ML1 – introduction

Machine Learning – Tools and applications for policy – Lecture 2 $\,$

Iman van Lelyveld – Michiel Nijhuis DNB Data Science Hub



ML1 – introduction

- 1. What is ML? Can we see OLS as a ML problem?
- 2. What is ML applied to?
- 3. The outlines of the ML approach
 - supervised vs. unsupervised learning
 - (hyper)parameters and models
 - gradient descent and grid search
 - pre-processing features

Outline

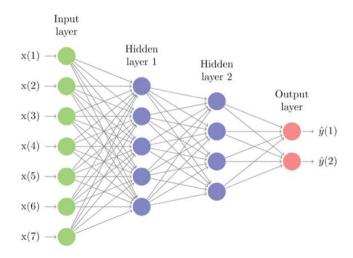
ML1 – introduction
Defining ML/AI
The ML approach
OLS with Gradient Descent
Scaling, normalization and standardization
The workhorse: the logit activation function
Ensuring robustness and measure performance



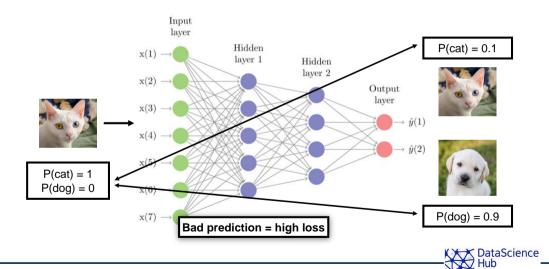
- How does optimization work? Part 1 (Brandon Rohrer) (link)
- How does optimization work? Part 2 (Brandon Rohrer) (link)
- How does optimization work? Part 3 (Brandon Rohrer) (link)
- How does optimization work? Part 4 (Brandon Rohrer) (link)

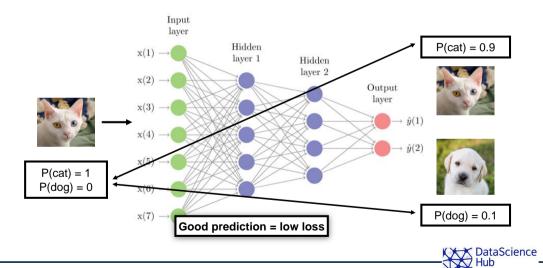
The field of study that gives computers the ability to learn without being explicitly programmed. (Samuel (1967))

A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E. (Mitchell (1997))









Outline 7

ML1 – introduction

Defining ML/AI

The ML approach

OLS with Gradient Descen

Scaling, normalization and standardization

The workhorse: the logit activation function

Ensuring robustness and measure performance

- 1. a problem
- 2. a data source
- 3. a model
 - e.g. logit, artificial neural networks
 - cost function: e.g. MSE
 - activation function
 - regularization scheme
- 4. an optimization algorithm
 - out of scope
 - Out of the box Python or R optimizers can get stuck in a local minimum
 - Initialization
- 5. validation & testing



- ML is an optimization problem, whose solution determines a set of model parameters $F(X, Y, \beta; \lambda)$
 - *Y* is the target/outcome variable that you want to predict with input data *X*
 - λ are model parameters depending on the input and the model class
 - Optimizing the objective/cost function $F(\cdot, \beta)$, wrt the parameters λ (training)

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- Although terminology in machine learning and econometrics varies, it often refers to the same concepts.
 - dependent variable, left-hand-side (LHS) ≡ Output, target response
 - independent variables, covariates, right-hand-side (RHS) \equiv Features, inputs



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- #features does not have to equal $\#X \rightarrow \text{higher-order features}$

We may define a cause to be an object, followed by another, and where all the objects similar to the first are followed by objects similar to the second. Or in other words where, if the first object had not been, the second never had existed (Hume (1748))

Social scientists know that large amounts of data will not overcome the selection problems that make causal inference so difficult (Grimmer, Justin (2015))

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• Econometrics:

- Classical conditional: if *X* occurred, then *Y* occurred
- Counterfactual: if *X* had not occurred, then *Y* would not have occurred
- Model and causality imply assumptions about the error term

Machine Learning:

- Prediction/classification
- "If it works, it works"



Feature	Role	Most common	Other examples in practice	
Cost function	1	*	Mean absolute error, Categorical cross entropy, Kullback-Leibler divergence, Cosine proximity, Hinge/Squared-Hinge, log-cosh	

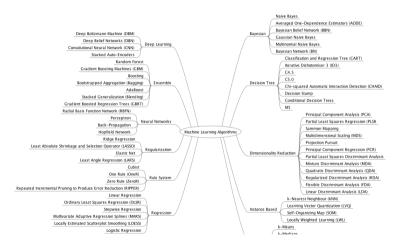
Feature	ature Role		Other examples in practice	
Cost function	Calculate penalty/error in prediction versus true output		Mean absolute error, Categorical cross entropy, Kullback-Leibler divergence, Cosine proximity,	
Activation Function	Achieve non-linear effect following each neuron (af- ter the weighted linear combination)	ReLU (intermediate layers), Linear (final layer in regression), Sigmoid (final layer in classification)	Hinge/Squared-Hinge, log-cosh Softmax/Softplus/Softsign, Leaky/Parametrized ReLU, tanh, Hard Sigmoid	

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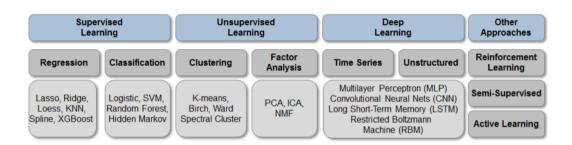
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Initialization Scheme	Initialize network weights	Xavier (including Glorot- Normal and Glorot- Uniform)		

Taxonomy of the methods









model class	S/US	reg/class	parametric	data size	norm	suited for	advantages	disadvantages
OLS	S	reg	yes	small - large	no	simple relations, hypothesis testing	interpretability, computability	inflexibility
Logit	S	class	yes	small - large	no	simple relations, hypothesis testing	interpretability, computability	inflexibility
naïve Bayes	S	class	no	small - large	no	simple benchmark	computability	independence assumption
k-NN	S	reg/class	no	small - medium	yes	clustered data, multiple regimes	flexibility	interpretability, COD
tree model	S	reg/class	no	small - large	no	complex relations, multiple regimes	flexibility, interpretability, computability	greedy, over-fitting
random forest	S	reg/class	no	small - medium	no	complex relations, multiple regimes	flexibility	computability, interpretability
artificial neural network (ANN)	S	reg/class	semi	mid - large	yes	complex relations, multiple scales	flexibility	computability, over-fitting, data hungry, interpretability
support vector machine (SVM)	S	reg/class	no	small - medium	yes	complex relations	flexibility, computability	over-fitting, interpretability
k-means	US		no	small - large	yes	feature extraction, stylised facts, structure	interpretability	interpretability, COD
hierarchical clustering analysis (HCA)	US		no	small - large	yes	feature extraction, stylised facts, structure	interpretability	interpretability, COD

Source: Chakraborty and A. Joseph (2017)



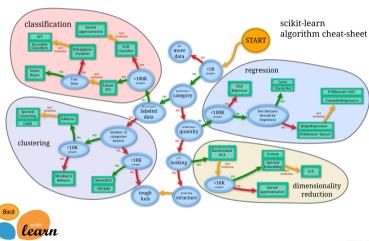
Which model to use?

Figure 41: Typical tasks and frequently used Machine Learning methods

Overation	Data Analasia Taskaina		
Question	Data Analysis Technique		
Given set of inputs, predict asset price direction	Support Vector Classifier, Logistic Regression,		
	Lasso Regression, etc.		
How will a sharp move in one asset affect other assets?	Impulse Response Function, Granger Causality		
Is an asset diverging from other related assets?	One-vs-rest classification		
Which assets move together?	Affinity Propagation, Manifold Embedding		
What factors are driving asset price?	Principal Component Analysis, Independent		
Is the asset move excessive, and will it revert?	Component Analysis		
What is the current market regime?	Soft-max classification, Hidden Markov Model		
What is the probability of an event?	Decision Tree, Random Forest		
What are the most common signs of market stress?	K-means clustering		
Find signals in noisy data	Low-pass filters, SVM		
Predict volatility based on a large number of input variables	Restricted Boltzmann Machine, SVM		
What is the sentiment of an article / text?	Bag of words		
What is the topic of an article/text?	Term/InverseDocument Frequency		
Counting objects in an image (satellite, drone, etc)	Convolutional Neural Nets		
What should be optimal execution speed?	Reinforcement Learning using Partially Observed Markov Decision Process		
Source: LP Morgan Macro ODS			

Source: J.P.Morgan Macro QDS







- classical Y = f(X)
- *labeled*: x_i is matched with y_i . Alternatively, it is known what class y_i belongs to (eg employed/unemployed)

1. Classification problems

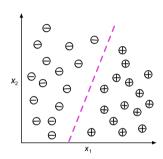
- is it a cat or a dog?
- output *Y* consists of a discrete set of outcomes which cannot be ordered. That is, each element of *Y* represents a class label
- economics: employment status of individuals (Y: employed or unemployed) from their communication or consumption habits $(X_{i...l})$ would be a typical classification problem. A learning method would return one of the two states/labels for each observation x_i

2. Regression problems

- match and return a continuous output variable

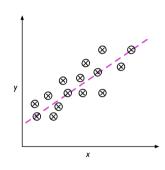


- Predict categorical class labels based on past observations
- Class labels are discrete unordered values which cannot be ordered.
- That is, each element of *Y* represents a class label and output *Y* consists of a discrete set of outcomes
- Examples
 - Binary: Email spam classification example or unemployment status
 - Multi-class: Handwritten digit classification example

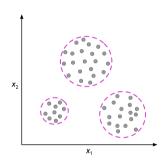




- Also a kind of supervised learning
- Prediction of continuous outcomes
- Predicting semester grades scores for students



- Finding structure in the data, e.g. clustering
- Objects within a cluster are "similar"
- Dealing with unlabeled data
- Can be used first to add labels to the observations or extract new features (e.g. cluster affiliation)
- Examples:
 - Cluster analysis
 - Latent Dirichlet Allocation (LDA)
 - Princeton Wiki
 - Wikipedia LDA entry
 - Sara Palin topics





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Scaling, normalization and standardization The workhorse: the logit activation function Ensuring robustness and measure performance



- The defining characteristic: taking the target variable *Y* as an input to the cost function
- Given a hypothesis h(X) representing the model, a commonly-used objective/cost function is the mean squared error (MSE). Problem takes the general form:

$$ERR(X,Y,\beta) \stackrel{\text{MSE}}{=} \frac{1}{m} \sum_{i=1}^{m} e_i^2 \equiv \frac{1}{m} \sum_{i=1}^{m} \left(h(x_i,\beta) - y_i \right)^2 \xrightarrow{\text{optimisation}} \beta$$

• The more data we have, the more complicated $h(\dot)$ can be: logit can be exchanged for a deep neural network

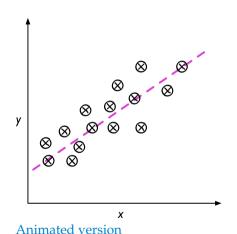
See "How does optimization work" in Knowledge clips.

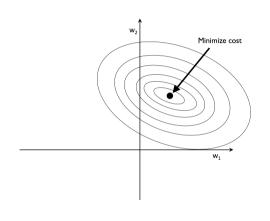


• Linear regression:

$$h(X) = X \cdot \beta \quad \to \quad ERR(X, Y, \beta) = \frac{1}{m} \sum_{i=1}^{m} (x_i \cdot \beta - y_i)^2$$

- With the Gauss-Markov theorem, the best linear unbiased estimator (BLUE) has the closed-form solution: $\beta^* = (X^T X)^{-1} X^T Y$
- BLUE assumptions might be violated → optimisation
- Algorithms look for a trade-off between computational cost and accuracy





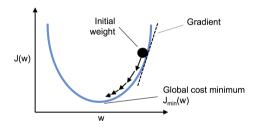


• We apply this procedure a number of times, and in each iteration we update the weights s.t.

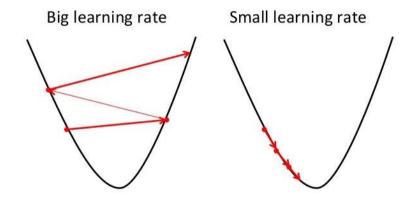
$$w_i := w_i - \alpha \frac{\delta J(w)}{\delta_i}$$

where α is a value in (0;1] called the learning rate

- The algorithm is called the (batch) gradient descent (GD)
- The loss (and therefore the gradient) is computed over all observations

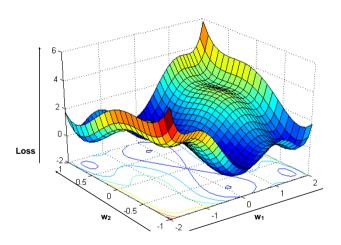


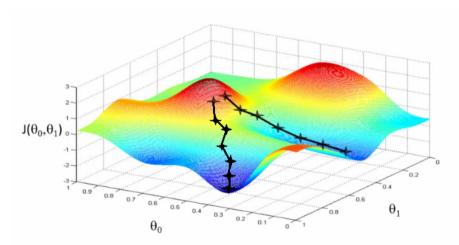




Source: What are Hyperparameters









- Large dataset with millions of data points ("big data") then batch gradient descent costly
- Remember that 'batch' actually means all the data so we need to compute the error for the entire dataset ...
- ... to take one step towards the global minimum!

$$\Delta \mathbf{w} = \eta \sum_{i} \left(y^{(i)} - \phi(z^{(i)}) \right) \mathbf{x}^{(i)}.$$

SGD updates the weights incrementally for each training sample

$$\Delta \mathbf{w} = \eta \left(y^{(i)} - \phi(z^{(i)}) \right) \mathbf{x}^{(i)}.$$

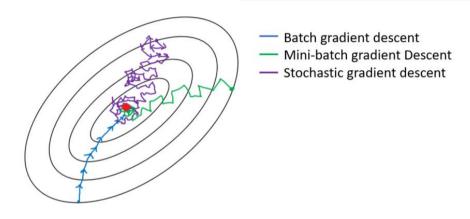


SGD details

- Approximation of gradient descent
- Faster convergence because of frequent weight updates
- Important to present data in random order
- Learning rate often gradually decreased (adaptive learning rate)
- Can be used for online learning
- Middle ground between SGD and batch GD is known as mini-batch learning
 - E.g. 50 examples at a time
 - Can use vector/matrix operations rather than loops as in SGD
 - Vectorized operations highly efficient
- SGD is mini-batch with n = 1



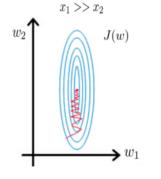
Batch size



Source: Andrew Ng – Coursera Course



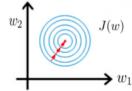
Gradient descent without scaling



Gradient descent after scaling variables

 $0 \le x_1 \le 1$

$$0 \le x_2 \le 1$$



- So Input preprocessing is important but may crucially affect a learners' performance
- Imagine we have two features
 - Feature x_1 : $1 < x_1 < 10$
 - Feature x_2 : $1 < x_2 < 100000$
- Algorithm will likely focus on optimizing w_2 as this will produce the largest changes in e.g. error/loss function. K-nearest neighbor (KNN, Euclidean distance) and Artificial Neural Networks (ANN) will be dominated by x_2 . (Both KNN and ANN discussed later)
- Two common approaches
 - Normalization
 - Standardization
 - with μ and σ
 - \triangleright with σ
- See an excellent article by Jeff Hale with JPNB. Note differences in definition.



Normalization refers to the rescaling of the features to a range of [0, 1]. To normalize the data, we apply the min-max scaling to each feature column, where the new value $x_{norm}^{(i)}$ of a sample $x^{(i)}$ is calculated as follows:

$$x_{norm}^{(i)} = \frac{x^{(i)} - \mathbf{x}_{min}}{\mathbf{x}_{max} - \mathbf{x}_{min}}$$

Here, $x^{(i)}$ is a particular sample/observation, x_{min} is the smallest value in a feature column, and x_{max} the largest value, respectively.

• Standardization (i.e., z-scores) centers the columns at mean = 0 and std = 1

$$x_{std}^{(i)} = \frac{x^{(i)} - \mu_x}{\sigma_x}$$

or similarly rescale with just σ

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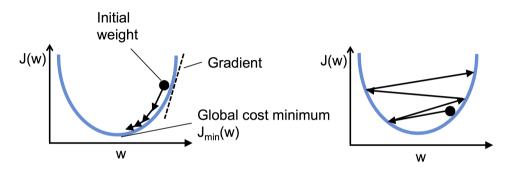
- While normalization gives us values in a bounded interval, standardization can be more practical
 - Many ML algorithms initialize the weights to zero
 - Feature columns take the form of a normal distribution
 - This makes it easier to learn the weights
- Standardization encodes useful info about outliers
- Normalization in contrast scales the data to a fixed range

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- Standardization encodes useful info about outliers
- Normalization in contrast scales the data to a fixed range
- Binning can also address non-linearity/heterogeneity (Finlay (2014))
- Other ways to achieve approximate standard normality: log or a Box DataScience transformation (cf A. C. Joseph et al. (2014))



- Learning rate too high: error becomes larger (overshoots global min)
- Learning rate too low: takes many epochs to converge
- Feature normalization

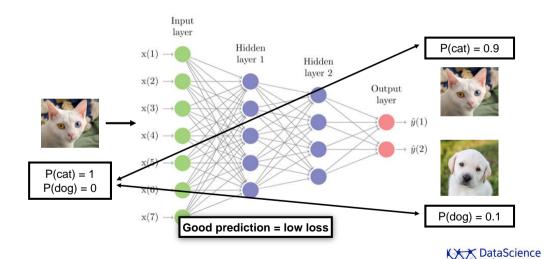


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- This is a "go to" model for classification
- Designed for binary classification but can be extended to multi-class
- Odds ratio

$$\frac{p}{(1-p)}$$

Where p is the probability of the positive class (class label y = 1). E.g. the probability that a patient has a certain disease.

• Logit function

$$logit(p) = \log \frac{p}{(1-p)}$$

• We model the logit function as a linear combination of features (dot product of feature values and weights)

$$logit(p(y = 1|\mathbf{x})) = w_0x_0 + w_1x_1 + \dots + w_mx_m = \sum_{i=0}^m w_ix_i = \mathbf{w}^T\mathbf{x}.$$

Where $p(y = 1|\mathbf{x})$ s the conditional probability that a particular sample belongs to class 1 given its features \mathbf{x}

• This is equivalent to expressing *p* as

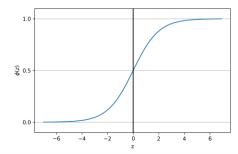
$$p(y=1|\mathbf{x}) = \frac{1}{1 + e^{-\mathbf{w}^T\mathbf{x}}}$$



• Logistic function (aka sigmoid function)

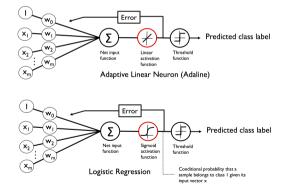
$$\phi(z) = \frac{1}{1 + e^{-z}}.$$

• S-shaped curve





- In Adaline, we used the "identity" function as the activation function
- In logistic regression, we use instead use the "sigmoid" function





Sigmoid logistic
$$y = \frac{1}{1+e^{-z}}$$

Tanh hyperbolic tangent
$$y = tanh(z)$$

ReLU Rectified Linear Unit
$$y = max(z, 0)$$

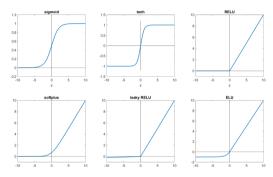
Softplus
$$y = log(1 + e^z)$$

Elu

Leaky Regularization
$$y = max(z, \alpha z), 0 < \alpha < 1$$

 α typically a small number e.g. 0.01

$$y = \begin{cases} z & z > 0 \\ \alpha(e^z - 1) & z \le 0 \end{cases}$$



Many more functions can be found here



- Output of the sigmoid often interpreted as probability
- E.g. $P(y = 1 | \mathbf{x}; \mathbf{w}) = 0.8$
- Probability can be converted to a binary outcome (quantizer)

$$\hat{y} = \begin{cases} 1 & \text{if } \phi(z) \ge 0.5\\ 0 & \text{otherwise} \end{cases}$$

• Which is equivalent to the following

$$\hat{y} = \begin{cases} 1 & \text{if } z \ge 0.0\\ 0 & \text{otherwise} \end{cases}$$

• For many applications (e.g. weather forecasting, default), we want the probability

• Previously we minimized the sum-squared-error cost function

$$J(\mathbf{w}) = \frac{1}{2} \sum_{i} \left(\phi(z^{(i)}) - y^{(i)} \right)^{2}$$

- Now we need to derive the cost function for logistic regression
- Define the likelihood *L*

$$L(\mathbf{w}) = P(\mathbf{y}|\mathbf{x}; \mathbf{w}) = \prod_{i=1}^{n} P(y^{(i)}|x^{(i)}; \mathbf{w})$$

$$L(\mathbf{w}) = \prod_{i=1}^{n} \left(\phi(z^{(i)}) \right)^{y^{(i)}} \left(1 - \phi(z^{(i)}) \right)^{1 - y^{(i)}}$$

Maximize the likelihood function

$$L(\mathbf{w}) = P(\mathbf{y}|\mathbf{x};\mathbf{w})$$

$$L(\mathbf{w}) = \prod_{i=1}^{n} \left(\phi(z^{(i)}) \right)^{y^{(i)}} \left(1 - \phi(z^{(i)}) \right)^{1 - y^{(i)}}$$

• In practice easier to deal with the natural log of this equation

$$l(\mathbf{w}) = \log L(\mathbf{w})$$

$$l(\mathbf{w}) = \sum_{i=1}^{n} \left[y^{(i)} \log \left(\phi(z^{(i)}) \right) + \left(1 - y^{(i)} \right) \log \left(1 - \phi(z^{i()}) \right) \right]$$

Easier to take derivative + fewer numerical underflow issues



• Rewrite likelihood as a cost function

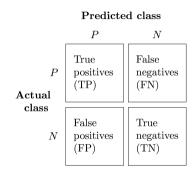
$$J(\mathbf{w}) = \sum_{i=1}^{n} \left[-y^{(i)} \log \left(\phi(z^{(i)}) \right) - \left(1 - y^{(i)} \right) \log \left(1 - \phi(z^{i()}) \right) \right]$$

- which can now be minimized using gradient descent
- Derivation in the slide at the end





- Great that we have methods to come to a prediction or a classification but how do we know if this makes any sense?
- In "ML2 the basics" we will discuss in detail:
 - -
 - Bias-Variance
 - Overfitting
 - Test-Train
 - Confusion matrix
 - ...





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- ...

False Negatives!



- Great that we have methods to come to a prediction or a classification but how do we know if this makes any sense?
- In "ML2 the basics" we will discuss in detail:
 - Bias-Variance
 - Overfitting
 - Test-Train
 - Confusion matrix

- ...

False Positives!



Summary 48

In this lecture we covered:

- 1. a few examples of how ML is applied in other fields
- 2. how an Ordinary Least Squares (OLS) model can be seen as a ML model
- 3. how updating weights leads to learning
- 4. Discussed the relevance of normalizing or standardizing
- 5. Worked our way up to the workhorse of classification the logit model

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