problem above. Use the variable *b* to represent the number of pieces of candy in one bag.

A) $5b+3=\frac{1}{2}(3b+2)$

B) $5b+3=3b+2$

C) $\frac{1}{2}(5b+3)=3b-2$

D) $\frac{1}{2}(3b+3)=5b-2$

15. Solve for *a:* $-\frac{a}{5}<\frac{35}{8}$

16. A student is solving the inequality $\frac{5}{2}+7x \le 4x-\frac{7}{2}$. If she combines like terms, which of the following inequalities could she see?

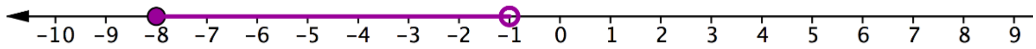
A) $-6 \le 3x$
B) $3x \le -6$
C) $\frac{19x}{2} \le \frac{x}{2}$
D) $x \le \frac{-6x}{7}$

17. Solve for *x*. $5\left[2(3-x)-1\right] \le 27$

18. Solve for *h*.
$h + 3 > 12$ *or* $3 - 2h > 9$

A) $h < 3$ or $h > -3$
B) $h > 9$ or $h > -3$
C) $h > -9$ or $h < 3$
D) $h > 9$ or $h < -3$

19. Which of the following compound inequalities represents the graph on the number line below?



A) $-8 \geq x > -1$
B) $-8 \leq x < -1$
C) $-8 \leq x > -1$
D) $-8 \geq x < -1$

20. Which of the expressions below is equal to the expression $\sqrt{25a^4b^5}$ when written using a rational exponent?

A) $5a^2b^2\sqrt{b}$

B) $(25a^4b^5)^{\frac{1}{2}}$

C) $(25a^4b^5)$

D) $25(a^4b^5)^{\frac{1}{2}}$

21. Which one of the following problem and answer pairs is incorrect?

A) Problem: $\sqrt{16} \bullet \sqrt{25}$ Answer: 20

B) Problem: $\sqrt{16} \bullet \sqrt{x^2}$ Answer: $4|x|$

C) Problem: $\sqrt[3]{x} \bullet \sqrt[3]{y^2}$ Answer: $\sqrt[3]{xy^2}$

D) Problem: $\sqrt{20} \bullet \sqrt[3]{y}$ Answer: $\sqrt[3]{20y}$

22. Divide and simplify. $\frac{\sqrt{27x^9}}{\sqrt{x^4}}$, $x > 0$

A) $3x^2\sqrt{3x}$

B) $\frac{3x^4\sqrt{3x}}{x^2}$

C) $x^2\sqrt{x}$

D) $\sqrt{27x^5}$

23. Add. $10\sqrt[3]{4}+4\sqrt[4]{3}+\sqrt[4]{3}+4\sqrt[3]{4}$

A) $14\sqrt[3]{4}+5\sqrt[4]{3}$

B) $5\sqrt[3]{4}+14\sqrt[4]{3}$

C) $19\sqrt[3]{7}$

D) $19\sqrt[4]{3}$

24. Subtract and simplify. $2\sqrt{50}-4\sqrt{8}$

A) $-2\sqrt{3}$

B) $-2\sqrt{42}$

C) $2\sqrt{2}$

D) $8\sqrt{2}$

25. Multiply and simplify. $\sqrt{10}\left(\sqrt{10}-\sqrt{5}\right)$

A) $10-5\sqrt{2}$
B) 10
C) $5\sqrt{2}$
D) $\sqrt{100}-\sqrt{50}$

26. Multiply and simplify. $(4\sqrt{x}+3)(2\sqrt{x}-1),\ x\geq 0$

A) 7
B) $8x+2\sqrt{x}-3$
C) $8x^2+2x+3$
D) $4x+8\sqrt{x}-3$

27. Solve. $\sqrt{3x+22}=4$

A) $x=2$
B) $x=\frac{16}{3}$
C) $x=-2$
D) $x=-6$

28. Solve. $x-3=\sqrt{4x+9}$

A) x = 3, 0
B) x = 0, 10
C) x = 0
D) x = 10

29. Simplify. $\sqrt{-50}$

A) 5
B) $-5\sqrt{2}$
C) 5*i*
D) $5i\sqrt{2}$

30. Which is the real part of the complex number −35 + 9*i*?

A) 9
B) −35
C) 35
D) 9 and −35

31. Subtract. (5 + 3*i*) – (3 – *i*)

A) 2 + 4*i*
B) 6
C) 2 + 2*i*
D) 8 + 2*i*

32. Multiply and simplify. (3*i*)(−*i*)

A) 3
B) −3
C) 3*i*
D) $-3i^2$

33. Multiply. $(9 + i)(9 - i)$

A) $82 + 18i$
B) $80 - 18i$
C) 80
D) 82

34. Simplify. $12 \div 10i$

A) $\frac{6}{5}$
B) $-\frac{6}{5}i$
C) $\frac{6}{5}i$
D) $-\frac{6}{5}$

35. Simplify. $(10 + 6i) \div (5 - 3i)$

A) $\frac{16}{17}+\frac{30}{17}i$
B) $2 - 2i$
C) $2+\frac{15}{4}i$
D) $\frac{16}{17}+60i$

36. The formula for compounding interest annually is
$A = P(1 + r)^t$, where A is the balance after t years, when P is the principal (initial amount invested) and r is the interest rate.

Find the interest rate r if R3,000 is invested and grows to R3,307.50 after 2 years.

37. A ball is launched upward at 48 feet/second from a platform that is 100 feet high. The equation giving its height t seconds after launch is $h = -16t^2 + 48t + 100$. The ball will shoot up to 136 feet high, then begin to come back down. About how long will the ball take to get to that maximum height?

A) 1.5 seconds
B) 3.6 seconds
C) 4.4 seconds
D) This problem cannot be solved.

38. Find the domain of the rational expression $\frac{5x}{2x+8}$.

A) all real numbers except −4
B) all real numbers except 4
C) all real numbers except 0
D) all real numbers

39. Simplify the rational expression below.

$$\frac{2x^2+13x+15}{2x^2+23x+30}$$

[Note: While the domain and excluded values of a rational expression are important, you will not always be asked to find them when simplifying a rational expression. In this expression, the domain is all real numbers except $\frac{3}{2}$ and -10.]

A) $\frac{14}{25}$

B) $\frac{x+5}{x+10}$

C) $\frac{2x}{3}$

D) $\frac{1}{2}$

40. Multiply, and express the product as a simplified rational expression.

$$\frac{y^2-4}{y^2-2y}\bullet\frac{y}{y^2+10y+16}, \; y \neq -8, -2, 0, 2,$$

A) $\frac{2}{y^2+26y}$

B) $\frac{y}{y(y+8)}$

C) $\frac{(y-2)y}{(y^2-2y)(y+8)}$

D) $\frac{1}{y+8}$

41. Find the quotient and express as a simplified rational expression.

$$\frac{(y+2)(y+1)}{4} \div \frac{y+2}{y+1}$$, domain is all real numbers except −2 and −1.

A) $\frac{(y+2)^2}{4}$

B) $\frac{(y+1)^2}{4}$

C) $\frac{y^2+y+1}{4}$

D) $\frac{y^2+3y+3}{4y+4}$

42.
Subtract, and state the difference in simplest form. $\frac{x^2}{x-5} - \frac{25}{x-5}$, x ≠ 5

A) $\frac{x^2-25}{x-5}$

B) $x + 5$

C) $x - 5$

D) 5

43. Add. State the sum in simplest form. $\dfrac{x}{x+4}+\dfrac{3}{x-3}$

A) $\dfrac{x(x-3)+3(x+4)}{(x+4)(x-3)}$

B) $\dfrac{x+3}{2x+1}$

C) $\dfrac{1}{x}$

D) $\dfrac{x^2+12}{(x+4)(x-3)}$

44. Simplify. $\dfrac{1+\dfrac{2}{3x}}{\dfrac{2}{x^2}+\dfrac{3}{x}}$

A) 1

B) $\dfrac{x}{3}$

C) $\dfrac{3x^2+2x}{6+9x}$

D) $\dfrac{x^2+2x}{2+3x}$

45. Give the excluded values for $\frac{7}{p+2}+\frac{5}{p-2}=\frac{10p-2}{p^2\text{-}4}$. Do not solve.

A) $\frac{2}{10}$
B) 2
C) −2, 2
D) −2, 2, 4

46. Solve the equation $\frac{4}{m}=\frac{3}{m-2}$, $m \neq$ 0 or 2

A) m = 2
B) no solution
C) m = 8

47. Mari and Liam can each wash a car and vacuum its interior in 2 hours. Zach needs 3 hours to do this same job alone. If Zach, Liam, and Mari work together, how long will it take them to clean a car?

A) 20 minutes
B) 45 minutes
C) 1.2 hours
D) 1 hour

48. Solve for k, the constant of variation, in an inverse variation problem where $x = 5$ and $y = 25$.

49. The water temperature in the ocean varies inversely with the depth of the water. The deeper a person dives, the colder the water becomes. At a depth of 1,000 meters, the water temperature is 5º Celsius. What is the water temperature at a depth of 500 meters?

Note: You are told that $y = \frac{k}{x}$ is an inverse relationship, and that the water temperature (y) varies inversely with the depth of the water (x).

50. Find the greatest common factor of $56xy$ and $16y^3$.

A) 8
B) $8y$
C) $16y$
D) $8xy^3$

51. Factor $8a^6 - 11a^5$.

A) $88(a^6 - a^5)$
B) $8a(a^5 - 3)$
C) $a^5(a - 1)$
D) $a^5(8a - 11)$

52. Factor $10ab + 5b + 8a + 4$.

A) $(2a + 1)(5b + 4)$
B) $(5b + 2a)(4 + 1)$
C) $5(2ab + b + 8a + 4)$
D) $(4 + 2a)(5b + 1)$

53. Jess is trying to use the grouping method to factor the trinomial $v^2 - 10v + 21$. How should she rewrite the central b term, $-10v$?

A) $+7v + 3v$
B) $-7v - 3v$
C) $-7v + 3v$
D) $+7v - 3v$

54. Factor $3x^2 + x - 2$.

A) $(3x + 2)(x - 1)$
B) $(3x - 2)(x + 1)$
C) $(3x + 1)(x - 2)$
D) $(3x - 1)(x + 2)$

55. Factor $125x^3 + 64$.

A) $(5x + 64)(25x^2 - 125x + 16)$
B) $(5x + 4)(25x^2 - 20x + 16)$
C) $(x + 4)(x^2 - 2x + 16)$
D) $(5x + 4)(25x^2 + 20x - 64)$

56. Using the difference of cubes, identify the product of $3(x - 3y)(x^2 + 3xy + 9y^2)$.

A) $x^3 - y^3$
B) $3x - 81y$
C) $3x^3 + 81y^3$
D) $3x^3 - 81y^3$

3.1 Exercise C: Equations and Inequalities – Solution

	Solution
1.	Subtract 10 from both sides of the equation. Subtracting 10 from each side of the equation yields an equivalent equation with the variable isolated to give the solution: x + 10 – 10 = 65 – 10, so x = 55.
2.	Add $\frac{1}{4}$ to both sides of the equation. Adding $\frac{1}{4}$ to each side of the equation yields an equivalent equation and isolates the variable: $x-\frac{1}{4}+\frac{1}{4}=\frac{7}{2}+\frac{1}{4}$ and $x=\frac{14}{4}+\frac{1}{4}$, so $x=\frac{15}{4}$.
3.	Add $\frac{3a}{2}$ to the left side, and add $-3a$ to the right side. Correct. Adding unequal amounts to the left and to the right will unbalance the equation, and you will no longer be able to solve accurately for a.
4.	C) 2y + 6 = 7 Correct. Since the distributive property allows us to distribute the multiplication of an entire expression to each of the terms of the expression separately, 2y + 6 = 7 is correct.
5.	You can solve this equation by multiplying both sides by 4, since $4 \bullet \frac{1}{4}=1$. The resulting equation, $a+3=4(2-a)$ can be rewritten as $a+3=8-4a$, and then $5a=5$. You find that a = 1.
6.	B) There is no solution to the equation. Correct. Whenever you end up with a false statement like −10 = 10 it means there is no solution to the equation.
7.	C) There are an infinite number of solutions. Correct. When you evaluate the expressions on either side of the equals sign, you get $8y+3=8y+3$. If you were to move the variables to the left side and the constants to the right, you would end up with 0 = 0. Since you have a true statement, the equation is true for all values of y.
8.	We need to find Amanda's age and her father's age. Let a = Amanda's age $2a$ = Father's age $a+2a=66$

	$a + 2a = 66$ $\dfrac{3a}{3} = \dfrac{66}{3}$ $a = 22$ Amanda's age = 22 Father's age = $2a = 2(22) = 44$ Amanda's father's age is double Amanda's age and their sum is 66. The solutions make sense. Amanda is 22 years old, and her father is 44 years old.
9.	The problem asks for an algebraic expression for the rental cost of the stump grinder for any number of hours. An expression will have terms, one of which will contain a variable, but it will not contain an equal sign. Look at the values in the problem: R26 = one-time fee R48 = per-hour fee Think about what this means, and try to identify a pattern. 1 hr rental: R26 + R48 2 hr rental: R26 + R48 + R48 3 hr rental: R26 + R48 + R48 + R48 Notice that the number of “+ R48” in the problem is the same as the number of hours the machine is being rented. Since multiplication is repeated addition, you could also represent it like this: 1 hr rental: R26 + R48(1) 2 hr rental: R26 + R48(2) 3 hr rental: R26 + R48(3) Now let’s use a variable, *h*, to represent the number of hours the machine is rented. Rental for *h* hours: 26 + 48*h* The rental cost for *h* hours is 26 + 48*h*.
10.	26 + 48*h*, where *h* = the number of hours. $26 + 48h$, where $h =$ the number of hours. $26 + 48h = 290$ $26 + 48h = 290$ $-26 \qquad -26$ $48h = 264$ $\dfrac{48h}{48} = \dfrac{264}{48}$ $h = 5.5$ hours

	Does $26 + 48\,(5.5) = 290$? $26 + 264 = 290$ $290 = 290$ You found that $h = 5.5$. Since the machine cannot be rented for part of an hour, the landscaper can only rent the machine for 5 hours and will have some money left over. The landscaper can rent the machine for 5 hours.
11.	The problem asks for how many packages of paper towels Gina can purchase. The paper towels cost R1.25 per package. Gina has R60 to spend on paper towels. Let p = the number of packages of paper towels. $1.25p = 60$ $\frac{1.25p}{1.25} = \frac{60}{1.25}$ $p = 48$ Does $1.25p = 60$? $1.25(48) = 60$ $60 = 60$ Gina can purchase 48 packages of paper towels.
12.	The problem asks how many candles are contained in one package. Levon bought 5 packages and 3 single candles. Maria bought 4 packages and 11 single candles. Let c = the number of candles in one package. $5c + 3$ $4c + 11$ $5c + 3 = 4c + 11$ $5c + 3 = 4c + 11$ $-4c \qquad -4c$ $1c + 3 = 11$ $-3 \qquad -3$ $1c = 8$ $c = 8$ $5c + 3 = 4c + 11$ $5(8) + 3 = 4(8) + 11$ $40 + 3 = 32 + 11$ $43 = 43$ There are 8 candles in one package of candles.
13.	The problem asks how many coins are in each machine. One machine has 30 dollar bills and a bunch of dimes.

	Another machine has 38 dollar bills and a bunch of nickels—the same number of coins as the first machine. Let *c* = the number of coins in each machine. $30+0.10c$ $38+0.05c$ $30+0.10c=38+0.05c$ $\begin{array}{rcl} 30+0.10c & = & 38+0.05c \\ -0.05c & & -0.05c \\ \hline 30+0.05c & = & 38 \\ -30 & & -30 \\ \hline 0.05c & = & 8 \\ c & = & 160 \end{array}$ *Check your solution*. Substitute 160 for *c* in the original equation $30+0.10c=38+0.05c$ $30+0.10(160)=38+0.05(160)$ $30+16=38+8$ $46=46$ There are 160 coins in each machine.
14.	C) $\frac{1}{2}(5b+3)=3b-2$ Correct. The amount of candy that Albert has can be represented by 5*b* + 3, and the amount of candy Bryn has can be represented by 3*b* – 2. Since Bryn has half as much as Albert, the final equation is $\frac{1}{2}(5b+3)=3b-2$.
15.	$a>-\frac{175}{8}$ Correct. By multiplying both sides by -5 and flipping the inequality sign from < to >, you found that $a>-\frac{175}{8}$.
16.	B) $3x\le -6$ Correct. You correctly combined like terms. $\frac{5}{2}+7x\le 4x-\frac{7}{2}$ becomes $7x-4x\le -\frac{7}{2}-\frac{5}{2}$, which is the same as $3x\le -6$.
17.	$x\ge -\frac{1}{5}$ Correct. Evaluating the interior parentheses first, you find that $5[2(3-x)-1]=5[6-2x-1]=5[5-2x]=25-10x$. Solving for *x*, you subtract 25 from both sides and then divide by -10, which requires you to flip the inequality sign from ≤ to ≥.
18.	D) *h* > 9 or *h* < −3 Correct. Solving each inequality for *h*, you find that *h* > 9 or *h* < −3.

19.	−8 ≤ x < −1 Correct. The selected region on the number line lies between −8 and −1and includes -8, so *x* must be greater than or equal to −8 and less than −1.
20.	B) $(25a^4b^5)^{\frac{1}{2}}$ Correct. Taking the square root is the same as raising the quantity under the radical to a power of $\frac{1}{2}$.
21.	D) Problem: $\sqrt{20}\bullet\sqrt[3]{y}$ Answer: $\sqrt[3]{20y}$ Correct. The two radicals have different roots, so you cannot multiply the product of the radicands and put it under the same radical sign. So, this problem and answer pair is incorrect.
22.	A) $3x^2\sqrt{3x}$ Correct. Using what you know about quotients, you can rewrite the expression as $\sqrt{\frac{27x^9}{x^4}}$, simplify it to $\sqrt{27x^5}$, and then pull out perfect squares. The simplified form is $3x^2\sqrt{3x}$.
23.	A) $14\sqrt[3]{4}+5\sqrt[4]{3}$ Correct. When adding radical expressions, you can combine like radicals just as you would add like variables. $10\sqrt[3]{4}+4\sqrt[4]{3}+\sqrt[4]{3}+4\sqrt[3]{4}=14\sqrt[3]{4}+5\sqrt[4]{3}$.
24.	C) $2\sqrt{2}$ Correct. Rewriting $2\sqrt{50}-4\sqrt{8}$ as $2\sqrt{25\bullet 2}-4\sqrt{4\bullet 2}$, you found that $10\sqrt{2}-8\sqrt{2}=2\sqrt{2}$.
25.	A) $10-5\sqrt{2}$ Correct. Multiplying $\sqrt{10}$ by $\sqrt{10}$ and $-\sqrt{5}$, you find it is equal to $\sqrt{100}-\sqrt{50}$, or $10-5\sqrt{2}$.
26.	B) $8x+2\sqrt{x}-3$ Correct. Using the FOIL method, you find that the product of the binomials is $8\bullet\sqrt{x}\bullet\sqrt{x}-4\sqrt{x}+6\sqrt{x}-3$, which simplifies to $8x+2\sqrt{x}-3$.
27.	C) $x=-2$ Correct. Squaring both sides, you find $\left(\sqrt{3x+22}\right)^2=(4)^2$ becomes $3x+22=16$, so $3x=-6$ and $x=-2$.

28.	D) $x = 10$ Correct. Solving the equation, you find that squaring both sides results in $x^2 - 6x + 9 = 4x + 9$, which simplifies to $x^2 - 10x = 0$. Although this equation produces x values of 0 or 10, 0 is extraneous since it does not make the original equation true.
29.	D) $5i\sqrt{2}$ Correct. $\sqrt{-50} = \sqrt{25}\sqrt{2}\sqrt{-1} = 5\sqrt{2}i = 5i\sqrt{2}.$
30.	B) −35 Correct. In a complex number $a + bi$, the real part is a. In this case, $a = -35$, so the real part is −35.
31.	A) $2 + 4i$ Correct. Distributing the subtraction to the second complex number gives $5 + 3i – 3 + i$. Rearranging to put like terms together gives $5 – 3 + 3i + i$, and combining like terms gives $2 + 4i$.
32.	A) 3 Correct. $(3i)(-i) = 3(-1)(i)(i) = -3i^2 = -3(-1) = 3$.
33.	D) 82 Correct. $(9 + i)(9 – i) = 81 – 9i + 9i – i^2 = 81 – i^2 = 81 – (-1) = 81 + 1 = 82$.
34.	B) $-\frac{6}{5}i$ Correct. After simplifying the numerical parts of $\frac{12}{10i}$, you still have to rationalize the denominator because i is left in the denominator: $\frac{12}{10i} = \frac{6}{5i} \bullet \frac{2}{2} = \frac{6}{5i} = \frac{6}{5i} \bullet \frac{i}{i} = \frac{6i}{5i^2} = \frac{6i}{5(-1)} = -\frac{6}{5}i$.
35.	A) $\frac{16}{17} + \frac{30}{17}i$ Correct. Write as a rational expression and multiply both numerator and denominator by the complex conjugate of the divisor: $\frac{10+6i}{5-3i} \bullet \frac{5+3i}{5+3i} = \frac{50+30i+30i+18i^2}{25-9i^2} = \frac{50+60i-18}{25+9} = \frac{32+60i}{34}$. This simplifies to $\frac{16}{17} + \frac{30}{17}i$.
36.	$A = P(1 + r)^t$ $A = 3{,}307.50$ $t = 2$ $P = 3{,}000$ $3{,}307.50 = 3{,}000(1 + r)^2$

	$\frac{3{,}307.50}{3{,}000} = (1+r)^2$ $1.1025 = (1+r)^2$ $\pm\sqrt{1.1025} = 1+r$ ±1.05 = 1 + r ±1.05 – 1 = r 1.05 – 1 = r, or −1.05 – 1 = r r = 0.05 or −2.05 The interest rate is 0.05, or 5%.
37.	A) 1.5 seconds Correct. When the height is 136 feet, the equation becomes 136 = $-16t^2 + 48t + 100$. Subtracting 136 from both sides gives 0 = $-16t^2 + 48t - 36$. Factoring out −4 results in the simpler equation 0 = $4t^2 - 12t + 9$. Using the Quadratic Formula gives $t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$. This simplifies to $t = \frac{12 \pm \sqrt{0}}{8} = 1.5$. (Since the discriminant is 0, there is only one solution.) The ball will reach its maximum height in 1.5 seconds.
38.	A) all real numbers except −4 Correct. When $x = -4$, the denominator is 2(−4) + 8 = −8 + 8 = 0. Division by 0 is undefined, so the domain must exclude $x = -4$.
39.	B) $\frac{x+5}{x+10}$ Correct. The rational expression can be simplified by factoring the numerator and denominator as $\frac{(2x+3)(x+5)}{(2x+3)(x+10)}$. Since $\frac{2x+3}{2x+3} = 1$, simplify the expression to $\frac{x+5}{x+10}$.
40.	D) $\frac{1}{y+8}$ Correct. Factoring the numerators and denominators, you get $\frac{(y-2)(y+2)}{y(y-2)} \bullet \frac{y}{(y+8)(y+2)}$. Regrouping, you get $\frac{y-2}{y-2} \bullet \frac{y+2}{y+2} \bullet \frac{y}{y} \bullet \frac{1}{y+8} = \frac{1}{y+8}$.
41.	B) $\frac{(y+1)^2}{4}$ Correct. Division is the same as multiplication by the reciprocal, so this problem can be written: $\frac{(y+2)(y+1)}{4} \bullet \frac{y+1}{y+2} = \frac{(y+1)^2}{4}$.

42.	B) $x + 5$ Correct. Since there is a common denominator, subtract the numerators to get $\frac{x^2-25}{x-5}$. The numerator can be factored and a common factor of (x – 5) is present in numerator and denominator. $\frac{(x-5)(x+5)}{x-5}=\frac{x-5}{x-5}\bullet(x+5)=x+5$.
43.	D) $\frac{x^2+12}{(x+4)(x-3)}$ Correct. First find a common denominator, (x + 4)(x – 3), and rewrite each addend using that denominator: $\frac{x(x-3)}{(x+4)(x-3)}+\frac{3(x+4)}{(x+4)(x-3)}$. Multiply and add the numerators: $\frac{x^2-3x+3x+12}{(x+4)(x-3)}$.
44.	B) $\frac{x}{3}$ Correct. The common denominator of all terms in both numerator and denominator is $3x^2$. Multiplying the expression by $\frac{3x^2}{3x^2}$ gives $\frac{3x^2+2x}{6+9x}$. The numerator and denominator in this expression have a common factor of (3x + 2), so the correct answer is $\frac{x}{3}$.
45.	C) −2, 2 Correct. −2 and 2, when substituted into the equation, result in a 0 in the denominator. Since division by 0 is undefined, both of these values are excluded from the solution.
46.	C) $m = 8$ Correct. Multiplying both sides of the equation by the common denominator gives $\frac{4}{m}\bullet m(m-2)=\frac{3}{m-2}\bullet m(m-2)$, so $4m-8=3m$. The correct answer is $m = 8$.
47.	B) 45 minutes Correct. According to the formula, $r=\frac{W}{t}$. Mari and Liam each have a rate of $\frac{1}{2}$ car in one hour, and Zach's rate is $\frac{1}{3}$ car in one hour. Working together, they have a rate of $\frac{1}{2}+\frac{1}{2}+\frac{1}{3}$, or $\frac{4}{3}$. W is one car, so the formula becomes $\frac{4}{3}=\frac{1}{t}$. This means $3t\bullet\frac{4}{3}=\frac{1}{t}\bullet 3t$, so $4t=3$, and $t=\frac{3}{4}$. It takes three-quarters of an hour, or 45 minutes, to clean one car.

48.	$y=\frac{k}{x}$ $25=\frac{k}{5}$ $5\bullet 25=\frac{k}{5}\bullet 5$ $125=\frac{5k}{5}$ $125 = k$ The constant of variation, *k*, is 125.
49.	$y=\frac{k}{x}$ $temp=\frac{k}{depth}$ $5=\frac{k}{1{,}000}$ $1{,}000\bullet 5=\frac{k}{1{,}000}\bullet 1{,}000$ $5{,}000=\frac{1{,}000k}{1{,}000}$ $5{,}000=k$ $temp=\frac{k}{depth}$ $temp=\frac{5{,}000}{500}$ At 500 meters, the water temperature is 10º C.
50.	B) $8y$ Correct. The expression $56xy$ can be factored as $2 \bullet 2 \bullet 2 \bullet 7 \bullet x \bullet y$, and $16y^3$ can be factored as $2 \bullet 2 \bullet 2 \bullet 2 \bullet y \bullet y \bullet y$. They have the factors $2 \bullet 2 \bullet 2$ and y in common. Multiplying them together will give you the GCF: $8y$.
51.	D) $a^5(8a – 11)$ Correct. The values 8 and 11 share no common factors, but the GCF of a^6 and a^5 is a^5. So you can factor out a^5 and rewrite the polynomial as $a^5(8a – 11)$.
52.	A) $(2a + 1)(5b + 4)$ Correct. The polynomial $10ab + 5b + 8a +4$ can be grouped as $(10ab + 5b) + (8a +4)$. Pulling out common factors, you find: $5b(2a + 1) + 4(2a + 1)$. Since $(2a + 1)$ is a common factor, the factored form is $(2a + 1)(5b + 4)$.
53.	B) $−7v – 3v$ Correct. Because the *c* term is positive and the *b* term is negative, both terms should be negative. Check: using the integers −7 and −3, $−7 + −3 = −10$ and $−7 \bullet −3 = 21$, so this provides both terms $−10v$ and 21 correctly.

54.	B) $(3x - 2)(x + 1)$ Correct. The product of $(3x - 2)(x + 1)$ is $3x^2 + x - 2$.
55.	B) $(5x + 4)(25x^2 - 20x + 16)$ Correct. $5x$ is the cube root of $125x^3$, and 4 is the cube root of 64. Substituting these values for a and b, you find $(5x + 4)(25x^2 - 20x + 16)$.
56.	D) $3x^3 - 81y^3$ Correct. Recognizing that this expression is in the form $(a - b)(a^2 + ab + b^2)$, you find $a = x$ and $b = 3y$. This means that the resulting $a^3 - b^3$ monomial is $x^3 - 27y^3$. It also needs to be multiplied by the coefficient 3: $3x^3 - 81y^3$.

4 Exercise D: Recall LU2 Activities

1. Solve $12.5 + x = -7.5$.

2. Solve $x + 10 = -65$.

3. Solve $3x = 24$.

4. Solve $\frac{1}{2}x = 8$

5. Solve $-\frac{7}{2} = \frac{x}{10}$

6. Solve $3y + 2 = 11$

7. Solve $3x + 5x + 4 - x + 7 = 88$

8. Solve for y.
$-20y + 15 = 2 - 16y + 11$

9. Solve $3y + 10.5 = 6.5 + 2.5y$.

10. Solve for a.
$4(2a + 3) = -3(a - 1) + 31$

11. Solve $\frac{1}{2}x - 3 = 2 - \frac{3}{4}x$

12. Solve $\frac{1}{2}(2 + a) = \frac{3a + 4}{3^2}$

13. Solve for *y*.
8*y* = 2[3(*y* + 4) + *y*]

14. Check that *x* < *2* is the solution to *x* + 3 < 5.

15. Solve for *x*.
$\frac{15}{2} + x > -\frac{37}{4}$

16. Check that $x \leq -2$ is the solution to $x - 10 \leq -12$

17. Solve for *x*.
$-\frac{1}{3}x > -12x$

18. Check that *p* < 6 is the solution to the inequality 4*p* + 5 < 29.

19. Solve for *x*.
$\frac{1}{3}(x + 3) + \frac{1}{2} > -\frac{9}{2}$

20. Solve for *x*.
2(3x – 5) ≤ 4x + 6

21. Solve for *a*.

$$\frac{2a-4}{6}<2$$

22. Solve for *d*.

$$\frac{3}{5}(2d-5)\le 4\left(7-\frac{1}{5}d\right)$$

23. Solve for *p*.

$|2p-4| = 26$

24. Solve for *w*.

$3|4w-1|-5=10$

25. Solve for *x*.

$$|x+3|>4$$

26. Solve for *y*.

$$3|2y+6|-9<27$$

27. Solve for *x*.

$|2x+3|+9\le 7$

28. Identify the domain of the expression.

$$\frac{3x+2}{x-4}$$

29. Identify the domain of the expression.

$$\frac{x+7}{x^2+8x-9}$$

30. Identify the domain of the expression.

$$\frac{5x^2}{25x}$$

31. Simplify and state the domain for the expression.

$$\frac{x+3}{x^2+12x+27}$$

32. Simplify and state the domain for the expression.

$$\frac{x^2+10x+24}{x^3-x^2-20x}$$

33. Solve the equation $\frac{x+5}{8}=\frac{7}{4}$.

34. Solve the equation $\frac{x}{3}+1=\frac{4}{3}$.

35. Myra takes two hours to plant 50 flower bulbs. Francis takes three hours to plant 45 flower bulbs. Working together, how long should it take them to plant 150 bulbs?

36. Joe and John are planning to paint a house together. John thinks that if he worked alone, it would take him three times as long as it would take Joe to paint the entire house. Working together, they can complete the job in 24 hours. How long would it take each of them, working alone, to complete the job?

37. Find the greatest common factor of 210 and 168.

38. Find the greatest common factor of $25b^3$ and $10b^2$.

39. Factor $4ab + 12a + 3b + 9$

40. Factor $x^2 + 2x + 5x + 10$

41. Factor $2x^2 – 3x + 8x – 12$.

42. Factor $3x^2 + 3x – 2x – 2$.

43. Factor $7x^2 – 21x + 5x – 5$.

44. Factor $25b^3 + 10b^2$.

45. Factor $81c^3d + 45c^2d^2$.

46. Multiply $(x + 2)(x + 5)$.

47. Factor $x^2 + 7x +10$

48. Factor $x^2 + 5x + 6$.

49. Factor $3x^3 - 3x^2 - 90x$.

50. Factor $-4h^2 + 11h + 3$

51. Factor $9x^2 - 24x + 16$

52. Factor $x^2 - 14x + 49$.

53. Factor $9x^2 - 4$.

54. Factor $x^3 + 8y^3$.

55. Factor $16m^3 + 54n^3$.

56. Factor $8x^3 - 1{,}000$.

57. Solve $(x + 4)(x - 3) = 0$ for x.

58. Solve for a: $5a^2 + 15a = 0$.

59. Solve for r: $r^2 - 5r + 6 = 0$.

60. The area of a rectangular garden is 30 square feet. If the length is 7 feet longer than the width, find the dimensions.

61. Solve $5b^2 + 4 = -12b$ for b.

62. A small toy rocket is launched from a 4-foot pedestal. The height (h, in feet) of the rocket t seconds after taking off is given by the formula $h = -2t^2 + 7t + 4$. How long will it take the rocket to hit the ground?

63. Simplify. $\sqrt{144}$

64. Simplify. $-\sqrt{81}$

65. Simplify. $\sqrt{63}$

66. Simplify. $\sqrt{2{,}000}$

67. Approximate $\sqrt{50}$ and also find its value using a calculator.

68. Simplify. $\sqrt{9x^6y^4}$

69. Simplify. $\sqrt{49x^{10}y^8}$

70. Simplify. $\sqrt{a^3b^5c^2}$

71. Simplify. $\sqrt[3]{8}$

72. Simplify. $\sqrt[3]{125}$

73. Simplify. $\sqrt[3]{32m^5}$

74. Simplify. $\sqrt[3]{-27x^4y^3}$

75. Simplify. $\sqrt{100x^2y^4}$

76. Write $\sqrt[4]{81}$ as an expression with a rational exponent.

77. Express $(2x)^{\frac{1}{3}}$ in radical form.

78. Express $4\sqrt[3]{xy}$ with rational exponents

79. Simplify. $(36x^4)^{\frac{1}{2}}$

80. Simplify. $\sqrt[3]{a^6}$

81. Simplify. $\sqrt[4]{81x^8y^3}$

82. Simplify. $\dfrac{10b^2c^2}{c\sqrt[3]{8b^4}}$

83. Simplify. $\sqrt{18} \bullet \sqrt{16}$

84. Simplify. $\sqrt[3]{x^5y^2} \bullet 5\sqrt[3]{8x^2y^4}$

85. Simplify. $\sqrt{12x^3} \bullet \sqrt{3x}$, $x \geq 0$

86. Simplify.
$2\sqrt[4]{16x^9} \bullet \sqrt[4]{y^3} \bullet \sqrt[4]{81x^3y}$,
$x \geq 0$, $y \geq 0$

87. Simplify. $\sqrt{\dfrac{48}{25}}$

88. Simplify. $\sqrt[3]{\dfrac{640}{40}}$

89. Simplify. $\dfrac{\sqrt[3]{640}}{\sqrt[3]{40}}$

90. Simplify. $\dfrac{\sqrt{30x}}{\sqrt{10x}}$, $x > 0$

91. Simplify. $\dfrac{\sqrt[3]{24xy^4}}{\sqrt[3]{8y}}$, $y \neq 0$

92. Add. $3\sqrt{11} + 7\sqrt{11}$

93. Add. $5\sqrt{2}+\sqrt{3}+4\sqrt{3}+2\sqrt{2}$

94. Add. $3\sqrt{x}+12\sqrt[3]{xy}+\sqrt{x}$

95. Add and simplify. $2\sqrt[3]{40}+\sqrt[3]{135}$

96. Add and simplify. $x\sqrt[3]{xy^4}+y\sqrt[3]{x^4y}$

97. Subtract. $5\sqrt{13}-3\sqrt{13}$

98. Subtract. $4\sqrt[3]{5a}-\sqrt[3]{3a}-2\sqrt[3]{5a}$

99. Subtract and simplify. $5\sqrt[4]{a^5b}-a\sqrt[4]{16ab}, where\ a\geq 0\ and\ b\geq 0$

100 Solve. $\sqrt{x}-3=5$

101 Solve. $\sqrt{x+8}=3$

102 Solve. $1+\sqrt{2x+3}=6$

103 Simplify. $\sqrt{-4}$

104 Simplify. $\sqrt{-18}$

105 Simplify. $-\sqrt{-72}$

106 Add. $(-3 + 3i) + (7 - 2i)$

107 Subtract. $(-3 + 3i) - (7 - 2i)$

108 Multiply. $(3i)(2i)$

109 Simplify. $-24i \div 6$

110 Simplify. $(56 - 8i) \div (14 + 10i)$

Learning Unit 3: Sets, Sequences and Functions	
Material used for this learning unit: • Module Manual Learning Unit 3; • Workbook; • IIE Learn.	**My notes**

1 Exercise A: Learn Activities

Complete the following activities on Learn:

1. Activity 3.1.1: Mathematical statements and logic
2. Activity 3.2.1: Sets and set operations
3. Activity 3.3.1: Sequences
4. Activity 3.4.1: Functions

1.1 Exercise A: Learn Activities – Solutions

1. Activity 3.1.1: Mathematical statements and logic
   1.1. C
   1.2. D
   1.3. A
   1.4. D
   1.5. D

2. Not provided

3. Activity 3.3.1: Sequences
   3.1. D
   3.2. C
   3.3. A
   3.4. D
   3.5. C

4. Activity 3.4.1: Functions
   4.1. D
   4.2. A
   4.3. B
   4.4. C
   4.5. B

2 Exercise B: Recall LU3 Activities

1. Consider the statement:

   If Bob gets a 90 on the final, then Bob will pass the class.

   Is the statement an implication? Motivate your answer.

2. Decide which of the following statements are true and which are false. Briefly explain.

   A. $0 = 1 \rightarrow$ 1 1
   B. $1 = 1 \rightarrow$ most horses have 4 legs
   C. If 8 is a prime number, then the 7624th digit of π is an 8.
   D. If the 7624th digit of π is an 8, then $2 + 2 = 4$

3. Prove: If two numbers a and b are even, then their sum $a + b$ is even.

4. True or false: If you draw any nine playing cards from a regular deck, then you will have at least three cards all of the same suit. Is the converse true?

5. Suppose I tell Sue that if she gets a 93% on her final, then she will get an A in the class. Assuming that what I said is true, what can you conclude in the following cases:

   1. Sue gets a 93% on her final.
   2. Sue gets an A in the class.
   3. Sue does not get a 93% on her final.
   4. Sue does not get an A in the class.

6. Rephrase the implication, “if I dream, then I am asleep” in as many different ways as possible. Then do the same for the converse.

7. Describe each of the following sets both in words and by listing out enough elements to see the pattern.

   A. $\{x : x + 3 \in \mathbb{N}\}$.
   B. $\{x \in \mathbb{N} : x + 3 \in \mathbb{N}\}$.
   C. $\{x : x \in \mathbb{N} \vee -x \in \mathbb{N}\}$.

8. Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6\}$, $C = \{1, 2, 3\}$ and $D = \{7, 8, 9\}$. Determine which of the following are true, false, or meaningless.

A. $A \subset B$.

B. $B \subset A$.

C. $B \in C$.

D. $\emptyset \in A$.

E. $\emptyset \subset A$.

F. $A < D$.

G. $3 \in C$.

H. $3 \subset C$.

I. $\{3\} \subset C$.

9. Let $A = \{1, 2, 3\}$. Find $P(A)$.

10. Find the cardinality of $A = \{23, 24, \ldots, 37, 38\}$.

11. Find the cardinality of $B = \{1, \{2, 3, 4\}, \varnothing\}$.

12. If $C = \{1, 2, 3\}$, what is the cardinality of $\mathcal{P}(C)$?

13. Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6\}$, $C = \{1, 2, 3\}$ and $D = \{7, 8, 9\}$. If the universe is $U = \{1, 2, \ldots, 10\}$, find:

1. $A \cup B$.
2. $A \cap B$.
3. $B \cap C$.
4. $A \cap D$.
5. $\overline{B \cup C}$.
6. $A \setminus B$.
7. $(D \cap \overline{C}) \cup \overline{A \cap B}$.
8. $\emptyset \cup C$.
9. $\emptyset \cap C$.

14. Can you find the next term in the following sequences?

A. $7, 7, 7, 7, 7, \ldots$

B. $3, -3, 3, -3, 3, \ldots$

C. $1, 5, 2, 10, 3, 15, \ldots$

D. $1, 2, 4, 8, 16, 32, \ldots$

E. $1, 4, 9, 16, 25, 36, \ldots$

F. $1, 2, 3, 5, 8, 13, 21, \ldots$

G. $1, 3, 6, 10, 15, 21, \ldots$

H. $2, 3, 5, 7, 11, 13, \ldots$

I. $3, 2, 1, 0, -1, \ldots$

J. $1, 1, 2, 6, \ldots$

15. Find a_6 in the sequence defined by $a_n = 2a_{n-1} - a_{n-2}$ with $a_0 = 3$ and $a_1 = 4$.

16. Use the formulas $T_n = \frac{n(n+1)}{2}$ and $a_n = 2^n$

to find closed formulas for the following sequences.

A. (b_n): 1, 2, 4, 7, 11, 16, 22, ...
B. (c_n): 3, 5, 9, 17, 33, ...
C. (d_n): 0, 2, 6, 12, 20, 30, 42, ...
D. (e_n): 3, 6, 10, 15, 21, 28, ...
E. (f_n): 0, 1, 3, 7, 15, 31, ...
F. (g_n) 3, 6, 12, 24, 48, ...
G. (h_n): 6, 10, 18, 34, 66, ...
H. (j_n): 15, 33, 57, 87, 123, ...

17. Which functions are surjective?

1. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n$.
2. $g : \{1, 2, 3\} \to \{a, b, c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$.
3. $h : \{1, 2, 3\} \to \{1, 2, 3\}$ defined as follows:

$$\begin{tikzcd} 1 \arrow[dr] & 2 \arrow[dl] & 3 \arrow[d] \\ 1 & 2 & 3 \end{tikzcd}$$

18. Which functions are injective (i.e., one-on-one)?

1. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n$.
2. $g : \{1, 2, 3\} \to \{a, b, c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$.
3. $h : \{1, 2, 3\} \to \{1, 2, 3\}$ defined as follows:

$$\begin{tikzcd} 1 \arrow[dr] & 2 \arrow[dl] & 3 \arrow[d] \\ 1 & 2 & 3 \end{tikzcd}$$

19. Consider the function $f: \{1, 2, 3, 4, 5, 6\} \to \{a, b, c, d\}$ given by

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & a & b & c & c & c \end{pmatrix}.$$

Find the complete inverse image of each element in the codomain.

20. Consider the function $g : Z \to Z$ defined by $g(n) = n^2 + 1$. Find $g^{-1}(1)$, $g^{-1}(2)$, $g^{-1}(3)$ and $g^{-1}(10)$.

3 Exercise C: Sets

The following activities are a compilation of selected mathematics exercises from mathematics specialist networks, websites, and resources. Completing the activities on your own and referring to the solution to check your work should adequately help you master the concepts and prepare for the assessments.

1. Find the intersection *A*∩*B*, union *A*∪*B* and differences *A*-*B*, *B*-*A* of sets *A*, *B* if :

a) $A=\{1,3,5,7,9\}$
$B=\{2,3,4,5,6,8\}$

b) $A=\{-1,0,2,3,5,7\}$
$B=\{0,1,2,3,4,5,6\}$

c) $A=\{2,4,5,6,8\}$
$B=\{-3,0,2,3,4,5,6,7\}$

d) $A=\{-10,-7,-4,-3,-2,-1\}$
$B=\{-9,-8,-7,-6,-4,-2\}$

e) $A=\{-7,-6,-5,-3,-2,1,3,5,6,9\}$
$B=\{-9,-8,-6,-4,-1,0,2,3,6,8\}$

f) $A=\{-10,-7,-4,-3,-2,-1\}$
$B=\{-9,-8,-7,-6,-4,-2\}$

g) $A=\{-6,-4,-2,0,1,3,5,7\}$
$B=\boldsymbol{N}$

h) $A=\boldsymbol{Z}$
$B=\boldsymbol{Q}$

i) $A=\boldsymbol{Z}$
$B=\langle -2;5\rangle$

j) $A=\{a,d,e,f,i,j,k,m,q,r,w\}$
$B=\{c,d,f,j,k,p,q,r,s,t,y\}$

2. Consider the sets $A=\{0,1,2,3,4,5,6,7,8,9\}$, $B=\{1,4,6,7,10,14\}$, $C=\{3,5,6,7,9\}$, $D=\{0,2,4,6,8\}$. Find the sets :

a) $A \cap C$

b) $B \cup D$

c) $A \cup B$

d) $C \cap D$

e) $(B \cap C) \cap D$

f) $A \cap (C \cup D)$

g) $A - D$

h) $A - C$

i) $C - B$

j) $(B \cup C) - D$

k) $D - (B \cap C)$

l) $(A - D) \cup C$

3. Find the intersections $A \cap B$, $A \cap C$, $B \cap C$, unions $A \cup B$, $A \cup C$, $B \cup C$ and differences A-B, A-C, B-A, B-C, C-A, C-B of sets A, B, C if :

a) $A = \{2,3,4,5,7,8,9\}$

$B = \{1,2,3,4,6,7,9\}$

$C = \{3,4,5,8,9\}$

b) $A = \{0,2,3,5,6,7\}$

$B = \{1,3,4,7,8\}$

$C = \{-3,-1,0,3,4,5\}$

c) $A = \{0,2,4,6,8\}$

$B = \{2,4,5,7,8,9\}$

$C = \boldsymbol{N}$

d) $A = \{-8,-6,-4,-2,0,2,4,6,8\}$

$B = \langle -3;3 \rangle$

$C = \boldsymbol{Z}$

3.1 Exercise C: Sets – Solutions

1.

a)
$A \cap B = \{3,5\}$
$A \cup B = \{1,2,3,4,5,6,7,8,9\}$
$A - B = \{1,7,9\}$
$B - A = \{2,4,6,8\}$

b)
$A \cap B = \{0,2,3,5\}$
$A \cup B = \{-1,0,1,2,3,4,5,6,7\}$
$A - B = \{-1,7\}$
$B - A = \{1,4,6\}$

c)
$A \cap B = \{2,4,5,6\}$
$A \cup B = \{-3,0,2,3,4,5,6,7,8\}$
$A - B = \{8\}$
$B - A = \{-3,0,3,7\}$

d)
$A \cap B = \{-7,-4,-2\}$
$A \cup B = \{-10,-9,-8,-7,-6,-4,-3,-2,-1\}$
$A - B = \{-10,-3,-1\}$
$B - A = \{-9,-8,-6\}$

e)
$A \cap B = \{-6,3,6\}$
$A \cup B = \left\{\begin{matrix}-9,-8,-7,-6,-5,-4,-3, \\ -2,-1,0,1,2,3,5,6,8,9\end{matrix}\right\}$
$A - B = \{-7,-5,-3,-2,1,5,9\}$
$B - A = \{-9,-8,-4,-1,0,2,8\}$

f)
$A \cap B = \{-7,-4,-2\}$
$A \cup B = \{-10,-9,-8,-7,-6,-4,-3,-2,-1\}$
$A - B = \{-10,-3,-1\}$
$B - A = \{-9,-8,-6\}$

g)
$A \cap B = \{1,3,5,7\}$
$A \cup B = \{-6,-4,-2,0\} \cup \boldsymbol{N}$
$A - B = \{-6,-4,-2,0\}$
$B - A = \boldsymbol{N} \setminus \{1,3,5,7\}$

h)
$A \cap B = \boldsymbol{Z}$
$A \cup B = \boldsymbol{Q}$
$A - B = \emptyset$
$B - A = \boldsymbol{Q} \setminus \boldsymbol{Z}$

i)
$A \cap B = \{-2,-1,0,1,2,3,4,5\}$
$A \cup B = \boldsymbol{Z} \cup \langle -2;5 \rangle$
$A - B = \boldsymbol{Z} \setminus \{-2,-1,0,1,2,3,4,5\}$
$B - A = \langle -2;5 \rangle \setminus \{-2,-1,0,1,2,3,4,5\}$

j)
$A \cap B = \{d,f,j,q,r\}$
$A \cup B = \{a,c,d,e,f,i,j,k,m,p,q,r,s,t,w,y\}$
$A - B = \{a,e,i,k,m,w\}$
$B - A = \{c,k,p,s,t,y\}$

2.

a)	$\{3,5,6,7,9\}$	**e)**	$\{6\}$	**i)**	$\{3,5,9\}$
b)	$\{0,1,2,4,6,7,8,10,14\}$	**f)**	$\{0,2,3,4,5,6,7,8,9\}$	**j)**	$\{1,3,5,7,9,10,14\}$
c)	$\{0,1,2,3,4,5,6,7,8,9,10,14\}$	**g)**	$\{1,3,5,7,9\}$	**k)**	$\{0,2,4,8\}$
d)	$\{6\}$	**h)**	$\{0,1,2,4,8\}$	**l)**	$\{1,3,5,6,7,9\}$

3.

a)

$A \cap B = \{2,3,4,7,9\}$

$A \cap C = \{3,4,5,8,9\}$

$B \cap C = \{3,4,9\}$

$A \cup B = \{1,2,3,4,5,6,7,8,9\}$

$A \cup C = \{2,3,4,5,7,8,9\}$

$B \cup C = \{1,2,3,4,5,6,7,8,9\}$

$A - B = \{5,8\}$

$A - C = \{2,7\}$

$B - A = \{1,6\}$

$B - C = \{1,2,6,7\}$

$C - A = \emptyset$

$C - B = \{5,8\}$

c)

$A \cap B = \{2,4,8\}$

$A \cap C = \{2,4,6,8\}$

$B \cap C = \{2,4,5,7,8,9\}$

$A \cup B = \{0,2,4,5,6,7,8,9\}$

$A \cup C = \{0\} \cup \boldsymbol{N} = \boldsymbol{N_0}$

$B \cup C = \boldsymbol{N}$

$A - B = \{0,6\}$

$A - C = \{0\}$

$B - A = \{5,7,9\}$

$B - C = \emptyset$

$C - A = \boldsymbol{N} \setminus \{2,4,6,8\}$

$C - B = \boldsymbol{N} \setminus \{2,4,5,7,8,9\}$

b)

$A \cap B = \{3,7\}$

$A \cap C = \{0,3,5\}$

$B \cap C = \{3,4\}$

$A \cup B = \{0,1,2,3,4,5,6,7,8\}$

$A \cup C = \{-3,-1,0,2,3,4,5,6,7\}$

$B \cup C = \{-3,-1,0,1,3,4,5,7,8\}$

$A - B = \{0,2,5,6\}$

$A - C = \{2,6,7\}$

$B - A = \{1,4,8\}$

$B - C = \{1,7,8\}$

$C - A = \{-3,-1,4\}$

$C - B = \{-3,-1,0,5\}$

d)

$A \cap B = \{-2,0,2\}$

$A \cap C = \{-8,-6,-4,-2,0,2,4,6,8\}$

$B \cap C = \{-3,-2,-1,0,1,2,3\}$

$A \cup B = \{-8,-6,-4,4,6,8\} \cup \langle -3;3 \rangle$

$A \cup C = \boldsymbol{Z}$

$B \cup C = \boldsymbol{Z} \cup \langle -3;3 \rangle$

$A - B = \{-8,-6,-4,4,6,8\}$

$A - C = \emptyset$

$B - A = \langle -3;3 \rangle \setminus \{-2,0,2\}$

$B - C = (-3;3) \setminus \{-2,-1,0,1,2\}$

$C - A = \boldsymbol{Z} \setminus \{-8,-6,-4,-2,0,2,4,6,8\}$

$C - B = \boldsymbol{Z} \setminus \{-3,-2,-1,0,1,2,3\}$

4 Exercise D: Sequences

The following activities are a compilation of selected mathematics exercises from mathematics specialist networks, websites, and resources. Completing the activities on your own and referring to the solution to check your work should adequately help you master the concepts and prepare for the assessments.

1. Determine the *n*th term of the sequence:

a) $2, 4, 6, 8, 10, \ldots$

b) $1, 3, 5, 7, 9, \ldots$

c) $99, 199, 299, 399, 499, \ldots$

d) $3, -5, 7, -9, 11, \ldots$

e) $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \ldots$

f) $1, 4, 9, 16, 25, \ldots$

g) $0, 2, 6, 12, 20, \ldots$

h) $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots$

i) $6, 12, 20, 30, 42, \ldots$

j) $\frac{2}{3}, \frac{3}{2\times4}, \frac{4}{3\times5}, \frac{5}{4\times6}, \frac{6}{5\times7}, \ldots$

k) $0, \frac{1}{3}, 0, \frac{1}{3}, 0, \ldots$

l) $-\frac{1}{2}, \frac{2}{5}, -\frac{3}{8}, \frac{4}{11}, -\frac{5}{14}, \ldots$

2. Find the sum of the first five terms of the sequence given by the recurrence relation:

a) $a_{n+1} = a_n - 1.5$

$a_1 = 3.5$

b) $a_{n+1} = \dfrac{1}{a_n} - 1$

$a_1 = 5$

c) $a_{n+1} = \dfrac{a_n^2 - n^2}{2}$

$a_1 = 1$

d) $a_{n+1} = \sqrt{a_n}$

$a_1 = 65{,}536$

e) $a_{n+1} = 2^n \times (n^2 - 2a_n)$

$a_1 = 1$

f) $a_{n+1} = 1 - |1 - a_n| - a_n$

$a_1 = 2$

g) $a_{n+2} = \dfrac{a_{n+1}}{a_n}$

$a_1 = -3; \quad a_2 = -6$

h) $a_{n+2} = a_{n+1} - 2a_n + n^2$

$a_1 = 4; \quad a_2 = 10$

i) $a_{n+2} = (a_n - a_{n+1})^2 - 10$

$a_1 = 5; \quad a_2 = 1$

j) $a_{n+2} = \dfrac{1}{a_n} - \dfrac{1}{a_{n+1}}$

$a_1 = 0{,}5; \quad a_2 = 4$

k) $a_{n+2} = \dfrac{2a_{n+1}}{a_n} - 1$

$a_1 = 1; \quad a_2 = -3$

l) $a_{n+3} = a_n + 2a_{n+1} + 3a_{n+2}$

$a_1 = 3; \quad a_2 = 2; \quad a_3 = 1$

3. Find out whether the given sequence is an arithmetic sequence. If so, find the first term and the difference of the arithmetic sequence and determine whether the sequence is increasing or decreasing:

a) $a_n = 3$

b) $a_n = n$

c) $a_n = 5 - 2n$

d) $a_n = \frac{1}{n}$

e) $a_n = 2n + 3$

f) $a_n = \frac{5n - 1}{3}$

g) $a_n = n^2 + 3$

h) $a_n = \frac{1 - n}{3}$

i) $a_n = \frac{n + 1}{2n}$

j) $a_n = (-1)^n \times 5$

k) $a_n = 1 - (-1)^n$

l) $a_n = n^2 + 2n - 1$

4. Find the terms a_2, a_5 and a_7 of the arithmetic sequence if you know:

a)	$a_1 = 4;\ d = 3$	**g)**	$a_1 = -5;\ d = -2$	**m)**	$a_{24} = -112;\ s_5 = -480$
b)	$a_4 = 35;\ d = 0$	**h)**	$a_1 = 0;\ d = -1.5$	**n)**	$s_n = \frac{1}{2}(7n - 3n^2)$
c)	$a_1 = -15;\ s_{42} = 2{,}520$	**i)**	$a_1 = -100;\ a_{100} = 78.2$	**o)**	$s_n = n^2 + 2n$
d)	$a_1 = 125;\ d = -2$	**j)**	$a_{17} = -25;\ d = 3$	**p)**	$s_n = 5n - 4n^2$
e)	$a_1 = 5;\ s_{10} = -220$	**k)**	$a_1 = -5;\ d = 2$	**q)**	$s_n = 6n^2 - 7n$
f)	$a_3 = 12;\ d = 1$	**l)**	$a_{12} = 25;\ a_{26} = -10$	**r)**	$s_n = -2n^2 - 3n$

5. Find the sum s_5, s_{12} and s_{20} of the arithmetic sequence if you know:

a)	$s_n = 12n - 3n^2$	**g)**	$a_8 = 11;\ d = 4$	**m)**	$a_1 = 12;\ a_{16} = 57$
b)	$a_5 = 15;\ a_{10} = 40$	**h)**	$a_6 = -10;\ a_{10} = 8$	**n)**	$s_n = 6n^2 - 7n$
c)	$a_4 = 7;\ a_8 = -1$	**i)**	$a_3 = 150;\ a_9 = 0$	**o)**	$a_4 : a_6 = -1;\ a_2 a_8 = -9$
d)	$a_3 = 1;\ d = -0.5$	**j)**	$a_4 = -3;\ d = 2$	**p)**	$a_1 a_4 = -8;\ a_9 : a_3 = -8$
e)	$a_1 = -2;\ a_9 = 2$	**k)**	$a_{35} = -16;\ d = -3$	**q)**	$a_9 : a_2 = 3;\ a_3 - a_6 = 6$
f)	$a_5 = 7;\ a_9 = 11$	**l)**	$a_3 = -5;\ a_7 = -7$	**r)**	$a_7 - a_3 = 8;\ a_2 a_7 = 75$

6. We put a few numbers between numbers 12 and 48 so that all the numbers together now form the increasing finite arithmetic sequence. The sum of all entered numbers is 330. What is the difference of the arithmetic sequence?

7. Put 7 numbers between the numbers 3 and 43 so that they all together form an arithmetic sequence. What are the numbers?

8. Find the sum of

a) the first *n* consecutive odd numbers
b) the first *n* consecutive even numbers

9. In the cinema with a capacity of 1,656 people is 105 seats in the last row. How many rows are in the cinema, if in each lower row is always 3 seats less than in the previous row?

10. The student needs to study 313 pages of math textbook for an exam he has in 14 days. If the first day he studied 35 pages and each following day he studied two pages less than the previous day, how many pages will he have to handle during the day before the exam?

11. The production volume of this year amounted of 90 product units. How big should be the annual increase to have the production volume doubled in 4 years?

12. Of how many percent we have to increase the production each year in order to have the production increased by 60% in five years?

4.1 Exercise D: Sequences – Solutions

1.

a) $a_n = 2n$

b) $a_n = 2n - 1$

c) $a_n = 100n - 1$

d) $a_n = (-1)^{n+1} \cdot (2n+1)$

e) $a_n = \frac{n+1}{n}$

f) $a_n = n^2$

g) $a_n = n\,(n-1)$

h) $a_n = \left(\frac{2}{3}\right)^{n-1}$

i) $a_n = (n+1)\,(n+2)$

j) $a_n = \frac{n+1}{n\,(n+2)}$

k) $a_n = \frac{1+(-1)^n}{6}$

l) $a_n = (-1)^n \cdot \frac{n}{3n-1}$

2.

a) $s_5 = 2.5$

b) $s_5 = -\frac{2\,777}{2\,340}$

c) $s_5 = -\frac{67}{8}$

d) $s_5 = 65{,}814$

e) $s_5 = 13{,}927$

f) $s_5 = 0$

g) $s_5 = -7.5$

h) $s_5 = -6$

i) $s_5 = 98$

j) $s_5 = \frac{173}{18}$

k) $s_5 = -\frac{155}{21}$

l) $s_5 = 66$

3.

a) sequence is arithmetic
$a_1 = 3;\ d = 0;$ it's constant

b) sequence is arithmetic
$a_1 = 1;\ d = 1;$ it's increasing

c) sequence is arithmetic
$a_1 = 3;\ d = -2;$ it's decreasing

d) sequence is not arithmetic

e) sequence is arithmetic
$a_1 = 5;\ d = 2;$ it's increasing

f) sequence is arithmetic
$a_1 = \frac{4}{3};\ d = \frac{5}{3};$ it's increasing

g) sequence is not arithmetic

h) sequence is arithmetic
$a_1 = 0;\ d = -\frac{1}{3};$ it's decreasing

i) sequence is not arithmetic

j) sequence is not arithmetic

k) sequence is not arithmetic

l) sequence is not arithmetic

4.

a) $a_2 = 7$, $a_5 = 16$, $a_7 = 22$

b) $a_2 = 35$, $a_5 = 35$, $a_7 = 35$

c) $a_2 = -\frac{465}{41}$, $a_5 = -\frac{15}{41}$, $a_7 = \frac{285}{41}$

d) $a_2 = 123$, $a_5 = 117$, $a_7 = 113$

e) $a_2 = -1$, $a_5 = -19$, $a_7 = -31$

f) $a_2 = 11$, $a_5 = 14$, $a_7 = 16$

g) $a_2 = -7$, $a_5 = -13$, $a_7 = -17$

h) $a_2 = -1.5$, $a_5 = -6$, $a_7 = -9$

i) $a_2 = -98.2$, $a_5 = -92.8$, $a_7 = -89.2$

j) $a_2 = -70$, $a_5 = -61$, $a_7 = -55$

k) $a_2 = -3$, $a_5 = 3$, $a_7 = 7$

l) $a_2 = 50$, $a_5 = 42.5$, $a_7 = 37.5$

m) $a_2 = -\frac{2000}{21}$, $a_5 = -\frac{2048}{21}$, $a_7 = -\frac{2080}{21}$

n) $a_2 = -1$, $a_5 = -10$, $a_7 = -16$

o) $a_2 = 5$, $a_5 = 11$, $a_7 = 15$

p) $a_2 = -7$, $a_5 = -31$, $a_7 = -47$

q) $a_2 = 11$, $a_5 = 47$, $a_7 = 71$

r) $a_2 = -9$, $a_5 = -21$, $a_7 = -29$

5.

a)	$s_5 = -15$ $s_{12} = -288$ $s_{20} = -960$	**g)**	$s_5 = -45$ $s_{12} = 60$ $s_{20} = 420$	**m)**	$s_5 = 90$ $s_{12} = 342$ $s_{20} = 810$
b)	$s_5 = 25$ $s_{12} = 270$ $s_{20} = 850$	**h)**	$s_5 = -117.5$ $s_{12} = -93$ $s_{20} = 205$	**n)**	$s_5 = 115$ $s_{12} = 780$ $s_{20} = 2{,}260$
c)	$s_5 = 45$ $s_{12} = 24$ $s_{20} = -120$	**i)**	$s_5 = 750$ $s_{12} = 750$ $s_{20} = -750$	**o)**	$s_5 = \pm 10$ $s_{12} = \mp 18$ $s_{20} = \mp 110$
d)	$s_5 = 5$ $s_{12} = -9$ $s_{20} = -55$	**j)**	$s_5 = -25$ $s_{12} = 24$ $s_{20} = 200$	**p)**	$s_5 = \pm 10$ $s_{12} = \mp 102$ $s_{20} = \mp 410$
e)	$s_5 = -5$ $s_{12} = 9$ $s_{20} = 55$	**k)**	$s_5 = 400$ $s_{12} = 834$ $s_{20} = 1{,}150$	**q)**	$s_5 = -45$ $s_{12} = -192$ $s_{20} = -480$
f)	$s_5 = 25$ $s_{12} = 102$ $s_{20} = 250$	**l)**	$s_5 = -25$ $s_{12} = -81$ $s_{20} = -175$	**r)**	$s_5 = 35 \vee s_5 = -65$ $s_{12} = 168 \vee s_{12} = -72$ $s_{20} = 440 \vee s_{20} = 40$

6. $d = 3$

7. **a)** a = 18 cm; b = 24 cm; c = 30 cm
b) a = 9 cm; b = 12 cm; c = 15 cm

8.

9. There are 23 rows of seats in the cinema.

10. During the day before the exam the student will have to handle 14 pages of math textbook.

11. 18.92 %

12. 6.8 years

5 Exercise E: Functions

The following activities are a compilation of selected mathematics exercises from mathematics specialist networks, websites, and resources. Completing the activities on your own and referring to the solution to check your work should adequately help you master the concepts and prepare for the assessments.

1. Determine two properties of each of the following functions:

   Examples of properties:

   - domain of a function, range of a function
   - function is/is not one-to-one function
   - continuous/discontinuous function
   - even/odd function
   - etc.

a) $y = -3$

b) $y = x$

c) $y = 2x$

d) $y = 4x + 2$

e) $y = -2x - 4$

f) $y = 0.5x$

g) $y = \frac{1}{4}x + 1$

h) $y = -x + 5$

i) $y = -\frac{3}{5}x$

j) $y = -x - 1$

k) $y = -3x + 0.5$

l) $y = \frac{-1}{3}x - \frac{-2}{3}$

m) $y = \frac{2x}{3} - 7$

n) $y = 1 - x$

o) $y = 5 - 2x + 1$

p) $y - 2 = -x + 2$

q) $2x + 4y - 6 = 0$

r) $y - x = \frac{3}{2}$

s) $x + y - 1 = 10 - x$

t) $2x + 5y + 8 = 12 - 4x$

u) $y = |x|$

v) $y = 2|x|$

w) $y = |2x| + 2$

x) $y = |-x + 1|$

y) $y = -2|4x - 8| + 3$

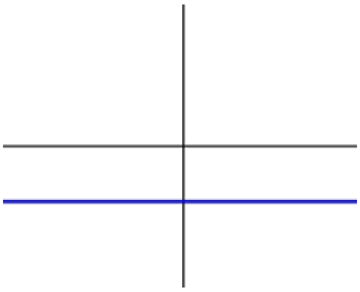
z) $y = |x + 3| - |x - 3|$

5.1 Exercise E: Functions – Solutions

1. Any two properties (you do not need to master and know all the properties, however, a range of properties have been given for information purposes) Your prescribed material (MM) mentions and indicates the properties that should be mastered. You are also not expected to draw and interpret any graphs in this section:

a) $D(f)$: $(-\infty;\infty)$

$R(f)$: $\{-3\}$



function:

is not one-to-one function

is continuous

is even

is not periodic

is bounded

x–axis int.: $\varnothing$

y–axis int.: $[0;-3]$

local min.: $\varnothing$

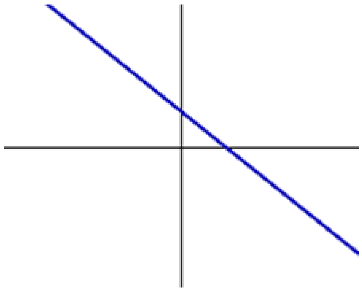
local max.: $\varnothing$

increasing: $\varnothing$

decreasing: $\varnothing$

n) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is neither even nor odd

is not periodic

is unbounded

x–axis int.: $[1;0]$

y–axis int.: $[0;1]$

local min.: $\varnothing$

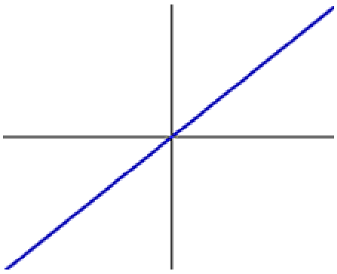
local max.: $\varnothing$

increasing: $\varnothing$

decreasing: $(-\infty;\infty)$

b) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is odd

is not periodic

is unbounded

x–axis int.: $[0;0]$

y–axis int.: $[0;0]$

local min.: Ø

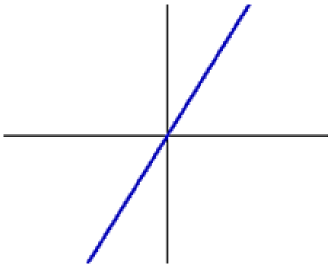
local max.: Ø

increasing: $(-\infty;\infty)$

decreasing: Ø

c) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is odd

is not periodic

is unbounded

x–axis int.: $[0;0]$

y–axis int.: $[0;0]$

local min.: Ø

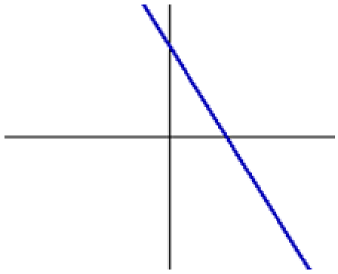
local max.: Ø

increasing: $(-\infty;\infty)$

decreasing: Ø

o) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is neither even nor odd

is not periodic

is unbounded

x–axis int.: $[3;0]$

y–axis int.: $[0;6]$

local min.: Ø

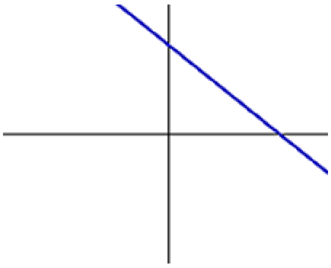
local max.: Ø

increasing: Ø

decreasing: $(-\infty;\infty)$

p) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is neither even nor odd

is not periodic

is unbounded

x–axis int.: $[4;0]$

y–axis int.: $[0;4]$

local min.: Ø

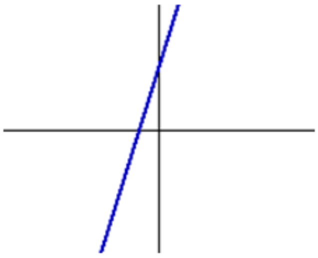
local max.: Ø

increasing: Ø

decreasing: $(-\infty;\infty)$

d) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:
is one-to-one function
is continuous
is neither even nor odd
is not periodic
is unbounded

x–axis int.: $\left[-\frac{1}{2};0\right]$

y–axis int.: $[0;2]$

local min.: $\varnothing$

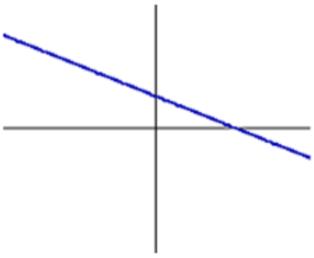
local max.: $\varnothing$

increasing: $(-\infty;\infty)$

decreasing: $\varnothing$

q) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:
is one-to-one function
is continuous
is neither even nor odd
is not periodic
is unbounded

x–axis int.: $[3;0]$

y–axis int.: $\left[0;\frac{3}{2}\right]$

local min.: $\varnothing$

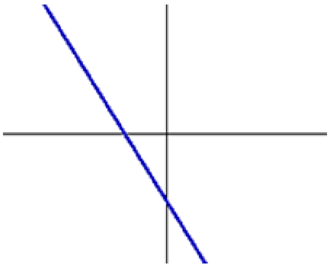
local max.: $\varnothing$

increasing: $\varnothing$

decreasing: $(-\infty;\infty)$

e) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:
is one-to-one function
is continuous
is neither even nor odd
is not periodic
is unbounded

x–axis int.: $[-2;0]$

y–axis int.: $[0;-4]$

local min.: $\varnothing$

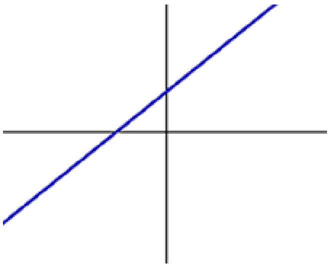
local max.: $\varnothing$

increasing: $\varnothing$

decreasing: $(-\infty;\infty)$

r) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:
is one-to-one function
is continuous
is neither even nor odd
is not periodic
is unbounded

x–axis int.: $\left[-\frac{3}{2};0\right]$

y–axis int.: $\left[0;\frac{3}{2}\right]$

local min.: $\varnothing$

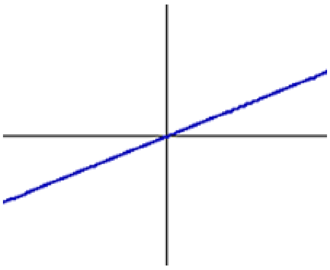
local max.: $\varnothing$

increasing: $(-\infty;\infty)$

decreasing: $\varnothing$

f) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is odd

is not periodic

is unbounded

x–axis int.: $[0;0]$

y–axis int.: $[0;0]$

local min.: Ø

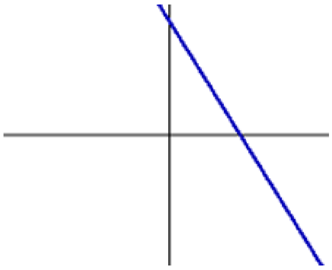
local max.: Ø

increasing: $(-\infty;\infty)$

decreasing: Ø

s) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is neither even nor odd

is not periodic

is unbounded

x–axis int.: $\left[\frac{11}{2};0\right]$

y–axis int.: $[0;11]$

local min.: Ø

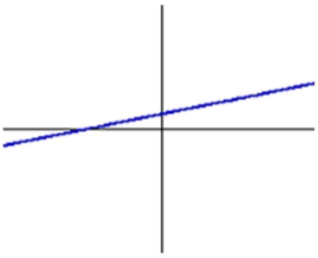
local max.: Ø

increasing: Ø

decreasing: $(-\infty;\infty)$

g) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is neither even nor odd

is not periodic

is unbounded

x–axis int.: $[-4;0]$

y–axis int.: $[0;1]$

local min.: Ø

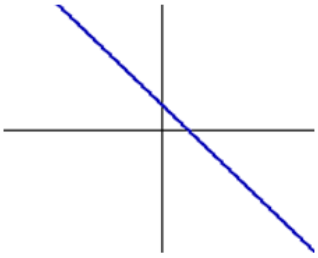
local max.: Ø

increasing: $(-\infty;\infty)$

decreasing: Ø

t) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is neither even nor odd

is not periodic

is unbounded

x–axis int.: $\left[\frac{2}{3};0\right]$

y–axis int.: $\left[0;\frac{4}{5}\right]$

local min.: Ø

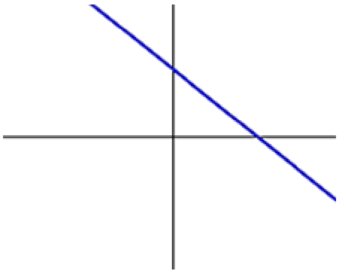
local max.: Ø

increasing: Ø

decreasing: $(-\infty;\infty)$

h) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is neither even nor odd

is not periodic

is unbounded

x–axis int.: $[5;0]$

y–axis int.: $[0;5]$

local min.: $\emptyset$

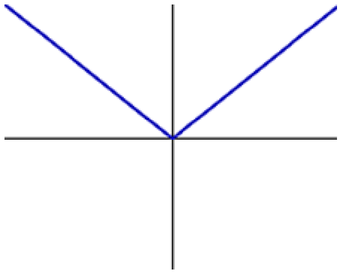
local max.: $\emptyset$

increasing: $\emptyset$

decreasing: $(-\infty;\infty)$

u) $D(f)$: $(-\infty;\infty)$

$R(f)$: $\langle 0;\infty)$



function:

is not one-to-one function

is continuous

is even

is not periodic

is bounded below

x–axis int.: $[0;0]$

y–axis int.: $[0;0]$

local min.: $[0;0]$

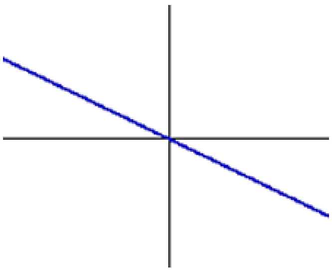
local max.: $\emptyset$

increasing: $(0;\infty)$

decreasing: $(-\infty;0)$

i) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is odd

is not periodic

is unbounded

x–axis int.: $[0;0]$

y–axis int.: $[0;0]$

local min.: $\emptyset$

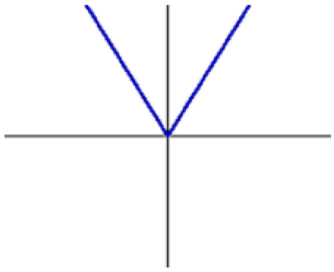
local max.: $\emptyset$

increasing: $\emptyset$

decreasing: $(-\infty;\infty)$

v) $D(f)$: $(-\infty;\infty)$

$R(f)$: $\langle 0;\infty)$



function:

is not one-to-one function

is continuous

is even

is not periodic

is bounded below

x–axis int.: $[0;0]$

y–axis int.: $[0;0]$

local min.: $[0;0]$

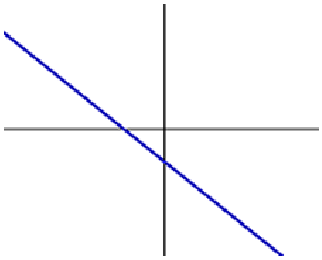
local max.: $\emptyset$

increasing: $(0;\infty)$

decreasing: $(-\infty;0)$

j) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is neither even nor odd

is not periodic

is unbounded

x–axis int.: $[-1;0]$

y–axis int.: $[0;-1]$

local min.: $\varnothing$

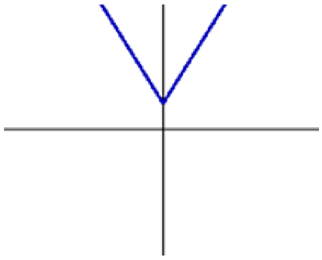
local max.: $\varnothing$

increasing: $\varnothing$

decreasing: $(-\infty;\infty)$

w) $D(f)$: $(-\infty;\infty)$

$R(f)$: $\langle 2;\infty)$



function:

is not one-to-one function

is continuous

is even

is not periodic

is bounded below

x–axis int.: $\varnothing$

y–axis int.: $[0;2]$

local min.: $[0;2]$

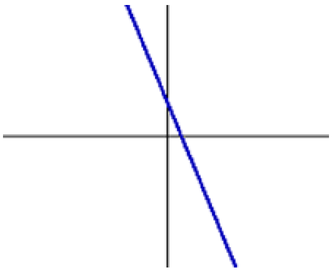
local max.: $\varnothing$

increasing: $(0;\infty)$

decreasing: $(-\infty;0)$

k) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is neither even nor odd

is not periodic

is unbounded

x–axis int.: $\left[\frac{1}{6};0\right]$

y–axis int.: $\left[0;\frac{1}{2}\right]$

local min.: $\varnothing$

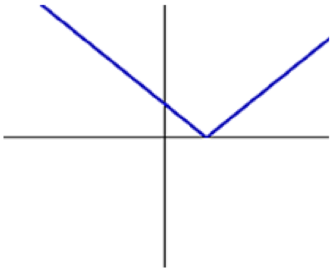
local max.: $\varnothing$

increasing: $\varnothing$

decreasing: $(-\infty;\infty)$

x) $D(f)$: $(-\infty;\infty)$

$R(f)$: $\langle 0;\infty)$



function:

is not one-to-one function

is continuous

is neither even nor odd

is not periodic

is bounded below

x–axis int.: $[1;0]$

y–axis int.: $[0;1]$

local min.: $[1;0]$

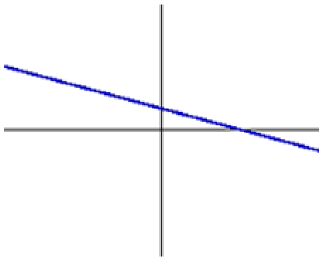
local max.: $\varnothing$

increasing: $(1;\infty)$

decreasing: $(-\infty;1)$

l) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is neither even nor odd

is not periodic

is unbounded

x–axis int.: $\left[2;0\right]$

y–axis int.: $\left[0;\dfrac{2}{3}\right]$

local min.: Ø

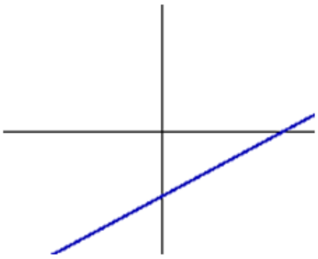
local max.: Ø

increasing: Ø

decreasing: $(-\infty;\infty)$

m) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;\infty)$



function:

is one-to-one function

is continuous

is neither even nor odd

is not periodic

is unbounded

x–axis int.: $\left[\dfrac{21}{2};0\right]$

y–axis int.: $\left[0;-7\right]$

local min.: Ø

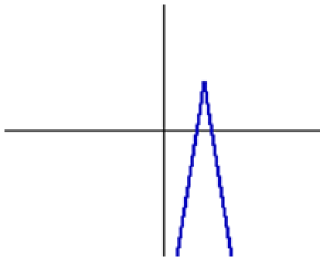
local max.: Ø

increasing: $(-\infty;\infty)$

decreasing: Ø

y) $D(f)$: $(-\infty;\infty)$

$R(f)$: $(-\infty;3\rangle$



function:

is not one-to-one function

is continuous

is neither even nor odd

is not periodic

is bounded above

x–axis int.: $\left[\dfrac{13}{8};0\right]$; $\left[\dfrac{19}{8};0\right]$

y–axis int.: $\left[0;-13\right]$

local min.: Ø

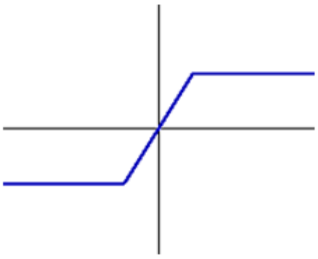
local max.: $\left[2;3\right]$

increasing: $(-\infty;2)$

decreasing: $(2;\infty)$

z) $D(f)$: $(-\infty;\infty)$

$R(f)$: $\langle -6;6\rangle$



function:

is not one-to-one function

is continuous

is odd

is not periodic

is bounded

x–axis int.: $\left[0;0\right]$

y–axis int.: $\left[0;0\right]$

local min.: Ø

local max.: Ø

increasing: $(-3;3)$

decreasing: Ø

Learning Unit 4: Counting	
Material used for this learning unit: • Module Manual Learning Unit 4; • Workbook; • IIE Learn.	**My notes**

1 Exercise A: Learn Activity 4.1.1

Complete Activity 4.1.1: Concepts in counting on Learn.

1.1 Exercise A: Learn Activity 4.1.1 – Solutions

Definition	**Term**
If event A can occur in m ways, and event B can occur in n disjoint ways, then the event "A or B" can occur in m + n ways.	[a] Additive Principle
If event A can occur in m ways, and each possibility for A allows for exactly n ways for event B, then the event "A and B" can occur in m x n ways.	[b] Multiplicative Principle
Given two sets A and B, if $A \cap B = \emptyset$ (that is, if there is no element in common to both A and B), then $\lvert A \cup B \rvert = \lvert A \rvert + \lvert B \rvert$	[c] Additive Principle (with sets)
Given sets A and B, we can form the set $A \times B = \{(x,y): x \in A \wedge y \in B\}$ to be the set of all ordered pairs (x,y) where x is an element of A and y is an element of B. We call $A \times B$ the Cartesian product of A and B.	[d] Cartesian Product
Given two sets A and B, we have $\lvert A \times B \rvert = \lvert A \rvert \times \lvert B \rvert$.	[e] Multiplicative Principle (with sets)
For any finite sets A and B, $\lvert A \cup B \rvert = \lvert A \rvert + \lvert B \rvert - \lvert A \cap B \rvert$.	[f] Cardinality of a union (2 sets)
For any finite sets A, B, and C, $\lvert A \cup B \cup C \rvert = \lvert A \rvert + \lvert B \rvert + \lvert C \rvert - \lvert A \cap B \rvert - \lvert A \cap C \rvert - \lvert B \cap C \rvert + \lvert A \cap B \cap C \rvert$.	[g] Cardinality of a union (3 sets)
There is no way for A and B to both happen at the same time.	[h] Disjoint events

2 Exercise B: Recall LU4 Activities

1. How many two letter "words" start with either A or B? (A **word** is just a strings of letters; it doesn't have to be English, or even pronounceable.)

2. How many two letter words start with one of the 5 vowels?

3. Suppose you are going for some Milkshake. You can pick one of 6 milkshake choices, and one of 4 toppings. How many choices do you have?

4. How many license plates can you make out of three letters followed by three numerical digits?

5. How many functions $f : \{1, 2, 3, 4, 5\} \to \{a, b, c, d\}$ have?

6. Suppose you own 9 shirts and 5 pairs of pants.

A. How many outfits can you make?
B. If today is half-naked-day, and you will wear only a shirt or only a pair of pants, how many choices do you have?

7. Let $A = \{1, 2\}$ and $B = \{3, 4, 5\}$. Find $A \times B$.

8. An examination in three subjects, Algebra, Biology, and Chemistry, was taken by 41 students. The following table shows how many students failed in each single subject and in their various combinations:

Subject:	A	B	C	AB	AC	BC	ABC
Failed:	12	5	8	2	6	3	1

How many students failed at least one subject?

9. You decide to give away your video game collection so to better spend your time studying advance mathematics. How many ways can you do this? Provided:

   A. You want to distribute your 3 different PS4 games among 5 friends, so that no friend gets more than one game?
   B. You want to distribute your 8 different 3DS games among 5 friends?
   C. You want to distribute your 8 different SNES games among 5 friends, so that each friend gets at least one game?

   In each case, model the counting question as a function counting question.

10. How many functions $f\colon \{1, 2, 3, 4, 5\} \to \{a, b\}$ are surjective?

11. How many functions $f\colon \{1, 2, 3, 4, 5\} \to \{a, b, c\}$ are surjective?

12. Write representations for the following. If it is not possible to do the representation, put "Impossible".
    A. Represent 101 with 7 bits
    B. Represent 28 with 10 bits
    C. Represent 7 with 3 bits
    D. Represent 18 with 4 bits
    E. Represent 28232 with 16 bits

13. It is really useful to know roughly how many bits you will need to represent a certain value. Think about the following scenarios, and choose the best number of bits out of the options given.

    You want to ensure that the largest possible number will fit within the number of bits, but you also want to ensure that you are not wasting space.

    1. Storing the day of the week
       a) 1 bit
       b) 4 bits
       c) 8 bits
       d) 32 bits

    2. Storing the number of people in the world
       a) 16 bits
       b) 32 bits
       c) 64 bits
       d) 128 bits

    3. Storing the number of roads in New Zealand
       a) 16 bits
       b) 32 bits
       c) 64 bits
       d) 128 bits

    4. Storing the number of stars in the universe
       a) 16 bits
       b) 32 bits
       c) 64 bits
       d) 128 bits

14. What would the decimal values be for the following, assuming that the first bit is a sign bit?
    A. 00010011
    B. 10000110
    C. 10100011
    D. 01111111
    E. 11111111

15. Determining the Two's Complement:

    What would be the two's complement representation for the following numbers, using 8 bits? Follow the process given in this section, and remember that you do not need to do anything special for positive numbers.
    A. 19
    B. -19
    C. 107
    D. -107
    E. -92

16. Convert the following Two's Complement numbers to decimal.
    A. 00001100
    B. 10001100
    C. 10111111

17. Answer the following questions:

    A. How would you represent “science” in ASCII?
    B. How would you represent "Wellington" in ASCII? (note that it starts with an upper-case “W”)
    C. How would you represent “358” in ASCII (it is three characters, even though it looks like a number)
    D. How would you represent "Hello, how are you?" (look for the comma, question mark, and space characters in ASCII table)

18. What is the largest number that can be represented with 32 bits? (In both decimal and binary).

19. The largest number in Unicode that has a character assigned to it is not actually the largest possible 32-bit number -- it is 00000000 00010000 11111111 11111111. What is this number in decimal?

20. Most numbers that can be made using 32 bits do not have a Unicode character attached to them -- there is a lot of wasted space. There are good reasons for this, but if you had a shorter number that could represent any character, what is the minimum number of bits you would need, given that there are currently around 120,000 Unicode characters?

21. How many hosts will each segment support if the Class B address 132.2.0.0 is subnetted with a 29-bit mask?

22. Given the IP address 172.16.12.54, with a mask of 255.255.255.240, which of the following are valid host addresses on the same subnet?

A. 172.16.12.64
B. 172.16.12.57
C. 172.16.12.49
D. 172.16.12.48
E. 172.16.12.63
F. 172.16.12.45

3 Exercise C: IP Addressing

1. Convert the following from binary to decimal:

   A. 11111111
   B. 1000101
   C. 0110001
   D. 1110000
   E. 00011011

2. Convert the following from decimal to binary

   A. 255
   B. 50
   C. 123
   D. 224
   E. 114

3. Which class is each of the following IP address belongs to?

A. 219.21.56.0
B. 95.0.21.90
C. 119.18.45.0
D. 220.200.23.1
E. 10.250.1.1

4. For each of the following IP addresses, identify the network portion and the host portion. Write down the default subnet mark for each address:

A. 17.45.222.45
B. 192.200.15.0
C. 217.21.56.0
D. 218.155.230.14
E. 126.201.54.231

5. Using the IP address and subnet mask shown write out the network address:

A. 199.20.150.35 ______________________________ 255.255.255.0
B. 223.69.230.250 ______________________________ 255.255.0.0
C. 192.149.24.191 ______________________________ 255.255.255.0

6. Using the IP address and subnet mask shown write out the host address:

A. 203.20.35.215 ______________________________ 255.255.255.0
B. 10.10.10.10 ______________________________ 255.0.0.0
C. 191.55.165.135 ______________________________ 255.255.255.0